

GENERATION OF UNCERTAINTY BOUNDARY
FOR ARCASONDE 1A TEMPERATURE SENSOR SYSTEM

Progress Report under
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DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF UTAH
SALT LAKE CITY, UTAH



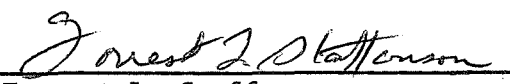
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ABSTRACT

A method of computing error bars, i.e., a running estimate of the overall uncertainty, for the temperature profile from a thermistor-type meteorological rocketsonde is presented. The resultant uncertainty is derived from estimated uncertainties in the parameter values assumed in the mathematical data correction relations. The "uncertainty boundary" is defined and the method of combining uncertainties is discussed.

A computer program in FORTRAN V is developed which computes corrections and uncertainties for real flight data. Simulated flight data is generated and used for illustration. Nominal values and uncertainty estimates for the parameters are those associated with the ARCASONDE 1A film-mounted thermistor sensor system.

Quantitative results reveal the relative sensitivity of the corrected temperature to the various parameters. Sensor improvement with the use of a radiation shield is illustrated in terms of reduced uncertainty boundary.

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GLOSSARY

a	= albedo
A	= total body surface
c	= specific heat of bead, wire, or film
c_p	= specific heat of air
D	= diameter of bead, wire, or thickness of film
D_{es}	= distance between the earth and the sun
$E_{b\lambda}(T)$	= plank radiant energy spectral distribution function for the source in $d\Omega$ at temperature T
$f_{i,j}$	= geometric factor for i th body with respect to j th source
g	= gravitational acceleration
h	= convective heat transfer coefficient
I_j	= radiant emittance of j th source
k	= thermal conductivity of the body
ℓ	= length of the wire
m	= mass of the parachute and the rocketsonde
q	= radiation heat input
r	= recovery factor
R	= radius of the earth
R_s	= radius of the sun
S	= solar constant
T	= temperature of bead, wire, or film
T_{air}	= temperature of the air
T_m	= measured value of T_b

T_s	= sensor temperature
T'_{wb}	= temperature gradient at the bead wire junction
T_r	= recovery temperature
v	= volume of bead, wire, or film
V	= air speed
W	= electric power dissipation
α	= long-wave absorptivity
α_s	= solar absorptivity
$\alpha_{i, j}$	= absorptivity of ith body with respect to jth source
α_λ	= spectral absorptivity
$\bar{\alpha}_j$	= mean absorptivity relative to the jth source
β_j	= radiation input perturbation factor
ϵ	= emissivity
ϵ_λ	= spectral emissivity
Ω	= solid angle subtended by the environment at the source
θ	= angle between sensor surface element dA and the direction toward $d\Omega$
λ	= radiation wavelength
σ	= Boltzmann constant
σ_{p_ℓ}	= standard deviation of parameter p_ℓ
η_{p_ℓ}	= mean value of parameter p_ℓ
ρ	= mass density of the sensor
$\mu_{\ell, m}$	= correlation coefficient between p_ℓ and p_m

Subscript

i = 1 bead

i = 2 wire

i = 3 film

j = 1 sun

j = 2 albedo

j = 3 long-wave radiation from the earth

j = 4 sonde parts in view of the sensor as a long-wave source

b bead

w wire

f film

fm Mylar part of the film

fs silver part of the film

I. INTRODUCTION

In order to increase the altitude capability of rocketsonde atmospheric temperature sensors, many different sensor configurations have been considered and developed.

Evaluation of sensor performance is necessary in order to compare sensors and improve the sensor system. This study employs the mathematical modeling approach to the theoretical study of immersion-type thermometer sensors used in current meteorological rocketsonde systems.

Figure 1 represents the system block diagram of a temperature measurement system. Input to the system is air temperature, T_{air} , which is transformed to the temperature of the sensor, T_s . The value of T_s is different from T_{air} due to heat flux associated with the following error sources:

1. Radiation
2. Aerodynamic heating
3. Heat conduction
4. Self heating
5. Thermal time lag

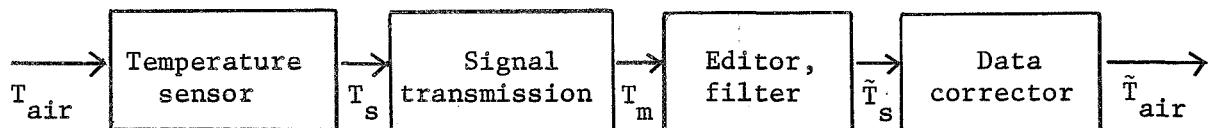


Fig. 1. Temperature measurement system.

T_s is then converted to the frequency of a blocking oscillator and relayed to a ground station where the signal is detected and recorded. This temperature, T_m , in general, contains measurement noise. Commonly, the data is filtered and edited to reject noise and spurious values and to obtain an improved representation, $\tilde{T}_s$, of the sensor temperature. Finally, computed corrections based on physical knowledge of the sensor and environment leads to the corrected air temperature, $\tilde{T}_{air}$.

Physical knowledge of the sensor system is embodied in a mathematical model of the form

$$T_s = f \left(T_{air}, \underline{p} \right) \quad (1)$$

where $\underline{p}$ is a vector notation representing the set of parameters used in the thermistor heat equation. From Eq. 1, an inverse function is derived and is used for data reduction using parameter estimates $\tilde{\underline{p}}$.

$$\tilde{T}_{air} = f^{-1} \left(\tilde{T}_s, \tilde{\underline{p}} \right) \quad (2)$$

The better system is generally considered as the system which has the smaller difference between T_s and T_{air} . However, if accurate information is known about each parameter, $\tilde{T}_{air}$ can be computed with small uncertainty by Eq. 2, regardless of the difference between T_s and T_{air} . The better system is actually the one which has the smaller

uncertainty in $\tilde{T}_{\text{air}}$.

In the following, a method is developed for computing the uncertainty in $\tilde{T}_{\text{air}}$ due to parameter uncertainties, and is applied to the ARCASONDE 1A meteorological rocketsonde temperature sensor.

II. DESCRIPTION OF THE ARCASONDE 1A TEMPERATURE MEASUREMENT SYSTEM

In the following two sections, a mathematical model of ARCA-sonde 1A temperature sensor and data correction system will be derived. The configuration of the ARCASONDE 1A sensor is shown in Fig. 2. Detail dimensions are listed in Table 1, page 27.

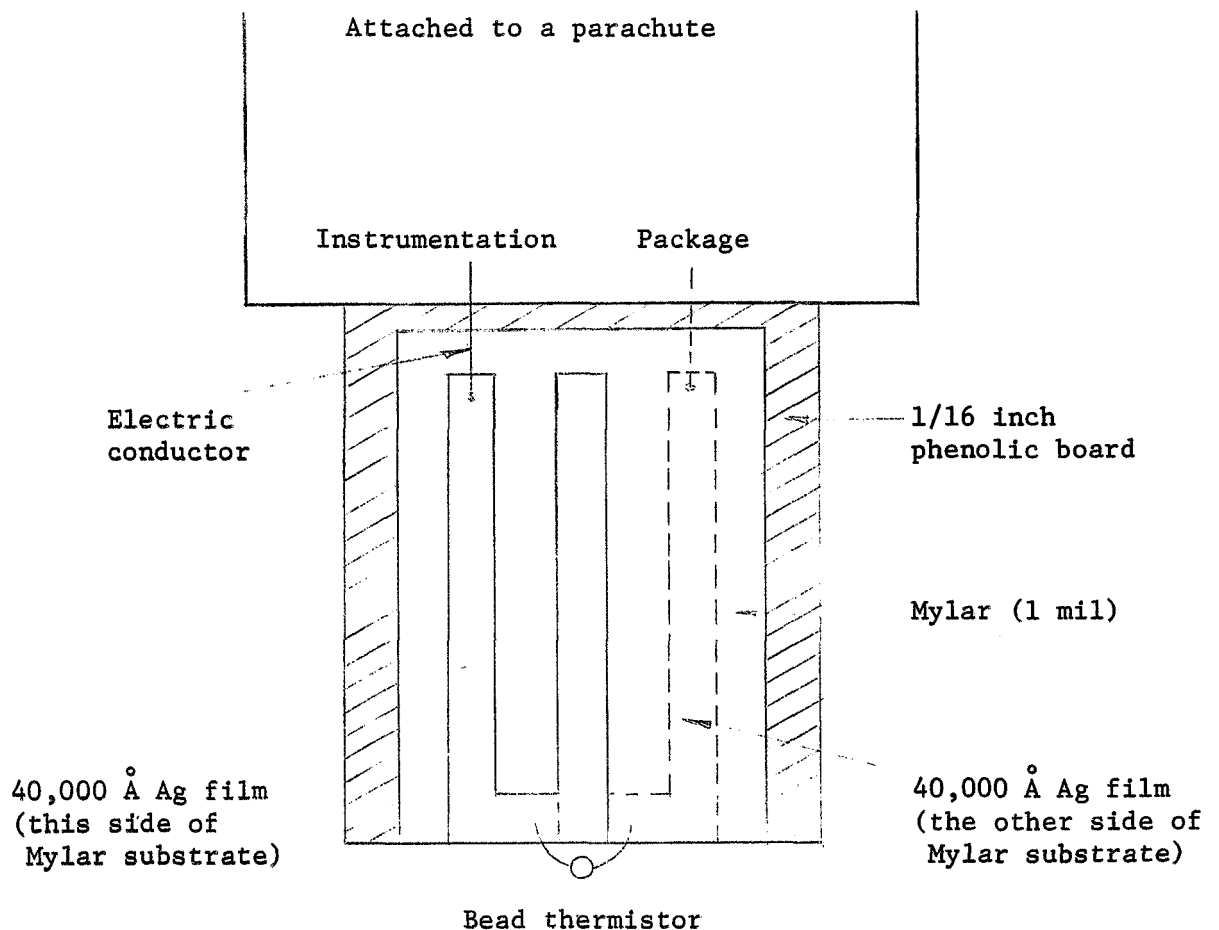


Fig. 2. ARCASONDE 1A thermistor sensor.

2.1 Mathematical Model of ARCASONDE 1A Sensor System

The temperature which is read by the electronic circuit in the sonde is that of the thermistor bead, i.e., T_s of Fig. 1 is T_b in the present discussion. The bead temperature is influenced by heat conduction from the wire, which is proportional to the temperature gradient in the wire at the point of contact with the bead. Using the bead and film temperatures as boundary conditions, the temperature gradient in the wire can be obtained.

The heat balance equations for the bead, wire, and film are:

(Bead)

$$(\rho cv)_b \frac{\partial T_b}{\partial t} = h_b A_b \left(T_{air} + r_b \frac{V^2}{2c_p} - T_b \right) + q_b A_b - A_b \sigma \epsilon_b T_b^4 + W_b$$

$$+ 2k_w \frac{\pi}{4} D_w^2 T'_{wb}$$
(3.a)

(Wire)

$$(\rho cv)_w \frac{\partial T_w}{\partial t} = h_w A_w \left(T_{air} + r_w \frac{V^2}{2c_p} - T_w \right) + q_w A_w - A_w \sigma \epsilon_w T_w^4 + W_w$$

$$+ k_w v_w \frac{\partial^2 T_w}{\partial x^2}$$
(3.b)

(Film)

$$\begin{aligned}
 (\rho cv)_f \frac{\partial T_f}{\partial t} = & h_f A_f \left(T_{\text{air}} + r_f \frac{V^2}{2c_p} - T_f \right) + q_f A_f - A_f \sigma \epsilon_f T_f^4 + W_f \\
 & + k_f v_f \left(\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right)
 \end{aligned}
 \tag{3.c}$$

where subscript b, w, f indicate bead, wire, and film, respectively,

where

ρ = mass density of an element

c = specific heat of an element

v = volume of an element

h = convective heat transfer coefficient

T'_{wb} = temperature gradient at the bead and the wire junction

r = recovery factor

V = relative speed of air

c_p = specific heat of air

q = radiation heat input

A = surface of an element

W = self heating

k = thermal conductivity

σ = Stefan-Boltzmann constant

D_b, D_w, D_f = diameter of bead, diameter of wire, thickness of film, respectively

ϵ = emissivity

These equations are coupled by conductive boundary conditions at surfaces of contact.

The radiation environment is considered in four parts, designated by the subscript $j = 1, \dots, 4$, according to radiant heat sources as follows:

- $j = 1$ direct solar illumination
- $j = 2$ indirect solar illumination
- $j = 3$ earth radiation
- $j = 4$ sonde radiation

The radiation input to a surface element of the sensor system is represented by

$$q_i = \sum_{j=1}^4 \alpha_{i,j} f_{i,j} I_j$$

where

α = radiation absorptivity

f = geometric factor

I = radiant emittance

and subscript $i = 1, 2, 3$ indicates the sensor part (body) b, w, f , respectively. More complete discussion concerning radiation terms is given in Appendix D.

It is not practical to solve the nonlinear simultaneous partial differential equation. The following assumptions have been made

in order to simplify the computation:

1. Time constant of the wire is very small compared with that of the film and the bead. Therefore, we can assume

$$\frac{\partial T_w}{\partial t} = 0$$

2. Temperature of the film is not influenced by heat exchange with the wire.
3. Temperature distribution of the film is assumed to be uniform near the film-wire junction.

By the above assumptions, Eqs. 3.a, 3.b, and 3.c are simplified as follows:

(Bead and wire)

$$\frac{(\rho c D)_b}{6} \frac{dT_b}{dt} = h_b \left(T_{air} + r_b \frac{V^2}{2c_p} - T_b \right) + q_b + \frac{W_b}{A_b} - \sigma \epsilon_b T_b^4 + X \quad (4.a)$$

(Film)

$$\frac{(\rho c D)_f}{2} \frac{dT_f}{dt} = h_f \left(T_{air} + r_f \frac{V^2}{2c_p} - T_f \right) + q_f - \sigma \epsilon_f T_f^4 \quad (4.b)$$

where

$$X = H_k \left(K_1 T_{air} + P + Q T_f - T_b \right)$$

$$H_k = c_2 \lambda_w \coth \lambda_w \ell$$

$$K_1 = \frac{h_w (1 - \operatorname{sech} \lambda_w \ell)}{h_w + 4\sigma \varepsilon_w T_{aw}^3}$$

$$P = \frac{(1 - \operatorname{sech} \lambda_w \ell) \left(h_w r_w \frac{V^2}{2c_p} + 3\sigma \varepsilon_w T_{aw}^4 \right)}{h_w + 4\sigma \varepsilon_w T_{aw}^3}$$

$$Q = \operatorname{sech} \lambda_w \ell$$

$$\lambda_w = \left(\frac{4(h_w + 4\sigma \varepsilon_w T_{aw}^3)}{(kD)_w} \right)^{1/2}$$

$$c_2 = \frac{(kD)_w D_w}{2D_b^2}$$

These constitute a set of two simultaneous differential equations representing the sensor system.

2.2 Data Correction System

From Fig. 1, the desired air temperature, T_{air} , is obtained by correcting the data, $\tilde{T}_b$, representing the sensor temperature, T_b . The required mathematical expressions are obtained from Eq. 4.a and 4.b, which expressed in one-step finite difference form are:

$$\tilde{T}_{\text{air}}^i = \frac{1}{\tilde{h}_b^i + \tilde{H}_k^i \tilde{K}_l^i} \left[\frac{(\tilde{\rho c D})_b}{6} \tilde{T}_b^i + \left(\tilde{h}_b^i + 4\sigma\tilde{\epsilon}_b (\tilde{T}_b^i)^3 + \tilde{H}_k^i \right) \tilde{T}_b^i \right. \\ \left. - \left(\tilde{h}_b^i \tilde{r}_b^i \frac{(\tilde{v}^i)^2}{2c_p} + 3\sigma\tilde{\epsilon}_b (\tilde{T}_b^i)^4 + \frac{\tilde{W}_b^i}{A_b} + \tilde{q}_b^i + \tilde{H}_k^i \tilde{p}^i \right) - \tilde{H}_k^i \tilde{Q}^i \tilde{T}_f^i \right] \quad (5.a)$$

$$\tilde{T}_f^i = \left[1 - \frac{2\Delta t}{(\tilde{\rho c D})_f} \left(\tilde{h}_f^{i-1} + \sigma\tilde{\epsilon}_f (\tilde{T}_f^{i-1})^3 \right) \tilde{T}_f^{i-1} \right] \\ + \frac{2\Delta t}{(\tilde{\rho c D})_f} \left(\tilde{q}_f^{i-1} + \tilde{h}_f^{i-1} \tilde{r}_f^{i-1} \frac{(\tilde{v}^{i-1})^2}{2c_p} \right) + \tilde{h}_f^{i-1} \frac{2\Delta t}{(\tilde{\rho c D})_f} \tilde{T}_{\text{air}}^{i-1} \quad (5.b)$$

where superscript i indicates the time, and

$$\tilde{H}_k^i = \tilde{c}_2 \tilde{\lambda}_w^i \coth \tilde{\lambda}_w^i \tilde{\ell}$$

$$\tilde{K}_1^i = \frac{\tilde{h}_w^i (1 - \operatorname{sech} \tilde{\lambda}_w^i \tilde{\ell})}{\tilde{h}_w^i + 4\sigma\tilde{\epsilon}_w (\tilde{T}_{\text{air}}^i)^3}$$

$$\tilde{P}^i = \frac{(1 - \operatorname{sech} \tilde{\lambda}_w^i \tilde{\ell}) \left(\tilde{h}_w^i \tilde{r}_w^i \frac{(\tilde{V}^i)^2}{2c_p} + 3\sigma\tilde{\epsilon}_w (\tilde{T}_{\text{air}}^i)^4 \right)}{\tilde{h}_w^i + 4\sigma\tilde{\epsilon}_w (\tilde{T}_{\text{air}}^i)^3}$$

$$\tilde{Q}^i = \operatorname{sech} \tilde{\lambda}_w^i \tilde{\ell}$$

$$\tilde{\lambda}_w^i = \left\{ \frac{4(\tilde{h}_w^i + 4\sigma\tilde{\epsilon}_w (\tilde{T}_{\text{air}}^i)^3)}{(\tilde{k}\tilde{D})_w} \right\}^{1/2}$$

$$\tilde{c}_2 = \frac{(\tilde{k}\tilde{D})_w \tilde{D}_w}{2\tilde{D}_b^2}$$

$$\Delta t = \text{sampling interval of } \tilde{T}_s \text{ (uniform)}$$

Assume the entire sensor system is initially uniform, i.e., $\tilde{T}_f^0 = \tilde{T}_b^0$.

Solution proceeds in time steps by computing alternately, T_f^i , then T_{air}^i . A schematic representation of the data correction process, together with the simulated sensors as it was programmed for use in this study, is shown in Fig. 3.

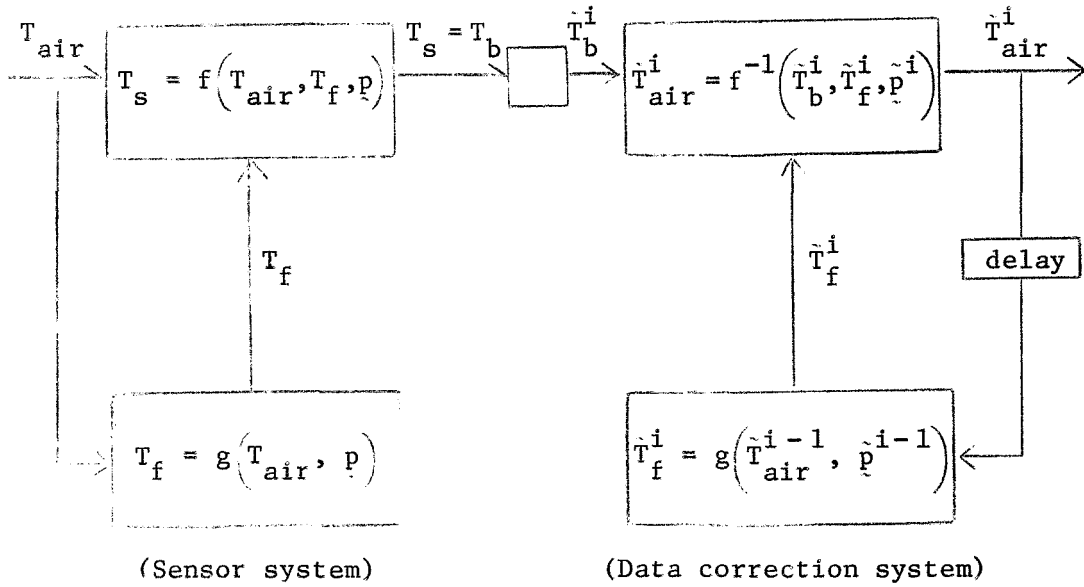


Fig. 3. Complete system description of ARCASONDE 1A measurement system.

III. PROPAGATION OF UNCERTAINTIES

A method of generating the uncertainty boundaries for the corrected air temperature function, $\tilde{T}_{\text{air}}$, discussed in the previous section is developed as follows. Successful assessment of variance in $\tilde{T}_{\text{air}}$ will require error examination in four categories:

1. Dimensions and properties (manufacturing variability).
2. Data handling (measurement errors).
3. Environmental parameters.
4. Approximations in the thermal analysis (model errors).

