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PROJECT TECHNICAL REPORT
MSC/TRW TASK KM-205

SLP OWS CONTROL SYSTEM DIGITAL SIMULATION REQUIREMENTS

NAS 9-8166

30 JANUARY 1970

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

N70-24250

Prepared by
Control System Analysis Section
Guidance and Control Systems Department
Electronics Systems Laboratory

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ILLUSTRATIONS

1. INTRODUCTION

This report presents the equations and logic to be programmed for an all digital simulation of the AAP orbital workshop (OWS) pointing control system (PCS) and thrust attitude control system (TACS). Development of this simulation is a subtask of Task KM-205. Analytical studies to investigate the performance of the OWS attitude control systems will be carried out as part of Task KM-205 using this simulation.

At the time that these simulation requirements were assembled, some areas of the OWS control system software were not completely defined. To work around this problem, it was established that the program would be developed in a modular form such that the final software could be incorporated without disturbing the overall program structure. In many of these undefined areas, subroutines have been developed which anticipate the software to be incorporated in the onboard computer (on the basis of the physical principles involved and any available documentation). Changes to the simulation will be made as the software to be programmed in the onboard computer becomes finalized.

A block diagram of the OWS attitude control systems (ACS) is shown in Figure 1-1. The portions of the OWS ACS which are modelled in this simulation include the CMG subsystem, the thrust attitude control system (TACS) jets, the pointing control system (PCS) and TACS software in the ATM digital computer, the star tracker, the vehicle rate gyros, and the acquisition sun sensors. The experiment pointing control (EPC) subsystem and the launch vehicle digital computer (LVDC) are not modelled.

The operational modes of the OWS ACS which have been modelled include all the possible modes of operation described in Reference 2, the Interface Program Requirements Document (IPRD).

A general purpose input/output processor will be used for this program. All parameters named in Section 4 will be available for printout or plotting. The frequency of printout and plotting will be specified seperately and will be some multiple of the fast loop frequency. More details on the input/output processor will be given in the user's manual.

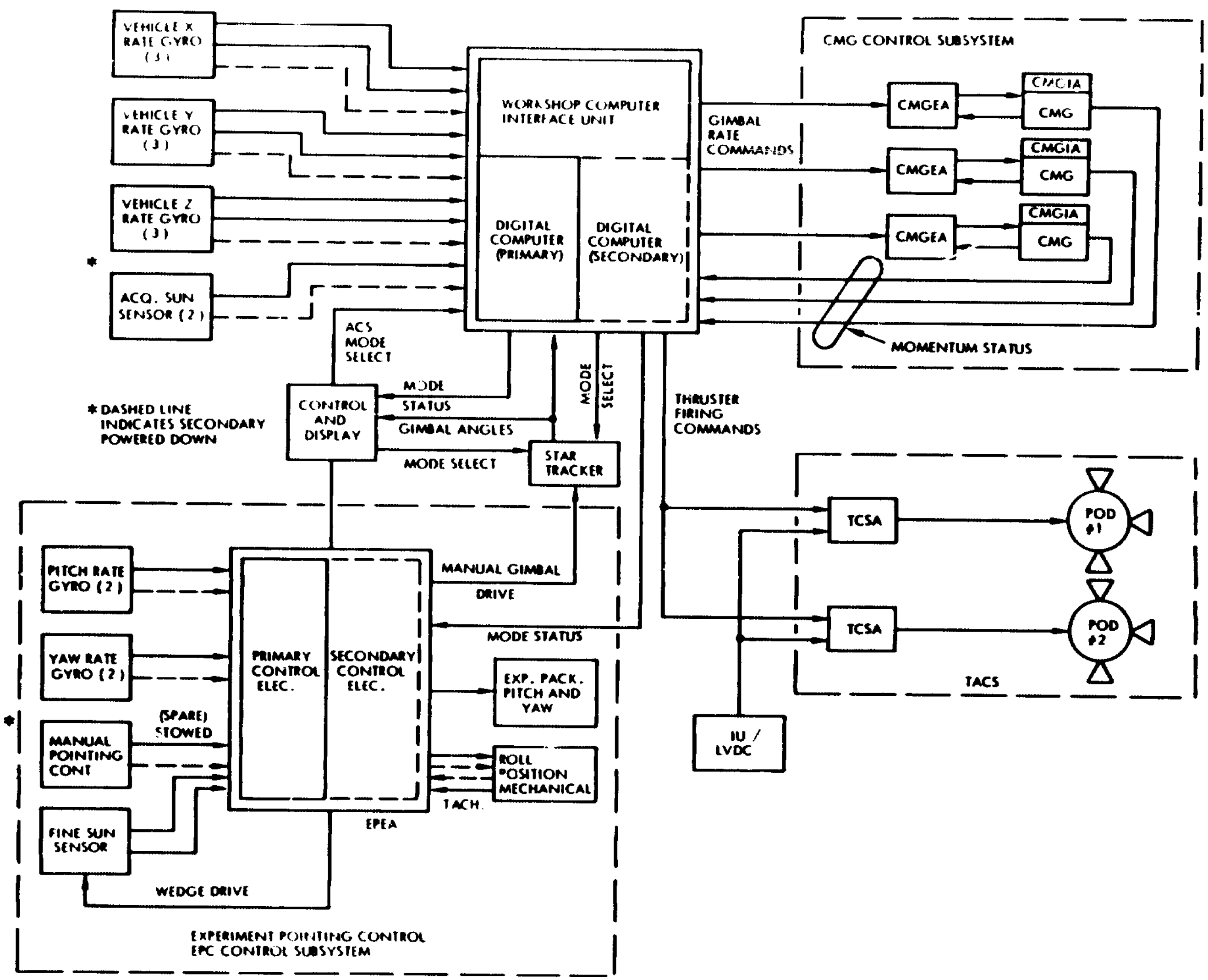

Figure 1-1. Saturn V Workshop Attitude Control System

2. COORDINATE SYSTEMS AND SPECIAL DEFINITIONS

This section provides the definitions of the coordinate systems used in the simulation and discusses the required matrix transformations between them. Figures 2-1 through 2-8 illustrate the relationships between pertinent pairs of coordinate frames.

2.1 s-SYSTEM

The s-system (X_s, Y_s, Z_s) is earth centered with X_s perpendicular to the ecliptic plane and positive in the direction of the celestial north pole, and Z_s pointing at the sun.

2.2 p-SYSTEM

The p-system (X_p, Y_p, Z_p) is an earth-centered system with X_p along the earth spin axis and positive towards the earth north pole. Z_p is the projection of Z_s onto the equatorial plane.

2.3 e-SYSTEM

The e-system (X_e, Y_e, Z_e) is earth centered and earth fixed, i.e., it rotates with the earth. X_e lies along the earth spin axis and is positive toward the earth north pole. Z_e lies in the equatorial plane and is positive through 0-degree longitude.

2.4 o-SYSTEM

The o-system (X_o, Y_o, Z_o) is earth centered with X_o perpendicular to the orbit plane, positive in the direction of the orbit momentum vector. Z_o lies along the intersection of the orbit plane and the equatorial plane, positive in the direction of the ascending node.

2.5 γ-SYSTEM

The γ-system (X_γ, Y_γ, Z_γ) is earth-centered inertial system with $X_\gamma = X_s$ and Z_γ parallel to a line from the sun to the vernal equinox.

2.6 v-SYSTEM

The v-system (X_v, Y_v, Z_v) is centered at the vehicle center of mass with Z_v along the vehicle radius vector, positive toward the center of the earth and X_v parallel to X_o and positive in same direction as + X_o.

2.7 u-SYSTEM

The u-system (X_u, Y_u, Z_u) is centered at the vehicle center of mass with Z_u pointing at the sun and X_u in the orbit plane, positive in the direction of the vehicle velocity vector at orbit noon. This system is the nominal solar inertial reference system.

2.8 b-SYSTEM

The b-system (X_b, Y_b, Z_b) is centered at the center of the vehicle multiple docking adapter (MDA) with X_b along the vehicle longitudinal axis, positive through the CSM. The Z_b axis lies along the centerline of the Apollo telescope mount (ATM) and is positive away from the ATM. The b-system is initially aligned to the v-system via three successive rotations: ϕ_{bv}, θ_{bv}, and ψ_{bv} about X_v, Y_v, and Z_v, respectively. Nominally, while in the solar inertial attitude, ϕ_{bv} is the central angle in the orbit plane between the orbital assembly (OA) and orbit noon, $\theta_{bv} = \beta$, and ψ_{bv} is the angle between X_b and the orbit plane. A positive β occurs when Z_u and X_o lie on the same side of the orbit plane (i.e., when the sun and orbit momentum vector lie on opposite sides of the orbit plane).

2.9 b′-SYSTEM

This system (X_b', Y_b', Z_b') represents the body coordinate system used by the onboard computer calculations in determining the vehicle inertial orientation from star tracker measurements prior to a local vertical maneuver. The b′-system which is ideally aligned with the b-system, is used to simulate star tracker misalignments. It is assumed that the nominal star tracker zero reference is along $+Y_b$, so that misalignments are incorporated by orienting the b′ system away from the b-system by the two successive rotations: $\Delta\phi_s$ about X_b' and $\Delta\psi_s$ about Z_b'.

2.10 i-SYSTEM

The i-system (X_i, Y_i, Z_i) is earth centered with $X_i = X_o$ and $+ Z_i$ pointing towards the sun (Z_i is the projection of Z_s on the orbit plane).

2.11 SPECIAL DEFINITIONS

In the detailed requirements which follow, the symbol $[A]_{ef}$ is used to indicate the transformation from the e-system to the f-system, where e and f represent any of the above coordinate systems. For example,

$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = [A]_{ps} \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix}$$

The notations $[\theta_x]$, $[\theta_y]$, and $[\theta_z]$ are employed to represent the transformations

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}, \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix},$$

and $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, respectively.

The notation $A_{ef}(i, j)$ refers to the element in the i^{th} row and j^{th} column of $[A]_{ef}$.

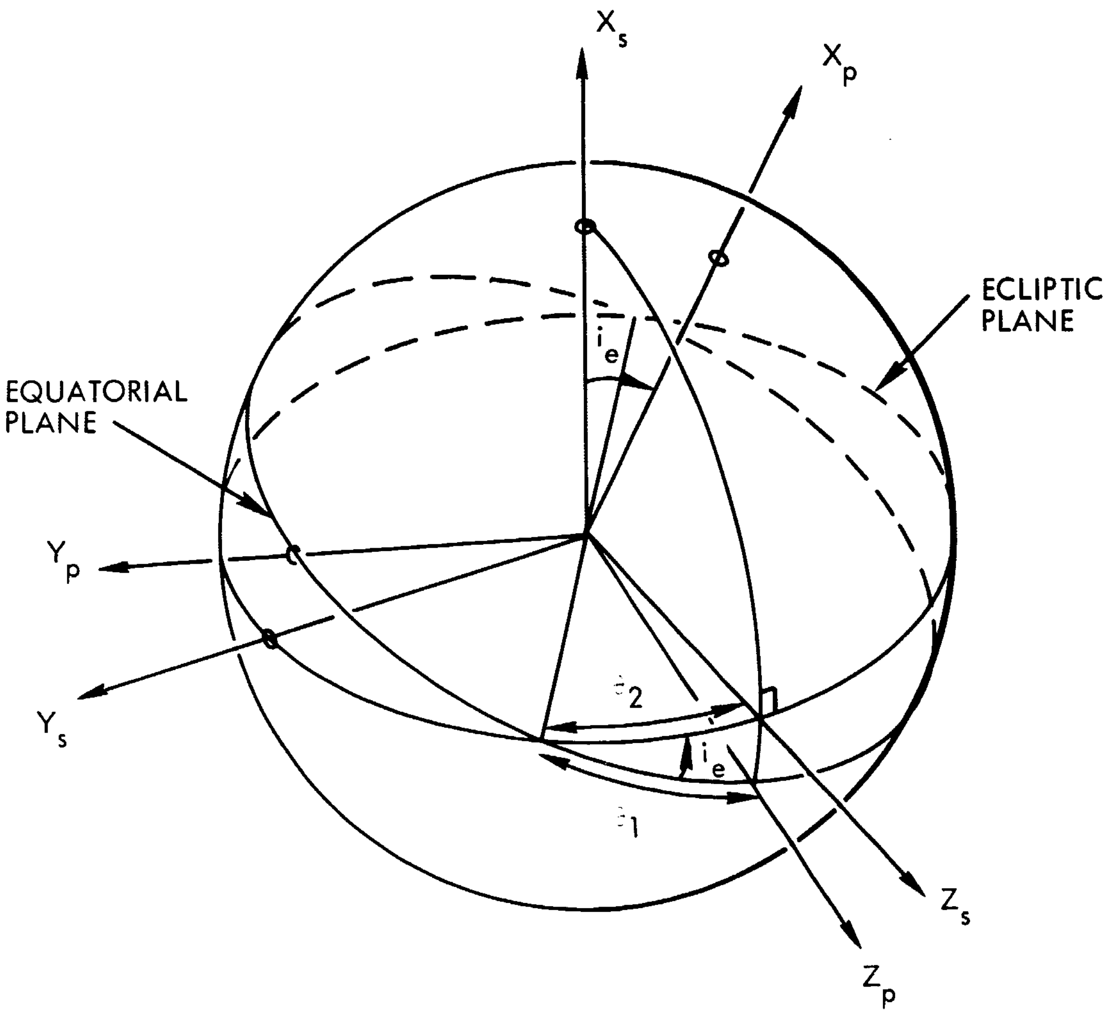

Figure 2-1. Definition of Solar and Equatorial Coordinate Systems (s-System and p-System)

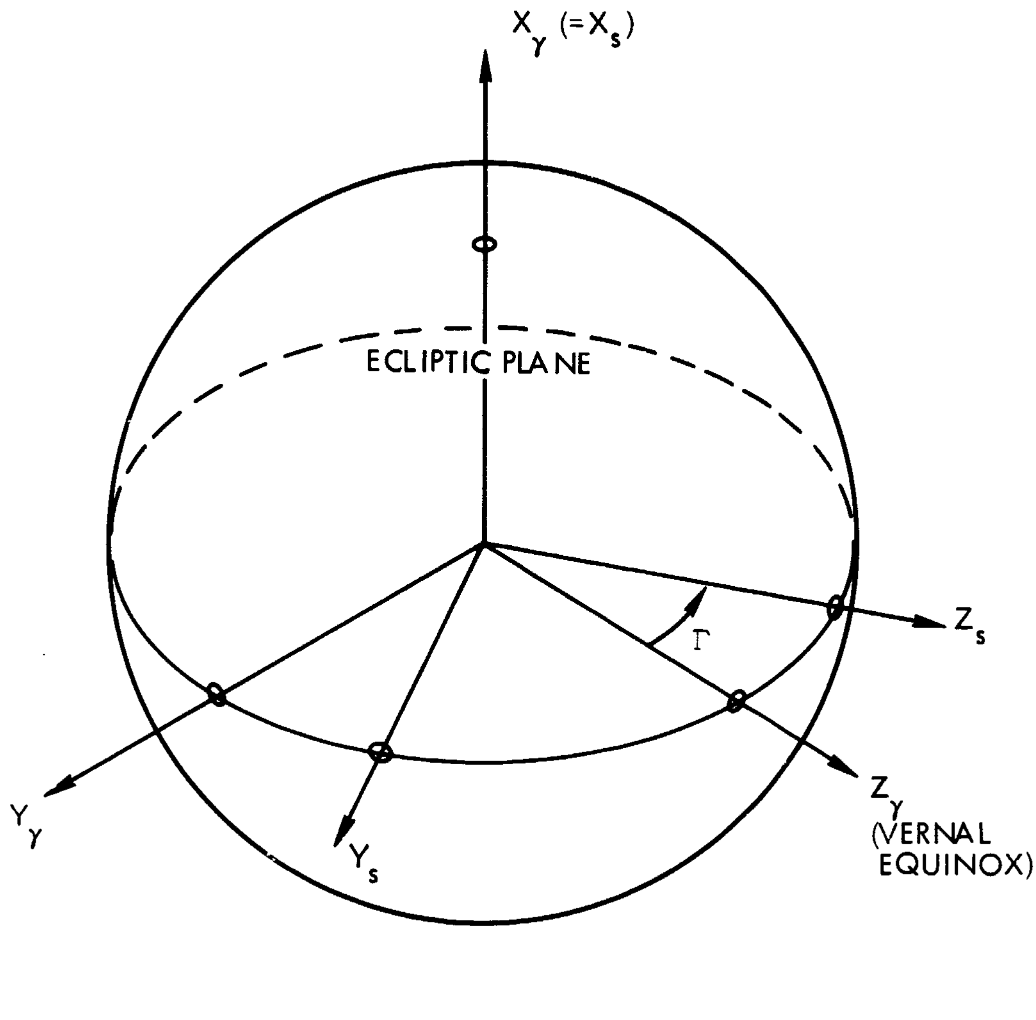

Figure 2-2. Definition of the γ-System

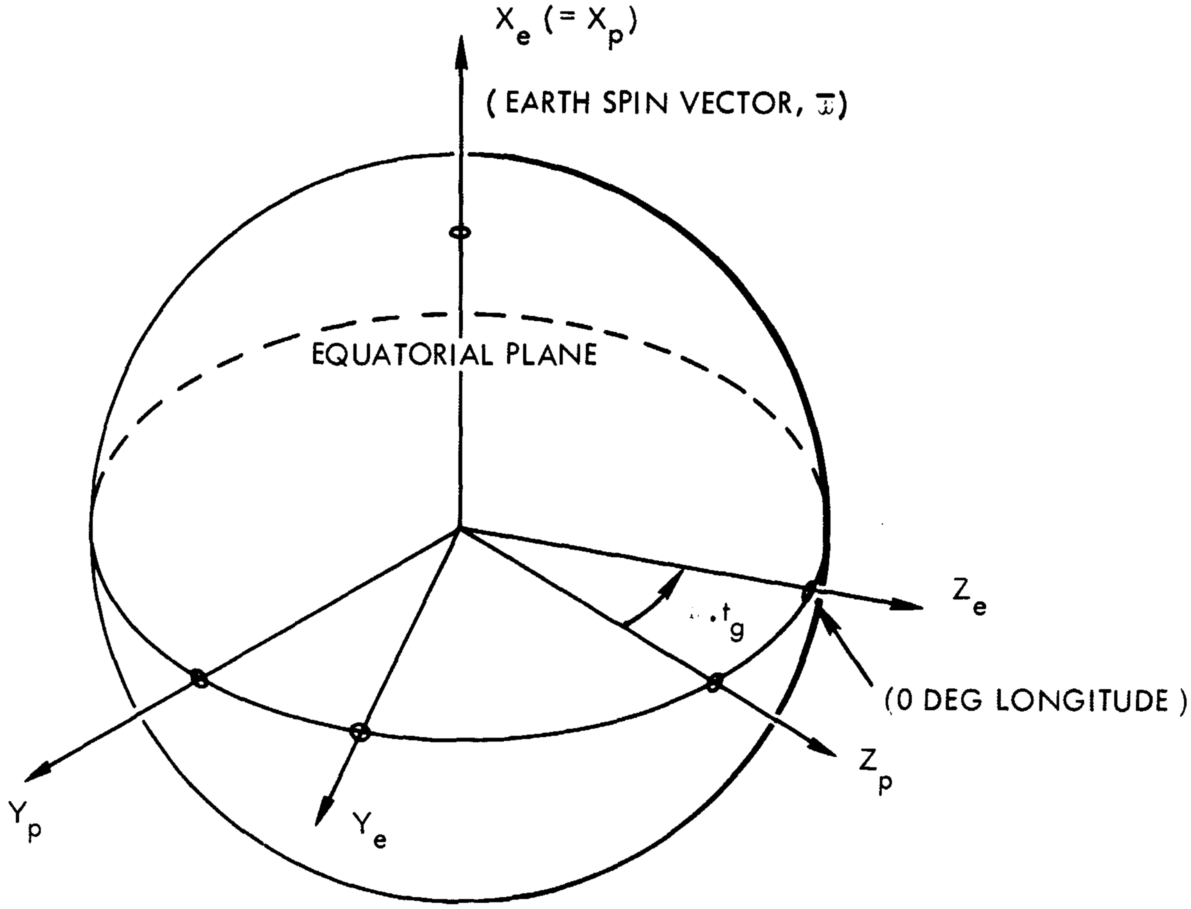

Figure 2-3. Definition of Earth-Fixed Coordinate System (e-System)

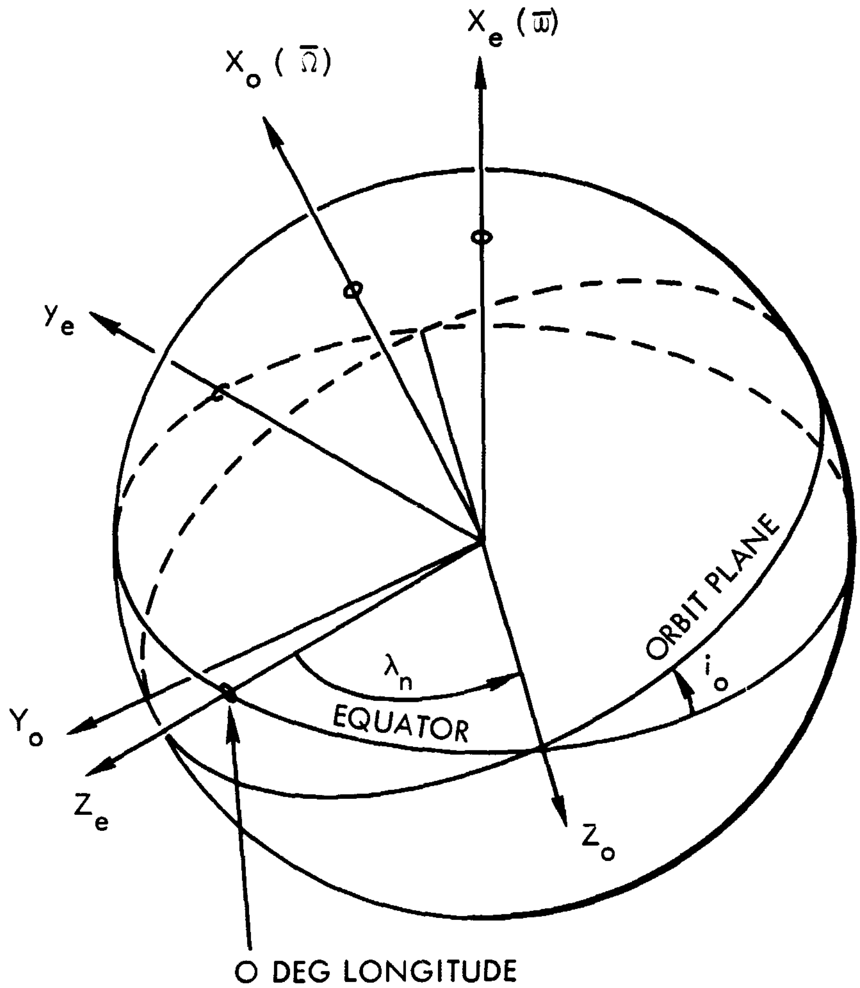

Figure 2-4. Definition of Orbit Coordinate System (o-System)

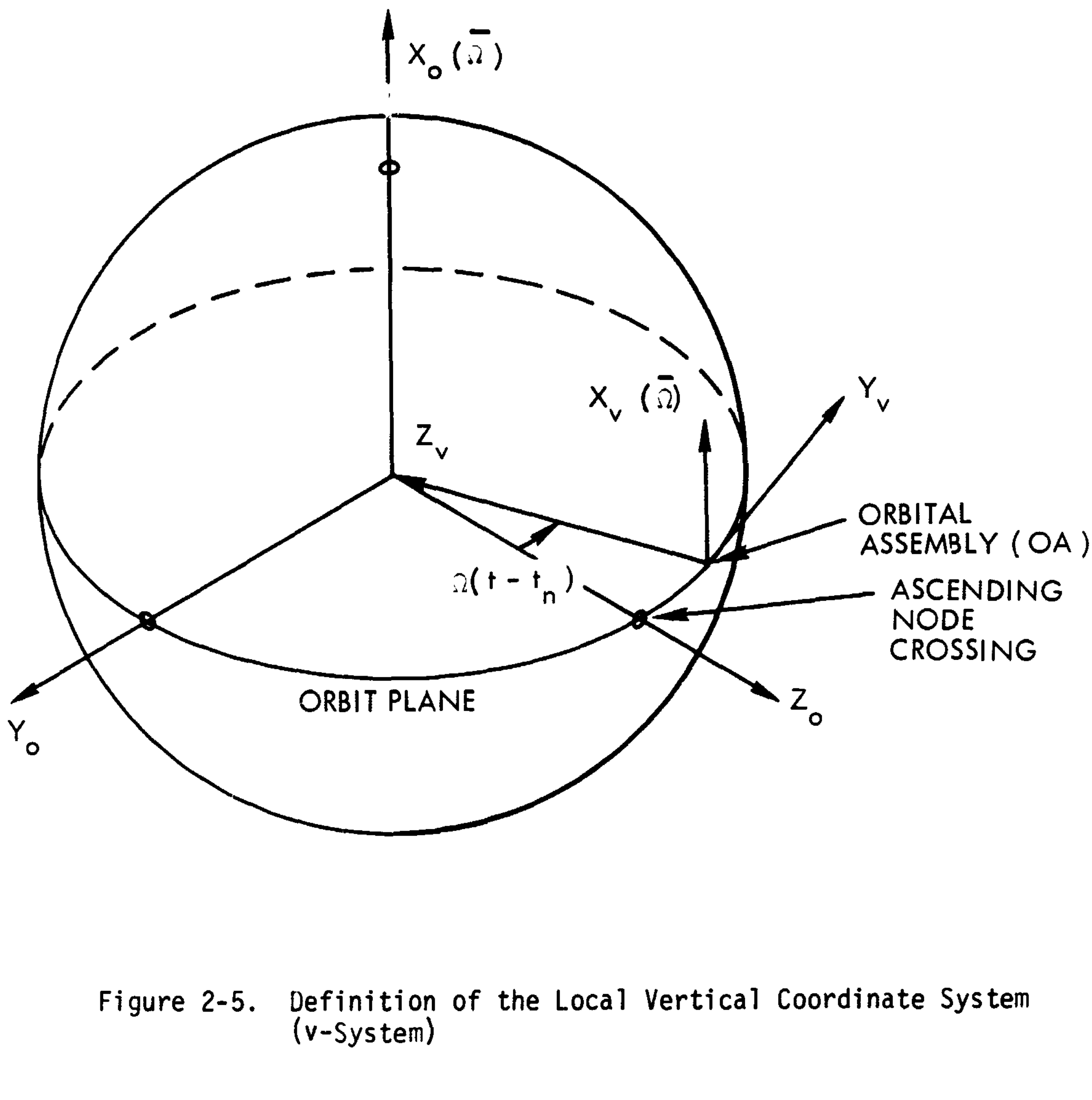

Figure 2-5. Definition of the Local Vertical Coordinate System (v-System)

