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EFFECTS OF TEMPERATURE-PROFILE VARIATION ON REFRACTION OF SOUND BY JET FLOW

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16. Abstract

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EFFECTS OF TEMPERATURE-PROFILE VARIATION ON REFRACTION OF SOUND BY JET FLOW

By Jay C. Hardin
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SUMMARY

In this analysis, the techniques of geometrical acoustics are employed to study the effect of variation of the jet-temperature profile upon propagation of sound in the jet. It is shown that the temperature profile of the jet can be shaped to cause the jet flow to act as a channel for the sound. Further, a simple analytical expression which predicts the occurrence of such phenomena in a nonspreading jet is obtained. This expression also seems to be approximately valid for a spreading jet.

INTRODUCTION

The phenomenon of refraction of sound by temperature and velocity gradients in the medium through which the sound propagates has long been recognized. (See ref. 1.) In certain instances, these gradients form ducts which can trap and channel the sound over large distances. (See ref. 2.) Because of considerable interest in the noise of aerodynamic jets, this result suggests that modification of the velocity and/or temperature profiles of a jet might favorably alter the directivity pattern of the sound without increasing its intensity.

A recent experimental study (ref. 3) has shown that the propagation of sound in a jet flow is, in fact, highly dependent upon the temperature and velocity of the jet. Further, while the intensity of the jet noise has been found to be a function of both the magnitude and shear of the mean velocity (ref. 4), it has also been found to be practically independent of temperature (refs. 5 and 6). Thus, it may be possible to achieve beneficial alterations in the jet-temperature profile without increasing the production of noise by the jet.

An experimental study in which such refraction was observed is reported by Grande (ref. 7). In this study a very cold nitrogen jet was used to study the propagation of sound from a source placed in the jet as well as from the jet itself. The results indicate that the sound from both the source and the jet is refracted inward and obtains a maximum intensity along the jet axis instead of the heart-shaped directivity pattern normally obtained in jet flows.

In this analysis, the propagation of sound in a jet flow is examined. The wave equation for a finite, spreading jet is complex (ref. 8) and has not yet yielded usable results. For this reason, the techniques of geometric acoustics have been employed, although they are not strictly applicable. This inapplicability is due to the fact that the use of such techniques requires the gradient of the speed of sound to be small with respect to the ratio of the speed of sound to the wavelength of the sound (ref. 9). This condition occurs only for the high-frequency components of the jet noise. However, increasing the frequency of the sound introduced into a jet causes an outward rotation of the peak of the directivity pattern (ref. 3). Thus, if it can be shown that the high-frequency components can be channeled, there is some reason to believe that the lower frequency components may also be channeled.

SYMBOLS

A	constant in equation (27)
a	speed of sound
a_a	ambient speed of sound
$a' = \frac{a}{a_a}$	
b_i	constants, where $i = 1,2,3$ (eq. (30))
c_i	constants, where $i = 1,2,3$ (eq. (31))
d	jet diameter
F	function defined by equation (12)
$f(y)$	function defined by equation (19)
$\vec{i}, \vec{j}, \vec{k}$	unit coordinate vectors
l,m,n	direction cosines
M_a	ambient Mach number

R	universal gas constant
r	radial coordinate
T_a	ambient temperature
T_H	high-temperature-core profile
T_j	jet temperature
T_L	low-temperature-core profile
$\bar{T}_s$	space average temperature deviation
t	time
$t' = \frac{2a_a t}{d}$	
u	component of velocity along X-axis
$\vec{u}$	velocity vector
$u' = \frac{u}{a_a}$	
$\vec{v}$	disturbance velocity vector
X,Y,Z	jet axes
x,y,z	Cartesian coordinate system
x_o,y_o	initial position coordinates
x',y',z'	nondimensional Cartesian coordinate system
y_c	singular value y at which ray becomes horizontal
$y_c' = \frac{2y_c}{d}$	

y^*	constant in equation (28)
α	constant which determines shape of profile (eq. (24))
β	constant which determines rate at which jet spreads (eq. (24))
γ	constant (see eq. (27))
δ_1	constant (see eq. (27))
δ_2	constant (see eq. (28))
θ	angle defined in figure 2
$\vec{\eta}$	unit vector normal to the wave front
λ_c	constant (see eq. (18))
μ	adiabatic constant
σ	spread variable
ϕ	spread angle

THEORETICAL ANALYSIS

Equations of Geometric Acoustics for a Spreading Jet

A cylindrical jet with the geometry shown in figure 1 is considered. The jet is assumed to have a velocity profile

$$\vec{u} = u(x,r)\,\vec{i} \tag{1}$$

where

$$r = \left(y^2 + z^2\right)^{1/2}$$

which neglects the small radial component of the velocity field and any turbulence which may arise in the jet. Likewise, the jet temperature is given by

$$T_j = T_j(x,r) \tag{2}$$

in degrees absolute. Thus, the speed of sound at any point in the jet is

$$a(x,r) = \left[\mu R T_j(x,r)\right]^{1/2} \tag{3}$$

where R is the universal gas constant and μ is the adiabatic constant.

An acoustic disturbance which arises in the jet propagates outward from its source with a ray velocity, obtained from reference 10, of

$$\vec{v} = a\vec{\eta} + \vec{u} \tag{4}$$

where $\vec{\eta} = l\vec{i} + m\vec{j} + n\vec{k}$ is a unit vector normal to the wave front. The change in this vector along a ray, given in reference 10, is

$$\frac{d\vec{\eta}}{dt} = \nabla a + \left(\nabla\vec{u}\right) \cdot \vec{\eta} - \vec{\eta}\left(\vec{\eta} \cdot \nabla a + \vec{\eta} \cdot \left(\nabla\vec{u}\right) \cdot \vec{\eta}\right) \tag{5}$$

For the velocity given in equation (1) and the sound speed given in equation (3), equation (4) yields

$$\frac{dx}{dt} = u + la \tag{6}$$

$$\frac{dy}{dt} = ma \tag{7}$$

$$\frac{dz}{dt} = na \tag{8}$$

and equation (5) yields

$$\frac{dl}{dt} + l\frac{\partial u}{\partial x} + \frac{\partial a}{\partial x} = lF \tag{9}$$

$$\frac{dm}{dt} + l\frac{\partial u}{\partial y} + \frac{\partial a}{\partial y} = mF \tag{10}$$

$$\frac{dn}{dt} + l\frac{\partial u}{\partial z} + \frac{\partial a}{\partial z} = nF \tag{11}$$

where

$$F = l\left(l \frac{\partial u}{\partial x} + \frac{\partial a}{\partial x}\right) + m\left(l \frac{\partial u}{\partial y} + \frac{\partial a}{\partial y}\right) + n\left(l \frac{\partial u}{\partial z} + \frac{\partial a}{\partial z}\right) \tag{12}$$

Now, a ray which starts at the origin of the coordinate system is considered. By treating the ray and the X-axis as vectors and by noting that any two intersecting vectors define a plane, the plane defined by these two vectors may be assumed, with complete generality, to be the XY-plane by a simple rotation of the coordinate system. Then

$$l(0) = \cos \theta \qquad m(0) = \sin \theta \qquad n(0) = 0$$

where the angle θ is defined in figure 2. Thus, equations (8) and (11) may immediately be integrated to yield

$$z(t) = 0 \qquad n(t) = 0$$

Therefore, a ray which starts in a plane will remain in that plane, and the three-dimensional problem has been reduced to two dimensions. The governing equations become equation (6),

$$\frac{dx}{dt} = u + la$$

equation (7),

$$\frac{dy}{dt} = ma$$

and

$$\frac{dl}{dt} = -\left(lm \frac{\partial u}{\partial x} + m \frac{\partial a}{\partial x} - l^2 \frac{\partial u}{\partial y} - l \frac{\partial a}{\partial y}\right)m \tag{13}$$

$$\frac{dm}{dt} = \left(lm \frac{\partial u}{\partial x} + m \frac{\partial a}{\partial x} - l^2 \frac{\partial u}{\partial y} - l \frac{\partial a}{\partial y}\right)l \tag{14}$$

with the initial conditions

$$x(0) = y(0) = 0 \qquad l(0) = \cos \theta \qquad m(0) = \sin \theta$$

This set of coupled, nonlinear, partial differential equations is complex but can readily be solved numerically. Examples of such solutions are given in a later section.