Uncertainty associated with error in the basic mathematical model of the sensor is assumed negligible.

3.1 Uncertainty Definition

The corrected air temperature is a function of a set of parameters, $\tilde{p}$.

$$\tilde{T}_{\text{air}} = f(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \dots, \tilde{p}_n) \quad (6)$$

Estimated value for the parameters are obtained from:

1. Specific laboratory measurements (such as for sensor emissivity, wire length).
2. In-flight measurement (such as for air speed).
3. Past experimental data (such as for earth albedo).

4. Theoretical calculations (such as for geometric factors, convective coefficients).

Suppose the error in a parameter estimate is defined as the difference between the estimated value and the true value of the parameter. Suppose further, that estimated values for a given flight are derived independently by many sufficiently qualified investigators. The estimated value, and therefore the error, would exhibit some statistical distribution, analogous to the results from a multiple sample experiment. The standard deviation and the mean of the estimated value may be defined as they are for a multiple sample experiment. If $\tilde{p}_i$ is the estimated value of a parameter from many different investigators, then the mean and standard deviation is defined as

$$\eta_{p_i} = \int_{-\infty}^{+\infty} p_i f(p_i) dp_i = \tilde{p} \quad (7)$$

$$\sigma_{p_i}^2 = \int_{-\infty}^{+\infty} (p_i - \eta_{p_i})^2 f(p_i) dp_i \quad (8)$$

where $f(p_i)$ is a density function of p_i .

If η_{p_i} and σ_{p_i} are given for each parameter in Eq. 6, $\eta_{T_{air}}$ and $\sigma_{T_{air}}$ can be computed. If T_{air} is distributed normally, one can expect to find, or can be confident of finding, true T_{air} lying

in the interval, $T_{\text{air}} \pm 2\sigma_{T_{\text{air}}}$, 95.45 percent of the time. It is expected that, in fact, T_{air} tends to be distributed normally because of the central limit theorem. The uncertainty boundary (or simply "uncertainty") is defined as $\pm 2\sigma$ of the nominal value.

3.2 Uncertainty Distribution

Though the distribution of the estimated value of a parameter may be conceptually defined, it is really known quantitatively for the parameters in the present discussion. For some parameters the uncertainty is given in the form of $p_i \pm \Delta p_i$. If only limiting values are known, the worst case method is sometimes used. The worst case method is a nonstatistical approach that employs the possible extremes of the parameters.

$$\Delta T_{\text{air}} = \sum_{i=1}^n \left| \left(\frac{\partial f}{\partial p_i} \right)_{p_i} \right| \Delta p_i \quad (9)$$

This usually gives an unrealistically pessimistic result. Even if they are strongly correlated, the assumption that the algebraic signs of all terms are the same is not statistically justified.

In order to obtain a more realistic uncertainty boundary, the concept of "uncertainty distribution" will be introduced. Kline and McClintock [9, 1953] applied this concept to describe uncertainties in single-sample experiments.

Suppose the variable p_i is expressed by $p_i \pm \Delta p_i$, where the value Δp_i is an estimation. An equivalent expression is the following:

$$P(p_i \leq p_i + \Delta p_i) \approx 1$$
(10)

$$P(p_i \leq p_i - \Delta p_i) \approx 0$$

If we define $F(X) = P\left\{p_i \leq X\right\}$, the above expression can be shown graphically as in Fig. 4.

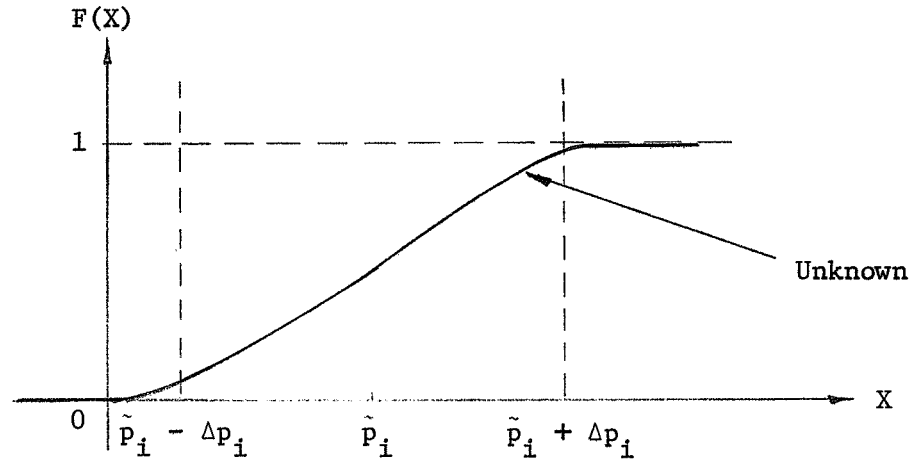


Fig. 4. Pseudo distribution curve.

Even if, between $p_i - \Delta p_i$ and $p_i + \Delta p_i$, there is no information, a

pseudo distribution may be imagined.

The corresponding pseudo density function may then be defined as $f(X) = dF(X)/dX$. One might feel that the pseudo distribution is

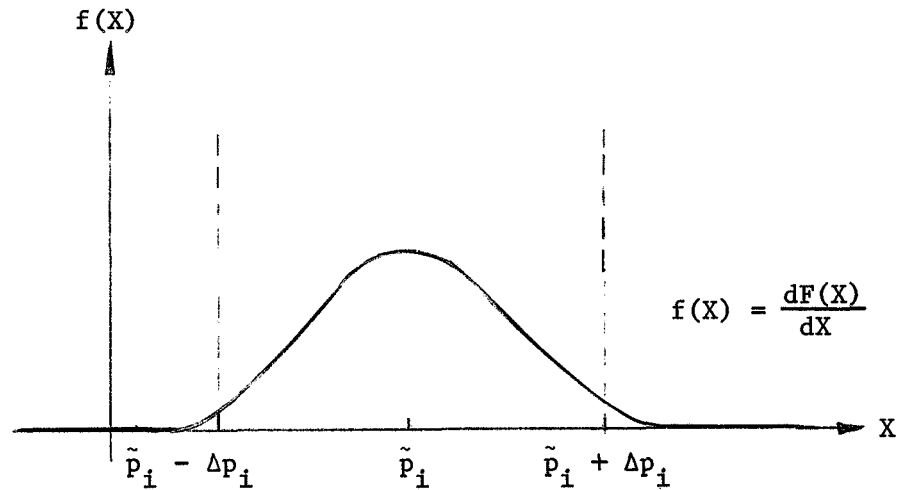


Fig. 5. Pseudo density function.

uniform for some parameters. For other parameters, however, intuition may suggest the errors are more likely near the center than near the ends of their respective ranges. Therefore, one might attempt to simulate this feeling by assuming the density function to be approximately normal, and Δp_i to be $2\sigma_{p_i}$ [7, Eisenhart 1963].

3.3 Propagation of Uncertainties

Now that we have estimated (or imagined) σ_{p_i} , the next step is to relate to the σ_{p_i} in accordance with $\sigma_{T_{air}}$, Eq. 6. Papoulis [10, 1965] discussed an approximation for this relation for the case of two

parameters. The idea is easily extended to n parameters (Appendix A).

$$\sigma_{T_{air}}^2 = \sum_{i=1}^n \left(\frac{\partial f}{\partial p_i} \right)_{p_i} \sigma_{p_i}^2 + \sum_{i=1} \sum_{\substack{j=1 \\ i \neq j}} \left(\frac{\partial f}{\partial p_i} \right) \left(\frac{\partial f}{\partial p_j} \right) \frac{\text{cov}(p_i, p_j)}{\sigma_{p_i} \sigma_{p_j}} \quad (11)$$

where

$$\begin{aligned} \text{cov}(p_i, p_j) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (p_i - \tilde{p}_i)(p_j - \tilde{p}_j) f(p_i, p_j) dp_i dp_j \\ &= \mu_{ij} \sigma_{p_i} \sigma_{p_j} \end{aligned}$$

The validity of the above equation depends on the nonlinearity of $f(p_1, p_2, \dots, p_n)$ and the distribution of the parameters, but the approximation error within a sufficiently small range of the parameters is assumed to be small compared with the estimation error in the σ_{p_i} .

Thorough mathematical study of the approximation error is beyond the scope of the present discussion, but Monte-Carlo simulation studies indicate the error is small in the ARCASONDE 1A system for a considerably large range of the parameters.

IV. UNCERTAINTY BOUNDARY OF ARCASONDE 1A SYSTEM

The method of generating the uncertainty boundary in $\hat{T}_{air}$, which was discussed in Chapter III, will be applied to the ARCASONDE 1A system which was introduced in Chapter II.

4.1 Expression of Uncertainty Boundary

Equations 5.a and 5.b can be expressed in the following form:

$$T_{air}^i = f^i \left(p_j^k \right) \quad \begin{matrix} j = 1, \dots, n \\ k = 1, \dots, i \end{matrix} \quad (12)$$

where the input parameters include the sensor temperature $\hat{T}_b^k$. The subscript j denotes the particular parameter, and k denotes the time.

Applying Eq. 11 to Eq. 12 gives

$$\sigma_{T_{air}}^2 = \sum_{j=0}^i \sum_{m=1}^n \left(\frac{\partial f^i}{\partial p_m^j} \right)^2 \sigma_{p_m^j}^2 + \sum_{j=0}^i \sum_{k=0}^i \sum_{m=1}^n \sum_{\ell=1}^n \left(\frac{\partial f^i}{\partial p_m^j} \right) \left(\frac{\partial f^i}{\partial p_\ell^k} \right) \frac{\text{cov}(p_m^j, p_\ell^k)}{\sigma_{p_m^j} \sigma_{p_\ell^k}} \quad (13)$$

$\ell \neq m, j \neq k$

The term $\left(\frac{\partial f^i}{\partial p_m^j} \right)$ is called the sensitivity coefficient [16, Tomovic 1962] and contains information about the system. $\sigma_{p_m^j}$ and $\text{cov} \left(p_m^j, p_\ell^k \right)$ are

independent of the sensor system and the values are estimated according to conditions at each point in time.

4.2 Sensitivity Analysis

In order to obtain the value of $\sigma_{T_{air}}^i$, the sensitivity coefficients have to be computed for each parameter at each preceding time point. Therefore, at each time i , we have to compute $i \times n$ sensitivity coefficients. One might consider this a very time consuming process for a system of many parameters and over many points in time, but actually $\frac{\partial f^i}{\partial p_\ell^j} \rightarrow 0$ when $(i - j) \rightarrow \infty$, therefore, older terms become negligible.

The sensitivity coefficient $\frac{\partial T_{air}^i}{\partial p_\ell^j}$ indicates the effect on T_{air} at time i of variation in p_ℓ at time j . As shown in Fig. 6, an error

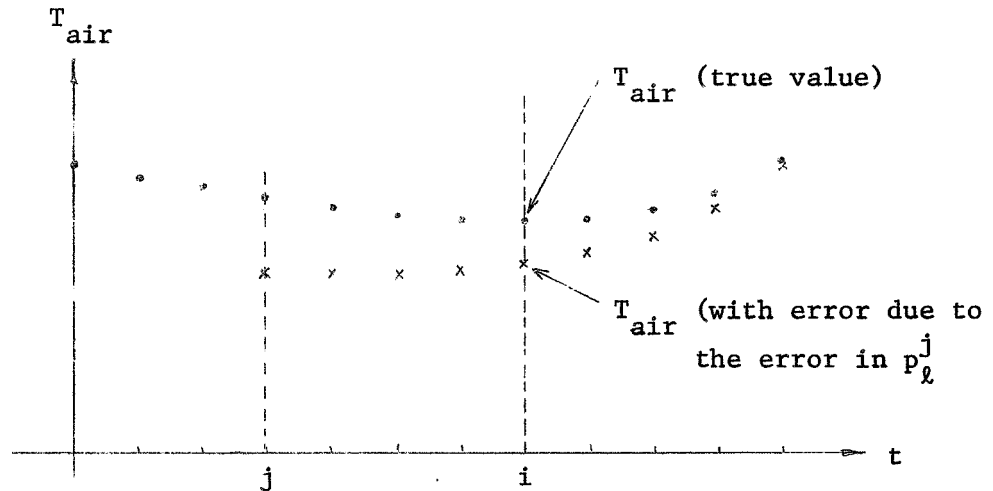


Fig. 6. Graphical description of $\frac{\partial T_{air}^i}{\partial p_\ell^j}$.

in p_ℓ at time j will have a diminishing effect on succeeding values of T_{air} .

Sensitivity coefficients are computed as follows. Equations 5.a and 5.b may be written in the form

$$T_{\text{air}}^i = f(p^i, T_f^i) \quad (14.a)$$

$$T_f^i = g(p^{i-1}, T_f^{i-1}, T_{\text{air}}^{i-1}) \quad (14.b)$$

where

$$p^i = \begin{bmatrix} p_1^i \\ p_2^i \\ \vdots \\ p_n^i \end{bmatrix}$$

Assume the sensitivity coefficients have been computed up to the time $(i - 1)$. Let already computed quantities be enclosed in parentheses. Only the bracketed quantities need be computed at the i th point in time. For the parameter, p_ℓ ;

$$\underline{j = i}$$

$$\frac{\partial T_{\text{air}}^i}{\partial p_{\ell}^j} = \frac{\partial T_{\text{air}}^i}{\partial p_{\ell}^j} = \left[\frac{\partial f}{\partial p_i} \right]$$

Expression for the $\frac{\partial f^i}{\partial p_i}$ for the ARCASONDE 1A are listed in Appendix B.

$$\underline{j = i - 1}$$

$$\frac{\partial T_{\text{air}}^i}{\partial p_{\ell}^j} = \frac{\partial T_{\text{air}}^i}{\partial p_{\ell}^{i-1}} = \left[\frac{\partial f^i}{\partial T_f^i} \right] \left[\frac{\partial T_f^i}{\partial p_{\ell}^{i-1}} \right]$$

where

$$\frac{\partial f^i}{\partial T_f^i} = \left[\frac{\partial f^i}{\partial G^i} \right] \left[\frac{\partial G^i}{\partial T_f^i} \right] + \left[\frac{\partial f^i}{\partial T_f^i} \right]$$

$$\frac{\partial T_f^i}{\partial p_{\ell}^{i-1}} = \left[\frac{\partial g^i}{\partial p_{\ell}^{i-1}} \right] + \left[\frac{\partial g^i}{\partial T_{\text{air}}^{i-1}} \right] \left(\frac{\partial T_{\text{air}}^{i-1}}{\partial p_{\ell}^{i-1}} \right)$$

Computation of $\frac{\partial f^i}{\partial G^i}$ and $\frac{\partial G^i}{\partial T_f^i}$ are shown in Appendix B.

$$\underline{j = i - 2}$$

$$\frac{\partial T_{\text{air}}^i}{\partial p_{\ell}^j} = \frac{\partial T_{\text{air}}^i}{\partial p_{\ell}^{i-2}} = \left(\frac{\partial f^i}{\partial T_f^i} \right) \left[\frac{\partial T_f^i}{\partial p_{\ell}^{i-2}} \right]$$

$$\frac{\partial T_f^i}{\partial p_{\ell}^{i-2}} = \left(\frac{\partial g^i}{\partial T_{\text{air}}^{i-1}} \right) \left(\frac{\partial T_{\text{air}}^{i-1}}{\partial p_{\ell}^{i-2}} \right) + \left[\frac{\partial g^i}{\partial T_f^{i-1}} \right] \left(\frac{\partial T_f^{i-1}}{\partial p_{\ell}^{i-2}} \right)$$

Computation of $\frac{\partial g^i}{\partial T_f^{i-1}}$ is shown in Appendix B.

$$\underline{j = i - 3}$$

$$\frac{\partial T_{\text{air}}^i}{\partial p_{\ell}^j} = \frac{\partial T_{\text{air}}^i}{\partial p_{\ell}^{i-3}} = \left(\frac{\partial f^i}{\partial T_f^i} \right) \left[\frac{\partial T_f^i}{\partial p_{\ell}^{i-3}} \right]$$

$$\frac{\partial T_f^i}{\partial p_{\ell}^{i-3}} = \left(\frac{\partial g^i}{\partial T_{\text{air}}^{i-1}} \right) \left(\frac{\partial T_{\text{air}}^{i-1}}{\partial p_{\ell}^{i-3}} \right) + \left(\frac{\partial g^i}{\partial T_f^{i-1}} \right) \left(\frac{\partial T_f^{i-1}}{\partial p_{\ell}^{i-3}} \right)$$

$$\underline{j = 0}$$

$$\frac{\partial T_f^i}{\partial p_\ell^0} = \left(\frac{\partial f^i}{\partial T_f^i} \right) \left(\frac{\partial T_f^i}{\partial p_\ell^0} \right)$$

$$\frac{\partial T_f^i}{\partial p_\ell^0} = \left(\frac{\partial g^i}{\partial T_{air}^{i-1}} \right) \left(\frac{\partial T_{air}^{i-1}}{\partial p_\ell^0} \right) + \left(\frac{\partial g^i}{\partial T_f^{i-1}} \right) \left(\frac{\partial T_f^{i-1}}{\partial p_\ell^0} \right)$$

Notice that only simple multiplication is needed for derivatives with respect to parameter values older than $i - 3$.

The correlation coefficient in time for a truly constant parameter p_ℓ is $\mu_\ell = 1$. The contribution of p_ℓ to $\sigma_{T_{air}^i}^i$, $\sigma_{T_{air}(p_\ell)}^i$ is

$$\sigma_{T_{air}(p_\ell)}^i = \frac{\partial T_{air}^i}{\partial p_\ell} \sigma_{p_\ell}$$

For constant parameters, the above procedure for computing the sensitivity coefficients is simplified by solving a set of difference equations.

From Eqs. 14.a and 14.b

$$\frac{\partial T_{\text{air}}^i}{\partial p_\ell} = \frac{\partial f^i}{\partial p_\ell} + \frac{\partial f^i}{\partial T_f^i} \frac{\partial T_f^i}{\partial p_\ell}$$

$$\frac{\partial T_f^i}{\partial p_\ell} = \frac{\partial g}{\partial p_\ell} + \frac{\partial g}{\partial T_f^{i-1}} \frac{\partial T_f^{i-1}}{\partial p_\ell} + \frac{\partial g}{\partial T_{\text{air}}^{i-1}} \frac{\partial T_{\text{air}}^{i-1}}{\partial p_\ell}$$

Substituting $U^i = \frac{\partial T_{\text{air}}^i}{\partial p_\ell}$ and $V^i = \frac{\partial T_f^i}{\partial p_\ell}$,

$$U^i = \frac{\partial f^i}{\partial p_\ell} + \frac{\partial f^i}{\partial T_f^i} V^i \quad (15.a)$$

$$V^i = \frac{\partial g^i}{\partial p_\ell} + \frac{\partial g^i}{\partial T_f^{i-1}} V^{i-1} + \frac{\partial g^i}{\partial T_{\text{air}}^{i-1}} U^{i-1} \quad (15.b)$$

which is a special case of the more general procedure given above

(Appendix C). The required computation is obviously much less for constant parameters.

4.3 Estimated Nominal Values and Uncertainties of Parameters

Sensor Properties

A list of input parameters for the ARCASONDE 1A temperature sensor is presented in Table 1. Since the film is a composition of silver and Mylar, properties of both must be used. Subscript fm, fs indicate the Mylar and silver parts of the film, respectively. Subscript f indicates the effective value.

- A. D_b, D_w, D_{fm}, D_{fs} (diameter of bead, wire, and the thickness of the film). Uncertainty in these quantities is due to manufacturing variability and imperfections in shape. The uncertainty is estimated to be 10 percent.
- B. $(\rho c)_b, (\rho c)_w, (\rho c)_{fm}, (\rho c)_{fs}, (\rho c)_f$ (density times heat capacity). The effective value of ρc for the film is given by

$$(\rho c)_f = \frac{D_s (\rho c)_s + D_m (\rho c)_m}{D_s + D_m}$$

The uncertainties for $(\rho c)_b, (\rho c)_w$ are estimated to be 5 percent due to lack of knowledge of composition, and 10 percent for $(\rho c)_f$ due to an estimated variability in Mylar thickness.

TABLE 1

Uncertainties and Nominal Value of Parameters

	Nominal Value	Uncertainty	Reference
D_b	$.28 \times 10^{-3}$ m (11 mil)	10%	Drews [4]
D_w	$.25 \times 10^{-4}$ m	10%	Drews
D_{fm}	$.25 \times 10^{-4}$ m		Drews
D_{fs}	$.4 \times 10^{-5}$ m		Drews
D_f	$.31 \times 10^{-4}$ m	7%	
$(\rho c)_b$	1.95×10^6 J/m ³ - °K	5%	Wright [19]
$(\rho c)_w$	2.79×10^6 J/m ³ - °K	5%	Wright
$(\rho c)_{fs}$	2.45×10^6 J/m ³ - °K		Weast [18]
$(\rho c)_{fm}$	1.84×10^6 J/m ³ - °K		Dupont [5]
$(\rho c)_f$	1.96×10^6 J/m ³ - °K	10%	
k_w	30.98 watt/m - °K	5%	Weast
k_{fm}	.152 watt/m - °K	5%	Dupont
k_{fs}	408 watt/m - °K	5%	Weast
α_b	.11	25%	Thompson [15]
α_w	.10	10%	Thompson
α_{fs}	.02		Weast
α_{fm}	.80		Drews
α_f	.65	50%	
α_{sb}	.16	40%	Thompson
α_{sw}	.19	50%	Thompson
α_{sfs}	.07		Weast
α_{sfm}	.06		Drews
α_{sf}	.22	50%	
ℓ	3.2×10^{-3} m	50%	Drews

- C. k_w, k_{fm}, k_{fs}, k_f (thermal conductivity). The uncertainty for k is estimated to be 5 percent because of the impurity of materials and experimental error in published data [11, Powell, Ho, Liley]. Effective value of k is k_{fs} because $(kD)_s \gg (kD)_m$.
- D. $\alpha_b, \alpha_w, \alpha_{fs}, \alpha_{fm}$ (absorptivity of long wave radiation). The effective absorptivity of the Mylar-exposed side of a silver-plated region is taken as

$$\alpha = \alpha_m + \left[\alpha_s + \alpha_m (1 - \alpha_s) \right] (1 - \alpha_m)$$

which assumes the reflectivity of the Mylar to be small.

