X-552-70-104
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NASA TM X- 63883

DERIVATION AND TESTS OF THE GODDARD COMBINED GEOPOTENTIAL FIELD (GSFC 1.70-C)

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JANUARY 1970

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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INTRODUCTION

The potential of the earth at a point with spherical coordinates (r,ϕ,λ), in an earth centered rotating coordinate system may be given by

$$U = \frac{\mu}{r}\left\{1 + \sum_{\ell=2}^{\infty} \sum_{m=0} \left(\frac{a_e}{r}\right)^{\ell} P_{\ell,m}(\sin\phi)\,(C_{\ell,m}\cos m\lambda + S_{\ell,m}\sin m\lambda)\right\}. \tag{1}$$

This form of the potential is the recommended form for axially asymmetric (zonal, tesseral, and sectorial harmonics) cases, Reference 1. In equation (1), μ is the product of the gravitational constant, G, times the mass of the earth, M, and a_e is the mean equatorial radius of the earth. The Legendre associated function, $P_{\ell,m}(\sin\phi)$, is defined by

$$P_{\ell,m}(x) = \frac{1}{2^{\ell}\,\ell!}\,(1 - x^2)^{m/2}\,\frac{d^{\ell+m}(x^2-1)^{\ell}}{dx^{\ell+m}}. \tag{2}$$

Instead of using the spherical harmonic coefficients $C_{\ell,m}$ and $S_{\ell,m}$, the fully normalized coefficients $\bar{C}_{\ell,m}$ and $\bar{S}_{\ell,m}$ have been adopted for this work. They are related to $C_{\ell,m}$ and $S_{\ell,m}$ coefficients by

$$\begin{aligned}\bar{C}_{\ell,m} &= N_{\ell,m}\,C_{\ell,m}\\ \bar{S}_{\ell,m} &= N_{\ell,m}\,S_{\ell,m}\end{aligned} \tag{3}$$

where

$$N_{\ell,m} = \left[\frac{K(2\ell+1)(\ell-m)!}{(\ell+m)!}\right]^{-1/2}.$$

and where $K = 1$ for $m = 0$, and $K = 2$ for $m \neq 0$.

Combined geopotential fields based upon four individual solutions for the geopotential have been published, Reference 2. The first such field, C, appearing in Reference 2 was obtained by taking an arithmetic mean of these four solutions which were obtained from satellite tracking data. The second field, CA, was obtained by taking field C and combining it with terrestrial gravity data. In this paper Professor Kaula stated that an arithmetic mean of different solutions from satellite data is superior to any single solution and a combination of satellite and terrestrial solutions, such as solution CA, should be superior to either solution alone. He also suggested that such a combination should be made when more solutions are available.

In this paper we will form such a combination solution based upon eleven complete solutions and four partial solutions for the geopotential. This solution is not completely analogous to either of the solutions C or CA of Kaula. It is an arithmetic mean of individual solutions as is solution C; however, the individual solutions are made up of determinations based upon satellite data and ones based upon both satellite and terrestrial data. Thus, this combined solution is an arithmetic mean of solutions of the type C and CA.

INPUT GEOPOTENTIAL FIELDS

The fields used in obtaining the combined geopotential field are of various types. They include fields obtained solely from satellite data. These satellite determined fields include ones obtained from optical observations, Doppler observations, and both optical and Doppler observations. There are also fields represented which were determined from both satellite and terrestrial gravity data. In addition to these two types of fields, various sets of coefficients determined from satellites in resonant orbits were used in obtaining the combined field. The particulars concerning these input fields are contained in Table 1.

Table 1
Gravity Fields used in the Combination Solution

		Derivation of Fields		
Gravity Field	Reference	No. of Satellites	Data Types	Comments
NWL 5E-6	4	3	Navy Doppler	Complete to (7,6)
APL 3.5	5	5	Navy Doppler	Complete to (8,8)
SAO M-1	3	16	Optical	Complete to (8,8)
Köhnlein	6	16	Optical & Terrestrial	Complete to (15,15)
Kaula UCLA 656	7	9	Navy Doppler, Optical & Terrestrial	Complete to (8,8)
Kaula CA	2		Navy Doppler, Optical & Terrestrial	Complete to (7,2)
Kaula K-8	2			Complete to (7,5)
SAO B6.1	8	24	Optical, Range & Range Rate	Complete to (16,16)
SAO B13.1	9	24	Optical, Range, Range Rate & Terrestrial	Complete to (16,16)
Rapp 1967	10	16	Optical & Terrestrial	Complete to (14,14)
Rapp 1968	11	16	Optical & Terrestrial	Complete to (14,14)
Wagner	12	3	Mean Kepler Elements	Resonant Coefficients (2,2), (3,3)
Gaposchkin & Veis	13	2	Optical	Resonant Coefficients (13,12), (14,12), (15,12)
APL	14	3	Navy Doppler	Resonant Coefficients (13,13), (15,13), (17,13)
Goddard	15	1	Optical & Range	Resonant Coefficient (14,13) to be used with APL coefficients in April 1968 for GEOS II

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The result of taking the arithmetic mean of these fields appears in Table 2. Note however, that the combined field, GSFC 1.70-C, so obtained was truncated after the terms of fifteenth degree and order. If terms of higher degree were averaged, there would be only a few contributors. A variety of tests conducted using this combined field are discussed in the next section. Tests were made with the remaining geodetic constants (zonals, earth radius, etc.) equal to those adopted for the SAO 1966 Standard Earth, Reference 3. For convenience they are also listed in Table 2.