Channeling in an Axially Uniform Jet

Some insight into the ray solutions is gained in an analytical sense by neglecting the axial or X-dependence of the velocity and temperature profiles. Equations (6) and (7) remain unchanged

$$\frac{dx}{dt} = u + l a$$

$$\frac{dy}{dt} = ma$$

and equations (13) and (14) become

$$\frac{dl}{dt} = l m\left(l \frac{du}{dy} + \frac{da}{dy}\right) \tag{15}$$

$$\frac{dm}{dt} = -l^2\left(l \frac{du}{dy} + \frac{da}{dy}\right) \tag{16}$$

When $m \neq 0$, equations (7) and (15) combine to yield

$$\frac{dl}{dy} - \frac{1}{a}\frac{da}{dy} l = \frac{1}{a}\frac{du}{dy} l^2 \tag{17}$$

Equation (17) is known as a Bernoulli differential equation (see ref. 11) and admits the solution

$$l(y) = \frac{a(y)}{\lambda_c - u(y)} \tag{18}$$

where, by the initial conditions,

$$\lambda_c = \frac{a(0)}{\cos \theta} + u(0)$$

If for all y, $-1 < l(y) < 1$, then since $l^2 + m^2 = 1$, $m \neq 0$ for any y and the solution (18) is valid everywhere. In addition, since $m = \sin \theta$, the condition that $m \neq 0$ implies that the ray never becomes horizontal and cannot be trapped in the jet. This implication can also be seen from equation (7). Since $m \neq 0$, dy/dt never vanishes and $y(t)$ must be monotonically increasing or decreasing. Thus, for a ray to be channeled, $l(y)$, given by equation (18), must be equal to ± 1 at some y.

Suppose that at $y = y_c$, the ray becomes horizontal. Then, since the velocity and temperature profiles are even functions of y, the ray will also become horizontal at $y = -y_c$. Then from equations (6) and (7), the equation for the rays becomes

$$\left(\frac{dy}{dx}\right)^2 = \frac{\left(1 - l^2\right)a^2}{(u + la)^2} = f(y) \tag{19}$$

If $a(y)$ and $u(y)$ are given, then $f(y)$ is completely determined. The class of equations exemplified by equation (19) has been studied extensively and admits solutions in terms of elementary, elliptic, or hyperelliptic functions, depending upon the power of y in $f(y)$. (See ref. 11.) However, for general $f(y)$, certain statements may be made about the nature of the integral curves. (See ref. 12.) One statement which is relevant to this discussion follows:

If an integral curve starts in the fundamental strip $-y_c < y < y_c$, it cannot pass out of that strip. Thus, channeling will occur for this curve. In fact, the initial condition that the ray start at the origin of coordinates may be relaxed somewhat to yield the general principle:

Let there be given a velocity $u(y)$ and sound speed $a(y)$ which are even functions of y. Then if a ray starts at any point $\left(x_o,y_o\right)$ with initial normal vector in the XY-plane, a necessary and sufficient condition for the ray to be channeled is that there exists a $y_c > 0$ such that $y_c > y_o > -y_c$ and

$$l\left(y_c\right) = \frac{a\left(y_c\right)}{\lambda_c - u\left(y_c\right)} = \pm 1$$

where $\lambda_c = \frac{a\left(y_o\right)}{\cos\theta} + u\left(y_o\right)$ and $\cos\theta$ is the x-component of the initial normal vector.

It might be noted that if the initial normal vector does not lie in the XY-plane, the two-dimensional character of this analysis does not apply, and the rays would be helical in nature.

NUMERICAL ANALYSIS

If the nondimensionalized variables $x' = \frac{2x}{d}$, $y' = \frac{2y}{d}$, $a' = \frac{a}{a_a}$, $u' = \frac{u}{a_a}$, $t' = \frac{2a_a t}{d}$, where d is the diameter of the jet and a_a is the ambient speed of sound, are used, then equations (6), (7), (13), and (14) become the nondimensional set

$$\frac{dx'}{dt'} = u' + l a' \tag{20}$$

$$\frac{dy'}{dt'} = m a' \tag{21}$$

$$\frac{dl}{dt'} = -\left(l m \frac{\partial u'}{\partial x'} + m \frac{\partial a'}{\partial x'} - l^2 \frac{\partial u'}{\partial y'} - l \frac{\partial a'}{\partial y'}\right) m \tag{22}$$

$$\frac{dm}{dt'} = \left(l m \frac{\partial u'}{\partial x'} + m \frac{\partial a'}{\partial x'} - l^2 \frac{\partial u'}{\partial y'} - l \frac{\partial a'}{\partial y'}\right) l \tag{23}$$

with the initial conditions

$$x'(0) = y'(0) = 0 \qquad l(0) = \cos\theta \qquad m(0) = \sin\theta$$

In order to approach these equations numerically, it is necessary that the velocity and temperature profiles be known. In an experimental situation, these profiles could be determined with probes. However, in the examples in this analysis, particular mathematical expressions were used for these profiles. These expressions were chosen because they were convenient computationally and illustrative of conditions occurring in an actual jet. However, the qualitative results are in no way dependent upon the particular expressions chosen.

Example 1

Velocity and temperature profiles.- For the velocity profile, it is assumed that

$$u'(x',y') = \frac{M_a}{\sqrt{1 + \beta x'}} \exp\left(-\frac{\alpha^2 y'^2}{1 + \beta x'}\right) \tag{24}$$

This expression was employed because it is similar to that found in a normal spreading jet. Here, M_a is the Mach number of the flow in the center of the jet-exit plane with respect to the ambient speed of sound. The constant α determines the shape of the profile and the constant β determines the rate at which the jet spreads. This velocity profile is shown in figure 3 for several values of $\beta x'$. Note that the radius of the jet pipe corresponds to $y' = 1$, that is, $\alpha y' = \alpha$.

A measure of the rate at which the jet spreads may be obtained by noting that 99 percent of the velocity variation will occur in $-3\sigma < y' < 3\sigma$, where

$$\sigma = \frac{1}{\alpha}\left(\frac{1 + \beta x'}{2}\right)^{1/2} \tag{25}$$

Thus, the rate of spread is related to $d\sigma/dx'$. The spread angle ϕ is defined by

$$\tan \phi = \left(\frac{3d\sigma}{dx'}\right)_{x'=0} = \frac{3\beta}{2\sqrt{2}\alpha} \tag{26}$$

For the temperature profile, two variations will be considered, one having a high-temperature core and the other a low-temperature core. Comparison of these will allow evaluation of the effects of altering the temperature profile on the refraction of sound by the jet.

Variation 1.- High-temperature core:

$$T_H(x',y') = T_a + \frac{A}{\sqrt{1 + \gamma x'}} \exp\left(- \frac{\delta_1 y'^2}{1 + \gamma x'}\right) \tag{27}$$

This profile is shaped like the velocity profile obtained from equation (24). Thus, the maximum temperature is on the jet axis, the state which normally occurs in a jet.

Variation 2.- Low-temperature core:

$$T_L(x',y') = T_a + \frac{2A}{\sqrt{1 + \gamma x'}} \exp\left[- \frac{\delta_2{}^2\left(y'^2 + y^{*2}\right)}{1 + \gamma x'}\right] \cosh \frac{2\delta_2 y' y^*}{1 + \gamma x'} \tag{28}$$

This profile is shown in figure 4 for several values on $\gamma x'$. At the exit plane, it reaches a maximum at some point removed from the jet axis.

For the purpose of comparison, the space average temperature deviation at corresponding cross sections has been required to be equal for both low- and high-temperature-core profiles. This condition

$$\bar{T}_S(x') = \int_{-\infty}^{\infty} \left[T_H(x',y') - T_a\right] dy' = \int_{-\infty}^{\infty} \left[T_L(x',y') - T_a\right] dy'$$

yields

$$\delta_2 = 2\delta_1$$

Ray solutions.- In figures 5 and 6, the ray solutions $y'(x')$ of equations (20) to (23) are shown for $0^\circ \leqq \theta \leqq 90^\circ$ in steps of 6° for the parametric set

$M_a = 0.887$	$\alpha^2 = 1.6$	$\beta = 0.0$	$A = 2.76T_a$
$\gamma = 0.0$	$\delta_1^{\ 2} = 3.0$	$\delta_2^{\ 2} = 12.0$	$y^* = 0.8$

This parametric set corresponds to a jet with a maximum exit velocity of 1000 ft/sec (305 m/sec) and a maximum temperature of 1500° F (1089° K). Because β and γ are both zero, this jet does not spread. The velocity and low-temperature-core profiles for this jet are shown in figures 3(a) and 4(a), respectively. In figure 5, where the high-temperature-core profile was employed, only the singular ray for $\theta = 0^\circ$ was channeled. However, in figure 6, the low-temperature-core profile channeled all rays for which $\theta \leqq 42^\circ$. The channeled rays are periodic and never escape from the channel.

