NASA TECHNICAL NOTE

NASA TN D-5772

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NASA TN D-5772

ATTITUDE REACQUISITION OF A GRAVITY-GRADIENT CONTROL-MOMENT-GYRO STABILIZED SPACECRAFT

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1970

1. Report No.
NASA TN D-5772

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle ATTITUDE REACQUISITION OF A GRAVITY-GRADIENT CONTROL-MOMENT-GYRO STABILIZED SPACECRAFT

5. Report Date
April 1970

6. Performing Organization Code

7. Author(s)
Clark E. Stohl and Bruce E. LeRoy

8. Performing Organization Report No.
E-5493

9. Performing Organization Name and Address
Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio 44135

10. Work Unit No.
704-00

11. Contract or Grant No.

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, D.C. 20546

13. Type of Report and Period Covered
Technical Note

14. Sponsoring Agency Code

15. Supplementary Notes
APPENDIX A - ANALOG SIMULATION - Robert R. Lovell and Clark E. Stohl

16. Abstract

Impulsive or time-decaying disturbances of sufficient magnitude will cause a gravity-gradient control-moment-gyro stabilized spacecraft to tumble. A procedure for operating a thruster system by ground command to perform reacquisition was functionally designed and evaluated on an analog computer. When the disturbance has ceased, an on-board thruster system is used to aline the total angular momentum vector of the spacecraft in an orientation that allows the control-moment gyros to damp all spacecraft motion into a single axis. This single-axis rate can then be directly despun. For a spacecraft in a Sun-synchronous orbit, a procedure was found to shorten the time for attitude reacquisition by using a solar array for crude attitude information.

17. Key Words (Suggested by Author(s))
Attitude control
Control-moment gyros
Gravity gradient

18. Distribution Statement
Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	30	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

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SUMMARY

Impulsive or time-decaying disturbances of sufficient magnitude will cause a gravity-gradient control-moment-gyro stabilized spacecraft to tumble. On reacquisition to the proper stable attitude, the mission can be continued if the experiments are in working order.

A procedure for operating a thruster system by ground command to perform reacquisition was functionally designed and evaluated on an analog computer. When the disturbance has ceased, an on-board thruster system is used to aline the total angular momentum vector of the spacecraft to an orientation that allows the control-moment gyros to damp all spacecraft motion into a single axis. This single-axis rate can then be directly despun.

For a spacecraft in a Sun-synchronous orbit, a procedure was found to shorten the time for attitude reacquisition by using a solar array for crude attitude information.

An analog computer simulation of a specific spacecraft showed that the procedure can be used to reacquire a spacecraft in sufficient time to allow the mission to continue. The optimization of performance and time to reacquire, which could be provided by a closed-loop reacquisition system, is sacrificed to obtain the increased reliability and reduction in expense afforded by an open-loop ground command system.

INTRODUCTION

A spacecraft using gravity-gradient and control-moment-gyro (CMG) stabilization will tend to orient one axis along the local vertical and a second axis perpendicular to the orbit plane. Spacecraft systems capable of producing short-term torques of sufficient magnitude can cause the spacecraft to tumble, and power may be lost if a solar array is used as the primary power source.

There are a number of closed-loop systems using three axis rate and position sensors that would perform a reacquisition. However, these systems usually are complex and expensive. This report presents the study of a system that uses the principle that a rigid body with internal damping will achieve a final rotation about the principal axis of maximum moment of inertia regardless of its initial axis of rotation. With the use of thrusters capable of providing torques about two of the three principal body axes and telemetered CMG gimbal angles, a reacquisition can be performed by ground command.

The reacquisition scheme was demonstrated and evaluated by using an analog computer simulation of the SERT II spacecraft and its control systems. The equations of motion and the analog program were developed by Robert R. Lovell and Clark E. Stohl. This simulation is discussed in appendix A. The SERT II mission is a 6-month orbital life test of a mercury-bombardment ion engine and is attitude stabilized by a gravity-gradient control-moment-gyro system. The SERT II orbit is Sun synchronous; that is, the nodal precession rate of the orbit matches the average angular velocity of the Earth about the Sun. The primary power source is a solar array fixed to the spacecraft.

SYMBOLS

c_i	i^{th} control-moment-gyro damping constant, (N)(m)(sec)
E	error in continuous-rotation equations, dimensionless
$E_{K_{CMG_i}}$	rotational kinetic energy of i^{th} control-moment gyro, where $i = 1, 2$, J
$E_{K_{SC}}$	rotational kinetic energy of spacecraft excluding control-moment gyros, J
$E_{K_{total}}$	total rotational kinetic energy, J
e_i	Euler or quaternion parameters, where $i = 1, 2, 3, 4$
$\overline{H}$	angular momentum vector, (N)(m)(sec)
$\overline{H}_{CMG_i}$	angular momentum vector of i^{th} control-moment gyro, where $i = 1, 2$, (N)(m)(sec)
$\overline{H}_{SC}$	angular momentum of spacecraft excluding control-moment gyros, (N)(m)(sec)
$\overline{H}_{total}$	total angular momentum of spacecraft, (N)(m)(sec)
h_i	angular momentum of i^{th} control-moment-gyro rotor, (N)(m)(sec)
I_g	inertia tensor of control-moment-gyro rotor
I_{ii}	spacecraft principal moments of inertia, where $i = x, y, z$, (kg)(m^2)

I'_{ij}	components of inertia tensor of spacecraft with respect to an orthogonal axis system x', y', z' rotating with respect to body axes, where $i = x, y, z$ and $j = x, y, z$, (kg)(m^2)
I_{ij_g}	i, j components of inertia tensor I_g of control-moment-gyro rotor, where $i = x, y, z$; $j = x, y, z$, (kg)(m^2)
IA_i	i^{th} control-moment-gyro input reference axis
K	gain in error loop for Euler or quaternion equations, dimensionless
$M_{CMG_{j_i}}$	torques due to i^{th} control-moment gyro about j^{th} body axis, where $i = 1, 2$; $j = x, y, z$, (N)(m)
M_{GG_i}	torques due to gravity gradient about i^{th} axis, where $i = x, y, z$, (N)(m)
M_{S_i}	torques due to solar pressure about i^{th} axis, (N)(m)
M_{T_i}	torques due to pneumatic thruster system applied about i^{th} axis, where $i = x, y, z$, (N)(m)
OA_i	i^{th} control-moment-gyro output reference axis
r_c	magnitude of radius vector from center of Earth to spacecraft, m
t	time, sec
μ	Earth gravitational constant, m^3/sec^2
σ_i	control-moment-gyro gimbal angle, where $i = 1, 2$, rad
τ_i	i^{th} control-moment-gyro programed torquer torque, (N)(m)
φ_{g_i}	rotation angle (mounting angle) of i^{th} gyro spin axis in y-z plane to form V-configuration, $i = 1, 2$, deg
$\overline{\omega}$	column matrix of spacecraft rates, rad/sec
$\overline{\omega}^T$	transpose of rate column matrix, rad/sec
$\overline{\omega}_g$	column matrix of control-moment-gyro rotor rotational rates, rad/sec
$\overline{\omega}_g^T$	transpose of column matrix of control-moment-gyro rotor rotational rates, rad/sec
ω_i	spacecraft principal body rates, where $i = x, y, z$, rad/sec
$\omega_{i'}$	body rates about rotating axis system x', y', z', rad/sec
ω_{i_g}	rotational rate of gyro rotor about i^{th} gyro axis, where $i = x, y, z$, rad/sec
ω_o	orbit angular rate, rad/sec

ω_o^* average orbit angular rate, rad/sec

ω_{spin} spin axis rate of control-moment-gyro rotor referenced to control-moment-gyro gimbal, rad/sec

$\omega_{y_g}^*$ amount of gyro rotor rate due to rate of motion of control-moment-gyro gimbal, rad/sec

PROBLEM DESCRIPTION

For spacecraft using gravity-gradient and CMG torques for stabilization, short-term or time-decaying disturbance torques may cause the spacecraft to tumble without destroying the experiments or supporting equipment. It is assumed that the mission is of sufficient duration and the experiments are such that an immediate attitude reacquisition is unnecessary. This permits consideration of simpler methods of reacquisition than provided by an on-board, closed-loop reacquisition system.

