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FINAL REPORT
SATURN V-LAUNCHER-UMBILICAL TOWER VIBRATION ANALYSIS

Vol. I
RESPONSE TO GROUND WIND EXCITATION

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## FOREWORD

The work described in this report was performed by Lockheed Missiles \& Space Company, Huntsville Research \& Engineering Center, for the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration under Contract NAS8-21301.

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The final report for "Saturn V-Launcher-Umbilical Tower Vibration Analysis" consists of two volumes as follows:

Vol I : Response to Ground Wind Excitation
Vol II: A Computer Program for Analysis of the Vibrational Characteristics of Large Linear Space Frames

SUMMARY

This report describes methods for calculating the vibrational characteristicand the ground-wind induced response of the Saturn V-Launcher-Uwhlical tower combination. The general features of the mathematical mudel are discussed and the techniques used to solve eigenproblems and response problems are described.

A digital program was prepared to implement the formulation. Input to the program consists of concise descriptions of displacement functions used to characterize vehicle and tower motions, damping coefficients, forcing function descriptions, etc. Program output includes complete solution information with optional SC 4020 plots of the response


Results (both eigenproblem and response solutions) obtained for the Saturn 501 configuration are presented.

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Section 1 m INTRODUCTION

As illustrated on Fig. 1, the system is modeled as a non-uniform beam, representing the vehicle, connected by a dashpot system to a structural framework, representing the tower. The dashpot system is located near the top of the vehicle and acts both in the vehicle-tower plane and normal to it, as shown on Fig. 2.


Fig. 1 - Stractural Model


Fig. 2 - Damper System Configuration

both the vehicle and the tower are characterized by the coefficients of several displacement functions. Base flexibilities can be included as integral parts of the displacement functions and do not appear explicitly as coordinates. The formulation can easily be extended to include the effects of elastic support of the launcher itself in order to study the system's dynamics during transport on the crawler. In the interest of brevity in explaining the methods being used, however, the launcher motion components are deleted in the following discussions.

After writing the various energy terms associated with the system components as quadratic forms in the generalized coordinates, Lagrange's equation is used to obtain a set of linear vibration equations of the usual matrix form:

$$
\begin{equation*}
\mathrm{M} \ddot{q}+\mathrm{D} \dot{\mathrm{q}}+\mathrm{K}_{\mathrm{q}}=\mathrm{Q} \tag{1-1}
\end{equation*}
$$

Two classes of problems are then considered: (1) determination of the system damped modes and frequencies; and (2) calculation of system response to forcing functions representing ground wind excitation. *The damped modes, which are discussed in detail in Section 2.3, are quite different from undamped modes. Accordingly, the usual approach taken in calculating the transient response of very lightly damped structures (i.e., expansion in terms of free vibration modes) is not taken. Instead, Eq. (1-1) is integrated directly. However, a knowledge of the system damped modes and frequencies is of considerable value in solving the response problem, as described in Section 2.4.

A discussion and user instructions are included for the computer program developed to implement the theoretical formulation.

Results obtained with the fueled and unfueled configurations of the Saturn 501 are also included.

## Section 2 * THEORETICAL APPROACH

### 2.1 MATHEMATICAL MODEL

The vehicle is modeled as a Timoshenko beam. The coordinates representing its motion in the vehicle-tower plane are $\xi_{\mathrm{vl}}^{1}, \xi_{\mathrm{v} 2}^{1}$...(see Fig. 3). Motion in the plane containing the vehicle and tower centerlines will be termed direction-1 motions. Tower motions in direction lare represented by the coordinates $\xi_{t 1}^{1}, \xi_{\mathrm{t} 2}^{1}$. ... Direction-2 motions are normal to direction 1.



Fig. 3 - Coordinates
The i-direction motions of points on the vehicle and tower are, respectively:

$$
\begin{align*}
& u_{v}^{i}(x)=\sum_{j} \xi_{v j}^{i} G_{v j}^{i}(x) \\
& \text { and } \\
& u_{t}^{i}(x)=\sum_{j} \xi_{t j}^{i} G_{t j}^{i}(x), \tag{2-1}
\end{align*}
$$

where x corresponds to the position coordinate of elevation above the launcher.

In the digital program which has been developed to implement this formulation, displacement functions (the $G^{\prime} s$ ) are undamped free vibrational mode shapes. Vehicle cantilevered modal functions were calculated with the Lockheed-developed tandem beam digital program described in Ref. 1. Freevibration cantilever modes of the tower were calculated using the digital program described in Volume II of this report for computing the vibrational characteristics of large undamped linear space frames.

Other coordinates provided for are $\Phi$, the coefficient of a tower twist function; and $\Psi$, the direction-2 displacement component of the end of the dashpot boom relative to the point of attachment of the boom to the tower. Accordingly, the direction-2 motion of the end of the boom is

$$
\begin{equation*}
\sum_{j} \xi_{\mathrm{tj}}^{2} G_{\mathrm{tj}}^{2}(\overline{\mathrm{x}})+\Phi \mathscr{\mathscr { F }}(\overline{\mathrm{x}})+\Psi \tag{2-2}
\end{equation*}
$$

where $\bar{x}$ corresponds to the level of attachment of the boom to the tower, and $\mathscr{F}$ is the tower twist function (e.g., the first cantilever torsional mode).

It will be convenient to define two generalized coordinate vectors, $q_{1}$ and $\mathrm{q}_{2}$ as follows:

$$
\mathrm{q}_{1}=\left\{\begin{array}{c}
1  \tag{2-3}\\
\xi_{\mathrm{vl}} \\
\cdot \\
\cdot \\
\cdot \\
\xi_{\mathrm{t} 1} \\
\cdot \\
\cdot
\end{array}\right\}, \text { and } \quad \mathrm{q}_{2}=\left\{\begin{array}{c}
\xi_{\mathrm{vl}}^{2} \\
\cdot \\
\cdot \\
\cdots \\
\xi_{\mathrm{t} 1} \\
\cdot \\
\cdot \\
\cdot \\
\Phi \\
\Psi
\end{array}\right\} .
$$

The kinetic and potential energies of the system, which are discussed in more detail in Section 2.2 may be written, respectively, as:

$$
T=\frac{1}{2}\left[\dot{q}_{1}^{*} M_{1} \dot{q}_{1}+\dot{q}_{2}^{*} M_{2} \dot{q}_{2}\right]
$$

and

$$
\begin{equation*}
\mathrm{V}=\frac{1}{2}\left[\mathrm{q}_{1}^{*} \mathrm{~K}_{1} \mathrm{q}_{1}+\mathrm{q}_{2}^{*} \mathrm{~K}_{2} \mathrm{q}_{2}\right] \tag{2-4}
\end{equation*}
$$

The dissipation function, as discussed in Section 3, is of the form

$$
\begin{equation*}
F=\frac{1}{2}\left[\dot{\mathrm{q}}_{1}^{*} \mathrm{D}_{1} \dot{\mathrm{q}}_{1}+\dot{\mathrm{q}}_{2}^{*} \mathrm{D}_{2} \dot{\mathrm{q}}_{2}\right] . \tag{2-5}
\end{equation*}
$$

For a system of this type, the generalized force associated with a coordinate $\eta_{\mathrm{i}}$ is equal to

$$
\begin{equation*}
\frac{d}{d t}{\frac{\partial T}{\partial \eta_{i}}}+\frac{\partial F}{\partial n_{i}}+\frac{\partial V}{\partial n_{i}} \tag{2-6}
\end{equation*}
$$

Accordingly, from Equations (2-4) and (2-5),

$$
\begin{align*}
& M_{1} \ddot{q}_{1}+D_{1} \dot{q}_{1}+K_{1} q_{1}=Q_{1}  \tag{2-7}\\
& \text { and } \\
& M_{2} \ddot{q}_{2}+D_{2} \dot{q}_{2}+K_{2} q_{2}=Q_{2} \tag{2-8}
\end{align*}
$$

which is Eq. (1-1) written in uncoupled form - the first equation for motion in the vehicle-tower plane, and the second equation for direction-2 motion which involves twisting of the tower and deformation of the damper boom in addition to vehicle and tower bending.

### 2.2 ENERGY TERMS

Quadratic forms from which the elements of the coefficient matrices of Eq. (2-7) and (2-8) may be identified are discussed below.