Since this jet does not spread, it must follow the analysis developed previously for an axially uniform jet. The existence of a y_c such that

$$l\left(y_c\right) = \frac{a\left(y_c\right)}{\lambda_c - u\left(y_c\right)} = \pm 1 \tag{29}$$

was determined to be a necessary and sufficient condition for channeling. In figure 7, the points y_c' for which this condition is met are shown as functions of the initial angle θ for the parametric set previously mentioned. Thus, the jet will actually channel all rays for which $\theta \leqq 43^\circ$.

Finally, in figures 8 and 9, the same parametric set has been employed except that $\beta = 0.188$ and $\gamma = 0.2$. The use of these values causes the jet to spread in the axial direction. Here the spread angle ϕ, given by equation (26), is approximately 10°. The a, b, c, d, and e parts of figures 3 and 4 show relevant profiles which are approximately 0.0, 2.5, 5.0, 7.5, and 10.0 jet radii from the jet-exit plane, respectively. In figure 8, where the high-temperature-core profile was utilized, again only the $\theta = 0^\circ$ ray was channeled. In figure 9, it can be seen that the low-temperature-core profile again initially channeled all rays for which $\theta \leqq 42^\circ$. Thus, it appears that equation (29) is approximately valid for spreading jets as well. However, the rays for the spreading jet are not periodic and ultimately escape from the channel.

Example 2

Velocity and sound-speed profiles.- In this example, the velocity and speed of sound are taken as polynomials since this is a simple method of describing experimental data. Knowledge of the sound speed is equivalent to knowledge of the temperature profile since they are directly related by equation (3). For the velocity profile, it is assumed that

$$u'(x',y') = \begin{cases} M_a + b_1 y'^2 + b_2 y'^4 + b_3 y'^6 & (-1 \leqq y' \leqq 1) \\ 0 & (\text{Otherwise}) \end{cases} \tag{30}$$

and for the speed of sound,

$$a'(x',y') = \begin{cases} 1 + c_1 y'^2 + c_2 y'^4 + c_3 y'^6 & (-1 \leqq y' \leqq 1) \\ 1 & (\text{Otherwise}) \end{cases} \tag{31}$$

This jet does not spread, but spreading could be allowed by supposing the b_i's, c_i's, and the bounds on y' to be functions of x'.

Ray solutions.- In figure 10, the polynomial velocity and sound-speed profiles are shown for the parametric set

$M_a = 0.89$ $\quad b_1 = -5.96$ $\quad b_2 = 12.14$ $\quad b_3 = -7.07$

$c_1 = 6.87$ $\quad c_2 = -13.32$ $\quad c_3 = 6.45$

These parameters correspond to a jet with maximum velocity 1000 ft/sec (305 m/sec), which attains a maximum temperature of 1500° F (1089° K) in an off-axis position.

In figures 11 to 13, the ray solutions produced by these profiles are shown for various initial conditions. Figure 11 depicts the rays for $-90^\circ \leqq \theta \leqq 90^\circ$ in steps of 6° for the initial condition $x'(0) = y'(0) = 0$. All rays for which $-24^\circ \leqq \theta \leqq 24^\circ$ are channeled. In figure 12, the ray solutions are displayed for the initial condition $x'(0) = 0$, $y'(0) = 0.316$. For these conditions, only those rays for which $-12^\circ \leqq \theta \leqq 12^\circ$ were channeled. Figure 13 shows the ray solutions when $x'(0) = 0$, $y'(0) = 0.706$. No rays are channeled under these conditions. Finally, in figure 14, the singular values y'_c are shown as a function of θ for the initial conditions in which rays were channeled. For $y'(0) = 0$, all rays for which $-29^\circ \leqq \theta \leqq 29^\circ$ would be channeled and for $y'(0) = 0.316$, all rays for which $-16^\circ \leqq \theta \leqq 16^\circ$ would be channeled.

CONCLUDING REMARKS

In this analysis, the techniques of geometrical acoustics have been employed to study the effect of variation of the jet-temperature profile upon propagation of sound in the jet. It has been shown that the temperature profile of the jet can be shaped to cause the jet flow to act as a channel for the sound. Further, a simple analytical expression which predicts the occurrence of such phenomena in a nonspreading jet has been obtained. This expression also seems to be approximately valid for a spreading jet.

From this analysis, the conclusion that jet noise may be favorably affected by varying the jet-temperature profile can be drawn. However, it should be mentioned that no allowance for changes in the production of noise by the jet has been included; that is, the mixing process has been considered to be independent of the temperature gradient. It is conceivable that altering the jet-temperature profile could increase the noise of the jet and ultimately negate any favorable effects of channeling the sound. Further, the important question of the thrust efficiency of such an engine has not been evaluated.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 30, 1970.

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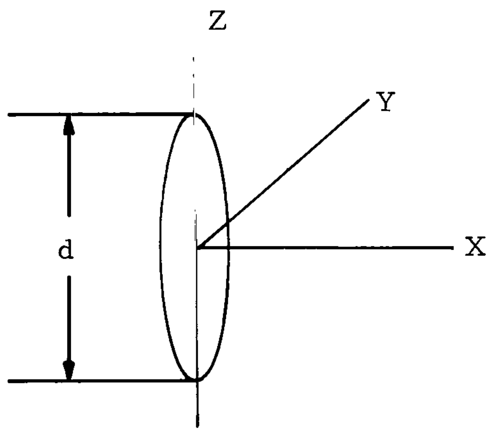


Figure 1.- Geometry of the jet.

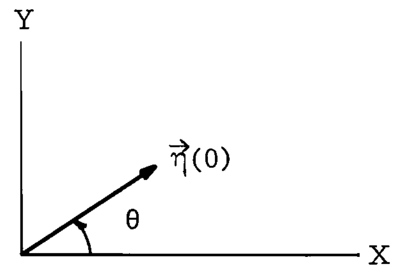
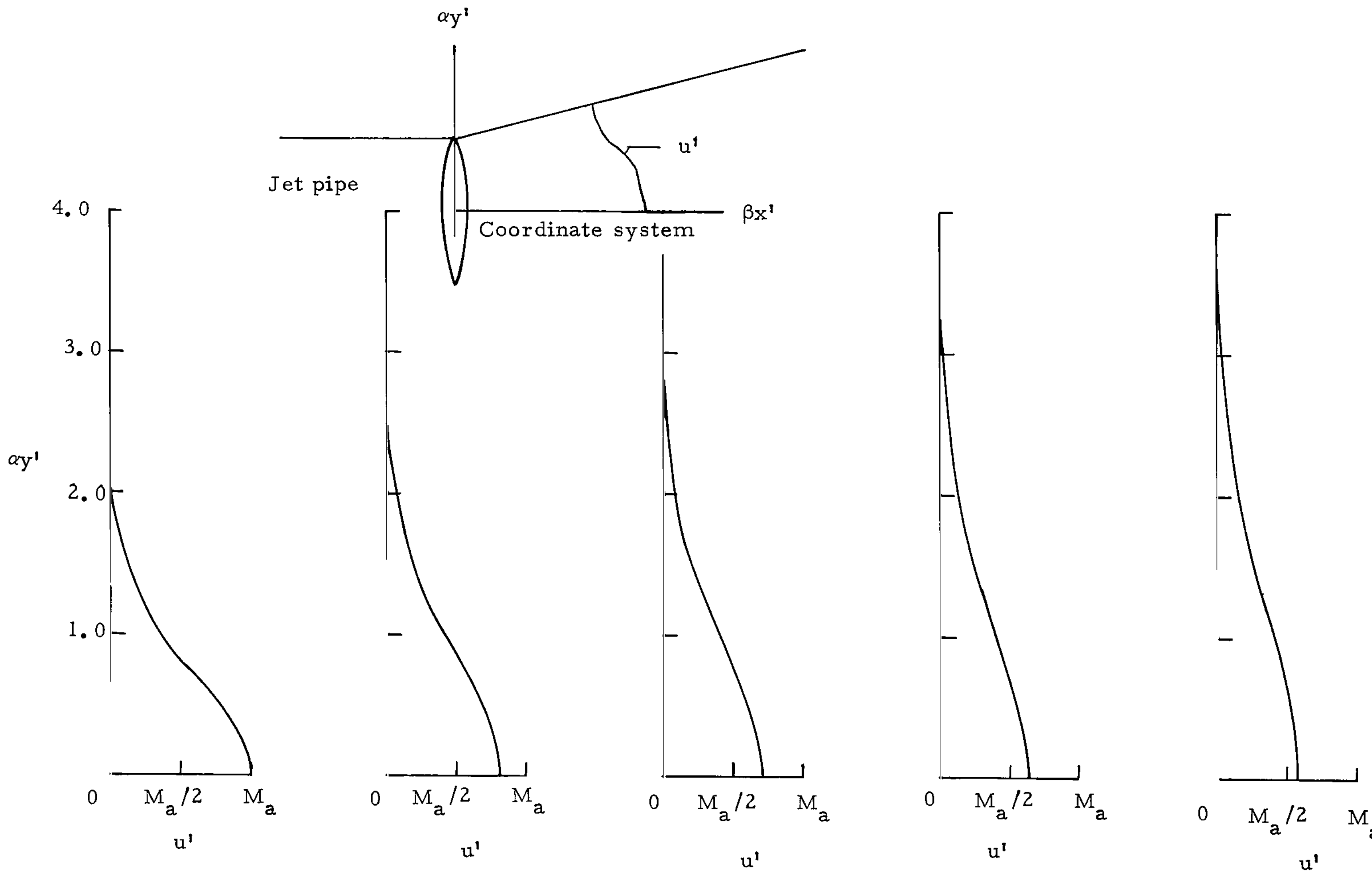


Figure 2.- Initial configuration.



