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We wish to investigate the implications of a new upper limit on the sideral anisotropy of the cosmic rays. After correcting for the influence of the interplanetary magnetic field, Elliot et al.[1] find that cosmic rays of energies $\gtrsim 10^{11}$ev have an anisotropy less than 2×10^{-4}. This value seems to conflict[2] with estimates of the cosmic-ray lifetime of 1 to 2×10^{6} years, since for such an anisotropy, the time required for escape from the disk is about 3×10^{7} years.

It has been suggested[3] that this discrepancy could be due to fluctuations resulting from the statistical nature of the cosmic-ray source distribution which at the present just happens to give an extremely low anisotropy at the earth. However, from a detailed statistical study, in which we have treated the galactic cosmic ray sources as random events in both space and time, we find that within reasonable probabilistic limits, the discrepancy between isotropy and escape lifetime of 1 to 2×10^{6} years cannot be accounted for solely by fluctuations.

Within this statistical discrete-source model, we have considered the alternate possibility that the streaming velocity is indeed as low as suggested by the observed lack of anisotropy. In this case, because of the long residence time of the cosmic rays in the galaxy, the lifetimes of the various nuclei are determined principally by nuclear interactions and not by escape. In particular, the proton lifetime is much longer than the lifetimes of the heavier nuclei. The apparent discrepancy between isotropy and lifetime can then be readily understood, since the anisotropy measurements refer to the total cosmic ray flux which consists mainly of protons, while the lifetime estimates are based principally on the amount of matter traversed in interstellar space by nuclei in the medium group. In this respect, the statistical model does not greatly differ from the steady state models which were used in previous calculations. However, when the combined effects of the statistical model and a low streaming velocity are simultaneously considered, several of the measurable properties of the cosmic rays, and in particular the abundances of nuclei heavier than iron, will exhibit large fluctuation which must be taken into account in order to interpret the observational data.

In this letter, we wish to present the outline and the results of calculations of the fluxes, interactions, lifetimes, and anisotropies of the various chemical components of the cosmic rays in a statistical discrete-source model. We shall present the details of these computations in a separate paper.

We have assumed that the cosmic ray sources are supernova explosions confined to a disk of thickness 200 pc with a random spatial and temporal distribution normalized to a local rate which corresponds to 1 supernova per 100 years in the galaxy. We treat each supernova as a point source in space and time and we assume for simplicity that cosmic ray propagation in interstellar space can be characterized by 3-dimensional isotropic diffusion with a constant mean free path λ. We take into account escape from the disk by introducing a characteristic time, $\tau_e = 3d^2/(2\lambda c)$, where $d \approx 500$ pc is the distance in the plane along which cosmic rays are conveyed to the surface of the disk by the random walk of the field lines[3], and escape from the galaxy. We also take into account nuclear disintegration by introducing a destruction lifetime, $\tau_d = (cn_H\sigma_d)^{-1}$, where n_H is the hydrogen density in interstellar space and σ_d is the destruction cross section. We have used $n_H = 1\ cm^{-3}$, and $\sigma_d = 30$ mb for protons, and $\sigma_d = 50\ A^{2/3}$ mb for the heavier nuclei. We limit our discussion to relativistic nuclei, so that ionization losses are negligible.

The energy density at the earth resulting from a supernova at a distance r_i and of age t_i, in the form of a particular nuclear component whose destruction lifetime is τ_d, is given by

$$w_i = W_i \exp[-3r_i^2/(4\lambda c t_i)][(4\pi/3)\lambda c t_i]^{-3/2} \exp[-t_i(\tau_d^{-1}+\tau_e^{-1})] \quad (1)$$

where W_i is the total energy released by the supernova in particles of the kind considered. We assume that the values of W_i are the same for all supernovae and we denote the total cosmic ray energy from one supernova by W_{SN}.