Constraints

Although an immediate reacquisition is unnecessary, the time available to perform a reaquisition is a primary constraint on the procedure. The reacquisition must be performed soon enough that thermal damage to spacecraft components is avoided. Missions such as SERT II, which rely on solar energy as the primary power source, must have battery power for operation while the solar panels are disoriented from the Sun. Since operable control-moment gyros, telemetry, and command systems are required for the procedure, the amount of battery energy available defines the upper limit on the time available for reacquisition.

The reacquisition procedure as described in this report assumed that the torque which caused the disorientation has ceased prior to the initiation of the procedure and that the spacecraft has assumed a steady-state tumble.

Axis System

The axis system used throughout is as follows: The principal body axes of the spacecraft form an orthogonal axis set defined as x, y, and z. The y-axis is that of the maximum moment of inertia, and the z-axis is the axis of minimum moment of inertia. As shown in figure 1, when the spacecraft is acquired under gravity-gradient control-moment-gyro stabilization, the x-axis is defined as the roll axis and points positively in

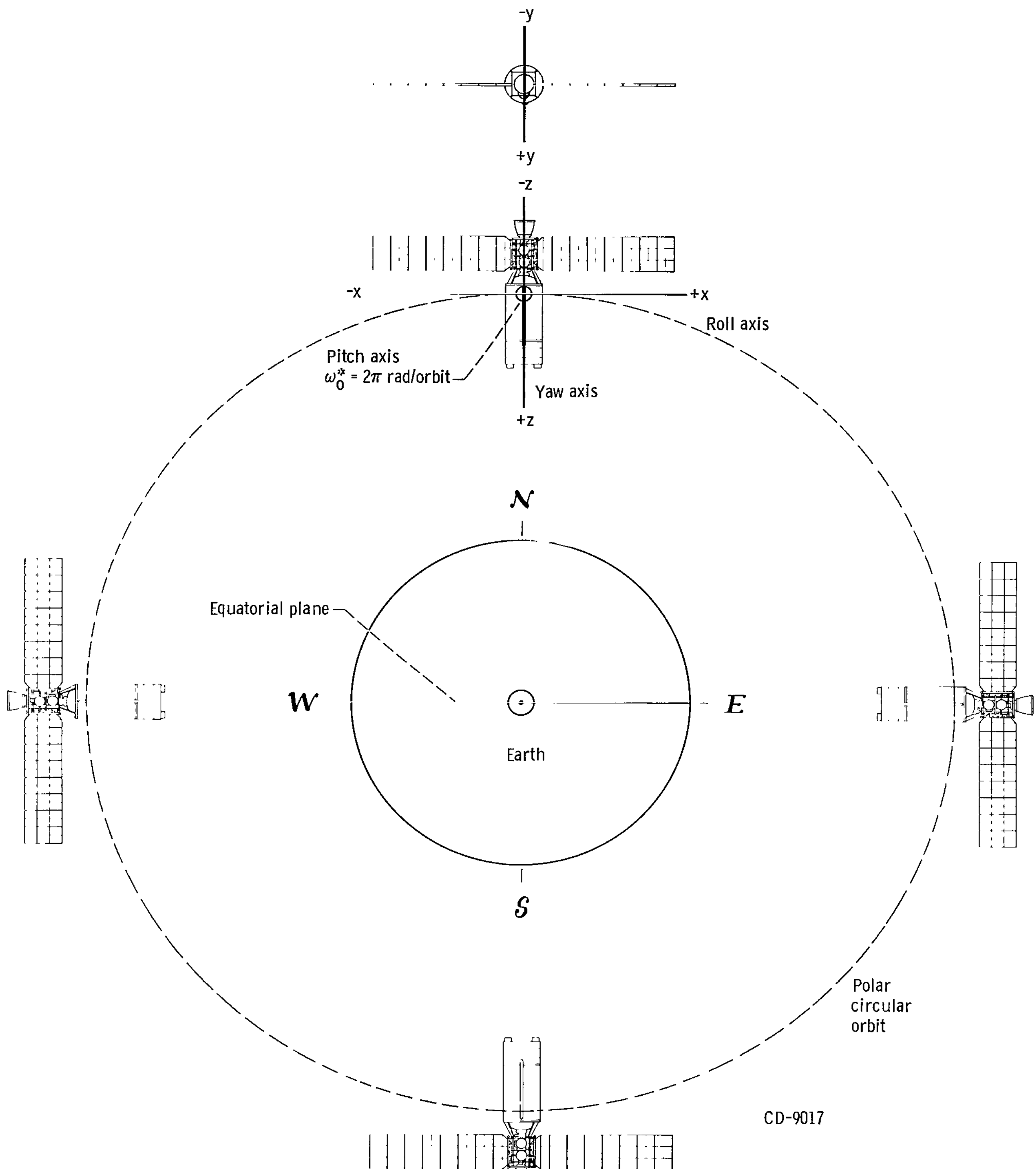

Figure 1. - SERT II spacecraft-vehicle coordinate system in orbit viewed from the Sun for spring launch and sunset orbit injection.

the direction of the tangential velocity vector. The z or yaw axis points to the center of the Earth, and the y or pitch axis is parallel to and in the opposite sense from the orbit angular momentum vector. The SERT II spacecraft conforms to this axis system and is used as an example to demonstrate the operation of the reacquisition procedure. SERT II spacecraft parameters are given in table I.

The orientation of the CMG axes with respect to the body axes is shown in figure 2. SERT II CMG parameters are given in table II.

A basic analysis of gravity-gradient control-moment-gyro stabilization is not treated in this report but can be found in references 1 and 2.

TABLE I. - SERT II SPACECRAFT AND ORBIT PARAMETERS

Inertia, (kg)(m^2)	
Roll, I_{xx}	9.3009×10^3
Pitch, I_{yy}	1.17428×10^4
Yaw, I_{zz}	2.8689×10^3
Spacecraft mass, kg	1.4288×10^3
Semimajor axis, m	7.378368×10^6
Eccentricity	0.005
Inclination, deg	99.1
Thruster system	
Number of thrusters	4
Thrust per thruster, N	0.1423
Pitch torque per pair of thrusters, (N)(m)	0.8108
Yaw torque per pair of thrusters, (N)(m)	0.1952

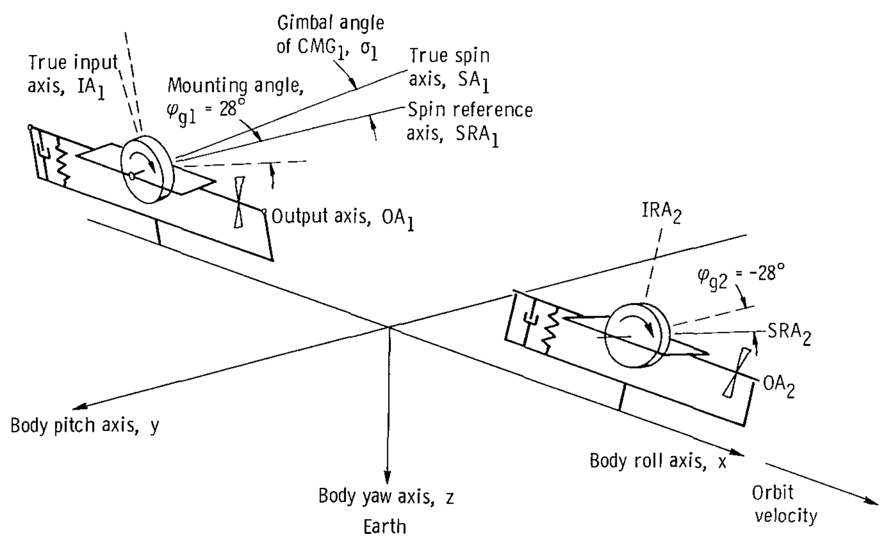

Figure 2. - Control-moment-gyro axes in body axis system. Gimbal stops are located ±30° from spin reference axis. Gimbal angle of CMG is a rotation about output axis and is referenced to spin reference axis.

TABLE II. - CONTROL-MOMENT-GYRO PARAMETERS

Angular momentum, h, (N)(m)(sec)	4.5
Damping constant, c, (N)(m)(sec)	2.25 to 4.5
Spring constant[a]	0.0
Torquer torque, (N)(m)	2.11×10^{-3}
Torque fade[a], (N)(m)/deg	8.7×10^{-6}
Stiction, (N)(m)	1.63×10^{-5}
Drift, (N)(m)	3.25×10^{-5}
Mounting angles, φ_g, deg	
Spin reference axes location from negative pitch axis in yaw-pitch plane	±28
Gimbal stops, deg from spin reference axes	±30

[a]Torquer fade acts as a very light spring in the CMG. Since spring constant is essentially zero, CMG behaves as rate-intergrating gyro with a large angular momentum.