The kinetic energy of the vehicle is

$$
\begin{align*}
T_{v} & =\frac{1}{2} \sum_{i=1}^{2} \int_{0}^{L}\left[m_{v}(x)\left\{\sum_{j} \xi_{v j}^{i} G_{v j}^{i}(x)\right\}^{2}\right. \\
& \left.+\rho_{v}^{i}(x)\left\{\sum_{j} \xi_{v j}^{i} H_{v j}^{i}(x)\right\}^{2}\right] d x \tag{2-9}
\end{align*}
$$

where $m_{v}$ and $\rho_{v}$ are, respectively, vehicle mass and effective mass moment of inertia per unit length; and $H$ is the cross-section rotation associated with $G$.

Where $E I_{v}$ and $G A_{v}$ are, respectively, vehicle bending and shear stiffness, the potential energy associated with vehicle deformation is:

$$
\begin{align*}
V_{v} & =\frac{1}{2} \sum_{i=1}^{2} \int_{0}^{L}\left[E I_{v}\left\{\sum_{j} \xi_{v j}^{i}\left[H_{v j}^{i}(x)\right]^{1}\right\}^{2}\right. \\
& \left.+G A_{v}\left\{\sum_{j} \xi_{v j}^{i}\left(\left[G_{v j}^{i}(x)\right]^{1}-H_{v j}^{i}(x)\right)\right\}^{2}\right] d x \tag{2-10}
\end{align*}
$$

The prime in the above equation indicates differentiation with respect to the position coordinate $x$.

As discussed in Section 2.1 the displacement functions used to represent vehicle motion are free vibration cantilever and "fixed-pinned" mode shapes, substitution of equations $(\angle-y)$ and $(\angle-1 U)$ into equation $(\angle-0)$, theretore, indicates that the diagonal terms of M and K (Equations (2-7) and (2-8)) as sociated with the coordinate $\xi_{\mathrm{vj}}^{\mathrm{i}}$ are simply the generalized mass and generalized stiffness, respectively, of the vehicle function $G_{v j}^{i}$. Also, from orthogonality relations it is evident that the off-diagonal terms of $M$ and $K$ that represent coupling between coordinate functions with identical boundary conditions are zero. Since the generalized mass and stiffness of each vehicle modal function are standard output of the digital program described in Ref. 1, Eqs. (2-9) and (2-10) are only used to compute the off-diagonal terms of $M$ and $K$ representing coupling between cantilever and "fixed-pinned" coordinate functions.

A complete description of the kinetic and potential energy computations associated with the coordinates representing tower motion is discussed in detail for a general framework in Volume II of this report. In the interest of brevity, these computations are not discussed here. As in the case of the vehicle, the diagonal terms of $M$ and $K$ associated with the coordinate $\xi_{t j}^{i}$ are the generalized mass and stiffness, respectively, of the tower coordinate function $G_{i j}^{i}$. Since
orthogonal free vibration cantilever and torsional mode shapes are used to represent tower motion, the off-diagonal terms of $M$ and $K$ that represent coupling among the tower coordinates are zero.

Since the launcher is assumed rigid, coupling between the coordinates representing vehicle motion and the coordinates representing tower motion does not exist in the kinetic and potential energy matrices, $M$ and $K$, of Eq. (2-7) and (2-8).

Using a quasi-static displacement function based on a direction-2 force applied at the vehicle end of the damper boom, the strain energy in the damper boom is $\frac{1}{2} k_{b} \Psi{ }^{2}$, where $k_{b}$ is the boom spring constant associated with $\Psi$, the direction-2 deformation relative to the tower, as shown on Fig. 4. The boom is assumed rigid in direction-1. The kinetic energy of the boom may be written as a quadratic form in $\xi_{\mathrm{t} 1}^{2}, \xi_{\mathrm{t} 2}^{2}, \ldots, \Phi, \Psi$, based on the quasi-static deformation function usod in calculating $k_{0}$.


Fig. 4 - Damper Boom Deformation Relative to Tower

Where $\bar{x}$ is the position coordinate corresponding to the location of the damper, and $c$ is the damping constant of a single dashpot, the dissipation function is

$$
F=F^{1}+F^{2}
$$

where

$$
\left.\begin{array}{rl}
F^{1}= & \frac{1}{2} c\left\{\left[\sum_{j} \dot{\xi}_{v j}^{1}\right.\right. \\
G_{v j}^{1}(\bar{x})
\end{array}\right],
$$

and

$$
\begin{gather*}
F^{2}=\frac{1}{2} c\left\{\left[\sum_{j} \dot{\xi}_{v j}^{2} G_{v j}^{2}(\bar{x})\right]-\left[\sum_{j} \dot{\xi}_{t j}^{2} G_{t j}^{2}(\bar{x})\right]\right. \\
-\dot{\Psi}-\dot{\Phi} \neq(\bar{x})\} \tag{2-11}
\end{gather*}
$$

Note that the damping matrices $D^{1}$ and $D^{2}$ corresponding to the above equations contain no zero elements. "Modal damping" terms corresponding to the vehicle and tower displacement functions may be added to the above functions.

### 2.3 MODES AND FREQUENCIES

Although the primary objective of the present study is the determination of the response of the system to ground-wind induced forces, the free-vibration characteristics are also of interest. For either the direction-l or direction-2 problem of Eqs. (2-7) and (2-8), the free vibration problem is of the following form (dropping the subscripts for brevity):

$$
\begin{equation*}
M \ddot{q}+D \dot{q}+K q=0 \tag{2-12}
\end{equation*}
$$

Assuming solutions of the form

$$
\begin{equation*}
q=\text { constant } \cdot e^{r t} Z \tag{2-13}
\end{equation*}
$$

the characteristic equation is

$$
\begin{equation*}
\left(r^{2} M+r D+K\right) Z=0 \tag{2-14}
\end{equation*}
$$

A standard Lockheed subprogram (Muller routine) is available for calculating the roots of Eq. (2-14).

Where a and pare scalars,

$$
\begin{equation*}
r=a+i p, \tag{2-15}
\end{equation*}
$$

and the vector $Z$ is also generally complex.

Standard subroutines are also available for calculating solutions to sets of complex simultaneous linear algebraic equations. Accordingly, once a particular root, $r$, has been calculated, the corresponding vector. 7 , may $h$ cunputeu unectiy irom eq. ( $\angle-14$ ) by prescribing the value of one element of $Z$ (e.g., $z_{1}=1+i \cdot o$ ) to define a set of $n-1$ equations in $n-1$ unknowns ( $n=$ order of the system).

It is significant to note that if a particular $r$ and $Z$ constitute a solution of Eq. (2-14), then their conjugates. $\vec{r}$ and $\bar{Z}$, also together form a solution. This may be shown, for example, by letting $Z=X+i Y(X$ and $Y$ real) and substituting both solutions (first $r, Z$, then $\bar{r}, \bar{Z}$ ) into Equation (2-14) and observing that they yield identical results.

The vector $Z$ may be written as:

$$
z=\left\{\begin{array}{c}
z_{1} e^{i \phi_{1}}  \tag{2-16}\\
\cdot \\
z_{j} e^{i \phi_{j}} \\
\cdot \\
z_{n} c^{i \phi_{n}}
\end{array}\right\}
$$

Accordingly, using Equations (2-13) and (2-14), the generalized displacement of the system may be written as

$$
q=A e^{a t}\left\{\begin{array}{lll}
z_{1} e^{i \phi} & e^{i p t}  \tag{2-17}\\
\cdot & & \\
\cdot & i \phi_{j} & \\
z_{j} e^{i p t} & e^{i p t} \\
\cdot & & \\
\cdot & i \phi_{n} & \\
z_{n} e^{i p t} & e
\end{array}\right\}
$$

where A is an arbitrary constant.

