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DRIFT OF NONINTERACTING

CHARGED PARTICLES IN A

SIMPLE GEOMAGNETIC FIELD*

by

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I. Introduction

The corotation of plasmas within the plasmasphere has been observed [Carpenter, 1966] and the existence of a potential difference across the magnetosphere in the dawn-dusk meridian of several tens of kilovolts is consistent with various geophysical observations [Brice, 1967]. The motion of charged particles in model electric and magnetic fields have been considered analytically by Alfvén [1939], and computationally by Taylor and Hones [1965] and by Kavanagh et al., [1968]. It is of value to analyze complex configurations requiring computational solutions in terms of simplified models which have analytic solutions. The purpose of this paper is to solve analytically the drift motion of noninteracting charged particles, mirroring equatorially in a rotating magnetic dipole-field, across which is superimposed a uniform electric field.

II. Theory

The drift velocity of a particle with charge q moving in a static magnetic and electric field is given by

$$(1) \qquad \bar{v}_{drift} = \{[-\bar{\nabla}(q\Phi+\mu B) + \bar{f}_c] \times \bar{B}\}/(qB^2)$$

where Φ is the electric field potential, μ is the magnetic moment of the particle, B is the magnetic field, and f_c is the centrifugal force owing to the motion of particles along field lines with a finite radius of curvature R_c where $f_c = mv_{\parallel}^2/R_c$. By restricting the analysis to situations where this centrifugal force is zero, such as in the equatorial plane of a dipole field, or in a uniform field, the drift velocity must be perpendicular to the gradient of $(q\Phi + \mu B)$ as well as perpendicular to B. By definition the gradient of $(q\Phi + \mu B)$ is perpendicular to the surface upon which $(q\Phi + \mu B)$ is constant. Therefore the drift path of particle is on a surface of $(q\Phi + \mu B)$ = constant. In the geomagnetic field, the requirements that $f_c = 0$ and $\bar{v}_d \perp \bar{B}$ restrict the particle's drift path to the line of $(q\Phi + \mu B)$ = constant in the equatorial plane as shown by Alfvén [1939].

The first invariant I_1, where $I_1 = p_\perp^2/B$ [Alfvén and Fälthammer, 1963], is assumed to be constant during the motion. Since I_1 is proportional to $\gamma\mu$, the magnetic moment μ is nearly constant so long as the fractional change in γ during the particle motion is much less than 1 where $\gamma = (1-(v/c)^2)^{-\frac{1}{2}}$. The drift paths, energization and compression of charged particles moving equatorially in an electromagnetic field

$$B = B_o r^{-3} \tag{2}$$
$$\Phi = V_o r^{-1} + V_1 r \sin\phi$$

are analyzed assuming μ is a constant.

The expression $V_o r^{-1}$ describes the equatorial corotation potential of a rotating magnetic dipole when surrounded by a tenuous plasma [Hones and Bergeson, 1965]. The expression $V_o r^{-1}$ does not describe the equatorial corotation potential in a distorted dipole field. For example, if a rotating dipole field is confined to a sphere of radius R_s (by a diamagnetic plasma) the equatorial corotation potential is $V_o r^{-1}(1 - (r/R_s)^3)$, where V_o is the same as in the 'dipole' form. The 'dipole' form of the corotation potential has been used in the analysis and should be reasonably appropriate for the geomagnetic field within 8 to 10 R_E. V_o equals $-(\Omega_E R_E) B_o R_E$ = -91 kv in the earth's magnetic field where B_o = .31 gauss and R_E = 6374 km.

The expression $V_1 r \sin\phi$ describes a uniform electric field. By defining $\phi = 0°$ at geomagnetic noon, an electric field E, directed from dawn to dusk, has V_1 = [-6.37 E(mv/m)]kv. Such an electric field could be produced by the convection of field lines toward the subsolar point of the magnetosphere as in Axford and Hines [1964].

A given drift path may be identified by the radius, r_o, and the angle, $\phi = \pm 90°$, at which the path crosses the dawn-dusk meridian(s). On this basis the general drift-path equation becomes

(3)
$$r^4 \sin\phi - r^3\left[\left(\frac{V_o}{V_1}\right)^2\left(\frac{\mu}{12\mu_*}\right)r_o^{-3} + \left(\frac{V_o}{V_1}\right)r_o^{-1} \pm r_o\right] + r^2\left(\frac{V_o}{V_1}\right) + \left[\left(\frac{V_o}{V_1}\right)^2\left(\frac{\mu}{12\mu_*}\right)\right] = 0$$

(4) where $\mu_* = qV_o^2/12V_1B_o$.

The most significant points in the equatorial plane are those where the drift velocity is zero. Such points satisfy

(5) $$\bar{\nabla}\,(q\Phi + \mu B) = 0$$

and will be denoted by r_*. If $V_1 \neq 0$, such points exist only along the dawn-dusk meridian. The general solution of r_* is given by

(6) $$r_*^2 = (V_o/\pm 2V_1)\;[1 \pm \sqrt{1 \pm (\mu/\mu_*)}\,].$$

This equation is illustrated in Figure 1.

Equations (3) and (6) show that the drift paths and the points of zero velocity are completely specified by two ratios, (V_o/V_1) and (μ/μ_*). If all distances are scaled by S, where

(7) $$S = |V_o/V_1|^{\frac{1}{2}},$$

the motion is specified entirely by the ratio (μ/μ_*).

One other useful relationship, in lieu of an exact solution of the drift path, is the relationship between the location of a particle at infinity and the radius, r_o, at which it drifts across the dawn-dusk meridian, $\phi = \pm 90°$. The location at infinity is given by X_∞ where $X_\infty = \mathrm{Lim}\,[r(\phi)\sin\phi]$ as $\sin\phi$ approaches zero or r approaches infinity. In this limit the general drift-path equation (eq. 3) reduces to

$$(8) \qquad X_\infty = \pm r_o + [(V_o/V_1)^2(\mu/12\mu_*)r_o^{-3} + (V_o/V_1)r_o^{-1}].$$

For every value of X_∞ there is a corresponding r_o. The reverse is not necessarily true since some values of r_o denote closed drift paths which do not go to infinity. This relationship may be advantageously used to solve for the minimum radius, r_{min}, of the innermost open drift-path. The innermost open drift-path must pass through a point of zero drift-velocity. The value of X_∞ is easily found for the open path 'through' this point of zero drift-velocity by setting r_o equal to r_* and choosing the appropriate sign of $\pm 90°$. By knowing that this same innermost open drift-path also passes through the point of minimum radius on the opposite side, with $r_o = r_{min}$ with the same value of X_∞ and by noting that drift paths do not cross, the value of r_{min} may be readily determined (by trial and error) for a given (μ/μ_*).