Initial Conditions

If a force-free, rigid body has three unequal moments of inertia, stable rotations are possible only about two of the principal axes. Rotation about the principal body axis of intermediate moment of inertia is unstable (ref. 3). A rotation about the axis of maximum moment of inertia is referred to as a pitch cone, while a rotation about the axis of

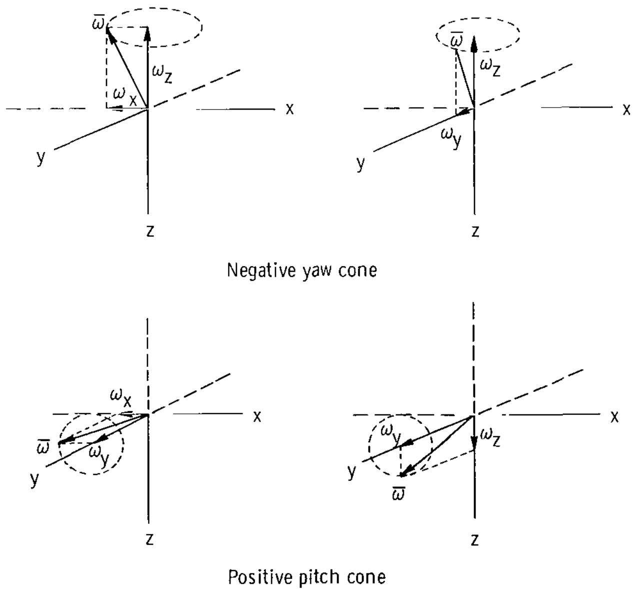

Figure 3. - Total rate vector referenced to body principal axes for tumbling spacecraft.

minimum moment of inertia is called a yaw cone. In a pitch cone, the pitch rate is always some nonzero value, while the yaw and roll rates may be zero or may oscillate about zero. Similarly in a yaw cone, the yaw rate is always some nonzero value, while the pitch and roll rates may be zero or may oscillate about zero (ref. 4). This principle is illustrated in figure 3.

It is assumed that the disturbance which caused the spacecraft to tumble has ceased before the reacquisition procedure is initiated. Allowing for polarities of the nonzero rate, the spacecraft is initially tumbling in either a positive or negative yaw cone or a positive or negative pitch cone. For study purposes, the body rates are limited to a maximum of 5° per second.

REACQUISITION PROCEDURE

The reacquisition procedure encompasses all necessary operations to take the spacecraft from any of the possible initial conditions to a state of stable operation in the proper orientation under gravity-gradient, control-moment-gyro control. Reacquisition is accomplished by two independent sequential operations.

The first operation (despin) includes all steps necessary to bring the spacecraft body

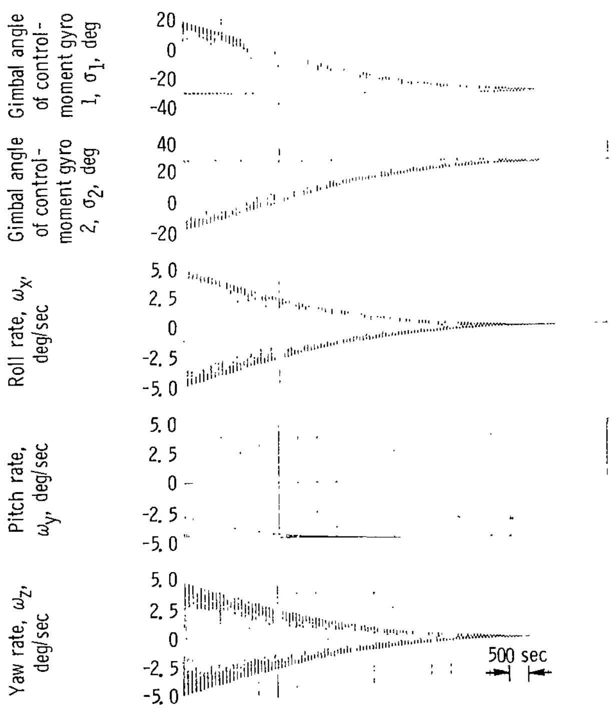

Figure 4. - Damping of yaw and roll rates in negative pitch cone.

rates under control. The control-moment gyros (CMG's) are used to recognize the spacecraft tumble mode. The on-board thruster system is used to force all other cones into a negative pitch cone, and the resulting pure negative pitch rate is then set to the desired value using the CMG's for rate information (see fig. 4).

The second operation (attitude orientation) includes all necessary steps to orient the spacecraft properly under gravity-gradient CMG control. The essential step is to reduce the inertial pitch rate to zero. For the SERT II mission, a procedure was devised to shorten the time to attitude acquisition by using the solar array for crude attitude orientation. Reacquisition is considered complete for SERT II when the maximum solar array deviation from the Sun line is damped to an angle small enough to support the essential spacecraft power requirements.

Despin

The despin portion of the procedure relies on the damping of the CMG's to force the spacecraft to assume the minimum kinetic energy state, while conserving angular momentum. Appendix B explains that, when the spacecraft is tumbling and the CMG's are free to cause internal damping, the spacecraft will seek a steady-state tumble with a pure negative pitch rate. This single-axis rate is easily controlled by ground command of on-board thrusters.

The CMG's cause internal damping when their gimbals move through a damping fluid. Since the CMG angular momentum attempts to aline with the spacecraft angular momentum, it may be seen with the aid of figure 2 that only for a negative pitch cone are the CMG's free to damp. If the dominant coning rate is a yaw rate or a positive pitch rate for the mounting and stop angles chosen, the CMG's are forced to their mechanical stops. Thus, to achieve internal damping, the on-board thrusters are used to force all other cones into a negative pitch cone.

Tumble mode recognition. - In the four possible spacecraft coning conditions, the predominant nonzero rate determines how the CMG's behave. With the aid of figure 2 and the following table, the coning conditions may be recognized from the CMG gimbal angles, σ_1 and σ_2:

Cone	Gimbal angle, deg	
	σ_1	σ_2
Positive yaw	-30	-30
Negative yaw	+30	+30
Positive pitch	+30	-30
Negative pitch	$-28 < \sigma_1 < -10$	$10 < \sigma_2 < 28$

The CMG gimbal angles given for the negative pitch cone are those for steady-state negative pitch rate. Since the CMG's are damping in this cone, the gimbal angles are not restricted to the foregoing limits while damping occurs.

As with all stabilizing systems of this type, certain minimum body rates must be achieved before tumbling can be sustained. For the SERT II parameters (table I), the ratio of yaw rate to pitch rate necessary for tumbling is approximately the same as the ratio of the pitch inertia to the yaw inertia. A pitch cone cannot be sustained for pitch rates less than about $\pm 0.1^{\circ}$ per second about the average orbit pitch rate ($\omega_0^* = -0.057^{\circ}$/sec) for zero roll and yaw rates. Similarly, a yaw cone cannot be sustained for yaw rates less than about $\pm 0.4^{\circ}$ per second for zero roll rate and pitch rate equal to orbit pitch rate.

Positive-pitch-cone despin. - A positive-pitch-cone tumble may be forced to a negative-pitch-cone tumble in a simple manner. The procedure is to command the thrusters to produce a negative pitch torque until both CMG gimbal angles change polarity. For a positive pitch cone, the CMG gimbal angles are $\sigma_1 = 30^{\circ}$ and $\sigma_2 = -30^{\circ}$. As the pitch rate changes polarity, the CMG gimbal angles also change polarity. Thus, when $\sigma_1 < 0.0^{\circ}$ and $\sigma_2 > 0.0^{\circ}$, the pitch rate is negative, and a negative pitch cone is assured.

Yaw cone despin. - Forcing a yaw cone to a negative pitch cone is the most difficult maneuver to perform. The procedure relies most heavily on thrust timing and CMG gimbal angle signals, and errors can easily be made.

Ideally, just enough yaw torque is applied to the spacecraft to force the yaw rate to zero. Provided that there is sufficient pitch axis energy when the yaw rate is near zero, the spacecraft begins a pitch cone tumble, as illustrated in figures 5 and 6.