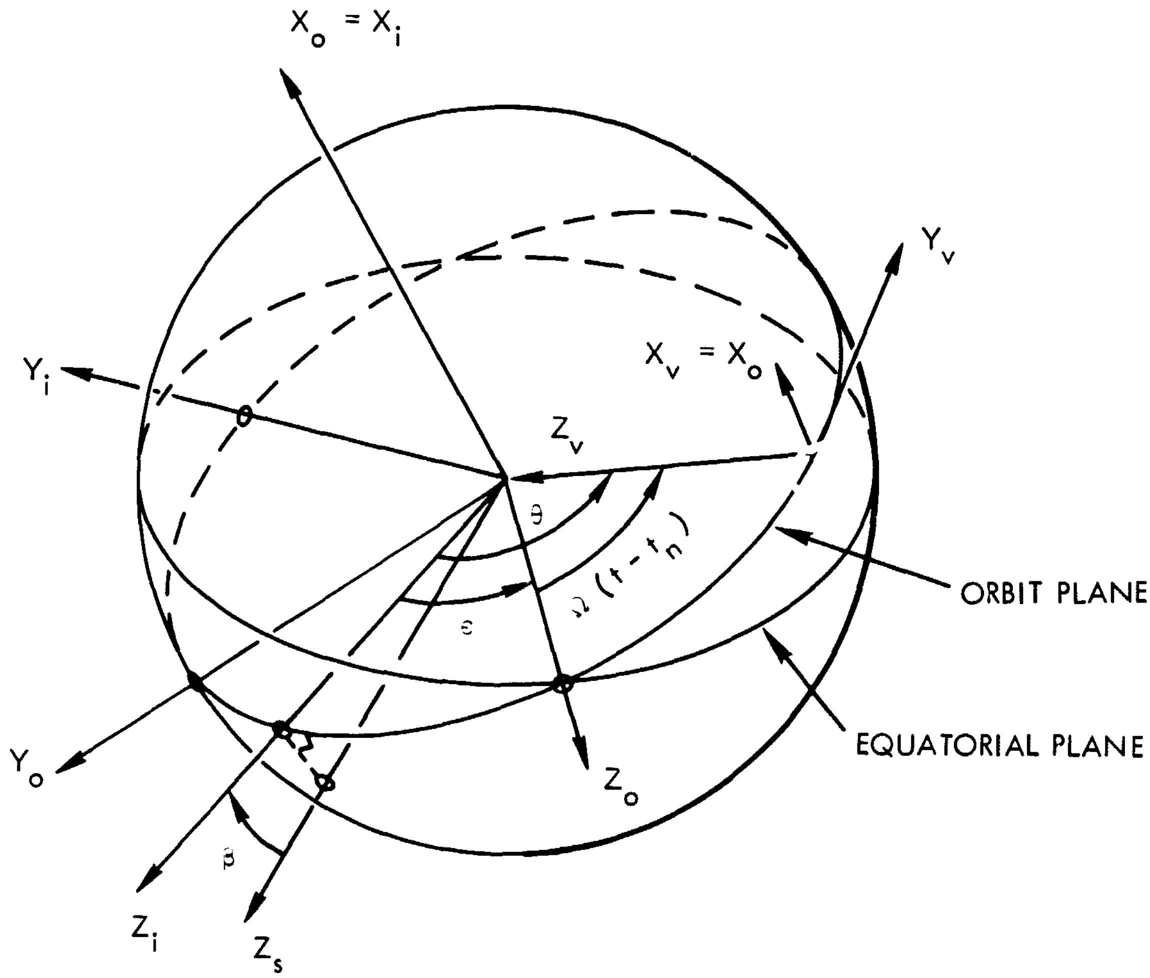

Figure 2-6. Defintion of the i-System

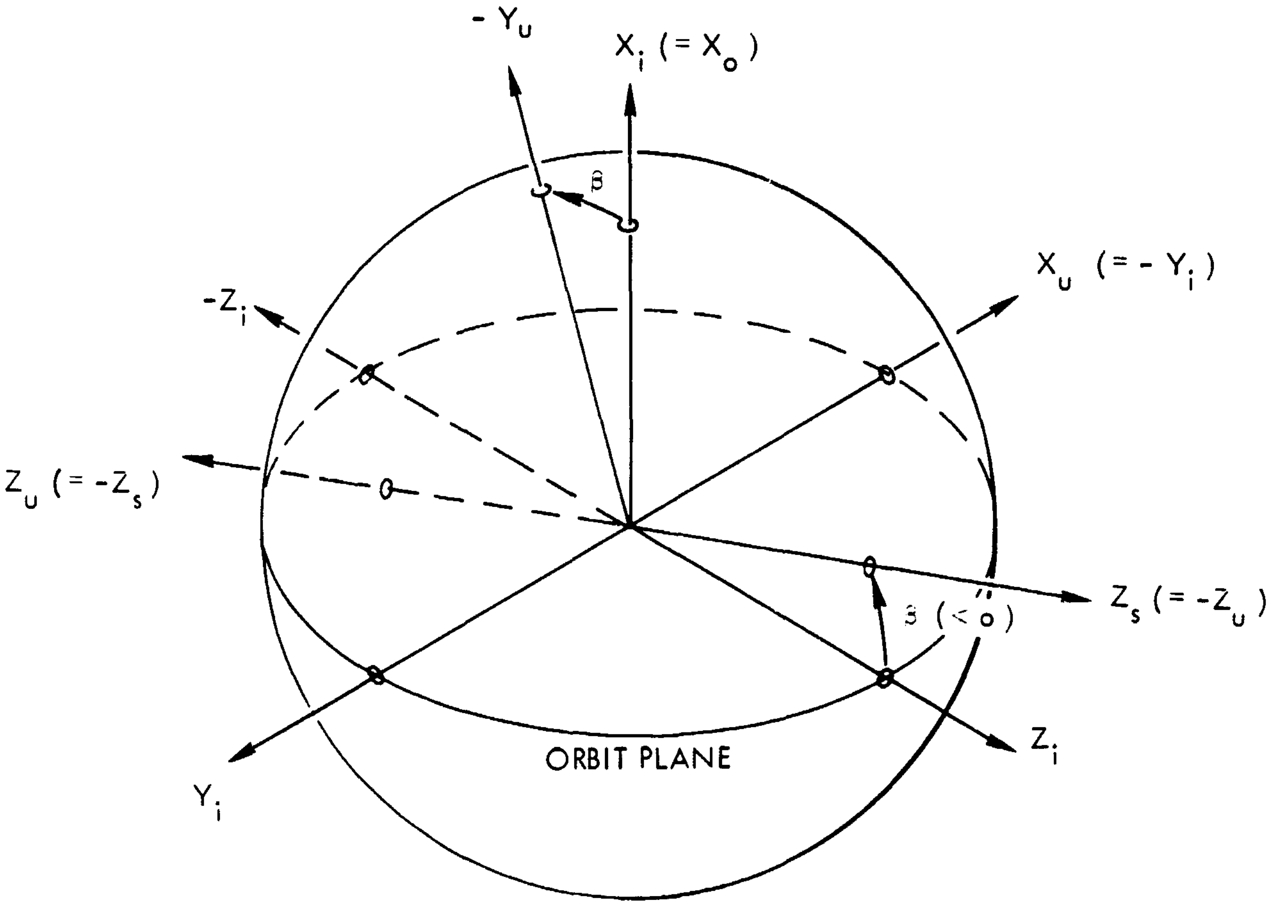

Figure 2-7. Definition of the Solar Inertial Coordinate System (u-System)

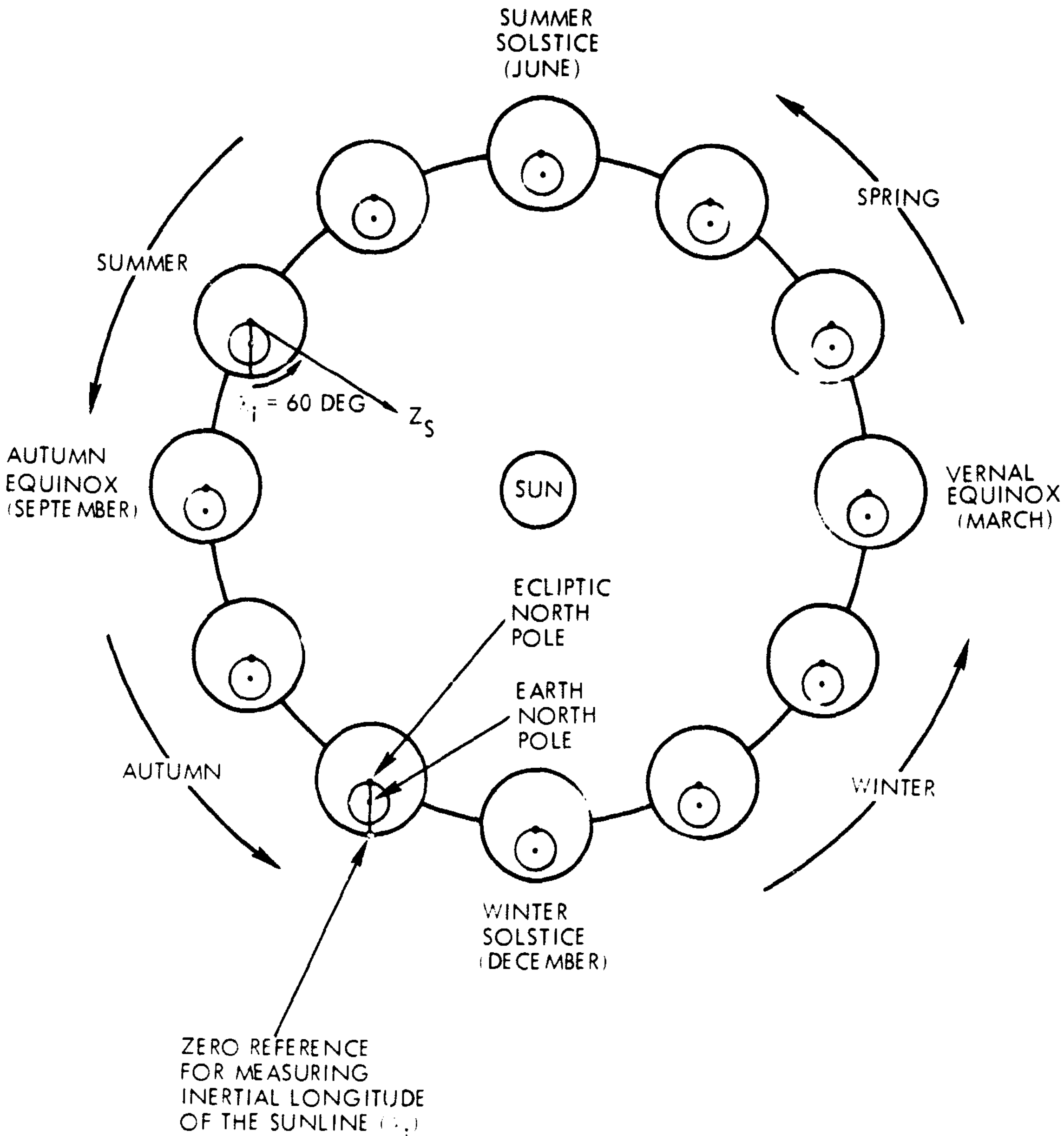

Figure 2-8. Definition of Inertial Longitude of the Sunline, λ_i

3. DISCUSSION OF EQUATIONS AND LOGIC TO BE PROGRAMMED

Appendixes A through H describe the detailed requirements for the orbital workshop simulation. An explanation of the functions performed in each of these appendixes is given in this section. All parameters identified in this section and Appendixes A through I are defined in Section 4.

3.1 APPENDIX A - PROGRAM TIMING FLOW DIAGRAM

In this appendix, the computations to determine the slow loop integration step size are detailed and the overall program timing is illustrated.

The fast loop, which contains the control systems, sensors, and some dynamics and other associated computations, is performed every Δt seconds (real time). The slow loop integration step size, which is computed on the basis of disturbance torque magnitude as reflected in body rates, is variable and ranges from Δt to TBD seconds. Phasing checks are also made in the fast loop to determine if any mode or parameter change is required.

3.2 APPENDIX B - SLOW LOOP ENVIRONMENT

Included within this appendix are the equations used to generate aerodynamic torques, gravity gradient torques, the latitude and longitude coordinates of vehicle position, vehicle orbit position, and matrix transformations associated with vehicle attitude. Some of the initialization is also specified in this appendix. The inertia matrix is inverted, the magnitude of the velocity vector is determined, and orbital rate is also computed. The initial body-to-local-vertical transformation and the initial values for other orbital parameters, such as the time of the last ascending node crossing, are also computed. In addition, the star tracker misalignment transformation is generated and the quaternions are initialized.

The slow loop is also entered periodically (some multiple of fast loop period) to update orbit position, β angle, slowly varying torques such as gravity gradients and aerodynamic torques, vehicle latitude and longitude coordinates, and the transformation between various coordinate systems.

Aerodynamic torques are determined on the basis of the computed vehicle angle of attack and tables of aerodynamic coefficients stored as functions of angle of attack. The diurnal bulge (i.e. variations in atmospheric density due to solar radiation) is taken into account.

Gravity gradient torques are computed in body coordinates on the basis of the vehicle inertias and elements of the body-to-local-vertical rotation matrix cosines.

Latitude and longitude are computed on the assumption of a circular orbit and a spherical earth. Orbital precession and earth rotation are taken into account. Orbit position is computed with the assumption of a circular orbit also.

3.3 APPENDIX C - FAST LOOP ENVIRONMENT

In Appendix C, the fast loop equations are specified. The first computation made determines external disturbance torques sucn as crew motion docking and venting. Then this torque is summed with the other external torques acting on the vehicle including control torque. Following this, orbit position is updated. The Euler equations, which specify the vehicle motion, and the quaterions, which specify vehicle attitude, are then integrated simultaneously to obtain vehicle angular rate and vehicle attitude relative to the inertial reference frame. The angles which the sun vector makes with the body X- and Y-axes are computed for sun sensor inputs if these angles are required (i.e., solar inertial daylight mode).

3.4 APPENDIX D - SENSORS

In this appendix, the sensor subroutines are called as required. The Rate Gyro Subroutine 23 is entered each pass through the fast loop. The vehicle rates are processed through the rate gyro model to determine measured rates.

The Sun Sensor Subroutine 24 is entered only in the solar inertial daylight mode. The sun vector angles are processed through the sun sensor model to determine measured sun angles.

3.5 APPENDIX E - CONTROL SYSTEMS

In this appendix, the mode logic which is present in the ATM DC is modelled. The flags which specify the mode of control are tested each pass through the fast loop and appropriate control subroutines are called on the basis of these flags. The first test made is for an inertial hold and maneuver mode. If this is the mode of control, it is further determined whether the maneuver to be defined is an auto maneuver or a momentum management maneuver and the appropriate subroutine is called. This is only done once per maneuver and the new maneuver flag, ITNM, is then set equal to zero.

The pointing control system control (PCS) mode status is then checked. If the PCS is in standby, further PCS mode checks and all PCS control functions are bypassed. If the PCS is in the cage mode, the Gimbal Caging Subroutine 5 is called and the commanded control moment gyroscope (CMG) gimbal drive rates are computed for use in the CMG dynamics. If the PCS is in the solar inertial mode, the Orbit Darkness Subroutine 6 is entered to determine if the vehicle is in daylight. Following this the Pseudo-Axis Maneuver subroutine is entered to determine if a Y-axis momentum sampling is required. A mode switch will occur if darkness has just been entered on this fast loop pass. A momentum management maneuver will be called for unless inhibited. The Solar Inertial Daylight Rate and Attitude Error subroutine is then entered to determine vehicle error signals prior to entering the PCS Control Law subroutine.

If the PCS is in the local-vertical mode, the Local Vertical Rate and Attitude Error subroutine is called and vehicle error signals are generated prior to entering the PCS Control Law subroutine. If the PCS is in the inertial hold and maneuver mode, the vehicle errors are generated in the Auto Maneuver Rate and Attitude Error Subroutine 14 prior to entering the PCS Control Law subroutine.

CMG gimbal drive commands are generated in the PCS Control Law subroutine and then processed through the CMG Dynamics subroutine to obtain the control torque. The CMG gimbal positions are then determined by integrating the gimbal rates in the CMG Gimbal Integration subroutine. The total momentum stored in the CMG system is then determined in subroutine 10. If the momentum exceeds some allowable limit, then the appropriate flags for a TACS momentum dump are set.

After all the PCS mode checks, the TACS control mode status is checked. If the TACS is in standby, all further TACS mode checks and control functions are bypassed. If the TACS is in the inertial hold and maneuver mode, subroutine 14 is called to generate attitude and rate error signals. If the TACS is in the local vertical mode, subroutine 8 is called to generate attitude and rate error signals. The TACS phase plane and jet select logic subroutine is implemented with these errors to determine if jet firings are required. The TACS control torque is then computed in subroutine 20, and the TACS fuel consumed is computed in subroutine 22. If the TACS is in the momentum dump mode, subroutine 21 is called to calculate the jet impulse required.

3.6 APPENDIX F - INITIALIZATION

In this appendix, certain computations are performed which are required only once for the control section. The portion of the orbit which is in darkness is determined. If the vehicle is always in daylight, a flag is set. The portion of the orbit to be used for gravity gradient dumping is also set. An angular increment used to determine whether a momentum sampling is required for use in the Pseudo-Axis Subroutine 12 is also computed. The TACS impulse which has been consumed is initialized. Several other dependent parameters which are used in the Momentum Management and Pseudo-Axis subroutines are computed in this section.

3.7 APPENDIX G - CONTROL SYSTEM SUBROUTINES

In this appendix, all the subroutines associated with the TACS and PCS are detailed. These subroutines contain all the ATM DC, TACS, and PCS control equations, and the TACS jet and CMG dynamics models and will be called as required each pass through the fast loop. The subroutines to be implemented each pass are specified in the control mode checks (Appendix E) or are called by other subroutines in this section. Most of these equations are in control system areas which are not well defined at this time.

Therefore, the description of these equations will be supplied at a later date. For the most part References 1, 2, 3 and 4 were used to define the PCS and TACS. Where other information was required References 5 and 6 were consulted.

3.8 APPENDIX H - SENSOR SUBROUTINES

This appendix describes the models used to simulate the transfer functions and input - output characteristics of the sensors (acquisition sun sensors and rate gyros). The sensor responses are represented by difference equations reduced from the Z-transforms of the transfer functions (with some simplifications made). The rate gyro saturation is represented by an output rate limit. The sun sensor nonlinearities have been determined by fairing in straight lines over the input-output characteristic curves.

3.9 APPENDIX I - OUTPUT PARAMETER COMPUTATIONS

This appendix contains the necessary computations to generate output parameters which differ from variables being generated in the fast and slow loops of the program. The only parameters of this nature identified to date are pointing errors. The angles which will be used to determine pointing error are adequately defined in Appendix I.

4. PARAMETER DEFINITION LIST

All the symbols used in the equations presented in the appendixes and the text are listed and defined in this section. The programmed equations assume all parameters are in English units, that is, in feet, pounds, and seconds. If other units are required, the conversion factor between the specified units and the foot-pound-second units is a required input for the input processor. Angular inputs are all expected to be in degrees. The input processor then converts these parameters to radians. More details on the input format will be given in the user's manual.

Parameter	Definition
$[A]$	a 3 x 3 control input matrix
a	quaternion defining orientation of body axes relative to the p-system
$[A]^{*}_{b'u}$	transformation relating the b′-system to the u-system, used in the auto maneuver routine
$[A]^{**}_{b'u}$	transformation defined by three successive rotations, ϕ_f, θ_f, and ψ_f, which specifies the desired final orientation of the b′-system relative to the u-system resulting from an auto maneuver to an arbitrary inertial attitude
$[A]_{b'uf}$	transformation used in auto maneuver calculations which relates the b′-system at maneuver termination (desired) to the u-system
$[A]_{b'ui}$	transformation used in auto maneuver calculations which relates the b′-system at the beginning of the maneuver to the u-system
$[A]_c$	transformation used in auto maneuver calculations which relates initial orientation to final orientation
$[A]_{ef}$	direction cosine matrix which rotates the e-system into the f-system (e or f = s,p,e,v,b, i,b′,u)
A_i	coefficients (momentum management) (i = 2,3)
a_{ii}	ii^{th} element of $[A]_{b\gamma}$

Parameter	Definition
a'_{ij}	coefficients (momentum management) (i = 2,3; j = 2,3)
A_{ref}	aerodynamic reference area
$*A_{rg}$	rate gyro characteristic constant
*ASLOP	parameter used in defining nonlinearity of sun sensors
$*A_{ss}$	sun sensor characteristic constant
*ASUNI	parameters used in defining nonlinearity of sun sensors (I = 1,2,3)
*ATH	constant used for curve fit of thrust versus total accumulated impulse
[B]	a 3 x 3 control input matrix
b	quaternion defining orientation of body axes relative to the p-system
B_{ii}	ii element of the matrix [B]
B'_{ij}	coefficients (momentum management) (i = 2,3; j = 1,2,3)
$*B_o$	a positive constant used in PCS control law implementation
$*B_{o1}$	a positive constant used in PCS control law implementation
*BPPI	abscissa in I^{th} phase plane break point I = 1, 2, ..., 10)
*BPRI	ordinate in I^{th} phase plane break point I = 1, 2, ..., 10)
$*B_{rg}$	rate gyro characteristic constant
$*B_{ss}$	sun sensor characteristic constant
*BTH	constant used for curve fit of thrust versus total accumulated impulse
b_1, b_2, b_3	elements of the third column of the direction cosine matrix $[A]_{vb}$
[C]	a 3 x 3 matrix related to [A] and [B]
c	quaternion defining orientation of body axes relative to the p-system

Parameter	Definition
c_{ii}	ii element of [C]
**c_l, c_m, c_n	aerodynamic moment coefficients about aero reference point
c_l', c_m', c_n'	aerodynamic moment coefficients about vehicle cm
c_{m1}, c_{m2}	moment management maneuver rate direction cosines
*c_o	positive constant used in PCS control law implementation
$\hat{c}_p$	unit vector in body coordinates defining axis of rotation during auto maneuver
c_{pi}	X, Y, and Z components of $\hat{C}_p$ (i = 1,2,3)
*C_{rg}	rate gyro characteristic constant
*c_{ss}	sun sensor characteristic constant
*CTH	constant used for curve fit of thrust versus total accumulated impulse
C_{tion}	computed TACS jet on time to dump CMG momentum about i-axis (i = X,Y,Z)
**c_y, c_z, c_a	aerodynamic normal force coefficients
D	value of determinant (momentum management)
d	quaternion defining orientation of body axes relative to the p-system
**D_{ref}	aerodynamic reference length
*D_{rg}	rate gyro characteristic constant
*DTH	constant used for curve fit of TISP versus total accumulated impulse
$\|e\|$	rss of the four quaternions a,b,c, and d
*ETH	constant used for curve fit of TISP versus total accumulated impulse
E_x, E_y, E_z	equal to E_1, E_2 and E_3, respectively

Parameter	Definition
E_1, E_2, E_3	phase plane points $(\theta_{\varepsilon x}, \dot{\theta}_{\varepsilon x})$, $(\theta_{\varepsilon y}, \dot{\theta}_{\varepsilon y})$, and $(\theta_{\varepsilon z}, \dot{\theta}_{\varepsilon z})$, respectively
f_{ij}	element from i^{th} row and j^{th} column of $[\Lambda]$
$*\bar{F}_j$	unit vector in direction of applied force from jet j
F_{m1}	a flag to indicate the second momentum management maneuver has yet to be initiated
*FTH	constant used for curve fit of TISP versus total accumulated impulse
G	$-3\Omega^2$
GN	the sum of the g_i (i = 1, 2, 3) coefficients
$**g_1$, g_2, g_3	coefficients used in total CMG stored momentum computation to indicate if a CMG is operational ($g_i = 1$) or inoperative ($g_i = 0$)
$\{H\}$	CMG momentum vector, with components H_x, H_y, and H_z
*h	orbit altitude measured from the earth's surface (n mi)
$*H_a$	allowable limit on stored momentum in CMG system
$\{H_i\}$	projection of $\{H\}$ on i^{th} outer gimbal plane, with components H_{i1}, H_{i2}, and H_{i3}
$\{\tilde{H}_i\}$	approximation to the vector $\{H_i\}$, with components $\tilde{H}_{i1}$, $\tilde{H}_{i2}$, and $\tilde{H}_{i3}$
$\{h_i\}$	unit vector along the i^{th} outer gimbal momentum component
H_{mm}	computed momentum available for momentum management maneuvers

Parameter	Definition
H_{mp}	computed momentum available for CMG auto maneuvers
$*H_o$	momentum capacity of each CMG, 2000 ft-lb/sec
$*H_r$	a positive constant used in PCS control law implementation
H_t	total momentum stored in CMG system
H_{tam}	magnitude of total CMG stored momentum which is allowed to be used for momentum management maneuvers
H_{tap}	magnitude of total CMG stored momentum which is allowed to be used for auto maneuvers
H_{tmax}	magnitude of total momentum stored in the CMG system
H_{tm}	magnitude of the maximum expected gravity gradient momentum accumulation used to determine allowable auto maneuver rate
H_{tp}	magnitude of the maximum expected gravity gradient momentum accumulation used to determine allowable auto maneuver rate
H_{xb}, H_{yb}, H_{zb}	bias components of stored momentum in body coodinates
H_{xd}, H_{yd}, H_{zd}	components of momentum to be dumped by TACS
$**H_{xdb}$, H_{ydb}, H_{zdb}	desired components of momentum bias
H_{xmax}, H_{ymax}, H_{zmax}	the maximum accumulated gravity gradient momentum expected during an orbit used to determine momentum available for CMG vehicle maneuvers
H_{yn}	momentum stored in the body Y-axis at the sunset terminator
H_{yn+1}	momentum stored in the body Y-axis at the sunrise terminator
H_{yr}	angular momentum removed from the Y-axis CMG (pseudo-axis maneuver)
IAMM	set equal 1 if momentum management maneuver to be performed
**IAUTOM	set equal 1 if auto maneuver to be performed

Parameter	Definition
**IARBI	set equal 1 if auto maneuver is to an arbitrary inertial attitude
IDARK	set equal 1 if vehicle in darkness
IDAYL	set equal 1 if vehicle always in daylight
**IDELTA	flag set to indicate a delta maneuver from ZLV is to be defined
IDELV	flag set to indicate a delta maneuver from ZLV is in progress
*i_e	inclination of equatorial plane relative to ecliptic plane
**IGGI	flag set 1 to inhibit GG momentum dumping
IICR	flag set 1 if either an auto maneuver residual angular increment or incremental change in attitude is to be added to the present attitude error
*$[I_{ij}]$	vehicle inertia tensor relative to the b-system (i,j = X, Y, Z)
IKMM	flag set to indicate momentum management maneuvers have been defined
IMPC	instantaneous total accumulated impulse
IMPCO	initial value of IMPC
*i_o	inclination of orbit plane relative to equatorial plane
**IPCAGE	set equal 1 if PCS in gyro cage mode
**IPIHM	set equal 1 if PCS in inertial hold and maneuver mode
**IPSI	set equal 1 if PCS in solar inertial hold mode
**IPSTB	set equal 1 if PCS in standby mode
**IPZLV	set equal 1 if PCS in LV hold mode
**ITIHM	set equal 1 if TACS in inertial hold and maneuver mode
ITMOD	set equal 1 if TACS is dumping CMG momentum

Parameter	Definition
**ITNM	set equal 1 if new auto maneuver is to be defined
**ISTAR	set equal 1 if star tracker computation is to be performed
ITSON	set equal 1 if TACS momentum dumping in progress
**ITSTB	set equal 1 if TACS in standby mode
**ITZLV	set equal 1 if TACS in LV hold mode
**I_x, I_y, I_z	estimated values of principal moments of inertia stored in control digital computer
IXON	set equal 1 if TACS X-axis jets dumping momentum
IYON	set equal 1 if TACS Y-axis jets dumping momentum
IZON	set equal 1 if TACS Z-axis jets dumping momentum
**IZLV	set equal 1 if auto maneuver is to ZLV
JFLAG(I)	jet-on flag for jet I (I = 1, 2, ...6)
*K_{b1}	a positive constant used in PCS control law implementation
*K_g	a gain constant used in computing gimbal rate drive during caging
K_h	a positive variable used in PCS control law implementation
*K_i	a positive constant used in PCS control law implementation
*K_i'	a positive constant used in PCS control law implementation
*K_m	a positive constant used in PCS control law implementation
KJET	parameter to indicate if a jet is firing

Parameter	Definition
$*K_{rgi}$	rate gyro output bias and scale factor constants corresponding to each body axis (i = 1, 2, ..., 6)
$*K_{rt}$	a positive constant used in PCS control law implementation
$*K_{ss1}$	sun sensor output scale factor about Y-axis
$*K_{ss2}$	sun sensor output bias about Y-axis
$*K_{ss3}$	sun sensor output scale factor about X-axis
$*K_{ss4}$	sun sensor output bias about X-axis
$*K_1$, K_2, K_3, K_4	constants used in computation of air density
$*K_{1\alpha}$, $K_{2\alpha}$, $K_{3\alpha}$	a number related to the i^{th} main diagonal component of $[K_\alpha]$
$*K_{1\beta}$, $K_{2\beta}$, $K_{3\beta}$	main diagonal elements of $[K_\beta]$
$*K_{1\Omega}$, $K_{2\Omega}$, $K_{3\Omega}$	main diagonal elements of $[K_\Omega]$
$*K_{1\theta}$, $K_{2\theta}$, $K_{3\theta}$	main diagonal elements of $[K_\theta]$
$[K_\alpha]$	a 3 x 3 weighting matrix used in PCS control law implementation
$[K_\alpha^2]$	a 3 x 3 matrix related to $[K_\alpha]$ used in PCS control law implementation
$[K_\beta]$	a 3 x 3 weighting matrix used in PCS control law implementation
$[K_\beta^2]$	a 3 x 3 matrix related to $[K_\beta]$ used in PCS control law implemention
$[K_\Omega]$	a 3 x 3 weighting matrix used in PCS control law implementation
$[K_\theta]$	a 3 x 3 weighting matrix used in PCS control law implementation

Parameter	Definition
$\{M_c\}$	total torque command vector
$\{M_{cd}\}$	desired torque command vector
$\{M_{cf}\}$	filtered torque command vector
M_{cfi}	i^{th} component of the vector $\{M_{cf}\}$
$\{M_{cfi}\}$	projection of $\{M_{cf}\}$ on i^{th} outer gimbal plane
M_{cfij}	j^{th} component of the vector $\{M_{cfi}\}$ ($j = 1, 2, 3$)
$^*M_{cflim}$	filtered torque command limit, a positive constant
NJET	total jet-on time during TACS computation cycle, Δt
$\overline{P}_j$	position vector of jet j, with components P_{xj}, P_{yj}, P_{zj}, ($j = 1,2,\ldots,6$)
PFLAG(I)	minimum impulse flag ($I = 1, 2, \ldots, 6$)
*PQRLIM	limit on rate gyro output for each gyro
Pt	rate error coordinate corresponding to the position error on those positions of the phase plane switching lines having non-zero slope
q_p	dynamic pressure
$\overline{R}$	vehicle angular rate vector, with components P, Q, and R
$\dot{\overline{R}}$	time rate of change of $\overline{R}$
*r	radius of the earth
$\overline{R}_c$	commanded body rate vector, with components P_c, Q_c, and R_c
[RG]	matrix defining the measured vehicle rates for use in determining the required CMG torque
$^*\overline{r}_j$	position vector of jet j, relative to vehicle c. m.