In the geomagnetic field (V_o/V_1) is positive if $\phi = 0°$ at noon and therefore

$$S = 3.8\ R_E/\sqrt{E(mv/m)} \text{ and}$$

(9)

$\mu_* = [-3.5z/E(mv/m)]$ eV/gamma so that

$$(r_*/S)^2 = \pm\frac{1}{2}[1 \pm \sqrt{1 \pm (\mu/\mu_*)}]$$

(10) $$(X_\infty/S) = \pm(r_o/S) + [(\mu/12\mu_*)(S/r_o)^3 + (S/r_o)] \text{ and}$$

$$(r/S)^4\sin\phi - (r/S)^3(X_\infty/S) + (r/S)^2 + (\mu/12\mu_*) = 0.$$

The following discussion will be in terms of S and (μ/μ_*) assuming $\phi = 0°$ is chosen so that (V_o/V_1) is positive.

III. Drift Paths

The equatorial plane may usually be divided into an inner and outer region. In the inner region, particle drift-paths are closed and quasi-circular whereas in the outer region the particle drift-paths are open to infinity. For protons with $\mu \ll |\mu_*|$, the inner region may be subdivided into an inner circular-zone and an outer ring-zone based on the clockwise and counterclockwise motion in the 'gradient-dominated' and 'corotation-dominated' zones respectively. As μ increases to $|\mu_*|$, the 'corotation-dominated' zone shrinks and vanishes for $\mu > |\mu_*|$ as indicated in Figure 1. The peculiar manner in which this 'corotation-dominated' region vanishes will be illustrated in a later paragraph.

For electrons the gradient and corotation drift are in the same direction so that the transition from $\mu = 0$ (Figure 2a) through $\mu = .5\mu_*$ (Figure 2b), $\mu = \mu_*$ (Figure 3a) to $\mu = 8\mu_*$ (Figure 3b) is smooth and regular as shown also by the dashed line in Figure 1. The corotation drift assists the gradient drift so that the innermost-drift path is larger than it would have been (the dashed line) for similar particles in a nonrotating magnetic field [Alfvén, 1939]. The Alfvén solution is valid for $\mu \gg |\mu_*|$ and the drift paths of protons and electrons are reflections of each other across the noon-midnight meridian since the corotation force becomes negligibly small.

For protons the gradient and corotation drift are opposed. In Figure 4a for $|\mu/\mu_*| = 8$, the innermost open drift-path is smaller than it would have been (the dashed line) for similar particles moving in a nonrotating magnetic field. In Figure 4b for $\mu = |\mu_*|$ the size of the 'gradient-dominated' zone is greatly decreased by the opposing corotation drift which is just strong enough to cancel the gradient and uniform E field drift-velocities at $r_* = .71$ S at $\phi = +90°$. In Figure 5 for $\mu = .5|\mu_*|$ the 'corotation-dominated' zone expands while the 'gradient-dominated' region continues to decrease. In order to form the $\mu = 0$ configuration illustrated in Figure 2a, the 'gradient-dominated' and 'corotation-dominated' regions must merge as μ decreases. The 'jaws' of the 'corotation-dominated' zone close about the 'gradient-dominated' zone when $\mu \simeq .34|\mu_*|$ forming a closed inner region. Note that, for protons with $.34 \leq \mu/|\mu_*| \leq 1$, the two regions of closed drift paths are separated and enclosed by the region of open drift paths.

IV. Charge-Separation and Current Asymmetries

Drift paths are difficult to observe and we are usually dealing with quasi-neutral plasmas. The following two figures illustrate the superposition of the innermost open drift-paths for protons and electrons having the same magnetic moments.

In Figure 6a $|\mu/\mu_*| = 8$. Consider first the Alfvén solutions, illustrated as dashed lines, in which two symmetric regions of net charge would tend to develop: a positive region at dusk and a negative region at dawn. This charge separation was proposed as the source of field-aligned currents by Dessler, Freeman and myself [Schield et al., 1969]. The only way to introduce any asymmetry was to utilize particles with two different magnetic moments. However the corotation field introduces an asymmetry which increases the size of the positively charged region while decreasing the size of the negatively charged region. Observation of the direction and location of the neutralizing field-aligned currents should identify the sign and size of the regions of net charge.

This model also results in an asymmetrical ring current with the asymmetry located on the dusk meridian. The electric fields produce no net current since they drift both protons and electrons in the same direction. The only currents are the gradient drift and diamagnetic currents, both of which depend upon the plasma location and number density.

Alfvén [1939] showed that the equatorial number density is proportional to the magnetic field intensity along the drift path. For plasma on open drift paths in a dipole field the diamagnetic current equals the gradient drift current and both are proportional to μr^{-4}. Thus any asymmetry in the minimum radii of the innermost open drift-paths will produce an asymmetric ring current. This asymmetry is the dark region in this figure. The fact that the neutralizing charges may have negligible energies should not decrease the net current by more than a factor of 2.

In Figure 6b $|\mu/\mu_*| = 1$ and the positively charged region completely encloses the jointly forbidden region. Furthermore the degree of asymmetry has increased. In summary this model predicts that the corotation potential will produce dawn-dusk asymmetries in the regions of net charge and in the ring current for particles on open drift paths. Both these effects should vanish for $|\mu/\mu_*|>>1$.