The appropriate amount of yaw torque may be sensed if the CMG's are properly oriented. For the SERT II spacecraft, the mounting angles and gimbal stops are such that during a yaw cone one CMG has its true input axis nearly alined with the negative yaw axis. In this orientation, the CMG provides an excellent indication of a yaw rate polarity change since pitch rate input to this CMG is negligible. As the yaw rate changes polarity, this CMG moves rapidly from its stop. The CMG lag is small for the initial motion

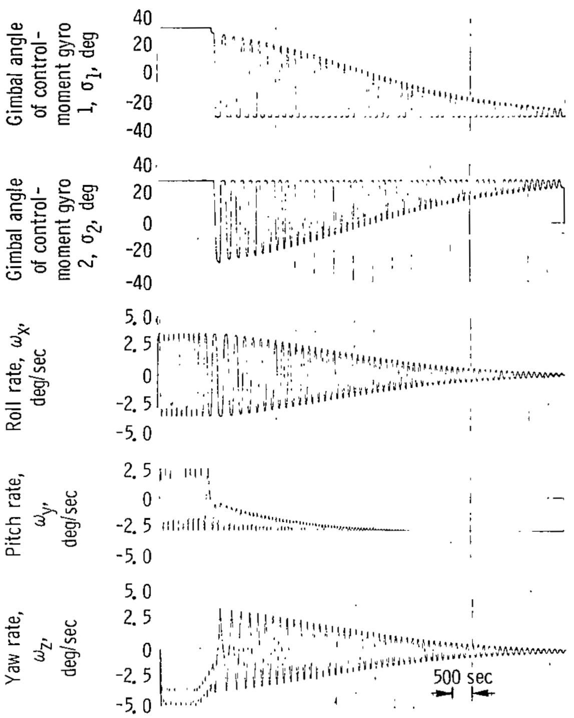

Figure 5. - Application of yaw torque to force spacecraft negative yaw cone to negative pitch cone. Two thrusters giving pure positive yaw torque were fired until 100 seconds after gimbal angle of control-moment gyro 2 came off stop.

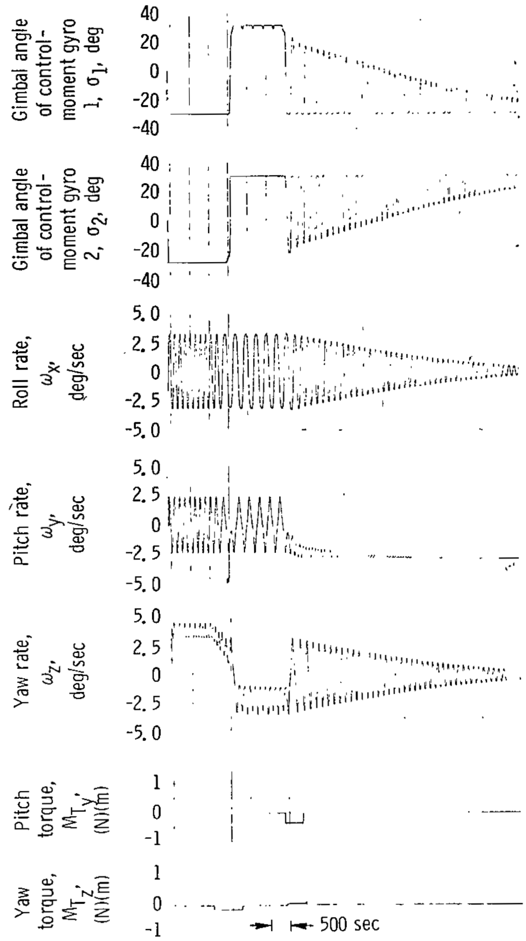

Figure 6. - Application of yaw torque to force spacecraft positive yaw cone to negative pitch cone. Negative-yaw thrusters were fired until 100 seconds after gimbal angle of control-moment gyro 1 came off stop. Note that second yaw cone has yaw rate less than that of first yaw cone. Thruster giving positive yaw torque and negative pitch torque was then fired until negative pitch cone was achieved.

from the stop and does not significantly affect the use of CMG's in determining a yaw rate polarity change.

The procedure is to fire the appropriate thrusters to force the yaw rate through zero. The thrusters are to be shut down as soon as the CMG, with its true input axis alined with the negative yaw axis, moves a detectable distance from its stop.

Negative-pitch-cone despin. - Once a negative pitch cone has been achieved, the CMG's damp the roll and yaw rates to zero. The time required to damp the rates to near zero for SERT II was approximately one orbit or 105 minutes. This time was nearly invariant for all negative pitch rates in the range of consideration. The amount of roll and yaw rate damping depends on the CMG oscillation frequency, and the CMG oscillation frequency depends largely on the yaw rate oscillation frequency. The yaw rate oscillation frequency depends on the pitch rate magnitude.

When the roll and yaw rates are damped to zero, the spacecraft is left with a pure negative pitch rate. The spacecraft now behaves as a gyro with its pitch axis orientation fixed in inertial space. By bringing the initial pitch rate to zero, the spacecraft will eventually be gravity gradient CMG stabilized in the desired attitude. To bring the pitch rate to 0.0° per second, pitch rate information is required. For the CMG mounting and stop angles chosen, the CMG gimbal angles provide this information. Each CMG provides an independent estimate of the negative pitch rate. Thus, at each sampling time, the pitch rate estimates may be averaged to reduce error in the final estimate. For the parameters in tables I and II, figure 7 shows the steady-state CMG response to a constant pitch rate. For pitch rates between -0.1° and -0.5° per second, the gimbal angles may be used to estimate directly the pitch rate. The thruster pulse size required to bring the rate to 0.0° per second may then be calculated.

For negative pitch rates greater than 0.5° per second in magnitude, the error in determining the rate from the CMG gimbal angles can be quite large. Thus, no accurate

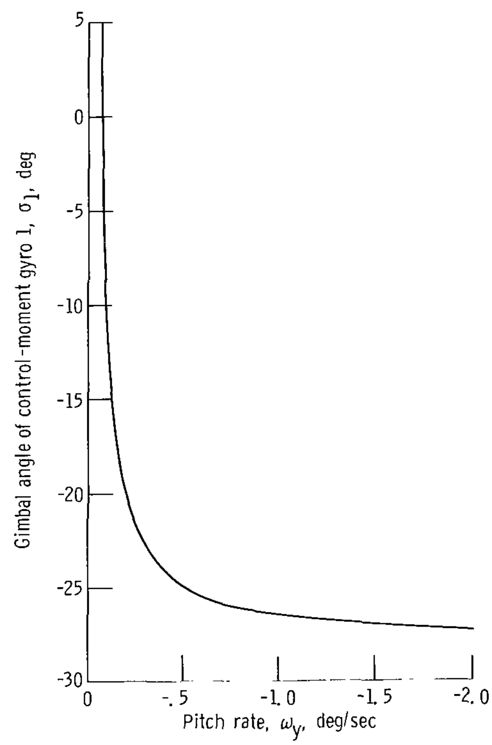

Figure 7. - Control-moment-gyro gimbal angle as function of pitch rate.

value of the required thruster pulse size may be determined. In this case, the positive-pitch-torque thrusters are fired until the end of the pass (approaching loss of communication with the spacecraft) or until $\sigma_1 \geq -20^\circ$ or $\sigma_2 \leq 20^\circ$. If the end of the pass is reached before this gimbal angle criterion is met, a delay sufficient for transients to die out (15 min for SERT II) should be allowed before thrusting again, since it is possible that the rate range of 0.0° to -0.5° per second was achieved but was not apparent because of CMG transient lags.

If the gimbal angle criterion is used to turn off the thrusters, the spacecraft is left tumbling with a pitch rate in the range $\omega_{y,1} \leq \omega_y \leq \omega_{y,2}$. The values of $\omega_{y,1}$ and $\omega_{y,2}$ are determined by the CMG characteristics, sampling time, reaction time, and thruster shutdown time. With the use of the analog simulation, $\omega_{y,1} = +0.4^\circ$ per second, and $\omega_{y,2} = +0.75^\circ$ per second for SERT II. A precalculated negative-pitch torque pulse may now be applied to bring the pitch rate to the range of -0.1° to -0.5° per second. The pitch rate can then be brought to 0.0° per second as described in the beginning of this section.

