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# INDUCED PRESSURE AND TEMPERATURE GRADIENTS IN A PLASMA AS A RESULT OF WEAK ELECTRIC FIELDS

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16. Abstract

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# INDUCED PRESSURE AND TEMPERATURE GRADIENTS IN A PLASMA AS A RESULT OF WEAK ELECTRIC FIELDS

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## SUMMARY

Computations are presented of the induced pressure and temperature gradients resulting from the application of weak electric fields to plasmas. Two methods are employed to solve an approximate model of the corresponding electron Boltzmann equation: (1) the direct approach consisting of an exact solution through quadratic terms in the electric field, and (2) Everett's technique for closing out the macroscopic equations of change with Grad's 13-moment velocity distribution function. Comparisons between the two sets of results indicate substantial differences for most interparticle interaction potentials, especially with regard to the induced temperature gradient; hence, the Grad 13-moment approximation may not accurately describe some of the fundamental aspects of plasma transport phenomena.

## INTRODUCTION

A general assumption of many authors (see ref. 1 for listing) over the past 12 years is that Grad's 13-moment velocity distribution function accurately describes nonequilibrium plasmas when it is used to close out the macroscopic equations of change. Since, however, the Grad approximation is specifically an expansion (ref. 1) in the eigenfunctions of Boltzmann's binary elastic collision operator for Maxwellian molecules (that is, inverse fourth-power interparticle interaction potentials), a rapid convergence rate is not guaranteed for more general potentials; consequently, additional justification is required for most plasmas of practical interest. Previous studies (refs. 2 and 3) of the convergence properties of Grad-like calculations have shown that although the residual heat flux, the entropy density, the collisional production rate of entropy, and second-order contributions to the electron pressure tensor are predicted with reasonable accuracy (within 26 percent) for spatially homogeneous systems, third-order contributions to the electron current density and the phase angles by which the current density and the heat flux lag an applied oscillating electric field are off by factors up to 4 in the limits of very hard or very soft particles.

The purpose of the present research is to extend the aforementioned convergence studies to spatially inhomogeneous plasmas. In particular, the radial pressure and temperature gradients induced in a plasma by the application of an axial electric field are computed by using two methods to solve an approximate model of the corresponding electron Boltzmann equation. These methods are (1) the direct approach consisting of an exact solution through quadratic terms in the electric field, and (2) Everett's technique (ref. 4) for closing out the macroscopic equations of change with Grad's 13-moment velocity distribution function. A wide range of effective interparticle interaction potentials is employed in both sets of calculations with the following results: Whereas significant differences in the pressure gradients occur only for particles which are much harder or much softer than the Maxwellian variety, the Everett-like predictions of the temperature profiles are very poor even for Maxwellian molecules. Hence, a feature which was never observed in the previous studies appears in the present investigation.

## SYMBOLS

| | |
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| $A$ | ratio of collision integrals |
| $b$ | impact parameter |
| $\vec{c}_e$ | electron particle velocity relative to laboratory frame of reference |
| $C_1,C_2,C_3$ | transport coefficients |
| $e$ | magnitude of electron charge |
| $\vec{E}$ | applied electric field |
| $f_e$ | electron velocity distribution function |
| $f_e^{(o)}$ | Maxwellian contribution to electron distribution function |
| $i,j$ | indices |
| $\hat{i},\hat{j},\hat{k}$ | unit vector in x-, y-, and z-direction, respectively, of Cartesian coordinate system |
| $\vec{j}$ | electron current density |

| | |
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| $k$ | Boltzmann's constant |
| $m_e$ | electron particle mass |
| $n_e$ | electron number density |
| $n_e^o$ | spatially homogeneous contribution to electron number density |
| $n_h$ | number density of heavy particles |
| $p_e$ | electron partial pressure relative to laboratory frame of reference |
| $p_e^o$ | spatially homogeneous contribution to electron partial pressure |
| $p'_e$ | electron partial pressure relative to electron frame of reference |
| $\overset{o}{\overleftrightarrow{P}}_e$ | traceless electron pressure tensor relative to laboratory frame of reference |
| $\overset{o}{\overleftrightarrow{P}}{}'_e$ | traceless electron pressure tensor relative to electron frame of reference |
| $\vec{q}$ | heat flux |
| $\vec{Q}$ | dimensionless residual heat flux |
| $r$ | radial distance |
| $R_{ij}$ | integral defined by equation (12) |
| $t$ | time |
| $T_e$ | electron temperature relative to laboratory frame of reference |
| $T_e^o$ | spatially homogeneous contribution to electron temperature |
| $T'_e$ | electron temperature relative to electron frame of reference |
| $\vec{u}$ | reduced electron particle velocity relative to electron frame of reference |

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| $\overleftrightarrow{U}$ | unit tensor |
| $\vec{v}_e$ | electron diffusion velocity |
| x | integration variable |
| x,y,z | Cartesian coordinates |
| X | dimensionless pressure derivative |
| $\vec{X}_e$ | electron body force per unit mass |
| Y | dimensionless temperature derivative |
| $\vec{\beta}_0$ | reduced electron diffusion velocity |
| $\vec{\beta}_1$ | reduced heat flux relative to laboratory frame of reference |
| $\vec{\beta}_1'$ | reduced heat flux relative to electron frame of reference |
| $\vec{\beta}_i$ | general nondimensional flux |
| $\vec{\gamma}$ | reduced electron particle velocity relative to laboratory frame of reference |
| $\epsilon$ | azimuthal angle for collisions |
| $\xi$ | effective interparticle interaction parameter |
| $\sigma$ | electrical conductivity |
| $\tau$ | collision time |
| $\phi_i$ | ith-order electron perturbation function |
| $\chi$ | scattering or deflection angle |

Notation:

Primed quantities in collision integrals denote conditions after a collision, as opposed to unprimed quantities which denote conditions before a collision.