V. Magnetospheric Particles and Fields

In the geomagnetic field $\mu_* = [-.004z/E(mv/m)]$ keV/γ and $S = 3.8\ R_E/\sqrt{E(mv/m)}$ where E is the magnitude of the uniform electric field. If E is between .1 and one mv/m then $|\mu_*/z|$ is between .04 and .004 keV/γ while S is between 3.8 and 12 R_E. A one keV proton or electron in a 20γ field such as the earth's magnetic tail has a magnetic moment of .05 keV/γ, whereas a 1 keV electron in a 100γ field at $\sim 7\ R_E$ has a magnetic moment of .01 keV/γ. Excluding the inner-zone protons having energies of ~ 40 MeV and moments of ~ 3 keV/γ and ionospheric particles having thermal energies and moments of 10^{-5} keV/γ, many equatorially mirroring particles have magnetic moments which are slightly greater than $|\mu_*/z|$. Thus the effects of corotation should be quite important in the magnetosphere if field lines do indeed remain equipotentials. One test of this hypothesis is to see if the observed geomagnetic-storm asymmetry can be explained satisfactorily.

VI. Geomagnetic Storm Asymmetry

The local time dependence of the geomagnetic storm variation has been attributed to an asymmetric ring current consisting of low energy protons located near the dusk meridian at $\sim 4\ R_E$ [Cummings, 1966]. The direct injection of 'ring current' protons in the evening-midnight quadrant of the magnetosphere has been observed by Frank [1969]. And although the plasma sheet protons are in general more energetic than the plasma sheet electrons this is not sufficient to deduce that the westward drift of the protons injected near local midnight will indeed form an asymmetric ring current. Electron observations in the dawn sector will be necessary to determine whether it is the difference in their initial conditions or in their drift path that is responsible for the resultant asymmetric ring-current.

Assuming that there is no significant difference in the initial conditions of the injected protons and electrons, the net current is proportional to r^{-4} since the diamagnetic current density and the drift current density are equal. The magnetic field produced by such a plasma symmetrically located beyond a radius r_s would be proportional to $2\pi\, r_s^{-3}$ for a given sector of latitude. $(\Delta B(0) = 2\pi \int_{r_s}^{\infty} j(r)\,dr\,d\lambda)$. A 180° partial ring-current centered on the dusk meridian located radially between r_{min} and r_s should produce a field which is proportional to $\pi(r_{min}^{-3} - r_s^{-3})$ when measured on the dusk meridian of the earth's surface. Cummings [1966] has shown that for a partial ring-current located at 4 R_E the magnetic field measured on the dawn meridian is less than 10% that measured on the dusk meridian. Therefore the ratio of the dusk to dawn disturbance fields is $\simeq 1 + (\frac{1}{2})[(r_s/r_{min})^3 - 1]$. For $E = .15$ mv/m; $S = 10\ R_E$ and $|\mu_*| = 24$ eV/γ. Assuming the ring current particles all have magnetic moments of $|\mu_*|$ which means an energy of ~ 12 keV for protons at 4 R_E the minimum radius of the partial ring current r_{min} is $\sim 3.2\ R_E$ and the inner radius of the symmetric ring current r_s is $\sim 4.6\ R_E$. On this basis the ratio of the dusk to dawn disturbance fields as measured at the earth should be a factor of two. This result is in excellent agreement with the example presented in Figure 1 of Cummings [1966].

In the Alfvén limit of $\mu >> |\mu_*|$ there is no ring current asymmetry neglecting any differential source or loss mechanisms. This model neglects any charge-separation electric fields and assumes the plasma does not saturate or distort the dipole field. The analysis of only equatorially-mirroring particles is not believed to be restrictive since particles mirroring at high latitudes may behave similarly [Hones, 1966].

Acknowledgements

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Figure Captions

Figure 1. Location of points of zero-drift velocity r_* (in units of S where $S = \sqrt{V_o/V_1}$) as a function of $(\pm\mu/\mu_*)$ where $\mu_* = (qV_o^2/12B_oV_1)$, where the symbol $\pm$ designates $\phi = \pm 90°$ respectively and where $\phi = 0°$ is chosen so that (V_o/V_1) is positive. This figure illustrates the solution of equation 10a. The circles represent the Alfvén solution ignoring corotation where $L = |\mu B_o/qV_1|^{\frac{1}{4}} = |\mu/12\mu_*|^{\frac{1}{4}} S$. As expected the corotation increases r_* for electrons $(qV_o>0)$ and decreases r_* for protons $(qV_o<0)$ as illustrated for $(\pm\mu/\mu_*) > 0$. For $\mu<<|\mu_*|$, $(r_*/S)^2 \simeq \pm(\mu/2\mu_*)$ for the gradient-drift dominated solution and $(r_*/S)^2 \simeq 1 + (\mu/4\mu_*)$ for the corotation-drift dominated solution at $\phi = 90°$. As μ increases to $|\mu_*|$ for protons $(qV_o<0)$, the corotation-dominated region shrinks to a single point and disappears for $\mu > |\mu_*|$. For $\mu>>|\mu_*|$, $(r_*/S)^2 \simeq \pm\sqrt{\pm(\mu/\mu_*)}\,/\pm 2$.

Figure 2a. Equipotentials of the electric field which are the drift paths for particles with $(\mu/\mu_*) \simeq 0$. The darker region denotes the central core of quasi-circular drift paths with $r_o \leq \oplus .5S$. The drift paths through r_* at $\phi = +90°$ satisfy $(r/S) = [(1 \pm \sqrt{1 - \sin\phi})/ \sin\phi]$ while the drift path through $r_o = 1$, $\phi = -90°$ is given by $(r/S) = (-\sin\phi)^{-\frac{1}{2}}$.

Figure 2b. Electron drift paths for $\mu = .5\mu_*$. The dashed line illustrates Alfvén's solution ignoring corotation. The dark line represents the innermost open drift-path and the shaded region illustrates the difference produced by the corotation field.

Figure 3. Electron drift paths for $\mu = \mu_*$ and $\mu = 8\mu_*$. See Figure 2b for description. Note that the difference between the actual solution (the solid line) and the Alfvén solution (the dashed line) decreases as (μ/μ_*) increases and the corotation drift is less influential.