Eq. (2-17) may be re-written as

$$
q=A e^{a t}\left\{\begin{array}{ccc}
z_{1} & {\left[\cos \left(p t \cdot \dot{\phi}_{2}\right)\right.} & i \sin \left(1 \cdot \dot{\phi}_{2}\right]  \tag{2-18}\\
z_{j} & {\left[\cos \left(p t+\phi_{j}\right)\right.} & \left.+i \sin \left(p t+\phi_{j}\right)\right] \\
& \cdot \\
\vdots & \cdot
\end{array}\right\}
$$

Since $\bar{r}$ and $\bar{Z}$ are also necessarily solutions of Eqs. (2-14), we also have the solution:
$q=A e^{a t}\left\{\begin{array}{c}z_{1} e^{-i \phi_{1}} e^{-i p t} \\ \cdot \\ z_{j} e^{-i \phi_{j}} e^{-i p t}\end{array}\right\}=A e^{a t}\left\{\begin{array}{c}z_{1}\left[\cos \left(-p t-\phi_{1}\right)+i \sin \left(-p t-\phi_{1}\right)\right] \\ \cdots \\ z_{j}\left[\cos \left(-p t-\phi_{j}\right)+i \sin \left(-p t-\phi_{j}\right)\right. \\ \cdots \\ \cdots\end{array}\right\}(2-19)$

Adding Eqs. $(2-18)$ and $(2-19)$ yields entirely real solutions of the form:

$$
q=2 A e^{a t}\left\{\begin{array}{cc}
z_{1} & \cos \left(p t+\phi_{1}\right)  \tag{2-20}\\
\cdot \\
z_{j} & \cos \left(p t+\phi_{j}\right) \\
\cdot \\
z_{n} \cos \left(p t+\phi_{n}\right)
\end{array}\right\}
$$

From the above equation it is apparent that damped modes are quite different in character from undamped modes. Although all elements of the generalized coordinate vector "oscillate" at the same frequency, a distinct phase angle is associated with each element. Accordingly, for general types of response problems the pursuit of means of using coefficients of damped -igenfunctions as generalized coordinates (analogous to the procedure commonly, used in undamped analysis) does not appear promising.

### 2.4 INTEGRATION METHOD

Determination of the system response involves integration of matrix equations of the form

$$
\begin{equation*}
\mathrm{M} \ddot{\mathrm{q}}+\mathrm{D} \dot{\mathrm{q}}+\mathrm{Kq}=\mathrm{Q} \tag{2-21}
\end{equation*}
$$

where $Q$ depends upon the motion of the system. The numerical integration method described below is one which has been found well-suited for several similar problems. Matrix series expansions of the vectors $q$ and $\dot{q}$ in powers of the time increment $\triangle$ are:

$$
\begin{aligned}
& q(t+\Delta)=q(t)+\Delta \dot{q}(t)+\frac{\Delta^{2}}{2} \ddot{q}(t)+\ldots . . \\
& \dot{q}(t+\Delta)=\dot{q}(t)+\Delta \ddot{q}(t)+\frac{\Delta^{2}}{2} \ddot{q}(t)+\ldots(2-22)
\end{aligned}
$$

$$
2-10
$$

Equation (2-21) may be re-written as

$$
\begin{equation*}
\ddot{q}=A \dot{q}+B \underline{q}+\eta \tag{2-23}
\end{equation*}
$$

where

$$
\begin{equation*}
A=-M^{-1} D, \quad B=-M^{-1} K \text {, and } \eta=M^{-1} Q . \tag{2-24}
\end{equation*}
$$

From Eq. (2-23), higher derivatives of $q$ may be expressed in terms of $q, \dot{q}$ and $\eta$.

$$
\begin{aligned}
& \dot{\ddot{q}}=A \ddot{q}+B \dot{q}+\dot{\eta} \\
&=A(A \dot{q}+B q+\eta)+B \dot{q}+\dot{\eta} \\
&=\left(A^{2}+B\right) \dot{q}+A B q+A \eta+\dot{\eta} \\
& \because \because-\left(A^{2}: B\right)(A \dot{q}:=q \cdot M, A D \dot{q}+A \dot{\eta}+\ddot{\eta} \\
&=\left[\left(A^{2}+B\right) A+A B\right] \dot{q}+\left(A^{2}+B\right) B q+\left(A^{2}+B\right) \eta+A \dot{\eta}+\ddot{\eta}
\end{aligned}
$$

etc. In general,

$$
(n)=R_{n} q+P_{n} \dot{q}+P_{n-1} \eta+P_{n-2} \dot{\eta}+\ldots+P_{1}^{(n-2)}
$$

Since

$$
\begin{gathered}
(n+1) \\
q
\end{gathered}=R_{n} \dot{q}+P_{n}(A \dot{q}+B q+\eta)+P_{n-1} \dot{\eta}+P_{n-2} \ddot{\eta}+\ldots+P_{1} n,
$$

The recursion formulae for $P$ and $R$ are

$$
\begin{align*}
& P_{n+1}=P_{n} A+R_{n} \\
& R_{n+1}=P_{n} B \tag{2-26}
\end{align*}
$$

beginning with

$$
P_{1}=I \text { (identity matrix) }
$$

and

$$
\begin{equation*}
R_{1}=0 \text { (zero matrix). } \tag{2-27}
\end{equation*}
$$

Substitution of Eq. (2-25) into (2-22) yields:

$$
\begin{align*}
q(t+\Delta)= & q(t)+\Delta \dot{q}(t) \\
& +\frac{\Delta^{2}}{2}\left[R_{2} q(t)+P_{2} \dot{q}(t)+P_{1} \eta(t)\right] \\
& +\frac{\Delta^{3}}{3!}\left[R_{3} q(t)+P_{3} \dot{q}(t)+P_{2} \eta(t)+P_{1} \dot{\eta}(t)\right] \\
& +\frac{\Delta^{4}}{4!}\left[R_{4} q(t)+P_{4} \dot{q}(t)+P_{3} \eta(t)+P_{2} \dot{\eta}+P_{1} \ddot{\eta}(t)\right]+\ldots \\
\dot{q}(t+\Delta)= & \\
& +\Delta \dot{q}^{\prime}(t) \\
& +\frac{\Delta^{2}}{2}\left[R_{2} q(t)+P_{2} \dot{q}(t)+P_{1} \eta(t)\right] \\
& \left.+\frac{\Delta^{3}}{3!}\left[R_{4} q(t)+P_{3} \dot{q}(t)+P_{2} \eta(t)+P_{1} \dot{q}(t)+P_{3} \eta(t)\right]+P_{2} \dot{\eta}(t)+P_{1} \ddot{\eta}(t)\right]+\ldots \tag{2-28}
\end{align*}
$$

or, for an $\ell$-term expansion,

$$
\begin{align*}
\left\{\begin{array}{c}
q(t+\Delta) \\
\dot{q}(t+\Delta)
\end{array}\right\} & {\left[\begin{array}{lll}
W_{11} & W_{12} \\
W_{21} & W_{22}
\end{array}\right]\left\{\begin{array}{c}
q(t) \\
\dot{q}(t)
\end{array}\right\} } \\
& +\left[\begin{array}{lll}
N_{10} & N_{11} \ldots \\
N_{20} & N_{21} \ldots & N_{1, \ell-2} \\
& \ldots & N_{2, \ell-2}
\end{array}\right]\left\{\begin{array}{c}
Q(t) \\
\dot{Q}(t) \\
\cdot \\
\cdot \\
(\ell-2) \\
Q(t)
\end{array}\right\} \tag{2-29}
\end{align*}
$$

where

$$
\begin{aligned}
W_{11} & =I+\sum_{n=2}^{n} \frac{\Delta^{n}}{n!} R_{n}, \quad W_{12}=\sum_{n=1}^{\mu} \frac{\Delta^{n}}{n!} P_{n} \\
W_{21} & =\sum_{n=1}^{\ell-1} \frac{\Delta^{n}}{n!} R_{n+1},
\end{aligned} W_{22}=I+\sum_{n=1}^{\ell-1} \frac{\Delta^{n}}{n!} P_{n+1}, ~ l l
$$

and, for $j=0,1,2, \ldots, \quad \ell-2$,

$$
\left.\begin{array}{l}
N_{1 j}=\left[\sum_{n=j+2}^{\ell} \frac{\Delta^{n}}{n!}\right. \\
P_{n-j-1}
\end{array}\right] M^{-1}, ~\left[\begin{array}{cc}
\sum_{2 j}^{l-1} & \frac{\Delta^{n}}{n!}  \tag{2-30}\\
N_{n-j+1} & P_{n-j}
\end{array}\right] M^{-1} .
$$

Since the $W$ and $N$ matrices are not functions of time, they need be evaluated only once (at the beginning of the solution process); provided that a constant time interval, $\Delta$, is used. Eq. (2-29) can then be used to calculate the solution step-by-step in time. One advantage of this method is that it permits economical use of high-order approximations, which allows relatively long time increments. That is, unless the higher derivatives of $Q$ become very complicated to evaluate, the time required to carry out an integration step using a 6 th order approximation (i.e., $\ell=6$ ) is typically only about $50 \%$ greater than the time required to effect one step of a 3 rd order approximation $(\ell=3)$; even less if higher derivatives of Q become negligibly small, as is frequently the case.

