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HEAT TRANSFER IN A CHANNEL WITH RANDOM VARIATIONS IN FLUID VELOCITY

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by Morris Perlmutter
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TECHNICAL PAPER proposed for presentation at
Fourth International Heat Transfer Conference
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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Morris Perlmutter
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Abstract

Heat transfer results have been obtained for randomly varying laminar flow in a heated parallel plate channel. Two types of heat transfer boundary conditions are considered: (a) The channel walls are at constant temperature; or (b) a specified uniform heat flux is transferred at the walls. The heat transfer behavior is obtained along the entire length of the channel. The temperature distributions are two-dimensional but a slug flow velocity profile is assumed. The results show significantly higher mean wall temperatures and, for short channels, lower wall heat fluxes when compared to the steady flow cases.

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INTRODUCTION

Although flows in internal flow systems are commonly assumed to be steady, velocity fluctuations are always present to some extent. These flow fluctuations may arise due to flow instabilities or from unsteadiness in the pumping system. It is of interest to determine the effect of the random velocity fluctuations on such things as wall heat transfer and wall temperatures which will also fluctuate and may be significantly different from the steady flow case.

Practically no analytical papers have dealt with the problem of heat transfer in a parallel plate channel with randomly fluctuating velocities. In Ref. 1 an analysis was carried out for a sinusoidally varying laminar slug flow in a channel. For the constant wall temperature boundary condition and constant properties the time average wall heat transfer was found not to change appreciably due to the oscillating flow velocity. These results were extended to the case for the two-dimensional velocity profile in Ref. 2 and similar results were found.

In Ref. 3 the heat conduction through a solid with randomly varying internal heat generation or randomly varying wall heat fluxes and wall temperature was analyzed. Since the heat conduction problem is linear a straightforward analysis yielded analytical solutions.

The present analysis treats the problem of heat transfer to a fluid with randomly varying velocity. Since this problem is nonlinear, it is difficult to find analytical solutions as in the linear case. The present method of analysis depends partially on model sampling or the Monte Carlo technique to obtain the solution to the present problem. The randomly varying velocity-time history is generated by randomly choosing velocity amplitudes and times between changes in velocity amplitude from the appropriate distributions. The velocity function so generated can be used in obtaining the wall heat transfer or temperature as a function of time using standard analytical methods. By repeating this process so as to obtain ensemble averages the various characteristics of interest of the system can be obtained.

Random Velocity

The fluid velocity is assumed constant across the channel but varying in time. Nondimensionalizing the velocity by the mean flow we can write

$$U(\theta) = 1 + \gamma U'(\theta) \tag{1}$$

The U' is the fluctuating component of velocity with a mean value of zero. The value of U' is constant for a time increment θ_δ and then U' changes to a new value. The amplitude factor γ is a parameter of the problem and determines the magnitude of the velocity fluctuation.

The times between changes of the velocity amplitude (θ_δ) are assumed independent events randomly distributed along the time axis with an average time of occurrence θ_a. The Poisson distribution (Refs. 4 and 5)

$$f(\theta,K) = \frac{e^{-\theta/\theta_a}(\theta/\theta_a)^K}{K!} \tag{2}$$

gives the distribution of K random events occurring independently during time θ for a given average time per event of θ_a. The occurrence of one event at the end of time θ_δ is given by first no event occurring during time θ_δ and then one event occurring during the incremental time $d\theta$. The probability of this occurrence is given by the product of the two probabilities

$$f(\theta_\delta) = f(\theta_\delta,0)f(\Delta\theta,1) = \frac{1}{\theta_a} e^{-\theta_\delta/\theta_a} \tag{3}$$

The distribution of θ_δ, the time between velocity changes, is thus given by $f(\theta_\delta)$. To pick a time between change in velocity θ_δ from the distribution $f(\theta_\delta)$ we proceed as shown in Ref. 4. Set R, the random number picked from a uniform distribution between zero and 1, equal to the cumulative distribution of $f(\theta)$.

$$R = \int_0^{\theta_\delta} f(\theta)d\theta = 1 - e^{-\theta_\delta/\theta_a} \tag{4}$$

Thus by having the computer generate R using a uniform random number generator we can solve for the random variable θ_δ, the time between changes in velocity from Eq. (4a). We can rewrite the above, using R = 1 - R, as

$$\theta_\varepsilon = -\theta_a \ln R \tag{4b}$$

After finding θ_δ the time to the change in value of U' a new value of U' must be randomly chosen from $f(U')$, the distribution of amplitudes of the fluctuating velocity component U'. In the present analysis we assume that U' takes the values of +1 or -1 with equal probability. The new value of U' is thus chosen at the end of time increment θ_δ by generating a new value of R. If R is less than 0.5, then U' is taken at +1 otherwise it is -1. In this manner an entire history of U can be generated (see Fig. 2). Using a similar procedure different distributions of amplitude of U' can be used as well as different distribution of time to change θ_δ to represent different randomly fluctuating velocities.