The energy density, lifetime and anisotropy of one nuclear component from all supernovae ($i = 1,\ldots,N$) are given by

$$w = \sum w_i \quad (2)$$

$$t = w^{-1} \sum_i t_i w_i \quad (3)$$

$$\delta = 3/2\, w^{-1}\left[\left(\sum w_i x_i/ct_i\right)^2 + \left(\sum w_i y_i/ct_i\right)^2 + \left(\sum w_i z_i/ct_i\right)^2\right]^{1/2} \quad (4)$$

where x_i, y_i, z_i are the coordinates (with the earth at the origin) of the i-th supernova. We have numerically evaluated equations (1) through (4) for H^1, O^{16}, Fe^{56}, and U^{238} nuclei and for a range of the scattering mean free path λ. Using a statistical treatment which we shall describe in detail in a subsequent paper, we have evaluated the medians, m, and the 1σ and 2σ levels of all quantities of interest. These levels for the proton anisotropy are plotted in Figure 1 as a function of λ. As can be seen, the upper limit[1] of 2×10^{-4}, can be best understood in terms of a low scattering mean free path, since, if $\lambda = 1$ pc corresponding to $\tau_e \approx 10^6$ years (for d = 500 pc), the probability is very low (<.02) that $\delta < 2 \times 10^{-4}$. In view of the fact that the anisotropy measurements only give an upper limit, it is most probable that $\lambda \lesssim 0.1$ pc. This value is about an order of magnitude smaller than that deduced from studies[5] of wave-particle interactions in interstellar space. According to Wentzel (private communication), however, these studies did not include the turbulence produced by low-energy cosmic rays, which, together with other uncertainties in the properties of the interstellar medium, could substantially decrease the effective mean free path.

The ratio of the light nuclei ($3 \le Z \le 5$) to the medium nuclei ($6 \le Z \le 8$) at relativistic energies is plotted as a function of λ in Figure 2. This ratio was computed by assuming that the former are produced by the fragmentation of the latter in interstellar space. The fragmentation cross section that we have used was 72 mb[6] and the destruction cross sections for the medium and light groups were 185 mb and 130 mb[6] respectively. The measured L/M ratio[7] at relativistic energies is about 0.25 ± 0.05. From Figure 2, we see that within the 1σ levels of the present model, this ratio is consistent with $10^{-2} \lesssim \lambda \lesssim 10^{-1}$pc.

Also shown in Figure 2 are the medians and the 1σ levels of the lifetimes of the H^1, O^{16} and U^{238} nuclei, together with the escape lifetime, τ_e. At large λ's the lifetimes are dominated by escape and are the same for all nuclei. At lower values of λ ($\lesssim 1$ pc) nuclear disintegrations become important and for differing nuclei yield different lifetimes all of which are less than τ_e. In the limit of very low λ's ($\lesssim 10^{-4}$), where all cosmic rays come from a single nearby source, the lifetimes of the various nuclei again approach a common value equal to the age of the source. The L/M ratio measures the medium group lifetime which, in the range 10^{-2}pc $\lesssim \lambda \lesssim 10^{-1}$pc is determined principally by nuclear destruction rather than escape. Moreover, for these values of λ, even the proton lifetime is dominated by inelastic processes and is much less than the escape lifetime. For the same reason, the lifetimes of the very high energy electrons ($\gtrsim$20 BeV) would be dominated by radiation losses and not by escape. The statistical properties of the present model have important effects on the electron spectrum at these energies and must be included in any interpretation of the electron measurements in terms of the spectra of these particles at the sources. In principle, the cosmic ray lifetimes could also be directly determined from the measured abundances of certain radioactive nuclei such as Be^{10}[8] and Mn^{53}[9]. At the present time, however, the measurements are not accurate enough for a positive determination.