The time to acquire gravity-gradient CMG stabilization after the pitch rate is brought to zero can be quite long. Depending on the initial orientation of the pitch axis, 10 or more orbits may elapse before the SERT II spacecraft is acquired as in figure 1. Available battery life on SERT II is too short to permit this, so a procedure was devised to shorten the time to attitude acquisition. The procedure is presented in detail in the section Attitude Orientation.

Despin anomalies. - The effects of two factors must be studied when a manual reacquisition system is considered. These are the amount of telemetry contact with the spacecraft and the delays introduced by the telemetry system in sending commands and in receiving and displaying data. They can seriously affect the achievement of a negative pitch cone.

The telemetry delays specifically affect yaw cones where action must be taken when the appropriate CMG signal is recognized. For the SERT II mission, the CMG signals are sampled once per minute, and the time required to shut down two thrusters may be as much as 40 seconds. Thus, the thrusters may not be shut down until 100 seconds after the yaw rate polarity change.

If the initial cone were a pure yaw cone (i.e., no pitch or roll rates), applying the yaw cone procedure given previously yields a new yaw cone. Allowing for the maximum delay in thruster shutdown, the new yaw cone has a yaw rate magnitude less than 0.5° per second for SERT II. If the initial cone were a yaw cone with nonzero pitch and roll rates, applying the yaw cone procedure may still produce a new yaw cone. This new yaw cone is the result of forcing the yaw rate through zero at the same time the pitch rate is zero. At this instant, the roll axis energy is dominant and would normally couple to a

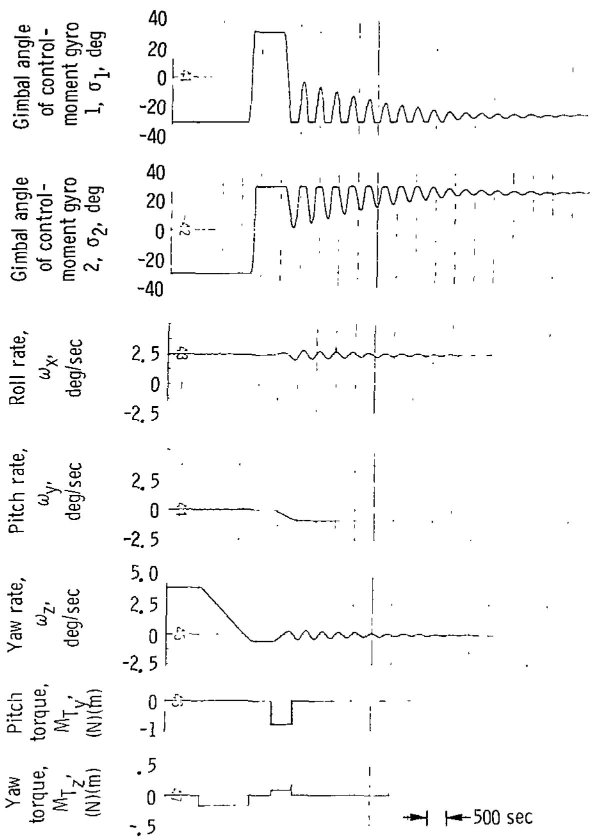

Figure 8. - Pure positive yaw cone to negative yaw cone to negative pitch cone.

pitch cone. But a delay in thruster shutdown may force the yaw rate to a magnitude sufficiently large to cause a new yaw cone.

In either of these cases, the new yaw cone may be recognized from the CMG signals. The procedure here is to fire the thruster that will produce both the appropriate yaw torque and a negative pitch torque. The criterion for thruster shutdown is CMG motion from a stop; however, the time of shutdown is not critical. In this manner, a negative pitch cone is assured. An example of this is shown in figure 8.

Premature thruster shutdown due to the end of a pass (approaching loss of communication with the spacecraft) may significantly affect the positive-pitch-cone procedure. If, at the time of forced shutdown, the pitch rate is small and the yaw rate magnitude is large, a yaw cone will result.

This yaw cone is handled in the same manner as all second yaw cones, by firing a single thruster to produce the appropriate yaw torque and a negative pitch torque. An example of this is shown in figure 9.

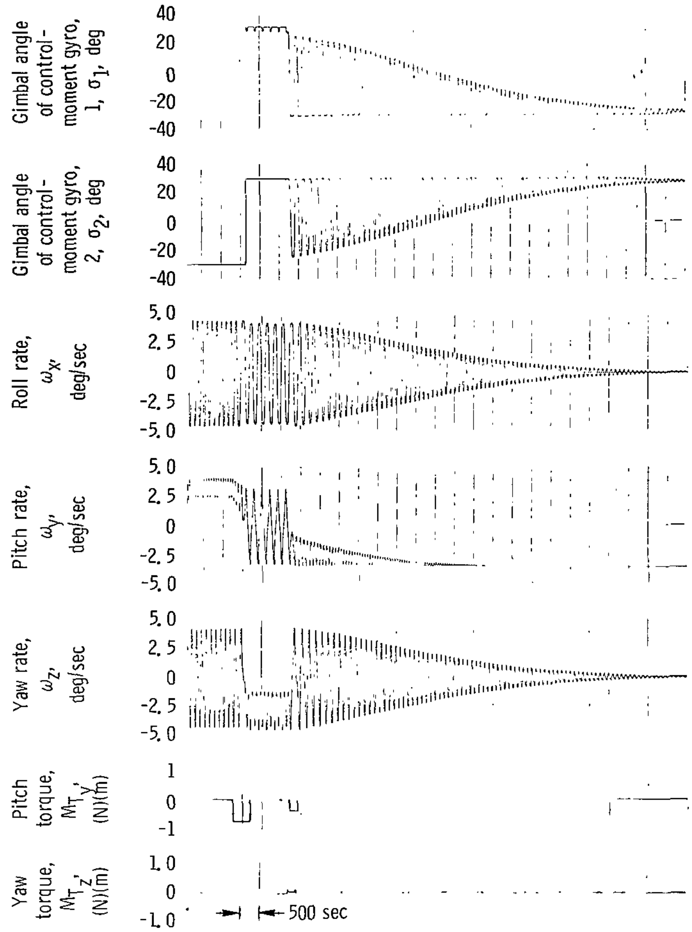

Figure 9. - Positive pitch cone to negative yaw cone to negative pitch cone. Negative-pitch-torque thrusters were fired until one gimbal angle changed polarity. Thrusters were shut down before second gimbal angle changed polarity. Thruster was fired giving positive yaw torque and negative pitch torque until negative pitch cone was achieved.

Attitude Orientation

The time to final attitude acquisition can be shortened for SERT II since the orbit is Sun synchronous. At the end of the despin procedure, the spacecraft pitch rate is set at -0.2° per second rather than at 0.0° per second. This rate spin stabilizes the spacecraft pitch axis in inertial space and is small enough that, for the parameters in table I, a 50-second positive-pitch-torque pulse will bring the rate to zero.

Since the SERT II solar array perpendicular is also the spacecraft pitch axis, the solar array voltage indicates the pitch axis orientation with respect to the Sun. If the solar array is in sunlight, the pitch rate is brought to zero to allow gravity-gradient CMG capture. For the SERT II orbit, the spacecraft Sun line may deviate from the orbit normal by as much as 20°. Thus, the pitch axis may be 110° from the desired orientation with the solar array in sunlight. For an initial pitch axis misalinement of 110°, the analog simulation shows that the pitch axis is misalined less than 60° from the orbit normal in 200 minutes, which is an acceptable time for the SERT II mission. An example of gravity-gradient capture can be seen in figure 10.