$(\partial f_e/\partial t)_c$ collisional time derivative of $f_e$

$\langle\ \rangle$ average over velocity space

## METHODS OF ANALYSIS

The approximate model of the electron Boltzmann equation employed in the present research for both the exact and the Everett-type solutions is one in which Meador's simplified collision model (ref. 5) replaces the more precise collision integrals of Chapman and Enskog (ref. 6). The basic assumption of the collision model is that an effective interparticle interaction parameter $\xi$, which appears formally as the exponent in a single type of inverse-power electron—heavy-particle interaction potential, can be chosen semiempirically to include the effects of electron-electron encounters and multiple heavy species. The advantage of the model is the ease with which the corresponding integrodifferential kinetic equation can be reduced to differential form and then solved.

Several examples of the reliability of this approach are given in reference 5 for plasmas involving electron-electron collisions; in addition, the parameter $\xi$ has a precise physical meaning for Lorentz plasmas – that is, slightly ionized gases or full ionization with large ionic charges. Consequently, the present results and comparisons should provide strong implications about the general accuracy of the Grad 13-moment approximation.

The first step in the exact method of solution is the assumption that the applied electric field $\vec{E}$ is sufficiently weak to permit a power series expansion of the electron distribution function in the form

$$f_e = f_e^{(o)}\left(1 + \sum_i \phi_i\right) \tag{1}$$

where $f_e^{(o)}$ is the Maxwellian contribution and $\phi_i$ is proportional to the ith power of E. This expression is substituted into the approximate Boltzmann equation and the coefficients of like powers of E are equated to obtain a sequence of differential relations, each of which is solved exactly for the corresponding $\phi_i$. If solutions are obtained in this manner through $\phi_n$, the resulting electron distribution function is called nth order and is said to be exact through terms containing $E^n$. Only second-order functions are considered in the present research.

An important feature of the exact method is that some members of the aforementioned sequence of differential relations will contain terms which do not appear in the analogous perturbation equations of Chapman and Enskog (ref. 6); in particular, $\phi_i$ may occur in the ith-order equation in terms other than those derived from the collision integrals. Some precedence for this character is found in reference 6 in connection with magnetic forces, the inference now being that more general causes may exist. In any case, the present equations never contain fewer contributions than those of Chapman and Enskog and thus represent a generalization of that technique.

Unlike the exact nth-order method, which gives the complete contribution to $f_e$ through $E^n$ terms, the Everett technique only partially describes the nth-order distribution function because the value of each term is affected by the number of moments employed. An unquestioned application of the Grad 13-moment approximation thus contains the implicit assumption that the contributions to $E^k$ terms $(k \leq n)$ from the higher moments are negligible. The validity of this assumption is investigated in the present research by a direct comparison of the induced pressure and temperature gradients computed by the two methods. Since the same collision model and the same power series expansion of $f_e$ are used in both sets of calculations, the exact results are obviously the correct standards in these comparisons.

A number of additional assumptions are made in the present research to facilitate the exact and Everett solutions to the approximate kinetic equations. These assumptions are reflected in the following plasma conditions: the heavy particles are infinitely massive and at rest relative to the laboratory, the ions are distributed in such a manner as to yield zero local charge densities, the applied and induced magnetic fields are zero (the latter in strict violation of Maxwell's equations, but consistent with a nonrelativistic treatment), all spatial inhomogeneities are induced by the application of a constant electric field along the axis of a cylindrical geometry, viscosity is neglected so that the axial electron diffusion velocity does not vary radially (through second order), and the system is independent of time.

## THE EXACT FIRST-ORDER SOLUTION

The derivations of plasma transport properties are usually based in some manner upon the electron Boltzmann equation (ref. 6)

$$\frac{\partial f_e}{\partial t} + \left(\frac{2kT_e^o}{m_e}\right)^{1/2} \vec{\gamma} \cdot \nabla f_e + \left(\frac{m_e}{2kT_e^o}\right)^{1/2} \vec{X}_e \cdot \frac{\partial f_e}{\partial \vec{\gamma}} = \left(\frac{\partial f_e}{\partial t}\right)_c \tag{2}$$

where $\vec{\gamma}$ is the reduced electron particle velocity defined by

$$\vec{\gamma} = \left(\frac{m_e}{2kT_e^o}\right)^{1/2} \vec{c}_e \tag{3}$$

and $T_e^o$ is the spatially homogeneous contribution to the electron temperature.

If the applied electric field and the electron velocity distribution function are expressed through second order as

$$\vec{E} = \hat{k}E \tag{4}$$

and

$$f_e = f_e^{(o)}\left(1 + \phi_1 + \phi_2\right) = n_e^o\left(\frac{m_e}{2\pi kT_e^o}\right)^{3/2} e^{-\gamma^2}\left(1 + \phi_1 + \phi_2\right) \tag{5}$$

the following simplification of equation (2) is obtained from the application of Meador's collision model (ref. 5) and the special assumptions given in the preceding section:

$$\left(\frac{2kT_e^o}{m_e}\right)^{1/2}\left[\gamma_x \frac{\partial\left(\phi_1 + \phi_2\right)}{\partial x} + \gamma_y \frac{\partial\left(\phi_1 + \phi_2\right)}{\partial y}\right] + \frac{eE}{m_e}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\left[2\left(1 + \phi_1 + \phi_2\right)\gamma_z - \frac{\partial\left(\phi_1 + \phi_2\right)}{\partial \gamma_z}\right]$$

$$= \frac{1}{f_e^{(o)}}\left(\frac{\partial f_e}{\partial t}\right)_c = -n_h\gamma\left(\frac{2kT_e^o}{m_e}\right)^{1/2}\int\left(\phi_1 + \phi_2 - \phi_1' - \phi_2'\right) b\, db\, d\epsilon \tag{6}$$

The only aspect of the collision model which appears explicitly at this stage in equation (6) is the assumption that the effective collision partners for electrons can be regarded as heavy scattering centers corresponding to a single species of heavy particles with number density $n_h$. Corrections for non-Lorentz gases, especially those involving electron-electron encounters, are discussed subsequently.