The digital routine developed to implement this method may readily be adapted to other types of problems by re-coding only the part of the program dealing with the forcing function. A general discussion of convergence, determination of efficient time increment length, etc., is beyond the scope of the present discussion; however, in present studies where the system natural frequencies are known from solutions of the damped eigenproblem, suitable ranges
 expansions ( $n=7$ ), typical choices of $\Delta$ are around one eighth of the period associated with the highest natural frequency of the mathematical model of the system.

### 2.5 FORCING FUNCTIONS

An approximate method of representing ground wind-induced forcing functions is outlined below. This method is similiar to the technique used in Ref. 3. In the following discussion it is assumed that the free-stream velocity is in either direction 1 or direction 2 , and that the wind velocity may vary with height. Other assumptions are:

- Only the vehicle is subject to time-varying loads.
- The steady state response of the vehicle will be normal to the free stream airflow direction and of the form $u(x) \sin \omega t$, where $\omega$ is either equal to or very nearly equal to the frequency of the first free-vibration mode in the response direction.
e At each location along the vehicle the flow field is quasi-two dimersional and the lift force per unit length is of the form
$\frac{1}{2} \rho v^{2} s\left\{C_{L} \cos \left(\omega_{a} t+\alpha\right)+F_{1} \sin \omega t+F_{v 2} \cos \omega t\right\}$
as shown on Fig. 5, where $v$ and $s$ are the local freestream velocity and diameter of the vehicle, respectively, and $\rho$ is the mass density of air. $C_{T}$ is a constant termed the coefficient of oscillatory lift on a stationary cylinder. $\alpha, F_{1}$, and $F_{2}$ are "known" functions of the response frequency, $\omega_{\text {, the }}$ frequency of vortex shedding, $\omega_{a}$, and the local vibration amplitude, $u(x)$.
$\frac{1}{2} \rho v^{2} s\left\{C_{L} \cos \left(\omega_{a} t+\alpha\right)+F_{1} \sin \omega t+F_{2} \cos \omega t\right\}=$ force/unit length


Fig. 5 - Aerodynamic Forces

According to Ref. 3, expressions for the terms appearing in Eq. (2-31) are:

$$
\begin{aligned}
& C_{L}=0.10 \longrightarrow .14 \text { (assumed } 0.10 \text { for this analysis); } \\
& \omega_{a}=0.54-\frac{\pi v}{s} \text {; } \\
& \alpha=\tan ^{-1}\left\{\frac{2 \omega_{a} \xi \omega}{\omega_{a}^{2}-\omega^{2}}\right\}, \\
& F_{1}=0.27 \pi \eta_{0}(.54 \times \cos \alpha) \text {; and } \\
& F_{2}=0.27 \pi \eta_{0}(.275 \pi+.54 r \sin \alpha) \text {, }
\end{aligned}
$$

where

$$
\begin{aligned}
& \eta_{0}=2 \frac{u(x)}{s}, \quad \text { and } \\
& r=1+\frac{.13}{n}
\end{aligned}
$$

In implementing these assumptions, the frequency and amplitude of the tip response of the vehicle was tracked and the actual response frequency and (local) amplitude was used to calculate $\alpha, F_{1}$ and $F_{2}$ along the vehicle. The success of this procedure rested, of course, upon the assumption that the response was very nearly of the simple form previously stated. The results of the calculations verified the assumption.

For purposes of calculating generalized forces, the vehicle was divided into four sectors, as shown on Fig。6. Within each sector the freestream velocity and properties were assumed constant, as were $\alpha, F_{1}$, and $F_{2}$, which were evaluated on the basis of the current response frequency and the average amplitude within the segment. Accordingly, where the subscript $j$ refers to the $j$-th segment, the $i$-th element of the generalized force vector was of the form:


Fig. 6 - Typical Subdivision of Vehicle into Segments for Purposes of Approximating Lift Force Distributions

$$
\sum_{j=1}^{4} \frac{1}{2} \rho v_{j}^{2} P_{j} r_{i j}
$$

where

$$
P_{j}=C_{L} \cos \left(\omega_{a} t+\alpha\right)_{j}+\left(F_{1} \sin \omega t+F_{2} \cos \omega t\right)_{j}
$$

and where $r_{i j}$ was the integral over the $j$-th segment of $s G_{i}(x) d x$. The generalized force vector was, then, of the form:
where

$$
\begin{gathered}
Q=\sum_{j=1}^{4} P_{j} S_{j}, \\
S_{j}=\frac{1}{2} \quad \rho v_{j}^{2}\left\{\begin{array}{c} 
\\
r_{1 j} \\
r_{2 j} \\
\vdots \\
r_{n j}
\end{array}\right\} .
\end{gathered}
$$

## Section 3

## COMPUTER PROGRAM

A computer program was developed to implement the formulation described in the preceeding section. The program is coded in Fortran IV and configured for execution on the IBM 7094 computer with 32 K core storage capacity. Only minor modifications would be necessary to configure the program for execution on another computer equipped with a Fortran IV compiler.

Detailed instructions for use of the program are included in the Appendix.

### 3.1 PROGRAM ORGANIZATION

The OVERLAY configuration of the program is illustrated on Fig. 7.


Fig. 7 - Overlay Configuration

Functions of the principal subroutines are briefly outlined below:

BLOCK DATA: Used for input of the basic problem definition including output options, damping information, solution options, etc.

INPUT: Reads the mass distribution of the vehicle and optionally prints out initial solution information. The mass distribution of the tower is defined by a DATA statement.

SHAPES: Reads the displacement functions used to represent vehicle and tower motions from cards and performs some preliminary integrations of these functions.

CALC: Constructs the matrices representing kinetic, potential, and dissipative energies and calls the proper eigensolution routine.

NDAMP: Solves the eigenproblem if no damping was included in the problem definition.

DAMPI: Solves the complex eigenproblem if damping was included in problem definition.

NUMINT: Performs numerical integration of the response to ground winds excitation.

TESTER: Tests and stnres the maximum amplitude of the tip response of the vehicle. If the amplitude varies as much as $5 \%$ from that used to compute the forcing function, a new forcing function is calculated.

FRCOEF: Reads forcing function definition from cards.

FORFCN: Computes the generalized forcing function at each time increment in the integration process.

GRAPH: Optionally generates time history plots of the response at 3 specified elevations along the vehicle and tower.

### 3.2 MODELING

The displacement functions used to represent vehicle and tower motions were free vibrational undamped modal functions. Cantilever mode shapes computed with the Lockheed/Huntsville developed tandem beam digital program, Ref. 1, were used to represent vehicle motions. Tower motions were characterized by cantilever modal functions computed with the frame dynamics program discussed in Volume II of this report.

All illustrations presented in this section were reproduced from SC 4020 plots that were automatically generated by the two programs mentioned above.

### 3.2.1 VEHICLE DISPLACEMENT FUNCTIONS

Figures 8 through 11 illustrate the first four cantilever modes of the fueled Saturn 501 configuration. Figures 12 through 15 illustrate the first


Since the vehicle properties are identical in the vehicle-tower plane and the plane normal to it, the same functions are used to represent vehicle motions in both direction 1 and direction 2.

For the results discussed in Section 4 all four cantilever functions were used to represent vehicle motion.
woer 11
FREOE世木 $=0.381$
(

Fig. 8 - First Cantilever Mode of Fueled Saturn 501

LMSC/HREC D148844-I

Fig. 9 - Second Cantilever Mode of Fueled Saturn 501


Fig. 1J- Third Cantilever Mocie of Fuel.ad Saturn 501

500 (4)


Fig. 11- Fourth Cantilever Mode of Fueled Saturn 501
togel 1
Ferevitiey = 0.584

(

Fig. 12 - First Cantilever Mode of Unfueled Saturn 501
no



Fig. 13 - Second Cantilever Mode of Unfueled Saturn 501


Fig. 14 - Third Cantilever Mode of Unfueled Saturn 501

LMSC/HREC D148844-I

## 1005 ( 6

VREEKTCy 9.863
(



Fig. 15 - Fourth Cantilever Mode of Unfueled Saturn 501

### 3.2.2 UMBILICAL TOWER DISPLACEMENT FUNCTIONS

Figures 16, 17 and 18 illustrate the undeformed structure and the first two modes, respectively, of the tower in direction 1 (the vehicle-tower plane). Figures 19,20 and 21 illustrate the direction 2 mode shapes. For each direction, both modal functions were used in representing tower motions to obtain the results discussed in Section 4.