The value of γ taken in the present analysis is always less than 1 so that the fluid velocity U is always positive. Since the average value of

U' is zero, $\langle U' \rangle = 1 \times (1/2) - 1 \times (1/2) = 0$, then the mean value of U is given by $\langle U \rangle = 1 + \gamma \langle U' \rangle = 1$. The standard deviation of the velocity is given by

$$\sigma_U = (\langle U^2 \rangle - \langle U \rangle^2)^{1/2} = \gamma \tag{5}$$

Since

$$\langle U^2 \rangle = \langle (1 + \gamma U')^2 \rangle = 1 + \gamma^2 \langle U'^2 \rangle + 2\gamma \langle U' \rangle = 1 + \gamma^2$$

The final term of interest is the normalized autocovariance of the velocity

$$C_U(\Delta) = \frac{\langle U(\theta)U(\theta + \Delta) \rangle - \langle U(\theta) \rangle^2}{\langle U^2(\theta) \rangle - \langle U(\theta) \rangle^2} \tag{6}$$

where Δ is a time increment added to θ.

If we let $f(\theta,0)$ be the probability that an event has not occurred in time θ in Eq. (2), then since there are only two possibilities, either an event has not occurred or has occurred in time θ, we can write

$$\langle U(\theta)U(\theta + \Delta) \rangle = \langle U^2 \rangle f(\Delta,0) + \langle U \rangle^2 [1 - f(\Delta,0)] \tag{7}$$

Then $C_U(\Delta)$, the normalized autocovariance, becomes

$$C_U(\theta) = \frac{e^{-\theta/\theta_a}(\langle U^2 \rangle - \langle U \rangle^2)}{\sigma_U^2} = e^{-\theta/\theta_a} \tag{8}$$

It can be seen that in the present case the mean, $\langle U \rangle = 1$, the mean deviation, $\sigma_U = \gamma$, and the normalized autocovariance, $C_U = e^{-\theta/\theta_a}$, contain all the parameters needed to describe the fluctuating velocity (Eqs. (1) and (3)). Knowing these quantities allow the generation of randomly fluctuating velocities by model sampling that have the correct statistical behavior as was done in the present case.

Analysis for Uniform Wall Temperature

The unsteady energy equation for flow in a parallel plate channel is (Fig. 1)

$$\frac{\partial t}{\partial \tau} + u(\tau) \frac{\partial t}{\partial x} = \alpha \frac{\partial^2 t}{\partial y^2} \tag{9a}$$

Viscous dissipation and axial conduction have been neglected and radial convection is assumed zero. Constant properties are assumed. The slug flow approximation has been made for the velocity. This equation for the uniform wall temperature case can be written in dimensionless form as

$$\frac{\partial T}{\partial \theta} + U(\theta) \frac{\partial T}{\partial X} = \frac{\partial^2 T}{\partial Y^2} \tag{9b}$$

using as the dimensionless temperature $T = (t - t_o)/(t_w - t_o)$ (see Fig. 1);

dimensionless axial distance from entrance of heated channel $X = (x\alpha)/(u_m a^2)$ and dimensionless time $\theta = (\tau\alpha/a^2)$; where u_m represents the mean flow in the channel.

The boundary conditions on Eq. (9) are

$T = 0$ at $X = 0$ for all θ and Y, constant entering temperature (10a)

$T = 1$ at $Y = \pm 1$ for all X and θ, constant wall temperature (10b)

$\frac{\partial T}{\partial Y} = 0$ at $Y = 0$ for all X and θ, symmetry (10c)

The method of solution follows the procedure given in Ref. 1. The solution for steady laminar slug flow in a channel with a constant wall temperature is given by

$$T_s = 1 - 2 \sum_{n=0}^{\infty} \frac{(-1)^n}{E_n} e^{-E_n^2 X} \cos E_n Y \tag{11}$$

where E_n are the eigenvalues $[n + (1/2)]\pi$.

For the case of randomly fluctuating velocity we assume a solution, as in Ref. 1, of the form

$$T = 1 - 2 \sum_{n=0}^{\infty} \frac{(-1)^n}{E_n} G_n(X\theta) \cos E_n Y \tag{12}$$

This form of the solution automatically satisfies the boundary conditions (10b) and (10c). Substitution of Eq. (12) into Eq. (9) gives

$$\frac{\partial G_n}{\partial \theta} + U \frac{\partial G_n}{\partial X} = -E_n^2 G_n \tag{13}$$

Using the method of characteristics we write the auxiliary equations as

$$d\theta = \frac{dX}{U} = \frac{dG_n}{-E_n^2 G_n} \tag{14}$$

The first two terms in Eq. (14) can be integrated to yield a set of curves or characteristic lines in the $x - \theta$ plane.

$$\int_{\theta_0}^{\theta} U(\theta) d\theta = \int_0^X dX = X \tag{15}$$

The characteristic lines of interest begin at $X = 0$ since it is at this point the boundary condition (10a) must be satisfied. Equating the first and last terms of Eq. (14) yields a relationship for G_n as a function of θ along the characteristic line.