(a) βx' = 0.

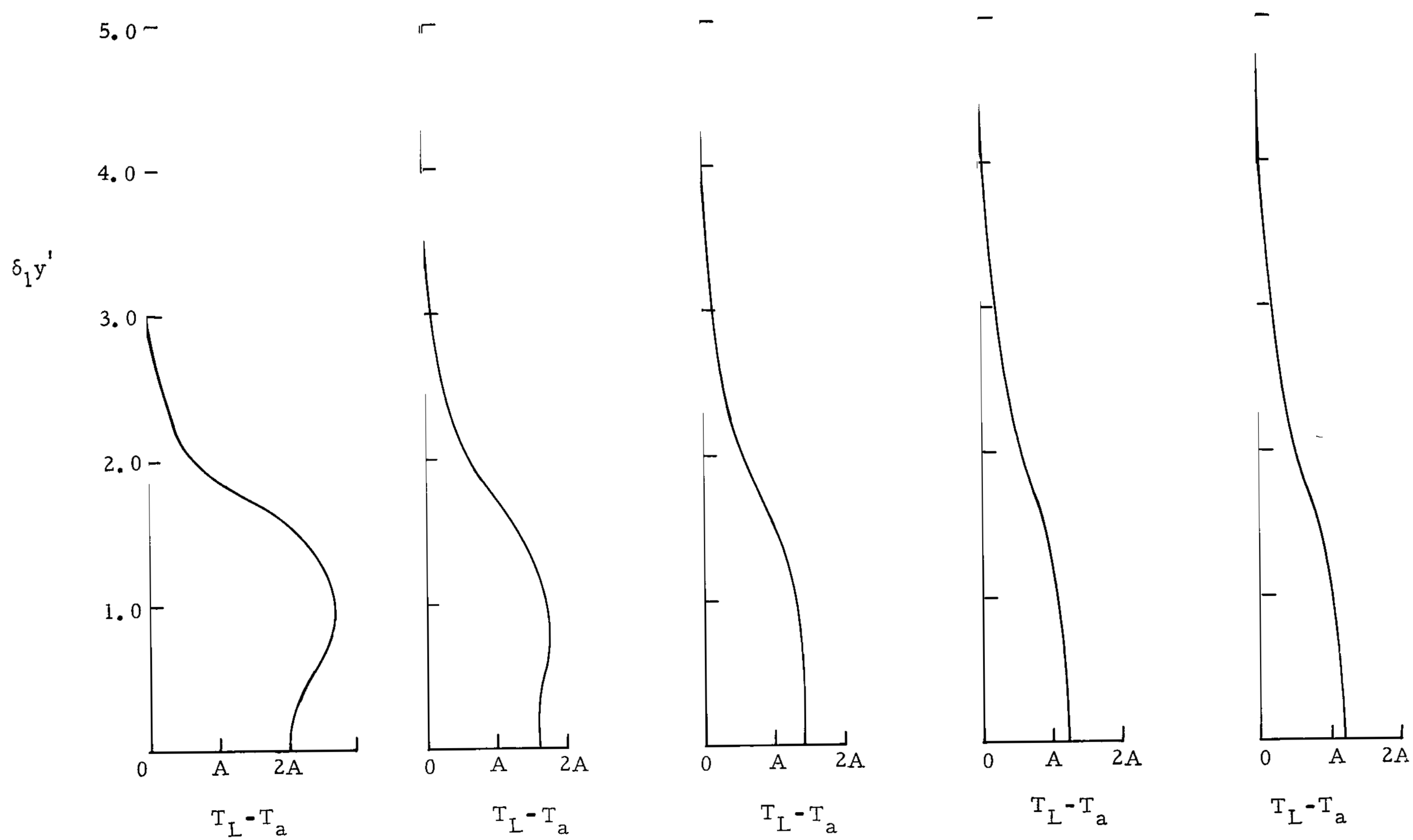
(b) βx' = 0.5.

(c) βx' = 1.0.

(d) βx' = 1.5.

(e) βx' = 2.0.

Figure 3.- Velocity profile.



(a) $\gamma x' = 0.$ (b) $\gamma x' = 0.5.$ (c) $\gamma x' = 1.0.$ (d) $\gamma x' = 1.5.$ (e) $\gamma x' = 2.0.$

Figure 4.- Low-temperature-core profile. $\delta_2 y^* = 1.0.$

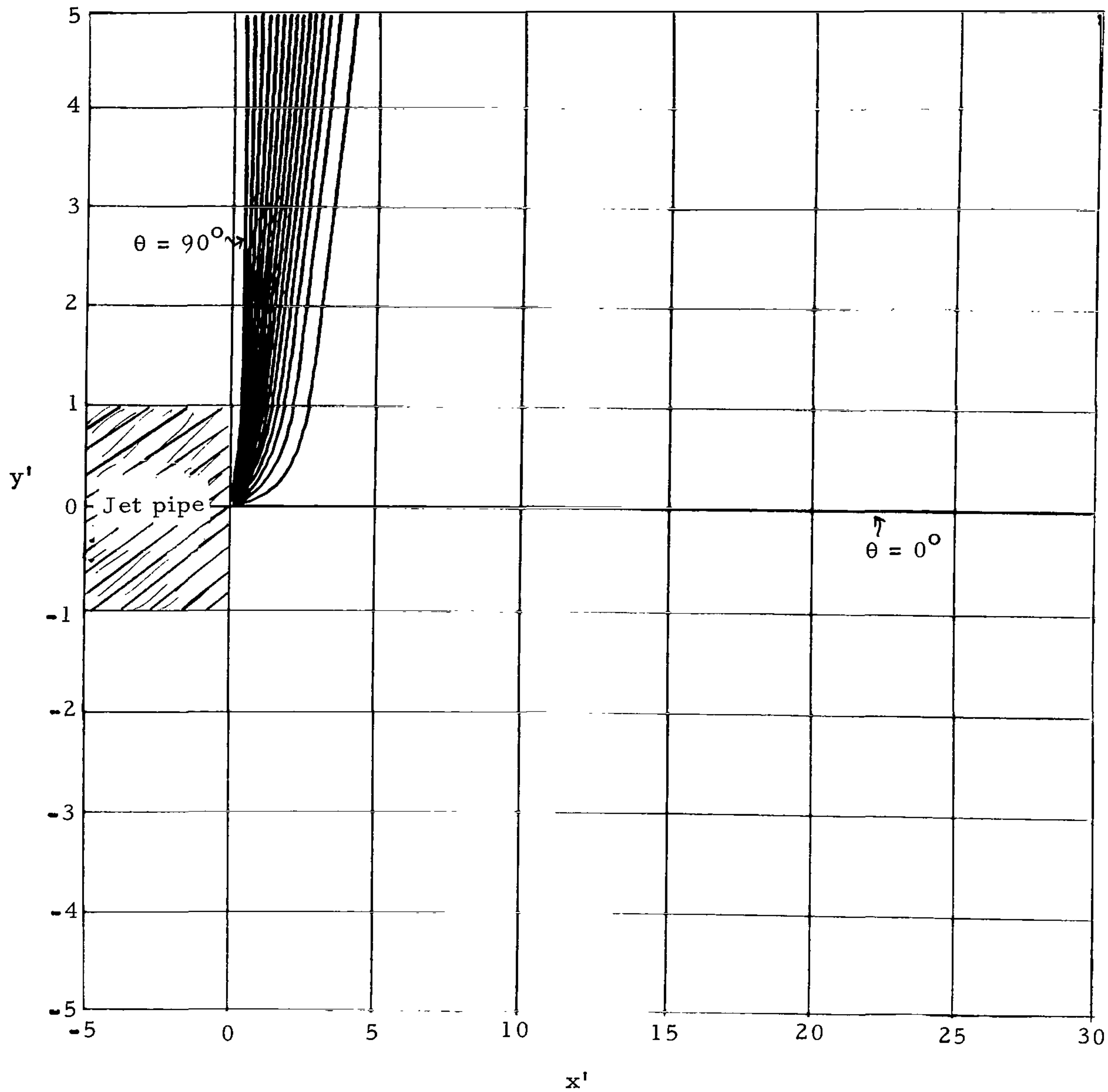


Figure 5.- Ray solutions (y'(x')) for high-temperature core. No spreading.