Similarly, the emissivity is assumed to be

$$\epsilon = \epsilon_m + (1 - \alpha_m) \left[(1 - \alpha_s) \epsilon_m + \epsilon_s \right]$$

which includes approximately the emission of the Mylar forward that emitted backward and reflected by the silver, and the emission of silver through the Mylar and, which incidentally, is the same value as the above absorptivity.

The effective emissivity of the film is computed by averaging that of the inner and outer film strips, and using their lengths as weighting factors.

Uncertainty in α_b is estimated to be about 25 percent

because of the condition of coating the sphere surface [15, Thompson 1966]. Uncertainty of ϵ_f is estimated to be 50 percent due to a nonuniform plastic coating of unknown composition over the silver.

- E. $\alpha_{sb}, \alpha_{sw}, \alpha_{sfs}, \alpha_{sfm}, \alpha_{sf}$ (absorptivity of short wave radiation). Uncertainty of α_{sb} and α_{sw} are assumed to be 40 percent due to the manufacturing variation in the surface [15, Thompson 1966]. The uncertainty in α_{sf} is estimated to be 50 percent due to the plastic coating.
- F. l (length of the wire). Uncertainty in the length of the wire is estimated to be about 50 percent due to manufacturing variation.

Convective Environment

Convective coefficient h and recovery factor r are computed using the interpolation formula [13, Staffanson and Alsaj 1968].

$$h = \frac{1}{\frac{1}{h_1} + \frac{1}{h_2}}, \quad r = r_1 + K_n \frac{r_2 - r_1}{Kn + Kn_0}$$

Subscript 1, 2 denote continuum and free molecular values, respectively. The uncertainty increases in the transition flow region where knowledge is least reliable. The uncertainty in recovery factor is estimated as shown in Fig. 7.

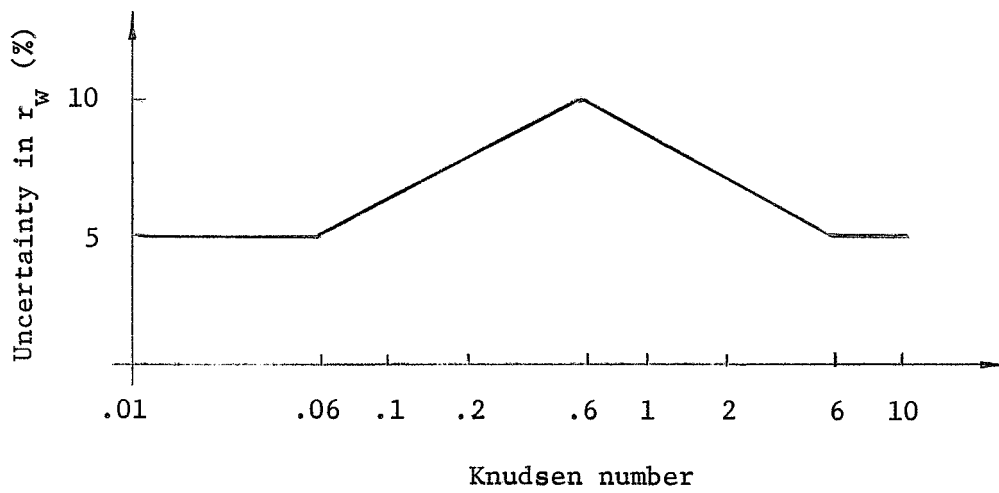
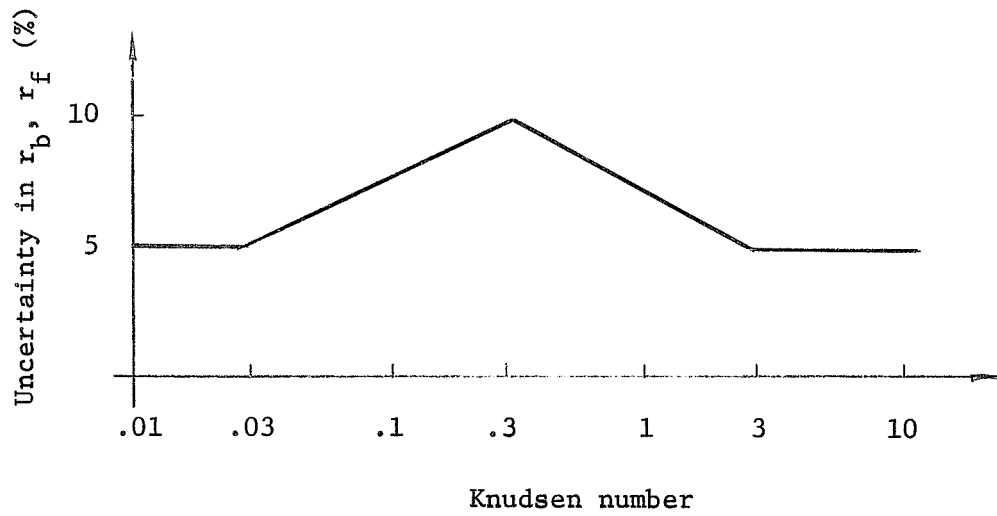


Fig. 7. Percent uncertainty of recovery factor.

Table 2 indicates the assumed uncertainty in h with respect to Reynolds number. Linear interpolation gives the uncertainty at any Reynolds number.

TABLE 2

Uncertainty of Convective Coefficient

Reynolds Number	Uncertainty (%)
$\leq 10^{-2}$	5%
10^{-1}	7.5%
1	10%
10	15%
10^2	18%
10^3	15%
10^4	10%
10^5	7.5%
$\geq 10^6$	5%

Radiation Environment

The geometric factor, $f_{i,j}$, is a quantity which depends on the shape of the sensor surfaces, the solid angle subtended by the source, and the orientation of the sensor relative to the source. The method of obtaining the values of $f_{i,j}$ is presented in Appendix D and the nominal values are presented in Table 3.

TABLE 3
Nominal Values and Uncertainties of the Geometric Factor

	Sphere	Cylinder	Plate
Sun	$.272 \times 10^{-5} \pm 100\%$	$.347 \times 10^{-5} \pm 100\%$	$.535 \times 10^{-5} \pm 100\%$
Albedo	$0.40 \pm 15\%$	$0.40 \pm 15\%$	$0.40 \pm 15\%$
Earth	$0.42 \pm 5\%$	$0.43 \pm 10\%$	$0.41 \pm 10\%$
Sonde	$0.087 \pm 3\%$	$0.108 \pm 3\%$	$0.044 \pm 3\%$

The radiant emittance, I_j (total radiant power emitted per unit area), of a solid surface depends on its emissivity, ϵ_λ , and its absolute temperature, T . I_j is computed in Appendix D. The nominal values of I_j are presented in Table 4.

TABLE 4
Nominal Values and Uncertainties of Radiant Emittance

Sun	$6.4558 \times 10^7 \text{ watt/m}^2 \pm 1\%$
Albedo	$460.7 \text{ watt/m}^2 \pm 36\%$
Earth	$233.8 \text{ watt/m}^2 \pm 20\%$
Sonde	$458 \text{ watt/m}^2 \pm 15\%$

A. $f_{1,1}$, $f_{2,1}$, $f_{3,1}$ (geometric factors with respect to the sun). Uncertainties of $f_{2,1}$ are 100 percent because there is, in general, no knowledge as to sensor aspect to the sun. The sensor might be completely in the shade or exposed "broadside" to the sun.

B. $f_{1,3}$, $f_{2,3}$, $f_{3,3}$ (geometric factors with respect to the earth long-wave radiation). As indicated in Appendix D, $f_{3,3}$ varies as the sensor rotates relative to the earth.

If we assume the parachute has less than 45° coning motion, the variation of $f_{2,3}$ and $f_{3,3}$ due to the motion is about 10 percent.

The geometric factor of a truly spherical bead does not depend on the motion of the parachute. However, an estimated 5 percent uncertainty in $f_{1,3}$ is assumed for deviations due to the presence of the wires and nonsphericity.

C. $f_{1,2}$, $f_{2,2}$, $f_{3,2}$ (geometric factor with respect to the earth albedo). The albedo geometric factor is dependent on the position of the sun and cloud distribution, as well as to sensor attitude. The uncertainties in $f_{1,2}$ are estimated to be 15 percent.

D. $f_{1,4}$, $f_{2,4}$, $f_{3,4}$ (geometric factor with respect to the sonde surfaces or shield). The errors in estimated nominal values, $f_{1,4}$, are estimated to be 3 percent.

E. I_1 , I_2 , I_3 , I_4 (radiant emittance). The uncertainty of

I_1 is due to the seasonal variation of the distance between the sun and the earth and is estimated to be 1 percent [8, Johnson 1954].

The uncertainty of I_2 is due to the cloud cover variability and is estimated to be 36 percent (see Appendix D).

The uncertainty of I_3 is due to variability of earth surface matter and temperature. The analysis of Tiros II (1960) data by Bandeen [2, 1961] gives about ± 20 percent variation in earth temperature at a given time over the North American continent. Assuming the uncertainty in the effective black body, temperature of the local region of the earth is 5 percent, uncertainty in I_3 is 20 percent.

The uncertainty of I_4 is due to the variability in the sonde temperature and emissivity. Uncertainty in I_4 is arbitrarily estimated to be 15 percent.

Self Heating

An estimation of the power dissipated by the thermistor is about 20×10^{-6} watts [4, Drews 1966]. The uncertainty is due to the temperature dependency of the power dissipation and is estimated to be 15 percent.

Data

Measurement error of T_b is estimated to be 2°K , and the uncertainty in T_b is assumed to be 10 percent. The uncertainty of the rela-

tive velocity, V , is about 5 percent based on the assumed angular motion about the center of mass trajectory recorded by the tracking radar.

Correlation Coefficient $\rho_{m, \ell}$

For the present study, all parameters are assumed that independent so $\text{cov}\left(p_m^j, p_\ell^k\right)$ and, therefore, $\rho_{m, \ell}$ are set equal to zero if $m \neq \ell$.

4.4 Simulation Study of ARCASONDE 1A System

A. Simulation

Sensor air flow is simulated by computing parachute motion based on the ballistic coefficient and drag [6, Eddy 1965] properties of the ARCASONDE system.

Total parachute-sonde mass $m = 2.33 \text{ kg}$

Parachute reference area $A = 16.4 \text{ m}^2$

The U.S. Standard Atmosphere (1962) will be used for density of the air, ρ .

Initial conditions of the parachute motion are arbitrarily chosen as

Initial altitude 70 km

Horizontal velocity 30 m/s

Vertical velocity -180 m/s

At every second, the altitude and the relative speed with respect to the air are computed. At each altitude, the properties of the air are obtained from the atmosphere table, which already has been read in, including T_{air} . Values of h and r are obtained, based on the given altitude and the air speed. Those values are used in Eq. 4.a and 4.b in order to obtain T_b .

The simulated T_b is used as input $\tilde{T}_b$ to the data corrector. Parameter values, p_j , used in the simulator are also used as input $\tilde{p}_j$ in the data corrector. Therefore, $\tilde{T}_{\text{air}}$ is equal to T_{air} . Now the objective of the study is to produce the uncertainty boundary of $\tilde{T}_{\text{air}}$.

The uncertainties in the parameters which were discussed in Section 4.3 are combined with Eq. 13, where sensitivity coefficients are computed by the method discussed in Section 4.2. The computational procedure is shown in Fig. 8.

Notice in Fig. 8 that part A corresponds to the "sensor system" in Fig. 2, and part B corresponds to "data reduction system" in Fig. 2.

B. Results

As shown in Fig. 9, the uncertainty boundary increases rapidly with altitude. This is due to the decrease of convective coupling with the air, while the radiation input remains constant.

As h_b and h_w decrease with altitude, there is more conductive heat flow from the film. The sensor is, therefore, more sensitive to the film temperature as altitude increases. This is clear if you compare the effect of h_b and h_f at 70 and 50 km in Table 5. The uncer-

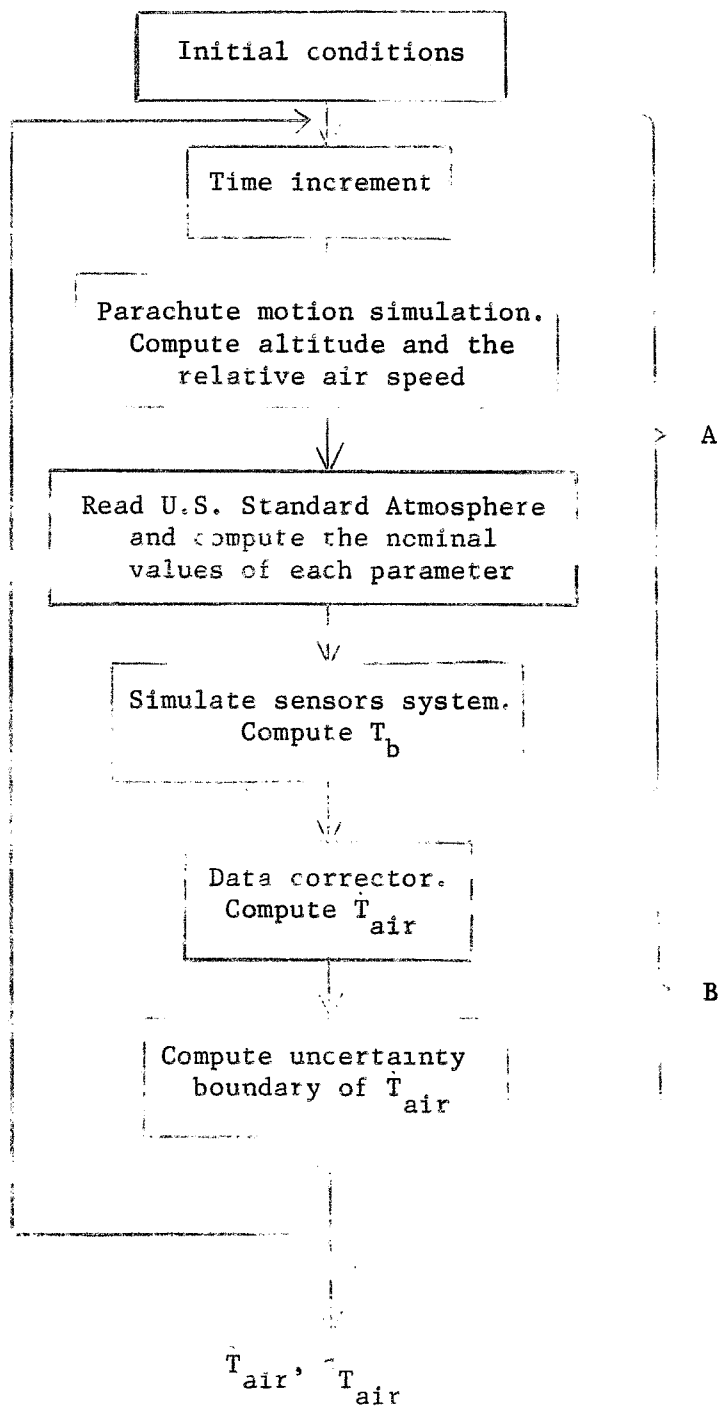


Fig. 8. Block diagram of simulation study.

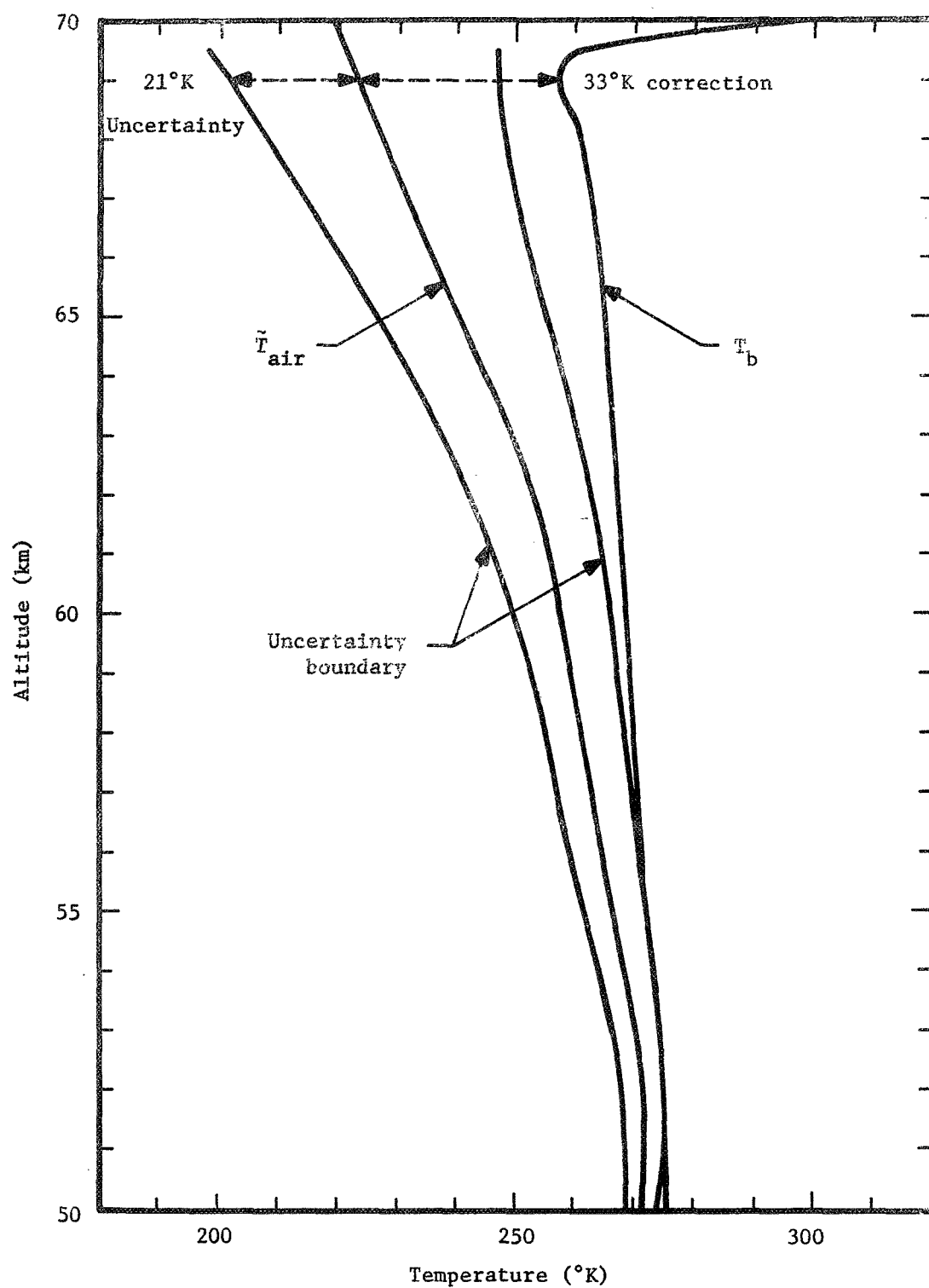


Fig. 9. Uncertainty boundary.