Parameter	Definition
$*\bar{R}_o$	initial value of $\bar{R}$
$\bar{R}_{rg}$	measured body rate vector, with components P_{rg}, Q_{rg}, and R_{rg}
[S]	matrix in the auto maneuver routine used in determining $\hat{C}_p$ for the case of $\gamma_c \geq 170$ degrees
*SF1	safety factor to reserve a fraction of the CMG momentum for transients during auto maneuvers
*SF2	safety factor to reserve a fraction of the CMG momentum for transients during moment management maneuvers
*SMALL	a flag used in the computation of the slow loop computation cycle to indicate if a reduction in Δt has been calculated
$*S_{rg}$	rate gyro response about X-axis
t	time from run initiation
$\bar{T}_a$	aerodynamic torque vector with components T_{ax}, T_{ay}, and T_{az}
*TATH	TACS thrust
$\bar{T}_{cj}$	command TACS torque by jet j, with components T_{xcj}, T_{ycj}, and T_{zcj} (j = 1, 2, ..., 6)
$\bar{T}_{cp}$	control torque, generated from PCS, with components T_{cpx}, T_{cpy}, and T_{cpz}
$\bar{T}_{ct}$	control torque, generated from TACS, with components T_{ctx}, T_{cty}, and T_{ctz}
TF	time at which auto maneuver is to be terminated
$\bar{T}_g$	gravity gradient torque vector with components T_{gx}, T_{gy}, and T_{gz}
t_g	instantaneous Greenwich time (referenced to noon)
$*t_{g0}$	Greenwich time at run initiation

Parameter	Definition
TI	time at which next momentum management maneuver is to be initiated
t_{io}	time to initiate a momentum management maneuver (i = 2, 3)
t_{if}	time to terminate a momentum management maneuver (i = 1, 2, 3)
*TISP	TACS specific impulse
$t_{(j+1)0}$	equals t_{io} (time to initiate a momentum management maneuver)
$t'_{(j+1)0}$	adjusted time to initiate a momentum management maneuver (j = 1, 2)
$*t_{I}$	minimum impulse time
t_n	time of the last crossing of the ascending node
TPC	TACS propellant consumed
$*T_{rg}$	rate gyro response about Y-axis
$**\bar{T}_r$	impulsive torque vector input in tabular form which represents crew motion, docking, venting, etc., with components T_{rx}, T_{ry}, and T_{rz}
$*t_{rui}$	abscissas in tabular input of T_{ru} versus time where u = x, y, z, i = 1, 2, 3
T_{slow}	time at which the slow loop was last entered
$*T_{xn}$, T_{yn}, T_{zn}	nominal torque components expected from TACS thrusters
T_{xon}, T_{yon}, T_{zon}	simulation time at which x, y, and z jets are shut off after TACS momentum dump
$*U_{rg}$	rate gyro response about Y-axis
U_1, U_2, U_3	elements of a vector in auto maneuver routine which lies along the unit vector $\hat{C}_p$
v	magnitude of vehicle orbital velocity ($\Omega \cdot [h + r]$)

Parameter	Definition
V_{xb}, V_{yb}, V_{zb}	components of vehicle velocity vector in body coordinates
**X_{cm}, Y_{cm}, Z_{cm}	components of vehicle center of mass in vehicle coordinates which are used in aerodynamic calculations
**XBAR, YBAR, ZBAR	coordinates of vehicle c.m. in body coordinates
*XYZ	scale factor used in the adjustment of the slow loop computation cycle
α	total angle of attack
$\{\alpha\}$	outer gimbal angle vector, with components α_1, α_2 and α_3
$\{\dot{\alpha}\}$	time rate of change of $\{\alpha\}$
$\{\dot{\alpha}_c\}$	commanded outer gimbal rate vector with components $\dot{\alpha}_{1c}$, $\dot{\alpha}_{2c}$, and $\dot{\alpha}_{3c}$
$\{\dot{\alpha}_{cd}\}$	desired outer gimbal rate command vector
$\{\dot{\alpha}_{ga}\}$	outer gimbal rate vector for gimbal avoidance computations, with components $\dot{\alpha}_{1ga}$, $\dot{\alpha}_{2ga}$, and $\dot{\alpha}_{3ga}$
$\dot{\alpha}_{ige}$	i^{th} outer gimbal rate error signal (i = 1, 2,3)
$\dot{\alpha}_{ilim}$	a quantity related to α_{1im} (i = 1, 2, 3)
*$\dot{\alpha}_{lim}$	outer gimbal rate limit
*α_{lm}	outer gimbal limit
**α_{1d}, α_{2d}, α_{3d}	desired CMG outer gimbal caging angles
β	angle formed by the vehicle radius vector from the center of the earth at orbit noon and the vector from the center of the earth to the sun; $\beta > 0$ when the sun and the orbit momentum vector lie on opposite sides of the orbit plane
*$\{\beta\}$	gimbal angle vector with components β_1, β_2, and β_3

Parameter	Definition
$\{\dot{\beta}\}$	time rate of change of $\{\beta\}$
$\{\dot{\beta}_c\}$	commanded inner gimbal rate vector, with components β_{1c}, β_{2c}, and β_{3c}
$\{\dot{\beta}_{cd}\}$	desired inner gimbal rate command vector
$\{\dot{\beta}_{ga}\}$	inner gimbal rate vector for gimbal avoidance computations, with components β_{1ga}, β_{2ga}, and β_{3ga}
$*\dot{\beta}_{lim}$	inner gimbal rate limit
$*\beta_{lm}$	inner gimbal limit
$**\beta_{1d}$, β_{2d}, β_{3d}	desired CMG inner gimbal caging angles
Γ	rotation about the ecliptic north pole which transforms the s-system into the γ-system
$[\gamma]$	a matrix defining the transformation from body to principal coordinates
γ_c	total angle of rotation used in auto maneuver routine
γ_{1j}	element from row 1 and column j of the transformation matrix from body to principal axes
ΔI_{ij}	$I_i - I_j$ $(i, j = x, y, z)$
Δ_m	magnitude of momentum stored in the CMG system in the same direction as the momentum change desired for a momentum management maneuver
Δ_p	magnitude of momentum stored in the CMG system in the same direction as the momentum change desired for an auto maneuver
$*\Delta t$	fast loop computation cycle
Δ_t	root sum square of ΔX, ΔY, and ΔZ

Parameter	Definition
$\Delta\theta$	fraction of the total orbit angle traversed in one fast loop cycle
ΔT_c	time duration of auto maneuver
$**\Delta_x, \Delta_y, \Delta_z$	desired incremental change in present attitude vector ($=\bar{\Delta}$), with rotation order equal X,Y,Z
$\Delta\lambda$	difference between longitude of ground track and longitude of ascending node
$\Delta\rho_j$	overlap condition indicator ($j = 1, 2$)
$*\Delta\phi_s$	the first of two successive rotations ($\Delta\phi_s$ about x and $\Delta\psi_s$ about z) of the b-system into the b′-system
$\Delta\psi$	angle about vehicle Z-axis through which vehicle must be rotated to compensate for Y-axis momentum accumulation expected to be due to principal X-axis being out of tne orbit plane
$*\Delta\psi_s$	the second of two successive rotations ($\Delta\phi_s$ about x and $\Delta\psi_s$ about z) of the b-system into the b′-system
$*\delta_{allow}$	time increment used to test the nearness of δt to δt_{max}. If δt is within δ_{allow} seconds of δt_{max} then, even though the angle criterion for expanding δt may have been satisfied, δt is not increased
$*\delta_{max}$	maximum allowable vehicle attitude change about any axis between any two successive slow loop computations. Used as a criterion for computing δt
δ_o	half angle cone subtended from orbit altitude, h
$*\delta t$	slow loop computation cycle
$*\delta t_{max}$	maximum allowable value of δt for a given run
ε	angle used in transforming from i-system to o-system

Parameter	Definition
ε_{ij}	computed momentum management maneuver angles (i = y, z and j =1, 2)
$\dot{\varepsilon}_{ij}$	momentum management maneuver rates (i = x, y, z; j = 1, 2, 3)
η_{tm}	the displacement of dump midnight due to x_b being out of the orbit plane
θ	orbit position angle measured from orbit noon
$*\theta_{bs}$	rotation about body Y-axis to represent a misalignment of the sun sensor with sun vector
θ_{bv}	angle relating the b-system to the v-system
$*\theta_{bvo}$	initial value of θ_{bv}
$**\theta_f$	the second of three Euler rotations defining the final desired vehicle orientation resulting from an auto maneuver to an arbitrary inertial attitude
θ_{1v}	orbit angle at which local vertical Z-mode is initiated
$*\theta_o$	initial value of θ
θ_p	updated value of θ used in fast loop calculations
θ_r	orbit position angle when darkness is entered (measured from orbit noon)
θ_{ss}	sun sensor output about Y-axis
θ_1, θ_2, θ_i	three successive rotations which define the orientation of the p-system relative to the s-system
$\overline{\theta}_\varepsilon$	attitude error vector to be used by control systems (from sun sensors outputs and integrated rate gyro outputs)
$\dot{\overline{\theta}}_\varepsilon$	attitude rate error vector to be used by control systems (from body mounted gyro outputs)
$[\Lambda]$	transformation from the b-system to the current reference system

Parameter	Definition
λ	longitude of ground track
$*\lambda_i$	inertial longitude of sun line (Figure 2-8)
λ_n	longitude of ascending node
$*\dot{\lambda}_n$	rate of change of λ_n
$*\lambda_{no}$	initial value of $\dot{\lambda}_n$
μ	latitude of ground track
$*\mu_g$	earth gravitational constant
ν_z	rotation about Z_b required to put the principal X-axis in the orbit plane
ρ_{air}	air density
$*\rho_{min}$	the minimum allowable momentum management dump half angle
$*\rho_n$	air density at orbit midnight
ρ_r	an angle used to calculate the orbit position from midnight where gg momentum dumping will be initiated or terminated
ϕ	bank angle, used in calculating aerodynamic torques
$*\phi_{bs}$	rotation about body X-axis to represent a misalignment of the sun sensor with sun vector
ϕ_{bv}	angle relating the b-system to the v-system
$*\phi_{bvo}$	initial value of ϕ_{bv}
$**\phi_f$	the first of three Euler rotations defining the final desired vehicle orientation resulting from an auto maneuver to an arbitrary inertial attitude
ϕ_{ss}	sun sensor output about the Y-axis

Parameter	Definition
ψ_{bv}	angle relating the b-system to the v-system
*ψ_{bvo}	initial value of ψ_{bv}
**ψ_f	the last of three Euler rotations defining the final desired vehicle orientation resulting from an auto maneuver to an arbitrary inertial attitude
Ω	magnitude of orbit rate vector, $\bar{\Omega}$
$\bar{\Omega}_b$	orbit rate vector in body coordinates, with components Ω_{xb}, Ω_{yb}, and Ω_{zb}
*ω	magnitude of $\bar{\omega}$
$\bar{\omega}$	earth spin rate vector
ω_l	auto maneuver rate limit
ω_{lp}	computed rate vector magnitude for CMG auto maneuvers
*ω_{lt}	auto maneuver rate magnitude for TACS maneuvers

*Input constant which is required only at program initialization

**Input constant which is required at program phasing points as well as program initialization

APPENDIX A

PROGRAM TIMING FLOW DIAGRAM

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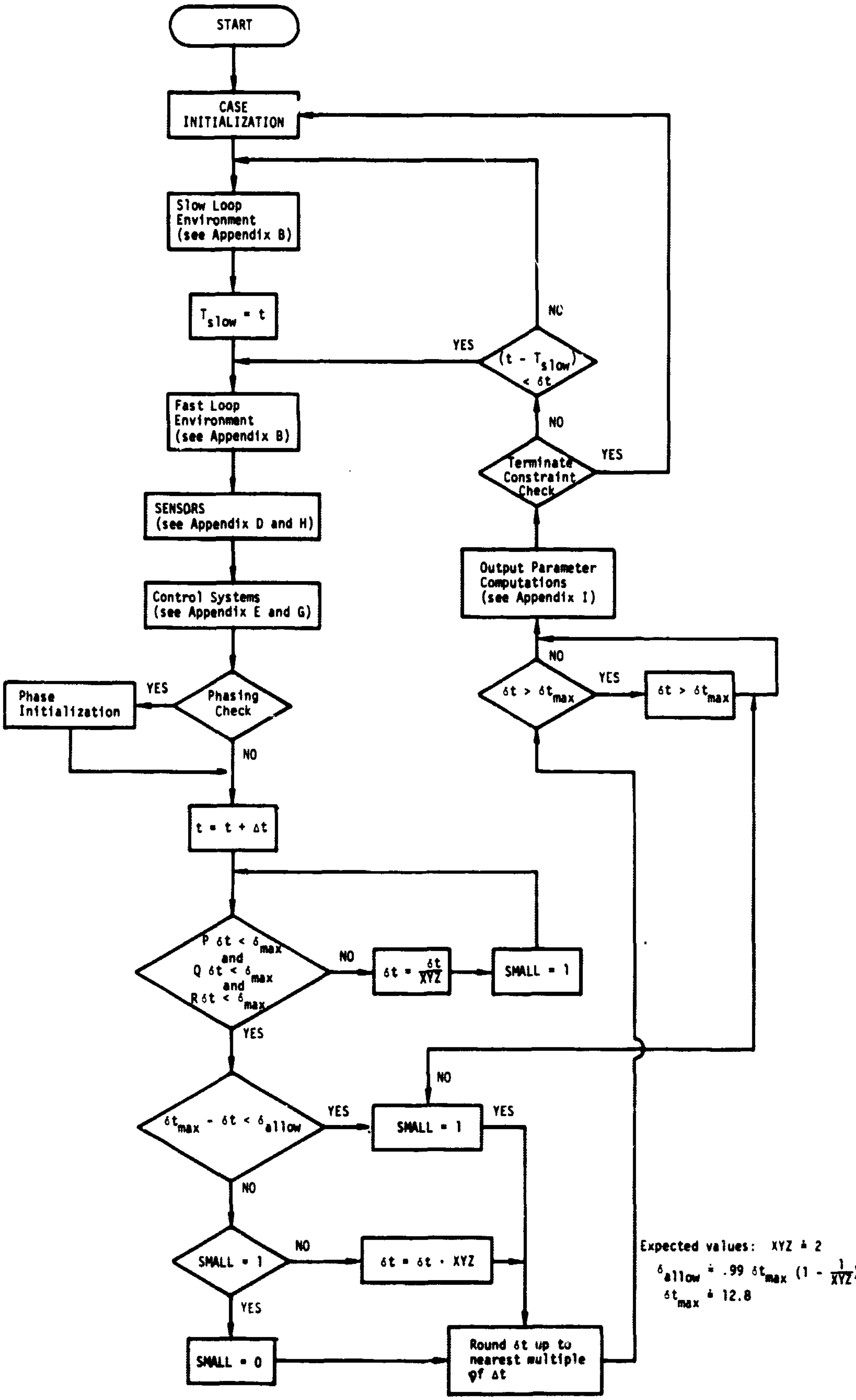

APPENDIX B

SLOW LOOP ENVIRONMENT

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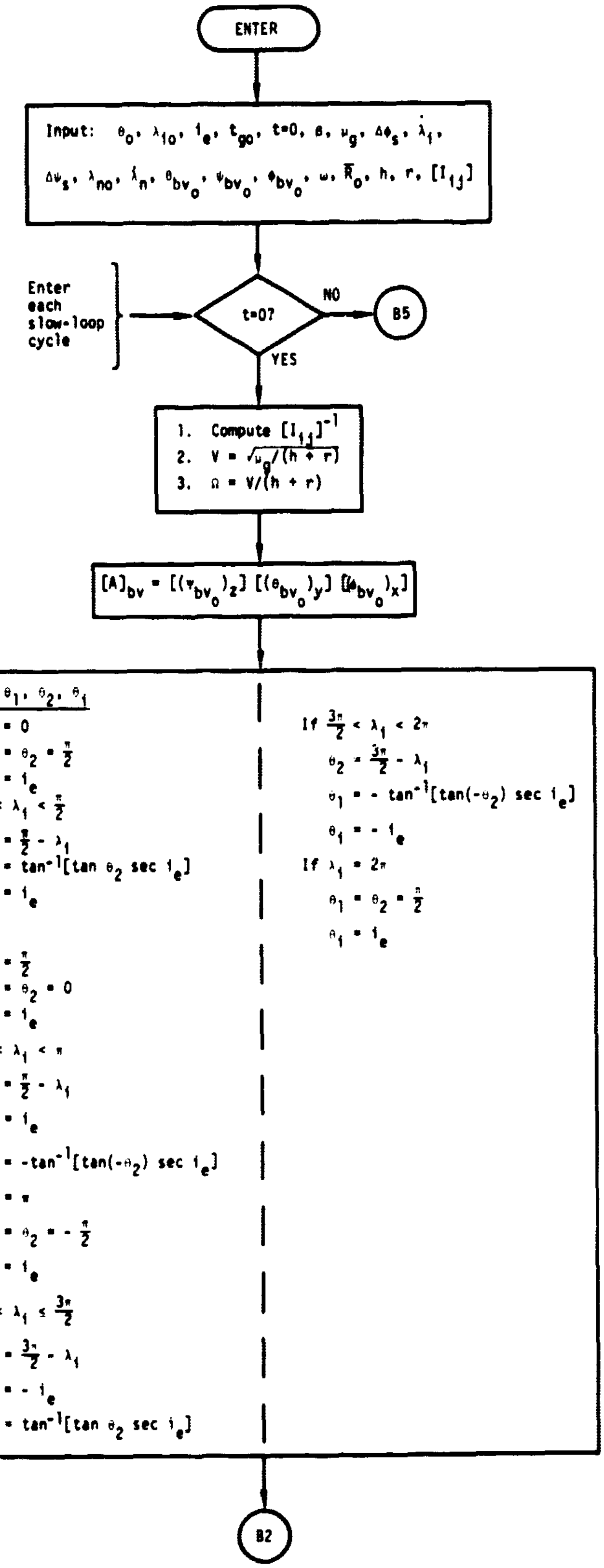


B-2

$$[A]_{ps} = [(-\theta_2)_x][(-\theta_1)_z][(\theta_1)_x]$$

$$[A]_{pe} = [(\omega \cdot t_g)_x]$$

$$[A]_{oe} = [(-\lambda_{no})_x][(-i_o)_z]$$

$$[A]_{os} = [A]_{ps} [A]'_{pe} [A]_{oe}$$

$$[A]_{iu} = \begin{bmatrix} 0 & -1 & 0 \\ -\cos\delta & 0 & -\sin\delta \\ \sin\delta & 0 & -\cos\delta \end{bmatrix}$$

$$[A]_{bb'} = [(\Delta\psi_s)_z][(\Delta\phi_s)_x]$$

$$\Gamma = \frac{3\pi}{2} + \lambda_i$$

$$[A]_{s\gamma} = [(-\Gamma)_x]$$

$$[A]_{p\gamma} = [A]_{s\gamma} [A]_{os}$$

Note: A'_{bb} is solved only once

Solve ϵ

$$\left.\begin{aligned} \sin\epsilon &= [A]_{os}(3,2)/\cos\beta \\ \cos\epsilon &= [A]_{os}(3,3)/\cos\beta \end{aligned}\right\}$$

where $[A]_{os}(ij)$ = element from i^{th} row and j^{th} column of $[A]_{os}$

$$\epsilon = \tan^{-1}\frac{\sin\epsilon}{\cos\epsilon}$$

$$0 \le \epsilon \le 2\pi$$

Solve t_n

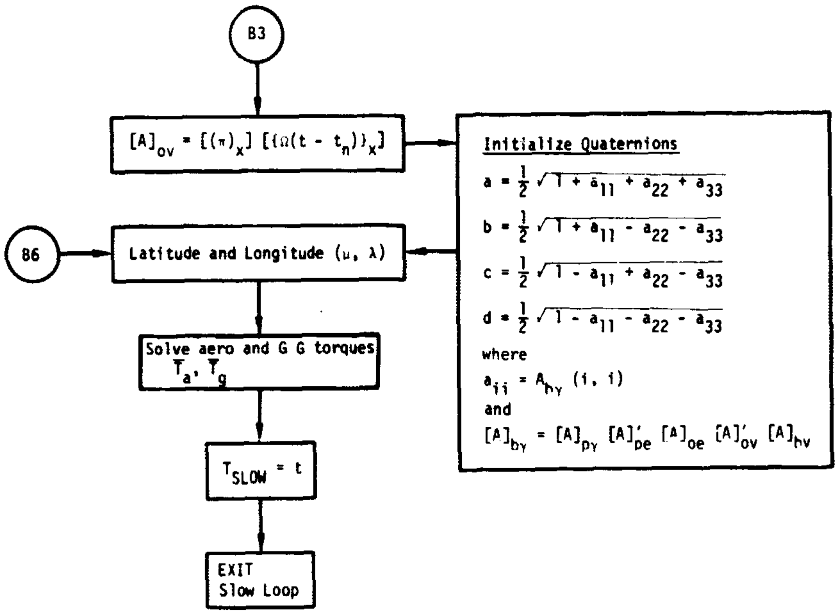
If $\epsilon > \theta_o$

$t_n = (\epsilon - \theta_o - 2\pi)/\Omega$

If $\epsilon \le \theta_o$

$t_n = (\epsilon - \theta_o)/\Omega$

B-3


B3
$[A]_{ov} = [(\pi)_x] [\{\Omega(t - t_n)\}_x]$
Initialize Quaternions
$a = \frac{1}{2}\sqrt{1 + a_{11} + a_{22} + a_{33}}$
$b = \frac{1}{2}\sqrt{1 + a_{11} - a_{22} - a_{33}}$
$c = \frac{1}{2}\sqrt{1 - a_{11} + a_{22} - a_{33}}$
$d = \frac{1}{2}\sqrt{1 - a_{11} - a_{22} - a_{33}}$
where
$a_{ii} = A_{h\gamma}(i, i)$
and
$[A]_{b\gamma} = [A]_{p\gamma} [A]'_{pe} [A]_{oe} [A]'_{ov} [A]_{hv}$
B6
Latitude and Longitude (μ, λ)
Solve aero and G G torques
$\bar{T}_a, \bar{T}_g$
$T_{SLOW} = t$
EXIT
Slow Loop

B-5

$t_g = t_{go} + t$

$[A]_{pe} = [(\omega\, t_g)_x]$

$\lambda_n = \lambda_{no} + \dot{\lambda}_n t, \; \lambda_i = \lambda_i + \dot{\lambda}_i t$

$\Gamma = \frac{3\pi}{2} + \lambda_i$

$[A]_{s\gamma} = [(-\Gamma)_x]$

$[A]_{p\gamma} = [A]_{s\gamma}\, [A]_{ps}$

Solve θ_1, θ_2, θ_i and $[A]_{ps}$ as in $t = 0$ calculations

$[A]_{oe} = [(-\lambda_n)_x][(-i_o)_z]$

$[A]_{os} = [A]_{ps}\, [A]'_{pe}\, [A]_{oe}$

$\beta = \sin^{-1}\, [-A_{os}\ (3,1)]$

$$[A]_{iu} = \begin{bmatrix} 0 & -1 & 0 \\ -\cos\beta & 0 & -\sin\beta \\ \sin\beta & 0 & -\cos\beta \end{bmatrix}$$

Note: $[(-i_o)_z]$ is solved only once

Solve ε as in $t = 0$ calculations.

If $t - t_n \geq \frac{2\pi}{\Omega}$

$t_n = t_n + 2\pi/\Omega$

otherwise, t_n remains unchanged.