Since the function $\phi_1$ is proportional to $E$ by the definition in the preceding section, so that the $m_e\vec{c}_e$ moment of $f_e^{(o)}\phi_1$ must give the linear form of Ohm's law (that is, the proportionality of the axial electron diffusion velocity to $E$), the assumption that the axial electron diffusion velocity does not vary radially through second order prevents $\phi_1$ from being $x$ and $y$ dependent in equation (6). The elimination of these derivatives and the third-order contributions $E\phi_2$ from equation (6) thus yields the complete second-order equation

$$\left(\frac{2kT_e^o}{m_e}\right)^{1/2}\left(\gamma_x \frac{\partial\phi_2}{\partial x} + \gamma_y \frac{\partial\phi_2}{\partial y}\right) + \frac{eE}{m_e}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\left[2\left(1 + \phi_1\right)\gamma_z - \frac{\partial\phi_1}{\partial\gamma_z}\right]$$

$$= -n_h\gamma\left(\frac{2kT_e^o}{m_e}\right)^{1/2}\int\left(\phi_1 + \phi_2 - \phi_1' - \phi_2'\right)b\ db\ d\epsilon \qquad (7)$$

the first-order form of which is

$$\frac{2eE}{m_e}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\gamma_z = -n_h\gamma\left(\frac{2kT_e^o}{m_e}\right)^{1/2}\int\left(\phi_1 - \phi_1'\right)b\ db\ d\epsilon \qquad (8)$$

Equation (8) can be solved by the assumption that $\phi_1$ has the form $\gamma_z$ multiplied by a function of $\gamma$. The result is

$$\phi_1 = -\frac{2e\tau R_{04}E}{m_e R_{13}}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\gamma^{(4/\xi)-1}\gamma_z \qquad (9)$$

if the $T_e^o$-dependent collision time

$$\tau = \frac{m_e\sigma}{e^2 n_e^o} \qquad (10)$$

and the collision integral (ref. 5)

$$\int\left(\gamma_z - \gamma_z'\right)b\ db\ d\epsilon = \frac{R_{13}}{n_h R_{04}\tau}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\gamma^{-4/\xi}\gamma_z \qquad (11)$$

are employed. The $R_{ij}$ integrals are defined by

$$R_{ij} = \int_0^\infty x^{(4i/\xi)+j}e^{-x^2}\,dx \qquad (12)$$

As explained in reference 5 and mentioned in the preceding section, the effective interaction parameter $\xi$ in equations (9), (11), and (12) formally appears as the exponent in a single type of inverse-power electron—heavy-particle interaction potential but can be chosen semiempirically to make the corresponding collision model reliable for some real plasmas with electron-electron encounters and multiple heavy species included.

Only for true Lorentz plasmas (slight ionization or fully ionized gases with large ionic charges), however, can $\xi$ have physical meaning.

The first-order electron current density $\vec{j}$ and the dimensionless residual heat flux $\vec{Q}$ are found from the velocity moments of equation (9) to satisfy

$$\vec{j} = -en_e^o \vec{v}_e = -en_e^o \left(\frac{2kT_e^o}{m_e}\right)^{1/2} \vec{\beta}_0 = \sigma \vec{E} \tag{13}$$

and

$$\vec{Q} = \vec{\beta}_1 - \frac{5}{2}\vec{\beta}_0 = -\frac{2(\xi + 1)e\tau}{\xi m_e}\left(\frac{m_e}{2kT_e^o}\right)^{1/2} \vec{E} - \frac{5}{2}\vec{\beta}_0 = \frac{2(\xi + 1)}{\xi}\vec{\beta}_0 - \frac{5}{2}\vec{\beta}_0 = -\hat{k}\frac{\xi - 4}{2\xi}\beta_0 \tag{14}$$

if the general nondimensional flux $\vec{\beta}_i$ is defined by

$$\vec{\beta}_i = \left\langle \gamma^{2i}\vec{\gamma}\right\rangle = \frac{1}{n_e}\int \gamma^{2i}\vec{\gamma} f_e \, d\vec{c}_e \tag{15}$$

The use of the spatially homogeneous contribution to the electron number density in equation (13) is dictated by the restriction to first order at this stage.

## THE EXACT SECOND-ORDER SOLUTION

Substitution into equation (7) of equation (9) and its velocity derivative

$$\frac{\partial \phi_1}{\partial \gamma_z} = -\frac{2e\tau R_{04}E}{m_e R_{13}\xi}\left(\frac{m_e}{2kT_e^o}\right)^{1/2} \gamma^{(4/\xi)-3}\left[\xi\gamma^2 - (\xi - 4)\gamma_z^2\right] \tag{16}$$

gives

$$\left(\frac{2kT_e^o}{m_e}\right)^{1/2}\left(\gamma_x \frac{\partial \phi_2}{\partial x} + \gamma_y \frac{\partial \phi_2}{\partial y}\right) + \frac{e^2\tau R_{04}E^2}{m_e k T_e^o R_{13}\xi}\gamma^{(4/\xi)-3}\left[\xi\gamma^2 - \left(2\xi\gamma^2 + \xi - 4\right)\gamma_z^2\right]$$
$$= -n_h\gamma\left(\frac{2kT_e^o}{m_e}\right)^{1/2}\int\left(\phi_2 - \phi_2'\right) b \, db \, d\epsilon \tag{17}$$

the solution of which provides the exact (through second order) expressions for the induced pressure and temperature gradients. Equation (17) differs from the usual second-order formulation of Chapman and Enskog in the explicit appearance of $\phi_2$ on the left side.