TABLE 5
Uncertainty Components

Altitude (km)	70	65	60	55	50
h_b	.27	.86-1	.63-1	.43-1	.30-1
r_b	.32	.33	.19	.11	.57-1
h_w	.14	.78-2	.37-2	.41-2	.25-2
r_w	.38	.32	.16	.76-1	.48-1
h_f	.35	.11	.29-1	.46-3	.50-2
r_f	.76-1	.51-1	.17-1	.53-2	.16-2
V	1.38	1.16	.56	.27	.13
T_b	3.47	2.34	1.71	1.37	1.19
$\dot{T}_b$	.23	.59-2	.12-2	.29-3	.18-2
$f_{1,1}$	.92	.53	.31	.19	.12
$f_{1,2}$	.15	.83-1	.48-1	.29-1	.19-1
$f_{1,3}$	.18-1	.10-1	.60-2	.36-2	.23-2
$f_{2,1}$	1.38	.77	.43	.24	.14
$f_{2,2}$	.17	.95-1	.53-1	.30-1	.17-1
$f_{2,3}$	.33-1	.18-1	.10-1	.57-2	.33-2
$f_{3,1}$	4.17	2.26	1.24	.65	.32
$f_{3,2}$	.33	.18	.99-1	.52-1	.26-1
$f_{3,3}$	.34	.18	.10	.53-1	.26-1
w_b	.40	.23	.13	.80-1	.51-1
α_b	.48-1	.43-1	.34-1	.26-1	.19-1
α_w	.41-1	.30-1	.21-1	.14-1	.93-2

TABLE 5
(continued)
Uncertainty Components

Altitude (km)	70	65	60	55	50
D_b	.19-1	.16	.94-1	.60-1	.31-1
$(\rho c)_b$	.40-1	.15-2	.42-2	.26-3	.93-3
D_w	.11	.24	.14	.93-1	.54-1
k_w	.41-1	.79-1	.49-1	.32-1	.19-1
ℓ	.66	1.18	.74	.48	.29
$(\rho c)_f$	.30	.35-2	.10-2	.97-3	.58-4
D_f	.21	.25-2	.70-3	.68-3	.40-4
$f_{1,4}$	.23-2	.15-2	.10-2	.71-3	.50-3
$f_{2,4}$	.25-2	.16-2	.11-2	.70-3	.44-3
$f_{3,4}$	.51-1	.29-1	.16-1	.85-2	.42-2
α_{sb}	.40	.27	.18	.13	.87-1
α_{sw}	.53	.34	.23	.15	.91-1
α_{sf}	7.59	4.37	2.42	1.26	.62
I_1	.11	.65-1	.37-1	.20-1	.10-1
I_2	2.30	1.35	.78	.43	.23
I_3	1.70	.98	.55	.29	.15
I_4	.28	.96	.92-1	.50-1	.25-1
resultant	23.98	14.06	8.36	4.98	3.20

tainty components listed in Table 5 are the terms

$$\left\{ \sum_{j=0}^i \left(\frac{\partial T_{\text{air}}^i}{\partial p_{\ell}^j} \right)^2 \sigma_{p_{\ell}^j}^2 \right\}^{1/2}$$

Quantities listed in Tables 5, 8, 9, and 10, pages 39, 46, 48, and 50, respectively, at 70 km are actually those computed at 69.5 km. Those computed at the initial altitude, 70 km, are those at ejection from the rocket and are not of interest here. Values tabulated, nevertheless, exhibit some effect of the initial transient, e.g., the small values at 70 km of ℓ , k_w , D_w .

The sensitivity to error in T_b (measurement error) approaches unity at low altitudes and increases rapidly at high altitudes. As the convective coefficients, h_i , decrease, sensitivity to air temperature decreases; i.e., a given variation in T_b corresponds to increasingly larger variations in T_{air} .

Notice that the uncertainty contribution associated with direct solar heating, $\alpha_s f_{i,1} I_1$, is large compared with other parameters. This suggests that a solar shield would significantly decrease the overall uncertainty. In the following, the ARCASONDE 1A sensor is assumed to be placed one radius deep into a cylindrical tube. The tube is oriented with its axis parallel to the flow, and with radius sufficiently large so that the boundary layer is away from the sensor.

C. With Shield

The purpose of the shield is to replace a sufficient part of the highly variable natural radiation environment with an environment whose influence on sensor temperature is both small enough and well enough known to enable precise correction of the sensor data.

A shield with a downward view half angle of $\theta = 45^\circ$ is used, and the inside of the shield is painted black. The black painted wall will minimize the effect of radiation arising from reflections within the shield, which would cause large uncertainties. The emissivity of the black paint is assumed to be 1, and the geometric factors are computed by the method discussed in Appendix D. The temperature of the shield is assumed to be $300 \pm 2^\circ\text{K}$. Input values are listed in Tables 6 and 7.

TABLE 6

Nominal Values and Uncertainties of the Geometric Factor (with Shield)

	Sphere	Cylinder	Plate
Sun	0	0	0
Albedo	.147 $\pm$ 15%	.16 $\pm$ 15%	.09 $\pm$ 15%
Earth	.147 $\pm$ 5%	.16 $\pm$ 10%	.09 $\pm$ 10%
Sonde	.853 $\pm$ 3%	.84 $\pm$ 3%	.91 $\pm$ 3%

TABLE 7

Nominal Values and Uncertainties of Radiant Emittance (with Shield)

Sun	0	
Albedo	$460.7 \text{ watt/m}^2 \pm 36\%$	
Earth	$233.8 \text{ watt/m}^2 \pm 20\%$	
Sonde	$458.0 \text{ watt/m}^2 \pm 2\%$	$T_{\text{shield}} = 300^\circ\text{K} \pm 2^\circ\text{K}$
	$221.0 \text{ watt/m}^2 \pm 4\%$	$T_{\text{shield}} = 250^\circ\text{K} \pm 2^\circ\text{K}$

Figure 10 shows the distinct improvement of performance at high altitude. Comparison of Tables 5 and 8 shows the increase of the effect of uncertainty in h . Increased heating due to the shield increases the sensitivity to h and to other parameters such as x . This suggests that a colder shield would significantly improve the sensor.

D. Cold Shield

The benefit of a cool shield is investigated by letting the shield temperature be 250°K . The corresponding value of I_4 is included in Table 7.

The results are especially significant at higher altitude as shown in Fig. 11. Notice in Table 8 that uncertainties due to ℓ , h are much smaller for the cold shield case.

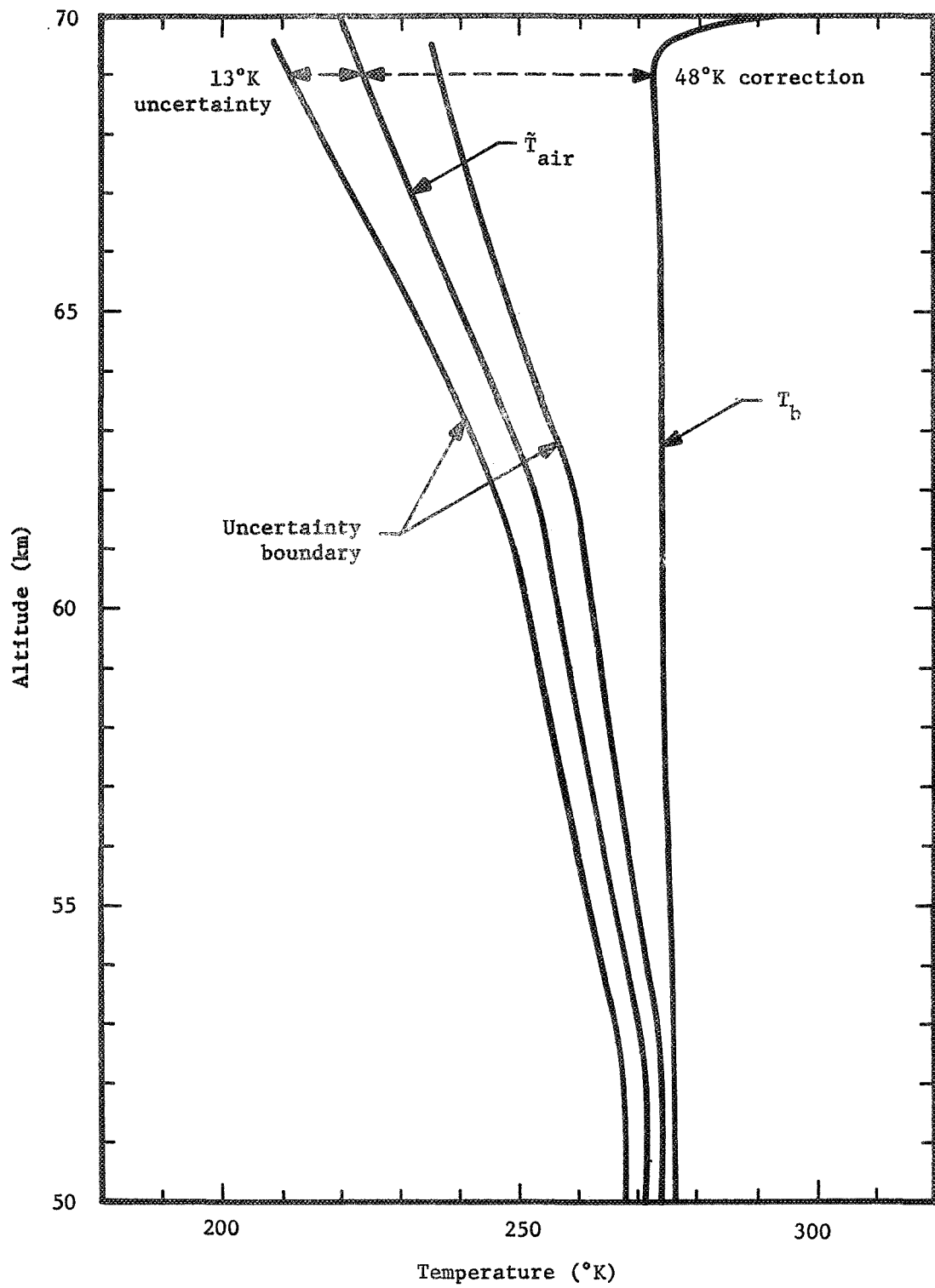


Fig. 10. Uncertainty boundary with a shield.

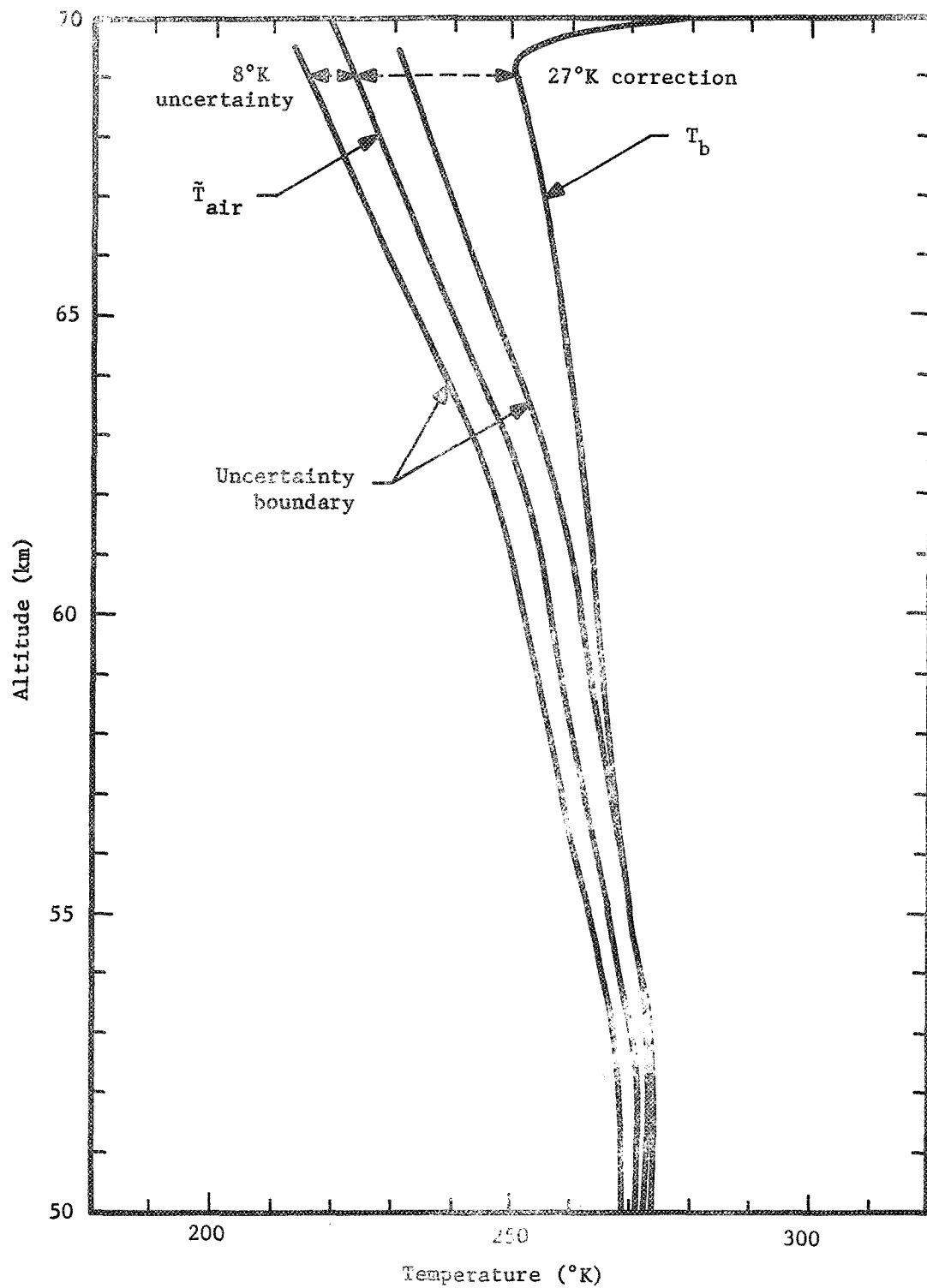


Fig. 11. Uncertainty boundary with cold shield.

TABLE 8
Uncertainty Components (with Shield)

Altitude (km)	70	65	60	55	50
h_b	.55	.31	.20	.12	.72-1
r_b	.32	.33	.19	.11	.57-1
h_w	.38	.19	.14	.91-1	.60-1
r_w	.38	.32	.16	.76-1	.48-1
h_f	.67	.35	.17	.84-1	.40-1
r_f	.74-1	.49-1	.16-1	.52-2	.16-2
V	1.38	1.16	.56	.27	.13
T_b	3.48	2.34	1.71	1.37	1.19
$\dot{T}_b$	.96-1	.94-3	.32-3	.18-3	.26-2
$f_{1,1}$	.0	.0	.0	.0	.0
$f_{1,2}$	.53-1	.30-1	.18-1	.11-1	.68-2
$f_{1,3}$	.62-2	.35-2	.21-2	.12-2	.79-3
$f_{2,1}$	.0	.0	.0	.0	.0
$f_{2,2}$	.68-1	.34-1	.21-1	.12-1	.69-2
$f_{2,3}$	.12-1	.67-2	.38-2	.21-2	.12-2
$f_{3,1}$	.0	.0	.0	.0	.0
$f_{3,2}$	.73-1	.40-1	.22-1	.11-1	.57-2
$f_{3,3}$	.73-1	.40-1	.22-1	.11-1	.57-2
w_b	.40	.23	.13	.80-1	.51-1
α_b	.60-1	.35-1	.23-1	.15-1	.10-1
α_w	.56-1	.37-1	.25-1	.16-1	.98-2

TABLE 8

(continued)

Uncertainty Components (with Shield)

Altitude (km)	70	65	60	55	50
α_f	3.47	2.13	1.05	.49	.23
D_b	.41	.17	.15	.10	.59-1
$(\rho c)_b$	.25-1	.51-3	.16-3	.12-3	.14-2
D_w	.62	.33	.28	.20	.12
k_w	.22	.12	.10	.73-1	.47-1
l	3.56	1.97	1.70	1.21	.79
$(\rho c)_f$	.13	.11-2	.17-3	.53-3	.17-3
D_f	.88-1	.76-3	.12-3	.37-3	.12-3
$f_{1,4}$	.23-1	.15-1	.10-1	.71-2	.49-2
$f_{2,4}$	.21-1	.13-1	.86-2	.55-2	.34-2
$f_{3,4}$	1.0	.57	.32	.17	.82-1
α_{sb}	.79-1	.51-1	.35-1	.24-1	.06-1
α_{sw}	.10	.64-1	.42-1	.27-1	.17-1
α_{sf}	.56	.32	.18	.93-1	.46-1
I_1	.0	.0	.0	.0	.0
I_2	.56	.33	.20	.11	.63-1
I_3	.36	.21	.12	.36-1	.33-1
I_4	.69	.40	.22	.12	.60-1
resultant	13.12	8.16	5.54	3.92	2.94

TABLE 9

Uncertainty Components (Cold Shield)

Altitude (km)	70	65	60	55	50
h_b	.12	.43-1	.26-1	.14-1	.42-2
r_b	.32	.33	.19	.11	.57-1
h_w	.17-1	.11	.80-1	.37-1	.38-1
r_w	.38	.32	.16	.76-1	.48-1
h_f	.22	.12-1	.32-1	.34-1	.23-1
r_f	.77-1	.51-1	.17-1	.53-2	.16-2
v	1.38	1.16	.56	.27	.13
T_b	3.47	2.33	1.71	1.37	1.19
$\dot{T}_b$	.29	.85-2	.19-2	.35-3	.12-2
$f_{1,1}$	.0	.0	.0	.0	.0
$f_{1,2}$	.53-1	.30-1	.18-1	.11-1	.70-2
$f_{1,3}$	.62-2	.35-2	.21-2	.12-2	.79-3
$f_{2,1}$	.0	.0	.0	.0	.0
$f_{2,2}$	.68-1	.38-1	.21-1	.12-1	.69-2
$f_{2,3}$	.12-1	.68-2	.38-2	.21-2	.12-2
$f_{3,1}$	.0	.0	.0	.0	.0
$f_{3,2}$	.76-1	.42-1	.23-1	.12-1	.58-2
$f_{3,3}$	.76-1	.42-1	.23-1	.12-1	.58-2
w_b	.40	.23	.13	.80-1	.51-1
α_b	.75-2	.95-2	.12-1	.11-1	.94-2
α_w	.34-1	.27-1	.20-1	.14-1	.90-2

TABLE 9

(continued)

Uncertainty Components (Cold Shield)

Altitude (km)	70	65	60	55	50
α_f	.81	.23	.42	.34	.21
D_b	.37-1	.24	.17	.12	.66-1
$(\rho c)_b$	.47-1	.19-2	.51-3	.36-3	.61-3
D_w	.21	.34	.26	.18	.11
k_w	.73-1	.12	.88-1	.63-1	.40-1
ℓ	1.14	1.75	1.33	.98	.64
$(\rho c)_f$	.40	.48-2	.14-2	.12-2	.21-5
D_f	.28	.34-2	.99-3	.83-3	.15-5
$f_{1,4}$	.11-1	.69-2	.49-2	.34-2	.23-2
$f_{2,4}$	.93-2	.60-2	.40-2	.26-2	.16-2
$f_{3,4}$	.52	.30	.17	.87-1	.42-1
α_{sb}	.73-1	.48-1	.34-1	.23-1	.16-1
α_{sw}	.94-1	.60-1	.41-1	.26-1	.16-1
α_{sf}	.61	.35	.19	.10	.49-1
I_1	.0	.0	.0	.0	.0
I_2	.59	.35	.21	.12	.65-1
I_3	.39	.23	.13	.68-1	.35-1
I_4	.72	.42	.23	.12	.62-1
resultant	8.60	6.64	4.72	3.56	2.78

TABLE 10

Comparison of Uncertainty Boundary

Altitude(km)	Without Shield($^{\circ}$ K)	With Hot Shield($^{\circ}$ K)	With Cold Shield($^{\circ}$ K)
70	23.98	13.12	8.60
65	14.06	8.16	6.64
60	8.36	5.54	4.72
55	4.98	3.92	3.56
50	3.20	2.94	2.78

V. CONCLUSIONS

The automatic computation of the running error of a measurement system, according to time-varying estimates of error in assumed parameter values, provides a useful quantitative basis for system evaluation and improvement. A plot of the uncertainty boundary about the nominal output value from a simulated system provides a clear graphical model of operational capability. The uncertainty envelope computed along with the reduction of real data provides the user with a convenient indicator of the quality of measurement results.

Results obtained for the current ARCASONDE 1A sensor indicate that the system uncertainty exceeds the magnitude of the correction for altitudes below about 53 km, so corrections tend to be meaningless for this sensor in the stratosphere and below. The uncertainty appears to remain greater than half the correction throughout the mesosphere (50-80 km). The chief contributing factors to the overall uncertainty are uncertainties in film absorptivity, emissivity, and aspect with respect to the sun. The contribution of an assumed 1°K error in acquiring the sensor temperature is the major contributor at low altitudes.

The hypothetical addition of a simple cylindrical shield to the ARCASONDE 1A sensor, while increasing its error, considerably improves its performance in terms of greater accuracy after corrections. Corrections may be needed to lower altitude, however, when a shield is used. The increased radiant heat input from the relatively hot

shield at 300°K limits the benefit of a shield. Reducing the shield temperature to 250°K decreases error flux into the sensor sufficient to considerably decrease sensitivity to h_1 and other parameters, as well as to direct solar parameters.

APPENDIX A

APPROXIMATION OF THE RESULTANT STANDARD DEVIATION

The following is a brief examination of the assumptions underlying the relation (Eq. 13) used in this dissertation to compute the overall uncertainty in the corrected temperature.

