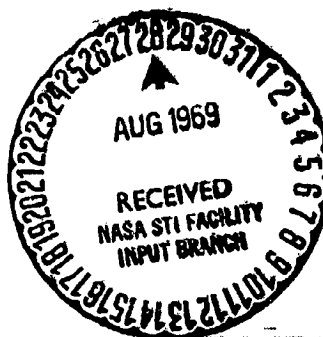


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**SIMILARITY AND CURVATURE EFFECTS  
IN POOL FILM BOILING**

by Robert C. Hendricks and Kenneth J. Baumeister  
Lewis Research Center  
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TECHNICAL PAPER proposed for presentation at  
Fourth International Heat Transfer Conference  
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**NATIONAL AERONAUTICS AND SPACE ADMINISTRATION**

## SIMILARITY AND CURVATURE EFFECTS IN POOL FILM BOILING

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NASA Lewis Research Center, Cleveland, Ohio

### Abstract

Cooldown film-boiling-heat transfer data from small diameter spheres (0.0397 to 1.27 cm) are presented, and the effects of curvature and similarities of the theoretical analyses for the flat plate, cylinder, and sphere are assessed. The data are in good agreement with theory but are significantly higher than predicted from the classic Bromley equation.

### INTRODUCTION

Film-boiling heat transfer from small diameter spheres is of fundamental interest in studying the effects of curvature, and of practical importance in engineering and biology. In nuclear reactors, small fission fragments (approximated by a sphere) can get "loose" and film boil in the coolant passages producing voids. In preservation of biological specimens and human organisms, the heat transfer rates of small, fluid globules must be accurately known to control the freezing process.

Film boiling heat transfer to a flat plate or a surface with no curvature has been studied by several authors (e.g., 1, 2), and the fundamental relation presented by Berenson (Ref. 1)

$$Nu_\ell = 0.424(Ra_\ell^*)^{1/4}$$

correlated much of the available data. Film boiling from a cylinder (single curved surface) has also been studied by several authors (e.g., 3, 4). Extending the previous work, Breen and Westwater (Ref. 4) empirically correlated much of the available cylinder data

$$Nu_\ell = (0.069\lambda_c/d + 0.59)(Ra_\ell^*/\lambda_c)^{1/4}$$

Compared with the classic Bromley correlation,  $[Nu_d = 0.62(Ra_d^*)^{1/4}]$ , ref. 3, this correlation predicts a significant increase in heat transfer for very small and very large diameter cylinders. Later, Baumeister and Hamill (Ref. 5) analyzed film boiling from the flat plate and the cylinder and were able to correlate available data. Their model extended the understanding of the film boiling process.

Film boiling from a sphere has been studied by references 6, 7, 8, 9, and 10. The analysis of Frederking and Clark (Ref. 10) correlates much of the available data for large-diameter spheres.

$$Nu_d = 0.14 Ra_d^{1/3}$$

Recently, Hendricks and Baumeister (Ref. 7) analyzed film boiling from both small and large spheres and were also able to correlate these data. However, a significant difference exists in the correlations for the small-sphere case. Thus the pur-

pose of this paper is to present experimental film-boiling heat transfer data for small spheres, test the analysis of reference 7, and also to assess the effects of curvature and similarities of the theoretical analysis for the flat plate, cylinder, and sphere. The analyses are outlined in the Appendix and the results given in table I.

### APPARATUS AND PROCEDURE

Cool down heat transfer data were taken on six different spheres with diameters ranging from 0.0397 to 1.27 cm. In each experiment, room temperature spheres were quenched in a bath of liquid nitrogen.

The smallest spheres (0.0397 and 0.0794 cm) were Tungsten-Carbide while the four remaining larger spheres (0.1585; 0.318; 0.397; and 1.27 cm) were copper. Tungsten Carbide was the only commercially available material for the small diameter spheres. The 0.0397, 0.0794, 0.1585, and 0.397 cm spheres were instrumented with 0.00126 cm diameter Chromel-Alumel thermocouple; while the 0.318 and 1.27 cm spheres had none. The heat transfer to the uninstrumented spheres was evaluated by measuring the quenching time (discussed in detail later). The couples were spot welded to the Tungsten-Carbide spheres and embedded into the copper spheres. The thermocouple output was recorded and reduced; a typical plot is shown in Figure 1. The accuracy of the recording system was within  $\pm 3^\circ$  K and the reproducibility of the data was within  $\pm 10\%$ .