The proton energy density at the earth, the composition ratios and the number of significant sources are shown in Table 1. Although we have considered all supernovae in the galaxy, because of the combined effects of nuclear disintegrations, escape, and propagation, only a relatively small number of sources contribute a major fraction of the cosmic ray flux at the earth at any one time. For protons, the number of significant sources M is a maximum at $\lambda \sim 3 \times 10^{-2}$, and for heavier nuclei, M is a maximum at higher values of λ. The maximum number of sources obviously leads to minimum fluctuations in the cosmic ray flux. For both high and low values of λ the ages and the number of significant sources approach a common limit for all nuclear components. At large λ's the cosmic rays are produced by a young and distant source, whereas at small λ's by an old and close one. In the intermediate range, the number of significant sources for different nuclei, like the lifetimes, are different and decrease with increasing nuclear mass. It is this property of the source distribution that is required to account not only for the apparent discrepancy between isotropy and lifetime but also for the interactions and abundances of the heavy nuclei.

A lower limit can also be placed on λ ($\gtrsim 10^{-2}$pc) since for $\lambda < 10^{-2}$pc M_O, M_{Fe}, and Mu would approach 1, and the path length distribution for relativistic O, Fe and U nuclei become the same, which is inconsistent with observed fragmentation ratios of products of medium and iron group nuclei[10], as well as the detection of nuclei with $Z \gtrsim 92$[11].

We suggest, therefore, that the diffusion mean free path is probably confined to the range $10^{-2}\text{pc} \lesssim \lambda \lesssim 10^{-1}\text{pc}$. The upper limit is determined by the observed isotropy of the cosmic rays; the lower limit is determined by the requirement that different nuclei have different path length distributions, and the range is consistent with the observed L/M ratio.

For the range 10^{-2}pc $\lesssim \lambda \lesssim 10^{-1}$pc the cosmic ray energy per supernova, W_{SN}, required to account for the observed energy density at the earth of 10^{-12} erg cm^{-3}, is between .3 to 1 x 10^{50} erg. This energy is well within the range of the estimated [12] cosmic-ray output of supernovae.

For the same range of λ, the fluctuations in the O/H, Fe/H and U/H ratios become large and make a comparison between the observed chemical composition of the cosmic rays and source compositions, based on models of nucleosynthesis, extremely difficult. The reason for this is our lack of knowledge of the distances and ages of the cosmic ray sources, although it has recently been suggested[13] that the pulsars may provide this information.

Nonetheless, the fluctuations shown in Table 1 directly reflect the expected spatial and temporal variations. At $\lambda = 3 \times 10^{-2}$pc, the $\pm$20% fluctuations in w_H give a measure of the time averaged variations and are consistent with cosmic-ray produced activities in meteorites[14]. Shorter term variations on the other hand would be larger and could be detectable in the Be^{10} activities in deep-sea sediments[15, 16]. Much larger time averaged fluctuations, however, are expected for the heavier nuclei. Fe could vary by about 100% and the variation in U may be several times larger. Much more extensive measurements, possibly based on the analysis of very heavy particle tracks in lunar samples (P.B. Price, private communication) are required to further investigate these potential fluctuations.

1. H. Elliot, T. Thambyahpillai, and P.S. Peacock, Imperial College of Science and Technology, preprint (1970).
2. E.N. Parker, Space Science Reviews, 9, 651 (1969).
3. F.C. Jones, NASA preprint X-641-69-401 (to be published.

4. J.R. Jokipii and E.N. Parker, Astrophys. J., 155, 799 (1969).

5. D.G. Wentzel, Astrophys. J., 156, 303 (1969).

6. F. Yiou, Ann. Phys., 3, 169 (1968).

7. C.E. Fichtel and D.V. Reames, Phys. Rev., 175, 1564 (1968).

8. M.M. Shapiro, R. Silverberg and C.H. Tsao, in Proceedings of the Eleventh International Conference on Cosmic Rays, Budapest, Hungary 1969 (to be published), OG88.

9. D.V. Reames, NASA preprint X-611-70-5 (to be published).

10. M.M. Shapiro and R.S. Silverberg, in Proceedings of the Eleventh International Conference on Cosmic Rays, Budapest, Hungary 1969 (to be published) OG87.