By using the collision integral

$$\int\left(\gamma_z^2 - \gamma_z'^2\right)b\ db\ d\epsilon = \frac{AR_{13}}{2n_h R_{04}\tau}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\gamma^{-4/\xi}\left(3\gamma_z^2 - \gamma^2\right) \tag{18}$$

derived in reference 2 and the definition

$$A = \frac{\int_0^\infty\left(1 - \cos^2\chi\right)b\ db}{\int_0^\infty\left(1 - \cos\chi\right)b\ db} \tag{19}$$

one easily confirms that the correct solution of equation (17) is

$$\phi_2 = \frac{2e^2\tau^2R_{04}^2E^2}{3Am_e kT_e^o R_{13}^2\xi}\gamma^{(8/\xi)-4}\left(\xi\gamma^2 - 2\right)\left(3\gamma_z^2 - \gamma^2\right) + \frac{R_{04}\sigma E^2}{p_e^o R_{13}\xi}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\gamma^{(4/\xi)-3}\left(\xi\gamma^2 - \xi - 2\right)\left(x\gamma_x + y\gamma_y\right) - \frac{r^2}{4\xi}\left(\frac{eE}{kT_e^o}\right)^2\gamma^{-2}\left(\xi\gamma^2 - \xi - 2\right) \tag{20}$$

where $r^2$ is $x^2 + y^2$.

In a manner analogous to the development in equation (14), the complete second-order formulation of the dimensionless residual heat flux is found from equations (5), (14), (15), and (20) to be

$$\vec{Q} = \vec{\beta}_1 - \frac{5}{2}\vec{\beta}_0 = \hat{i}\frac{x}{2p_e^o}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\vec{E}\cdot\vec{j} + \hat{j}\frac{y}{2p_e^o}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\vec{E}\cdot\vec{j} - \hat{k}\frac{\xi - 4}{2\xi}\beta_0 \tag{21}$$

Moreover, since the transverse components of the dimensionless electron diffusion velocity $\vec{\beta}_0$ are zero from equations (5), (9), (15), and (20), the ordinary heat flux $\vec{q}$ is obtained from equation (21) and the equation of state

$$p_e^o = n_e^o kT_e^o$$

as

$$\vec{q} = \frac{n_e^o m_e}{2}\left(\frac{2kT_e^o}{m_e}\right)^{3/2}\vec{\beta}_1 = \hat{i}\frac{x}{2}\vec{E}\cdot\vec{j} + \hat{j}\frac{y}{2}\vec{E}\cdot\vec{j} + \hat{k}\frac{2(\xi + 1)}{\xi}p_e^o v_e \tag{22}$$

This expression automatically satisfies the energy balance relation

$$\nabla \cdot \vec{q} = \vec{E} \cdot \vec{j} \tag{23}$$

The induced number-density and pressure derivatives are likewise derived from equation (20) by the following integrations:

$$\frac{\partial n_e}{\partial x} = \frac{\partial}{\partial x} \int f_e^{(o)} \phi_2 \, d\vec{c}_e = \frac{(\xi + 4) n_e^o x}{2\xi} \left(\frac{eE}{kT_e^o}\right)^2 \tag{24}$$

and

$$\frac{\partial p_e}{\partial x} = \frac{2p_e^o}{3\pi^{3/2}} \frac{\partial}{\partial x} \int e^{-\gamma^2} \gamma^2 \phi_2 \, d\vec{\gamma} = -\frac{(\xi - 4) p_e^o x}{6\xi} \left(\frac{eE}{kT_e^o}\right)^2 \tag{25}$$

whereupon

$$\frac{\partial T_e}{\partial x} = \frac{1}{n_e^o k} \frac{\partial p_e}{\partial x} - \frac{T_e^o}{n_e^o} \frac{\partial n_e}{\partial x} = -\frac{2(\xi + 2) T_e^o x}{3\xi} \left(\frac{eE}{kT_e^o}\right)^2 \tag{26}$$

The spatially homogeneous contributions to $n_e$ and $T_e$ are used in the coefficients of the derivatives in equation (26) to avoid terms containing $E^4$.

Similar expressions (with x replaced by y) hold for the y derivatives, so that equations (25) and (26) comprise the exact second-order induced pressure and temperature gradients mentioned in the Introduction as a purpose of the present research.

## GRAD 13-MOMENT APPROXIMATION

The final effort of the present research is the determination of the induced pressure and temperature gradients by using the Grad 13-moment approximation and Meador's collision model (ref. 5) to close out the macroscopic equations of change. Numerical comparisons with equations (25) and (26) should yield valuable insight into the applicability of this technique to nonequilibrium plasmas because a completely common framework (i.e., basic assumptions and expansions in powers of E) is provided for both methods.

If the reduced electron particle velocity relative to the electron frame of reference is defined by

$$\vec{u} = \left(\frac{m_e}{2kT_e}\right)^{1/2} \left(\vec{c}_e - \vec{v}_e\right) \tag{27}$$

Everett's formulation of the Grad 13-moment velocity distribution function through second-order terms can be written as

$$f_{e,\mathrm{Grad}} = n_e\left(\frac{m_e}{2\pi kT_e}\right)^{3/2} e^{-u^2}\left\{1 - \frac{\beta_0^2}{3}\left(2u^2 - 3\right) + \frac{4}{5}\left(u^2 - \frac{5}{2}\right)\left(\vec{\beta}_1 - \frac{5}{2}\vec{\beta}_0\right)\cdot\vec{u} + \frac{1}{p_e^o}\left[\overset{o}{\overleftrightarrow{P}}_e + \frac{2p_e^o\beta_0^2}{3}\left(\overleftrightarrow{U} - 3\hat{k}\hat{k}\right)\right] : \vec{u}\vec{u}\right\} \tag{28}$$

for the present problem (see the appendix). The term $-\frac{\beta_0^2}{3}\left(2u^2 - 3\right)$ within the braces in equation (28) and also the form of the tensor product follow from the conversions of the electron temperature and the traceless pressure tensor from the electron frame of reference to the laboratory frame of reference.