In Figure 1,  $t_{Leid}$ , represents the time for film boiling to cease, while  $t_Q$ , represents the time required for the sphere to reach the bath temperature. A sharp break in the time temperature curve can be seen in Figure 1 at  $t_{Leid}$ , and condition (1) follows:

$$t_{Leid} \approx t_Q \quad (1)$$

The measurement of  $t_Q$  will be shown to give a good first order estimate of the heat transfer coefficient. For the noninstrumented spheres, the quenching time was measured using a 1/100 second stopclock and coordinated visually. Each sphere was quenched ten times and an average time was taken. The deviation ranged from  $\pm 15\%$  for the 0.0397 cm sphere to less than 1% for the 1.27 cm diameter sphere.

### THERMOMETRY ERROR

The thermocouple conduction errors in measured surface temperature were estimated by modifying the analysis of reference 11, and found to be less than  $1^\circ$  K; no correction was applied to the data.

The sensible heat loss by conduction through the thermocouple wires was estimated using the cylindrical fin equation

$$q_{TC} = 2\pi r_w \sqrt{2h_L k_e r_w} (T_w - T_s) = 2\pi r_w \sqrt{k_e k_L Nu_L} (T_w - T_s) \quad (2)$$

In these calculations, values of  $k_e = 0.044$  cal/cm-sec-K and  $Nu_L = 1$  were assumed and  $q_{TC}$  altered the heat transfer coefficient as much as 25% for the 0.0397 cm sphere.

### EXPERIMENTAL HEAT TRANSFER COEFFICIENT

The energy equation for quenching a sphere is given as

$$-\rho c V D_t (T_w - T_s) = hA(T_w - T_s) + q_{TC} \quad (3)$$

where the bath is at the saturation liquid temperature  $T_s$ . At any instant the thermal capacitance ( $\rho c V$ ) and the wall temperature  $T_w$  are assumed uniform throughout the sphere. The latter assumption is justified because the value of the Biot modulus ( $B_i = hd/k$ ) is low. By rearranging equation (3), the film boiling heat transfer coefficient can be expressed as

$$h = -(\rho c V / A) D_t \ln(T_w - T_s) - q_{TC} / A(T_w - T_s) \quad (4)$$

Since the geometry and properties are known, the heat transfer coefficients were determined directly from equation (4) using values of  $D_t \ln(T_w - T_s)$  from the experimental data of Figure 1. The results are illustrated in Figure 2. For the 0.0397 and 0.0797 cm spheres two heat transfer paths were found. The reason for these paths is at present unknown, and attributed to uncertainties in the data.

#### Average Heat Transfer Coefficient

As seen in Figure 2, for  $\Delta T > 90^\circ \text{K}$  the heat transfer coefficients are reasonably constant. Consequently, assuming average properties and neglecting heat losses, an average heat transfer coefficient is found by solving equation (3). This gives

$$(T_w - T_s) / (T_{w_0} - T_s) = \exp(-t/\tau_c) = \exp(-thA/\rho c V) \quad (5)$$

where  $T_{w_0}$  is the initial sphere temperature and  $\tau_c$  is the sphere time constant. For a given thermal resistance ( $1/hA$ ) and capacity ( $\rho c V$ ), the system time constant  $\tau_c$  can be specified. To determine  $\tau_c$ , the average heat transfer coefficient,  $\bar{h}$ , was estimated from the data in reference 8. Assuming condition (1), the quench-time to the calculated-time-constant ratio  $t_Q/\tau_c$  was experimentally found to be 1.85; consequently a first-order estimate of the average heat-transfer coefficient can be made

$$\bar{h}_y = (\rho c d / \epsilon) / (t_Q / 1.85) \quad (6)$$

Without any attached instrumentation, simply observing the quenching time, equation (6) gave values for the heat transfer coefficient which were in good agreement with the values determined from equation (4).