11. P.H. Fowler, preprint (1969).

12. S. Colgate and R.H. White, Astrophys. J., 143, 626 (1966).

13. R.E. Lingenfelter, Nature, 224, 1182 (1969).

14. D. Lal, in Proceedings of the Ninth International Conference on Cosmic Rays, 1, 81 (1965).

15. B. Peters, Z. fur Physik, 148, 93 (1957).

16. B.S. Amin, D.P. Kharkar and D. Lal, Deep Sea Res., 13, 805 (1966).

Figure 1. Cosmic ray anisotropy as a function of the diffusion mean free path λ. The probability that for a given λ the anisotropy is less than the value corresponding to the $2\sigma+$, $1\sigma+$, m, $1\sigma-$, and $2\sigma-$ levels is 0.98, 0.83, 0.5, 0.17, and 0.02, respectively.

Figure 2. Cosmic ray lifetimes and the ratio of light nuclei to medium nuclei as a function of λ.

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Table 1. The proton energy density and the oxygen-, iron- and uranium-to-hydrogen ratios. The quantities M represent the smallest number of sources required to account for at least 50% of the energy densities at the earth. The center values represent medians and the error bars correspond to the 1 σ levels.

λ(pc)	w_H $\left(\frac{10^{-62} W_{SN}}{\mathrm{erg\ cm}}\right)$	O^{16}/H^1	Fe^{56}/H^1	U^{238}/H^1	M_H	M_O	M_{Fe}	M_U
10^{-4}	10^{+17}_{-6}	$(6^{+40}_{-5})\times 10^{-2}$	$(8^{+300}_{-8})\times10^{-4}$	$(3.6^{+5000}_{-3.6})\times10^{-8}$	2^{+0}_{-1}	1^{+0}_{-0}	1^{+0}_{-0}	1^{+0}_{-0}
10^{-3}	$7^{+5}_{-2.5}$	$(6^{+20}_{-4})\times 10^{-2}$	$(9^{+80}_{-8})\times10^{-3}$	$(1^{+100}_{-1})\times10^{-4}$	4^{+3}_{-2}	1^{+1}_{-0}	1^{+0}_{-0}	1^{+0}_{-0}
10^{-2}	$3^{+0.6}_{-0.3}$	$(1.5^{+1.5}_{-0.5})\times10^{-1}$	$(5^{+7}_{-2})\times10^{-2}$	$(6^{+40}_{-5})\times10^{-3}$	28^{+9}_{-20}	2^{+2}_{-1}	2^{+1}_{-1}	1^{+0}_{-0}
3×10^{-2}	$1.7^{+0.4}_{-0.3}$	$(2.5^{+1}_{-0.7})\times10^{-1}$	$(9^{+10}_{-2})\times10^{-2}$	$(2^{+8}_{-1})\times10^{-2}$	36^{+18}_{-19}	4^{+2}_{-2}	2^{+1}_{-1}	1^{+1}_{-1}
10^{-1}	$0.7^{+0.2}_{-0.1}$	$(4^{+1}_{-1})\times10^{-1}$	$(2^{+1}_{-0.7})\times10^{-1}$	$(8^{+10}_{-4})\times10^{-2}$	25^{+13}_{-19}	5^{+4}_{-3}	3^{+1}_{-2}	2^{+1}_{-1}
1	$(7^{+3}_{-2})\times10^{-2}$	$(8^{+0.3}_{-0.3})\times10^{-1}$	$(6^{+1}_{-0.6})\times10^{-1}$	$(4^{+1}_{-1})\times10^{-1}$	8^{+5}_{-5}	6^{+4}_{-4}	4^{+3}_{-2}	3^{+2}_{-2}
10	$(5^{+5}_{-2})\times10^{-2}$	$(9.5^{+0.5}_{-0.4})\times10^{-1}$	$(9^{+4}_{-3})\times10^{-2}$	$(8^{+1}_{-1})\times10^{-1}$	2^{+2}_{-1}	2^{+2}_{-1}	2^{+2}_{-1}	2^{+2}_{-2}