Fig. 16 - Undeformed View of Launcher Umbilical Tower, Direction 1


Fig. 17 - First Mode of Launcher Umbilical Tower, Direction 1


Fig. 18 - Second Mode of Launcher Umbilical Tower, Direction 1
viensofundeformed structure


Fig. 19 - Undeformed View of Launcher Umbilical Tower, Direction 2


Fig. 20 - First Mode of Launcher Umbilical Tower, Direction 2

$$
\begin{aligned}
& \text { HODE HUNBER } 2 \\
& \text { ITERATION NUABER } \\
& \text { FRECUERCY }=2.1142 \times 10^{+00} \text { CPS }
\end{aligned}
$$



Fig. 2I - Second Mode of Launcher Umbilical Tower, Direction 2

## Section 4

RESULTS

Complex eigenproblem solutions and ground wind-induced responses were generated with the computer program discussed in the previous section. Results were computed for fueled and unfueled configurations of Saturn 501. Eigenproblem solutions are presented in the form of Eq. $(2-20)$ where
$p=$ frequency,
$\mathrm{a}=$ damping factor,
$z_{i}=$ coefficient of the displacement function representing the i-th generalized coordinate, and
$\phi_{\mathrm{i}}=$ phase angle associated with the i-th generalized coordinate.
 the program.

According to the forcing function description discussed in Section 2.5, the most critical wind velocity for a uniform cylinder is that which creates a vortex shedding frequency equal to the first natural frequency of the cylinder. This velocity may be expressed as

$$
v=\frac{\omega s}{.54 \pi},
$$

where
$\omega=$ the first natural frequency of the cylinder, and
$s=$ the diameter of the cylinder.

For each configuration, two solutions were computed assuming the wind velocity to be constant over the entire vehicle. The two wind velocities were:
(1) the velocity required to critically excite Segments 1 and 2 (see Fig. 6) of the vehicle, and (2) the velocity required to critically excite Segmert 3 of the vehicle. The critical wind velocities summarized in Table 1 were computed using the vehicle modal properties presented in Section 3.2.1.

Table 1

CRITICAL WIND VELOCITIES FOR THE SATURN 501 VEHICLE

|  | Velocity 1 | Velocity 2 |
| :--- | :--- | :--- |
| Fueled Vehicle | $11.969 \mathrm{~m} / \mathrm{sec}$ | $7.792 \mathrm{~m} / \mathrm{sec}$ |
| Unfueled Vehicle | $19.810 \mathrm{~m} / \mathrm{sec}$ | $12.898 \mathrm{~m} / \mathrm{sec}$ |

$1 \%$ modal damping was included in each of the solutions.

The plotted response amplitudes are in meters.

### 4.1 FUELED VEHICLE

The following results were obtained for the fully-fueled Saturn 501. The first four cantilever modes illustrated on Figs. 8 through 11 were used to represent the motion of the vehicle. The modal displacement functions illustrated on Figs. 16 through 21 were used to characterize the motion of the tower.

Tables 2 and 3 summarize the first four damped modes of the system in directions 1 and 2, respectively. Figures 22 through 27 illustrate the following response solutions:

- Response of the vehicle with damper disconnected, wind velocity $=11.969 \mathrm{~m} / \mathrm{sec}$,
- Response of the vehicle and tower in direction 1 , wind velocity $=11.969 \mathrm{~m} / \mathrm{sec}$,
- Response of the vehicle and tower in direction 2, wind velocity $=11.969 \mathrm{~m} / \mathrm{sec}$,
- Response of the vehicle with damper disconnected, wind velocity $=7.792 \mathrm{~m} / \mathrm{sec}$,
- Response of the vehicle and tower in direction 1 , wind velocity $=7.792 \mathrm{~m} / \mathrm{sec}$, and
- Response of the vehicle and tower in direction 2, wind velocity $=7.792 \mathrm{~m} / \mathrm{sec}$.

Table 2
Eigenproblem Solution，Fully Fueled Vehicle，Direction 1

| DISPLACEMENT <br> FUNCTION | ```MODE 1 Frequency = 0.3224 cps Damping Factor =-.0377``` |  | ```MODE 2 requency = 0.4731 cps Damping Factor = -.5100``` |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Coefficient | $\phi(\mathrm{radian} s)$ | Coefficient | $\phi$（radians） |
| Vehicle <br> First Cantilever Mode | 0.26712 | 0.00000 | 0.00233 | 0.00000 |
| Vehicle <br> Second Cantilever Mode | 0.00245 | －1．95508 | 0.00178 | $-2.45495$ |
| Vehicle Third Cantilever Mode | 0.00097 | $-1.95524$ | 0.00062 | $-2.54246$ |
| Vehicle <br> Fourth Cantilever Mode | 0.00027 | 1.18697 | 0.00017 | 0.58167 |
| Tower First Cantilever Mode | 1.00000 | －1．92209 | 1.00000 | $-1.17574$ |
| Tecona Panter Cancer ivioue | 0．122土 | 2.17010 | 0． $0115 \%$ | U．ナフフロフ |
| DISPLACEMENT FUNCTION | MOD <br> Frequency Damping Fa | $\begin{aligned} & 3 \\ & 0.8998 \mathrm{cps} \\ & \text { tor }=-.0977 \end{aligned}$ | MOD <br> Frequency $=$ Damping Fa | $\begin{aligned} & 4 \\ & 1.5862 \mathrm{cps} \\ & \text { tor }=-.1234 \end{aligned}$ |
|  | Coefficient | $\phi$（radians） | Coefficient | $\phi$（radians） |
| Vehicle <br> First Cantilever Mode | 0.00330 | 0.00000 | 0.00056 | 0.00000 |
| Vehicle <br> Second Cantilever Mode | 0.64959 | －1．74032 | 0.00264 | －0．23038 |
| Vehicle <br> Third Cantilever Mode | 0.00624 | －3．04780 | 0.49738 | $-1.69466$ |
| Vehicle <br> Fourth Cantilever Mode | 0.00140 | 0.09208 | 0.001165 | 0.00048 |
| Tower <br> First Cantilever Mode | 1.00000 | 0.04667 | 0.14335 | －0．02442 |
| Tower <br> Second Cantilever Mode | .64663 | 0.10543 | 1.00000 | 0.06627 |

Table 3
Eigenproblem Solution, Fully Fueled Vehicle, Direction 2

| DISPLACEMENT FUNCTION | MODE 1 <br> Frequency $=0.3226 \mathrm{cps}$ Damping Factor $=-0357$ |  | $\begin{aligned} & \text { MODE } 2 \\ & \text { aency }=0.4338 \mathrm{cps} \\ & \text { ing_Factor }=-.5157 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Coefficient | $\phi$ (radians) | Coefficient | $\phi$ (radians) |
| Vehicle <br> First Cantilever Mode | 0.23936 | 0.00000 | 0.00333 | 0.00000 |
| Vehicle <br> Second Cantilever Mode | 0.00208 | -2.09375 | 0.00184 | -2.32598 |
| Vehicle <br> Third Cantilever Mode | 0.00082 | -2.09380 | 0.00067 | -2.40166 |
| Vehicle <br> Fourth Cantilever Mode | 0.00023 | $1.04846$ | 0.00018 | 0.72404 |
| Tower <br> First Cantilever Mode | 1.00000 | -2.05103 | 1.00000 | -1.05328 |
| Tower <br> Second Cantilever Mode | 0.15571 | 1.05143 | 0.12551 | 0.73195 |
| DISPLACEMENTFUNCTION | $\begin{gathered} \text { MODE } 3 \\ \text { Frequency }=0.8999 \\ \text { Damping Factor }=-.0980 \\ \hline \end{gathered}$ |  | $\begin{gathered} \text { MODE } 4 \\ \text { Frequency }=1.5862 \\ \text { Damping Factor }=-.1234 \end{gathered}$ |  |
|  | Coefficient | $\phi$ (radians) | Coefficient | $\phi$ (radians) |
| Vehicle <br> First Cantilever Mode | 0.00331 | 0.00000 | 0.00053 | 0.00000 |
| Vehicle <br> Second Cantilever Mode | 0.64917 | -1.72614 | 0.00250 | -0.23033 |
| Vehicle <br> Third Cantilever Mode | 0.00625 | $-3.04760$ | 0.47045 | -1.69134 |
| Vehicle <br> Fourth Cantilever Mode | 0.00140 | 0.09220 | 0.00110 | 0.00038 |
| Tower <br> First Cantilever Mode | 0.80368 | 0.04944 | 0.11365 | -0.02404 |
| Tower <br> Second Cantilever Mode | 1.00000 | 0.10279 | 1.00000 | 0.03470 |








Figure 22 - Response in Metere of Satharn 501 Fueled Configuration. Damper Disconnected, Wind Velocity $=11.97 \mathrm{~m} / \mathrm{sec}$.