Solve Orbit Angle

$\theta = \Omega(t - t_n) + \varepsilon$

$$[A]_{ov} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \left[\left\{\Omega(t-t_n)\right\}_x\right]$$

$$[A]_{bv} = [A]_{ov}\, [A]'_{os} [A]_{bs}$$

B-6

Aerodynamic and Gravity Gradient Torques

Input:

K_1, K_2, K_3, K_θ, A_{ref}, D_{ref}, ρ_n, X_{cm}, Y_{cm}, Z_{cm}, $C_\ell(\alpha,\phi)$, $C_m(\alpha,\phi)$,

$C_n(\alpha,\phi)$, $C_y(\alpha,\phi)$, $C_z(\alpha,\phi)$, $C_a(\alpha,\phi)$

Transform Velocity to Body Coordinates

$$\begin{bmatrix} V_{xb} \\ V_{yb} \\ V_{zb} \end{bmatrix} = [A]_{vb} \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix}$$

Compute Total Angle of Attack

$$\alpha = \cos^{-1} \frac{V_{xb}}{V}$$

$$0 \le \alpha \le \pi$$

Compute Bank Angle

$$\phi = \tan^{-1} \frac{V_{yb}}{V_{zb}}$$

$$0 < \phi < 2\pi$$

Compute Density

$$\rho_{air} = K_1 \left(1 + K_2 e^{-K_3\theta^2}\right)$$

$$\text{for } 2\pi - K_\theta < \theta \le 2\pi \text{ or } 0 \le \theta < K_\theta$$

$$\rho_{AIR} = \rho_N$$

for $K_\theta \leq \theta \leq 2\pi - K_\theta$

Compute Dynamic Pressure

$$q_p = 1/2\, \rho_{air} v^2$$

Compute Aero Moment Coefficients About Vehicle Center of Mass

$$C_m' = C_m + C_a \frac{Z_{cm}}{D_{ref}} - C_z \frac{X_{cm}}{D_{ref}}$$

$$C_n' = C_n - C_y \frac{X_{cm}}{D_{ref}} - C_a \frac{Y_{cm}}{D_{ref}}$$

$$C_\ell' = C_\ell + C_y \frac{Z_{cm}}{D_{ref}} + C_z \frac{Y_{cm}}{D_{ref}}$$

Compute Aerodynamic Torques

$$T_{ax} = q_p A_{ref} D_{ref} C_\ell'$$

$$T_{ay} = q_p A_{ref} D_{ref} C_m'$$

$$T_{az} = q_p A_{ref} D_{ref} C_n'$$

Compute Gravity Gradient Torques

$$G = -3\,\omega^2$$

$$T_{gx} = G[b_2\, b_3\, (I_{yy} - I_{zz}) - b_1\, b_2\, I_{xz} - (b_2^2 - b_3^2)\, I_{yz} + b_1\, b_3\, I_{xy}]$$

$$T_{gy} = G[b_1\, b_3\, (I_{zz} - I_{xx}) - b_2\, b_3\, I_{xy} - (b_3^2 - b_1^2)\, I_{xz} + b_1\, b_2\, I_{yz}]$$

$$T_{gz} = G[b_1\, b_2\, (I_{xx} - I_{yy}) - b_1\, b_3\, I_{yz} - (b_1^2 - b_2^2)\, I_{xy} + b_2\, b_3\, I_{xz}]$$

where

$$b_i = A_{vb}(i,3)$$

Latitude - Longitude

Compute Latitude, μ

If $\Omega\,(t - t_n) \leq \pi$

$$\mu = \sin^{-1}\,[\Omega(t - t_n)]\ \sin i_o$$

where

$0 \leq \mu \leq \pi/2$

If $\Omega\,(t - t_n) > \pi$

$$\mu = \sin^{-1}\,[\pi - \Omega\,(t - t_n)]\ \sin i_o$$

where

$-\frac{\pi}{2} \leq \mu \leq 0$

Compute Longitude, λ

If $\Omega\,(t - t_n) = 0$

$$\Delta\lambda = 0$$

If $0 < \Omega\,(t - t_n) \leq \frac{\pi}{2}$

$$\Delta\lambda = \cos^{-1}\left[\frac{1}{\cos\mu}\cos\Omega\,(t - t_n)\right]$$

$$0 < \Delta\lambda < \frac{\pi}{2}$$

If $\frac{\pi}{2} < \Omega\,(t - t_n) < \pi$

$$\Delta\lambda' = \cos^{-1}\left\{\frac{1}{\cos\mu}\cos\,[\pi - \Omega\,(t - t_n)]\right\}$$

$$0 < \Delta\lambda' < \frac{\pi}{2}$$

$$\Delta\lambda = \pi - \Delta\lambda'$$

If $\Omega\ (t - t_n) = \pi$

$$\Delta\lambda = \pi$$

If $\pi < \Omega\ (t - t_n) \leq \frac{3\pi}{2}$

$$\Delta\lambda' = \cos^{-1}\left\{\frac{-1}{\cos\mu} \cos\left[\Omega(t - t_n) - \pi\right]\right\}$$

$$0 < \Delta\lambda' < \frac{\pi}{2}$$

$$\Delta\lambda = \pi + \Delta\lambda'$$

If $\frac{3\pi}{2} < \Omega\ (t - t_n) < 2\pi$

$$\Delta\lambda' = \cos^{-1}\left\{\frac{-1}{\cos\mu} \cos\left[2\pi - \Omega\ (t - t_n)\right]\right\}$$

$$0 < \Delta\lambda' < \frac{\pi}{2}$$

$$\Delta\lambda = 2\pi - \Delta\lambda'$$

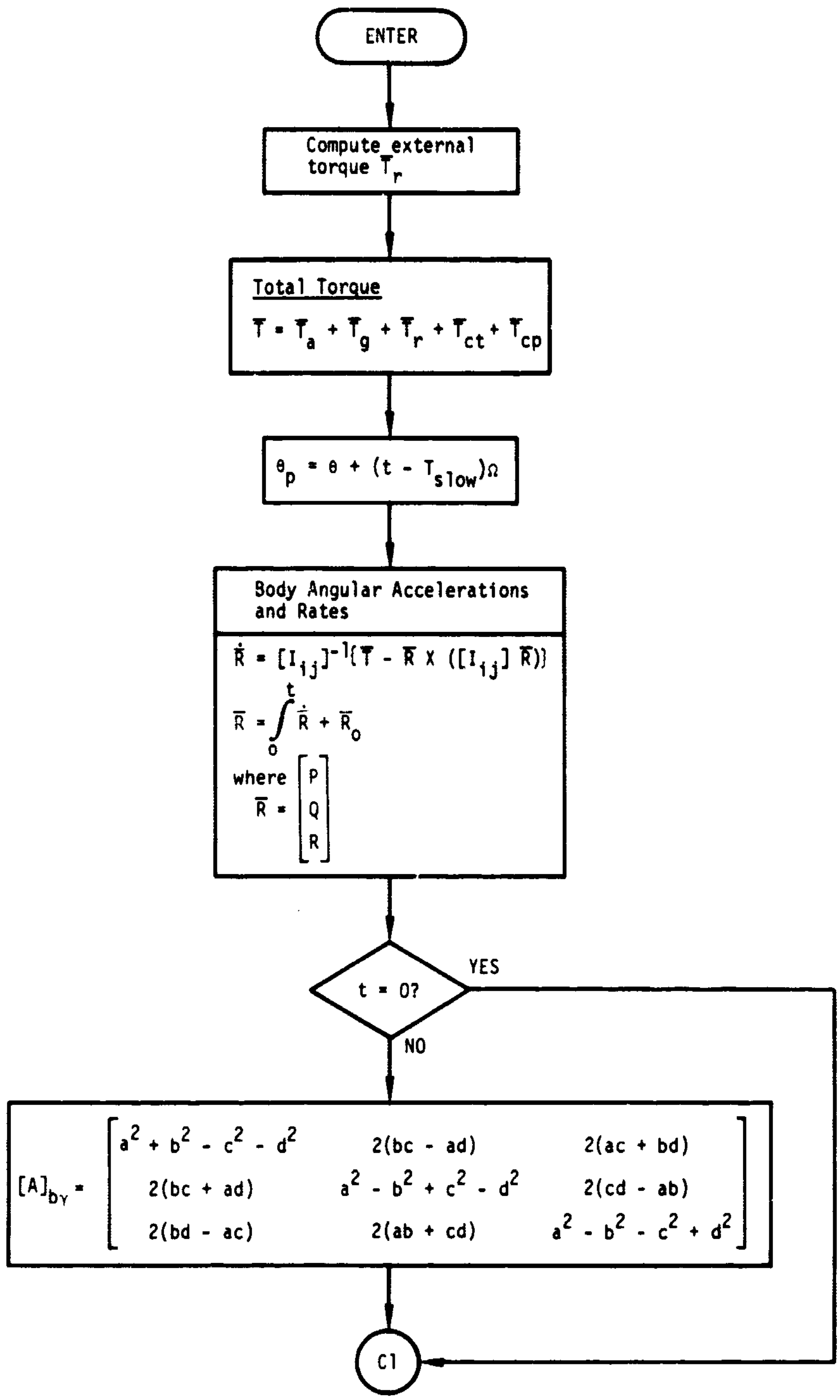
If $\Omega\ (t - t_n) = 2\pi$

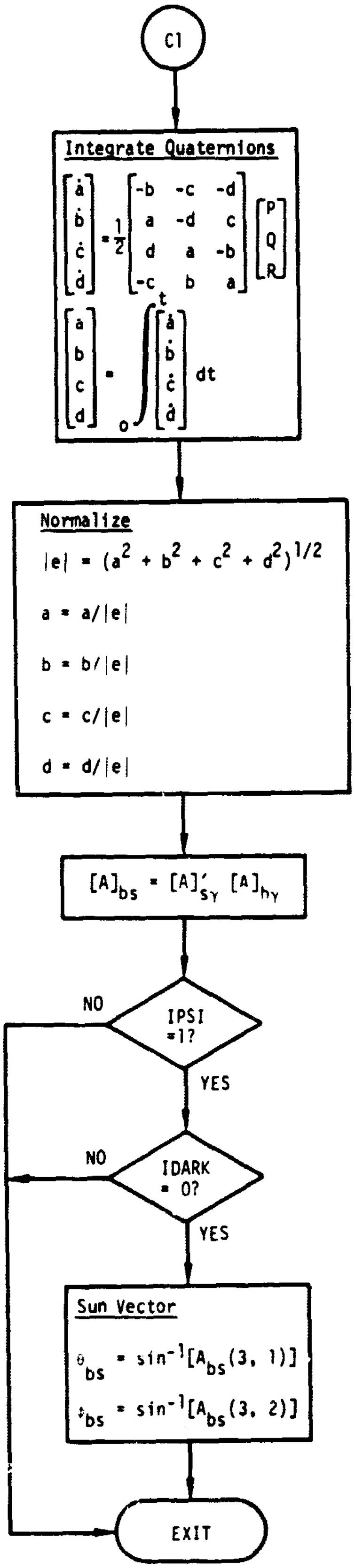
$$\Delta\lambda = 0$$

$$\lambda = \lambda_n + \Delta\lambda$$

APPENDIX C

FAST LOOP ENVIRONMENT

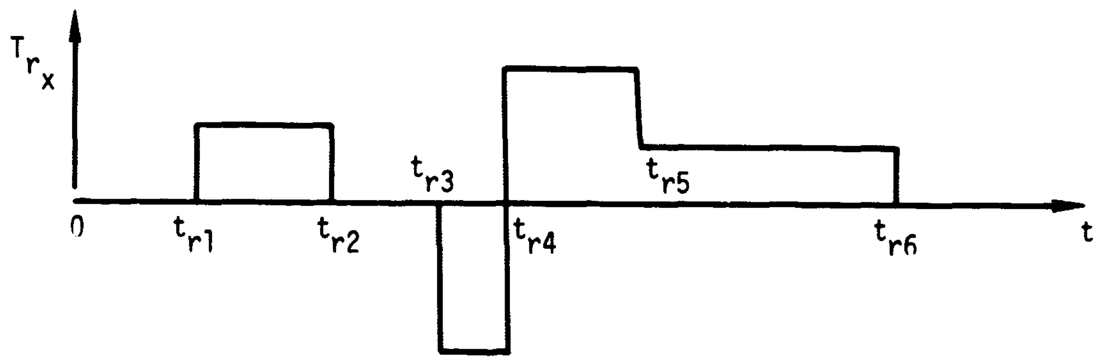

ENTER
Compute external torque $\overline{T}_r$
Total Torque
$\overline{T} = \overline{T}_a + \overline{T}_g + \overline{T}_r + \overline{T}_{ct} + \overline{T}_{cp}$
$\theta_p = \theta + (t - T_{slow})\Omega$
Body Angular Accelerations and Rates
$\dot{\overline{R}} = [I_{ij}]^{-1}(\overline{T} - \overline{R} \times ([I_{ij}]\, \overline{R}))$
$\overline{R} = \int_0^t \dot{\overline{R}} + \overline{R}_o$
where $\overline{R} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$
t = 0?
YES
NO
$[A]_{b\gamma} = \begin{bmatrix} a^2 + b^2 - c^2 - d^2 & 2(bc - ad) & 2(ac + bd) \\ 2(bc + ad) & a^2 - b^2 + c^2 - d^2 & 2(cd - ab) \\ 2(bd - ac) & 2(ab + cd) & a^2 - b^2 - c^2 + d^2 \end{bmatrix}$
C1


C1
Integrate Quaternions
$\begin{bmatrix}\dot a\\ \dot b\\ \dot c\\ \dot d\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-b & -c & -d\\ a & -d & c\\ d & a & -b\\ -c & b & a\end{bmatrix}\begin{bmatrix}P\\ Q\\ R\end{bmatrix}$
$\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix} = \int_0^t \begin{bmatrix}\dot a\\ \dot b\\ \dot c\\ \dot d\end{bmatrix} dt$
Normalize
$|e| = (a^2 + b^2 + c^2 + d^2)^{1/2}$
$a = a/|e|$
$b = b/|e|$
$c = c/|e|$
$d = d/|e|$
$[A]_{bs} = [A]'_{s\gamma}\,[A]_{h\gamma}$
NO
IPSI =1?
YES
NO
IDARK = 0?
YES
Sun Vector
$\theta_{bs} = \sin^{-1}[A_{bs}(3, 1)]$
$\phi_{bs} = \sin^{-1}[A_{bs}(3, 2)]$
EXIT

Impulsive Torques due to Crew Motion, Docking, Venting

$\bar{T}_r$ is input in tabular form versus time, t.

$\bar{T}_r$ shall be calculated from this table as having a constant value between any two successive tabulated time increments, t_{ri} and t_{ri+1}.



Example

APPENDIX D

SENSORS

2

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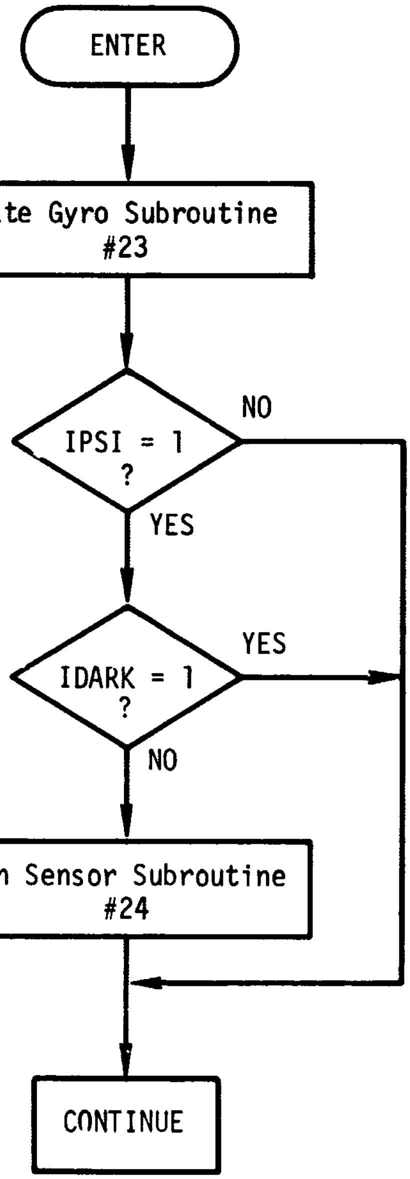

APPENDIX E

CONTROL SYSTEMS

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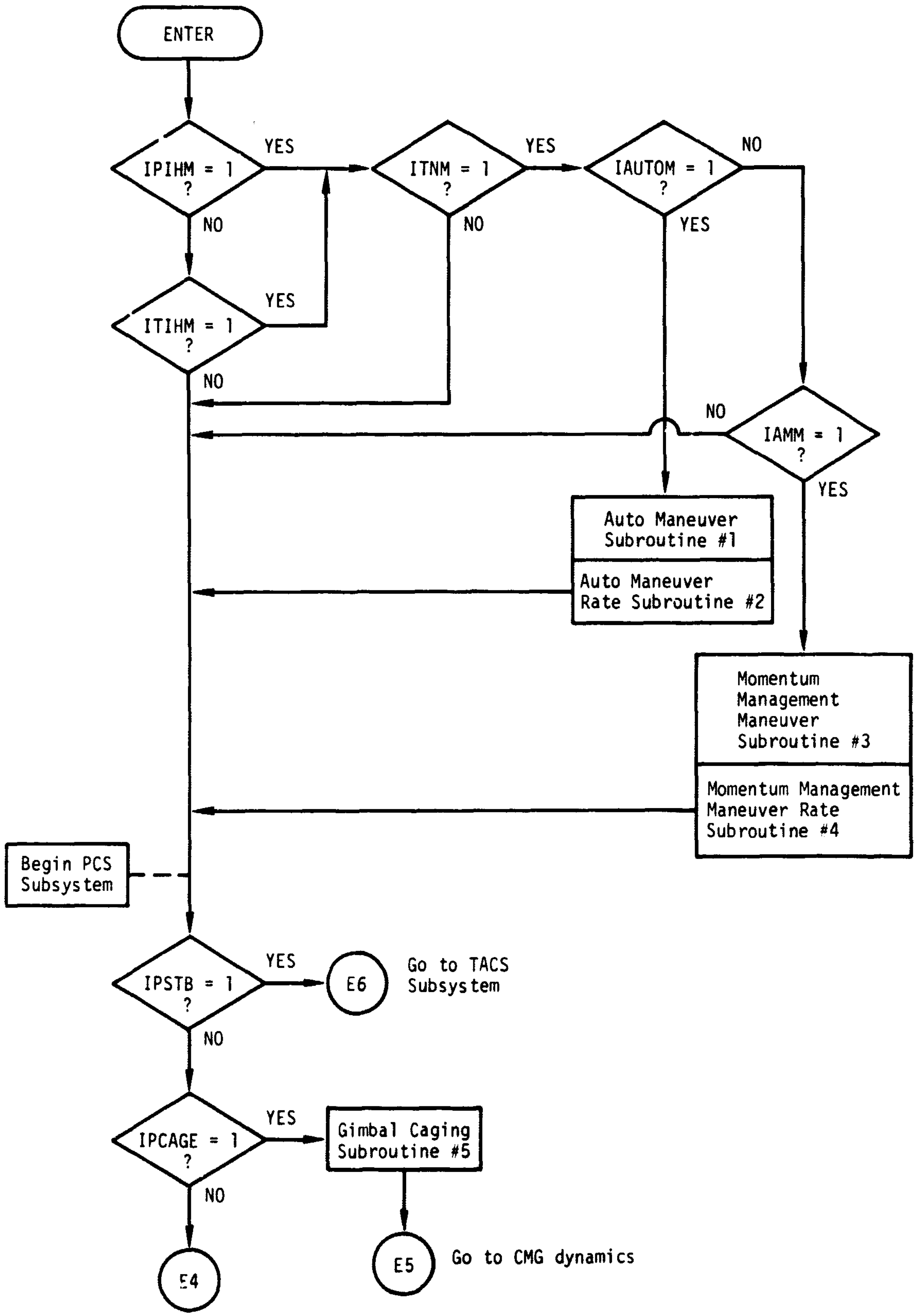
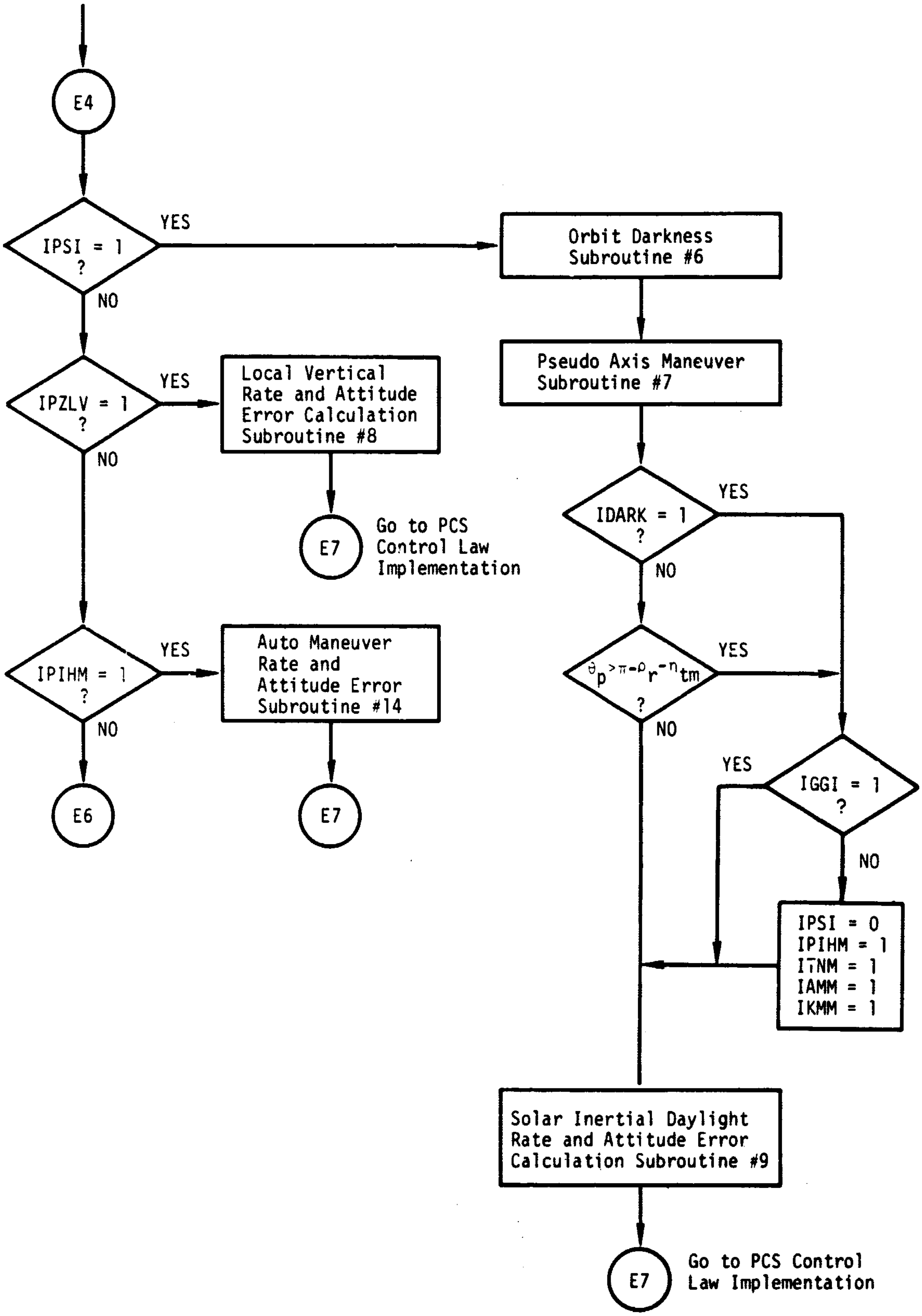
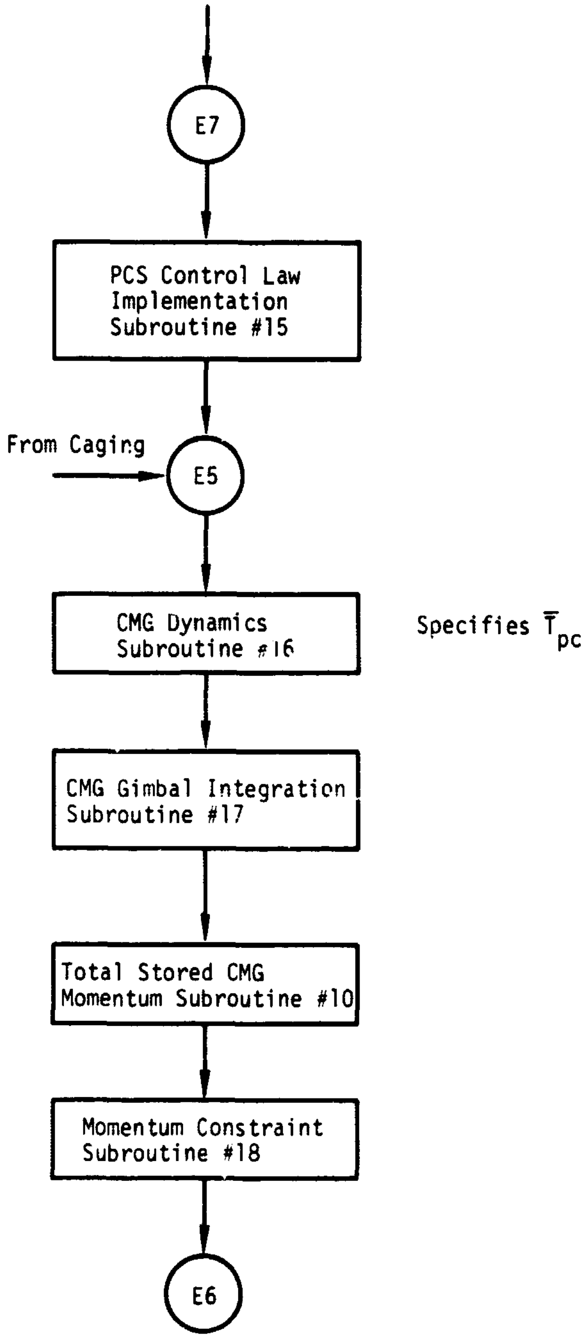
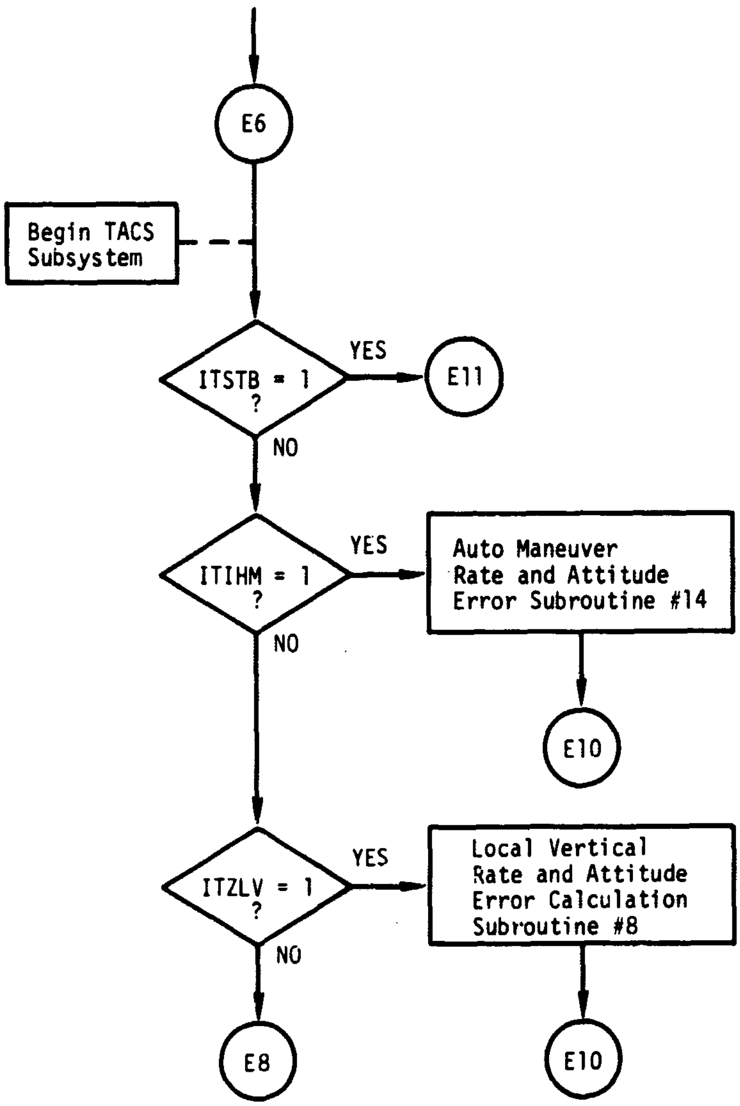
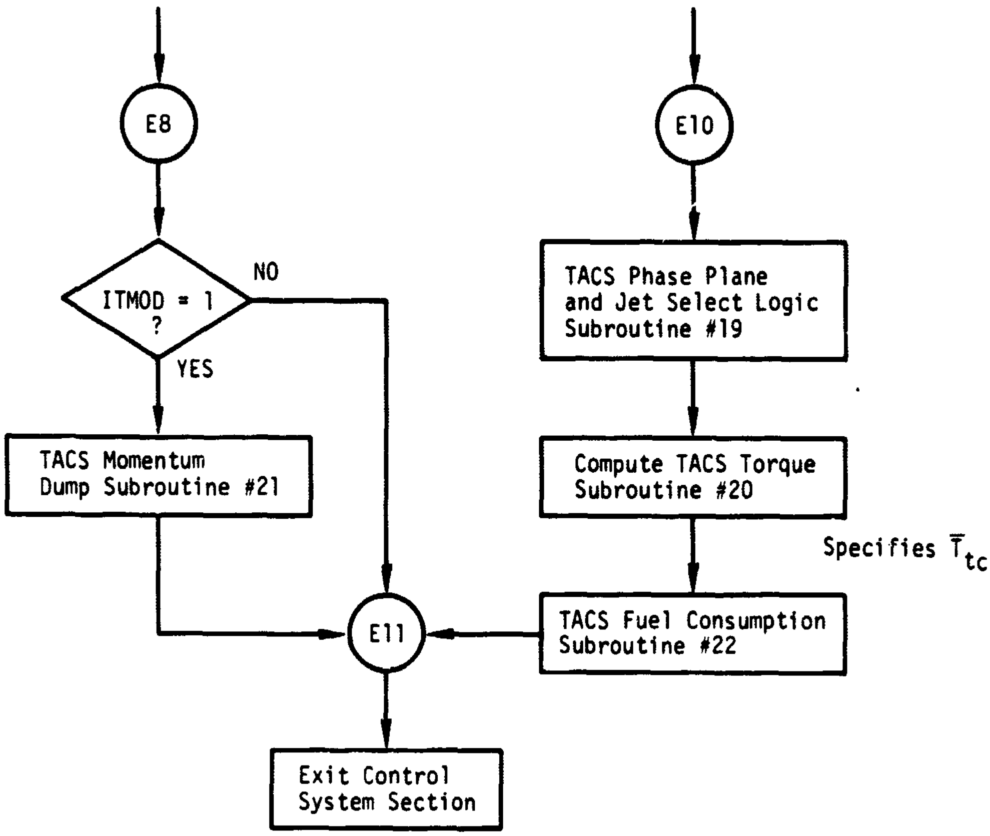