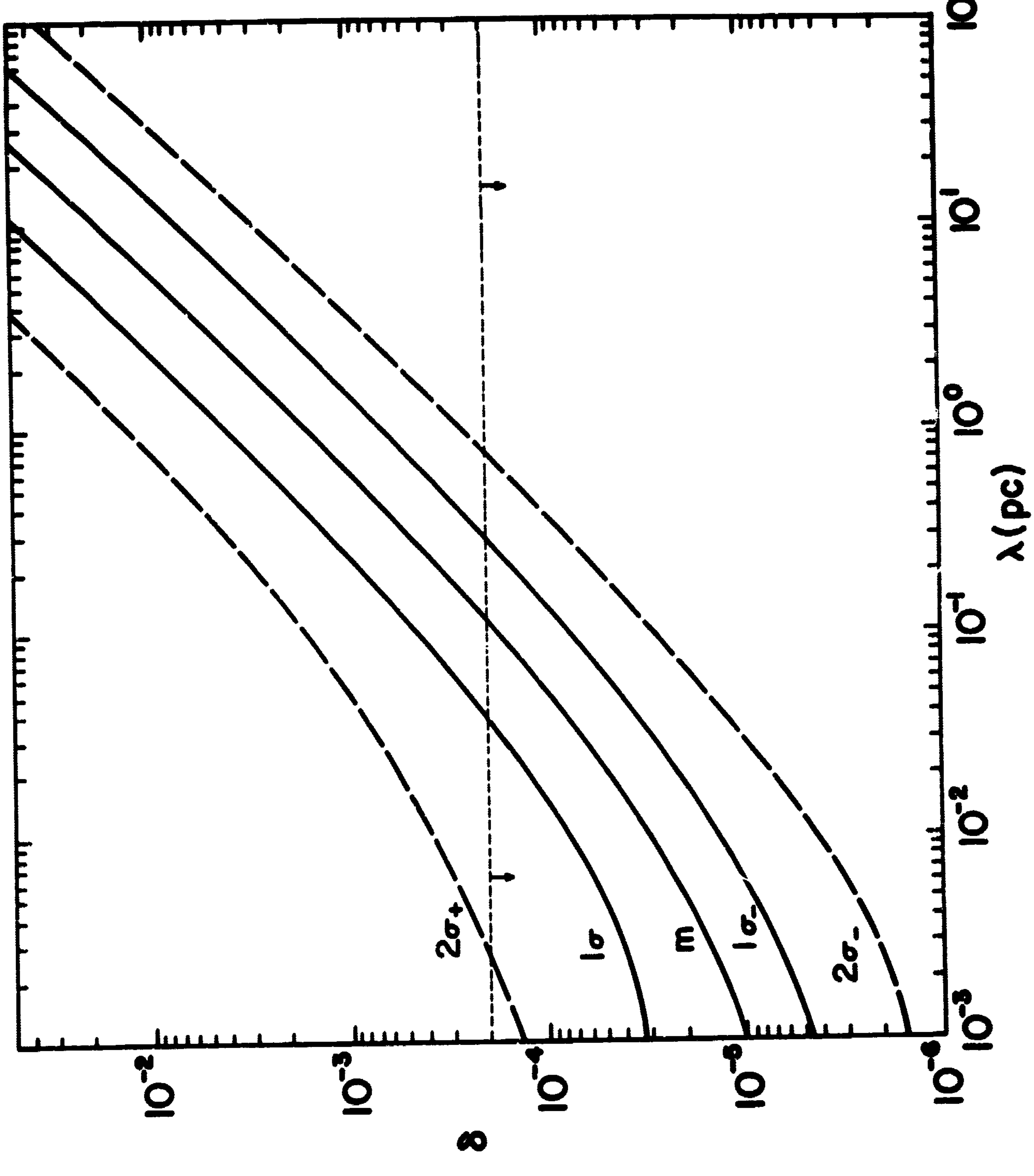

FIGURE 1

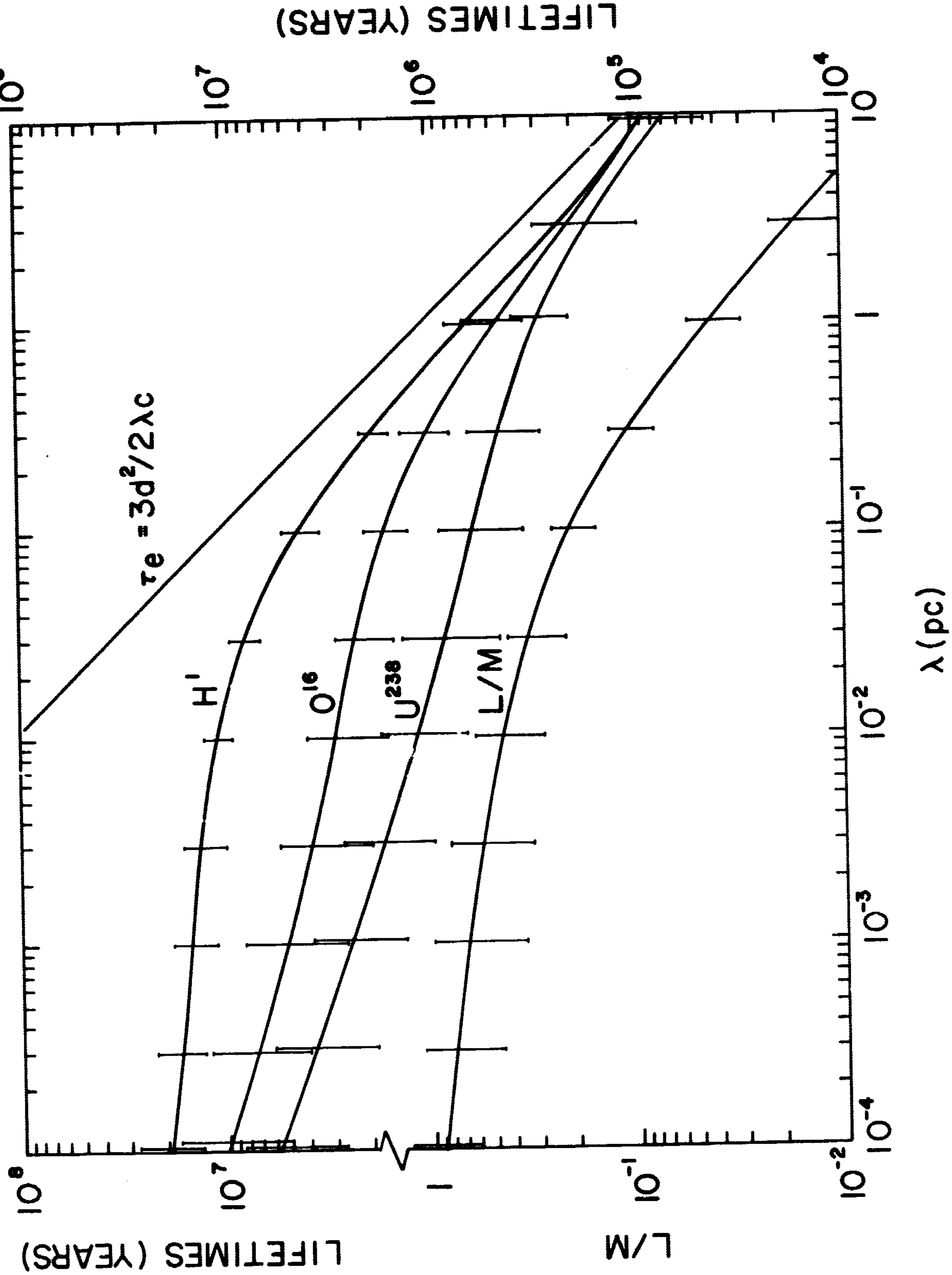
LIFETIMES (YEARS)
L/M
λ (pc)
τe = 3d²/2λc
H¹
O¹⁶
U²³⁸
L/M
10⁸
10⁷
10⁶
10⁵
10⁴
10⁻¹
1
10⁻²
10⁻³
10⁻⁴
10

FIGURE 2