Figure 4. Proton drift paths for $\mu = -8\mu_*$ and $\mu = -\mu_*$. The Alfvén solution of the innermost open drift-path is illustrated by the dashed line. The equatorial corotation potential is that of a negative charge at the earth's center which tends to 'attract' protons,

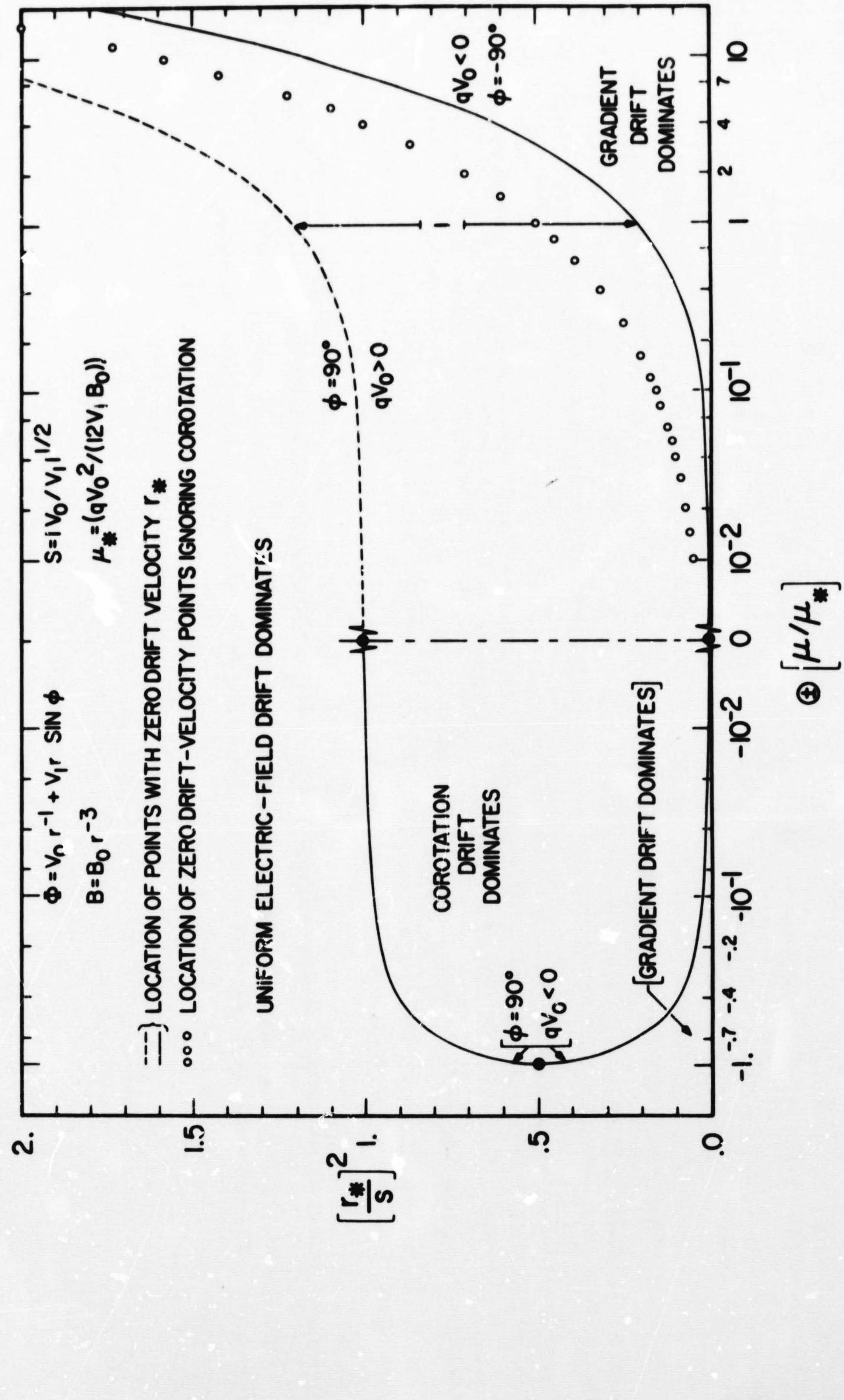
Figure 4. thus decreasing the size of the innermost open drift-path, while repelling electrons as shown in Figure 3. In Figure 4b there are two points of zero-drift velocity; r_{*c} where the corotation drift cancels the other drifts and r_{*g} where the gradient drift cancels the other drifts. Notice also that the particles near the earth are drawn in from $X_\infty \simeq 1$ rather than $X_\infty \simeq 0$ for $\mu >> |\mu_*|$.

Figure 5. Proton drift paths for $\mu = -.5\mu_*$. The central 'gradient-drift dominated' region is smaller than the corresponding Alfvénic forbidden region because of the corotation field which also produces a region of closed drift paths which do not encircle the earth. Notice that the particles nearest the earth are drawn from $X_\infty \simeq 1.50$. Calculations indicate that the jaws of the 'corotation dominated' region will enclose the 'gradient-drift dominated' region for $\mu \simeq -(\mu_*/3)$, and the drift paths will then correspond to those shown in Figure 2a as $(-\mu/\mu_*)$ approaches zero.

Figure 6a. Regions of charge separation and of current asymmetries for $\mu = \rho|\mu_*|$. Electrons on open drift paths cannot penetrate the lightly shaded region at +90°, but must drift around

Figure 6a. this forbidden zone. Protons on the other hand are excluded from the moderately shaded region at -90°. Both electrons and protons located on open drift paths are jointly excluded from the central unshaded region. Field-aligned currents would tend to neutralize these two regions of net charge; the positive region near ϕ = +90°, the negative at -90°. An asymmetric ring-current, illustrated by the heavily shaded crescent, will develop on the ϕ = +90° side owing to the asymmetry in the innermost open drift-paths between protons and electrons.

Figure 6b. Regions of charge separation and of current asymmetries for $\mu = |\mu_*|$. The region of net positive charge completely encircles the jointly forbidden region and the asymmetry of the ring current is enhanced considerably. The partial ring current is assumed to close in the ionosphere [Cummings, 1966].