$$\int_{\theta_0}^{\theta} d\theta = -\frac{1}{E_n^2} \int_1^{G_n} \frac{dG_n}{G_n} \tag{16}$$

$$\theta - \theta_o = - \frac{1}{E_n^2} \ln G_n \tag{17}$$

The boundary condition $G_n = 1$ at $\theta = \theta_o$ fixes the value of G_n at $X = 0$ since $\theta = \theta_o$ corresponds to the beginning of a characteristic line at $X = 0$. This fulfills boundary condition (10a). Substitution of Eq. (17) into Eq. (12) gives

$$T(\theta) = 1 - 2 \sum_{n=0}^{\infty} \frac{(-1)^n}{E_n} e^{-E_n^2(\theta-\theta_o)} \cos E_n Y \tag{18}$$

In a situation where the wall temperature is specified it is desirable to compute the heat transferred from the wall to the fluid. This can be found from the temperature distribution by applying Fourier's law

$$q = K\left(\frac{\partial t}{\partial y}\right)_{y=a} \tag{19}$$

Differentiating Eq. (18) and evaluating it at $Y = 1$ yields the wall heat flux

$$Q(\theta) = \frac{qa}{k(t_w - t_o)} = 2 \sum_{n=0}^{\infty} e^{-E_n^2(\theta-\theta_o)} \tag{20}$$

To solve for $Q(\theta)$ in Eq. (20) we have to find $\theta - \theta_o$ from Eq. (15). We can generate a random velocity $U(\theta)$ as discussed previously. This $U(\theta)$ can be used to give values of X as a function of θ where

$$X(\theta) = \int_0^{\theta} U(\theta) d\theta$$

This is plotted in Fig. 3. We would like to obtain values of the wall heat flux Q at various values of time, $Q(\theta)$, $Q(\theta + \Delta)$, . . . $Q(\theta + n\Delta)$ for a fixed position along the heated channel X. For $Q(\theta)$ the value of $(\theta - \theta_o)$ needed in Eq. (20) is obtained as shown in Fig. 3. A value of θ is picked. This defines a point along the X coordinate. Moving back along the X coordinate a distance X defines point θ_o as shown graphically in Fig. 3. To find $Q(\theta + \Delta)$ we need the value of $[(\theta + \Delta) - \theta_{o,\Delta}]$ given by the following equation

$$\int_{\theta_{o,\Delta}}^{\theta+\Delta} U(\theta) d\theta = X \tag{21}$$

This is found as before by finding a point along X in Fig. 3 corresponding to $\theta + \Delta$ moving back a distance X along X then finding $\theta_{o,\Delta}$. In this manner we can obtain ${}^1Q(\theta)$, ${}^1Q(\theta + \Delta)$, ${}^1Q(\theta + 2\Delta)$ . . . based on the randomly fluctuating velocity ${}^1U(\theta)$ where the 1 superscript denotes one member of an ensemble. This same procedure is repeated using a newly generated function of $U(\theta)$ for the second member of the ensemble by generating a new random velocity ${}^2U(\theta)$ and finding ${}^2Q(\theta)$, ${}^2Q(\theta + \Delta)$, ... This is done K times and the following ensemble averaged values are obtained; $\overline{Q}(\theta)$, the mean wall heat flux; the standard deviation of the wall heat flux;

$$S_Q = \left\{\left[\overline{Q^2(\theta)}\right] - \left[\overline{Q(\theta)}\right]^2\right\}^{1/2} \tag{22}$$

and the normalized autocovariance of the wall heat flux

$$C_Q(n\Delta) = \frac{\overline{Q(\theta)Q(\theta + n\Delta)} - [\overline{Q(\theta)}]^2}{S_Q^2} \tag{23}$$

These results have been evaluated and will be given later. The value of K was taken as 2000 after comparison of results for a K of 4000 showed negligible change.

Analysis for Uniform Wall Heat Flux Case

For this case the energy equation is the same as given in Eq. (9) except the dimensionless temperature is now $T^* = (t - t_o)k/qa$. The boundary conditions remain the same but Eq. (10c) becomes

$$\frac{\partial T^*}{\partial Y} = \pm 1 \quad \text{at} \quad Y = \pm 1 \quad \text{for all} \quad X \quad \text{and} \quad \theta \tag{24}$$

Following the analysis in Ref. 1, the steady slug flow solution for constant wall heat flux is given by

$$T_s^* = X + \frac{3Y^2 - 1}{6} - 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{(n\pi)^2} e^{-(n\pi)^2 X} \cos n\pi Y \tag{25}$$

For the present analysis we assume a solution of the form

$$T^* = F_o(\theta X) + \frac{3Y^2 - 1}{6} - 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{(n\pi)^2} F_n(\theta X) \cos n\pi Y \tag{26}$$