E4
IPSI = 1 ?
YES
NO
Orbit Darkness Subroutine #6
Pseudo Axis Maneuver Subroutine #7
IPZLV = 1 ?
YES
NO
Local Vertical Rate and Attitude Error Calculation Subroutine #8
E7
Go to PCS Control Law Implementation
IDARK = 1 ?
YES
NO
$\theta_p > \pi - \rho_r - \eta_{tm}$?
YES
NO
IPIHM = 1 ?
YES
NO
Auto Maneuver Rate and Attitude Error Subroutine #14
E6
E7
IGGI = 1 ?
YES
NO
IPSI = 0
IPIHM = 1
ITNM = 1
IAMM = 1
IKMM = 1
Solar Inertial Daylight Rate and Attitude Error Calculation Subroutine #9
E7
Go to PCS Control Law Implementation


E7
PCS Control Law Implementation Subroutine #15
From Caging
E5
CMG Dynamics Subroutine #16
Specifies $\overline{T}_{pc}$
CMG Gimbal Integration Subroutine #17
Total Stored CMG Momentum Subroutine #10
Momentum Constraint Subroutine #18
E6


E6
Begin TACS Subsystem
ITSTB = 1 ?
YES
NO
E11
ITIHM = 1 ?
YES
NO
Auto Maneuver Rate and Attitude Error Subroutine #14
E10
ITZLV = 1 ?
YES
NO
Local Vertical Rate and Attitude Error Calculation Subroutine #8
E8
E10


E8
ITMOD = 1 ?
NO
YES
TACS Momentum Dump Subroutine #21
E10
TACS Phase Plane and Jet Select Logic Subroutine #19
Compute TACS Torque Subroutine #20
Specifies $\bar{T}_{tc}$
TACS Fuel Consumption Subroutine #22
E11
Exit Control System Section

APPENDIX F

WCIU INITIALIZATION

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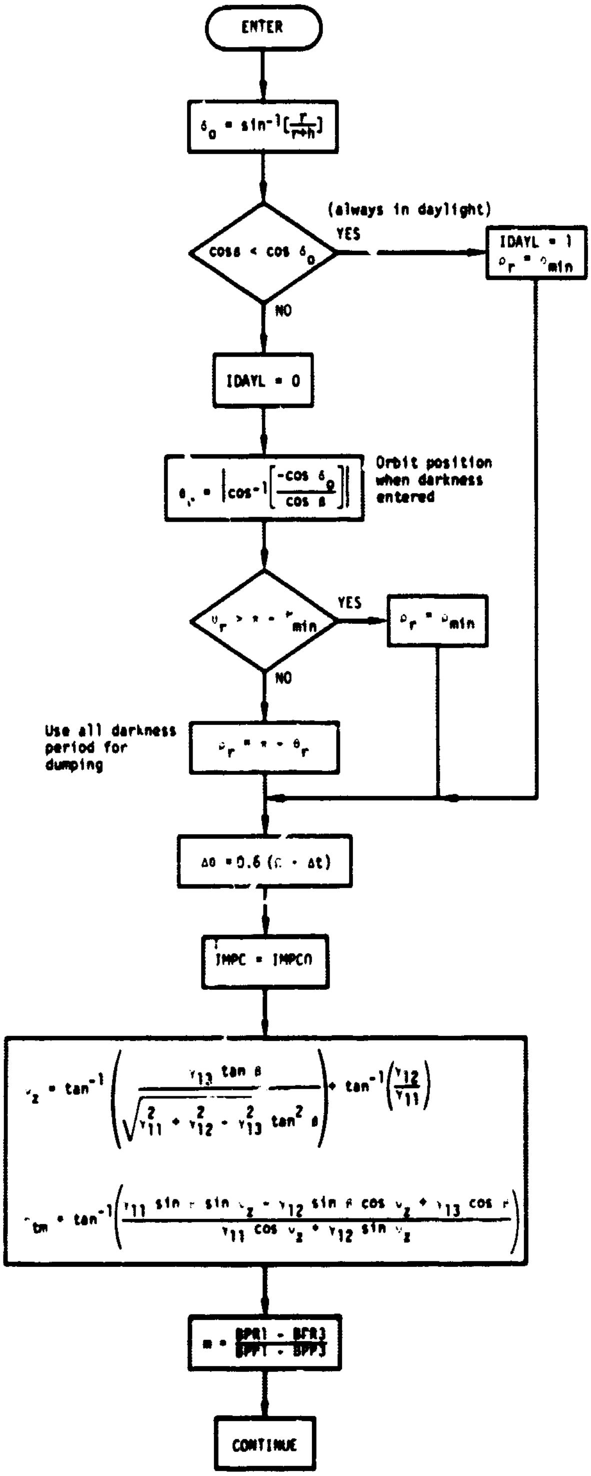
WCIU Initialization



Note: See also Slow Loop Computations and Fast Loop Computations.

APPENDIX G

CONTROL SYSTEM SUBROUTINES

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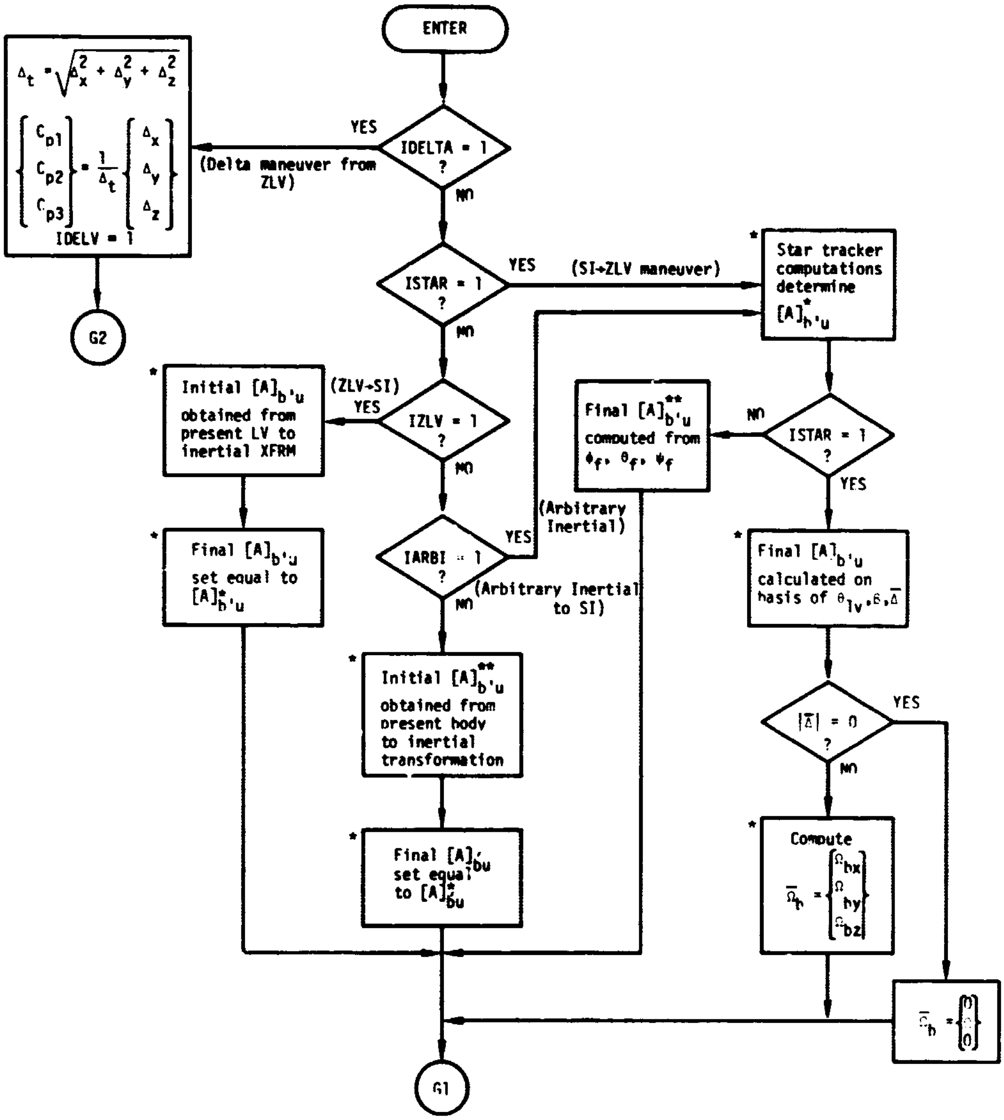
Auto Maneuver Subroutine #1.



*See page G-6

Star Tracker Computations

Compute present orientation relative to solar inertial attitude, $[A^*]_{b'ui}$, from off-nominal star tracker measurements.

$$[A]_{bu} = [A]_{iu} \, [(-\varepsilon)_x] \, [A]'_{os} \, [A]_{bs}$$

$$[A]_{b'ui} = [A]_{bu} \, [A]'_{bb'}$$

Set

$$[A]^{*}_{b'u} = [A]_{b'ui}$$

Final $[A]_{b'u}$ $(=[A]_{b'uf})$ Calculated on Basis of θ_{1v}, β, $\bar{\Delta}$

$$[A]_{b'uf} = [(\beta)_x] \, [(\theta_{1v})_y] \, [(-\Delta_x)_x] \, [(-\Delta_y)_y] \, [(-\Delta_z)_z]$$

Final $[A]_{b'u}$ $(= [A]_{b'uf})$ Computed from ϕ_f, θ_f, ψ_f

$$[A]_{b'uf} = [(\psi_f)_z] \, [(\theta_f)_y] \, [(\phi_f)_x]$$

Set

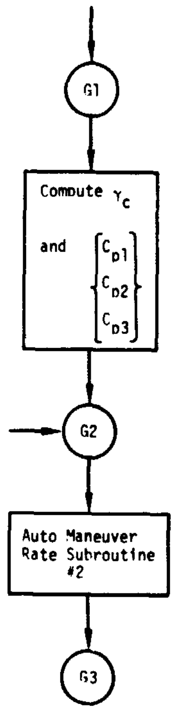
$$[A]^{**}_{b'u} = [A]_{b'uf}$$

Initial $[A]_{b'u}$ Obtained from Present LV

$$[A]_{b'ui} = [(\beta)_x] \, [(\theta_p)_y]$$

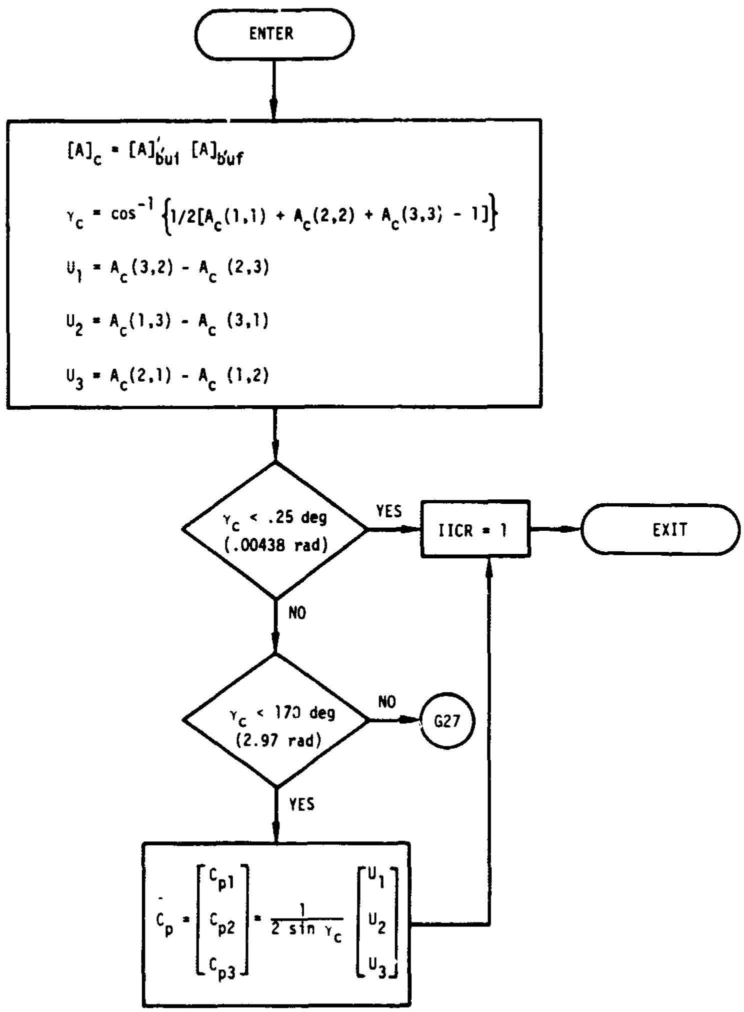
Compute $\bar{\Omega}_b$

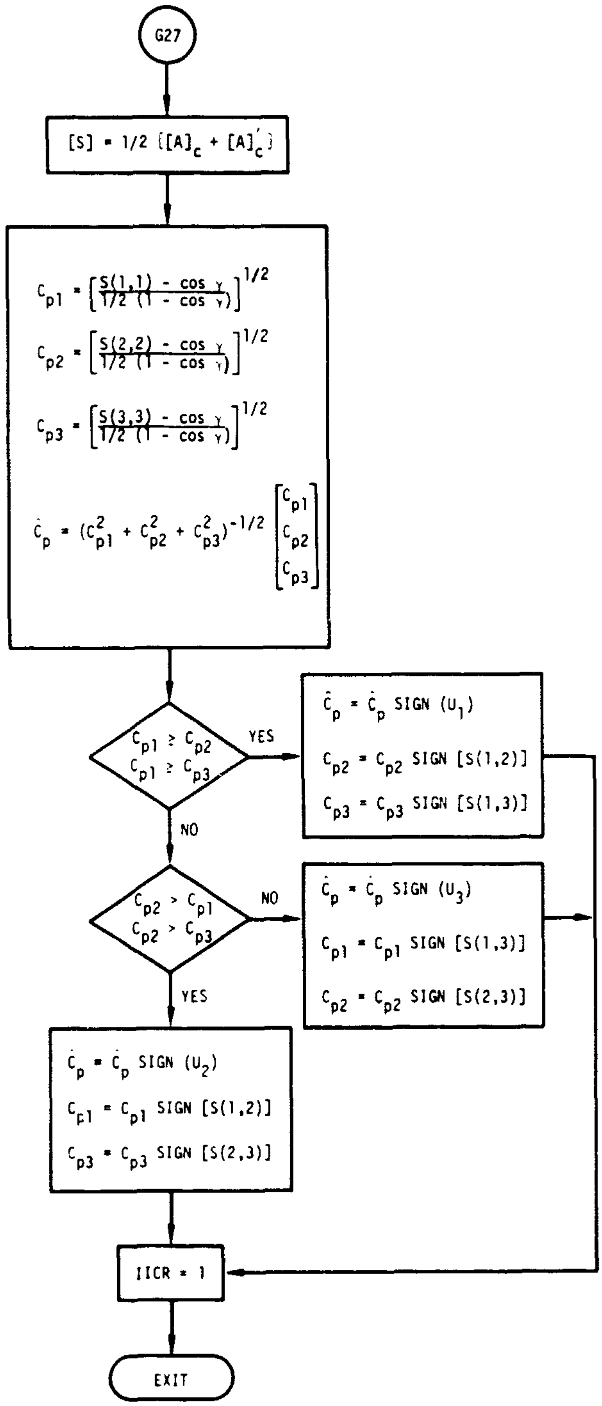
$$\bar{\Omega}_b = [(\Delta_z)_z] \, [(\Delta_y)_y] \, [(\Delta_x)_x] \begin{bmatrix} 0 \\ \Omega \\ 0 \end{bmatrix}$$

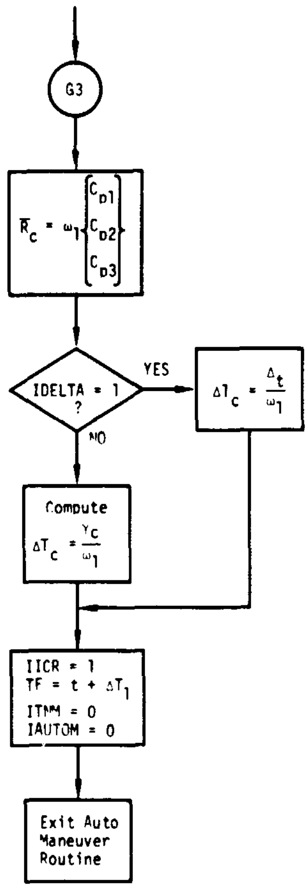


(see pages G-6 and G-7)

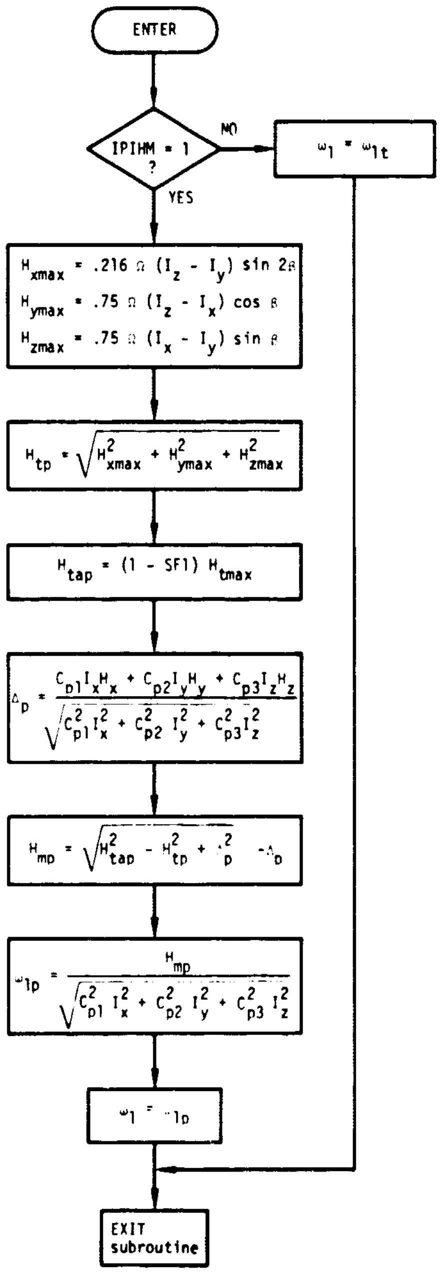
Compute γ_c and $\hat{C}_p$



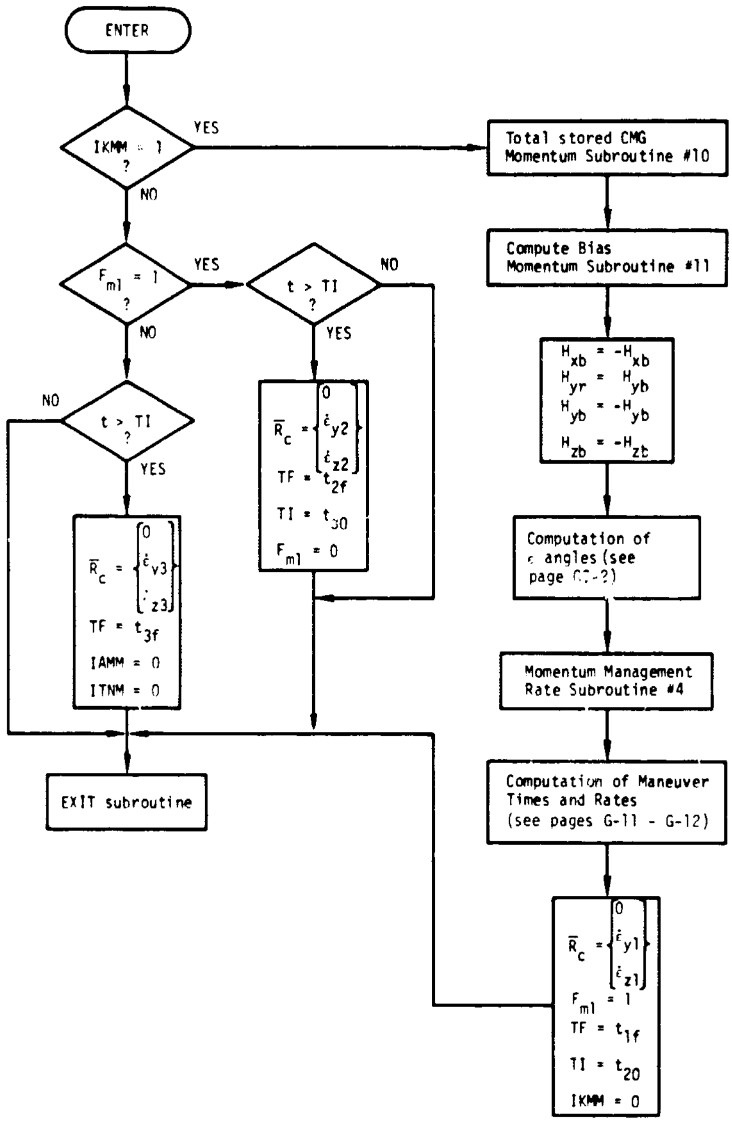

G27
$[S] = 1/2 \{[A]_c + [A]_c'\}$
$C_{p1} = \left[\frac{S(1,1) - \cos\gamma}{1/2\,(1 - \cos\gamma)}\right]^{1/2}$
$C_{p2} = \left[\frac{S(2,2) - \cos\gamma}{1/2\,(1 - \cos\gamma)}\right]^{1/2}$
$C_{p3} = \left[\frac{S(3,3) - \cos\gamma}{1/2\,(1 - \cos\gamma)}\right]^{1/2}$
$\dot{C}_p = (C_{p1}^2 + C_{p2}^2 + C_{p3}^2)^{-1/2} \begin{bmatrix} C_{p1} \\ C_{p2} \\ C_{p3} \end{bmatrix}$
$C_{p1} \geq C_{p2}$
$C_{p1} \geq C_{p3}$
YES
$\dot{C}_p = \dot{C}_p$ SIGN (U_1)
$C_{p2} = C_{p2}$ SIGN $[S(1,2)]$
$C_{p3} = C_{p3}$ SIGN $[S(1,3)]$
NO
$C_{p2} > C_{p1}$
$C_{p2} > C_{p3}$
NO
$\dot{C}_p = \dot{C}_p$ SIGN (U_3)
$C_{p1} = C_{p1}$ SIGN $[S(1,3)]$
$C_{p2} = C_{p2}$ SIGN $[S(2,3)]$
YES
$\dot{C}_p = \dot{C}_p$ SIGN (U_2)
$C_{p1} = C_{p1}$ SIGN $[S(1,2)]$
$C_{p3} = C_{p3}$ SIGN $[S(2,3)]$
IICR = 1
EXIT


G3
$\bar{R}_c = \omega_1 \begin{Bmatrix} C_{p1} \\ C_{p2} \\ C_{p3} \end{Bmatrix}$
IDELTA = 1
?
YES
$\Delta T_c = \frac{\Delta_t}{\omega_1}$
NO
Compute
$\Delta T_c = \frac{\gamma_c}{\omega_1}$
IICR = 1
TF = t + ΔT_1
ITMM = 0
IAUTOM = 0
Exit Auto
Maneuver
Routine

Auto Maneuver Rate Subroutine #2



Momentum Management Subroutine #3



Computation of ε Angles

$$\varepsilon_{y1} = A_2 \left(B'_{21} H_{xb} + B'_{22} H_{yb} + B'_{23} H_{zb}\right)$$

$$\varepsilon_{y2} = A_2 \left(-B'_{21} H_{xb} + B'_{22} H_{yb} + B'_{23} H_{zb}\right)$$

$$\varepsilon_{z1} = A_3 \left(B'_{31} H_{xb} + B'_{32} H_{yb} + B'_{33} H_{zb}\right)$$

$$\varepsilon_{z2} = A_3 \left(-B'_{31} H_{xb} + B'_{32} H_{yb} + B'_{33} H_{zb}\right)$$

$$B'_{21} = \frac{\sin \beta}{1 - \cos 2\rho_r} \quad , \quad B'_{31} = -\frac{\cos \beta}{1 - \cos 2\rho_r}$$

$$B'_{22} = \frac{a'_{33}}{D} \quad , \quad B'_{32} = \frac{a'_{23}}{D}$$

$$B'_{23} = -\frac{a'_{23}}{D} \quad , \quad B'_{33} = \frac{a'_{22}}{D}$$

$$D = a'_{22}\, a'_{33} + a'^{2}_{23}$$

$$a'_{22} = -\frac{1}{2}\left(1 + \cos 2\beta\right)\left(2\rho_r + \sin 2\rho_r\right) + \left(2\rho_r - \sin 2\rho_r\right)$$

$$a'_{23} = -\frac{1}{2} \sin 2\beta \left(2\rho_r + \sin 2\rho_r\right)$$

$$a'_{33} = \frac{1}{2}\left(1 - \cos 2\beta\right)\left(2\rho_r + \sin 2\rho_r\right) - \left(2\rho_r - \sin 2\rho_r\right)$$

$$A_2 = \frac{2}{3\Omega \cdot \Delta I_{xz}} \quad , \quad A_3 = \frac{2}{3\Omega \cdot \Delta I_{yx}}$$

$$\Delta I_{xz} = I_x - I_z \quad , \quad \Delta I_{yx} = I_y - I_x$$

Maneuver Times

Maneuver 1

$\rho_r \triangleq$ Orbital half angle used for momentum removal (rad)

$\Omega \triangleq$ Orbital angular velocity (rad/sec)

$$t_{1f} = t + \frac{1}{\omega_1}\sqrt{\varepsilon_{y1}^2 + \varepsilon_{z1}^2}$$

$\omega_1 \triangleq$ Vehicle angular rate limit (rad/sec)

Maneuver 2

$$t_{20} = \frac{\pi - \eta_{tm}}{\Omega} - \frac{1}{2\omega_1}\sqrt{(\varepsilon_{y1} - \varepsilon_{y2})^2 + (\varepsilon_{z1} - \varepsilon_{z2})^2}$$

$$t_{2f} = \frac{\pi - \eta_{tm}}{\Omega} + \frac{1}{2\omega_1}\sqrt{(\varepsilon_{y1} - \varepsilon_{y2})^2 + (\varepsilon_{z1} - \varepsilon_{z2})^2}$$

Maneuver 3

$$t_{30} = \frac{\pi - \eta_{tm} + \rho_r}{\Omega} - \frac{1}{\omega_1}\sqrt{\varepsilon_{y2}^2 + \varepsilon_{z2}^2}$$

$$t_{3f} = \frac{\pi - \eta_{tm} + \rho_r}{\Omega}$$

If $\Delta\rho_j$ is negative, the next maneuver initialization time must be delayed. The following equations apply.

$$\Delta\rho_j = \rho_r - \frac{\Omega}{\omega_1}\sqrt{\varepsilon_{yj}^2 + \varepsilon_{zj}^2} - \frac{1}{2}\frac{\Omega}{\omega_1}\sqrt{(\varepsilon_{y1} - \varepsilon_{y2})^2 + (\varepsilon_{z1} - \varepsilon_{z2})^2} \qquad (3\text{-}6)$$

where

$j = 1, 2$

This equation is valid for both the Y- and Z- axes. Thus, only one $\Delta\rho$ is required per maneuver, not one for each axis.