HAXIRUM AFPLITVOE $\quad .660 \times 10^{-08}$
Tower response ...........
THE IA SECOHOS
HAXIMUA AGPLITUOE $=1.6803 \times 20^{-0 E}$
GEGATIVE MESPOABE

POSITIVE REBPOABE
VEHICLE RESPONSE
MAXIMUN AMPLITUNE $=3.3175 \times 10^{-08}$
TOWER RESPONSE
...........

KAXIMUH AMPLITUDE $=1.8019 \times 10^{-0 E}$

HEgATIVE RESPOGSE $\qquad$
$\therefore$. 4
Positive resporge

VEATCLE RESPONSE …

toner response
Tite in ecconea
$\qquad$






Figure 23 - Direction 1 Response, Fueled Saturn 501 Vehicle, Wind Velocity $=11.97 \mathrm{~m} / \mathrm{sec}$.
NEGATIVE RESPOMSE

POSITIVE REsponse $\qquad$

YEHICLE RESPORSE ——＿m＿n＿m
MAKIHUA AMPLITUOE $=3.4032 \times 10^{-O E}$

OUER RESPOHSE
MAKIAUA AMPLITUDE $=\quad 1.0505510^{-08}$

WEGTIVE RESFOHSE
RESPONSEAT ELEVATION OS．E：S
POSITIVE REBPOtGE

VEHICLE RESPONPE … ．．．．
AAKI酸 ABPLITHEE $=1.9800250^{-0 日}$

H月会 in aecotas





Figure 2\% - Direction 2 Response. Fueled Saturn 501 Vehicle, Wind Velocity $=11.97 \mathrm{~m} / \mathrm{sec}$.

$\qquad$
$\qquad$









00000


Figure 25-Responge in Materg of Seturn 501 Fueled Configuration, Damper Disconnected, Wind Velocity $=7.79 \mathrm{~m} / \mathrm{sec}$.



NEGATIVE REGPOUSE $\qquad$
POSITIVE RESPOHSE
號


POSITIVE AESPOHEE $\qquad$





00000


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0
0
0
0


Figure 26 - Direction 1 Response, Fueled Saturn 501 Vehicle, Wind Velocity $=7.79 \mathrm{~m} / \mathrm{sec}$.

POQIIVE RESFOHzE $\qquad$


HEGATIVE RESPONGE $\qquad$
POSITIVE RE\&POREE $\qquad$


HEGATIVE REGPOAGE $\qquad$
POSITIVE RESPORGE




Figure 27 - Direction 2 Response, Fueled Saturn 501 Vehicle, Wind Velocity $=7.79 \mathrm{~m} / \mathrm{sec}$.

The following results were obtained for the unfueled Saturn 501 vehicle. The first four cantilever modes illustrated on Figs. 12 - fh rough 15 were used to represent vehicle motion. Tower motion was characterized by the mode shapes shown on Figs. 16 through 21.

Tables 4 and 5 summarize the first four damped modes of the system in directions 1 and 2, respectively. Figures 28 through 33 illustrate the following response solutions:

- Response of the vehicle with damper disconnected, wind velocity $=19.810 \mathrm{~m} / \mathrm{sec}$,
- Response of the vehicle and tower in direction l, wind velocity $=19.810 \mathrm{~m} / \mathrm{sec}$,
- Response of the vehicle and tower in direction 2 , wind velocity $=19.810 \mathrm{~m} / \mathrm{sec}$,
- Response of the vehicle with damper disconnected, wind velocity $=12.898 \mathrm{~m} / \mathrm{sec}$.
- Response of the vehicle and tower in direction 1 , wind velocity $=12.898 \mathrm{~m} / \mathrm{sec}$, and
- Response of the vehicle and tower in direction 2 , wind velocity $=12.898 \mathrm{~m} / \mathrm{sec}$.

Table 4
Eigenproblem Solution, Unfueled Vehicle, Direction 1

| DISPLACEMENT FUNCTION | MODE 1 <br> Frequency $=0.4775 \mathrm{cps}$ <br> Damping Factor $=-.5634$ |  | $\begin{aligned} & \text { MODE } 2 \\ & \text { Frequency }=0.5262 \mathrm{cps} \\ & \text { Damping Factor }=\Omega 468 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Coefficient | $\phi$ (radians) | Coefficient | $\phi$ (radians) |
| $\begin{gathered} \text { Vehicle } \\ \text { Eirst Cantilever Mode } \end{gathered}$ | 0.01384 | 0.00000 | 0.12121 | 0.00000 |
| Vehicle Second Cantilever Mode | 0.00052 | -0.89484 | 0.00029 | -0.37131 |
| $\begin{gathered} \text { Vehicle } \\ \text { Third Cantilever Mode } \end{gathered}$ | 0.00031 | 2.23265 | 0.00017 | 2.77062 |
| Vehicle Fourth Cantilever Mode | 0.00006 | 2.22657 | 0.00003 | 2.77352 |
| Tower First Cantilever Mode | 1.00000 | 0.52620 | 1.00000 | 2.60464 |
| cower <br> Second Cantilever Mode | 0.07905 | 2.25134 | 0.04334 | 2.77645 |
| DISPLACEMENT FUNCTION | MODE 3 <br> Frequency $=1.8368 \mathrm{cps}$ <br> Damping Factor $=-.1285$ |  | MODE 4 <br> Frequency $=1.8694 \mathrm{cps}$ Damping Factor $=-.0748$ |  |
|  | Coefficient | $\phi$ (radians) | Coefficient | $\phi$ (radians) |
| $\begin{gathered} \text { Vehicle } \\ \text { First Cantilever Mode } \\ \hline \end{gathered}$ | 0.00026 | 0.00000 | 0.00003 | 0.00000 |
| Vehicle <br> Second Cantilever Mode | 0.11958 | -1.57794 | 0.00071 | 0.41799 |
| Vehicle <br> Third Cantilever Mode | 0.00029 | 0.01216 | 0.00003 | -0.02036 |
| Vehicle <br> Fourth Cantilever Mode | 0.00004 | 0.01348 | 0.00000 | -0.00797 |
| Tower <br> First Cantilever Mode | 0.01536 | -0.00520 | 0.00152 | -0.00601 |
| Tower Second Cantilever Mode | 1.00000 | 0.56358 | 1.00000 | 1.42059 |

Table 5
Eigenproblem Solution, Unfueled Vehicle, Direction 2

| DISPLACEMENT FUNCTION | MODE 1 <br> Frequency $=0.4395 \mathrm{cps}$ Damping Factor $=-.5475$ |  | MODE 2 <br> Frequency $=0.5252$ <br> Damping Factor $=-.0667$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Coefficient | $\phi$ (radians) | Coefficient | $\phi$ (radians) |
| Vehicle First Cantilever Mode | 0.01230 | 0.00000 | 0.17454 | 0.00000 |
| $\begin{gathered} \text { Vehicle } \\ \text { Second Cantilever Mode } \end{gathered}$ | 0.00054 | 0.62618 | 0.00058 | -0.68235 |
| $\begin{gathered} \text { Vehicle } \\ \text { Third Cantilever Mode } \end{gathered}$ | 0.00032 | -2.50603 | 0.00034 | 2.45917 |
| Vehicle <br> Fourth Cantilever Mode | 0.00007 | -2.50068 | 0.00007 | 2.46099 |
| Tower First Cantilever Mode | 1.00000 | -0.79752 | 1.00000 | 2.32346 |
| Tower <br> Second Cantilever Mode | 0.13365 | -2.51738 | 0.14276 | 2.46455 |
| DISPLACEMENTFUNCTION | MODE 3 <br> Frequency $=1.83664 \mathrm{cps}$ <br> Damping Factor $=-.1277$ |  | MODE 4 <br> Frequency $=2.1133 \mathrm{cps}$ <br> Damping Factor $=-.0605$ |  |
|  | Coefficient | $\phi$ (radians) | Coefficient | $\phi$ (radians) |
| Vehicle <br> First Cantilever Mode | 0.00092 | 0.00000 | 0.00003 | 0.00000 |
| Vehicle Second Cantilever Mode | 0.45692 | $-1.63332$ | 0.00011 | 0.03818 |
| Vehicle <br> Third Cantilever Mode | 0.00105 | 0.01084 | 0.00005 | -0.01750 |
| Vehicle <br> Fourth Cantilever Mode | 0.00015 | 0.01220 | 0.00001 | -0.00339 |
| Tower <br> First Cantilever Mode | 0.04668 | -0.00480 | 0.00129 | -0.00514 |
| Tower <br> Second Cantilever Mode | 1.00000 | 0.08664 | 1.00000 | 1.47832 |





Figure 28-Response in Metere of Saturn 501 Unfueled Configuration, Damper Disconnected, Wind Velocity $=19.81 \mathrm{~m} / \mathrm{sec}$.