C-G69-88
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$S = |V_0/V_1|^{1/2}$
$B = B_0 r^{-3}$
$\mu_* = (qV_0^2/(12 V_1 B_0))$
LOCATION OF POINTS WITH ZERO DRIFT VELOCITY r_*
LOCATION OF ZERO DRIFT-VELOCITY POINTS IGNORING COROTATION
UNIFORM ELECTRIC-FIELD DRIFT DOMINATES
COROTATION DRIFT DOMINATES
GRADIENT DRIFT DOMINATES
GRADIENT DRIFT DOMINATES
$\phi = 90°$ $qV_0 < 0$
$\phi = 90°$ $qV_0 > 0$
$qV_0 < 0$ $\phi = -90°$
$\left[\frac{r_*}{S}\right]^2$
2.
1.5
1.
.5
0.
$\pm\left[\mu/\mu_*\right]$
-1.
-.7
-.4
-.2
-10^{-1}
-10^{-2}
0
10^{-2}
10^{-1}
1
2
4
7
10

Figure 1

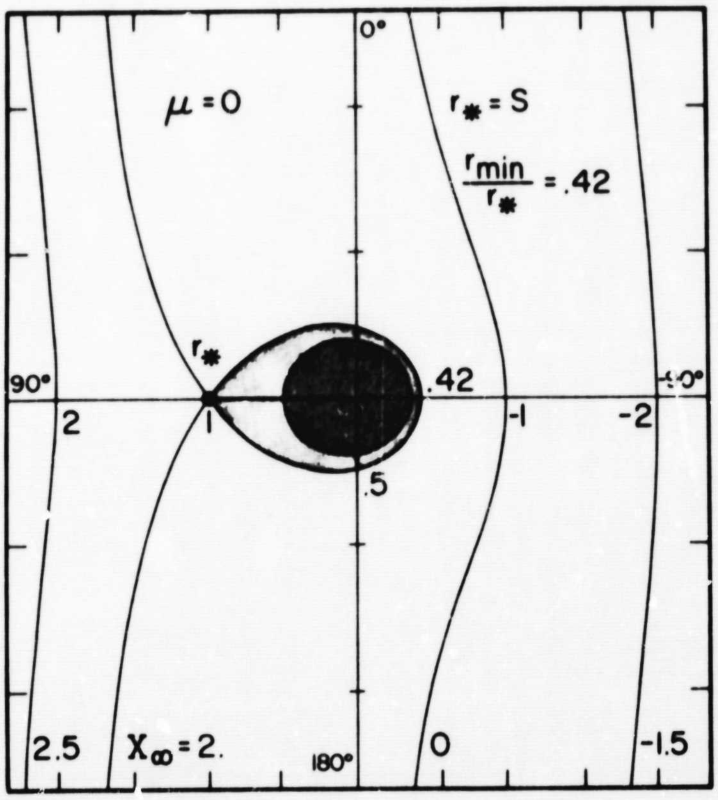


Figure 2a

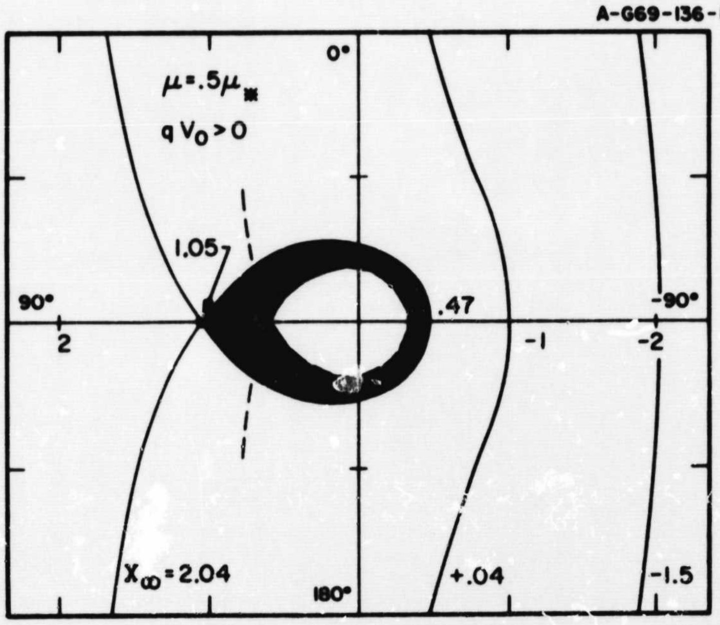
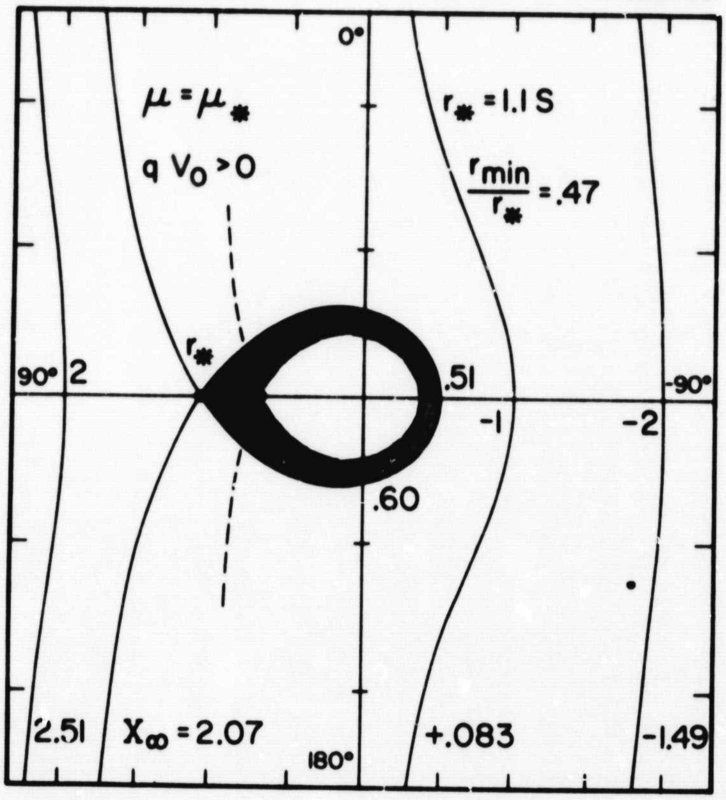