Substitution into the energy equation gives

$$\frac{\partial F_o}{\partial \theta} + U \frac{\partial F_o}{\partial X} = 1 \tag{27a}$$

$$\frac{\partial F_n}{\partial \theta} + U \frac{\partial F_n}{\partial X} = -(n\pi)^2 F_n \tag{27b}$$

Using the method of characteristics the auxiliary equations are

$$d\theta = \frac{dX}{U} = dF_o \tag{28a}$$

$$d\theta = \frac{dX}{U} = \frac{dF_n}{-(n\pi)^2 F_n} \tag{28b}$$

These can be solved to give

$$F_o = \theta - \theta_o \tag{29a}$$

$$F_n = e^{-(n\pi)^2(\theta - \theta_o)} \tag{29b}$$

where $\theta - \theta_o$ is given by Eq. (14). We can now write the solution for the wall temperature by letting $Y = 1$

$$T_w^* = (\theta - \theta_o) + \frac{1}{3} - 2 \sum_{n=1}^{\infty} \frac{1}{(n\pi)^2} e^{-(n\pi)^2(\theta-\theta_o)} \qquad (30)$$

Then following the procedure described earlier we can then find ${}^1T_w^*(\theta)$, ${}^1T_w^*(\theta + \Delta)$, ${}^1T_w^*(\theta + 2\Delta)$, . . . based on the randomly fluctuating velocity ${}^1U(\theta)$. This procedure is repeated and the following ensemble averages are obtained. The mean wall temperature, $\overline{T_w^*}(\theta)$; the standard deviation of the wall temperature,

$$S_T = \left\{\left[\overline{T_w^*(\theta)^2}\right] - \left[\overline{T_w^*}(\theta)\right]^2\right\}^{1/2} \qquad (31)$$

and the normalized autocovariance of the wall temperature

$$C_T(n\Delta) = \frac{\overline{T_w^*(\theta)T_w^*(\theta + n\Delta)} - \left[\overline{T_w^*}(\theta)\right]^2}{S_T^2}$$

These results have been evaluated and will be given later.

Limiting Solutions for Large Times Between Changes in Velocity

For very large values of θ_a, the average time between changes in velocity, the problem reduces to the case of heat transfer in steady flow, half of the time at velocity $U = 1 + \gamma$ and half the time at $U = 1 - \gamma$. This means that the wall temperature and wall heat transfer results will be given by the steady flow results half the time based on velocity $U = 1 + \gamma$ and the other half of the time based on $U = 1 - \gamma$. In Fig. 4 is a plot of the steady wall temperature versus X for the case of uniform wall heat flux. The wall temperature at the steady flow velocity $U = 1 + \gamma$ can be read off this curve at

$$X_{1+\gamma} = \frac{x\alpha}{(1 + \gamma)u_m a^2} = \frac{X}{1 + \gamma} \qquad (33)$$

So that to find the wall temperature at X when the velocity is $U = 1 + \gamma$ read the steady flow wall temperature at $X_{1+\gamma}$ which is equal to X divided by $1 + \gamma$. For the limiting case of $\theta_a \to \infty$ the average wall temperature for the uniform wall heat flux case is now given by

$$\overline{T^*_{w,\infty}} = (T^*_{w,1+\gamma})\left(\frac{1}{2}\right) + (T^*_{w,1-\gamma})\left(\frac{1}{2}\right) = \frac{1}{2}\,(T^*_{w,1+\gamma} + T^*_{w,1-\gamma}) \qquad (34)$$

We can find the limiting case for the fully developed region as follows. In the fully developed region for the steady wall heat flux case the steady wall temperature is given by $T^*_{w,s} = X + (1/3)$. We can then rewrite Eq. (34) for the fully developed regions as

$$\overline{T^*_{w,\infty,d}} = \frac{1}{2}\left[\left(\frac{X}{1 + \gamma} + \frac{1}{3}\right) + \left(\frac{X}{1 - \gamma} + \frac{1}{3}\right)\right] = \frac{X}{1 - \gamma^2} + \frac{1}{3} \qquad (35)$$

We can see from Eq. (35) that in the limiting case of large times between velocity changes in the fully developed region that for zero value of the fluctuating velocity, $\gamma = 0$ the mean wall temperature is the same as the steady state case, however, when the velocity fluctuation approaches the value of the mean flow, $\gamma \to 1$ so that the velocity will be close to zero part of the time then the mean wall temperature becomes very large and approaches infinity. The wall temperature standard deviation for the constant wall heat flux case for the limiting case of very large time between velocity changes is given following Eq. (31) as

$$S_{T,\infty} = \left[(T^*_{w,1+\gamma} - \overline{T^*_{w,\infty}})^2 \frac{1}{2} + (T^*_{w,1-\gamma} - \overline{T^*_{w,\infty}})^2 \frac{1}{2}\right]^{1/2} \tag{36}$$

This reduces to

$$S_{T,\infty} = \frac{T^*_{w,1-\gamma} - T^*_{w,1+\gamma}}{2} \tag{37}$$

We can find the limiting result in the fully developed region which is

$$S_{T,\infty,d} = \frac{x\gamma}{1 - \gamma^2} \tag{38}$$

We can see that the standard deviation of the wall temperature for θ_a very large in the fully developed region goes to zero as the amplitude of the velocity fluctuation γ go to zero. However, as $\gamma \to 1$ then $S_{T,\infty,d} \to \infty$.