### DISCUSSION OF THEORETICAL AND EXPERIMENTAL RESULTS

In the derivation of the heat-transfer coefficients outlined in the Appendix, the heat transfer coefficients (listed in table I) are found to be a function of two key similarity parameters  $l$  and Bond number  $B_o$ . The theoretical expressions for Nusselt number are plotted as a function of Bond number in Figure 3 for the flat plate, cylinder, and sphere along with the classical Bromley equation. As is evident, at low Bond number a significant enhancement in heat transfer due to curvature is predicted for

both the cylinder and the sphere. Using these same parameters, the experimental heat-transfer data for spheres are compared with theory in Figure 4.

It is apparent that the data significantly depart from the large sphere analysis of reference 10 as well as the classic Bromley relation and follow those trends established in Figure 3. However, while the analysis of reference 7 predicts the general data trend, the 0.0397 cm sphere data appear on the average to be about 25% higher than theory. Three factors which may contribute to this deviation are: (1) difficulties in assessing large time-temperature gradients, (2) significant heat loss through the thermocouple, and (3) uncertainty in the heat transfer data as illustrated in Figure 2 and the specific heat of tungsten carbide. The vertical scatter in the data of Figure 3 exhibits a dependence on temperature difference which is not accounted for by the theory. (This is also true for the flat plate and the cylinder.) Perhaps a variable property analysis would encompass these trends. In addition, data for a 10.16 cm diameter sphere (Ref. 8) and those collected in reference 7 are in agreement with the theory as seen in Figure 4. These data verify the theory for a Bond number range from 0.132 to 8700.

### SIMILARITY

Similarities exist between the flat plate, cylinder, and sphere in the method of modeling, governing equations, boundary conditions, constraints, and evolved similarity parameters. Sufficiently allied are these criteria as to suggest they form a solution basis for all pool film boiling problems (see Appendix for details).

Some physical similarities between the sphere and cylinder are noted in Figure 5. At small  $B_0$ , the vapor release is periodic and the field quiescent. The transition regimes are similar and note the analogous chaotic appearance at large  $B_0$ . Figure 6 shows the similar idealized vapor dome configurations chosen to model the phenomena. The vapor dome spacing parameter  $\lambda$  is related through hydrodynamic stability theory to  $B_0$ .

### CONCLUSIONS

(1) The theoretical analysis of reference 7 predicted a significant increase in the film-boiling heat-transfer coefficient for small-diameter spheres, compared with previous analysis for large-diameter spheres. Experimental film boiling data for small spheres ranging from 0.0397 to 1.27 cm confirm the theoretical prediction.

(2) The Bond number is shown to be a key correlating parameter in relating the enhancing effect of system curvature on the film boiling heat transfer coefficient. This enhancement becomes extremely pronounced at small Bond numbers (small spheres or cylinders) and approaches that of the flat plate at large Bond numbers. Equivalently, if the radii of curvature are small, or very large, compared with the vapor dome spacing parameter  $\ell$ , the effects of curvature augment the heat transfer over that predicted by the Bromley relation.

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## APPENDIX A

### General Method of Solution

The method of modeling and the theoretical solutions for film boiling from the flat plate, cylinder, and sphere are quite similar. In this Appendix we shall review the models, the governing equations, their solutions and seek similarity forms which will enable us to compare the effects of curvature on the film boiling process.

Consider the models of film boiling from the horizontal flat plate, horizontal cylinder,

and sphere as shown in Figure 6. As seen in Figure 6, a thin vapor gap thickness  $\delta$  and vapor dome are common to all the geometries. In all cases, the liquid evaporates and flows into domes which are assumed to form in a temporal periodic manner at regular intervals  $\lambda$  or  $\theta^*$ . One coordinate is eliminated through geometric symmetry thereby enabling one to employ the stream function in solving the momentum equations.