If the solar array is not in sunlight (no solar array output), the procedure calls for yawing the spacecraft 180°. To reduce cross-coupling effects, the spacecraft angular

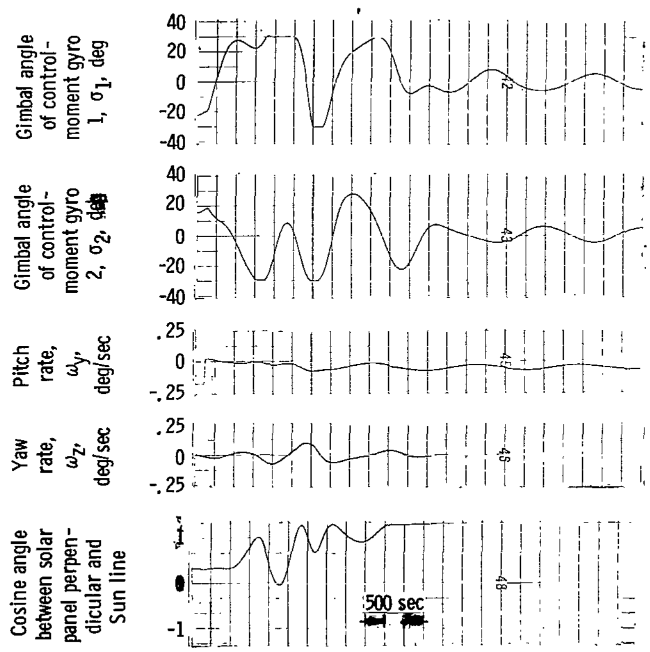

Figure 10. - Gravity-gradient capture.

momentum is reduced to near zero by giving the spacecraft a slight positive pitch rate. The spacecraft angular momentum for this pitch rate cancels the angular momentum of the CMG's. Then a positive- or negative-yaw-torque pulse is commanded, providing a yaw rate. After a 180° rotation, the spacecraft is again spin stabilized in pitch by increasing the pitch rate negatively to a sufficiently large magnitude. An opposite yaw impulse is not used since enough cross coupling occurs to cause a roll rate that must then also be damped to zero.

Since the orbit normal may deviate from the Sun line by 20°, it is possible for the pitch axis to be misalined from the orbit normal by only 70°, with the solar panels not in sunlight. In this case, when the foregoing maneuver is performed, the pitch axis is brought to a 110° misalinement. In this orientation, the solar panels may or may not register an output, depending on the solar array characteristics and the accuracy of the yaw maneuver. Because the gravity gradient can capture the spacecraft from a 110° misalinement, no further yaw maneuvers are performed, whatever the solar array output. Therefore, once the yaw and roll rates are damped to zero, the pitch rate is brought to zero to allow gravity-gradient capture.

Knowledge of the initial pitch rate and the moments of inertia of the spacecraft permit precalculation of the time sequence of pulses required to effect the attitude maneuver. There is no unique pulse pattern required. A larger yaw pulse, and, consequently, a higher yaw rate, requires a shorter elapsed time before initiating the stabilizing negative pitch impulse. However, a higher yaw rate also requires a larger subsequent negative pitch impulse for stabilization. In practice, there are upper and lower bounds on the the yaw rotation rate. Since it is difficult to reduce the spacecraft total angular momentum to exactly zero, small cross-coupling terms will exist. If the yaw rotation rate is too low, the attitude maneuver will take sufficient time that these small cross-coupling terms, in conjunction with the gravity-gradient torques, will distort the turnaround trajectory. On the other hand, if the yaw rotation rate is too large, the pitch acceleration is not large enough to yield the necessary pitch rate for stabilization within the time allowed, and the total angle of rotation will exceed the desired 180°.

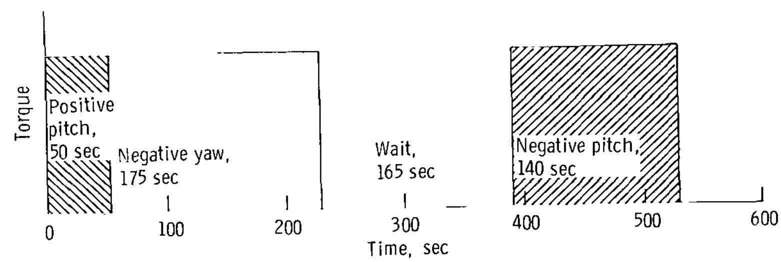

Figure 11. - Nine-minute turnaround pulse sequence.

In figure 11, a pulse sequence of 9 minutes performs this maneuver for the SERT II parameters in table I. Figure 12 presents data from the analog simulation showing a turnaround from an initial misalinement of 180° performed in 9 minutes.

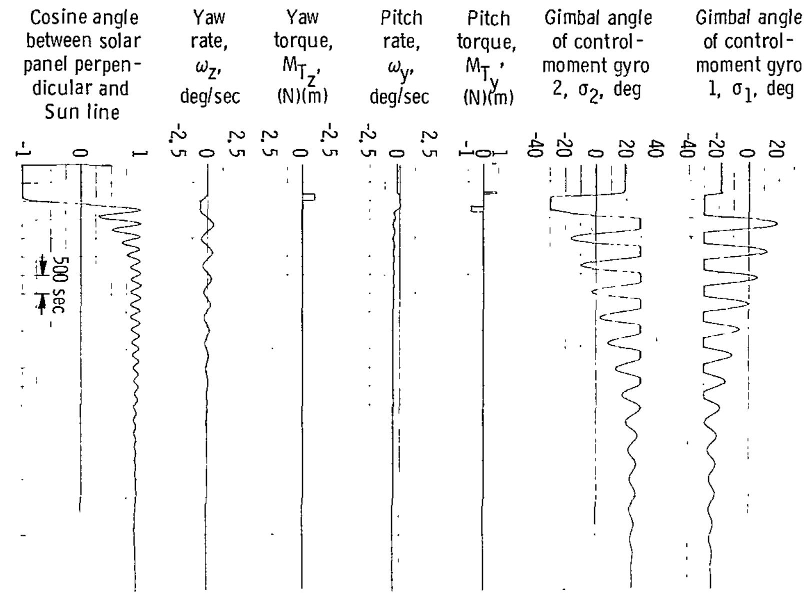

Figure 12. - Attitude maneuver.

ERRORS AND TOLERANCES

Time and Impulse Requirements

Time and impulse requirements are both functions of the spacecraft inertia distribution, torque levels, and magnitude and direction of the initial rate vector from which reacquisition is to be performed. Total reacquisition time is a function of ground station coverage or the time per orbit that the spacecraft is in telemetric contact with the ground.

The analog simulation showed that a worst-case reacquisition (where body angular rates were limited to 5°/sec) can be performed using an impulse of about 175 pound-seconds from the SERT II cold gas storage bottle. For the parameters given in table I, a negative pitch cone can be obtained in about 35 minutes of thrusting. About 100 minutes are required to damp the energy into the pitch axis. About 40 minutes are required to set the pitch rate to -0.2° per second. Of this 40 minutes, only 20 to 25 minutes are actual thrusting time.

If an attitude maneuver is necessary, 9 minutes are required to perform the maneuver, and 100 minutes are required to stabilize at the new attitude. About 2 minutes of thrusting are required to reduce the pitch rate to zero. Finally, 200 minutes are required to sufficiently stabilize the spacecraft at an attitude in which the solar panels may be reliably used as a power source.

Summing the foregoing times gives 486 minutes or 8.1 hours for a worst-case reacquisition of SERT II. However, this number assumes continuous telemetry contact with the spacecraft. In reality, continuous contact with the SERT II spacecraft is not possible, and the total time for a worst-case reacquisition is nearly tripled.

Instrumentation Requirements

If CMG mounting and stop angles are chosen correctly, only knowledge of the CMG gimbal angles is required to perform the reacquisition. The accuracy to which the CMG gimbal angles must be known depends on the pitch rate for which gravity-gradient CMG capture is possible. For the SERT II spacecraft and orbit parameters, bringing the pitch rate to $0.0 \pm 0.03^{\circ}$ per second assures capture. This rate tolerance allows about a 1° error in the CMG gimbal angles when determining the proper pulse size to bring the pitch rate to zero.

The attitude maneuver for shortening capture time described for SERT II is possible because of the Sun-synchronous orbit of the spacecraft. The additional information required is obtained by monitoring solar array output.

Effects of Errors and Tolerances

The control-moment-gyro gain h_i/c_i can vary as shown in table II. The gain variation results from the temperature dependence of the gyro damping constant c_i. The analog simulation shows that the time required to damp roll and yaw rates decreases as the CMG gain decreases.

Measurement of the pitch rate, as required in the procedure, is subject to errors in CMG mounting angles, torquer torque, torquer fade, and CMG angular momentum. Telemetry errors also affect this measurement whether the CMG's or some other device is used to measure the pitch rate.

The reacquisition procedure requires changing the pitch rate by a specified amount by thrusting for a precalculated length of time, which is determined from knowledge of the torque-to-inertia ratio. Therefore, errors in the knowledge of the moments of inertia or thrust level result directly in an error in the final rate.