A similar type of limiting argument would apply to the wall heat flux Q for the constant wall temperature boundary condition. As in Eq. (34) we can write for the case of very long times between velocity changes, $\theta_a \to \infty$,

$$\overline{Q_\infty} = Q_{1+\gamma} \frac{1}{2} + Q_{1-\gamma} \frac{1}{2} \tag{39}$$

where $Q_{1+\gamma}$ and $Q_{1-\gamma}$ are evaluated from the steady flow result as shown in Fig. 6 for Q at $x_{1+\gamma}$ or $x_{1-\gamma}$ as given by Eq. (33). We can similarly write the standard deviation of the wall heat flux for the limiting case as in Eq. (37)

$$S_{Q,\infty} = \frac{Q_{1+\gamma} - Q_{1-\gamma}}{2}$$

These limiting values are discussed in the section on results.

Results for Uniform Wall Heat Flux Case

The wall temperature results for the uniform wall heat flux case are shown in Fig. 4. The solid line shows the limiting solution of the wall temperature for the steady flow case. Mean wall temperatures are shown for two different amplitudes of flow velocity fluctuations $\gamma = 0.9$ and $\gamma = 0.5$. The γ of 0.5 means the fluctuating component of the velocity has a positive or negative amplitude 50 percent of the mean flow. These mean wall temperature results are considerably above the steady flow values. The larger value of γ giving results further above the steady flow case. The mean wall temperature results are also given for different values of θ_a, the average time between changes in the fluctuations in the velocity. These results indicate that for large values of θ_a the average value of the mean wall temperature fall further above the steady flow result. The limiting result of $\theta_a \to \infty$ are also shown. This limiting solution is the average of the wall temperatures for the steady flow case when the velocities are taken at the maximum value $1 + \gamma$, and the minimum value $1 - \gamma$.

Also shown in Fig. 4 are the standard deviations of the wall temperature S_T.

These results indicate the amplitude of the variation of the instantaneous wall temperature above and below the average wall temperature. The standard deviation is given in Fig. 4 as the difference in values between the open and the solid symbols. The standard deviation approaches zero as the average wall temperature approaches the steady flow value. Very large values of the standard deviation which indicate large fluctuations in amplitude of the wall temperature occur for large values of γ and θ_a as can be seen in the figure.

The wall temperature autocovariance results are shown in Fig. 5. For comparison the autocovariance for the velocity fluctuations are shown as a solid line. The slope of the autocovariance curves are indicative of the average time between changes in the wall temperature, the steeper the slope the shorter the time between changes in wall temperature. In Fig. 5 the slopes are smaller for the unsteady wall temperature than for the unsteady velocity, meaning that the average time between changes in wall temperature are somewhat longer than for the case of the fluid velocity. Also for larger values of X the average time between changes in wall temperature increase as can be deduced from Fig. 5.

Using the mean value, the standard deviation and the autocovariance results for the wall temperature, the wall temperature can be generated as a function of time in a similar manner as the velocity used in the present analysis was generated as discussed earlier.

Results for the Uniform Wall Temperature Case

The resulting mean wall heat flux for the constant wall temperature boundary condition is shown in Fig. 6. The solid line shows the resulting wall heat flux for the steady flow case. Mean wall heat fluxes are shown for two different amplitudes of the velocity fluctuations $\gamma = 0.9$ and $\gamma = 0.5$. The mean wall heat flux is significantly below the steady wall heat flux for heated channel lengths less than about X of 0.6. However, since the fluid must finally reach the temperature of the wall in the case of constant wall temperature boundary condition the mean wall heat flux for values of X greater than 0.6 must be above the steady value. These results indicate that for short heated channels there can be a significant decrease in heat transfer for unsteady flow. The mean wall heat flux results are also shown for different values of θ_a, the average time between changes in the velocity fluctuations. The results indicate that for larger values of γ and larger values of θ_a the mean wall heat flux falls further below the steady flow value for short channels. The limiting results for $\theta_a \to \infty$ are also shown. Also shown in Fig. 6 are the standard deviations of the wall heat flux S_Q. These results are given as the difference between the solid and the open points. The standard deviation becomes smaller as the mean wall heat flux comes closer to the steady flow case, i.e., small values of γ and θ_a.

The wall heat flux autocovariance results are shown in Fig. 7. For comparison the autocovariance for the velocity is shown as a solid line. The slopes of the heat flux results are smaller than for the velocity indicating a larger average time between changes in wall heat flux compared to the average time between changes in the velocity fluctuations. The slope was smaller for larger values of X than for smaller values of X.