### Basic Assumptions

For all geometries, the creeping flow assumptions are made, the gap thickness is taken as constant, and surface capillary waves, while influencing the boundary, do not significantly alter the heat transfer or the laminar nature of the flow. In considering the energy transport, radiation is neglected, the properties are evaluated at the film temperature ( $T_f = (T_w + T_g)/2$ ), and convective terms are neglected in comparison with the conductive terms in the energy equation. The convective term in the last assumption are usually accounted for by replacing the latent heat of vaporization with a modified heat of vaporization of the form

$$\lambda^* = \lambda_1 (1 + \frac{1}{2} C_p \Delta T / \lambda_1) \quad (7)$$

Justification for these assumptions is given in references 2, 5, and 7.

### Governing Equations and Boundary Condition

With the previous assumptions the governing momentum and energy equations can be written as

$$E^4 \psi = 0; \quad \nabla^2 T = 0 \quad (8)$$

The operators  $E^4$  and  $\nabla^2$  are given in references 2, 5, and 7 for each of the three coordinate systems under consideration, that is, for the flat plate, cylinder and sphere geometries. Reference 2 also presents a solution of equation (8) with a convective term left in the equation, resulting in a modified latent heat of vaporization, similar to equation (7).

For the boundary conditions, the tangential vapor velocity to the interface is assumed to be zero, the interface to be at constant temperature  $T_g$ , and the wall at a constant temperature  $T_w$ . Also, uniform vaporization is assumed at the liquid-vapor interface and, of course, zero velocity at the heater surface.

### Constraints

The above assumptions and boundary conditions do not lead to a tractable problem because the vapor gap thickness, interface velocity, and vapor dome spacing are unknowns. Thus, three additional constraints are required.

(1) Static Force Balance Constraint. - The average pressure  $\bar{P}_g$  on the vapor side of the liquid vapor interface results from the weight of the supported liquid, ambient pressure, and surface tension effects. This average pressure,  $\bar{P}_g$ , must be balanced by the vapor pressure field arising from the flow of vapor into the vapor dome



$$\int_A P_I dA = \bar{P}_s A \quad (9)$$

where  $P_I$  is the pressure evaluated at the liquid-vapor interface.

From this balance arises the two similarity parameters  $\ell$  and  $B_0$ , discussed in the body of the report.

(2) Interface Energy Balance Constraint. - The energy conducted across the vapor gap vaporizes the liquid and a balance must be maintained between the advancing evaporating interface and the incident energy. The energy balance may be written as

$$-\rho \lambda u_\delta = q = -k \partial T / \partial n |_I \quad (10)$$

where  $n$  represents the coordinate normal to the interface. In reference 2, the balance is augmented to include the effects of subcooling and radiation.

(3) Optimization Constraint. - In determining the vapor dome spacing parameters, the energy transfer or rate of entropy production is assumed to be a maximum

$$\partial \bar{h}_w / \partial L = 0; \partial^2 \bar{h}_w / \partial L^2 < 0 \quad (11)$$

where  $L$  represents a characteristic wavelength or dome spacing parameter and  $\bar{h}_w$  is the area weighted heat transfer coefficient;

$$\bar{h}_w = h_{fb}(A_{fb}/A_T) + h_D(A_D/A_T) \quad (12)$$

The hypothesis asserts that the characteristic wavelength adjusts the size and spacing of the vapor dome to optimize the rate of heat removal from the surface. The wavelengths predicted by this assumption compare favorably with the experimental data.

With these three constraints, the system is now determinant. The analytical solutions are listed in table I and shown in Figure 3 along with the general solution; Table I also shows some special limiting cases.

#### Heat Transfer Similarities

The solutions for the horizontal flat plate (Ref. 2), cylinder (Ref. 5), and the sphere (Ref. 7) are presented in table I in terms of Rayleigh numbers defined as,

$$Ra_d^* = \rho(\rho_L - \rho)g\lambda^* d^3 / (k\mu \Delta T) \quad (13)$$

$$Ra_\ell^* = \rho(\rho_L - \rho)g\lambda^* \ell^3 / (k\mu \Delta T) \quad (14)$$

The flat plate represents the limiting case of a surface with no curvature, while the horizontal cylinder has one finite curvature. Except for the slight difference in the constants of proportionality (0.424 compared with 0.373) the finite curvature of the wire increases the heat transfer by a factor

$$\left[ 1 + 9/\sqrt{6B_0} + 8/\left(3\sqrt{(6B_0)^3}\right) \right]^{1/4} \quad (15)$$

For small cylinders (small Bond numbers,  $B_o$ ) the heat transfer coefficient will be significantly affected by the curvature of the surface, likewise for the sphere. Figure 3 illustrates this quite graphically. At small Bond numbers, figures 3 and 4 show that the heat transfer coefficient from the sphere will be greater than that from an equivalent cylinder because the second component of curvature on the sphere further enhances the heat transfer.