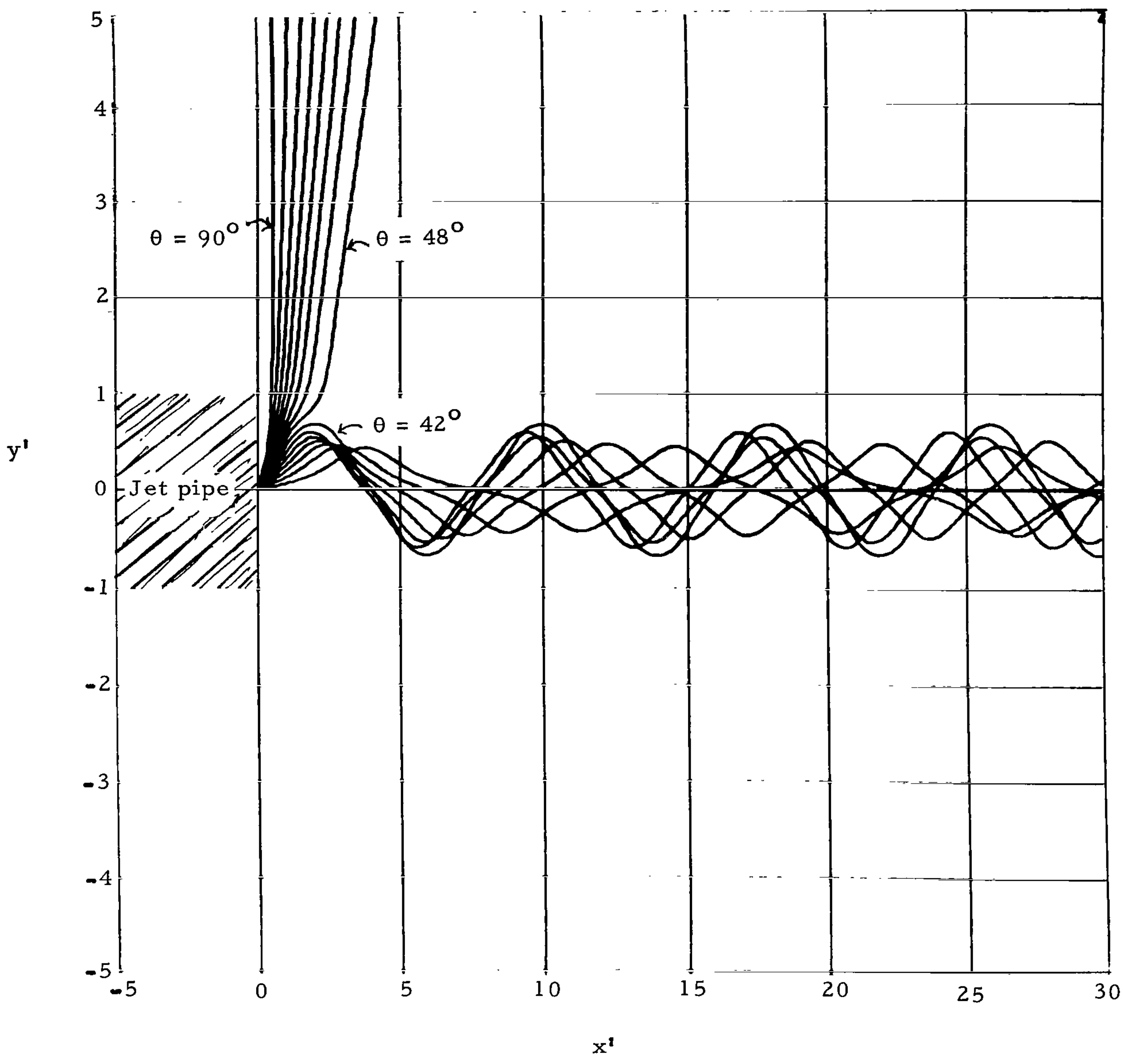


Figure 6.- Ray solutions (y'(x')) for low-temperature core. No spreading.

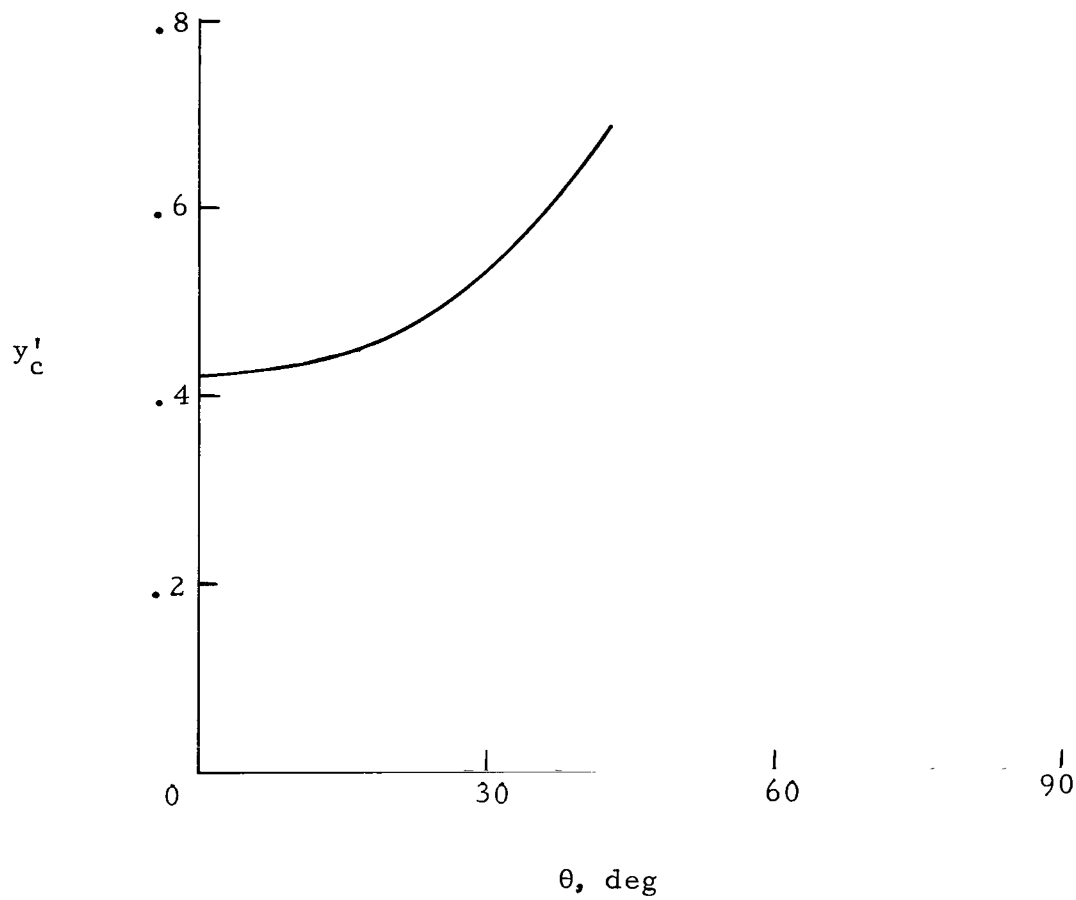


Figure 7.- Turning-point values.