Expansion of a function of n variables in Taylor's series

$$f(\underline{p}) = \sum_{k=0}^{\infty} \frac{1}{k!} \left[\sum_{j=1}^n \Delta p_j \frac{\partial}{\partial p_j} \right]^k f(\underline{p}) \quad (\text{A.1})$$

Neglecting third-order terms and higher,

$$\begin{aligned} &= f_0 + \sum_{j=1}^n \left. \frac{\partial f}{\partial p_j} \right|_0 \Delta p_j + \frac{1}{2} \sum_{j=1}^n \left. \frac{\partial^2 f}{\partial p_j^2} \right|_0 \Delta p_j \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n \left. \frac{\partial^2 f}{\partial p_i \partial p_j} \right|_0 \Delta p_i \Delta p_j \quad (i \neq j) \end{aligned} \quad (\text{A.2})$$

where Δp_j is the variation of the j th variable from its expected value.

$$\Delta p_j = p_j - \eta_j, \quad \eta_j = p_j|_0 = E[p_j]$$

The coefficients

$$f_0 = a, \left. \frac{\partial f}{\partial p_j} \right|_0 = b_j, \left. \frac{\partial^2 f}{\partial p_j^2} \right|_0 = c_j, \left. \frac{\partial^2 f}{\partial p_i \partial p_j} \right|_0 = d_{ij}$$

are independent of the variation. The expected value of f is then

$$E[f] = a + \frac{1}{2} \sum_{j=1}^n c_j \sigma_j^2 + \sum_{i=1}^n \sum_{j=1}^n d_{ij} \text{cov}(p_i, p_j) \quad (\text{A.3})$$

The terms of the variance, $\sigma_f^2 = E[f^2] - E^2[f]$ of $f(p)$ in the two-variable case are found as follows:

$$f = a + b_1 \Delta p_1 + b_2 \Delta p_2 + \frac{1}{2} c_1 \Delta p_1^2 + \frac{1}{2} c_2 \Delta p_2^2 + d_{12} \Delta p_1 \Delta p_2$$

$$E[f] = a + \frac{1}{2} c_1 \sigma_1^2 + \frac{1}{2} c_2 \sigma_2^2 + d_{12} \text{cov}(p_1, p_2)$$

$E^2(f)$ contains 10 terms and $E(f^2)$ has 19 terms. Their difference after cancellation contains 21 terms.

$$\begin{aligned}
\sigma^2 &= E(f^2) - E^2(f) = b_1^2 \sigma_1^2 + b_2^2 \sigma_2^2 + 2b_1 b_2 \operatorname{cov}(p_1, p_2) \\
&+ b_1 c_1 E(\Delta p_1^3) + b_2 c_2 E(\Delta p_2^3) \\
&+ (2b_1 d_{12} + b_2 c_1) E(\Delta p_1^2 \Delta p_2) \\
&+ (2b_2 d_{12} + b_1 c_2) E(\Delta p_1 \Delta p_2^2) \\
&+ \frac{1}{2} c_1^2 \left[E(\Delta p_1^4) - \sigma_1^4 \right] + \frac{1}{2} c_2^2 \left[E(\Delta p_2^4) - \sigma_2^4 \right] \\
&+ 2d_{12}^2 \left[E(\Delta p_1 \Delta p_2^2) - \operatorname{cov}^2(p_1, p_2) \right] \\
&+ \frac{1}{2} c_1 c_2 \left[E(\Delta p_1^2 \Delta p_2^2) - \sigma_1^2 \sigma_2^2 \right] \\
&+ c_1 d_{12} \left[E(\Delta p_1^3 \Delta p_2) - \sigma_1^2 \operatorname{cov}(p_1, p_2) \right] \\
&+ c_2 d_{12} \left[E(\Delta p_1 \Delta p_2^3) - \sigma_2^2 \operatorname{cov}(p_1, p_2) \right]
\end{aligned}$$

If, in addition to the implied initial assumptions that higher order terms in the Taylor's expansion are negligible, it is assumed that the distribution of the p_j 's are symmetrical so that $E(\Delta p_1^3)$ and $E(\Delta p_2^3)$ are small, that $c, d \ll b$, and that third and fourth-order moments are small, then the above rather lengthy expression reduces to

$$\sigma^2 = b_1^2 \sigma_1^2 + b_2^2 \sigma_2^2 + 2b_1 b_2 \text{ cov}(p_1, p_2)$$

The corresponding relation for the n-variable case is

$$\sigma_f^2 = \sum_{j=1}^n b_j^2 \sigma_j^2 + 2 \sum_{i=1}^n \sum_{j=1}^n b_i b_j \text{ cov}(p_i, p_j) \quad (i \neq j)$$

as can be seen by introducing additional variables from the beginning and dropping small terms. An indication of the validity of Eq. 13 was found by a Monte Carlo computation in which the $\tilde{p}$'s were generated as random variables. The resulting standard deviation of T_{air} , using normal and uniform distributions for the parameters, with $\sigma_j = \Delta p_j/2$ and $p_j - \Delta p_j < p_j < p_j + \Delta p_j$, respectively, is compared with $\sigma_{T_{\text{air}}}$ from Eq. 13 in Table A.1. Recall that uncertainty in T_{air} is comparable to twice the standard deviation. The tabulated results are those of the unshielded ARCASONDE 1A, discussed in Section 4.B. The data

corrector routine was repeated 300 times, using subroutine (RANDN) from the UNIVAC 1108 Math Pack as a source of random numbers. Correlation between parameters was small. The comparison is considered quite good indicating that the system equations accommodate the magnitude of the given variation is such that the magnitude of the given variation in parameters does not exceed limits of validity.

TABLE A.1

Comparison of Approximated and Simulated $\sigma_{T_{air}}$

Altitude	Approximation (°K)	Simulation (normal distribution) (°K)	Simulation (uniform distribution) (°K)
70	11.99	11.71	11.65
65	7.03	7.00	6.98
60	4.18	4.16	4.14
55	2.49	2.48	2.47
50	1.60	1.60	1.59

APPENDIX B

EXPRESSIONS FOR SENSITIVITY COEFFICIENTS

A. The $\frac{\partial f^i}{\partial p^i}$ are computed as follows. Superscript i is omitted in the right-hand members of the following statements since it is obvious.

$$\frac{\partial f^i}{\partial h_b^i} = \frac{1}{(h_b + H_K K_1)^2} \left[(h_b + H_K K_1) \left(T_b - r_b \frac{v^2}{2c_p} \right) - E1 \right]$$

where

$$E1 = \frac{(\rho c D)_b}{6} \dot{T}_b + \left(h_b + 4\sigma \epsilon_b T_b^3 + H_K \right) T_b$$

$$- \left(h_b r_b \frac{v^2}{2c_p} + 3\sigma \epsilon_b T_b^4 + q_b + H_K P \right) - H_K Q T_f$$

$$\frac{\partial f^i}{\partial r_b^i} = - \frac{h_b \frac{v^2}{2c_p}}{h_b + H_K K_1}$$

$$\begin{aligned}
\frac{\partial f^i}{\partial h_w^i} = & -E2 \frac{H_K \frac{\partial K_1}{\partial h_w} + K_1 \frac{\partial H_K}{\partial h_w}}{(h_b + H_K K_1)^2} + \frac{\frac{\partial H_K}{\partial h_w} (h_b + H_K K_1) - H_K \left(H_K \frac{\partial K_1}{\partial h_w} + K_1 \frac{\partial H_K}{\partial h_w} \right)}{(h_b + H_K K_1)^2} T_b \\
& - \frac{\left(\frac{\partial H_K}{\partial h_w} P + \frac{\partial P}{\partial h_w} H_K \right) (h_b + H_K K_1) - H_K P \left(H_K \frac{\partial K_1}{\partial h_w} + K_1 \frac{\partial H_K}{\partial h_w} \right)}{(h_b + H_K K_1)^2} \\
& - \frac{\left(\frac{\partial H_K}{\partial h_w} Q + \frac{\partial Q}{\partial h_w} H_K \right) (h_b + H_K K_1) - H_K Q \left(H_K \frac{\partial K_1}{\partial h_w} + K_1 \frac{\partial H_K}{\partial h_w} \right)}{(h_b + H_K K_1)^2} T_f
\end{aligned}$$

where

$$\frac{\partial \lambda_w}{\partial h_w} = \frac{2}{(kd)_w \lambda_w}$$

$$\frac{\partial H_K}{\partial h_w} = \left(\frac{\partial \lambda_w}{\partial h_w} \right) \left(-C_2 \lambda_w \ell \operatorname{cosech}^2 \lambda_w \ell + C_2 \coth \lambda_w \ell \right)$$

$$\begin{aligned}
\frac{\partial K_1}{\partial h_w} = & \frac{\left(h_w + 4\sigma \epsilon_w T_{aw}^3 \right) h_w \ell_w \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \left(\frac{\partial \lambda_w}{\partial h_w} \right)}{\left(h_w + 4\sigma \epsilon_w T_{aw}^3 \right)^2} \\
& + \frac{4\sigma \epsilon_w T_{aw}^3 \left(1 - \operatorname{sech} \lambda_w \ell \right)}{\left(h_w + 4\sigma \epsilon_w T_{aw}^3 \right)^2}
\end{aligned}$$

$$A(h_w) = 1 - \operatorname{sech} \lambda_w \ell$$

$$B(h_w) = h_w r_w \frac{V^2}{2c_p} + \frac{q_w}{A_w} + 3\sigma \epsilon_w T_{aw}^4$$

$$\frac{\partial P}{\partial h_w} = \frac{1}{\left(h_w + 4\sigma \epsilon_w T_{aw}^2\right)^2} \left[\left\{ A(h_w) r_w \frac{V^2}{2c_p} + B(h_w) \ell \operatorname{sech} \lambda_w \ell \right. \right. \\ \left. \left. \tanh \lambda_w \ell \left(\frac{\partial \lambda_w}{\partial h_w} \right) \right\} \left(h_w + 4\sigma \epsilon_w T_{aw}^3 \right) - A(h_w) B(h_w) \right]$$

$$\frac{\partial Q}{\partial h_w} = - \ell \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \left(\frac{\partial \lambda_w}{\partial h_w} \right)$$

$$E_2 = \frac{(\rho c D)_b}{6} \dot{T}_b + \left(h_b + 4\sigma \epsilon_b T_{ab}^3 \right) T_b - \left(h_b r_b \frac{V^2}{2c_p} + 3\sigma \epsilon_b T_{ab}^4 \frac{W}{A_b} + q_b \right)$$

$$\frac{\partial f^i}{\partial r_w^i} = - \frac{H_K}{h_b + H_K l} \frac{h_w \frac{V^2}{2c_p} \left(1 - \operatorname{sech} \lambda_w \ell \right)}{h_w + 4\sigma \epsilon_w T_{aw}^3}$$

$$\frac{\partial f^i}{\partial h_f^i} = 0$$

$$\frac{\partial f^i}{\partial r_f^i} = 0$$

$$\frac{\partial f^i}{\partial V^i} = - \frac{1}{h_b + H_K K_1} \left(h_b r_b \frac{V}{c_p} + H_K \frac{\partial P}{\partial V} \right)$$

where

$$\frac{\partial P}{\partial V} = \frac{h_w r_w V}{c_p} \left(\frac{1 - \operatorname{sech} \lambda_w \ell}{h_w + 4\sigma \epsilon_w T_{aw}^3} \right)$$

$$\frac{\partial f^i}{\partial T_b^i} = \frac{1}{(h_b + H_K K_1)^2} \left\{ \left[h_b + 4\sigma \epsilon_b T_{ab}^3 + \frac{\partial H_K}{\partial T_b} T_b + H_K - \frac{\partial H_K}{\partial T_b} P \right. \right. \\ \left. \left. - \frac{\partial P}{\partial T_b} H_K - T_f \left(H_K \frac{\partial Q}{\partial T_b} + Q \frac{\partial H_K}{\partial T_b} \right) \right] (h_b + H_K K_1) - E1 \left(\frac{\partial H_K}{\partial T_b} K_1 + \frac{\partial K_1}{\partial T_b} H_K \right) \right\}$$

where

$$\frac{\partial T_{aw}}{\partial T_b} = \frac{1}{2}$$

$$\frac{\partial \lambda_w}{\partial T_b} = \frac{12\sigma\epsilon_w T_{aw}^2}{\lambda_w (kd)_w}$$

$$\frac{\partial H_K}{\partial T_b} = C_2 \left\{ \frac{\partial \lambda_w}{\partial T_b} \coth \lambda_w \ell - \lambda_w \left(\operatorname{cosech}^2 \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial T_b} \right\}$$

$$\frac{\partial T_{aw}^3}{\partial T_b} = \frac{3}{2} T_{aw}^2$$

$$\frac{\partial K_1}{\partial T_b} = \frac{1}{\left(h_w + 4\sigma\epsilon_w T_{aw}^3 \right)^2} \left[\left(h_w + 4\sigma\epsilon_w T_{aw}^3 \right) \left(h_w \operatorname{sech} \lambda_w \ell \right. \right. \\ \left. \left. \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial T_b} - h_w \left(1 - \operatorname{sech} \lambda_w \ell \right) 4\sigma\epsilon_w \frac{\partial T_{aw}^3}{\partial T_b} \right]$$

$$\frac{\partial T_{aw}^4}{\partial T_b} = 2 T_{aw}^3$$

$$\begin{aligned}
\frac{\partial P}{\partial T_b} &= \frac{1}{\left(h_w + 4\sigma \varepsilon_w T_{aw}^3\right)^2} \left\{ \left[\left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial T_b} \right. \right. \\
&\quad \left. \left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma \varepsilon_w T_{aw}^4 \right) + 6 \left(1 - \operatorname{sech} \lambda_w \ell \right) \sigma \varepsilon_w T_{aw}^3 \right] \left(h_w + 4\sigma \varepsilon_w T_{aw}^3 \right) \\
&\quad \left. - \left(1 - \operatorname{sech} \lambda_w \ell \right) \left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma \varepsilon_w T_{aw}^4 \right) 6\sigma \varepsilon_w T_{aw}^2 \right\} \\
\frac{\partial Q}{\partial T_b} &= \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial T_b}
\end{aligned}$$

$$\frac{\partial f^i}{\partial T_b} = \frac{\rho c D_b}{6 \left(h_b + H_K K_1 \right)}$$

$$\frac{\partial f^i}{\partial f_{11}^i} = \frac{\partial f^i}{\partial q_b} I_1 \alpha_{11}$$

where

$$\frac{\partial f^i}{\partial q_b} = - \frac{1}{h_b + H_K K_1}$$

$$\frac{\partial f^i}{\partial f_{12}^i} = \frac{\partial f^i}{\partial q_b} I_2 \alpha_{12}$$

$$\frac{\partial f^i}{\partial f_{13}^i} = \frac{\partial f^i}{\partial q_b} I_3 \alpha_{13}$$

$$\frac{\partial f^i}{\partial f_{21}^i} = \frac{\partial f^i}{\partial q_w} I_1 \alpha_{21}$$

where

$$\frac{\partial f^i}{\partial q_w} = - \frac{H_K}{h_b + H_{K1}} \frac{1 - \operatorname{sech} \lambda_w \ell}{h_w + 4\sigma_{\epsilon_w} T_{aw}^3}$$

$$\frac{\partial f^i}{\partial f_{22}^i} = \frac{\partial f^i}{\partial q_w} I_2 \alpha_{22}$$

$$\frac{\partial f^i}{\partial f_{23}^i} = \frac{\partial f^i}{\partial q_w} I_3 \alpha_{23}$$

$$\frac{\partial f^i}{\partial f_{31}^i} = 0$$

$$\frac{\partial f^i}{\partial f_{32}^i} = 0$$

$$\frac{\partial f^i}{\partial f_{33}^i} = 0$$

$$\frac{\partial f^i}{\partial w_b^i} = -\frac{1}{h_b + H_K K_1} + \left(\frac{\partial f^i}{\partial T_b} \right) \left(\frac{\partial T_b}{\partial w_b^i} \right)$$

$$\frac{\partial f^i}{\partial \epsilon_b} = \frac{\partial f^i}{\partial q_b} \left(f_{13} I_3 + f_{14} I_4 \right) + \frac{\sigma T_b^4}{h_b + H_K K_1}$$

$$\frac{\partial f^i}{\partial \epsilon_w} = \frac{\partial f^i}{\partial H_K} \frac{\partial H_K}{\partial \lambda_w} \frac{\partial \lambda_w}{\partial \epsilon_w}$$

$$+ \frac{\partial f^i}{\partial K_1} \left(\frac{\partial K_1}{\partial \epsilon_w} + \frac{\partial K_1}{\partial \lambda_w} \frac{\partial \lambda_w}{\partial \epsilon_w} \right)$$

$$+ \frac{\partial f^i}{\partial P} \left(\frac{\partial P}{\partial \epsilon_w} + \frac{\partial P}{\partial \lambda_w} \frac{\partial \lambda_w}{\partial \epsilon_w} \right) + \frac{\partial f^i}{\partial Q} \frac{\partial Q}{\partial \lambda_w} \frac{\partial \lambda_w}{\partial \epsilon_w}$$

where

$$\frac{\partial f^i}{\partial H_K} = \frac{1}{\left(h_b + H_K K_1\right)^2} \left[\left(T_b - P - Q T_f\right) \left(h_b + H_K K_1\right) - E I K_1 \right]$$

$$\frac{\partial f^i}{\partial K_1} = \frac{1}{\left(h_b + H_K K_1\right)^2} - \left[E I H_K \right]$$

$$\frac{\partial f^i}{\partial P} = \frac{-H_K}{h_b + H_K K_1}$$

$$\frac{\partial f^i}{\partial Q} = \frac{-H_K T_f}{h_b + H_K K_1}$$

$$\frac{\partial H_K}{\partial \lambda_w} = C_2 \left(\coth \lambda_w \ell - \lambda_w \ell \operatorname{cosech}^2 \lambda_w \ell \right)$$

$$\frac{\partial K_1}{\partial \lambda_w} = \frac{h_w \ell \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell}{h_w + 4 \sigma \varepsilon_w T_{aw}^3}$$

$$\frac{\partial Q}{\partial \lambda_w} = - \ell \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell$$

$$\frac{\partial \lambda_w}{\partial \varepsilon_w} = \frac{8\sigma T_{aw}^3}{\lambda_w(kd)_w}$$

$$\frac{\partial K_1}{\partial \varepsilon_w} = \frac{-h_w (1 - \operatorname{sech} \lambda_w \ell) 4\sigma T_{aw}^3}{\left(h_w + 4\sigma \varepsilon_w T_{aw}^3\right)^2}$$

$$\begin{aligned} \frac{\partial P}{\partial \varepsilon_w} = & \frac{1 - \operatorname{sech} \lambda_w \ell}{\left(h_w + 4\sigma \varepsilon_w T_{aw}^3\right)^2} \left[3\sigma T_{aw}^4 \left(h_w + 4\sigma \varepsilon_w T_{aw}^3\right) \right. \\ & \left. - \left(h_w r_w \frac{v^2}{2c_p} + q_w + 3\sigma \varepsilon_w T_{aw}^4\right) 4\sigma T_{aw}^3 \right] \end{aligned}$$

$$\frac{\partial f^i}{\partial \varepsilon_f} = 0$$

$$\begin{aligned} \frac{\partial f^i}{\partial D_b} = & \frac{1}{\left(h_b + H_K K_1\right)^2} \left[\left(\frac{\rho c}{6} \dot{T}_b + T_b \frac{\partial H_K}{\partial D_b} - P \frac{\partial H_K}{\partial D_b} - Q T_f \frac{\partial H_K}{\partial D_b} \right) \right. \\ & \left. \left(h_b + H_K K_1 \right) - K_1 \frac{\partial H_K}{\partial D_b} E l \right] \end{aligned}$$

where

$$\frac{\partial C_2}{\partial D_b} = \frac{-(kD)_w D_w}{D_b^3}$$

$$\frac{\partial H_K}{\partial D_b} = \lambda_w \coth \lambda_w \ell \left(\frac{\partial C_2}{\partial D_b} \right)$$

$$\frac{\partial f^i}{(\rho c)_b} = \frac{D_b \dot{T}_b}{6 (h_b + H_K K_1)}$$

$$\frac{\partial f^i}{\partial D_w} = \frac{1}{(h_b + H_K K_1)^2} \left[\left\{ T_b \frac{\partial H_K}{\partial D_w} - H_K \frac{\partial P}{\partial D_w} - P \frac{\partial H_K}{\partial E_w} - T_f \left(\frac{\partial H_K}{\partial D_w} Q + \frac{\partial Q}{\partial h_w} H_K \right) \right\} \right. \\ \left. (h_b + H_K K_1) - E_l \left(\frac{\partial H_K}{\partial D_w} K_1 + \frac{\partial K_1}{\partial D_w} H_K \right) \right]$$

where

$$\frac{\partial C_2}{\partial D_w} = \frac{k_w D_w}{D_b^2}$$

$$\frac{\partial \lambda_w}{\partial D_w} = - \frac{\lambda_w}{2D_w}$$

$$\frac{\partial H_K}{\partial D_w} = \left(\frac{\partial C_2}{\partial D_w} \lambda_w + C_2 \frac{\partial \lambda_w}{\partial D_w} \right) \coth \lambda_w \ell - C_2 \lambda_w \ell \frac{\partial \lambda_w}{\partial D_w} \operatorname{cosech}^2 \lambda_w \ell$$

$$\frac{\partial K_1}{\partial D_w} = \frac{h_w}{h_w + 4\sigma \varepsilon_w T_{aw}^3} \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial D_w}$$

$$\frac{\partial P}{\partial D_w} = \frac{\left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma \varepsilon_w T_{aw}^4 \right)}{h_w + 4\sigma \varepsilon_w T_{aw}^3} \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial D_w}$$

$$\frac{\partial Q}{\partial D_w} = - \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial D_w}$$