If $\Delta\rho_j$ is negative, delay the next initial maneuver time so that

$$t'_{(j+1)o} = t_{(j+1)o} - \frac{1}{2\Omega} \Delta\rho_j \ .$$

Maneuver Rates

Maneuver 1

$$\dot{\varepsilon}_{y1} = \frac{\varepsilon_{y1}}{\sqrt{\varepsilon_{y1}^2 + \varepsilon_{z1}^2}}\,\omega_1$$

$$\dot{\varepsilon}_{z1} = \frac{\varepsilon_{z1}}{\sqrt{\varepsilon_{y1}^2 + \varepsilon_{z1}^2}}\,\omega_1$$

Maneuver 2

$$\dot{\varepsilon}_{y2} = \frac{(\varepsilon_{y2} - \varepsilon_{y1})}{\sqrt{(\varepsilon_{y1} - \varepsilon_{y2})^2 + (\varepsilon_{z1} - \varepsilon_{z2})^2}}\,\omega_1$$

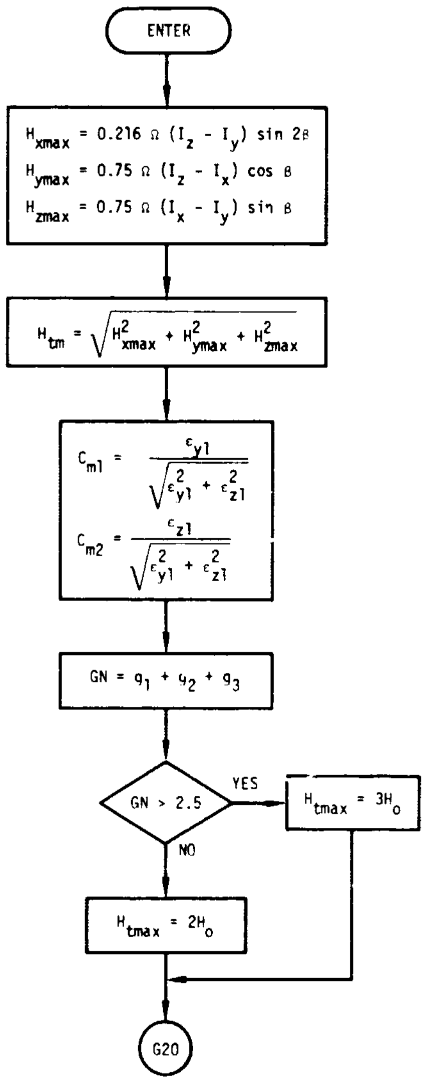
$$\dot{\varepsilon}_{z2} = \frac{(\varepsilon_{z2} - \varepsilon_{z1})}{\sqrt{(\varepsilon_{y1} - \varepsilon_{y2})^2 + (\varepsilon_{z1} - \varepsilon_{z2})^2}}\,\omega_1$$

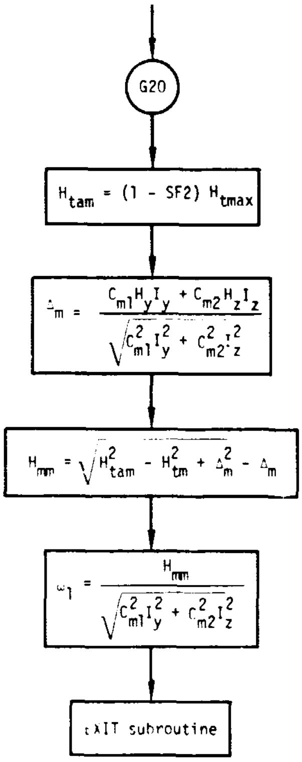
Maneuver 3

$$\dot{\varepsilon}_{y3} = \frac{-\varepsilon_{y2}}{\sqrt{\varepsilon_{y2}^2 + \varepsilon_{z2}^2}}\,\omega_1$$

$$\dot{\varepsilon}_{z3} = \frac{-\varepsilon_{z2}}{\sqrt{\varepsilon_{y2}^2 + \varepsilon_{z2}^2}}\,\omega_1$$

Momentum Management Rate Subroutine #4




G20
$H_{tam} = (1 - SF2)\, H_{tmax}$
$\Delta_m = \dfrac{C_{m1}H_yI_y + C_{m2}H_zI_z}{\sqrt{C_{m1}^2 I_y^2 + C_{m2}^2 I_z^2}}$
$H_{mm} = \sqrt{H_{tam}^2 - H_{tm}^2 + \Delta_m^2} - \Delta_m$
$\omega_1 = \dfrac{H_{mm}}{\sqrt{C_{m1}^2 I_y^2 + C_{m2}^2 I_z^2}}$
EXIT subroutine

Gimbal Caging Subroutine #5

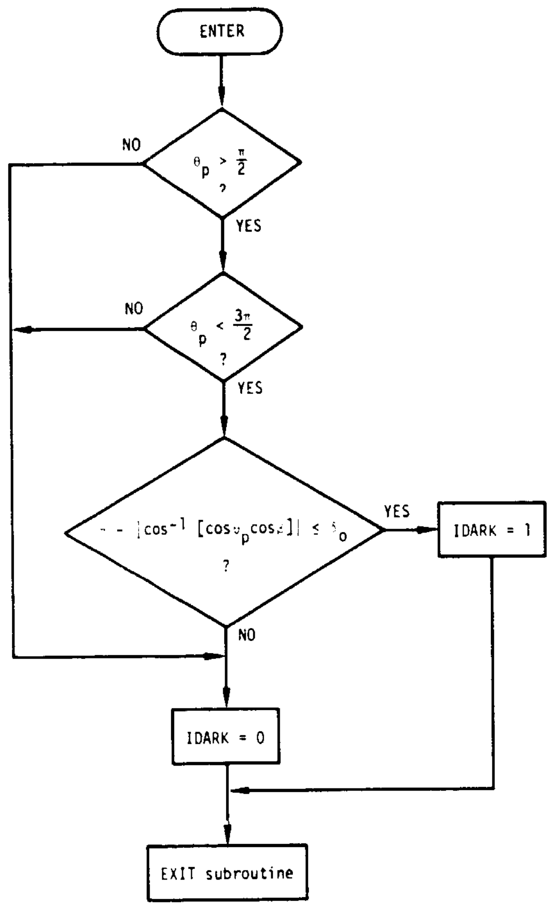
ENTER

$$\dot{\alpha}_1 = K_g (\alpha_{1d} - \alpha_1)$$
$$\dot{\alpha}_2 = K_g (\alpha_{2d} - \alpha_2)$$
$$\dot{\alpha}_3 = K_g (\alpha_{3d} - \alpha_3)$$

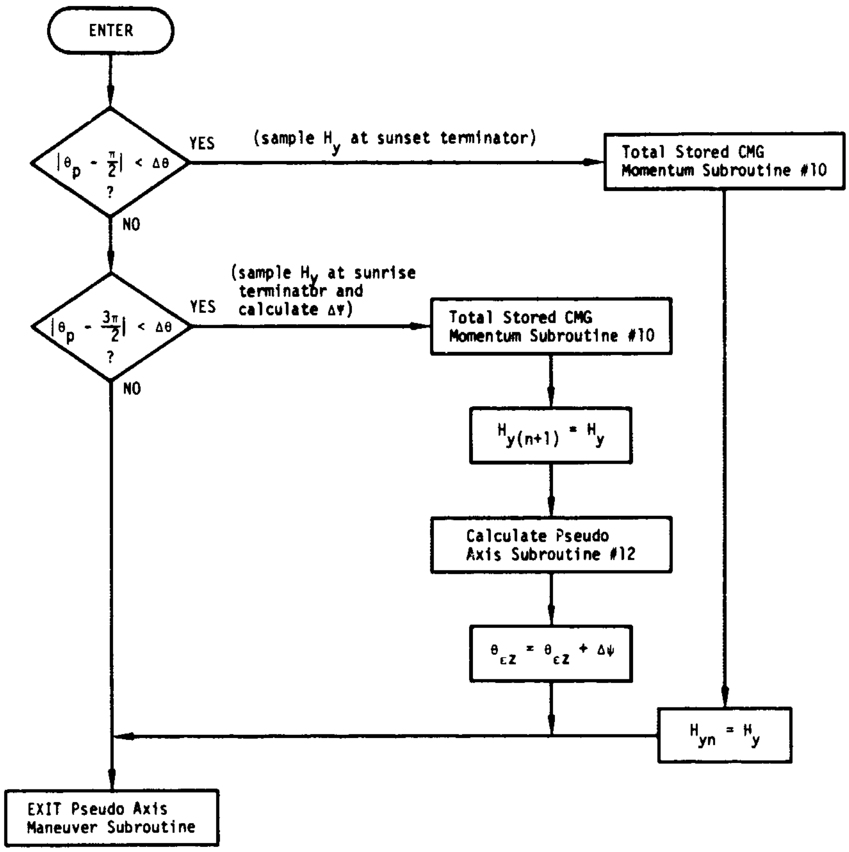
$$\dot{\beta}_1 = K_g (\beta_{1d} - \beta_1)$$
$$\dot{\beta}_2 = K_g (\beta_{2d} - \beta_2)$$
$$\dot{\beta}_3 = K_g (\beta_{3d} - \beta_3)$$

EXIT subroutine

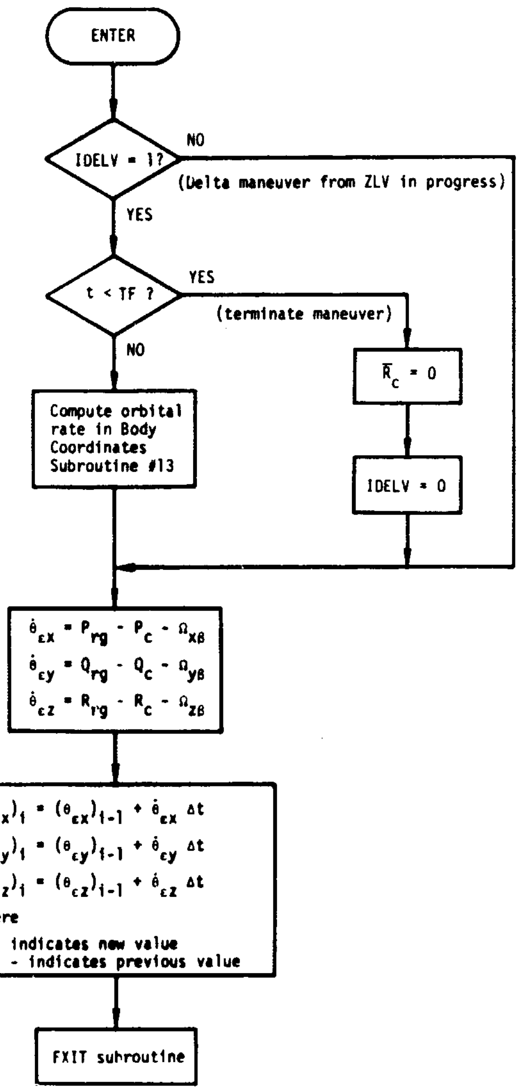
Orbit Darkness Subroutine #6



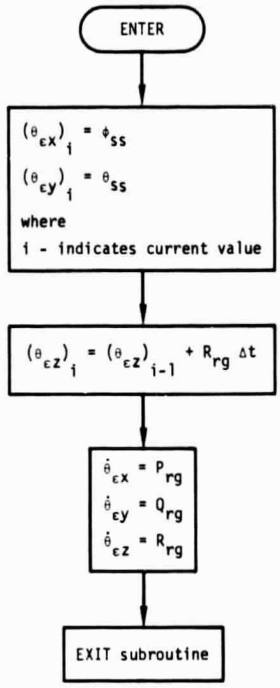
Pseudo Axis Maneuver Subroutine #7



Local Vertical Rate and Attitude Error Calculation Subroutine #8



Solar Inertial Daylight Rate and Attitude Error Calculation Subroutine #9



Total Stored CMG Momentum Subroutine #10

$$H_x = H_o \{g_1 \cos \alpha_1 \cos \beta_1 - g_2 \sin \beta_2 - g_3 \sin \alpha_3 \cos \beta_3\}$$

$$H_y = H_o \{-g_1 \sin \alpha_1 \cos \beta_1 + g_2 \cos \alpha_2 \cos \beta_2 - g_3 \sin \beta_3\}$$

$$H_z = H_o \{-g_1 \sin \beta_1 - g_2 \sin \alpha_2 \cos \beta_2 + g_3 \cos \alpha_3 \cos \beta_3\}$$

g_i $i = 1,2,3$	Constants set equal 1 or 0 depending upon the operation of the i^{th} CMG
α_i, β_i $i = 1,2,3$	Instantaneous value of the outer and inner CMG gimbal angles
H_o	Stored momentum in each CMG
$\left.\begin{matrix} H_x \\ H_y \\ H_z \end{matrix}\right\}$	Components of total stored momentum in the CMG system in body coordinates

Compute Bias Momentum Subroutine #11

Called by: Momentum Management Subroutine and TACS Momentum Dump Subroutine

$$H_{xb} = P_{rg}I_x + H_x - \frac{3}{8}\Omega(I_z - I_y)\sin 2\beta\,(\sin 2\theta_p - \sin 2\theta_o)$$
$$-H_{xdb}$$

$$H_{yb} = Q_{rg}I_y + H_y + \frac{3}{4}\Omega(I_z - I_x)\cos\beta\,(\cos 2\theta_p - \cos 2\theta_o)$$
$$-H_{ydb}$$

$$H_{zb} = R_{rg}I_z + H_z + \frac{3}{4}\Omega(I_x - I_y)\sin\beta\,(\cos 2\theta_p - \cos 2\theta_o)$$
$$-H_{zdb}$$

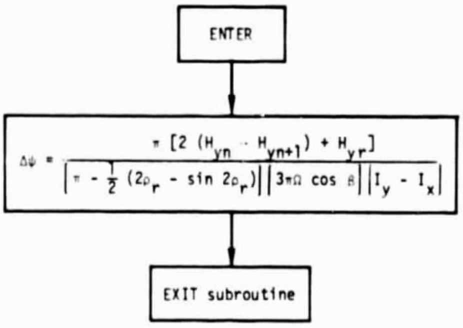
$\left.\begin{array}{l} H_{xb} \\ H_{yb} \\ H_{zb} \end{array}\right\}$ Components of bias momentum expected to be stored in the CMG system in body coordinates

$\left.\begin{array}{l} H_{x} \\ H_{y} \\ H_{z} \end{array}\right\}$ Components of total instantaneous momentum stored in CMG system in body coordinates (computed every cycle in compute total stored momentum subroutine)

$\left.\begin{array}{l} H_{xdb} \\ H_{ydb} \\ H_{zdb} \end{array}\right\}$ Desired components of momentum bias

Subroutine 12
Page 1

Calculate Pseudo Axis Subroutine #12



Orbit Rate in Body Coordinates Subroutine #13

ENTER

$$\begin{bmatrix} \Omega_{xb} \\ \Omega_{yb} \\ \Omega_{zb} \end{bmatrix} = \begin{bmatrix} \text{ZLV to body} \end{bmatrix} \begin{bmatrix} 0 \\ \Omega \\ 0 \end{bmatrix}$$

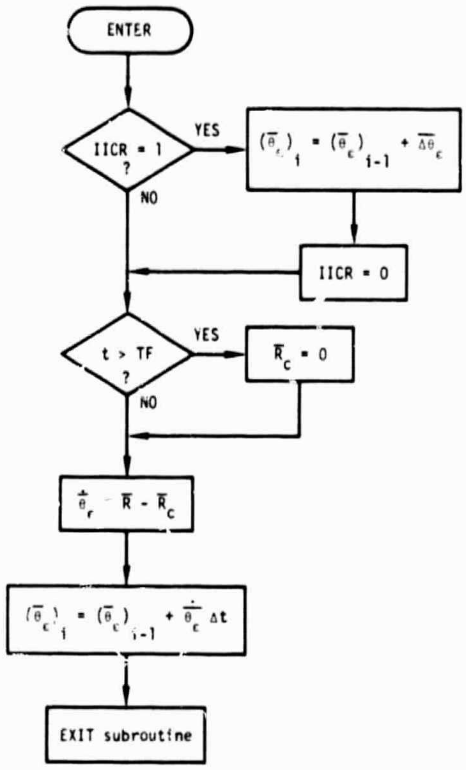
$$\begin{bmatrix} (\Delta_x)_i = (\Delta_x)_{i-1} + P_c \, \Delta t \\ (\Delta_y)_i = (\Delta_y)_{i-1} + Q_c \, \Delta t \\ (\Delta_z)_i = (\Delta_z)_{i-1} + R_c \, \Delta t \end{bmatrix}$$

EXIT subroutine

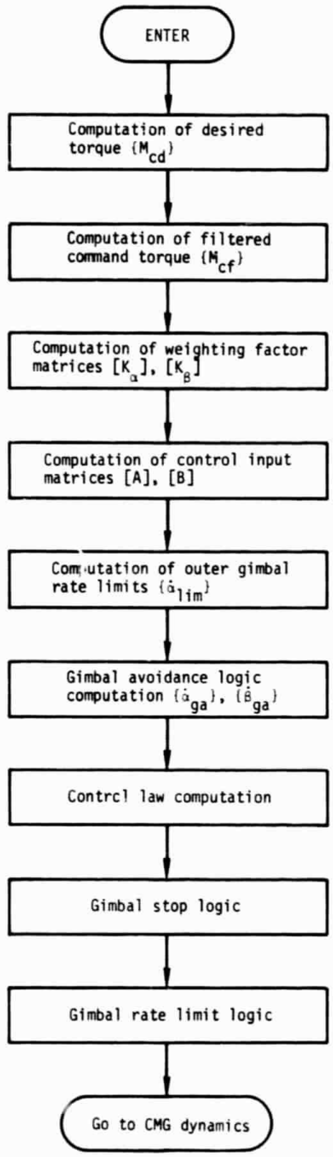
where

$$\begin{bmatrix} \text{ZLV to body} \end{bmatrix} = \begin{bmatrix} \left(1 - \frac{\Delta_z^2}{2}\right)\left(1 - \frac{\Delta_y^2}{2}\right) & \Delta_z & -\left(1 - \frac{\Delta_z^2}{2}\right)\Delta_y \\ -\left(1 - \frac{\Delta_x^2}{2}\right)\left(1 - \frac{\Delta_y^2}{2}\right)\Delta_z & \left(1 - \frac{\Delta_z^2}{2}\right)\left(1 - \frac{\Delta_x^2}{2}\right) & +\left(1 - \frac{\Delta_y^2}{2}\right)\Delta_x \\ -\left(1 - \frac{\Delta_x^2}{2}\right)\Delta_y & -\Delta_x\left(1 - \frac{\Delta_z^2}{2}\right) & \left(1 - \frac{\Delta_x^2}{2}\right)\left(1 - \frac{\Delta_y^2}{2}\right) \end{bmatrix}$$

Auto Maneuver Rate and Attitude Error Calculation Subroutine #14



PCS Control Law Implementation Subroutine #15



Detailed Equations for CMG Control Law Implementation

$$[RG] = \begin{bmatrix} 0 & -R_{rg} & Q_{rg} \\ R_{rg} & 0 & -P_{rg} \\ -Q_{rg} & P_{rg} & 0 \end{bmatrix}$$

The Total Stored CMG Momentum Subroutine #10 is called which specifies $\{H\}$.

Computation of Desired Torque $\{M_{cd}\}$

$$\{M_{cd}\} = [\gamma] \left\{ [K_\theta] \cdot \{\overline{\theta}_\varepsilon\} + [K_\Omega] \{\theta_\varepsilon\} + \frac{1}{H_o} [RG] \cdot \{H\} \right\}$$

where

$$[K_\theta] = \begin{bmatrix} K_{1\theta} & 0 & 0 \\ 0 & K_{2\theta} & 0 \\ 0 & 0 & K_{3\theta} \end{bmatrix}$$

$$[K_\Omega] = \begin{bmatrix} K_{1\Omega} & 0 & 0 \\ 0 & K_{2\Omega} & 0 \\ 0 & 0 & K_{3\Omega} \end{bmatrix}$$

Computation of Filtered Command Torque $\{M_{cf}\}$

At this time, skip this computation and let

$$\{M_{cf}\} = \{M_{cd}\}$$

A subprogram will be provided later to compute $\{M_{cf}\}$ from $\{M_{cd}\}$

For i = 1,3, if

$$|M_{cf_i}| > M_{cflim}$$

then

$$M_{cf_i} = \frac{M_{cf_i}}{|M_{cf_i}|} \cdot M_{cflim}$$

The total torque command is given by $\{M_c\} = \{M_{cf}\}$

<u>Computation of Weighting Factor Matrices $[K_\alpha]$, $[K_\beta]$</u>

The weighting matrices $[K_\alpha]$ and $[K_\beta]$ are given by

$$[K_\alpha] \equiv \begin{bmatrix} \frac{K_{1\alpha}}{\cos \beta_1} & 0 & 0 \\ 0 & \frac{K_{2\alpha}}{\cos \beta_2} & 0 \\ 0 & 0 & \frac{K_{3\alpha}}{\cos \beta_3} \end{bmatrix}$$

$$[K_\beta] \equiv \begin{bmatrix} K_{1\beta} & 0 & 0 \\ 0 & K_{2\beta} & 0 \\ 0 & 0 & K_{3\beta} \end{bmatrix}$$

The positive constants $K_{i\alpha}$ and $K_{i\beta}$ are used to define the relative weighting of the inner and outer gimbal rate components normal to the CMG momenta. In case of a CMG failure or temporary loss of control due to gimbal limiting, the corresponding constants $K_{i\alpha}$ or $K_{i\beta}$ could be set to zero by the control computer.

Presently $K_{i\alpha} = K_{i\beta} = 1$ if the i^{th} CMG is operational; otherwise, the constants are set to zero.

Computation of Control Input Matrices [A] and [B]

For the orthogonal configuration, the matrices [A] and [B] are given by

$$[A] = -\begin{bmatrix} \sin\alpha_1 \cos\beta_1 & 0 & \cos\alpha_3 \cos\beta_3 \\ \cos\alpha_1 \cos\beta_1 & \sin\alpha_2 \cos\beta_2 & 0 \\ 0 & \cos\alpha_2 \cos\beta_2 & \sin\alpha_3 \cos\beta_3 \end{bmatrix}$$

$$[B] = -\begin{bmatrix} \cos\alpha_1 \sin\beta_1 & \cos\beta_2 & -\sin\alpha_3 \sin\beta_3 \\ -\sin\alpha_1 \sin\beta_1 & \cos\alpha_2 \sin\beta_2 & \cos\beta_3 \\ \cos\beta_1 & -\sin\alpha_2 \sin\beta_2 & \cos\alpha_3 \sin\beta_3 \end{bmatrix}$$

$$[A] = [\gamma]\,[A]$$

$$[B] = [\gamma]\,[B]$$

Computation of Outer Gimbal Rate Limits

$$\dot{\alpha}_{ilim} = \frac{\dot{\alpha}_{lim}}{\cos\beta_i} \quad \text{for } i = 1, 3$$

where

$\dot{\alpha}_{lim}$ is a given number

Gimbal Avoidance Logic Computations

Let $\{H_i\}$ be the projection of $\{H\}$ on i^{th} outer gimbal plane and let $\{M_{cfi}\}$ be the projection of $\{M_{cf}\}$ on i^{th} outer gimbal plane.