Conclusion

Adding an unsteady axial component of velocity with a mean value of zero to slug flow in a heated channel can result in significantly higher mean wall temperature and, for short channels, lower wall heat fluxes. These effects were found to increase as the amplitude of the velocity fluctuations were increased and also as the average time between velocity fluctuations were increased.

The method of analysis consisted of generating a randomly fluctuating velocity as a function of time by picking amplitudes and times to change in amplitudes randomly from the appropriate distributions. These were used in the energy equation which were solved using standard methods of solution. This process was repeated so as to obtain ensemble averages. This method allows complex nonlinear stochastic problems to be readily solved that would be very difficult using more usual analytical procedures. The resulting mean value, standard deviation and autocovariance results for wall temperature and wall heat flux given in the present report can be used as in the case of the randomly fluctuating velocity to generate an approximate time history for the wall temperature and wall heat flux that could be used in the analysis of other problems such as temperature stress in materials.

The present method of analysis can be extended to different forms of the randomly fluctuating velocity.

Several assumptions were made that could have significant effects on the results including: no random fluctuations in velocity normal to the mean flow direction, no axial conduction and the slug flow velocity profile. Further analysis removing these restrictions would be of interest.

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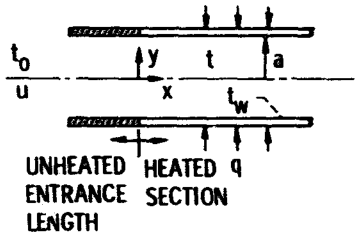

Figure 1. - Parallel plate channel.

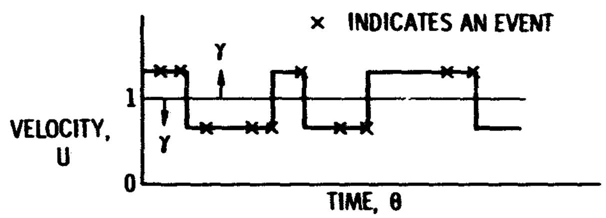

Figure 2. - Fluid velocity.

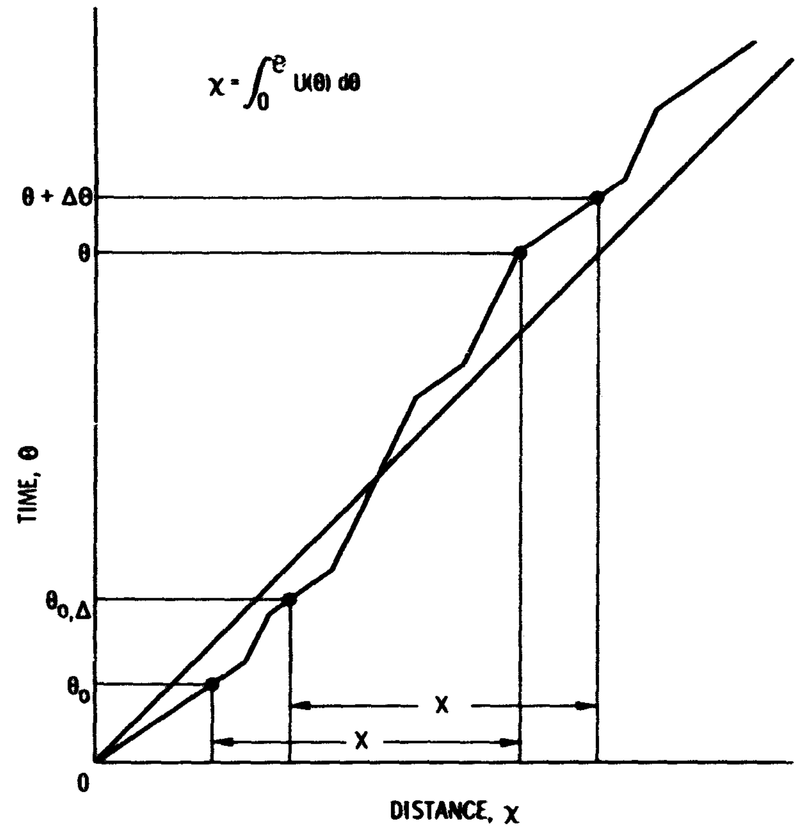

Figure 3. - Typical characteristic line plot.