### NOMENCLATURE

A	area	u, v	velocity components
$B_o$	Bond number $(d/l)^2$	V	volume
$C_p$	specific heat at constant pressure	$\delta$	vapor gap thickness
c	specific heat	$\rho$	density
d	diameter	$\sigma$	surface tension
$D_t$	derivative with respect to time	$\psi$	stream function
$E^4$	differential operator	$\theta^*$	reference angle (Fig. 6) - values tabulated in ref. 7
$G(B_o)$	function defined in reference 7	$\lambda^*$	modified latent heat of vaporization (eq. (11))
g	local acceleration	$\lambda_1$	latent heat
$g_c$	gravitational conversion constant	$\lambda_c$	characteristic wavelength
h	heat transfer coefficient	$\lambda$	wavelength
k	thermal conductivity	$\tau_c$	system time constant
$k_e$	effective thermocouple thermal conductivity, $\frac{1}{2}(\sqrt{k_{\omega_1}} + \sqrt{k_{\omega_2}})$	$\mu$	viscosity
l	dome spacing parameter, $\sqrt{g g_c / (\rho_L - \rho) g}$	<b>Subscripts:</b>	
L	characterizing length	d	diameter
Nu	Nusselt number	D	vapor dome
P	pressure	f	film conditions
q	heat flux	fb	film boiling
r	radius	I	interface
$R_o$	radius of sphere	l	based on characteristic length
$Ra^*$	modified Rayleigh number (Eq. (13))	L	liquid
t	time	Leid	Leidenfrost
$t_Q$	quenching time	s	saturation
T	temperature	T	total
$\Delta T$	temperature difference ( $T_w - T_s$ )	w	surface or wall
		$w_o$	initial surface or wall
		$\omega$	wire (thermocouple)
		$\delta$	evaporating interface
		$\gamma$	average, uninstrumented

Table I. - Theoretical Heat Transfer Expressions and Limiting Cases for  
Three Geometries, The Flat Plate, Cylinder, and Sphere

Nusselt number expression	Limiting cases
<u>Horizontal Flat Plate</u> $Nu_l = \frac{h_l}{k} = 0.424(Ra_l^*)^{1/4}$	A special limit in itself
<u>Horizontal Cylinder</u> $Nu_d = \frac{h_d}{k} = 0.373(Ra_d^* \sqrt{Bo})^{1/4} \left[ 1 + \frac{9}{\sqrt{6Bo}} + \frac{8}{3\sqrt{(6Bo)^3}} \right]^{1/4}$	<u>Large <math>Bo</math></u> — $Nu_d = 0.373(Ra_d^* \sqrt{Bo})^{1/4}$ <u>Small <math>Bo</math></u> $Nu_d = 0.00846 \left( \frac{Ra_d^*}{Bo} \right)^{1/4}$
<u>Sphere</u> $Nu_d = 2 + \frac{1}{4} \left[ \frac{-2Ra_d^* G(Bo)}{3} \right]^{1/4} + \left[ 0.177(Ra_d^* \sqrt{Bo})^{1/4} + \csc \theta^* \right] (1 + \cos \theta^*)$	<u>Large <math>Bo</math></u> $Nu_d = 0.35(Ra_d^* \sqrt{Bo})^{1/4}$ <u>Small <math>Bo</math></u> $Nu_d = 3 + (Ra_d^* \sqrt{Bo})^{1/4} \times \left[ \frac{0.71}{Bo^{1/8}} + 0.177 \right]$