Then

$$\{H_i\} = \begin{Bmatrix} 0 \\ H_{i2} \\ H_{i3} \end{Bmatrix},$$

$$\{M_{cfi}\} = \begin{Bmatrix} 0 \\ M_{cfi2} \\ M_{cfi3} \end{Bmatrix}$$

where (for the orthogonal configuration)

$H_{12} = H_y$

$H_{13} = H_x$

$H_{22} = H_z$

$H_{23} = H_y$

$H_{32} = H_x$

$H_{33} = H_z$

and

$M_{cf12} = -M_{cf2}$ (M_{cfi} = i^{th} component of $\{M_{cf}\}$)

$M_{cf13} = -M_{cf1}$

$M_{cf22} = -M_{cf3}$

$M_{cf23} = -M_{cf2}$

$M_{cf32} = -M_{cf1}$

$M_{cf33} = -M_{cf3}$

$$H_t = \sqrt{\{H\}^T \cdot \{H\}}$$

$$\{h_i\} = \begin{Bmatrix} 0 \\ -\sin\alpha_i \\ \cos\alpha_i \end{Bmatrix}$$

$$\{\tilde{H}_i\} = -K_m \cdot (1 - K_h \cdot H_t)\{M_{cfi}\} + K_n \cdot H_t \{H_i\}$$

where K_m is a positive constant and

$$K_h = \frac{1}{H_r} \text{ if } H_t \le H_r$$

otherwise,

$$K_h = \frac{1}{H_t}$$

$$\tilde{H}_i = \sqrt{\{\tilde{H}_i\}^T \cdot \{\tilde{H}_i\}}$$

$$\dot{\alpha}_{ige} = \frac{K_{rt} \cdot K_h}{\cos \beta_i}\left(\tilde{H}_i - \{h_i\}^T \cdot \{\tilde{H}_i\}\right)$$

where K_{rt} is a given number,

$$\text{sign } \dot{\alpha}_{iga} = \text{sign } (-\tilde{H}_{i2} \cos \alpha_i - \tilde{H}_{i3} \sin \alpha_i)$$

If

$$\tilde{H}_{i2} \cdot \sin \alpha_i > 0$$

then

$$\text{sign } \dot{\alpha}_{iga} = \text{sign } (-\tilde{H}_{i2})$$

$$\dot{\alpha}_{iga} = \dot{\alpha}_{ige} \cdot \text{sign } \dot{\alpha}_{iga}$$

If

$$|\dot{\alpha}_{iga}| > K_1' \, |\dot{\alpha}_{i1im}|$$

then

$$\dot{\alpha}_{iga} = K_1' |\dot{\alpha}_{i1im}| \cdot \text{sign } \dot{\alpha}_{iga}$$

where

K_1' is a positive constant ≤ 1.

$$\{\dot{\alpha}_{ga}\} = \begin{Bmatrix} \dot{\alpha}_{1ga} \\ \dot{\alpha}_{2ga} \\ \dot{\alpha}_{3ga} \end{Bmatrix}$$

$$\{\dot{\beta}_{ga}\} = - [B]^{-1} [A] \{\dot{\alpha}_{ga}\}$$

The following caution is exercised in the computation of $[B]^{-1}$:

$$|B_{ii}| \geq B_o$$

$$|\det [B]| \geq B_{o1}$$

If

$$|\det [B]| < B_{o1}$$

then

$$\dot{\alpha}_{iga} = - \frac{K_{b1} \sin \alpha_i}{\cos \beta_i}$$

and $\{\dot{\beta}_{ga}\}$ is computed by the above equation, where B_o, B_{o1}, K_{b1} are positive constants.

If

$$|\dot{\beta}_{iga}| > K_i \dot{\beta}_{1im}$$

then

$$\dot{\beta}_{iga} = K_i \dot{\beta}_{1im} \cdot \text{sign } \dot{\beta}_{iga}$$

where

K_i is a positive constant ≤ 1.

Control Law Computation

$$[K_\alpha^2] = [K_\alpha]^T \cdot [K_\alpha]$$

$$[K_\beta^2] = [K_\beta]^T \cdot [K_\beta]$$

$$[C] = [A]\,[K_\alpha^2]\,[A]^T + [B]\,[K_\beta^2]\,[B]^T$$

Note that [C] is a 3x3 symmetric matrix.

If

$$\text{rank } [C] < 3$$

then

$$|C_{ii}| \geq C_o$$

and

$$|\det [C]| \geq C_{o1}$$

where

C_o and C_{o1} are positive constants.

$$\{\dot{\alpha}_{cd}\} = -\,[K_\alpha^2]\,[A]^T\,[C]^{-1}\,\{M_c\}$$

$$\{\dot{\beta}_{cd}\} = -\,[K_\beta^2]\,[B]^T\,[C]^{-1}\,\{M_c\}$$

$$\{\dot{\alpha}_c\} = \{\dot{\alpha}_{cd}\} + \{\dot{\alpha}_{ga}\}$$

$$\{\dot{\beta}_c\} = \{\dot{\beta}_{cd}\} + \{\dot{\beta}_{ga}\}$$

Gimbal Stop Logic

The discrete inputs from the CMG hardware and gimbal rate commands implement the gimbal stop logic which sets

$$\left.\begin{aligned} \dot{\alpha}_{ic} &= 0 \\ K_{i\alpha} &= 0 \end{aligned}\right\} \quad \text{if } |\alpha_i| \geq \alpha_{1m} \text{ and } \dot{\alpha}_{ic} \text{ sign } \alpha_i > 0$$

and

$$\dot{\beta}_{ic} = 0$$

$$\text{if} \quad |\beta_i| > \beta_{lm} \text{ and } \dot{\beta}_{ic} \text{ sign } \beta_i > 0$$

$$K_{i\beta} = 0$$

where

$$\alpha_{lm} = 175 \text{ deg}$$

and

$$\beta_{lm} = 75 \text{ deg}$$

Gimbal Rate Limit Logic

The gimbal rate commands are then limited by the conditions

$$\dot{\alpha}_{ic} = \dot{\alpha}_{ilim} \text{ sign } \dot{\alpha}_i$$

$$\text{if } |\dot{\alpha}_i| \geq \dot{\alpha}_{ilim}$$

and

$$\dot{\beta}_{ic} = \dot{\beta}_{lim} \text{ sign } \dot{\beta}_i$$

$$\text{if } |\dot{\beta}_i| \geq \dot{\beta}_{lim}$$

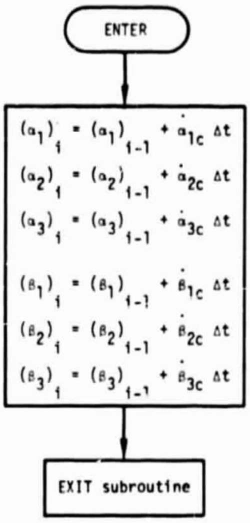
CMG Dynamics Subroutine #16

The reaction torque exerted on the spacecraft by the CMG system due to the commanded gimbal rates is given by

$$[R] = \begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix}$$

$$\{T_{cp}\} = - H_o \left([A]\{\dot{\alpha}_c\} + [B]\{\dot{\beta}_c\}\right) - [R]\{H\}$$

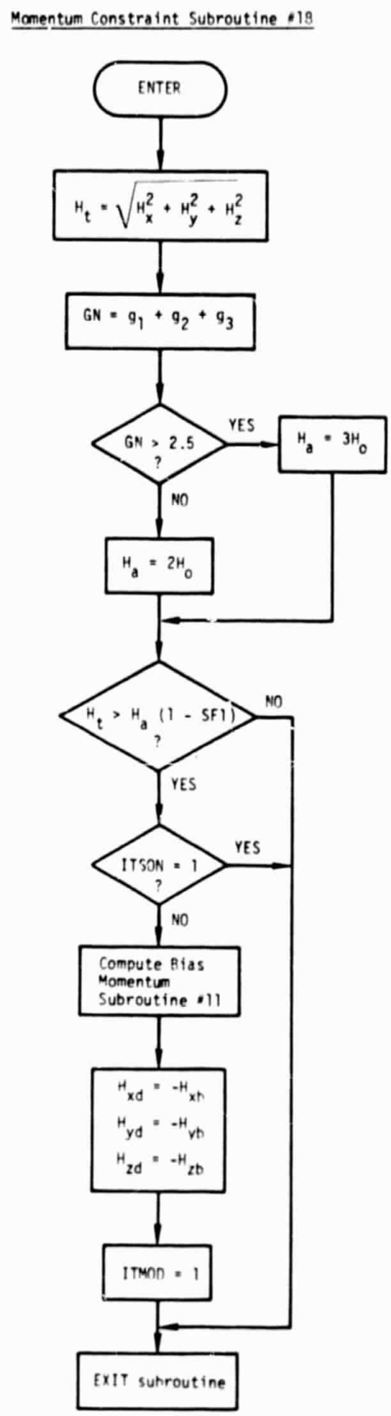
CMG Gimbal Integration Subroutine #17



Note: The subscripts i and i-1 refer to the current value and the value from the previous cycle, respectively.

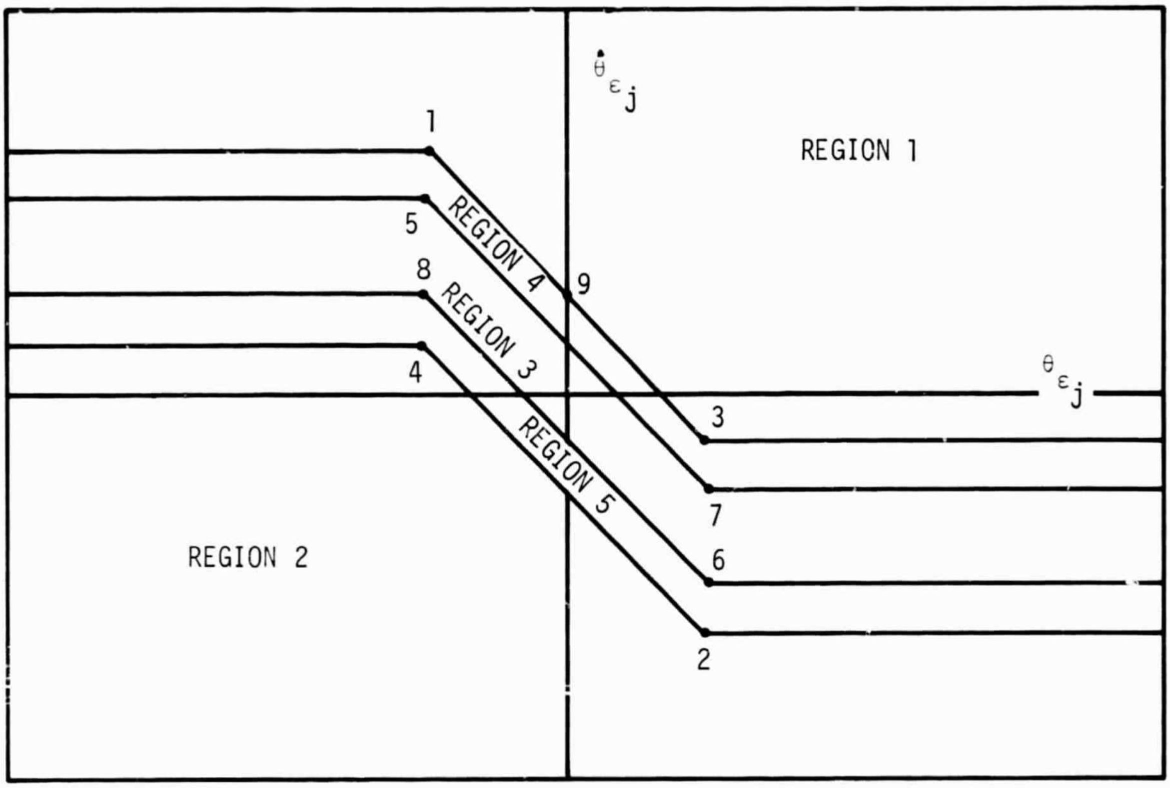
Subroutine 18

Page 1



TACS Phase Plane and Jet Select Logic Subroutine #19

TACS Phase Plane Section



Coordinates of Breakpoints

Point 1) (BPP1, BPR1)
Point 2) (BPP2, BPR2)
Point 3) (BPP3, BPR3)
Point 4) (BPP4, BPR4)
Point 5) (BPP5, BPR5)
Point 6) (BPP6, BPR6)
Point 7) (BPP7, BPR7)
Point 8) (BPP8, BPR8)
Point 9) (BPP9, BPR9)

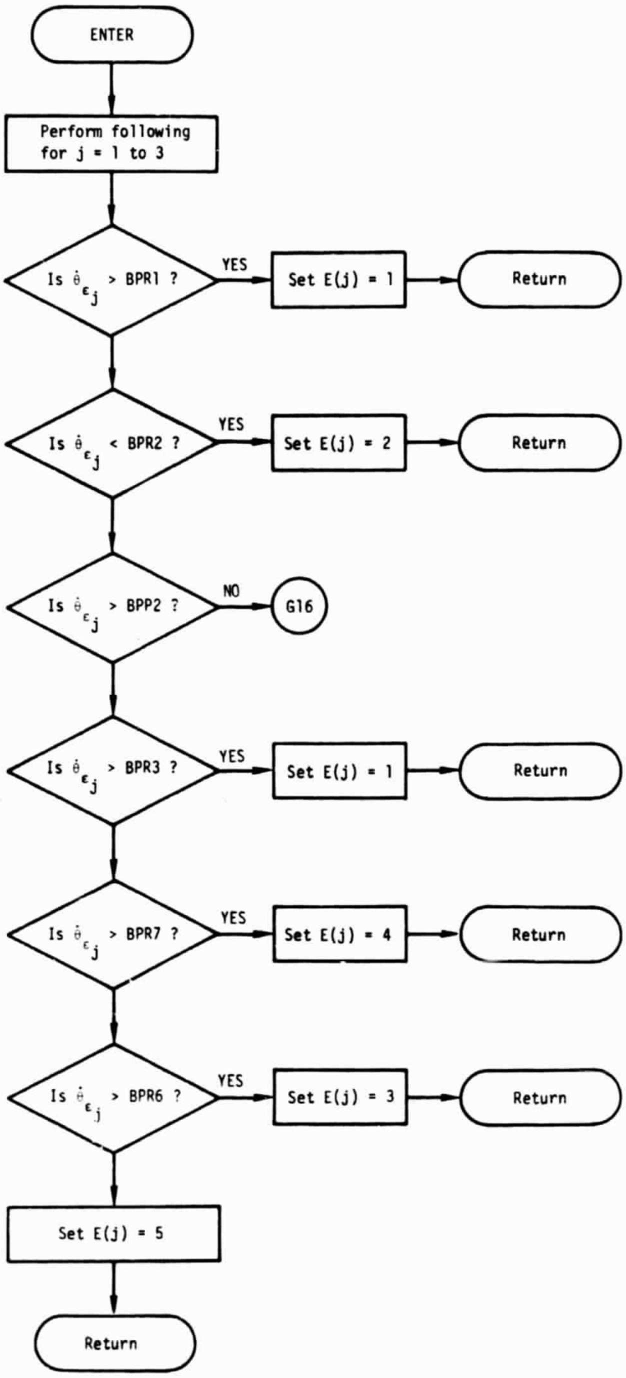
E(2) coordinates are ($\theta_{\varepsilon x}$, $\dot{\theta}_{\varepsilon x}$)

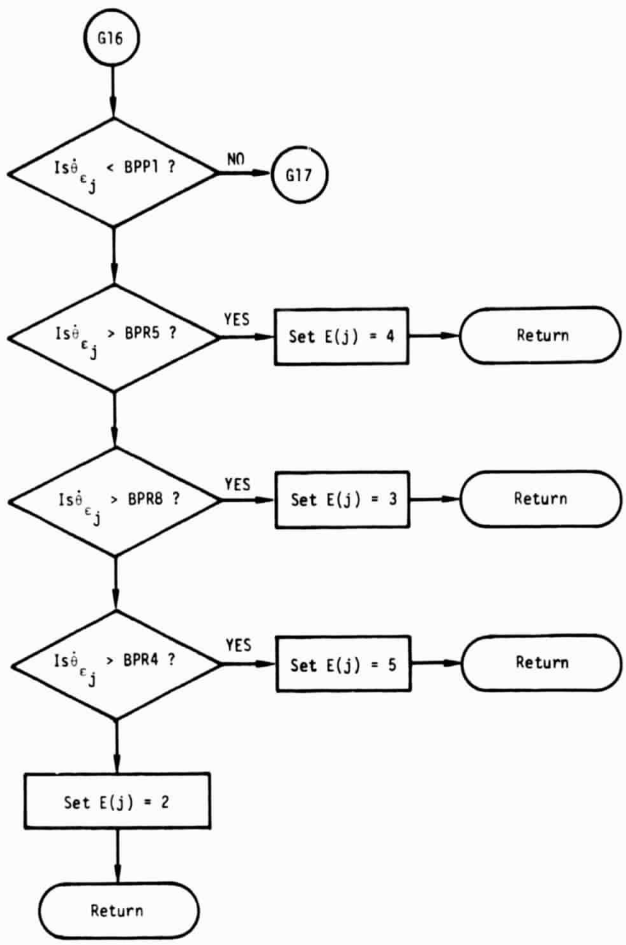
E(3) coordinates are ($\theta_{\varepsilon y}$, $\dot{\theta}_{\varepsilon y}$)

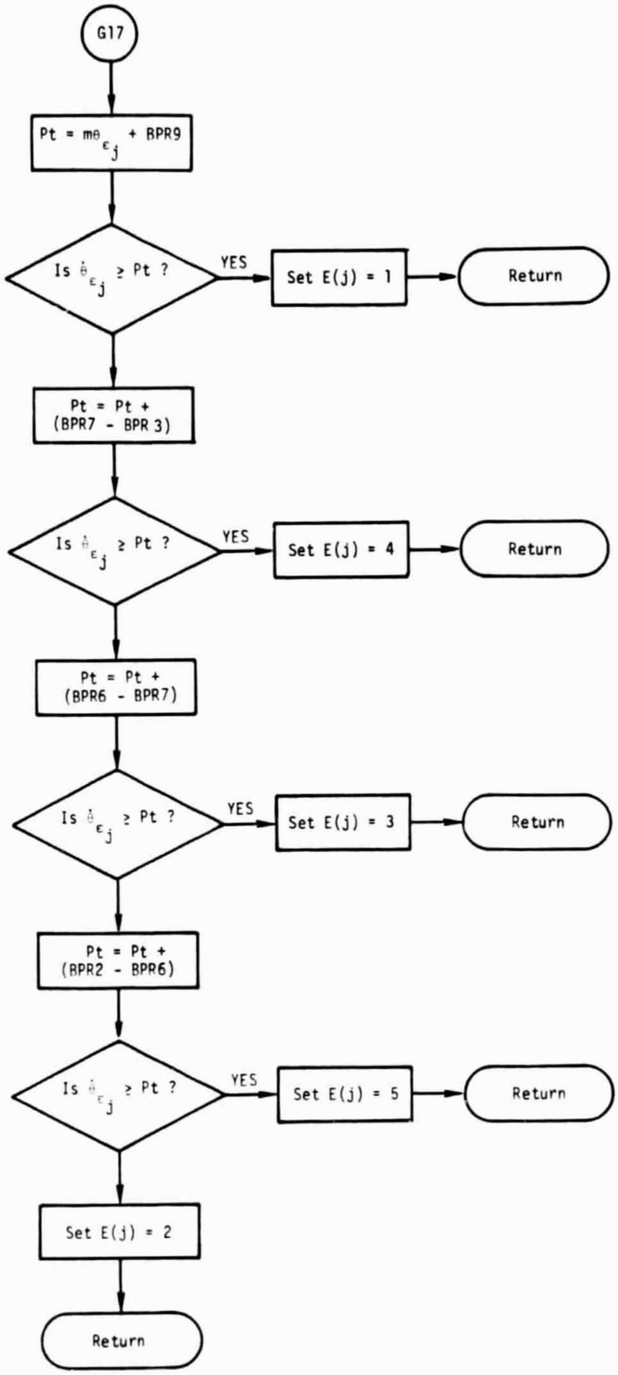
E(1) coordinates are ($\theta_{\varepsilon z}$, $\dot{\theta}_{\varepsilon z}$)

Subroutine 19

Page 2




G16
Is $\dot{\theta}_{\varepsilon_j}$ < BPP1 ?
NO
G17
Is $\dot{\theta}_{\varepsilon_j}$ > BPR5 ?
YES
Set E(j) = 4
Return
Is $\dot{\theta}_{\varepsilon_j}$ > BPR8 ?
YES
Set E(j) = 3
Return
Is $\dot{\theta}_{\varepsilon_j}$ > BPR4 ?
YES
Set E(j) = 5
Return
Set E(j) = 2
Return


G17
Pt = mθ_εj + BPR9
Is θ̇_εj ≥ Pt ?
YES
Set E(j) = 1
Return
Pt = Pt + (BPR7 - BPR 3)
Is θ̇_εj ≥ Pt ?
YES
Set E(j) = 4
Return
Pt = Pt + (BPR6 - BPR7)
Is θ̇_εj ≥ Pt ?
YES
Set E(j) = 3
Return
Pt = Pt + (BPR2 - BPR6)
Is θ̇_εj ≥ Pt ?
YES
Set E(j) = 5
Return
Set E(j) = 2
Return

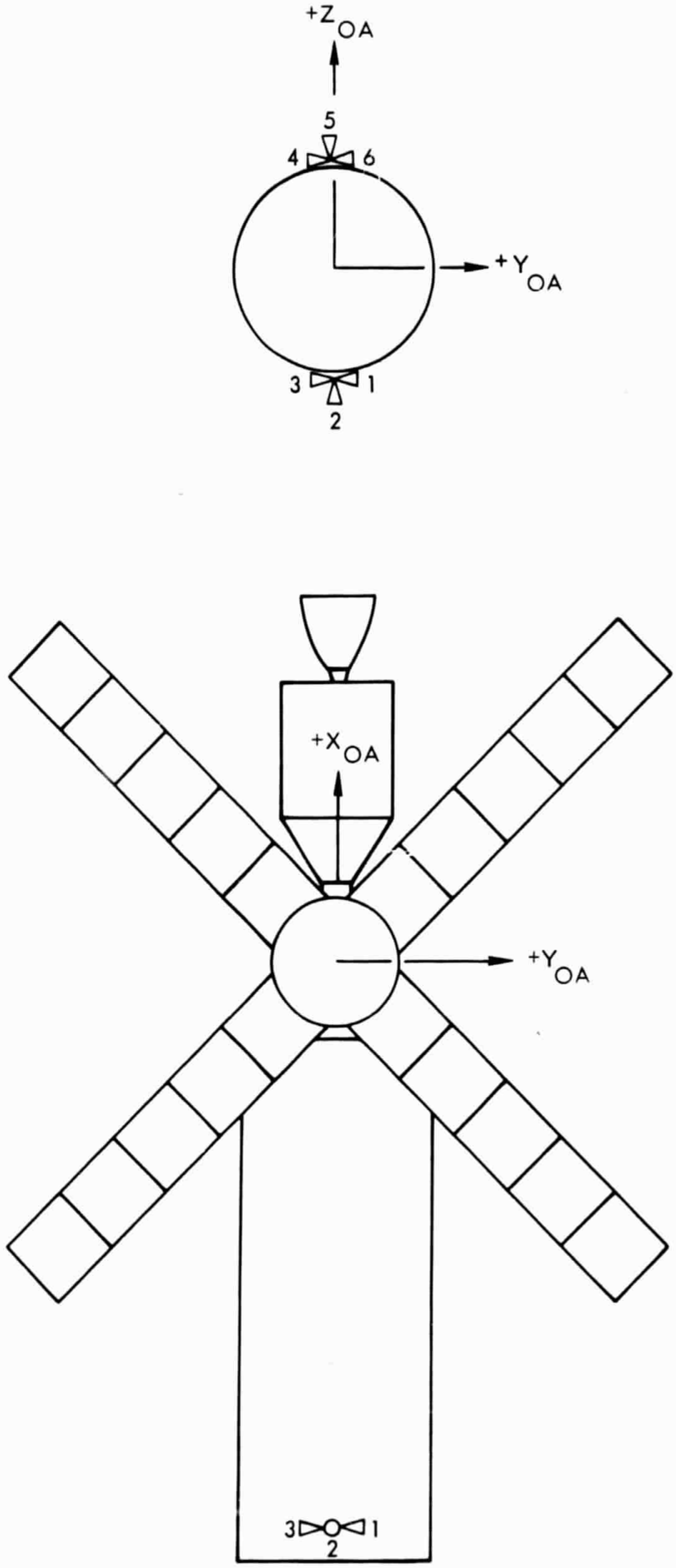


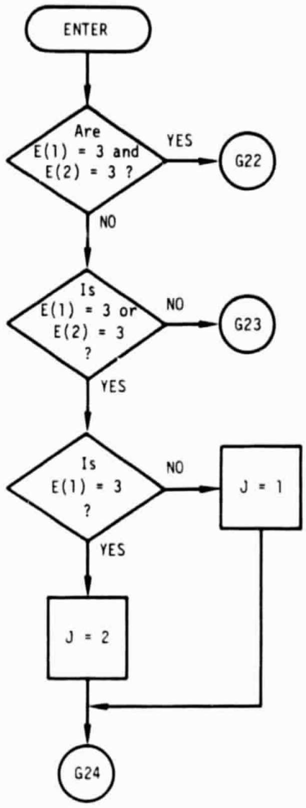
Figure G-1. TACS Jet Configuration

Jet Select Logic

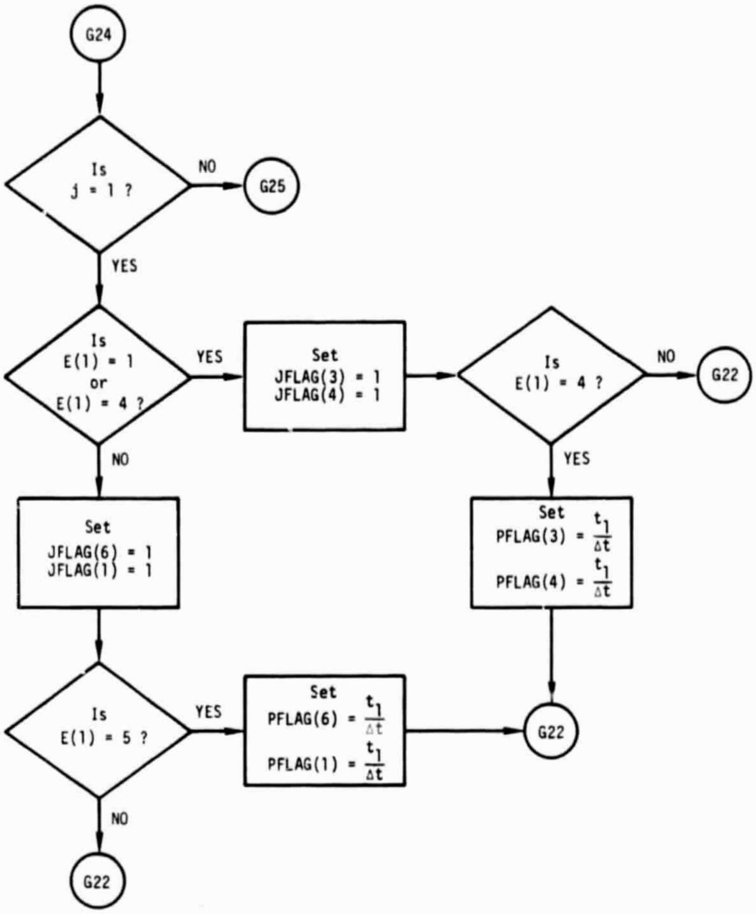
Region		Jet ON Flags JFLAG(I)						Minimum Impulse Flags PFLAG(I)					
$E_z = E(1)$	$E_x = E(2)$	1	2	3	4	5	6	1	2	3	4	5	6
1	1				X								
	2			X									
	3			X	X								
	4				X						X		
	5			X						X			
2	1	X											
	2						X						
	3	X					X						
	4	X						X					
	5						X						X
3	1	X			X								
	2			X			X						
	3												
	4	X			X			X			X		
	5			X			X			X			X
4	1				X						X		
	2			X						X			
	3			X	X					X	X		
	4				X						X		
	5			X						X			
5	1	X						X					
	2						X						X
	3	X					X	X					X
	4	X						X					
	5						X						X