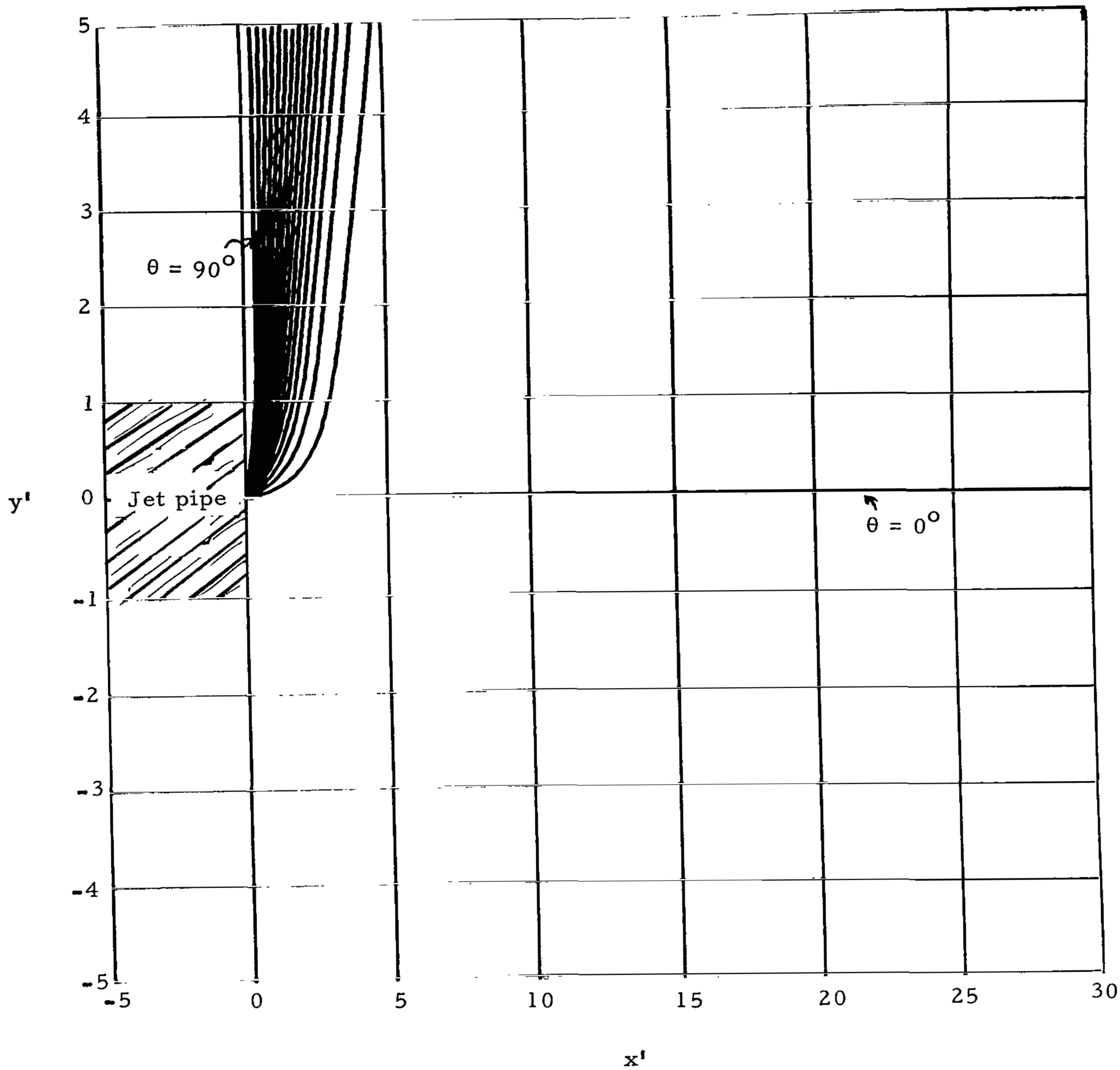


Figure 8.- Ray solutions (y'(x')) for high-temperature core. Spreading.

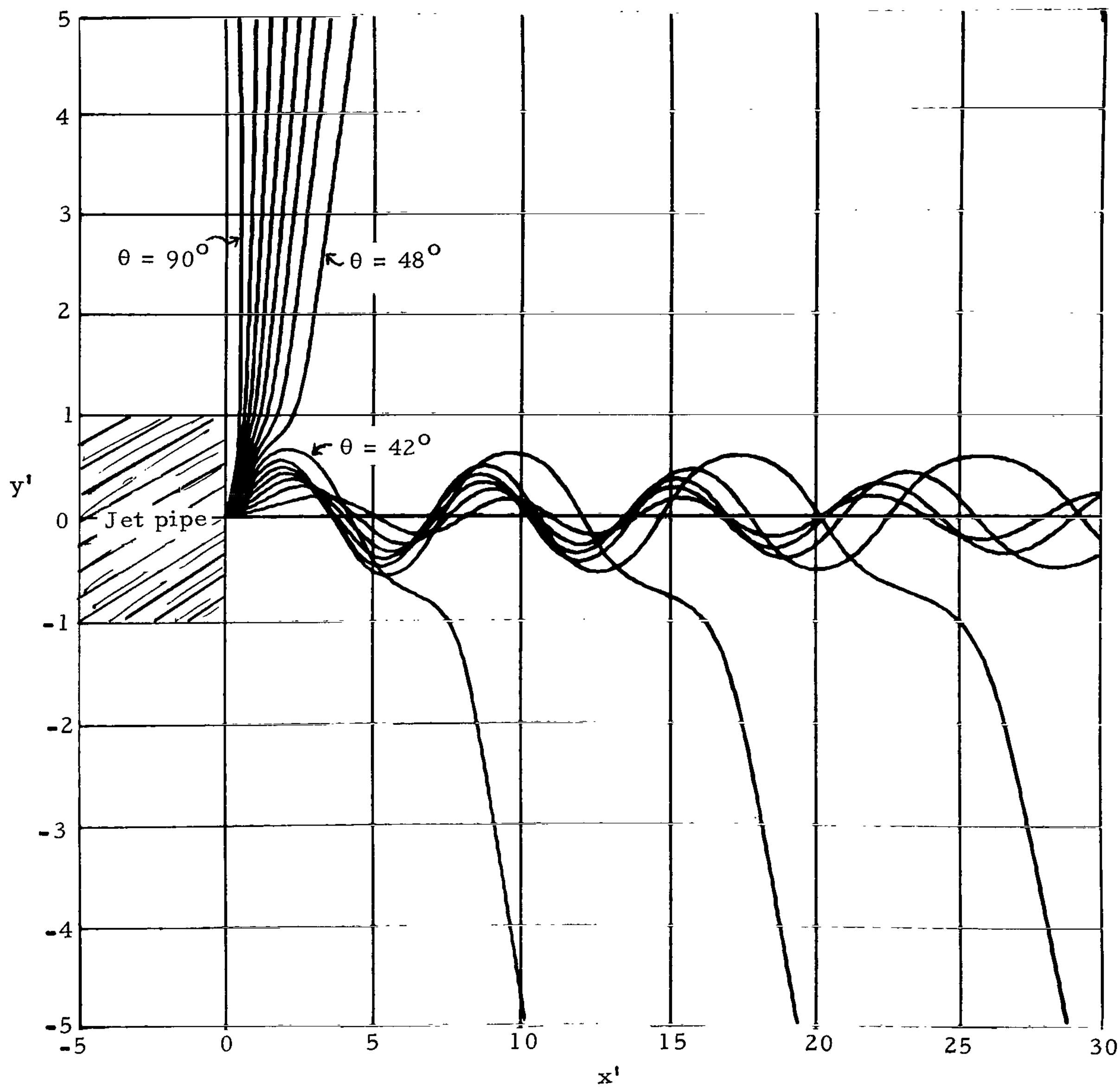
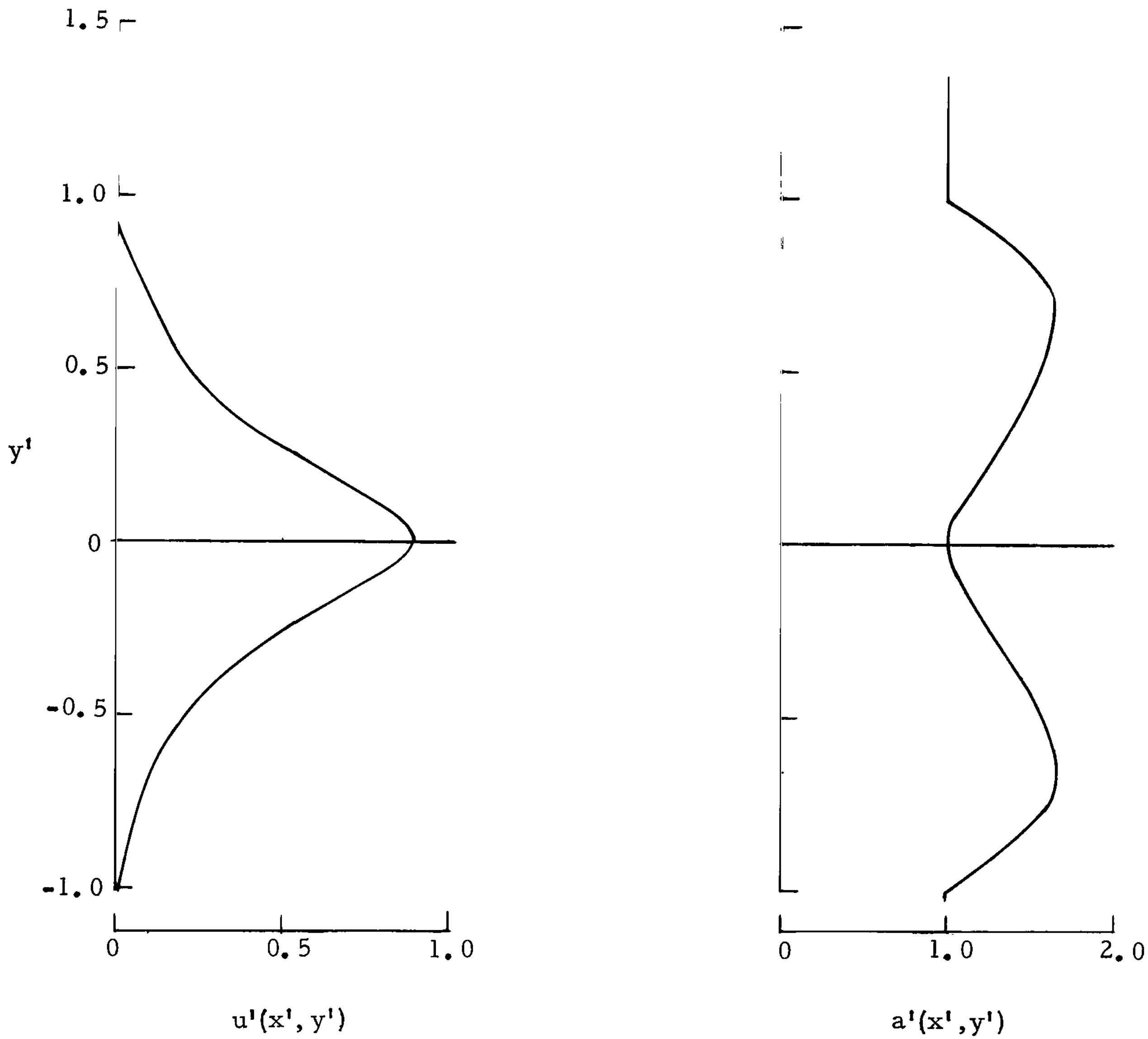