$$\frac{\partial f^i}{\partial k_w} = \frac{1}{(h_b + H_K K_1)^2} \left[\left\{ T_b \frac{\partial H_K}{\partial k_w} - H_K \frac{\partial P}{\partial k_w} - P \frac{\partial H_K}{\partial k_w} - T_f \left(\frac{\partial H_K}{\partial k_w} Q + \frac{\partial Q}{\partial k_w} H_K \right) \right\} \right. \\ \left. \left(h_b + H_K K_1 \right) - E1 \left(\frac{\partial H_K}{\partial k_w} K_1 + \frac{\partial K_1}{\partial k_w} H_K \right) \right]$$

where

$$\frac{\partial C_2}{\partial k_w} = \frac{d_w^2}{2d_b^2}$$

$$\frac{\partial \lambda_w}{\partial k_w} = -\frac{\lambda_w}{2k_w}$$

$$\frac{\partial H_K}{\partial k_w} = \left(\frac{\partial C_2}{\partial k_w} \lambda_w + C_2 \frac{\partial \lambda_w}{\partial k_w} \right) \coth \lambda_w \ell - C_2 \lambda_w \ell \frac{\partial \lambda_w}{\partial k_w} \operatorname{cosech}^2 \lambda_w \ell$$

$$\frac{\partial K_1}{\partial k_w} = \frac{h_w}{h_w + 4\sigma_{\varepsilon_w} T_{aw}^3} \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial k_w}$$

$$\frac{\partial P}{\partial k_w} = \frac{\left(h_w r_w \frac{v^2}{2c_p} + q_w + 3\sigma_{\varepsilon_w} T_{aw}^4 \right)}{h_w + 4\sigma_{\varepsilon_w} T_{aw}^3} \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial k_w}$$

$$\frac{\partial Q}{\partial k_w} = - \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial k_w}$$

$$\frac{\partial f^i}{\partial \ell} = \frac{1}{(h_b + H_K K_1)^2} \left[\left\{ (T_b - P) \frac{\partial H_K}{\partial \ell} - H_K \frac{\partial P}{\partial \ell} - T_f \left(\frac{\partial H_K}{\partial \ell} Q + \frac{\partial Q}{\partial \ell} H_K \right) \right\} (h_b + H_K K_1) \right. \\ \left. - E_1 \left(\frac{\partial H_K}{\partial \ell} K_1 + H_K \frac{\partial K_1}{\partial \ell} \right) \right]$$

where

$$\frac{\partial H_K}{\partial \ell} = - C_2 \lambda_w^2 \operatorname{cosech}^2 \lambda_w \ell$$

$$\frac{\partial K_1}{\partial \ell} = \frac{h_w r_w}{h_w + 4\sigma \epsilon_w T_{aw}^3} \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell$$

$$\frac{\partial P}{\partial \ell} = \frac{\left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma \epsilon_w T_{aw}^4 \right) \lambda_w \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell}{h_w + 4\sigma \epsilon_w T_{aw}^3}$$

$$\frac{\partial Q}{\partial \ell} = - \lambda_w \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell$$

$$\frac{\partial f^i}{\partial (\rho c)_f} = 0$$

$$\frac{\partial f^i}{\partial D_f} = 0$$

$$\frac{\partial f^i}{\partial f_{14}} = \frac{\partial f^i}{\partial q_b} \alpha_{14} \text{ } I_4$$

$$\frac{\partial f^i}{\partial f_{24}} = \frac{\partial f^i}{\partial q_w} \alpha_{24} \text{ } I_4 = \left(\frac{H_K}{h_b + H_K K_1} \right) \left(\frac{1 - \text{sech } \lambda_w \ell}{h_w + 4 \sigma \epsilon_w T_{aw}^3} \right) \alpha_{24} \text{ } I_4$$

$$\frac{\partial f^i}{\partial f_{34}} = 0$$

$$\frac{\partial f^i}{\partial \alpha_{11}} = \frac{\partial f^i}{\partial q_b} \left(f_{11} \text{ } I_1 + f_{12} \text{ } I_2 \right)$$

$$\frac{\partial f^i}{\partial \alpha_{21}} = \frac{\partial f^i}{\partial q_w} \left(f_{21} \text{ } I_1 + f_{22} \text{ } I_2 \right)$$

$$\frac{\partial f^i}{\partial \alpha_{31}} = 0$$

$$\frac{\partial f^i}{\partial I_1} = \frac{\partial f^i}{\partial q_b} f_{11} \alpha_{11} + \frac{\partial f^i}{\partial q_w} f_{21} \alpha_{21}$$

$$\frac{\partial f^i}{\partial I_2} = \frac{\partial f^i}{\partial q_b} f_{12} \alpha_{12} + \frac{\partial f^i}{\partial q_w} f_{22} \alpha_{22}$$

$$\frac{\partial f^i}{\partial I_3} = \frac{\partial f^i}{\partial q_b} f_{13} \alpha_{13} + \frac{\partial f^i}{\partial q_w} f_{23} \alpha_{23}$$

$$\frac{\partial f^i}{\partial I_4} = \frac{\partial f^i}{\partial q_b} f_{14} \alpha_{14} + \frac{\partial f^i}{\partial q_w} f_{24} \alpha_{24}$$

B. Computation of $\frac{\partial f^i}{\partial T_f^i}$. Grouping the set of quantities H_K^i, K_1^i, P^i, Q^i as elements of a vector G

$$G^i(T_f, p) = \begin{bmatrix} H_K^i(T_f^i, p^i) \\ K_1^i(T_f^i, p^i) \\ P^i(T_f^i, p^i) \\ Q^i(T_f^i, p^i) \end{bmatrix}$$

$$\frac{\partial f^i}{\partial T_f^i} = \frac{\partial f^i}{\partial G^i} \frac{\partial G^i}{\partial T_f^i}$$

where $\frac{\partial f^i}{\partial G^i}$ and $\frac{\partial G^i}{\partial T^i}$ are given as follows:

$$\frac{\partial f^i}{\partial G_{\sim}^i} = \left(\frac{\partial f}{\partial H_K}, \frac{\partial f}{\partial K_1}, \frac{\partial f}{\partial Q}, \frac{\partial f}{\partial P} \right)$$

$$\frac{\partial f}{\partial H_K} = \frac{1}{(h_b + H_K K_1)} \left[(T_b - P - Q T_f)(h_b + H_K K_1) - E I K_1 \right]$$

$$\frac{\partial f}{\partial K_1} = - \frac{1}{(h_b + H_K K_1)^2} \left[E I H_K \right]$$

$$\frac{\partial f}{\partial P} = - \frac{H_K}{h_b + H_K K_1}$$

$$\frac{\partial f}{\partial Q} = - \frac{H_K T_f}{h_b + H_K K_1}$$

$$\frac{\partial G^i}{\partial T_f^i} = \begin{bmatrix} \frac{\partial H_K}{\partial T_f} \\ \frac{\partial K_1}{\partial T_f} \\ \frac{\partial P}{\partial T_f} \\ \frac{\partial Q}{\partial T_f} \end{bmatrix}$$

$$\frac{\partial H_K}{\partial T_f} = C_2 \left(\coth \lambda_w \ell - \lambda_w \ell \operatorname{cosech}^2 \lambda_w \ell \right) \left(\frac{\partial \lambda_w^i}{\partial T_f^i} \right)$$

$$\frac{\partial K_1}{\partial T_f} = \frac{1}{\left(h_w + 4\sigma \epsilon_w T_{aw}^3 \right)^2} \left[\left(h_w + 4\sigma \epsilon_w T_{aw}^3 \right) \left(h_w \operatorname{sech} \lambda_w \ell \right. \right.$$

$$\left. \left. \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial T_f} - h_w \left(1 - \operatorname{sech} \lambda_w \ell \right) 6\sigma \epsilon_w T_{aw}^2 \right]$$

$$\frac{\partial P}{\partial T_f} = \frac{1}{\left(h_w + 4\sigma\epsilon_w T_{aw}^3\right)^2} \left\{ \left[\left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial T_f} \right. \right. \\ \left. \left. \left(h_w r_w \frac{v^2}{2c_p} + q_w + 3\sigma\epsilon_w T_{aw}^4 \right) + 6 \left(1 - \operatorname{sech} \lambda_w \ell \right) \sigma\epsilon_w T_{aw}^3 \right] \right. \\ \left. \left(h_w + 4\sigma\epsilon_w T_{aw}^3 \right) - \left(1 - \operatorname{sech} \lambda_w \ell \right) \left(h_w r_w \frac{v^2}{2c_p} + q_w \right. \right. \\ \left. \left. + 3\sigma\epsilon_w T_{aw}^4 \right) 6\sigma\epsilon_w T_{aw}^2 \right\}$$

$$\frac{\partial Q}{\partial T_f} = - \ell \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \left(\frac{\partial \lambda_w}{\partial T_f} \right)$$

where

$$\frac{\partial \lambda_w}{\partial T_f} = \frac{12\sigma\epsilon_w T_{aw}^2}{(kd)_w \lambda_w}$$

C. $\frac{\partial g^i}{\partial T_f^{i-1}}$, $\frac{\partial g^i}{\partial T_{air}^{i-1}}$, and $\frac{\partial g^i}{\partial p_\ell^{i-1}}$ are computed as follows, this case omitting superscript $i-1$ in the right-hand members.

$$\frac{\partial g^i}{\partial T_f^i - 1} = 1 - \frac{2\Delta t}{(\rho c D)_f} h_f - \frac{8\Delta t \sigma \epsilon_f}{(\rho c D)_f} T_f^3$$

$$\frac{\partial g^i}{\partial T_{air}^i - 1} = 2h_f \frac{\Delta t}{(\rho c D)_f}$$

$$\frac{\partial g^i}{\partial h_f^i - 1} = \frac{2\Delta t}{(\rho c D)_f} \left(r_f \frac{v^2}{2c_p} - T_f + T_{air} \right)$$

$$\frac{\partial g^i}{\partial r_f^i - 1} = \frac{2\Delta t}{(\rho c D)_f} h \frac{v^2}{2c_p}$$

$$\frac{\partial g^i}{\partial V} = \frac{2\Delta t}{(\rho c D)_f} h_f r_f \frac{v}{c_p}$$

$$\frac{\partial g^i}{\partial (\rho c)_f} = - \frac{E3}{(\rho c)_f^2 D_f}$$

$$\frac{\partial g^i}{\partial D_f} = - \frac{E3}{(\rho c)_f D_f^2}$$

where

$$E_3 = -2\Delta t \left(h_f + \sigma \epsilon_f T_f^3 \right) T_f + 2\Delta t \left(q_f + h_f r_f \frac{v^2}{2c_p} \right) + 2\Delta t h_f T_{air}$$

$$\frac{\partial g^i}{\partial f_{31}} = \left(\frac{\partial g^i}{\partial q_f} \right) \left(\alpha_{31} \ I_1 \right)$$

$$\frac{\partial g^i}{\partial f_{32}} = \left(\frac{\partial g^i}{\partial q_f} \right) \left(\alpha_{32} \ I_2 \right)$$

$$\frac{\partial g^i}{\partial f_{33}} = \left(\frac{\partial g^i}{\partial q_f} \right) \left(\alpha_{33} \ I_3 \right)$$

$$\frac{\partial g^i}{\partial f_{34}} = \left(\frac{\partial g^i}{\partial q_f} \right) \left(\alpha_{34} \ I_f \right)$$

where

$$\frac{\partial g^i}{\partial q_f} = \frac{2\Delta t}{(\rho cD)_f}$$

$$\frac{\partial g^i}{\partial \varepsilon_f} = - \frac{2\Delta t}{\rho c D_f} \sigma T_f^4 + \frac{\partial g^i}{\partial q_f} \left(f_{33} I_3 + f_{34} I_4 \right)$$

$$\frac{\partial g^i}{\partial I_1} = \frac{\partial g^i}{\partial q_f} f_{31} \alpha_{31}$$

$$\frac{\partial g^i}{\partial I_2} = \frac{\partial g^i}{\partial q_f} f_{32} \alpha_{32}$$

$$\frac{\partial g^i}{\partial I_3} = \frac{\partial g^i}{\partial q_f} f_{33} \alpha_{33}$$

$$\frac{\partial g^i}{\partial I_4} = \frac{\partial g^i}{\partial q_f} f_{34} \alpha_{34}$$

$$\frac{\partial g^i}{\partial \alpha_{31}} = \frac{\partial g^i}{\partial q_f} \left(f_{31} I_1 + f_{32} I_2 \right)$$

APPENDIX C

SENSITIVITY OF TIME INVARIANT PARAMETERS

The computation of the sensitivity coefficient for constant parameters is a special case of the computation for time variant parameters.

Suppose the system is described as

$$x^{i+1} = f^i(x^i, p^i)$$

and if $p^i \neq p^j$ when $i \neq j$, then

$$\frac{\partial x^{i+1}}{\partial p^i} = \frac{\partial f^i}{\partial p^i}$$

$$\frac{\partial x^{i+1}}{\partial p^{i-1}} = \frac{\partial f^i}{\partial x^i} \frac{\partial x^i}{\partial p^{i-1}}$$

$$\frac{\partial x^{i+1}}{\partial p^{i-2}} = \frac{\partial f^i}{\partial x^i} \frac{\partial x^i}{\partial x^{i-1}} \frac{\partial x^{i-1}}{\partial p^{i-2}}$$

$$\vdots$$

$$\frac{\partial x^{i+1}}{\partial p^0} = \frac{\partial f^i}{\partial x^i} \frac{\partial x^i}{\partial x^{i-1}} \frac{\partial x^{i-1}}{\partial x^{i-2}} \cdot \cdot \cdot \frac{\partial x^1}{\partial p^0}$$

Now if $p^i = p^j$ for $i \neq j$, then p^i and p^j are completely correlated, then

$$\sigma_{T_{air}^{i+1}} = \sum_{j=0}^i \frac{\partial X^{i+1}}{\partial p^j} \sigma_{p^j}$$

Assuming $\sigma_{p^j} = \sigma_p$

$$\begin{aligned} \sigma_{T_{air}^{i+1}} &= \sum_{j=0}^i \frac{\partial X^{i+1}}{\partial p^j} \sigma_p \\ &= \left(\frac{\partial f^i}{\partial p^i} + \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial p^{i-1}} \right. \\ &\quad + \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial X^{i-1}} \frac{\partial X^{i-1}}{\partial p^{i-2}} + \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial X^{i-1}} \frac{\partial X^{i-1}}{\partial X^{i-2}} \frac{\partial X^{i-2}}{\partial p^{i-3}} \\ &\quad \left. + \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial X^{i-1}} \frac{\partial X^{i-1}}{\partial X^{i-2}} \cdot \cdot \cdot \frac{\partial X^2}{\partial X^1} \frac{\partial X^1}{\partial p} \right) \sigma_p \\ &= \left(\frac{\partial f^i}{\partial p^i} + \frac{\partial f^i}{\partial X^i} \left\{ \frac{\partial f^{i-1}}{\partial p^{i-1}} + \frac{\partial f^{i-1}}{\partial X^{i-1}} \left[\frac{\partial X^{i-1}}{\partial p^{i-2}} + \frac{\partial X^{i-1}}{\partial X^{i-2}} \frac{\partial X^{i-2}}{\partial p^{i-3}} \right. \right. \right. \\ &\quad \left. \left. \left. + \cdot \cdot \cdot + \frac{\partial X^{i-1}}{\partial X^{i-2}} \cdot \cdot \cdot \frac{\partial X^2}{\partial X^1} \frac{\partial X^1}{\partial p} \right] \right\} \right) \sigma_p \end{aligned}$$

$$\begin{aligned}
&= \left(\frac{\partial f^i}{\partial p} + \frac{\partial f^i}{\partial X^i} \left\{ \frac{\partial f^{i-1}}{\partial p} + \frac{\partial f^{i-1}}{\partial X^{i-1}} \left[\frac{\partial X^{i-1}}{\partial p} + \frac{\partial X^{i-1}}{\partial X^{i-2}} \frac{\partial X^{i-2}}{\partial p} \right. \right. \right. \\
&\quad \left. \left. \left. + \dots + \frac{\partial X^{i-1}}{\partial X^{i-2}} \dots \frac{\partial X^2}{\partial X^1} \frac{\partial X^1}{\partial p} \right] \right\} \right) \sigma_p
\end{aligned}$$

The coefficient of σ_p is the solution of the following difference equation:

$$\frac{\partial X^{i+1}}{\partial p} = \frac{\partial f^i}{\partial p} + \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial p}$$

Therefore

$$\sigma_{T_{air}^{i+1}} = \frac{\partial X^{i+1}}{\partial p} \sigma_p = \sum_{j=0}^i \frac{\partial X^{i+1}}{\partial p^j} \sigma_p$$

APPENDIX D
RADIATION HEAT TRANSFER

The radiative heat input power to the sensor is given by the general expression,

$$Aq_R = \int_A \int_{\Omega} \int_{\lambda} \frac{\cos \theta}{\pi} \alpha_{\lambda} \epsilon_{\lambda} E_{b\lambda}(T) d\lambda d\Omega dA \quad (D.1)$$

where

α_{λ} = spectral absorptivity of the body

ϵ_{λ} = spectral emissivity of the source in $d\Omega$

$E_{b\lambda}(T)$ = plank radiant energy spectral distribution function for
the source in $d\Omega$ at temperature T

λ = solid angle subtended by the environment

A = total sensor surface area

θ = angle between sensor surface element dA and the direction
toward $d\Omega$

λ = radiation wavelength

Consider the four principal environmental radiation sources seen by
the sensor:

j = 1 sun

j = 2 earth and atmosphere as a source of reflected solar
radiation

j = 3 earth and atmosphere as a long wave source

j = 4 sonde parts (including shield) in view of the sensor

Assuming the radiant emittance

$$I_j = \int_0^{\infty} \epsilon_{\lambda j} E_{b\lambda}(T_j) d\lambda \quad (D.2)$$

and the mean absorptivity

$$\bar{\alpha}_j = \frac{\int_0^{\infty} \alpha_{\lambda j} \epsilon_{\lambda j} E_{b\lambda}(T_j) d\lambda}{I_j}$$

are independent of the angle (taken as an appropriate mean value, if necessary and practical, for this assumption), then the geometric factor, f_j

$$f_j = \frac{1}{A} \int_A \int_{\Omega_j} \cos \theta d\Omega_j \frac{d\Omega_j}{\pi} \quad (D.3)$$

may be calculated separately and treated as a multiplicative factor, and the radiation input term takes the form

$$q_R = \sum_j \bar{\alpha}_j f_j I_j \quad (D.4)$$

Geometric Factor f_j

- A. $f_{1,1}, f_{2,1}, f_{3,1}$ (geometric factor with respect to the sun). The solid angle subtended by the sun is $\pi R_s^2/D_{es}^2$, where R_s is the radius of the sun and D_{es} is the distance between the earth and the sun. Then, referring to Eq.

D.3

$$f_{i1} = \frac{R_s^2}{D_{es}^2} \int_A \cos \theta \, dA$$

The computed value of $f_{i,1}$ is listed in Table 3.

- B. $f_{1,4}, f_{2,4}, f_{3,4}$ (geometric factor with respect to the sonde). Figure D.1 shows $f_4(\theta_0)$ for the three shapes when the sonde surfaces occupy a "polar cap" with half-angle θ_0 as shown in the figure [14, Staffanson 1969]. The curves are given by

$$\begin{aligned} f_4 &= 0.5 \left(1 - \cos \theta_0 \right) \\ &= \left(\theta_0 - 0.5 \sin 2\theta_0 \right) / \pi \end{aligned}$$

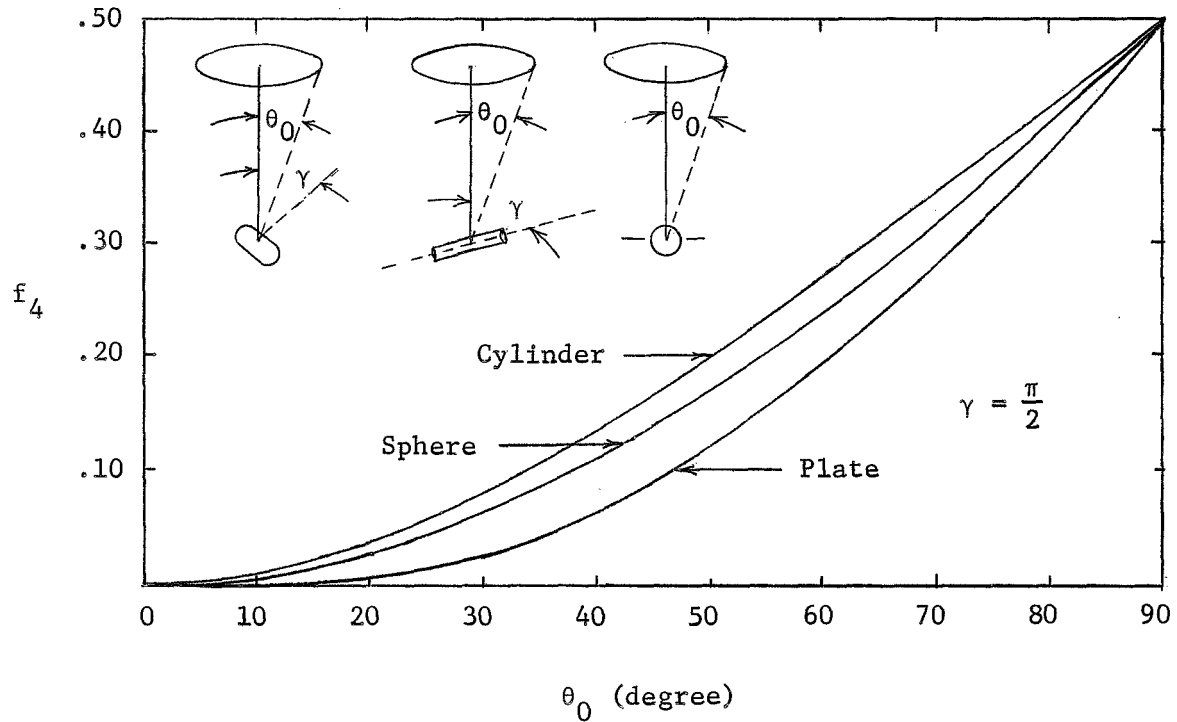


Fig. D.1. Geometric factor for the three sensor shapes of a circular region located 90 degrees from the sensor axis subtending half-angle θ_0 .

for the sphere and plate, respectively, and by numerical tables from Ballinger [1, 1960]. A half angle of 35° approximates that for the ARCASONDE 1A.