With $\overset{o}{\overleftrightarrow{P}}_e$ anticipated to be spatially homogeneous through the second-order level, which is subsequently confirmed in equation (35), the pertinent macroscopic equations of change are obtained as follows from the application of equation (28) to the $m_e c_{e,x}$ and $m_e c_e^2 c_{e,x}$ moments of equation (2):

$$\frac{\partial p_e}{\partial x} = m_e\int c_{e,x}\left(\frac{\partial f_e}{\partial t}\right)_c d\vec{c}_e = -n_h m_e\int c_e\left(c_{e,x} - c_{e,x}'\right)f_e b\, db\, d\epsilon\, d\vec{c}_e$$

$$= -\frac{4R_{13}n_e^o m_e\beta_{1,x}}{5\pi^{3/2}R_{04}\tau}\left(\frac{2kT_e^o}{m_e}\right)^{1/2}\int e^{-\gamma^2}\gamma_x^2\gamma^{1-(4/\xi)}\left(\gamma^2 - \frac{5}{2}\right)d\vec{\gamma}$$

$$= -\frac{(\xi - 4)R_{13}R_{-1,5}n_e^o m_e\beta_{1,x}}{5\xi R_{04}^2\tau}\left(\frac{2kT_e^o}{m_e}\right)^{1/2} \tag{29}$$

and

$$\frac{\partial}{\partial x}\left(n_e m_e\left\langle c_e^2 c_{e,x}^2\right\rangle\right) = m_e\int c_e^2 c_{e,x}\left(\frac{\partial f_e}{\partial t}\right)_c d\vec{c}_e = -n_h m_e\int c_e^3\left(c_{e,x} - c_{e,x}'\right)f_e b\, db\, d\epsilon\, d\vec{c}_e$$

$$= -\frac{4R_{13}n_e^o m_e\beta_{1,x}}{5\pi^{3/2}R_{04}\tau}\left(\frac{2kT_e^o}{m_e}\right)^{3/2}\int e^{-\gamma^2}\gamma_x^2\gamma^{3-(4/\xi)}\left(\gamma^2 - \frac{5}{2}\right)d\vec{\gamma}$$

$$= -\frac{(3\xi - 2)(3\xi - 4)R_{13}R_{-1,5}n_e^o m_e\beta_{1,x}}{5\xi^2 R_{04}^2\tau}\left(\frac{2kT_e^o}{m_e}\right)^{3/2} \tag{30}$$

The successive simplifications of the collision integrals in these expressions correspond to the use of Meador's collision model (ref. 5), the concepts of inverse collisions and microscopic reversibility (ref. 6), and equations (11) and (28). In addition, $\vec{u}$ is replaced with $\vec{\gamma}$ in the application of $f_{e,Grad}$ to the collision integrals, the distinction being unimportant here because it represents third-order contributions in view of the factor $\beta_{1,x}$.

The Grad value of $\beta_{1,x}$ is compelled by the solution of equation (23) to satisfy

$$\beta_{1,x} = \frac{x}{2p_e^o}\left(\frac{m_e}{2kT_e^o}\right)^{1/2} \vec{E} \cdot \vec{j} = \frac{kT_e^o \tau_{Grad} x}{2m_e}\left(\frac{m_e}{2kT_e^o}\right)^{1/2}\left(\frac{eE}{kT_e^o}\right)^2 \tag{31}$$

the second equality of which follows from equations (10) and (13) with the use of

$$\tau_{Grad} = \frac{R_{04}^2\left(13\xi^2 - 16\xi + 16\right)}{4R_{13}R_{-1,5}(3\xi - 2)\xi}\tau \tag{32}$$

from reference 3 to preserve a consistent treatment of the 13-moment approximation. Accordingly, equations (29) and (30), respectively, become

$$\frac{\partial p_e}{\partial x} = -\frac{(\xi - 4)\left(13\xi^2 - 16\xi + 16\right)p_e^o x}{40\xi^2(3\xi - 2)}\left(\frac{eE}{kT_e^o}\right)^2 \tag{33}$$

and

$$\frac{\partial}{\partial x}\left(n_e m_e\left\langle c_e^2 c_{e,x}^2\right\rangle\right) = -\frac{(3\xi - 4)\left(13\xi^2 - 16\xi + 16\right)n_e^o e^2 E^2 x}{20\xi^3 m_e} \tag{34}$$

A most important feature of equation (34) is that $\left\langle c_e^2 c_{e,x}^2\right\rangle$ is not one of the 13 moments; consequently, a distribution function must be employed to express this quantity in terms of the more familiar variables. The error thus introduced by the Grad 13-moment approximation is expected to be quite large by analogy with the results for higher moments calculated in reference 2, and it may even overshadow the errors originating from the use of the Grad approximation in the collision integrals. Only the latter discrepancies appear in studies of the 13 moments in spatially homogeneous plasmas or in the investigation of the induced pressure gradient in the present research.

Additional emphasis on the error inherent in the closing out of $\left\langle c_e^2 c_{e,x}^2\right\rangle$ with the Grad 13-moment approximation is provided by a study of equations (9) and (20) in the special case of Maxwellian particles ($\xi = 4$). In particular, since the traceless electron pressure tensor is calculated from equation (20) to be

$$\overset{o}{\overleftrightarrow{P}}_e = n_e m_e\left(\left\langle\vec{c}_e\vec{c}_e\right\rangle - \frac{1}{3}\left\langle c_e^2\right\rangle\overleftrightarrow{U}\right) = -\left(\hat{i}\hat{i} + \hat{j}\hat{j} - 2\hat{k}\hat{k}\right)\frac{4(3\xi + 4)R_{04}R_{22}p_e^o\beta_0^2}{15A\xi R_{13}^2} \tag{35}$$

the substitution of this expression and equations (13), (21), (24), and (26) into the exact second-order solution of equations (5), (9), and (20) yields

$$f_{e,exact}(\xi = 4) = f_{e,Grad} + f_e^{(o)}\beta_0^2\gamma^{-2}\left[\frac{4}{15A}\left(\gamma^2 - \frac{5}{2}\right)\left(3\gamma_z^2 - \gamma^2\right) - \frac{1}{5\beta_0^2}\left(4\gamma^4 - 20\gamma^2 + 15\right)\left(\beta_{1,x}\gamma_x\right.\right.$$
$$\left.\left. + \beta_{1,y}\gamma_y\right) + \frac{m_e r^2}{4kT_e^o\tau^2}\left(4\gamma^4 - 12\gamma^2 + 3\right)\right] \tag{36}$$

The function $f_{e,Grad}$ is given in equation (28).