FOUER RESPOHSE $\qquad$

IHE IA SECOHOE

NAXINUH AHPLITUOE $=2.1377 \times 10^{-0:}$
NEGATIVE RESPONGE
RESFONSEATELEVATION 105.000
POSITIVE RESPOESE $\qquad$
VEHICLE RESPONSE
HAXIHUH AHPLITUOE $=\quad 2.29 Y O K 20^{-01}$
TONER RESPONSE


MAXIHUK AHPLITUDE $=1.9205210^{-01}$
negative regpomse
RESTO ONSEATELEYATION SS.210
POSIIVE RESFOMSE

HAWIWUA AWPLITUOE $=$ OTAEXIO OE



HEGATIVE

$\qquad$

## vEMECLE AESPOHOE



IOLEA RESPOHsE ............
HAEIDU昜 AMPLITUOE = $1.0356810^{-08}$

REGATIUE RESPOGSE $\qquad$
POSITIVE RESFO䋨 $\qquad$


NEGATIVE RESPOPGE $\qquad$
POSIIIVE RESPOHES $\qquad$

VEMICLE RESPONSE -
WAXINUA ANPLITUOE $=$ S.EASOX:COE

OWE OEAPOHEE




00000



Figure 30 - Direction 2 Response, Unfueled Saturn 501 Vehicle, Wind Velocity $=19.81 \mathrm{~m} / \mathrm{sec}$.
NEHICLE REGONSE

TOWE KESPOASE ............
vehtcle pesporse
WAXIMUA AAPLITUOE $=2.0115 \times 10^{-01}$
tonar regponse ............
WAKIHUN AKPLITVOE $=0.0000 \times 10^{000}$


$\qquad$
$\qquad$
$\qquad$







Figure 31 - Response in Meterg of Saturn 501 Unfueled Configuration, Damper Disconnected, Wind Velacity $=12.90 \mathrm{~m} / \mathrm{sec}$.

VEAICL思 REGPOtG


NEGATIUE REPFOHEE $\qquad$
POSITIUE REAROABE $\qquad$


HEGATIVE NESPOREE $\qquad$
ROSIT:VE MESTOASE




 $\qquad$

## POsITIUE nEsponge

YEKICLE REGPOF\&





Figure 33 - Direction 2 Reoponsen Unfueled Saturn 501 Vehicie, Wint Velocity $=12.90 \mathrm{~m} / \mathrm{sec}$

## Section 5 <br> CONCLUSIONS

As a check of the ground wind-induced response analysis, a comparison was made with extrapolated wind tumnel test results reported in Ref. 4. According to Ref. 4 the tip response of a fueled Saturn 504 vehicle (first cantilever frequency $=.276 \mathrm{cps}$ ) exposed to a ground wind of $11.97 \mathrm{~m} / \mathrm{sec}$ is .172 meters. The steady state tip response of the fully-fueled Saturn 501 (first cantilever frequency $=.321 \mathrm{cps}$ ) with the damper disconnected and exposed to a ground wind of $11.97 \mathrm{~m} / \mathrm{sec}$ was computed by the program to be . 218 meters. The difference in response amplitudes is attributed to the physical differences between the two vehicle configurations.

The effect of the dashpot system on the ground wind-induced response of a particular vehicle can be ascertained from the results presented in Section 4 . In
 response with the damper connected was $78 \%$ less than that with the damper disconnected. This effect was typical among all response results.

Section 6
REFERENCES

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3. Rattayya, J. V. and L. P. Scott, "On Aerodynarnic Excitation and Peak Response of Typical Structures Exposed to Ground Winds," LMSC/HREC A783910, Lockheed Missiles \& Space Company, Huntsville, Ala., February 1967.
4. Marshall Space Flight Center Memorandum No. R-AERO-AU-68-63, 27 November 1968.

## APPENDIX

INPUT INSTRUCTIONS FOR THE GROUND WINDS EXCITATION ANALYSIS OF THE SATURN V - LAUNCHER UMBILICAL TOWER IN COMBINATION.

TABLE OF CONTENTS

| SECTION | GENERAL INSTRUCTIONS |
| :---: | :--- |
| 2 | VEHICLE MASS DISTRIBUTION |
| 3 | INITIAL DISPLACEMENTS AND VELOCITIES |
| 4 | VEHICLE OISPLACEMENT FUNCTIONS |
| 5 | TOWER DISPLACEMENT FUNCTIONS |
| 6 | GROUND WINDS DATA |
| 7 |  |

THE INPUT DATA SEQUENCE IS OUTLINED BELOW FOR THE DIRECTIONW 1 AND DIRECTION-2 SOLUTIONS TO A GIVEN CONFIGURATION BY STACKING DATA DECKS SEQUENTIALLY. ANY NUMBER OF CONFIGURATIONS MAY BE SOLVED IN A SINGLE EXECUTION.
A. BLOCK DATA INPUT
SEE SECTION 7 FOR DETAILS
VARIOUS INPUT, OUTPUT AND SOLUTION OPTIONS ARE DEFINED IN BLOCK DATA THESE VALUES MUST EE CONSISTENT AMONG ALL CONFIGURATIONS SOLVED DURING THIS EXECUTION.
THE FOLLOWING DATA IS STACKED SEQUENTIALLY FOR MULTIPLE SOLUTIONSO
Be VEHICLE MASS DISTRIEUTIONSEE SECTION 2 FOR DETAILS
THE DATA DEFINED BELOW IN $C$ THRU F IS READ IN FOR DIRECTIONS 1 AND 2 FOR EACH CONFYGURATIOR:
Ce INITIAL DISPLACEMENTS AND VELOCITIES FOR GROUND WINDS RESPONSE SEE SECTION 3 FOR DETAILS
De DISPLACEMENT FUNCTIONS USED TO REPRESENT VEHICLE MOTION SEE SECTION 4 FOR DETAILS
E. DISPLACEMENT FUNCTIONS USED TO REPRESENT TOUER MOTIONSEE SECTION 5 FOR DETATLS
F. FORCINE FUNCTION DESCRIPTION FOR GROUNO WINDS RESPONSESEE SECTION 6 FOR DETAILS

```
THE MASS OF THE VEHICLE IS DEFINED AT EACH VEHIGLEE STATION
LIST= (VEMASS(I)OI=1ONSI)
FORMAT(4E1808)
```

NSI IS THE TOTAL NUMBER OF VEHLCLE STATIONS ITS VALUE IS SET IN BLOCK DATA (SECTION 7) O VEMASS(I) IS THE MASS PER UNIT LENGTH OF THE VEHICLE AT STATION I.