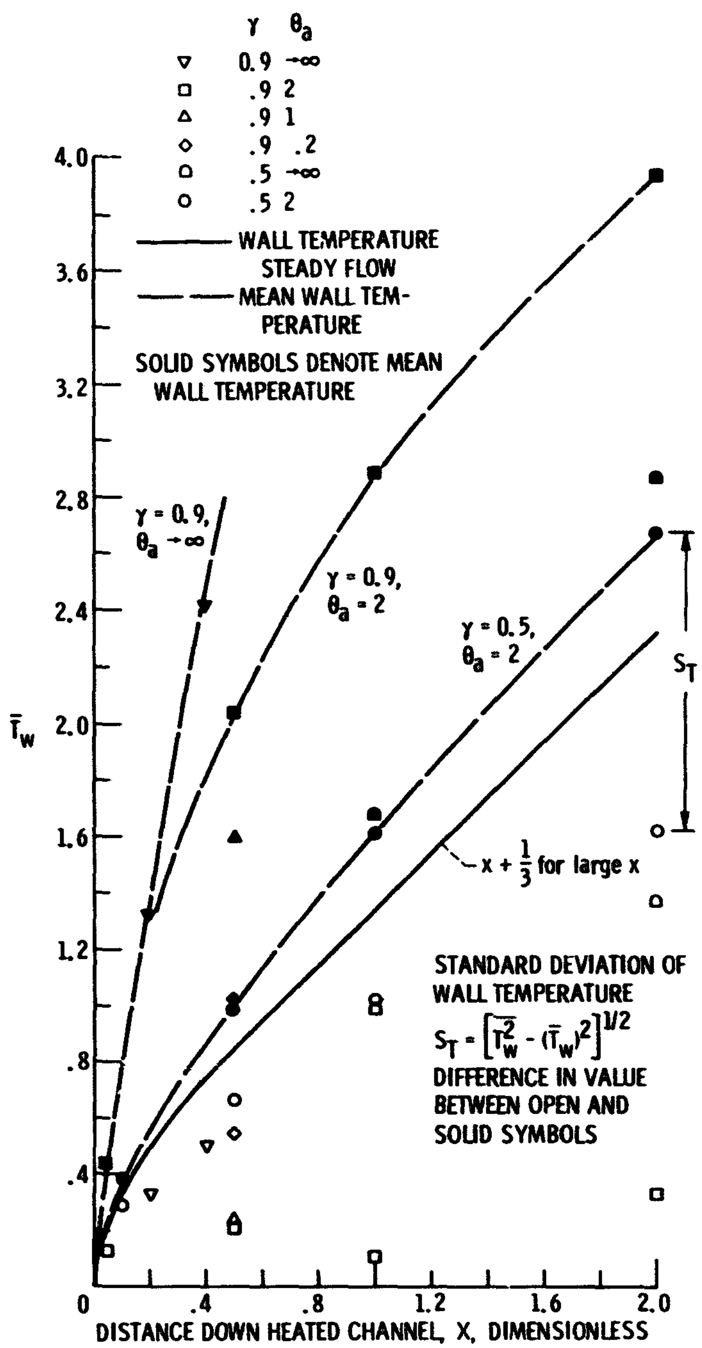

Figure 4. - Wall temperature mean, $\bar{T}_w$; and standard deviation, S_T; results for constant wall heat flux case.

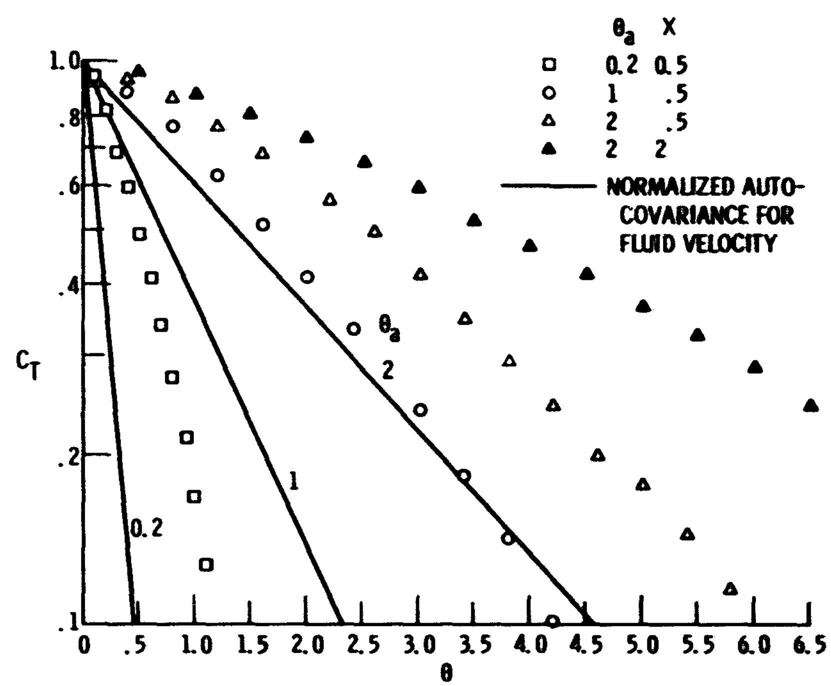

Figure 5. - Normalized auto covariance of wall temperature for constant wall heat flux case, $\gamma = 0.9$.