Although the terms in equation (36) which are additional to the Grad 13-moment function contribute neither to the direct calculation of the 13 moments nor to the closing out of the collision integrals in the corresponding macroscopic equations of change, the third term in the bracket does contribute to the closing out of the quantity $\partial\left(n_e m_e\left\langle c_e^2 c_{e,x}^2\right\rangle\right)/\partial x$ in equation (34). More specifically, the use of equation (36) on the left side of equation (34) yields the following second-order result with the aid of equations (10) and (13):

$$\begin{aligned}
\frac{\partial}{\partial x}\left(n_e m_e\left\langle c_e^2 c_{e,x}^2\right\rangle\right) &= \frac{\partial}{\partial x}\left[m_e\int c_e^2 c_{e,x}^2 f_{e,Grad}\, d\vec{c}_e + \frac{p_e^o\beta_0^2 r^2}{\pi^{3/2}\tau^2}\int e^{-\gamma^2}\gamma_x^2\left(4\gamma^4 - 12\gamma^2 + 3\right)\, d\vec{\gamma}\right] \\
&= \frac{\partial}{\partial x}\left[\frac{5kp_eT_e}{m_e} + \frac{e^2 n_e^o E^2 r^2}{2\pi^{3/2}m_e}\int e^{-\gamma^2}\gamma_x^2\left(4\gamma^4 - 12\gamma^2 + 3\right)\, d\vec{\gamma}\right] \\
&= \frac{5kT_e^o}{m_e}\left[n_e^o k\frac{\partial T_e}{\partial x} + \frac{\partial p_e}{\partial x} + \frac{p_e^o x}{5\pi^{3/2}}\left(\frac{eE}{kT_e^o}\right)^2\int e^{-\gamma^2}\gamma_x^2\left(4\gamma^4 - 12\gamma^2 + 3\right)\, d\vec{\gamma}\right] \\
&= \frac{5kT_e^o}{m_e}\left[n_e^o k\frac{\partial T_e}{\partial x} + \frac{\partial p_e}{\partial x} + \frac{4p_e^o x}{5}\left(\frac{eE}{kT_e^o}\right)^2\right]
\end{aligned} \tag{37}$$

Since the pressure gradient is zero from equation (33) when $\xi = 4$, the combination of equation (37) with equation (34) yields the following complete $\xi = 4$ temperature derivative:

$$\frac{\partial T_e}{\partial x} = -\left(\frac{1}{5} + \frac{4}{5}\right) T_e^o x \left(\frac{eE}{kT_e^o}\right)^2 \tag{38}$$

The first and second numerical fractions on the right side of this expression correspond, respectively, to the contribution from the collision integral (which is the same for both the Grad and the exact distribution functions if $\xi = 4$) and the contribution to $\partial\left(n_e m_e \left\langle c_e^2 c_{e,x}^2 \right\rangle\right)/\partial x$ from the last term in equation (36). In addition, the total value for $\partial T_e/\partial x$ agrees with the exact result in equation (26) when $\xi = 4$; hence, this analysis firmly establishes the importance of one of the terms neglected by the Grad 13-moment approximation.

Another interpretation of the last term in equation (36) is obtained from the following heat flux expression deduced from equations (30) and (37):

$$q_x \propto \frac{\partial}{\partial x}\left(n_e m_e \left\langle c_e^2 c_{e,x}^2 \right\rangle\right) = C_1 \frac{\partial T_e}{\partial x} + C_2 \frac{\partial p_e}{\partial x} + C_3 E^2 x \tag{39}$$

Except for the coefficient $C_3$, which is zero in the 13-moment approximation but is quite significant in the exact solution, equation (39) is the familiar transport relation with transport coefficients $C_1$ and $C_2$. Since the 13-moment values of $q_x$, $C_1$, and $C_2$ are fairly accurate, large errors in the temperature or pressure gradients (or both) must exist in that approximation to compensate for the serious error in $C_3$.

Finally, the use of only the Grad 13-moment distribution function to perform the indicated average on the left side of equation (34) gives

$$n_e^o k \frac{\partial T_e}{\partial x} + \frac{\partial p_e}{\partial x} = -\frac{(3\xi - 4)\left(13\xi^2 - 16\xi + 16\right) p_e^o x}{100\xi^3}\left(\frac{eE}{kT_e^o}\right)^2 \tag{40}$$

so that

$$\frac{\partial T_e}{\partial x} = -\frac{\left(13\xi^2 - 16\xi + 16\right)^2 T_e^o x}{200\xi^3(3\xi - 2)}\left(\frac{eE}{kT_e^o}\right)^2 \tag{41}$$

with the aid of equation (33). Equations (33) and (41) thus comprise the Grad 13-moment values of the induced pressure and temperature gradients mentioned in the Introduction as a purpose of the present research.