- C. $f_{1,3}$, $f_{2,3}$, $f_{3,3}$ (geometric factor for thermal radiation from the earth). The geometric factors associated with the earth long-wave radiation are computed by the

method used for $f_{i,4}$. Nominal value of $f_{i,4}$ is computed, based on $\gamma = \frac{\pi}{2}$ and the uncertainty is due to the fact that the parachute might have a coning motion which varies γ .

- D. $f_{1,2}$, $f_{2,2}$, $f_{3,2}$ (geometric factor with respect to earth albedo). Unlike the thermal geometric factor, the albedo geometric factor is dependent on the position of the sun. The values presented in Table 3 are based on the assumption that the sun is high enough to illuminate essentially all of the earth under the sensor.

Radiant Emittance I_j

Values for the radiant emittance, I_j , together with an estimated uncertainty for each source, j , are as follows:

$$A. \quad I_1 = \left(\frac{r_{es}}{r_s} \right)^2 S = 6.456 \times 10^7 \text{ watts/m}^2 \pm 1\%$$

In the calculation of I_1 , r_{es} is the mean earth-to-sun distance 9.29×10^7 mi, r_s is the radius of the sun 4.32×10^5 mi, and S is the solar constant,

$$S = \int_0^{\infty} \epsilon_{\lambda} E_{b\lambda} d\lambda = 1396 \text{ watts/m}^2 \pm 1\% \quad [8, \text{Johnson 1954}]$$

$$B. \quad I_2 = aS = 460.7 \text{ watts/m}^2 \pm 36\%$$

Here, a is the albedo of the earth. It has been estimated

that clouds can reflect back to space 50 percent or more of the solar flux and absorb another 20 percent. The portions of the earth covered with water reflect about 5 percent of the total radiation reaching them, and the land masses, on the average, reflect slightly more [12, Snoddy 1965].

Therefore, cloud cover becomes a very important factor in determining the earth's albedo. P. F. Clapp [3, 1962] presents cloud cover data using Tiro's nephanalysis, showing cloud cover for various seasons at different latitudes on the earth. By averaging cloudiness for the four seasons, assuming values of reflectance of clouds and the surface of the earth, curves of albedo versus latitude were obtained. From this information it was possible to make some estimation of the effect of cloud cover on albedo. Assuming that clouds reflect 50 percent and the surface of the earth reflects 5 percent, the average albedo is about 0.33, with a variability of ± 0.12 or 36 percent.

C. $I_3 = 233.8 \text{ watts/m}^2 \pm 20\%$

The earth's long-wave emittance depends on the surface temperature and it's emission characteristics. Neglecting details of the planet surface, it is possible to compute the average energy radiated by a planet using a thermal balance based on the solar radiation absorbed by the planet.

As the temperature of most planets do not vary appreciably over extended periods, it can be concluded that the thermally radiated energy is equivalent to the absorbed solar energy.

Using S as the solar heat flux per unit projected area of the planet (as seen from the sun), a as the planetary albedo, R as the planet radius, and I as the thermal energy radiated per average unit planet area and time, the energy balance is

$$(1 - a) S \pi R^2 = f \pi R^2 I$$

or

$$I = \frac{1 - a}{4} S$$

I_3 for the earth computed by this method equals 233.8 watts/m² $\pm$ 20%, using 36 percent variability in a . Actual measurements of earth long-wave radiation have been made by Tiros II (1960) and Tiros IV (1966). Bandeen [2, 1961] analyzed the Tiros II data, and the results fall within this 20 percent uncertainty.

D. $I_4 = 458 \text{ watts/m}^2 \pm 15, \pm 3\% \text{ (shield)}$

The radiant emittance of the sonde can be found by using the relation $I_4 = \sigma \epsilon T_4^4$. T_4 is assumed to be 300°K and

$\epsilon_4 = 1.0$. Small f_4 renders this magnitude essentially insignificant. In the case of the shield, however, f_4 is much larger, but T_4 is assumed measured to within 2°K , and the shield interior is assumed black, both by its coating and by the effect of reflections within its concave interior surfaces.

APPENDIX E

SIMULATION PROGRAM

Fortran V was used to program the simulation study discussed in Chapter IV. The organization of the programming is summarized in the flow diagram, Fig. E.1.

In the main program the environmental conditions are established, and thermal properties of the sensors are assigned. Initial conditions and uncertainties of each parameter are also stated in the main program.

Subroutine TRAJ generates the motion of the parachute with given initial conditions and parachute dimensions. Subroutine ATMO is called from TRAJ to find the necessary atmosphere conditions at a given altitude. Computation will be terminated when the parachute reaches to a lower limit in altitude.

Subroutine SIMULA is called and the temperature of the sensor is computed. Subroutine HANDR is called from SIMULA to calculate necessary values of h and r .

HANDR subprogram calls ATMO for necessary atmosphere conditions. INTRE and INTKN are called from HANDR to calculate uncertainties in h and r .

After SIMULA computes T_b , subroutine REDUCT is called and $\tilde{T}_{air}$ is computed. The uncertainty boundary of $\tilde{T}_{air}$ is also computed in REDUCT.

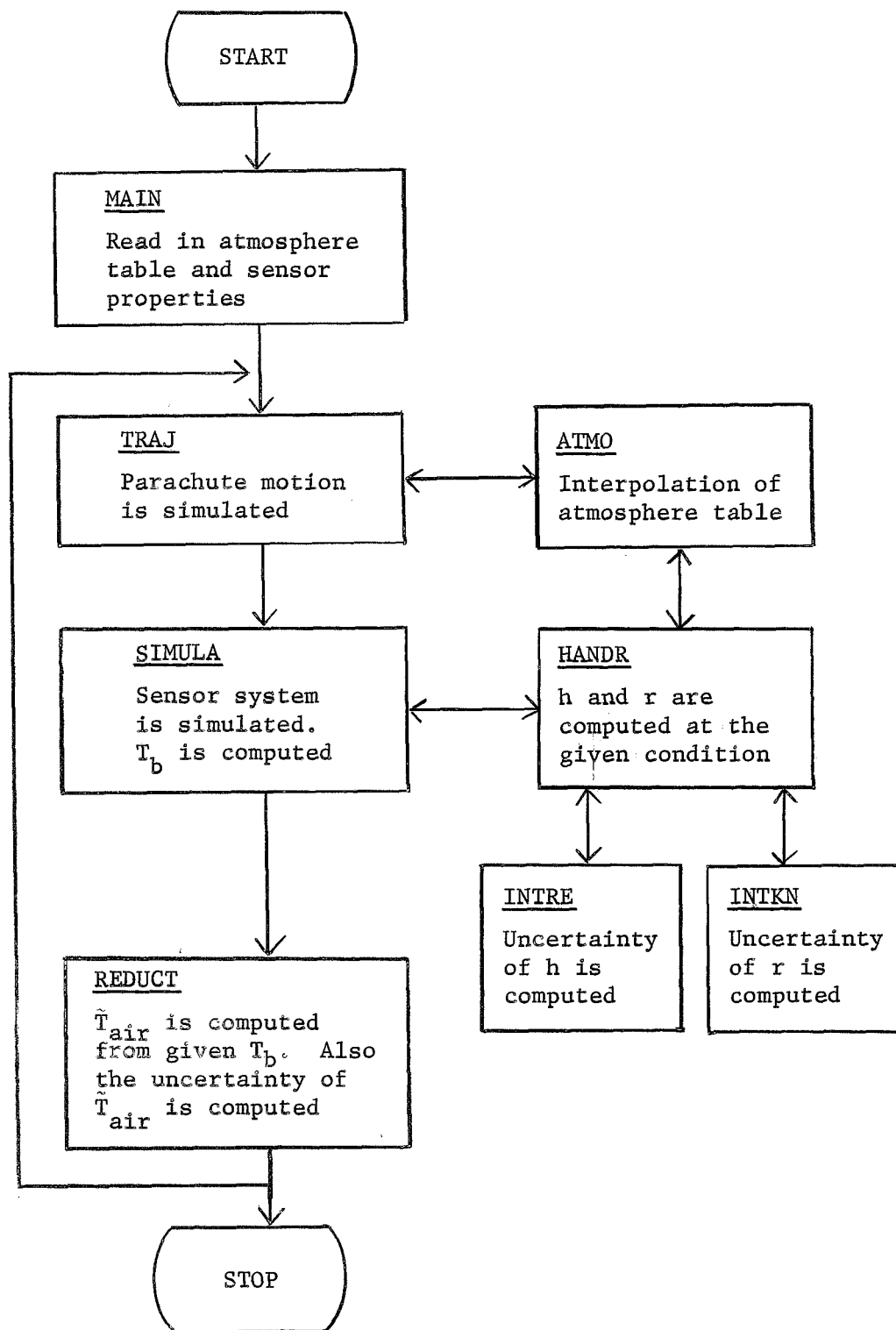


Fig. E.1. Block diagram.

MAIN PROGRAM

```

C.....INPUT QUANTITIES ARE READ IN
C.....LIST OF SYMBOLS
ZSR,ZHG,ZRW OPPENHEIMS TABLE
BESS BESSEL FUNCTION
IC,IS,IT,IF FLOW REGIME EXPRESSION
ALPHA,BEOMF ABSORPTIVITY,BEOMF GEOMETRIC FACTOR
D,RHOC,EPS DIAMETER,DENSITY,SPECIFIC HEAT,EMISSIVITY
W,ZND SELF HEATING,CONDUCTIVITY,DIAMETER
ZL,BETA WIRE LENGTH,PERTURBATION FACTOR
CLB,CLW,CLF CHARACTERISTIC LENGTH OF BEAD,WIRE,FILM
ZD,VVG,VHO INITIAL ALTITUDE,VERTICAL AND HORIZONTAL SPEED
AREA,MASS AREA AND MASS OF PARACHUTE
TB,TBD,TF TEMPERATURE OF BEAD,TIME DERIVATIVE,FILM
JACK,JAZZ,JOEY COUNTER
COMMON /ATHOS/ ALT(250), TENV(250), R(250), CS(250), ZND(250)
COMMON /MIND/ WK(250), WZ(250)
COMMON /RFAC/ ZSR(15), ZRB(15), ZRW(15)
COMMON /CLEW/ CLB, CLW, CLF
COMMON /REP/ IC, IS, IT, IF
COMMON /DACC/ ALPHA(3,4),BEOMF(3,4),FLUX(4),BETA(4)
COMMON /DCC/ EPS(3),RHOC(3),D(3),W(3),ZK(3)
COMMON /MMP/ MMP
COMMON /EIK/ TB,TBD,JACK,JOEY,TF,JAZZ
COMMON /UUU/ UNC(39),TF
1 FORMAT (I4,I)
2 FORMAT (3F10,3)
3 FORMAT (7F10,4)
4 FORMAT (2E15,5)
5 FORMAT (3E15,5)
6 FORMAT (4E15,5)
7 FORMAT (5E15,5)
C.....WIND STRUCTURE IS GIVEN
DO 10 I=1,131
WZ(I)=0.
HX(I)=0.
10 CONTINUE
C.....READ ATMOSPHERE TABLE AND THE SENSOR CHARACTERISTICS
READ 3, (ALT(I),TENV(I),R(I),CS(I),ZND(I),ZK(I),ZMPP(I), I=1,131)
READ 2, (ZSR(I), ZRB(I), ZRW(I), I=1,13)
READ 6, (BESS(I), I=1,32)
READ 1, IC, IS, IT, IF
READ 6, ((ALPHA(I,J), J=1,4), I=1,3)
READ 6, ((GEOMF(I,J), J=1,4), I=1,3)
READ 6, (FLUX(J), J=1,4)
READ 5, ID(1), I=1,3)
READ 5, (RHOC(I), I=1,3)
READ 5, (EPS(I), I=1,3)
READ 5, (W(I), I=1,3)
READ 5, (ZND(I), I=1,3)
READ 4, RO, ZL
MMP=1
READ 6, (BETA(J), J=1,4)
READ 5, CLB, CLW, CLF
READ 7, ZD, VVG, VHO, AREA, MASS
PRINT 100
DO 101 I=1,3
100 FORMAT (3H,3)HSONDE CONFIGURATION PARAMETERS:/:/
101 PRINT 102, I, (ALPHA(I,J), J=1,4)
102 FORMAT (1H,6H)ALPHA(I,4H,J),9X,4E15.5)
DO 103 I=1,3
103 PRINT 104, I, (GEOMF(I,J), J=1,4)

```

```

65* 104 FORMAT (1H,2HF(,11,4H,J),15X,4E15.5)
66* 105 PRINT 105, (FLUX(J), J=1,4)
67* 106 FORMAT (1H,5HF(,11,4H,J),15X,4E15.5)
68* 107 PRINT 106, (BETA(J), J=1,4)
69* 108 FORMAT (1H,8HBETA(J),12X,4E15.5)
70* 109 PRINT 107, (D(I), I=1,3)
71* 110 FORMAT (1H,5HD(,11,4H,J),15X,4E15.5)
72* C.....INITIAL CONDITIONS
73* 111 PRINT 108, (RHOC(I), I=1,3)
74* 112 PRINT 109, (EPS(I), I=1,3)
75* 113 PRINT 10, (W(I), I=1,3)
76* 114 PRINT 11, (ZND(I), I=1,3)
77* 115 PRINT 12, RO
78* 116 PRINT 13, (3HR0,17X,4E15.5)
79* C.....W/A IS COMPUTED
80* W(1)=3.1516
81* W(1)=3.1516
82* C.....UNCERTAINTIES FOR EACH PARAMETERS
83* DATA (UNC(1),I=1,167)/.028,0.01,05,.5,.075,.025,.5,.075,.05,.5/
84* DATA (UNC(1),I=17,25)/.075,.05,.075,.125,.05,.25,.05,.025,.05/
85* DATA (UNC(1),I=26,34)/.025,.25,.05,.035,.015,.015,.015,.20,.20/
86* C.....INITIAL CONDITIONS
87* 117 PRINT 14, (3HR0,17X,4E15.5)
88* 118 PRINT 15, ZL
89* 119 PRINT 16, ZL
90* 120 PRINT 17, ZL
91* 121 PRINT 18, ZL
92* 122 PRINT 19, ZL
93* 123 PRINT 20, ZL
94* 124 PRINT 21, ZL
95* 125 PRINT 22, ZL
96* 126 PRINT 23, ZL
97* 127 PRINT 24, ZL
98* 128 PRINT 25, ZL
99* 129 PRINT 26, ZL
100* 130 PRINT 27, ZL
101* 131 PRINT 28, ZL
102*

```

SUBROUTINE TRAJ

```

1* SUBROUTINE TRAJ
2* THE PARACHUTE MOTION IS SIMULATED
3* LIST OF SYMBOLS
4* C
5* VEL,ZOT TIME,TIME INCREMENT
6* VELOCITY,ALTITUDE
7* DIMENSION ZK(4),VVK(4),XK(4),VHK(4),XXX(1,500),YYY(1,500)
8* COMMON /PP/ Z0, VV0, VHO, AREA, MASS
9* COMMON /AP/ ZOT, TINF, RHO, V5, ANU, AK, AMP, VEL
10* COMMON /M/ VVU, HVV
11* COMMON /R/ RPP
12* COMMON /EIK/ TB, TBO, JACK, JOEY, TP, JAZZ
13* PRINT 1, AREA, MASS
14* IF 6.2,27H METERS SQUARED, AND MASS =,F5.2,10H KILOGRAMS)
15* PRINT 2, Z0
16* 2 FORMAT (1H ,5X,3SHAND INITIAL CONDITIONS: ALTITUDE =,F9.2,
17* 17H METERS)
18* PRINT 3, VV0, VHO
19* 3 FORMAT (1H ,30X,30HVERTICAL VELOCITY = ,F7.2,14H METERS/SECOND,
20* 12H, HORIZONTAL VELOCITY = ,F7.2,14H METERS/SECOND)
21* PRINT 4
22* 4 FORMAT (1H ,5X,26HAND A STANDAPD ATMOSPHERE:))
23* PRINT 5
24* 5 FORMAT (1H ,5X,24HAND A ZERO WIND PROFILE:))
25* H=0
26* A=200.
27* T=0.
28* X0=0.
29* V=SQRT(VV0**2+VHO**2)
30* H=1.
31* IN=0
32* ALTITUDE
33* 10 ZOT=Z0
34* TZOT=T0
35* INK=INKA+1
36* VVOT=VV0
37* XOT=X0
38* VHOT=VH0
39* DO 20 J=1,4
40* IATE=0
41* CALL ATMO (IATE)
42* IF (IATE.EQ. 1) GO TO 21
43* C.....RELATIVE WIND SPEED
44* VZ=VZ0:-VVU
45* VX=VHO:-HVV
46* TVEL=V
47* C.....TOTAL VELOCITY IS FOUND
48* VEL=SQRT(VZ**2+VX**2)
49* CE=(AREA/MASS)*(1.-ZOT*(5.33E-6))
50* G=9.80665-ZOT*(3.04E-6)
51* IF (J.NE. 1) GO TO 17
52* 6=0.5*RH0*VEL**2
53* XXX(1,INKA)=VEL
54* ZOT=ZOT/1000.
55* VVY(1,INKA)=ZOOT
56* ZK(J)=HVVUT
57* CALL SIMULA
58* VVK(J)=H*(-0.5*CARHO%VEL*VZ-G)
59* XK(J)=H*VHOT
60* VHK(J)=H*(-0.5*CARHO%VEL*VX)
61* I= (J.NE. 1) GO TO 18
62* -0.1=Z0+0.5*ZK(1)
63* VVG=VVH+0.5*VVK(1)

```

```

65* XOT=X0+0.5*VVK(1)
66* VHOT=VH0+0.5*VHK(1)
67* GO TO 20
68* 18 IF (J.NE. 2) GO TO 19
69* ZOT=Z0+0.5*ZK(2)
70* VVOT=VV0+0.5*VVK(2)
71* XOT=X0+0.5*VVK(2)
72* VHOT=VH0+0.5*VHK(2)
73* GO TO 20
74* 19 IF (J.NE. 3) GO TO 20
75* ZOT=Z0+ZK(3)
76* VVOT=VV0+VVK(3)
77* XOT=X0+XK(3)
78* VHOT=VH0+VHK(3)
79* 20 CONTINUE
80* TZ0=Z0
81* C.....NEXT ALTITUDE
82* Z0=Z0+ZK(1)+2.*ZK(2)+2.*ZK(3)+ZK(4))/6.
83* VV0=VV0+VVK(1)+2.*VVK(2)+2.*VVK(3)+VVK(4))/6.
84* X0=X0+XK(1)+2.*XK(2)+2.*XK(3)+XK(4))/6.
85* VHO=VH0+VHK(1)+2.*VHK(2)+2.*VHK(3)+VHK(4))/6.
86* IF (ABS(VH0).LT. 1.E-10) VHO=0.
87* V=SQRT(VV0**2+VHO**2)
88* IF (V.EQ. 100.) N=1
89* TAT=H
90* IF (N.EQ. 0) H=1.
91* IF (N.EQ. 0) GO TO 10
92* H=H/V
93* C.....COMPUTATION IS TERMINATED AT 50 KM
94* IF (ZOT.LT.50000.) GO TO 23
95* IF (INKA.GT.1000) GO TO 23
96* GO TO 10
97* 21 PRINT 22
98* 22 FORMAT (1H,25HATMOSPHERE TABLE EXCEEDED)
99* 23 RETURN
100* END
101*