CONCLUDING REMARKS

The procedure given in this report is an open-loop, ground-commanded method to reacquire the desired spacecraft attitude. The procedure consists of two sequential operations, despin and attitude orientation. In the despin operation, all other tumble cones are forced to a negative pitch cone, the control-moment gyros (CMG's) damp the roll and yaw rates to zero, and the resulting pure negative pitch rate is set to the desired value. In the attitude orientation operation, the pitch rate is reduced to zero if the solar panels are in sunlight. If the solar panels are not in sunlight, the on-board thruster system is used to yaw the spacecraft 180° and to spin stabilize the spacecraft at its new pitch axis attitude. After the roll and yaw rates are damped to zero, the pitch rate is reduced to zero.

An attractive feature of the procedure is that little information and no special instrumentation is required to perform the reacquisition of a gravity-gradient control-moment gyro-stabilized spacecraft. Since the procedure contains well-defined steps, the required telemetry system could be bypassed in favor of an on-board logic system to close the loop. For spacecraft having little telemetry contact with ground stations, an on-board system would greatly decrease the total time to reacquisition.

Methods to shorten the reacquisition time analogous to the turnaround used for SERT II could conceivably be found for orbits that are not "Sun synchronous." For instance, if the orbit were equatorial, instruments sensing the orientation of the pitch axis with respect to the Earth's magnetic field would provide essentially the same information as the SERT II solar panels.

The procedure as applied to the SERT II spacecraft was subjected to practical constraints. A flight-qualified pneumatic thruster system was available and used on this spacecraft. The telemetry system provided 1-minute samples of CMG gimbal angles, and thus, the thrust was required to be small enough that large changes in spacecraft parameters would not occur between samples. If continuous data were available, larger thrust sizes could be used, which would decrease the thrusting time required for the reacquisition.

The reacquisition procedure in theory has no upper limit on the initial rates from which it can reacquire. In practice, this limit is set by the time allowable for reacquisition, the total impulse available, and the destructive limits of the spacecraft.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 2, 1970,
704-00.

APPENDIX A

ANALOG SIMULATION

by Robert R. Lovell and Clark E. Stohl

An evaluation of the reacquisition procedure was performed using an analog simulation of the SERT II spacecraft. Two EAI 231 R-V analog computers were slaved together for the simulation. The memory and logic units incorporated in the 231 R-V computers allows simulation of nonlinearities and discontinuous characteristics.

The spacecraft was modeled as a rigid body with two control-moment gyros. The control-moment gyros were modeled as first-order systems with viscous damping, and spring restraints. In addition to the rigid-body equations of motion and the control-moment-gyro equations, the analysis considered the following:

(1) Continuous rotation equations

(2) Gravity-gradient torques

(3) Orbit eccentricity

(4) Solar pressure

(5) Control-moment-gyro nonlinearities such as gimbal stops, bearing stiction, and torquer fade

Figure 13 is a block diagram of the major parts of the simulation and is followed by a listing of the equations corresponding to the numbered program blocks. Solar pressure

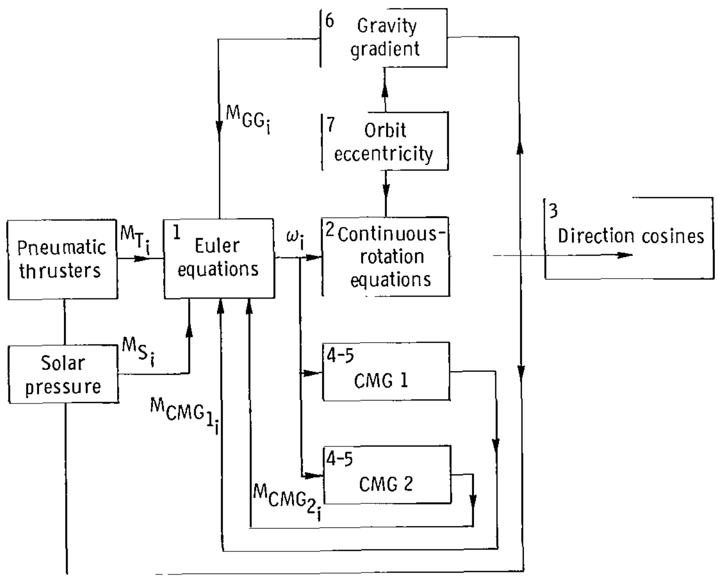

Figure 13. - SERT II analog simulation (i = x, y, z).

and control-moment-gyro nonlinearities are not given since these are mission peculiar items.

Euler Equations

The heart of the program is the Euler equations, which are integrated to determine the spacecraft rates from the sum of all torques acting on it.

$$\left.\begin{aligned}
\dot{\omega}_x &= \frac{I_{yy} - I_{zz}}{I_{xx}} \omega_y \omega_z + \frac{M_x^*}{I_{xx}} \\
\dot{\omega}_y &= \frac{I_{zz} - I_{xx}}{I_{yy}} \omega_x \omega_z + \frac{M_y^*}{I_{yy}} \\
\dot{\omega}_z &= \frac{I_{xx} - I_{yy}}{I_{zz}} \omega_x \omega_y + \frac{M_z^*}{I_{zz}}
\end{aligned}\right\} \tag{1}$$

where $M_i^* = \sum_i \text{Torques} \qquad i = x, y, z.$

Continuous Rotation Equations

A four-parameter system known as the quaternion or Euler parameters (ref. 5) was used to allow for continuous rotation of the spacecraft. These parameters are determined from the spacecraft rates and are combined to yield the direction cosines of the spacecraft attitude with respect to a reference coordinate system. The subscript r denotes a quasi-inertial axis system which is rotating at orbit rate such that the y axis is always perpendicular to the orbit plane and the z axis is always parallel to the local vertical.

$$\left.\begin{aligned}
\dot{e}_1 &= -e_4 \frac{\omega_{xr}}{2} - e_3 \frac{\omega_{yr}}{2} - e_2 \frac{\omega_{zr}}{2} + KEe_1 \\
\dot{e}_2 &= -e_3 \frac{\omega_{xr}}{2} + e_4 \frac{\omega_{yr}}{2} + e_1 \frac{\omega_{zr}}{2} + KEe_2 \\
\dot{e}_3 &= +e_2 \frac{\omega_{xr}}{2} + e_1 \frac{\omega_{yr}}{2} - e_4 \frac{\omega_{zr}}{2} + KEe_3 \\
\dot{e}_4 &= +e_1 \frac{\omega_{xr}}{2} - e_2 \frac{\omega_{yr}}{2} + e_3 \frac{\omega_{zr}}{2} + KEe_4
\end{aligned}\right\} \tag{2}$$

where

ω_{xr} $\omega_x - 2(e_1e_2 + e_3e_4)\omega_o$

ω_{yr} $\omega_y - (e_1^2 - e_2^2 + e_3^2 - e_4^2)\omega_o$

ω_{zr} $\omega_z - 2(e_2e_3 - e_1e_4)\omega_o$

K error loop gain

$E = 1 - e_1^2 - e_2^2 - e_3^2 - e_4^2$

Direction cosines. - The direction cosines relating the body axes to a quasi-inertial axis set are given by the following matrix:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} e_1^2 - e_2^2 - e_3^2 + e_4^2 & 2(e_1e_2 + e_3e_4) & 2(e_2e_4 - e_1e_3) \\ 2(e_3e_4 - e_1e_2) & e_1^2 - e_2^2 + e_3^2 - e_4^2 & 2(e_2e_3 + e_1e_4) \\ 2(e_1e_3 + e_2e_4) & 2(e_2e_3 - e_1e_4) & e_1^2 + e_2^2 - e_3^2 - e_4^2 \end{bmatrix} \begin{bmatrix} x_{QI} \\ y_{QI} \\ z_{QI} \end{bmatrix} \tag{3}$$

Control-moment-gyro equation of motion. - It was found that the gimbal inertia could be neglected in the simulation. Therefore, the equations of motion for the control-moment gyros are:

$$\dot{\sigma}_i = -\frac{K}{c_i \sigma_i} + \frac{h_i}{c_i}\left[(-\omega_y \epsilon_{32} - \omega_z \epsilon_{33})\cos \sigma_i - (-\omega_y \epsilon_{22} - \omega_z \epsilon_{23})\sin \sigma_i\right] + \frac{1}{c_i}\tau_i \tag{4}$$