## COMPARISON OF RESULTS

A convenient representation of the preceding results is through the following definitions of nondimensional pressure and temperature derivatives:

$$X = -\frac{1}{rp_e^o}\left(\frac{kT_e^o}{eE}\right)^2\frac{dp_e}{dr} \tag{42}$$

and

$$Y = -\frac{1}{rT_e^o}\left(\frac{kT_e^o}{eE}\right)^2\frac{dT_e}{dr} \tag{43}$$

Accordingly, from equations (25), (26), (33), and (41),

$$X_{exact} = \frac{\xi - 4}{6\xi} \tag{44}$$

$$X_{Grad} = \frac{(\xi - 4)\left(13\xi^2 - 16\xi + 16\right)}{40\xi^2(3\xi - 2)} \tag{45}$$

$$Y_{exact} = \frac{2(\xi + 2)}{3\xi} \tag{46}$$

and

$$Y_{Grad} = \frac{\left(13\xi^2 - 16\xi + 16\right)^2}{200\xi^3(3\xi - 2)} \tag{47}$$

Numerical calculations appropriate to equations (44) to (47) are presented in table I for a variety of effective interparticle interaction potentials ranging from the fully ionized Lorentz plasma ($\xi = 1$) to a gas of rigid spheres ($\xi = \infty$). The expectations discussed in the paragraph following equation (34) are clearly realized: Although the 13-moment values for the induced temperature gradient are quite poor over the entire range of $\xi$ (Maxwellian particles included), the errors in the induced pressure gradient are far more tolerable except for very soft or very hard force laws.

## A PROPOSED USE OF THE EXACT SOLUTION

The comparisons given in the present research are between the exact (through $E^2$ terms) and 13-moment solutions of an approximate Boltzmann equation. Although the collision model employed is exact for Lorentz plasmas and has been shown to be valid (ref. 5) in the first-order theory of real fully ionized gases (electron-electron collisions included), its applicability to second- and higher-order treatments of non-Lorentz plasmas has not been demonstrated. Consequently, the present exact results should not be regarded as the final answer in the non-Lorentz case but rather as an indicator of the magnitudes and sources of errors inherent in the 13-moment approximation.

More specifically, the importance in the exact solution (given by eq. (36)) of the terms which are additional to the 13-moment distribution function requires that a minimum of four new moments must be added to the original 13 moments to complete the moment representation of equation (36) and thereby derive a minimally acceptable Grad-like approximation for the present problem. This approximation can then be employed in the usual Everett manner to close out the macroscopic equations of change corresponding to the rigorous collision integrals instead of the present collision model. Since the exact solution of the approximate Boltzmann equation serves in this method only to establish the minimum number and types of moments to be incorporated in the Grad-like approximation, the limitations on the original collision model should have a trivial impact on the final results of the ensuing closing-out procedure.

The additional terms in equation (36) can, of course, be represented by more than four moments, in which case the proposed closing-out procedure will give improved results because the increased number of macroscopic equations of change will offer the rigorous collision integrals a greater opportunity to exert an influence.

Another important advantage of this proposed use of exact solutions concerns the forms of the additional terms in equation (36). If moments were added in the usual Grad fashion (employing Hermite polynomials) to improve the 13-moment approximation for the present problem, the convergence would be extremely slow because of the difficulty in describing by this method the $\gamma^{-2}$ dependence of one of the terms. The proposed technique introduces the $\gamma^{-2}$ dependence directly, so that a very rapid convergence is expected. Further improvements in the proposed Grad-like approximation can be obtained by using a more realistic $\xi$ in the exact solution, as opposed to equation (36) which is based on Maxwellian particles. This alteration may, in fact, be necessary if some of the important $\gamma$ dependencies in the exact solution are functions of the interaction potential.

## CONCLUDING REMARKS

Calculations through second order in the electric field have indicated that large errors can occur when the Grad 13-moment velocity distribution function is used to close out the macroscopic equations of change for a steady-state plasma. Much of the difficulty in computing the induced temperature gradient, the values of which are especially poor, can be traced to the failure of the 13-moment approximation to predict accurately an important higher moment appearing in the macroscopic equation for the heat flux. The pressure gradient, on the other hand, does not involve the closing out of such higher moments, so that the 13-moment predictions of this property are in much better agreement with the exact second-order values.

The present results, and those of previous research, suggest that some of the past studies employing the Grad 13-moment approximation perhaps should be reevaluated.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 19, 1970.

## APPENDIX

## TRANSFORMATION OF THE GRAD 13-MOMENT VELOCITY DISTRIBUTION FUNCTION

The purpose of this appendix is to derive equation (28) from the original Everett formulation (ref. 4) of the Grad 13-moment approximation, which is written as

$$f_{e,Grad} = n_e\left(\frac{m_e}{2\pi kT_e'}\right)^{3/2}\exp\left[-\frac{m_e}{2kT_e'}\left(\vec{c}_e - \vec{v}_e\right)^2\right]\left\{1 + \frac{4}{5}\left(\frac{m_e}{2kT_e'}\right)^{1/2}\left[\frac{m_e}{2kT_e'}\left(\vec{c}_e - \vec{v}_e\right)^2 - \frac{5}{2}\right]\vec{\beta}_1' \cdot \left(\vec{c}_e - \vec{v}_e\right) + \frac{1}{p_e'}\frac{m_e}{2kT_e'}\overset{o}{\overleftrightarrow{P}}{}_e' : \left(\vec{c}_e - \vec{v}_e\right)\left(\vec{c}_e - \vec{v}_e\right)\right\} \tag{A1}$$

Note that the primed quantities are measured relative to the electron frame of reference and that $n_e$ is the same in both the electron and the laboratory coordinate systems.