Figure 9.- Ray solutions (y'(x')) for low-temperature core. Spreading.



(a) Velocity profile.

(b) Sound-speed profile.

Figure 10.- Polynomial-velocity and sound-speed profiles.

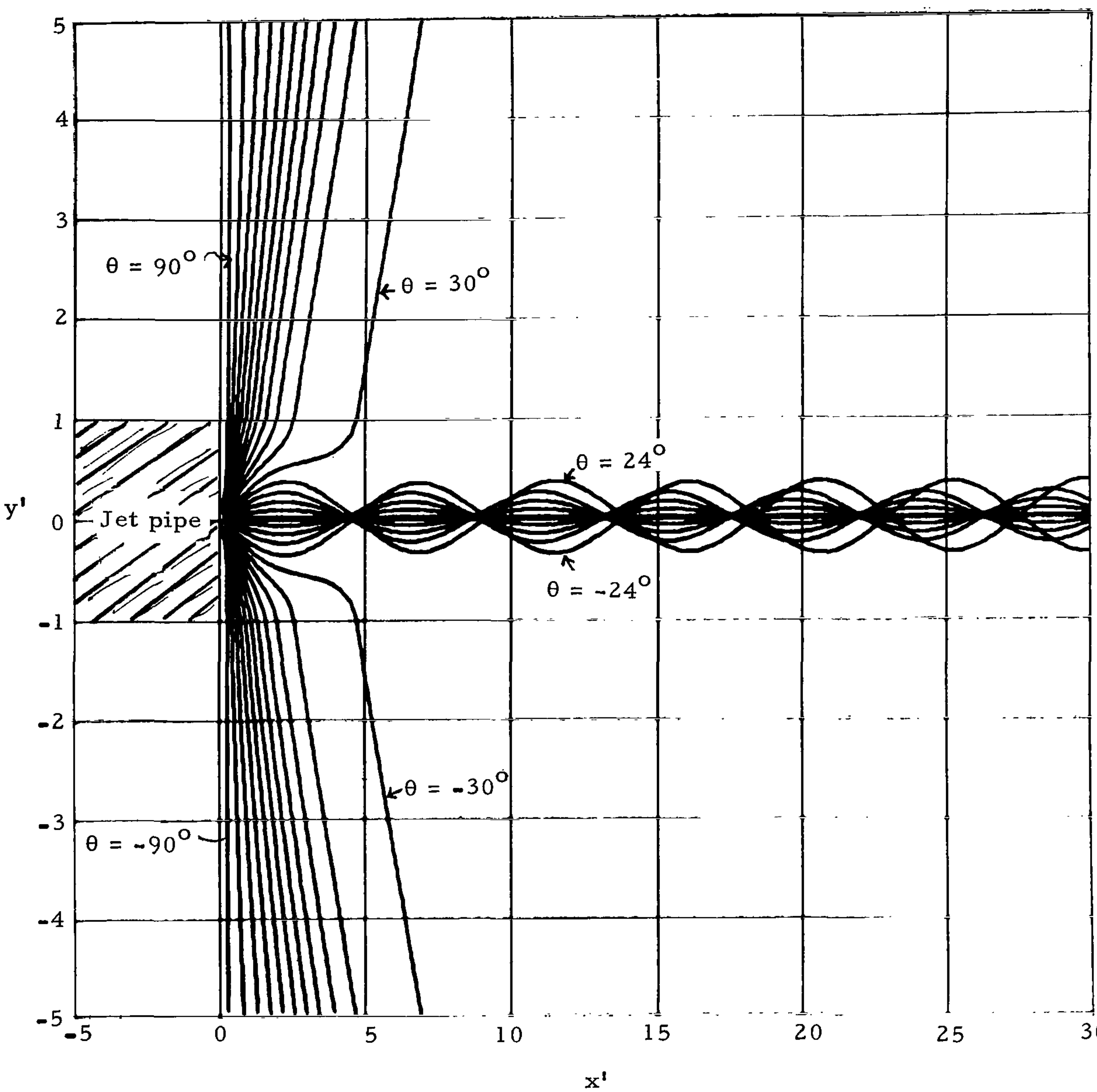


Figure 11.- Ray solutions (y'(x')) for polynomial profiles. x'(0) = y'(0) = 0.