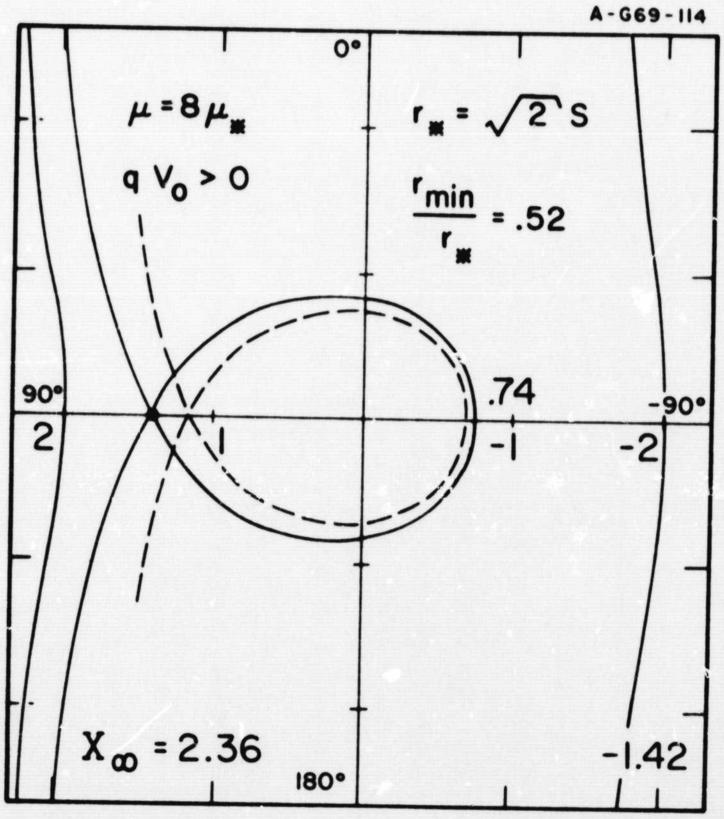
Figure 2b

A-G69-95-1



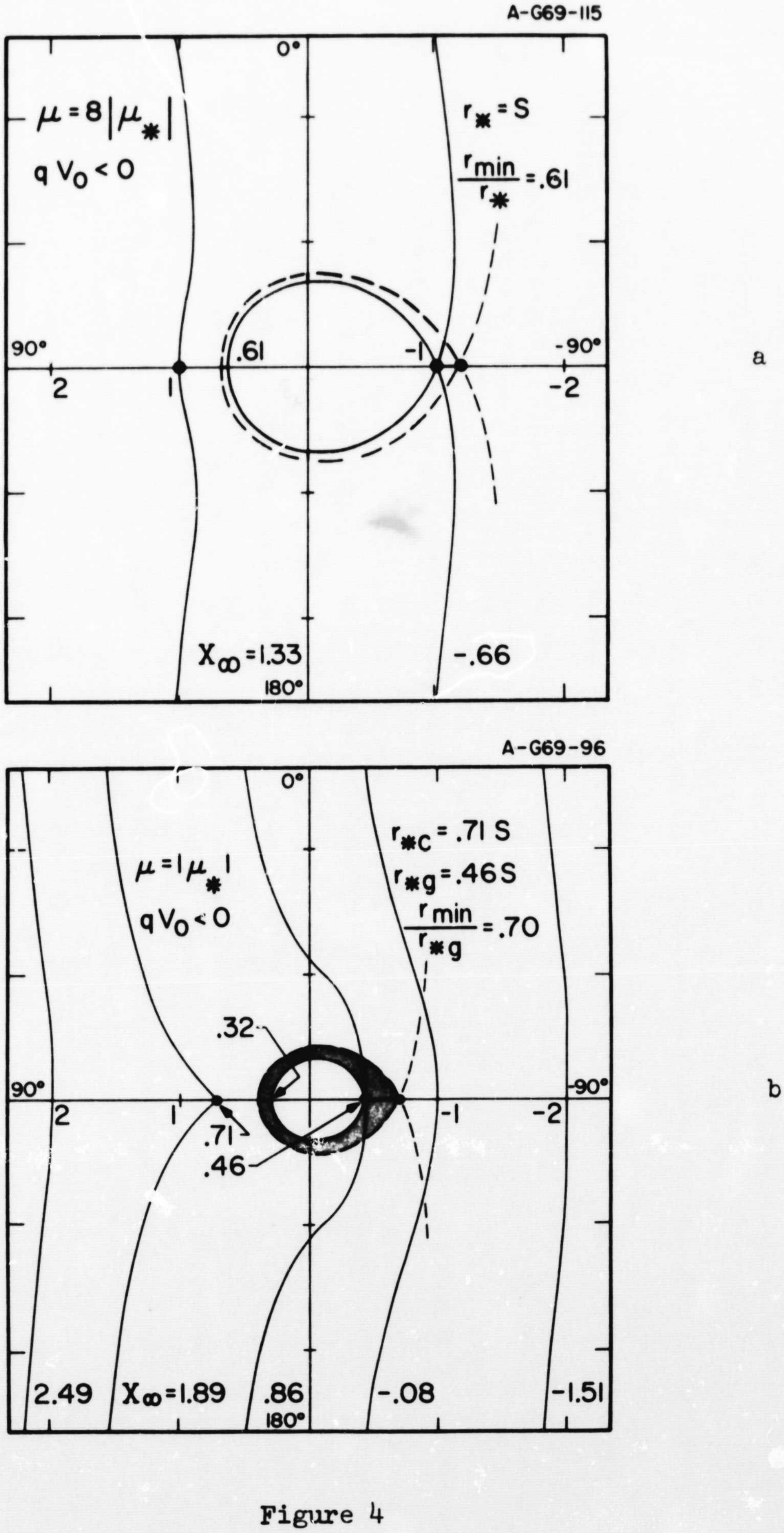
a

A-G69-114



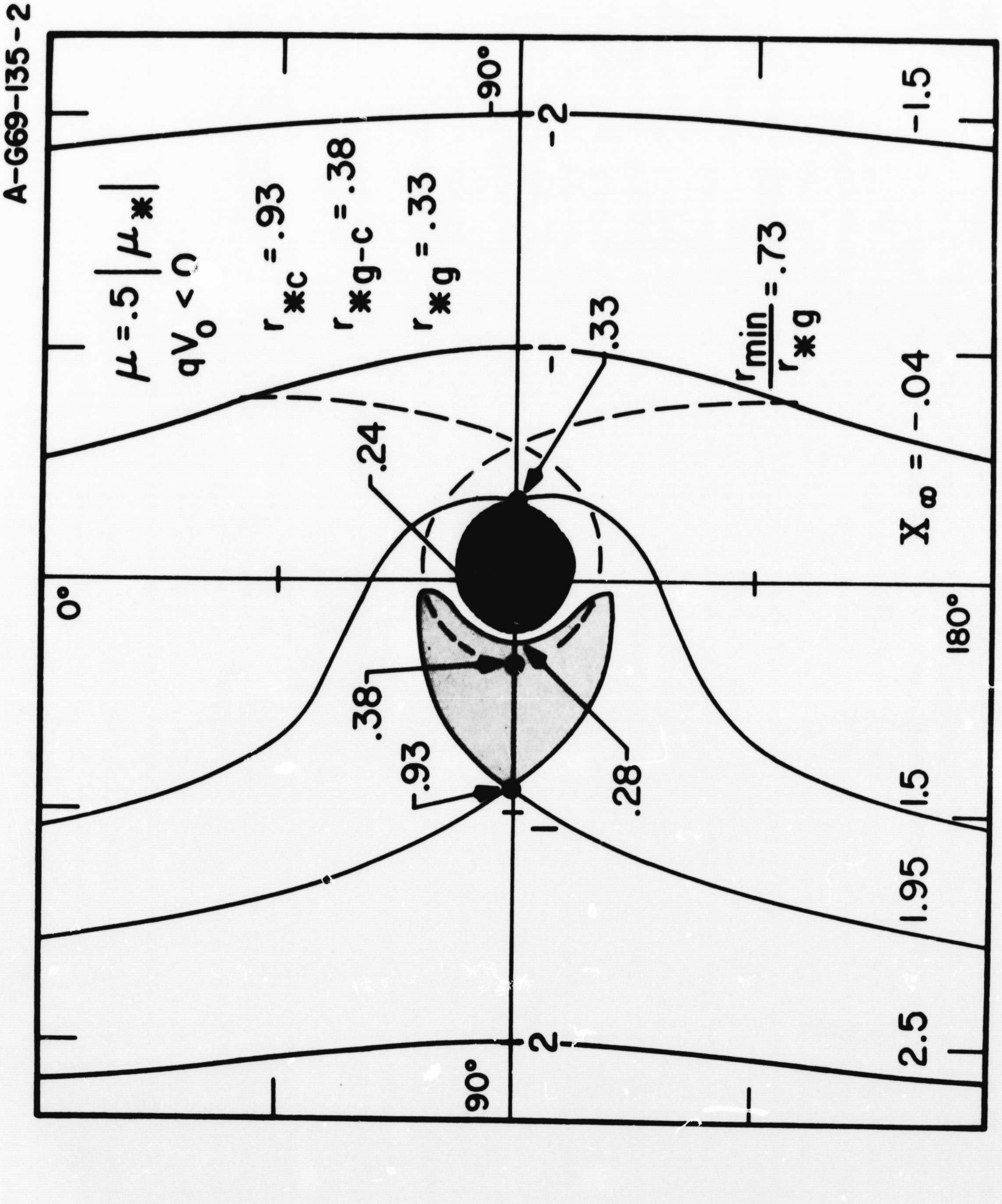
b

Figure 3



A-G69-115
0°
$\mu = 8|\mu_*|$
$qV_0 < 0$
$r_* = S$
$\frac{r_{min}}{r_*} = .61$
90°
-90°
.61
-1
2
1
-2
$X_\infty = 1.33$
-.66
180°
a
A-G69-96
0°
$r_{*c} = .71\,S$
$r_{*g} = .46\,S$
$\mu = 1|\mu_*|$
$qV_0 < 0$
$\frac{r_{min}}{r_{*g}} = .70$
.32
90°
-90°
2
1
-1
-2
.71
.46
2.49
$X_\infty = 1.89$
.86
-.08
-1.51
180°
b

Figure 4


A-G69-135-2
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$r_{*g-c} = .38$
$r_{*g} = .33$
$\frac{r_{min}}{r_{*g}} = .73$
$X_\infty = -.04$
-1.5
-2
1.5
1.95
2.5
2
.33
.24
.38
.93
.28
0°
90°
-90°
180°