The following relations are fundamental to the transformation from equation (A1) to equation (28) if terms higher than second order are neglected and if the differences $n_e - n_e^o$ and $T_e - T_e^o$ are proportional to $E^2$ $\left(\text{or } v_e^2\right)$ as in the present problem:

$$T_e' = \frac{m_e}{3k}\left\langle\left(\vec{c}_e - \vec{v}_e\right)^2\right\rangle = \frac{m_e}{3k}\left(\left\langle c_e^2\right\rangle - v_e^2\right) = T_e - \frac{2T_e^o}{3}\beta_0^2 \approx T_e\left(1 - \frac{2\beta_0^2}{3}\right) \tag{A2}$$

$$p_e' = n_e kT_e' = p_e + n_e k\left(T_e' - T_e\right) \approx p_e - \frac{2p_e^o}{3}\beta_0^2 \tag{A3}$$

$$\vec{\beta}_1' = \left(\frac{m_e}{2kT_e'}\right)^{3/2}\left\langle\left(\vec{c}_e - \vec{v}_e\right)^2\left(\vec{c}_e - \vec{v}_e\right)\right\rangle \approx \left(\frac{m_e}{2kT_e^o}\right)^{3/2}\left[\frac{1}{n_e^o}\int c_e^2\vec{c}_e f_e^{(o)}\left(\phi_1 + \phi_2 + \ldots\right) d\vec{c}_e - 2\left\langle\vec{c}_e\vec{c}_e\right\rangle \cdot \vec{v}_e - \left\langle c_e^2\right\rangle\vec{v}_e\right] = \vec{\beta}_1 - \frac{5}{3}\left(\frac{m_e}{2kT_e^o}\right)^{3/2}\left\langle c_e^2\right\rangle\vec{v}_e = \vec{\beta}_1 - \frac{5}{2}\vec{\beta}_0 \tag{A4}$$

and

$$\overset{o}{\overleftrightarrow{P}}{}_e' = n_e m_e\left[\left\langle\left(\vec{c}_e - \vec{v}_e\right)\left(\vec{c}_e - \vec{v}_e\right)\right\rangle - \frac{1}{3}\left\langle\left(\vec{c}_e - \vec{v}_e\right)^2\right\rangle\overleftrightarrow{U}\right] = n_e m_e\left(\left\langle\vec{c}_e\vec{c}_e\right\rangle - \vec{v}_e\vec{v}_e\right) - p_e'\overleftrightarrow{U}$$
$$\approx \overset{o}{\overleftrightarrow{P}}_e + \left(p_e - p_e'\right)\overleftrightarrow{U} - 2p_e^o\beta_0^2\hat{k}\hat{k} \approx \overset{o}{\overleftrightarrow{P}}_e + \frac{2p_e^o\beta_0^2}{3}\left(\overleftrightarrow{U} - 3\hat{k}\hat{k}\right) \tag{A5}$$

Equation (A5) anticipates the radial electron diffusion velocity to be zero.

If one further anticipates that $\overset{o}{\overleftrightarrow{P}}_e$ is second order, which is confirmed by the exact result of equation (35), equations (A1), (A4), and (A5) combine to yield

$$f_{e,Grad} = n_e\left(\frac{m_e}{2\pi kT_e'}\right)^{3/2}\exp\left[-\frac{m_e}{2kT_e'}\left(\vec{c}_e - \vec{v}_e\right)^2\right]\left\{1 + \frac{4}{5}\left(u^2 - \frac{5}{2}\right)\left(\vec{\beta}_1 - \frac{5}{2}\vec{\beta}_0\right)\cdot\vec{u}\right.$$
$$\left. + \frac{1}{p_e^o}\left[\overset{o}{\overleftrightarrow{P}}_e + \frac{2p_e^o\beta_0^2}{3}\left(\overleftrightarrow{U} - 3\hat{k}\hat{k}\right)\right] : \vec{u}\vec{u}\right\} \tag{A6}$$

Since $T_e$ differs from $T_e'$ by a second-order term, the introduction of $\vec{u}$ of equation (27) in the non-Maxwellian terms of equation (A6) is consistent with a second-order analysis because the vector and tensor factors are first order or higher.

Equation (28) is immediately obtained from equation (A6) when equation (A2) is employed to rewrite the Maxwellian contribution in the form

$$n_e\left(\frac{m_e}{2\pi kT_e'}\right)^{3/2}\exp\left[-\frac{m_e}{2kT_e'}\left(\vec{c}_e - \vec{v}_e\right)^2\right] \approx n_e\left(\frac{m_e}{2\pi kT_e}\right)^{3/2}e^{-u^2}\left(1 + \beta_0^2\right)\left(1 - \frac{2\beta_0^2}{3}u^2\right)$$
$$\approx n_e\left(\frac{m_e}{2\pi kT_e}\right)^{3/2}e^{-u^2}\left[1 - \frac{\beta_0^2}{3}\left(2u^2 - 3\right)\right] \tag{A7}$$

## REFERENCES


1. Grohs, Gerhard L.: Transport Properties of Partially Ionized Nonequilibrium Gases. 99900-6770-T0-00, TRW Systems Group, Dec. 1968.
2. Meador, Willard E.: A Critical Analysis of the Grad Approximation for Closing Out the Magnetohydrodynamic Equations for Plasmas. NASA TR R-325, 1969.
3. Meador, Willard E.: Third-Order Contributions to Electrical Conduction in Plasmas. NASA TN D-5477, 1969.
4. Everett, Willis L.: Generalized Magnetohydrodynamic Equations for Nonequilibrium Plasma Systems. Rarefied Gas Dynamics Vol. I, J. A. Laurmann, ed., Academic Press, 1963, pp. 1-25.
5. Meador, Willard E.: A Semiempirical Collision Model for Plasmas. NASA TR R-310, 1969.
6. Chapman, Sydney; and Cowling, T. G.: The Mathematical Theory of NonUniform Gases. Second ed., Cambridge Univ. Press, 1952. (Reprinted 1960.)


TABLE I.- NONDIMENSIONAL INDUCED PRESSURE AND TEMPERATURE GRADIENTS

| $\xi$ | X | | Y | |
|---|---|---|---|---|
| | 13 moment | Exact | 13 moment | Exact |
| 1 | -0.975 | -0.500 | 0.845 | 2.000 |
| 2 | -.113 | -.167 | .203 | 1.333 |
| 4 | .000 | .000 | .200 | 1.000 |
| $\infty$ | .108 | .167 | .282 | .667 |

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