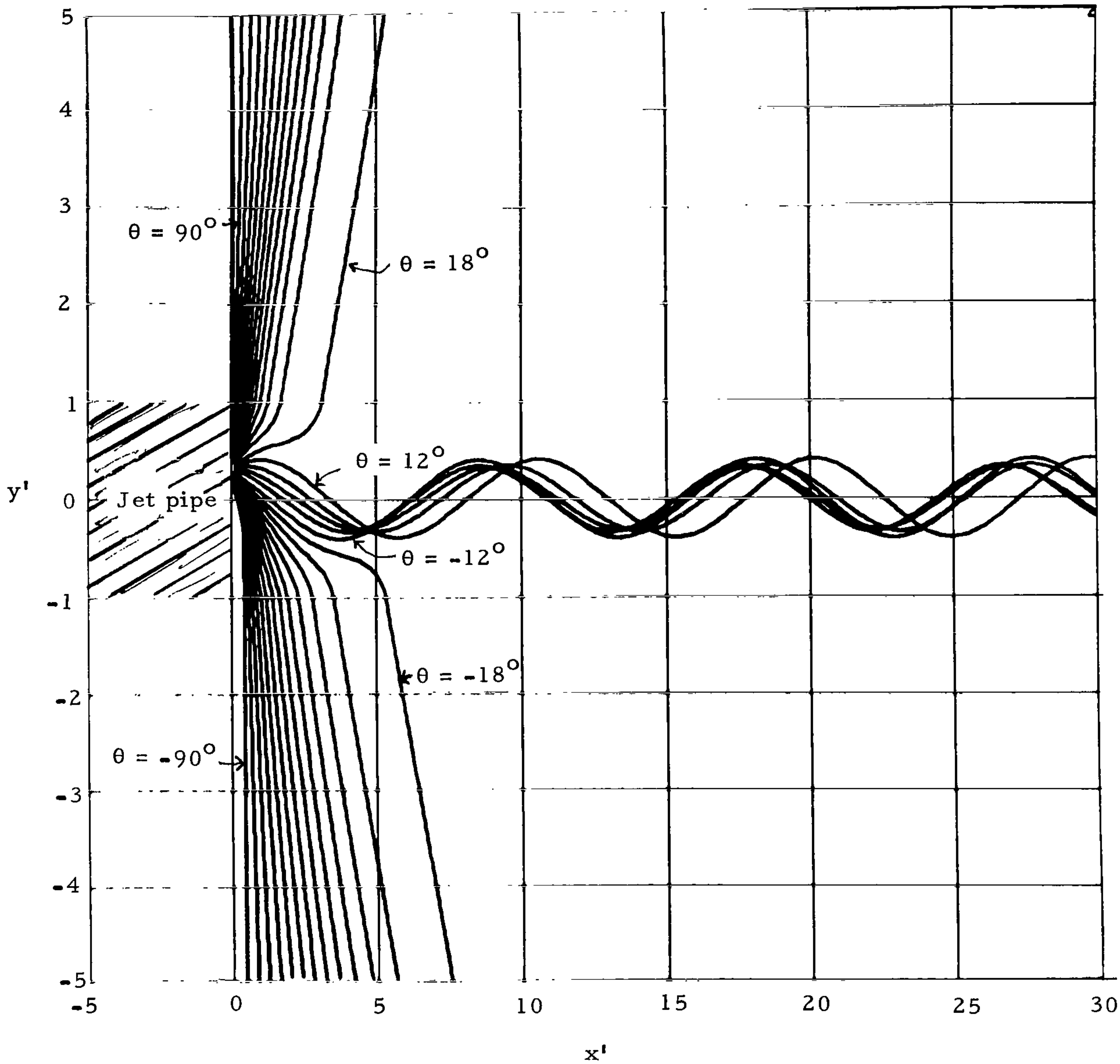


Figure 12.- Ray solutions (y'(x')) for polynomial profiles. x'(0) = 0; y'(0) = 0.316.

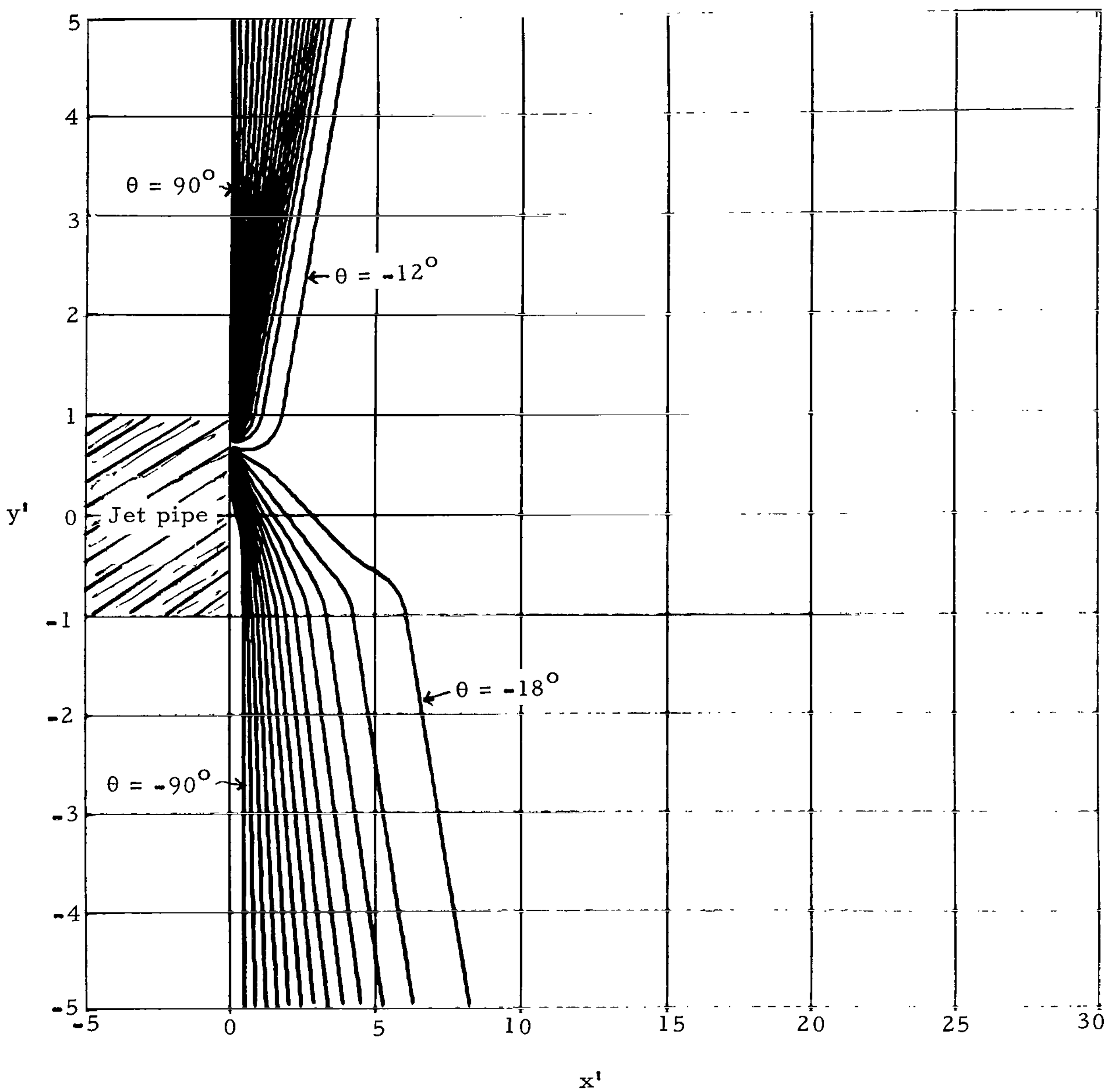


Figure 13.- Ray solutions (y'(x')) for polynomial profiles. x'(0) = 0; y'(0) = 0.706.

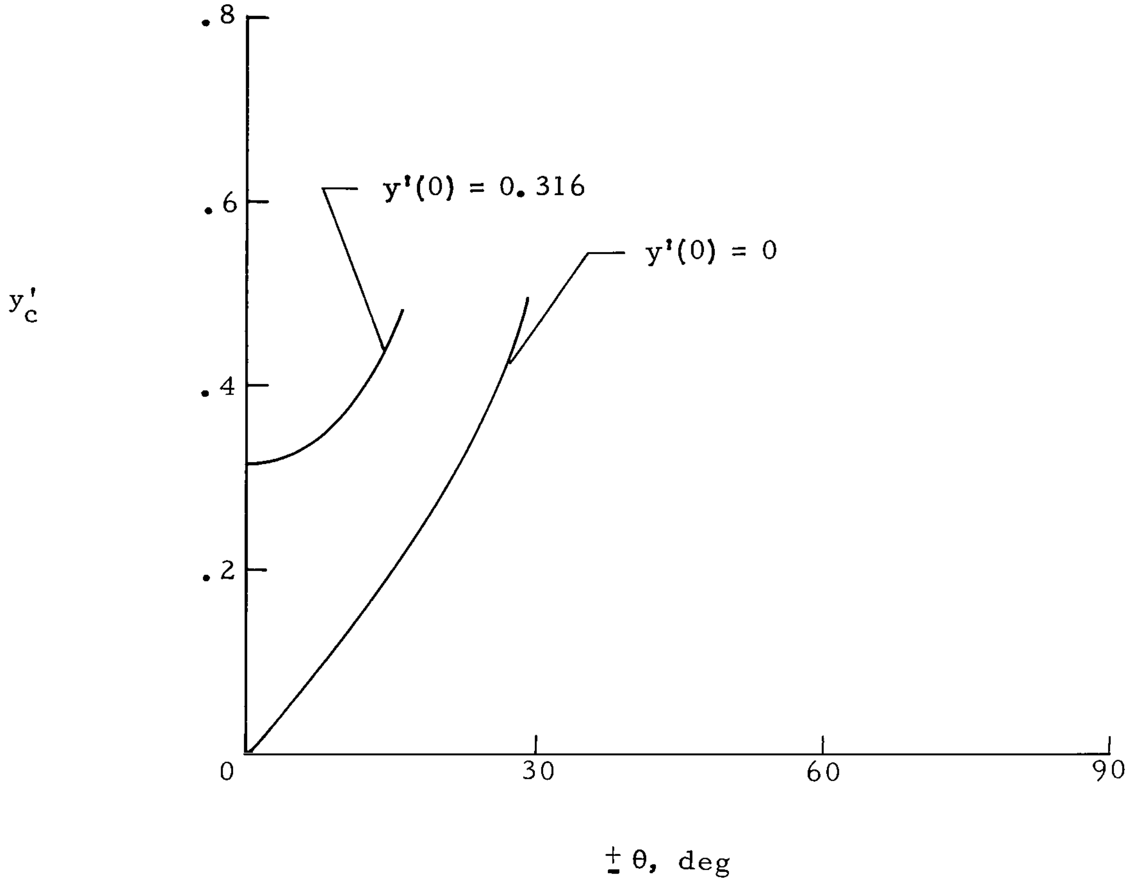


Figure 14.- Turning-point values.

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