Control-moment-gyro torques. - The components of the torque applied to the spacecraft by the control-moment gyros are

$$\left.\begin{aligned}
M_{CMG_{xi}} &= h_i\left[-\omega_y(-\epsilon_{23}\cos \sigma_i - \epsilon_{33}\sin \sigma_i) + \omega_z(-\epsilon_{22}\cos \sigma_i - \epsilon_{32}\sin \sigma_i)\right] \\
M_{CMG_{yi}} &= h_i\left[-\dot{\sigma}_i(\epsilon_{22}\sin \sigma_i - \epsilon_{32}\cos \sigma_i) + \omega_x(-\epsilon_{23}\cos \sigma_i - \epsilon_{33}\sin \sigma_i)\right] \\
M_{CMG_{zi}} &= h_i\left[-\dot{\sigma}_i(\epsilon_{23}\sin \sigma_i - \epsilon_{33}\cos \sigma_i) - \omega_x(-\epsilon_{22}\cos \sigma_i - \epsilon_{32}\sin \sigma_i)\right]
\end{aligned}\right\} \tag{5}$$

The terms ϵ_{ij} represent the direction cosines between the body axis system and the i^{th} gyro reference axis. For the "V"-configuration, these are

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi_{gi} & \sin \varphi_{gi} \\ 0 & -\sin \varphi_{gi} & \cos \varphi_{gi} \end{bmatrix}$$

where φ_{gi} is the rotation of the i^{th} gyro spin reference axis in the y-z plane to form the "V"-configuration.

Gravity-Gradient Torques

The gravity-gradient torques are determined from the attitude direction cosines:

$$
\left.\begin{aligned}
M_{GG_x} &= \frac{3\mu}{r_c^3}\left(I_{zz} - I_{yy}\right)2\left(e_2e_3 + e_1e_4\right)\left(e_1^2 + e_2^2 - e_3^2 - e_4^2\right) \\
M_{GG_y} &= \frac{3\mu}{r_c^3}\left(I_{xx} - I_{zz}\right)2\left(e_2e_4 - e_1e_3\right)\left(e_1^2 + e_2^2 - e_3^2 - e_4^2\right) \\
M_{GG_z} &= \frac{3\mu}{r_c^3}\left(I_{yy} - I_{xx}\right)4\left(e_2e_4 - e_1e_3\right)\left(e_2e_3 + e_1e_4\right)
\end{aligned}\right\} \tag{6}
$$

Orbit Eccentricity

The following equations relate the instantaneous orbit angular rate ω_o, the average orbit angular rate ω_o^*, and μ/r_c^3 for small eccentricity:

$$
\left.\begin{aligned}
\omega_o &= \omega_o^*(1 + 2e \cos \omega_o^* t) \\
\omega_o^* &= \sqrt{\frac{\mu}{a^3}} \\
\frac{\mu}{r_c^3} &= (\omega_o^*)^2 \left(1 + 3e \cos \omega_o^* t\right)
\end{aligned}\right\} \tag{7}
$$

where a = semimajor axis of orbit.

APPENDIX B

EFFECT OF INTERNAL DAMPING ON SERT II SPACECRAFT

A tumbling spacecraft with internal damping and freedom from external torques must assume the minimum kinetic energy state while conserving angular momentum. The following discussion shows that this minimum kinetic energy state for the SERT II spacecraft corresponds to a single-axis negative rate about the spacecraft pitch axis.

The spacecraft can be considered as three connected rigid bodies consisting of the main body and two control-moment gyros. When the spacecraft is tumbling, the gravity-gradient torques essentially integrate to zero over a cycle, and the spacecraft can be treated approximately as having no external torques; therefore, angular momentum is conserved. In this case, the control-moment gyros provide internal damping that dissipates the kinetic energy. As this energy is dissipated, the kinetic energy will seek a minimum level while conserving angular momentum:

$$E_{K_{total}} = E_{K_{CMG_1}} + E_{K_{CMG_2}} + E_{K_{SC}}$$

$$E_{K_{CMG}} = \frac{1}{2} \overline{\omega}_g \cdot \overline{H} = \frac{1}{2} \overline{\omega}_g^T I_g \overline{\omega}_g$$

$$E_{K_{CMG}} = \frac{1}{2} I_{xx_g} \omega_{x_g}^2 + \frac{1}{2} I_{yy_g} \omega_{y_g}^2 + \frac{1}{2} I_{zz_g} \omega_{z_g}^2 + I_{xy_g} \omega_{x_g} \omega_{y_g} + I_{xz_g} \omega_{x_g} \omega_{z_g} + I_{yz_g} \omega_{y_g} \omega_{z_g}$$

If y is the gyro wheel spin axis, then I_{xx_g}, I_{yy_g}, and I_{zz_g} are all of the same order of magnitude, and the cross-products of inertia are essentially zero. Also, ω_{y_g} can be considered $\omega_{y_g}^* + \omega_{spin}$. Therefore,

$$E_{K_{CMG}} = \frac{1}{2} I_{xx_g} \omega_{x_g}^2 + \frac{1}{2} I_{yy_g} \left(\omega_{y_g}^* + \omega_{spin}\right)^2 + \frac{1}{2} I_{zz_g} \omega_{z_g}^2$$

Since

$$\omega_{spin} \gg \omega_{x_g}$$

$$\omega_{spin} \gg \omega_{y_g}^*$$

and

$$\omega_{spin} \gg \omega_{z_g}$$

the only term of significance is that due to the ω_{spin}:

$$E_{K_{CMG}} = \frac{1}{2} I_{yy_g} \omega_{spin}^2$$

The kinetic energy of the CMG's can then be considered a constant.

The total kinetic energy is, therefore, given by the expression

$$E_{K_{total}} = I_{yy_g} \omega_{spin}^2 + E_{K_{SC}}$$

where

$$E_{K_{SC}} = \frac{1}{2} \overline{\omega}^T I \overline{\omega}$$

$$E_{K_{SC}} = \frac{1}{2} I'_{xx} \omega_{x'}^2 + \frac{1}{2} I'_{yy} \omega_{y'}^2 + \frac{1}{2} I'_{zz} \omega_{z'}^2 + I'_{xy} \omega_{x'} \omega_{y'} + I'_{xz} \omega_{x'} \omega_{z'} + I'_{yz} \omega_{y'} \omega_{z'}$$

x', y', and z' refer to any orthogonal axis set centered at the spacecraft center of mass. If the coordinate system is constrained to rotate so that the y'-axis always is parallel to the instantaneous axis of rotation, the foregoing expression reduces to

$$E_{K_{SC}} = \frac{1}{2} I'_{yy} \omega_{y'}^2 = \frac{1}{2} \frac{\left|\overline{H}_{SC}\right|^2}{I'_{yy}}$$

Since $E_{K_{CMG_1}} + E_{K_{CMG_2}}$ is a constant, $E_{K_{total}}$ is minimized when $E_{K_{SC}}$ is minimized. Therefore, $E_{K_{SC}}$ is minimized when y' is alined with the spacecraft pitch axis, the axis of maximum moment of inertia, and I'_{yy} equals I_{yy}. When $\left|\overline{H}_{SC}\right|$ is minimized, $E_{K_{SC}}$ is also minimized. Since the total angular momentum vector remains constant, $\overline{H}_{SC} = \overline{H}_{total} - \overline{H}_{CMG_1} - \overline{H}_{CMG_2}$ and $\overline{H}_{CMG_1}$ and $\overline{H}_{CMG_2}$ both have constant magnitudes with limited ranges of orientation. Therefore, H_{SC} is a minimum when the sum of $\overline{H}_{CMG_1}$ and $\overline{H}_{CMG_2}$ and $\overline{H}_{SC}$ and $\overline{H}_{total}$ are all alined. This is only possible when H_{SC} is alined with the negative spacecraft pitch axis, because of the limits on the angular freedom of the control-moment gyros.

Therefore, the state of minimum kinetic energy is that of a pure negative pitch rate with the angular momentum vectors of the CMG's held at equal and opposite angles relative to the pitch axis by the CMG torques. When the spacecraft is tumbling and the CMG's are free to cause internal damping, the spacecraft will seek a steady-state tumble with a pure negative pitch rate.

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