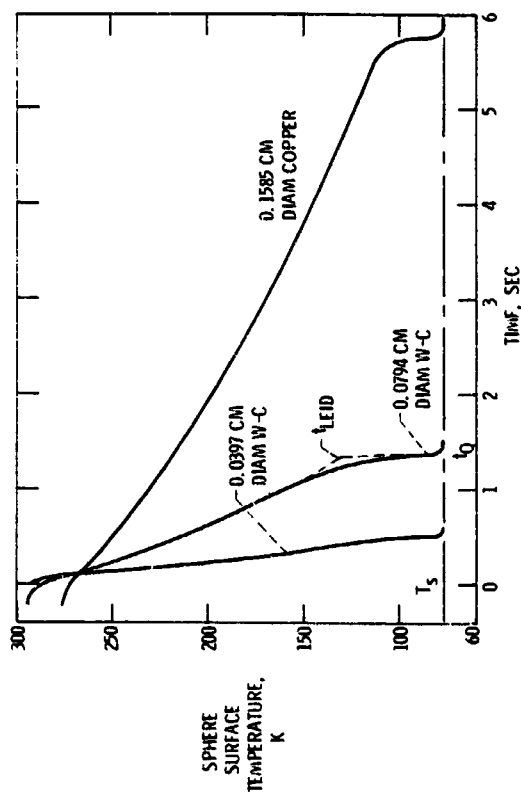


Figure 1. - Typical time-temperature histories for cooldown of spheres in saturated liquid nitrogen.

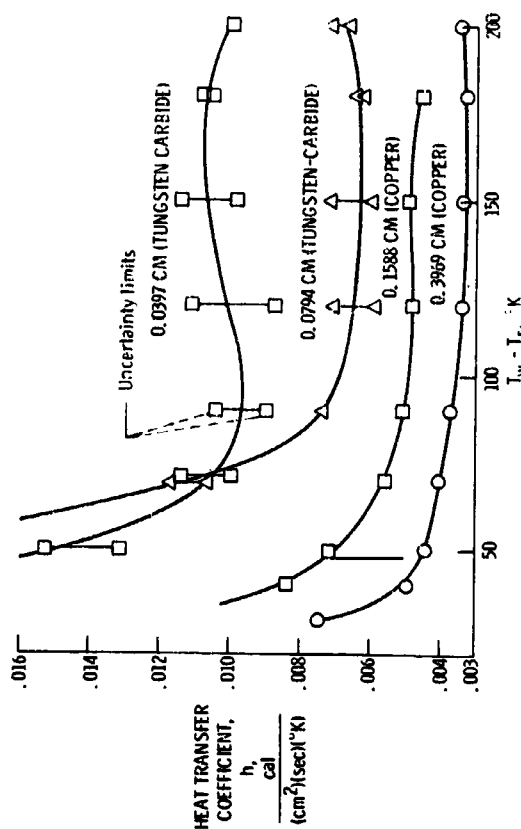


Figure 2. - Heat transfer coefficient as a function of temperature difference for four spheres.

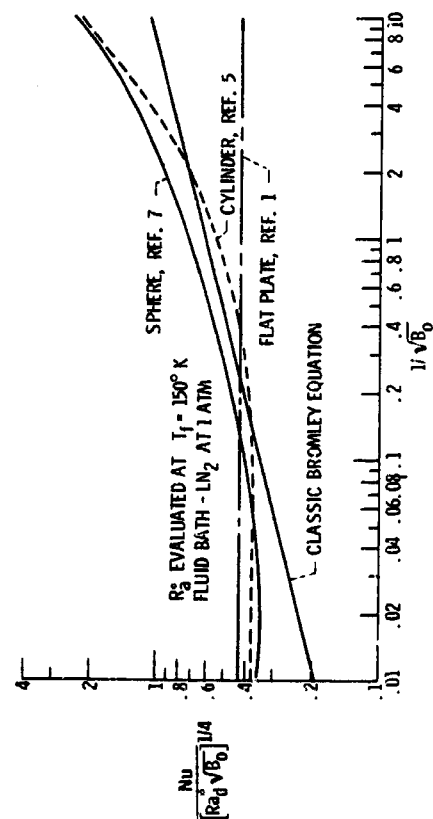


Figure 3. - Theoretical comparison of curvature effects on pool film boiling for the flat plate, cylinder, and sphere. The classic Bromley equation is for reference.