Figure 5

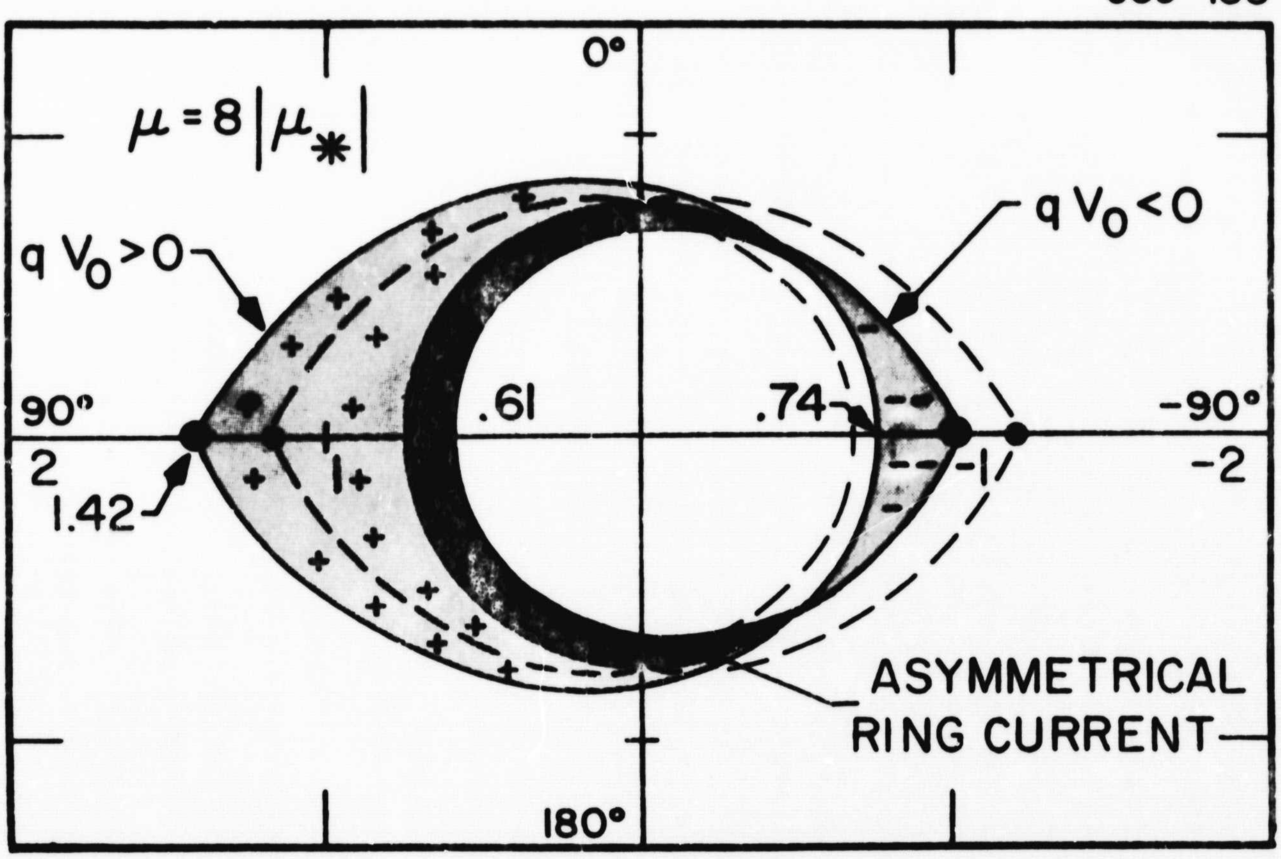

Figure 6a

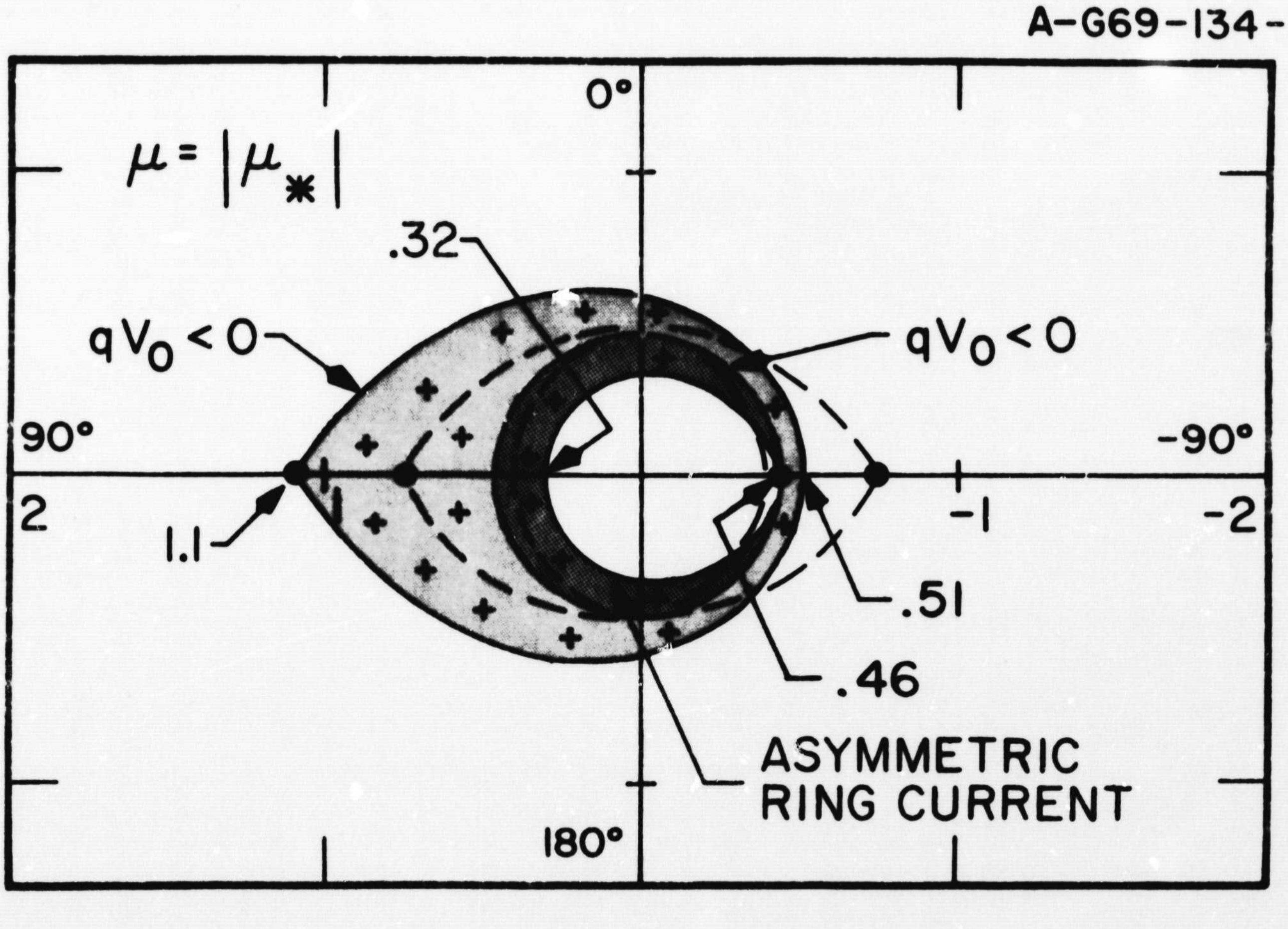

Figure 6b