```

SUBPOLYTANE ATMO

```

10 SUBROUTINE ATMO (IATE)
20 C.....LINEAR INTERPOLATION OF THE ATMOSPHERE TABLE IS DONE
30 COMMON /ATMOS/ ZK(1250), TENV(250), R(250), C5(250), ZMU(250)
40 COMMON /ATMOS/ ZK(1250), ZMFP(250)
50 COMMON /MND/ X(250), Z(250)
60 COMMON /AP/ ZOT, TINF, RMO, VS, AMU, AK, ANSP, VEL
70 COMMON /VW/ VW, MVV
80 COMMON /M/ MNP
90 IF (ZOT .LT. ALT(1) .OR. ZOT .GT. ALT(131)) GO TO 2
100 DO 1 I=1,131
110 DZ=ALT(I)-ZOT
120 IF (DZ .LT. 0.) GO TO 1
130 ERRDZ=(ALT(I)-ALT(I-1))
140 DIFFTE=TENV(I)-TENV(I-1)
150 DIFFER(I)=R(I)-1
160 DIFFCS=CS(I)-CS(I-1)
170 DIFFMU=ZMU(I)-ZMU(I-1)
180 DIFFRZ=ZK(I)-ZK(I-1)
190 DIFFMFP=ZMFP(I)-ZMFP(I-1)
200 DIFFRZ=ZK(I)-ZK(I-1)
210 DIFFRZ=ZK(I)-ZK(I-1)
220 CORRTE=ERRDZ/DIFFTE
230 CORRRE=ERRDZ/DIFFR
240 CORRC5=ERRDZ/DIFFCS
250 CORRMU=ERRDZ/DIFFMU
260 CORRRZ=ERRDZ/DIFFRZ
270 CORRMFP=ERRDZ/DIFFMFP
280 CORRZK=ERRDZ/DIFFRZ
290 CORRVX=ERRDZ/DIFFVX
300 TINF=TENV(I)-CORRTE
310 RMO=ZK(I)-CORRR
320 VS=CS(I)-CORRC5
330 AMU=ZMU(I)-CORRMU
340 AK=(ZK(I)-CORRR)*185.
350 ANFP=ZMFP(I)-CORRMFP
360 VV=ZK(I)-CORRVX
370 HV=ZK(I)-CORRVX
380 GO TO 3
390 1 CONTINUE
400 2 IATE=1
410 3 CONTINUE
420 RETURN
430 END

```


SUBROUTINE REDUCT

```

C.....SUBROUTINE REDUCT
C.....TEMPERATURE REDUCTION AND UNCERTAINTY BOUNDARY IS POUND
C.....LIST OF SYMBOLS
C.....NUMBER OF TOTAL PARAMETER, TIME VARIANT PARAMETER
C.....STANDARD DEVIATION OF EACH PARAMETER
C.....S-SSS SENSITIVITY COEFFICIENT WITH RESPECT TO T8
C.....DPL,SS SENSITIVITY COEFFICIENT WITH RESPECT TO TF
C.....STDV STANDARD DEVIATION OF REDUCED AIR TEMPERATURE
C.....TI REDUCED TEMPERATURE OF THE AIR
C.....DIMENSION RAD(3,4),SRA(3)
C.....COMMON /AP/ ZOT, TINF, RHO, VS, AMU, AK, AMPP, VEL
C.....COMMON /HR/ H(3), RECF(3)
C.....COMMON /DAC/ ALPHA(3,4),GEOMF(3,4),FLUX(4),BETA(4)
C.....COMMON /DIC/ EPS(3),RHO(3),D(3),N(3),ZND(3)
C.....COMMON /DOC/ R0,ZL,BESS(50)
C.....COMMON /DUB/ URC(39),TF
C.....COMMON /MAY/ RMP
C.....COMMON /EIK/ TB,TB0,JACK,JCEY,TP,JAZZ
C.....DIMENSION S(40),SII(40),SS(40,40),SSII(40,40),SSSII(40,
140),DSE(40)
C.....DIMENSION SAV(40,40),SAU(40),SAA(40),PGR(40)
C.....DIMENSION STP(40,40),STP*(40)
C.....NUMBER OF PARAMETER IS SET
C.....JT(49)
C.....NF=39
C.....NF=19
C.....DATA CP,P1,SIG/1000.,r3,1416,5.6687E-4/
C.....TB=TP
C.....IF LACK,EG,0, TF=TB
C.....DELTA=1.0
C.....TIME
C.....TET=DELTA
C.....RADIATION INPUT
C.....DO 2 I=1,3
C.....GRAT(1)=0.0
C.....DO 2 J=1,3
C.....RAD(1,J)=ALPHA(1,J)*GEOMF(1,J)*FLUX(J)*BETA(J)
C.....GRAT(1)=GRAT(1)+GRAT(1)
C.....GRAT(1)=GRAT(1)+GRAT(1)
C.....PRODUCTION OF STANDARD DEVIATION
C.....PR(1)=H(1)
C.....PR(2)=RECF(1)
C.....PR(3)=H(2)
C.....PR(4)=RECF(2)
C.....PR(5)=H(3)
C.....PR(6)=RECF(3)
C.....PR(7)=VEL
C.....PR(8)=TF
C.....PR(9)=TBD
C.....PR(10)=GEOMF(1,1)
C.....PR(11)=GEOMF(1,2)
C.....PR(12)=GEOMF(1,3)
C.....PR(13)=GEOMF(2,1)
C.....PR(14)=GEOMF(2,2)
C.....PR(15)=GEOMF(2,3)
C.....PR(16)=GEOMF(3,1)
C.....PR(17)=GEOMF(3,2)
C.....PR(18)=GEOMF(3,3)
C.....PR(19)=W(1)
C.....PR(20)=ALPHA(1,3)
C.....PR(21)=ALPHA(2,3)

```

```

658 PR(23)=D(1)
659 PR(24)=RHO(1)
660 PR(25)=D(2)
661 PR(26)=RHO(2)
662 PR(27)=SS
663 PR(28)=RHO(3)
664 PR(29)=D(3)
665 PR(30)=GEOMF(1,4)
666 PR(31)=GEOMF(2,4)
667 PR(32)=GEOMF(3,4)
668 PR(33)=ALPHA(1,1)
669 PR(34)=ALPHA(2,1)
670 PR(35)=ALPHA(3,1)
671 PR(36)=FLUX(1)
672 PR(37)=FLUX(2)
673 PR(38)=FLUX(3)
674 DO 9 H(1)=NP
675 SOVZ(40)=RMC(NP1)*PR(NP1)
676 SOVZ(16)=1.0
677 DO 11 I=1,6
678 SOVZ(10)=UNC(I)
679 CONTINUE
680 DO 11 CONTINUE
681 DO 11 CONTINUE
682 C.....PRELIMINARY COMPUTATION 1
683 TH=TB+TF/2
684 TT4SEN=SSISEPS(1)*(TB+3)
685 TT4SEN=SSISEPS(2)*(TB+3)
686 TT4SEN=SSISEPS(3)*(TB+3)
687 VCP=VEL*VEL/12.*CP
688 HRVSEN(1)=RECF(1)*VCP
689 HRVSEN(2)=RECF(2)*VCP
690 HRVSEN(3)=RECF(3)*VCP
691 C.....PRELIMINARY COMPUTATION 2
692 C=ZND(2140(21))/2.*RD(1)*D(1)
693 RAN=SSRT(RANM)
694 RAN=4.*H(2)+TT4SEN/2ND(2)
695 C.....PRELIMINARY COMPUTATION 3 HYPERBOLIC FUNCTION
696 EE=EXP(RAN*ZL)
697 EE=1./EE
698 FEECH2=/(EE+SEN)
699 PTANHPSECH=(EE+SEN)/2.
700 PCOTH=1./PTANH
701 PCOSEH2=/(EE+SEN)
702 C.....PRELIMINARY COMPUTATION 4
703 OI=1.*PSECH
704 OSE=HRV*GRAT(2)+TT3W
705 OSE=H(2)+TT4S
706 OSE=PCOSEHPTANH
707 OSE=RAN*ZL*PCOSECH*2
708 OOSE=OSE*2
709 C.....PRELIMINARY COMPUTATION 5
710 P=101*OI/OS
711 P=101*OI/OS
712 P=SECH
713 AK1=H(2)*OI/OS
714 OSE=H(1)+H(1)*NK1
715 OOSE=OSE*2
716 TONG=REHOC(1)*O(1)*TBD/6.
717 TONG=H(1)+TT4SEN+H(1)*TB
718 TONG=HRV*TT3W+GRAT(1)*H(1)*P
719 TONG=REHOC(1)*P
720

```

```

1305 07=TON2+TON3+TON4+TON5
1310 08=TON2+TON3+TON4+TON5
1315 09=TON2+TON3+TON4+TON5
1320 10=TON2+TON3+TON4+TON5
1325 11=TON2+TON3+TON4+TON5
1330 12=TON2+TON3+TON4+TON5
1335 13=TON2+TON3+TON4+TON5
1340 14=TON2+TON3+TON4+TON5
1345 15=TON2+TON3+TON4+TON5
1350 16=TON2+TON3+TON4+TON5
1355 17=TON2+TON3+TON4+TON5
1360 18=TON2+TON3+TON4+TON5
1365 19=TON2+TON3+TON4+TON5
1370 20=TON2+TON3+TON4+TON5
1375 21=TON2+TON3+TON4+TON5
1380 22=TON2+TON3+TON4+TON5
1385 23=TON2+TON3+TON4+TON5
1390 24=TON2+TON3+TON4+TON5
1395 25=TON2+TON3+TON4+TON5
1400 26=TON2+TON3+TON4+TON5
1405 27=TON2+TON3+TON4+TON5
1410 28=TON2+TON3+TON4+TON5
1415 29=TON2+TON3+TON4+TON5
1420 30=TON2+TON3+TON4+TON5
1425 31=TON2+TON3+TON4+TON5
1430 32=TON2+TON3+TON4+TON5
1435 33=TON2+TON3+TON4+TON5
1440 34=TON2+TON3+TON4+TON5
1445 35=TON2+TON3+TON4+TON5
1450 36=TON2+TON3+TON4+TON5
1455 37=TON2+TON3+TON4+TON5
1460 38=TON2+TON3+TON4+TON5
1465 39=TON2+TON3+TON4+TON5
1470 40=TON2+TON3+TON4+TON5
1475 41=TON2+TON3+TON4+TON5
1480 42=TON2+TON3+TON4+TON5
1485 43=TON2+TON3+TON4+TON5
1490 44=TON2+TON3+TON4+TON5
1495 45=TON2+TON3+TON4+TON5
1500 46=TON2+TON3+TON4+TON5
1505 47=TON2+TON3+TON4+TON5
1510 48=TON2+TON3+TON4+TON5
1515 49=TON2+TON3+TON4+TON5
1520 50=TON2+TON3+TON4+TON5
1525 51=TON2+TON3+TON4+TON5
1530 52=TON2+TON3+TON4+TON5
1535 53=TON2+TON3+TON4+TON5
1540 54=TON2+TON3+TON4+TON5
1545 55=TON2+TON3+TON4+TON5
1550 56=TON2+TON3+TON4+TON5
1555 57=TON2+TON3+TON4+TON5
1560 58=TON2+TON3+TON4+TON5
1565 59=TON2+TON3+TON4+TON5
1570 60=TON2+TON3+TON4+TON5
1575 61=TON2+TON3+TON4+TON5
1580 62=TON2+TON3+TON4+TON5
1585 63=TON2+TON3+TON4+TON5
1590 64=TON2+TON3+TON4+TON5
1595 65=TON2+TON3+TON4+TON5
1600 66=TON2+TON3+TON4+TON5
1605 67=TON2+TON3+TON4+TON5
1610 68=TON2+TON3+TON4+TON5
1615 69=TON2+TON3+TON4+TON5
1620 70=TON2+TON3+TON4+TON5
1625 71=TON2+TON3+TON4+TON5
1630 72=TON2+TON3+TON4+TON5
1635 73=TON2+TON3+TON4+TON5
1640 74=TON2+TON3+TON4+TON5
1645 75=TON2+TON3+TON4+TON5
1650 76=TON2+TON3+TON4+TON5
1655 77=TON2+TON3+TON4+TON5
1660 78=TON2+TON3+TON4+TON5
1665 79=TON2+TON3+TON4+TON5
1670 80=TON2+TON3+TON4+TON5
1675 81=TON2+TON3+TON4+TON5
1680 82=TON2+TON3+TON4+TON5
1685 83=TON2+TON3+TON4+TON5
1690 84=TON2+TON3+TON4+TON5
1695 85=TON2+TON3+TON4+TON5
1700 86=TON2+TON3+TON4+TON5
1705 87=TON2+TON3+TON4+TON5
1710 88=TON2+TON3+TON4+TON5
1715 89=TON2+TON3+TON4+TON5
1720 90=TON2+TON3+TON4+TON5
1725 91=TON2+TON3+TON4+TON5
1730 92=TON2+TON3+TON4+TON5
1735 93=TON2+TON3+TON4+TON5
1740 94=TON2+TON3+TON4+TON5
1745 95=TON2+TON3+TON4+TON5
1750 96=TON2+TON3+TON4+TON5
1755 97=TON2+TON3+TON4+TON5
1760 98=TON2+TON3+TON4+TON5
1765 99=TON2+TON3+TON4+TON5
1770 100=TON2+TON3+TON4+TON5
1775 101=TON2+TON3+TON4+TON5
1780 102=TON2+TON3+TON4+TON5
1785 103=TON2+TON3+TON4+TON5
1790 104=TON2+TON3+TON4+TON5
1795 105=TON2+TON3+TON4+TON5
1800 106=TON2+TON3+TON4+TON5
1805 107=TON2+TON3+TON4+TON5
1810 108=TON2+TON3+TON4+TON5
1815 109=TON2+TON3+TON4+TON5
1820 110=TON2+TON3+TON4+TON5
1825 111=TON2+TON3+TON4+TON5
1830 112=TON2+TON3+TON4+TON5
1835 113=TON2+TON3+TON4+TON5
1840 114=TON2+TON3+TON4+TON5
1845 115=TON2+TON3+TON4+TON5
1850 116=TON2+TON3+TON4+TON5
1855 117=TON2+TON3+TON4+TON5
1860 118=TON2+TON3+TON4+TON5
1865 119=TON2+TON3+TON4+TON5
1870 120=TON2+TON3+TON4+TON5
1875 121=TON2+TON3+TON4+TON5
1880 122=TON2+TON3+TON4+TON5
1885 123=TON2+TON3+TON4+TON5
1890 124=TON2+TON3+TON4+TON5
1895 125=TON2+TON3+TON4+TON5
1900 126=TON2+TON3+TON4+TON5
1905 127=TON2+TON3+TON4+TON5
1910 128=TON2+TON3+TON4+TON5
1915 129=TON2+TON3+TON4+TON5
1920 130=TON2+TON3+TON4+TON5
1925 131=TON2+TON3+TON4+TON5
1930 132=TON2+TON3+TON4+TON5
1935 133=TON2+TON3+TON4+TON5
1940 134=TON2+TON3+TON4+TON5
1945 135=TON2+TON3+TON4+TON5
1950 136=TON2+TON3+TON4+TON5
1955 137=TON2+TON3+TON4+TON5
1960 138=TON2+TON3+TON4+TON5
1965 139=TON2+TON3+TON4+TON5
1970 140=TON2+TON3+TON4+TON5
1975 141=TON2+TON3+TON4+TON5
1980 142=TON2+TON3+TON4+TON5
1985 143=TON2+TON3+TON4+TON5
1990 144=TON2+TON3+TON4+TON5
1995 145=TON2+TON3+TON4+TON5

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390* PRINT 520
391* PRINT 521, (PAR(NP1),NOISE,NR)
392* PRINT 521, (SAA(NP1),NP1,NR)
393* 13 CONTINUE
394* IF (JACK,NE,JOEY) 60 TO 515
395* DO 501 ITA2,NS
396* STMP(ITA2,STMP(ITA2,NS))
397* PRINT 491,ITA
398* PRINT 499,STMP(ITA), (STP(ITA,SA),IAT2,NR)
399* 501 CONTINUE
400* JAZZ=JAZZ+100
401* 515 CONTINUE
402* DO 53 I1=1,JIN
403* I1=I1+1
404* DO 53 IMO2,NS
405* SS11(IMO,11)=SS(IMO,I1)
406* SS11(IMO,11)=SS(IMO,I1)
407* 63 CONTINUE
408* 60 TO 95
409* 92 STDV=0.0
410* DO 93 NP1=1,NP
411* STOV=STOV+(S(NP1)+SDVZ(NP1))**2
412* 93 CONTINUE
413* STOV=STOV/STDV
414* DO 61 IMO2,NS
415* S11(IMO)=S(IMO)
416* 95 CONTINUE
417* IF (JACK,NE,JOEY) 60 TO 14
418* PRINT 210,STDV
419* 14 CONTINUE
420* C.....DATA REDUCTION
421* TREDUC=07/04
422* YG=TREDUC
423* Y1=TREDUC
424* C.....CALCULATION OF FILM TEMPERATURE
425* COL=2,*DELTA/(RHOC(3)*D(3))
426* PIG=1,-COL*(H(3)*SIGEPS(3))*(TF**3)
427* WOLF=COL*(H(3)*TG*HRVF+QNA(3))
428* TF=PIG+TF+WOLF
429* C.....SAVE PREVIOUS PARAMETER VALUES
430* TF1=TF
431* Y11=Y1
432* H11=H(3)
433* V11=VEL
434* RF11=RECF(3)
435* IF (JACK,NE,JOEY) 60 TO 15
436* PRINT 140,T1NF,TREDUC,TF
437* JOEY=JOEY+10
438* 15 CONTINUE
439* JACK=JACK+1
440* C.....COMPUTATION OF PARAMETER SHIFT
441* JEEP=JIN
442* IF (JACK,LE,JIN) JEEP=JACK
443* IF (JEEP,LE,1) 60 TO 52
444* DO 50 KEY=2,JEEP
445* KKEY=KEY-1
446* DO 50 NP1=1,NP
447* SDV(NP1,KEY)=SDV(NP1,KKEY)
448* 50 CONTINUE
449* 52 CONTINUE
450* DO 51 NP1=1,NP
451* SDV(NP1,1)=SDVZ(NP1)
452* 51 CONTINUE
453* 137 FORMAT(50H NOT CONVERGE
454* 210 FORMAT(1H ,19HSTANDARD DEVIATION=E15.6)

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455*
456*
457*

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210 FORMAT(50H PARAMETER DEVIATION
211 FORMAT(1H ,E15.6)
212 FORMAT(50H SENSITIVITY
213 FORMAT(1H ,E15.6)
214 FORMAT(1H ,E15.6)
215 FORMAT(1H ,E15.6)
216 FORMAT(1H ,E15.6)
217 FORMAT(1H ,E15.6)
218 FORMAT(1H ,E15.6)
219 FORMAT(1H ,E15.6)
220 FORMAT(1H ,E15.6)
221 FORMAT(1H ,E15.6)
222 FORMAT(1H ,E15.6)
223 FORMAT(1H ,E15.6)
224 FORMAT(1H ,E15.6)
225 FORMAT(1H ,E15.6)
226 FORMAT(1H ,E15.6)
227 RETURN
228 END

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SUBROUTINE INTRE
SUBROUTINE INTRE(REYN,UNCH)
DIMENSION APE(9),DPE(9)
P1,E=3
C.....UNCERTAINTY TABLE FOR DIFFERENT REYNOLDS NUMBER
DO 2 I=1,9
P=P*10.
2 APE(I)=P
DPE(1)=.05
DPE(2)=.075
DPE(3)=.10
DPE(4)=.15
DPE(5)=.18
DPE(6)=.15
DPE(7)=.10
DPE(8)=.075
DPE(9)=.05
IF(REYN.GT.APE(9)) 60 TO 50
IF(REYN.LT.APE(1)) 60 TO 50
C.....LINEAR INTERPOLATION
DO 10 NANA=2,9
NONENANA
IF(REYN.LT.APE(NANA)) 60 TO 12
10 CONTINUE
12 A2=APE(NON)
B2=DPE(NON)
A1=APE(NON-1)
B1=DPE(NON-1)
RCCC=REYN/A1
UNCH=B1+(B2-B1)*(RCCC)/(ALOG10(A2)-ALOG10(A1))
60 TO 51
50 UNCHED=.05
51 RETURN
END
35*
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44*
SUBROUTINE INTRN
SUBROUTINE INTRN(DKN,UNCR,INU)
DIMENSION BWA(5),BWB(5)
C.....UNCERTAINTY TABLE FOR DIFFERENT KNUDSEN NUMBER
IF(INU.LE.2) 60 TO 5
C.....FOR BEAD AND PLATE
BWA(1)=3.E-2
BWA(2)=0.1
BWA(3)=0.3
BWA(4)=1.0
BWA(5)=3.0
60 TO 7
C.....FOR WIRE
5 CONTINUE
BWA(1)=6.E-2
BWA(2)=0.2
BWA(3)=0.6
BWA(4)=2.0
BWA(5)=5.0
7 CONTINUE
BWB(1)=0.05
BWB(2)=0.075
BWB(3)=0.10
BWB(4)=0.075
BWB(5)=0.05
IF(DKN.GT.BWA(5)) 60 TO 50
IF(DKN.LT.BWB(1)) 60 TO 50
C.....LINEAR INTERPOLATION
DO 10 NANA=2,5
NONENANA
IF(DKN.LT.BWA(NANA)) 60 TO 12
10 CONTINUE
12 CONTINUE
A2=BWA(NON)
B2=BWB(NON)
A1=BWA(NON-1)
B1=BWB(NON-1)
DDD=DKN/A1
UNCR=B1+(B2-B1)*(ALOG10(DDD)/(ALOG10(A2)-ALOG10(A1)))
60 TO 51
50 CONTINUE
51 UNCRE=.05
51 CONTINUE
51 RETURN
END
45*
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