INITIAL DISPLACEMENTS AND VELOCITIES
SECTION 3

THIS DATA IS USED ONLY FOR THE NUMERICAL INTEGRATION SOLUTION OF THE GROUND WINDS RESPONSE THEREFORE IF SOLTYP=I NO CARDS APPEAR FOR THIS DATA. SOLTYP IS DEFINED IN BLOCK DATA (SECTION 7):

FORMAT (6E12.5)
NVEH AND NTOW ARE THE NUMBER OF GENERALIZED FUNCTIONS USED TO REPRESENT THE MOTIONS OF THE VEHICLE AND TOWER RESPECTIVELYO THESE VALUES ARE DEFINED AS PART OF THE ARRAY OPTION IN BLOCK DATA (SECTION 7) INTALQ(K) AND INTALV(K) ARE THE INITIAL VALUES OF THE COEFFICIENT OF THE KTH GENERALIZED COORDINATE AND THE FIRST DERIVATIVE OF THAT COEFFICIENT WITH RESPECT TO TIME RESPECTIVELY

```
THE FOLLOWING DATA IS READ IN SEQUENTIALLY FOR EACH GENERALIZED
FUNCTION REPRESENTING VEHLCLE MOTIONO
LIST= FREQUENCY
FORMAT(E1205)
IF THE FUNCTION IS A MODE SHAPE THE FREQUENCY IS READ IN AS RADISEC
IF THE FUNCTION IS A STATIC DISPLACEMENT FUNCTION. LET FREQUENCY=OO
LIST= (VEHY(I)OI=IONSI)
FORMATPGE12.5)
VEHY(I) IS THE DISPLACEMENT OF STAYION IO NSI IS DEFINED IN BLOCK OATA (SECTION 7) V VEHY(1) REFERS TO THE LOWERMOST STATIONO
IF FREQUENCY=0.0 THE FUNCTION IS ASSUMED TO BE THE RESULT OF A
STATIC LOADING FUNCTION WHICH IS READ IN NEXT IF FREQUENCYOGTOOO
THE FOLLOWING DATA IS NOT READ ING
LIST= (FORCY:I)&I=1&NS1)
FORMAT(6E12.5)
```

FORCY(i) IS THE LATERAL POINT FORCE AT STATION I

TOWER DISPLACEMENT FUNCTIONS
SECTION 5

```
all TOWER FREQUENCIES ARE READ IN TOGETHER.
LIST= (FREQUENCY(I):I=IONTOW)
FORMAT(6E12.6)
    IF THE ITH GENERALIZED FUNCTION IS A VIBRATIONAL MOOE FREQUENCY(I)
    IS ITS FREQUENCY IN RADISECG OTHERURSE LET FREOUENCY(L)=OGOE NTON
    IS THE TOTAL MUWER OF TOWER FUMCTYONS USEDO
```

the following data is read in sequentially for each generalized FUNCTION REPRESENTING TOWER MOTION.

LIST $=(\operatorname{TOWY}(I): I=1 \cdot 18)$
FORMAT(GE12.6)
TOWY(I) IS THE DISPLACEMENT OF STATION I。 TOWY(1) REFERS TO THE FIRST FLOOR ABOVE THE LAUNCHER EACH TOWER STATION REFERS TO A FLOOR.

If THE FREQUENCY OF THIS FUNCTION IS 0.0 . THE FUNCTION IS ASSUMED TO BE THE RESULT OF A STATIC LOADING FUNCTION WHICH IS READ IN NEXT. IF FREQUENCYOGT.O.O THE FOLLOWING DATA IS NOT READ INE

LIST= (FORCT(1):1=1.19)
FORMAT(6E12.6)
FORCT(I) is the lateral point force at station i.
ENERGY TERYS ARE NOW READ IN FOR EACH GENERALIZED TOWER FUNCTION AS FOLLOWS. THE FOLLOWING 2 GARDS ARE READ FOR EACH FUNCTIONE

```
LIST=(KE(IFCN:B:OI=1.NFCN)
```

FORMAT(5E12.8)
KE (1.J) IS THE COEFFICIENT PLACED IN THE ITH ROW AND JTH COLUMN OF THE MATRIX REPRESENTING TOWER KINETIC ENERGY. NFCN IS THE TOTAL NUMBER OF FUNCTIONS USED TO REPRESENT TOWER MOTION. NFCN IS DEFINED IN BLOCK DATA. SECTION 7. A SIMILIAR CARD REPRESENTING potential energy is read next as follows

LIST=(PE(IFCN.I)OI.1.NFCN)
FORMAT(5EI2.8)

THIS DATA IS USED ONLY FOR THE NUMERICAL INTEGRATION SOLUTION OF THE GROUND WINDS RESPONSE THEREFORE IF SOLTYP= $I$ NO GARDS APPEAR FOR THIS DATA SOLTYP IS DEFINED IN BLOCK DATA (SECTION T) THE FOLLOWING DATA IS USED TO DEFINE THE FORCING FUNCTION THIS FUNCTION IS ASSUMED TO BE NORMAL TO THE ACTUAL WIND DIRECTION
$1.15 T=N S E G$
FORMAT(I5)
NSEG IS THE NUMEER OF SEGMENTS INTO WHICH THE VEHICLE IS DIVIDED TO MODEL THE FORCING FUNCTION

```
LIST= (LENGTH(I):DIA(I):VEL(1)&I=10NSEG)
```

FORMAT (3E12.8)

```
LENGTH(I)= LENGTH OF SEGMENT I
DIA(I)= DIAMETER OF SEGMENT I
VEL(I)= WIND VELOCITY ACTING ON SEGMENT I
```

FORMAT(AE12.8)
$C L=$ COEFFICIENT OF OSCILLATORY LIFT. IT VARIES BETWEEN O. 10 AND 0.14 OMEGA = ASSUMED RESPONGE FREQUENCY OF THE VEHICLE IT SHOULD BE VERY NEARLY EQUAL TO THE FIRST NATURAL VEHBCLE CANTILEVERED FRE QUENCY: ITS UNITS ARE RAD/SEC.
RHO $=$ MASS DENSITY OF AIRO
XZEE = THE CRITICAL DAMPING RATIO OF THE FIRST FREE VIERATIONAL VEHICLE MODE. THE CRITICAL DAMPING CONSTANT OF A GIVEN MOOE IS 2.O*SQRT(GM*GS) WHERE GM AND GS ARE THE MODAL GENERALIZED MASS AND STIFFNESS RESPECTIVELY DIVIDING THE TOTAL STRUCTURAL DAMPING BY THIS VALUE GIVES XZEE.
the following data is input as block data

DATA(NAME (I). $1=1.12$ )/GHW11 . GHW12. 6 HW21 . GHW22 -GHU11 16 HU12 . GHUL1 GHUZ2 GHSTRN-1.6HSTRN-2.6HTWR-1. 6 HTWR-2
name is an array used to label portions of output
8 NS $1 / 421$. NS2/18/ *NST/443/.


IF OPTIONS 5 THRU 12 ARE NOT EQUAL. TO ZERO THE INTERPRETATION IS OPPOSITE TO THAT OF THE ABOVE MEANING

DATALIeLE /10500.115.82438/

LI=LENGTH OF VEHECLE LEELENGTH OF TOWER

DATA XH:CD1:CD2/99.21633.0.0 0000/
$X H=E L E V A T I O N$ OF DAMPER LOCATION CDI AND CD2 = DAMPING CONSTANTS IN DIRECTIONS 1 AND 2 RESPECTIVELY

OATA INDI:(XZEE(1). $1=1.5) / 100.01 .0 .01 .0 .01 .0 .010000 /$
INDI = O INORCATES NO MODAL DAMPINE FOR THE VENECLE
XZEE (I) = MODAL DAMPTNG COEFFICIENT OF VENECLE MODE

```
DATA SYIFKI/ +20OE+OS// STIFKE/ +1075E+06/
    STIFKI ANO STIFKZDDANPING FRANE STIFFNESS IN DIREGTIONS I AND 2%
    RESPECTIVELY
    QATA NTORQ / O/
    NTORQ=NUNBER OF TORSIONAL MODES INCLUDED
```



```
*/90%0.0.18/
    PHI DEFINES UP TO 3 TORSIONAL DISPLACEMENT FUNCTIONS
    GJ(1)= TORSIONAL STIFFNESS OF THE ITH FLOOR
    RHO(I): MASS MOMENT OF INERTIA OF FLLOOR I
    NSTAQ= NUMEER OF STATIONS (FLOORS) USED TO DEFINE TORSIONAL FGNSO
DATA NoDELTAOINCTOT / 8.0.05.1800 /
THIS DATA IS USED IN THE NUMERICAL INTEGRATION SOLUTION OF THE FORCED RESPONSE N=NUMBER OF TERMS USED FOR SERIES CONVERGENCE DELTA=TIME INCREMENT INCTOT=TOTAL NUMEER OR INCREMENTS DESIRED
```



```
THIS DATA DEFINES THE PLOTTED OUTPUT OF THE FORCED RESPONSE NSEC=NUMEER OF SECONDS FER PAGE OF PLOT PLTLOC(I) \#ELEVATION OF POINTS FOR WHICH RESPONSE PLOTS ARE DESIRED. \(1=3\)
DATA SOLTYP/3/
SOLTYPEI FOR EIGENVALUE SOLUTION ONLY
\(=2\) FOR NUMERICAL SOLUTION ONLY
\(=3\) FOR BOTH SOLUTIONS
```