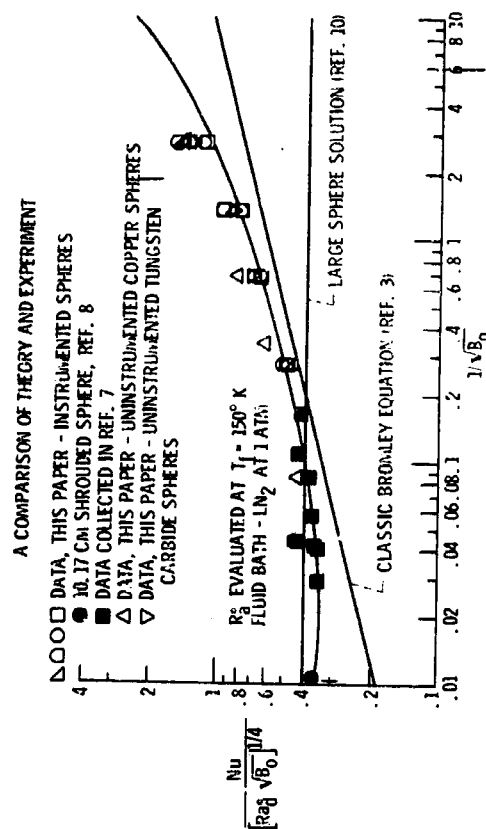


Figure 4. - Film boiling heat transfer data for spheres.

Film boiling  
from horizon-  
tal cylinders

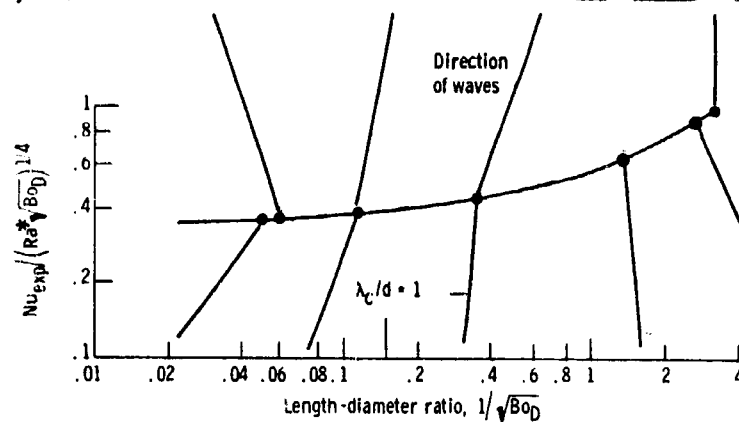
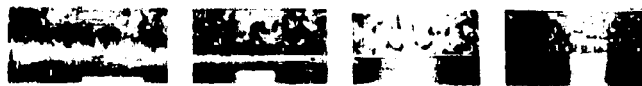
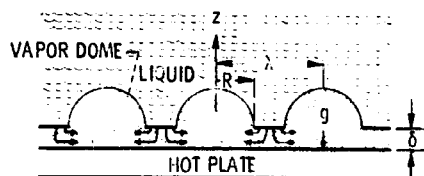
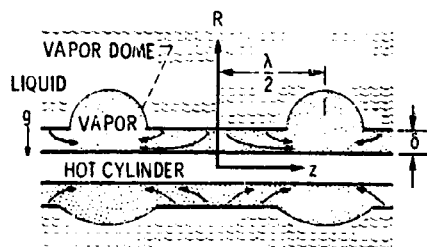


Figure 5. - Effect of geometry and Bond number for film boiling off curved surfaces, in liquid nitrogen.

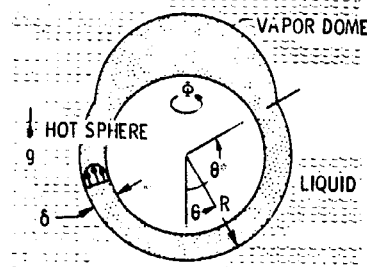
Film boiling  
from spheres



(a) HORIZONTAL FLAT PLATE.



(b) HORIZONTAL CYLINDER.



(c) SPHERE.

Figure 6. - Idealized model of film boiling from various geometries.