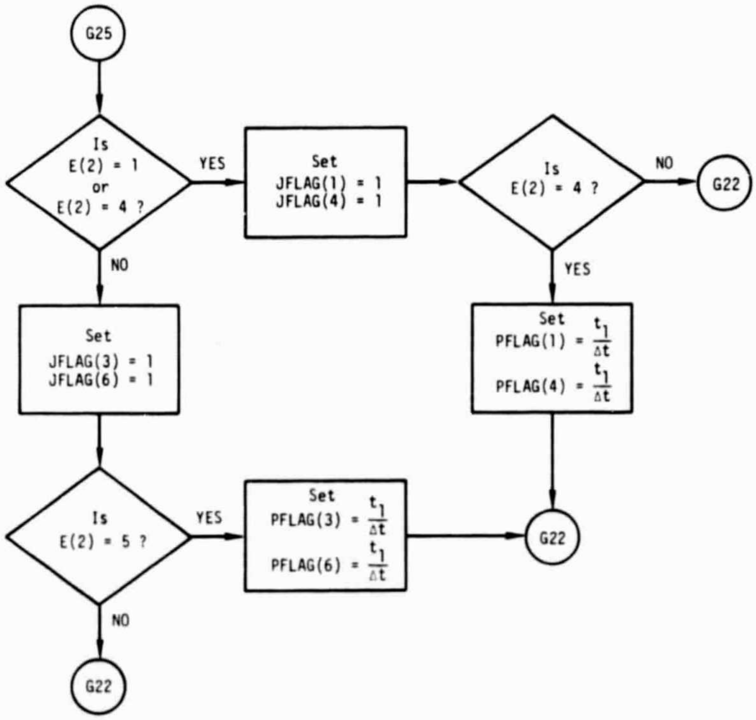
$E_y = E(3)$	1	2	3	4	5	6	1	2	3	4	5	6
1					X							
2		X										
3												
4					X						X	
5		X						X				

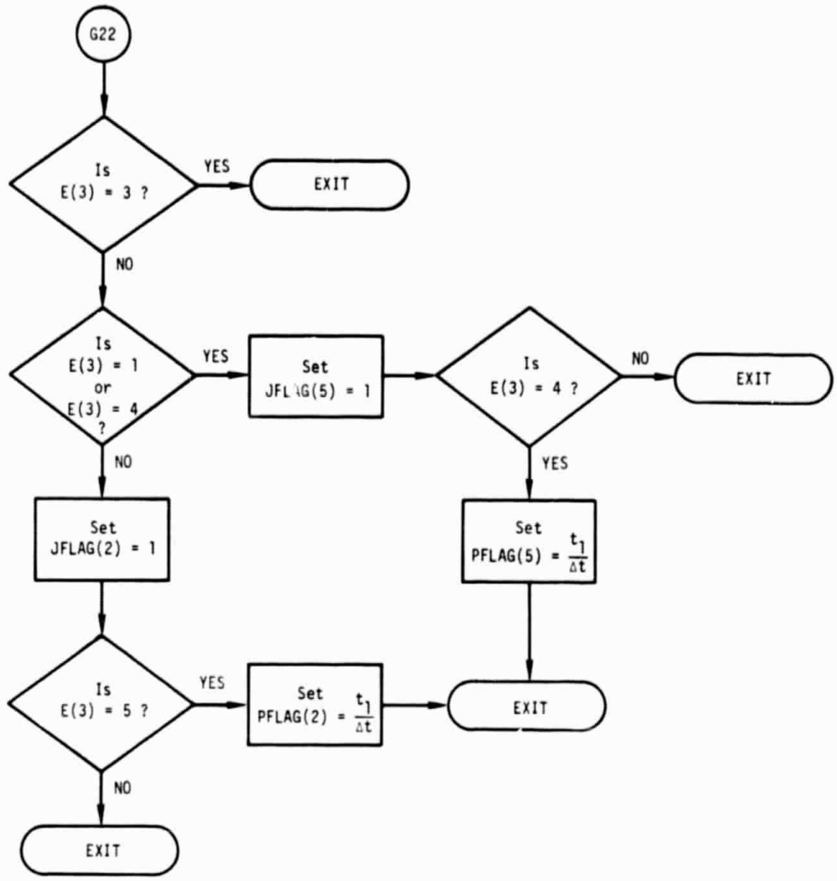
Jet Select Logic

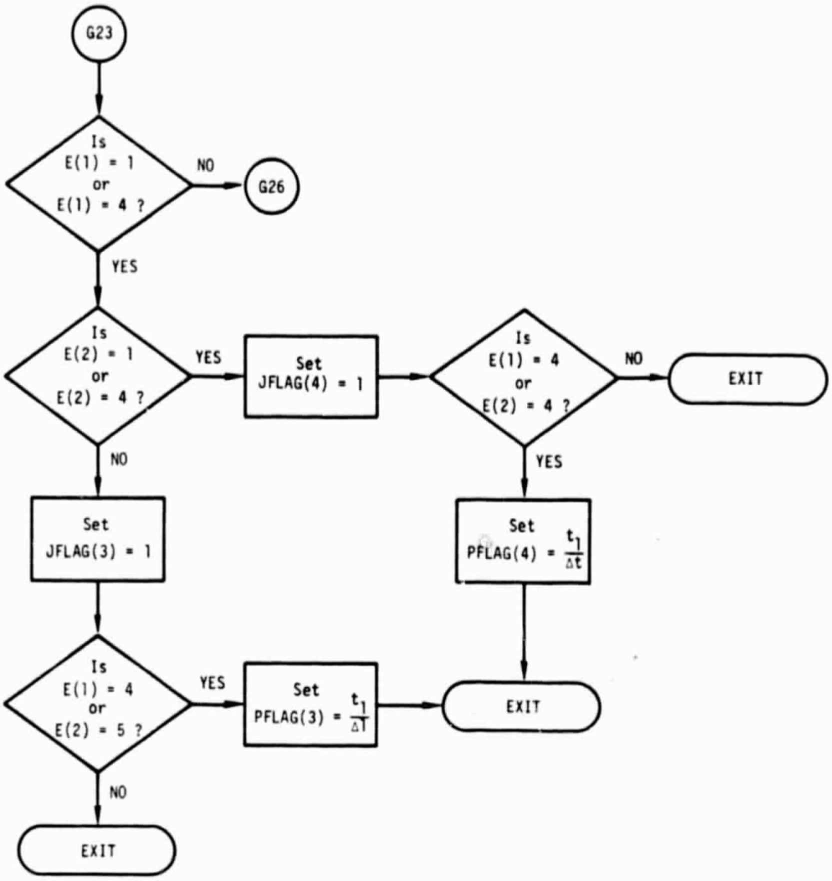


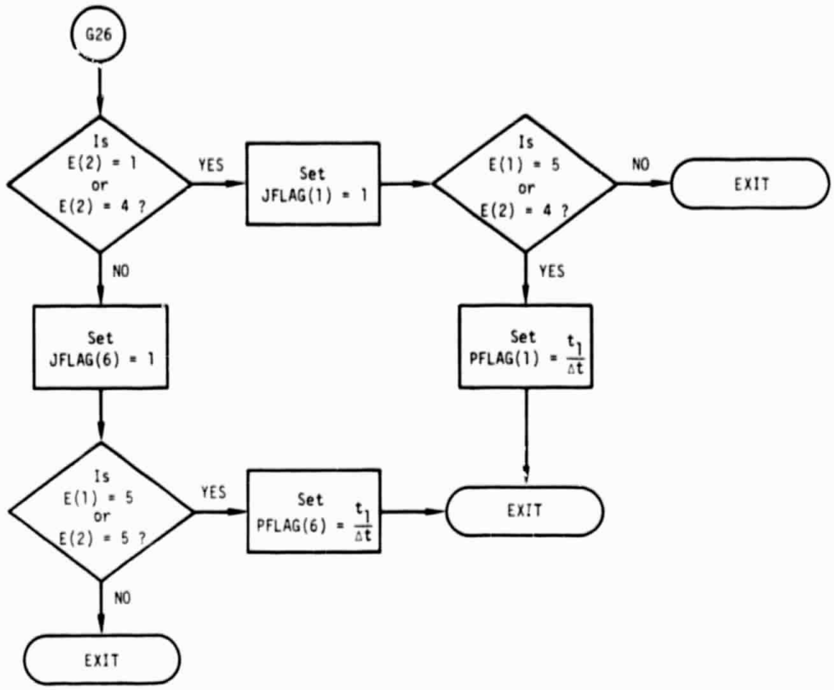
Subroutine 19
Page 8






G22
Is E(3) = 3 ?
YES
EXIT
NO
Is E(3) = 1 or E(3) = 4 ?
YES
Set JFLAG(5) = 1
Is E(3) = 4 ?
NO
EXIT
YES
Set PFLAG(5) = $\frac{t_1}{\Delta t}$
NO
Set JFLAG(2) = 1
Is E(3) = 5 ?
YES
Set PFLAG(2) = $\frac{t_1}{\Delta t}$
EXIT
NO
EXIT

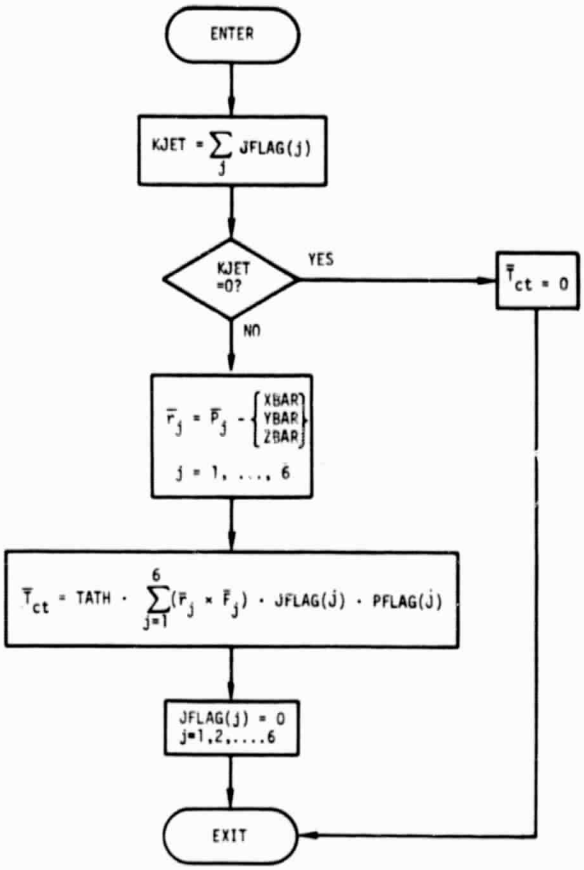

G23
Is
E(1) = 1
or
E(1) = 4 ?
NO
G26
YES
Is
E(2) = 1
or
E(2) = 4 ?
YES
Set
JFLAG(4) = 1
Is
E(1) = 4
or
E(2) = 4 ?
NO
EXIT
YES
Set
PFLAG(4) = $\frac{t_1}{\Delta t}$
NO
Set
JFLAG(3) = 1
Is
E(1) = 4
or
E(2) = 5 ?
YES
Set
PFLAG(3) = $\frac{t_1}{\Delta T}$
EXIT
NO
EXIT


G26
Is E(2) = 1 or E(2) = 4 ?
YES
Set JFLAG(1) = 1
Is E(1) = 5 or E(2) = 4 ?
NO
EXIT
YES
Set PFLAG(1) = $\frac{t_1}{\Delta t}$
NO
Set JFLAG(6) = 1
Is E(1) = 5 or E(2) = 5 ?
YES
Set PFLAG(6) = $\frac{t_1}{\Delta t}$
EXIT
NO
EXIT

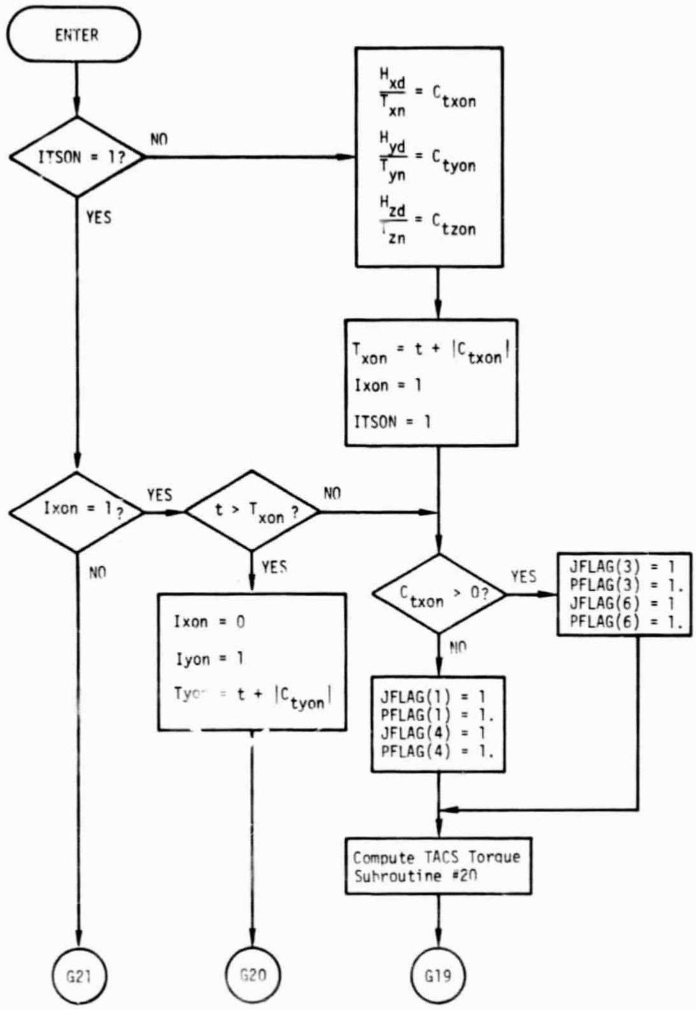
Subroutine 20

Page 1

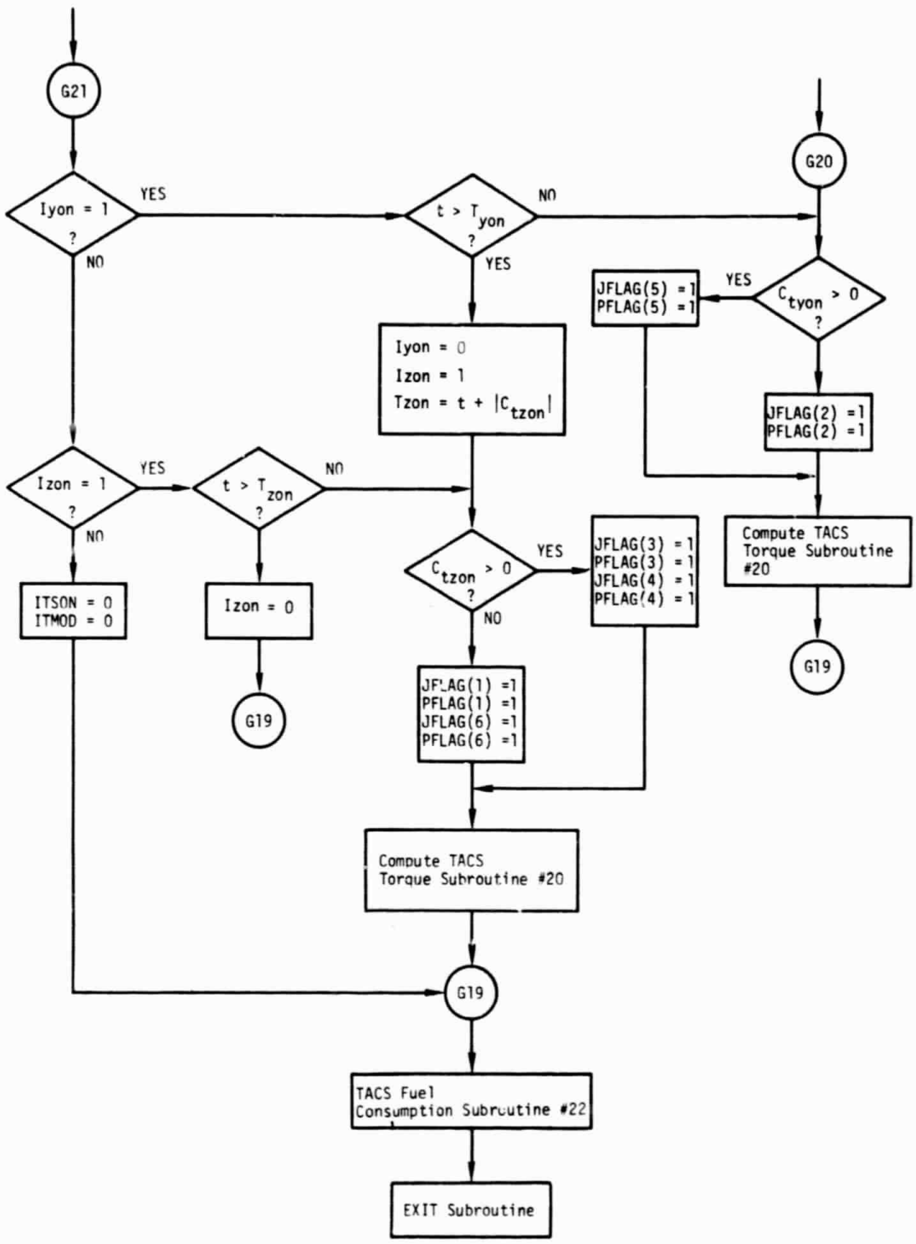
Compute TACS Torque Subroutine #20



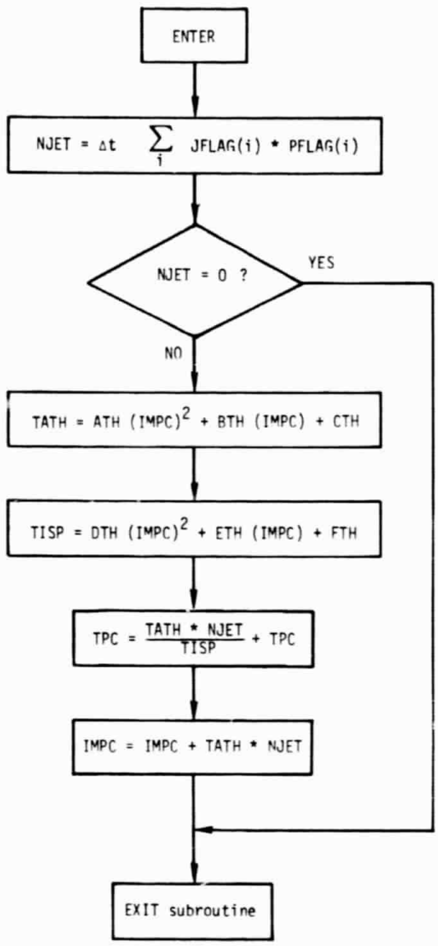
TACS Momentum Dump Subroutine #21



Subroutine 21
Page 2



TACS Fuel Consumption Subroutine #22



APPENDIX H

SENSOR SUBROUTINES

Simulation of Rate Gyro Subroutine #23

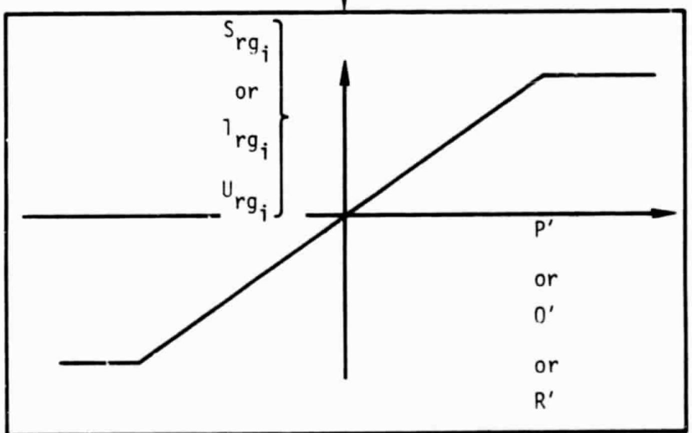
INPUT

$$P' = K_{rg_1} P + K_{rg_2}$$
$$Q' = K_{rg_3} Q + K_{rg_4}$$
$$R' = K_{rg_5} R + K_{rg_6}$$

$$P_{rg} = A_{rg} S_{rg_{i-1}} + B_{rg} S_{rg_{i-2}} + C_{rg} P_{rg_{i-1}} + D_{rg} P_{rg_{i-2}}$$

$$Q_{rg} = A_{rg} T_{rg_{i-1}} + B_{rg} T_{rg_{i-2}} + C_{rg} Q_{rg_{i-1}} + D_{rg} Q_{rg_{i-2}}$$

$$R_{rg} = A_{rg} U_{rg_{i-1}} + B_{rg} U_{rg_{i-2}} + C_{rg} R_{rg_{i-1}} + D_{rg} R_{rg_{i-2}}$$



Output P_{rg}, Q_{rg}, R_{rg}

Note: The notations i, i-1, and i-2 refer to the current value, the value calculated on the previous compute cycle, and the value calculated 2 compute cycles earlier.

Subroutine 24

Page 1

Simulation of Sun Sensor Subroutine #24

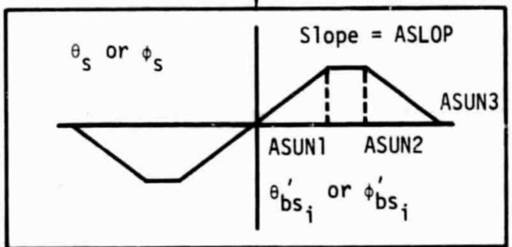
INPUT

↓

$$\theta'_{bs_i} = K_{ss_1}\,\theta_{bs_i} + K_{ss_2}$$

$$\phi'_{bs_i} = K_{ss_3}\,\phi_{bs_i} + K_{ss_4}$$

↓



↓

$$\theta_{ss_i} = A_{ss}\,\theta_{ss_{i-1}} + B_{ss}\,\theta_{gs_i} + C_{ss}\,\theta_{gs_{i-1}}$$

↓

$$\phi_{ss_i} = A_{ss}\,\phi_{ss_{i-1}} + B_{ss}\,\phi_{s_i} + C_{ss}\,\phi_{s_{i-1}}$$

↓

Output θ_{ss_i}, ϕ_{ss_i}

Note: The subscripts i and i-1 refer to the current value and the value calculated on the previous compute cycle, respectively.

APPENDIX I

OUTPUT PARAMETER CALCULATIONS

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Solve [Λ] Based on Current Mode Status

Mode	Value of [Λ]
Delta Maneuver from ZLV (IDELTA = 1)	$[(\Delta Z)_z][(\Delta Y)_y][(\Delta X)_x][\pi/2)_z][A]_{bv}$
ZLV (ISTAR = 1)	$[(\pi/2)_z][A]_{bv}$
Arbitrary Inertial Hold (IARBI = 1)	$[A]_{bu}^{\prime **}$
Solar Inertial (IPSI = 1)	$[A]_{iu}\ [(-\varepsilon)_x][A]_{os}^{\prime}\ [A]_{bs}$

Solve Vehicle Attitude Deviations from Current Reference

See Figure I-1 for a geometrical interpretation of the pointing errors.

f_{ij} = Element from i^{th} row and j^{th} column of [Λ]

$$\theta_x = \sin^{-1} (f_{12}^2 + f_{13}^2)^{1/2}$$

$0 \leq \theta_x < \pi/2$ for $f_{11} \geq 0$

$\pi/2 < \theta_x \leq \pi$ for $f_{11} < 0$

$$\theta_y = \sin^{-1} (f_{21}^2 + f_{23}^2)^{1/2}$$

$0 \leq \theta_y \leq \pi/2$ for $f_{22} \geq 0$

$\pi/2 < \theta_y \leq \pi$ for $f_{22} < 0$

$$\theta_z = \sin \left(f_{31}^2 + f_{32}^2\right)^{1/2}$$

$$0 \le \theta_z \le \pi/2 \qquad \text{for} \qquad f_{33} \ge 0$$

$$\pi/2 < \theta_z \le \pi \qquad \text{for} \qquad f_{33} < 0$$

$$\gamma_x = \tan^{-1}\left(\frac{-f_{12}}{f_{11}}\right) \qquad \beta_x = \tan^{-1}\left(\frac{f_{13}}{f_{11}}\right)$$

$$\alpha_y = \tan^{-1}\left(\frac{-f_{23}}{f_{22}}\right) \qquad \gamma_y = \tan^{-1}\left(\frac{f_{21}}{f_{22}}\right)$$

$$\alpha_z = \tan^{-1}\left(\frac{f_{32}}{f_{33}}\right) \qquad \beta_z = \tan^{-1}\left(\frac{-f_{31}}{f_{33}}\right)$$

Each of α_y, α_z, β_x, γ_x, and γ_z is in the range $[-\pi, \pi]$.

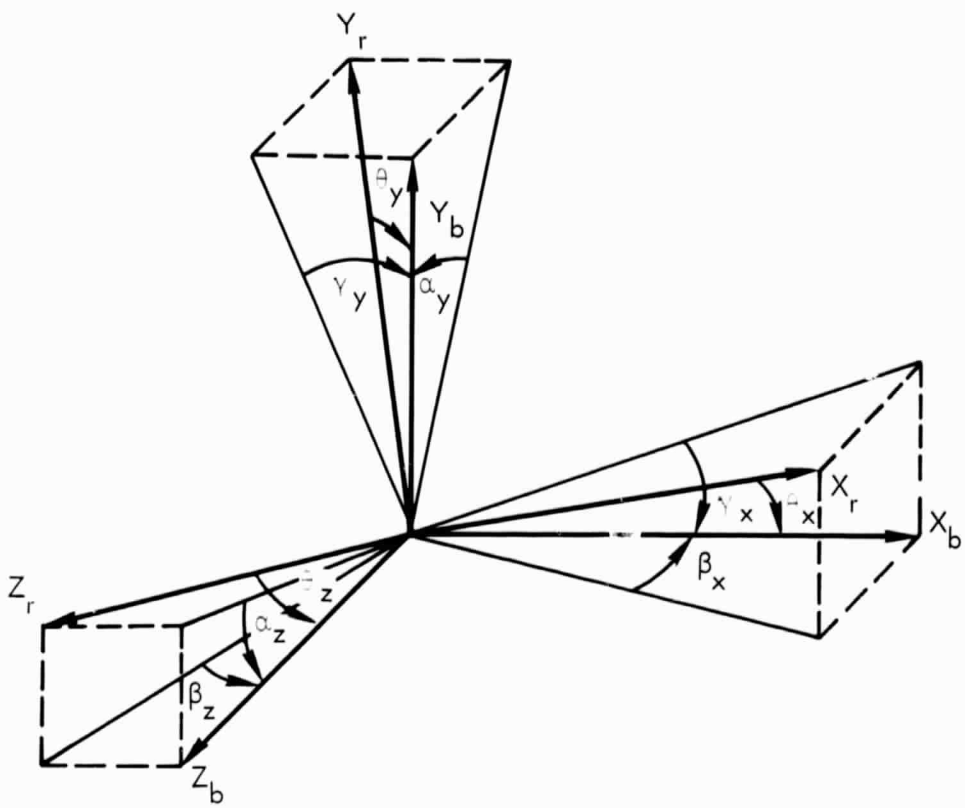

Figure I-1. Definition of Pointing Errors Between Body and Current Reference Coordinate System

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3. L. A. Morine and B. J. O'Connor, "A Description of the CMG and Its Application to Space Vehicle Control," AIAA Paper No. 67-589, August 1967.

4. William B. Chubb and Michael Epstein, "Application of Control Moment Gyros in the Attitude Control of the Apollo Telescope Mount," August 1968.

5. Peter R. Kurzhals and Charles T. Woolley, "Pseudo-Inverse Steering of a System for Inertial Experiment Pointing and Attitude Controls," Preliminary copy dated 7 August 1969 from Langley Research Center.

6. Lee M. Avis, "ATM Gravity Gradient Desaturation Digital Computer Simulation," Preliminary copy dated 26 May 1969 from Langley Research Center.