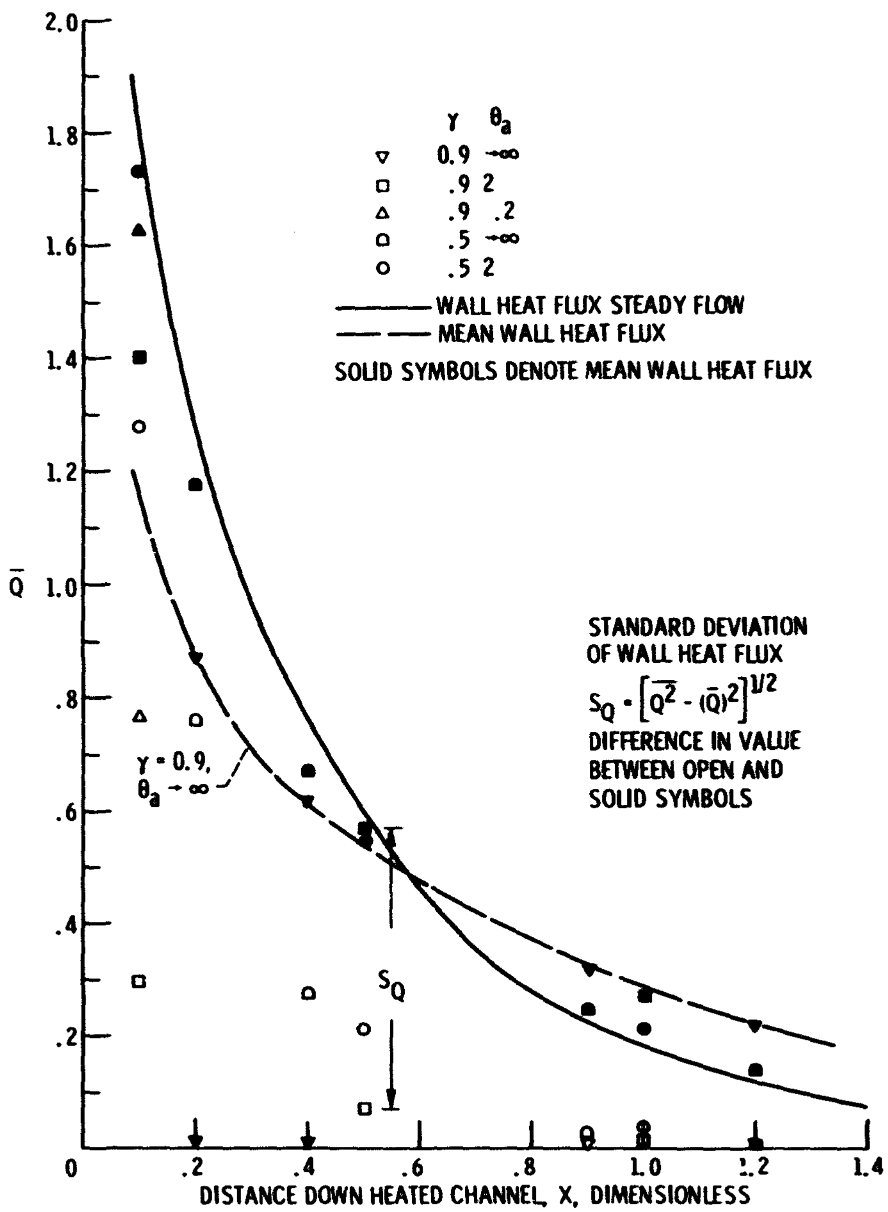

Figure 6. - Wall heat flux mean, $\bar{Q}$; and standard deviation, S_Q; for constant wall temperature case.

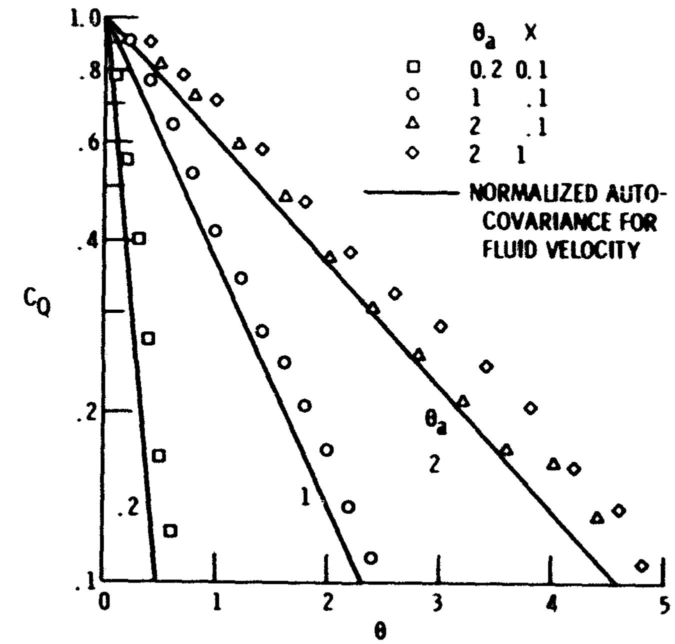

Figure 7. - Normalized autocovariance of wall heat flux for the constant wall temperature case, $\gamma = 0.9$.