Table 2

Earth Model

TESSERAL HARMONICS *

N	M	$\bar{C}$(N,M)	$\bar{S}$(N,M)
2	2	2.402	-1.370
3	1	1.891	0.219
3	2	0.834	-0.653
3	3	0.618	1.365
4	1	-0.550	-0.452
4	2	0.316	0.604
4	3	0.915	-0.121
4	4	-0.125	0.212
5	1	-0.037	-0.061
5	2	0.553	-0.248
5	3	-0.357	-0.012
5	4	-0.142	0.117
5	5	0.040	-0.501
6	1	-0.086	0.055
6	2	0.027	-0.329
6	3	0.051	0.079
6	4	-0.090	-0.459
6	5	-0.220	-0.501
6	6	-0.072	-0.261
7	1	0.159	0.039
7	2	0.324	0.089
7	3	0.206	-0.094
7	4	-0.224	-0.043
7	5	0.055	-0.040
7	6	-0.266	0.115
7	7	0.070	0.043
8	1	-0.047	0.028
8	2	0.059	0.020
8	3	-0.012	0.081
8	4	-0.094	0.025
8	5	-0.050	0.021
8	6	-0.038	0.259
8	7	0.040	0.025
8	8	-0.142	0.020
9	1	0.111	-0.014
9	2	0.003	-0.008
9	3	-0.034	-0.030
9	4	0.060	0.053
9	5	-0.044	-0.054
9	6	0.062	0.036
9	7	-0.055	-0.087
9	8	0.218	0.037
9	9	0.012	-0.025
10	1	0.075	-0.064
10	2	-0.067	-0.034
10	3	-0.017	-0.068
10	4	-0.050	-0.125
10	5	-0.045	0.006
10	6	-0.076	-0.060
10	7	0.071	-0.013
10	8	0.079	-0.097
10	9	0.040	-0.044
10	10	0.051	-0.064
11	1	-0.034	-0.027
11	2	0.057	-0.083
11	3	-0.023	-0.103
11	4	-0.000	-0.019
11	5	0.041	-0.006

N	M	$\bar{C}$(N,M)	$\bar{S}$(N,M)
11	6	-0.044	-0.073
11	7	0.012	-0.101
11	8	0.160	0.014
11	9	-0.016	-0.044
11	10	-0.046	-0.040
11	11	0.083	0.012
12	1	-0.079	-0.047
12	2	0.004	0.045
12	3	0.073	-0.011
12	4	-0.039	-0.096
12	5	-0.002	0.065
12	6	-0.065	0.024
12	7	0.004	0.026
12	8	0.009	0.073
12	9	-0.073	0.043
12	10	-0.002	0.005
12	11	-0.029	-0.011
12	12	-0.025	-0.012
13	1	-0.017	0.011
13	2	-0.031	-0.031
13	3	-0.067	0.024
13	4	-0.025	0.054
13	5	0.083	-0.077
13	6	-0.038	0.054
13	7	0.016	-0.007
13	8	0.047	-0.025
13	9	-0.010	0.009
13	10	0.055	-0.071
13	11	-0.038	0.009
13	12	-0.030	0.072
13	13	-0.098	0.055
14	1	-0.017	0.044
14	2	-0.028	0.009
14	3	0.109	-0.021
14	4	0.032	-0.043
14	5	0.068	-0.039
14	6	-0.033	-0.011
14	7	0.051	-0.016
14	8	-0.032	-0.056
14	9	-0.027	0.054
14	10	0.009	-0.070
14	11	0.081	0.024
14	12	0.017	-0.027
14	13	0.028	0.051
14	14	-0.040	-0.009
15	1	-0.015	-0.057
15	2	0.004	-0.109
15	3	-0.033	0.030
15	4	0.023	0.074
15	5	0.022	0.008
15	6	0.050	-0.053
15	7	0.075	0.053
15	8	-0.030	0.031
15	9	-0.040	0.045
15	10	0.038	0.022
15	11	0.013	0.072
15	12	-0.048	0.032
15	13	-0.045	-0.033
15	14	0.008	-0.013
15	15	-0.005	-0.010

*UNIT EQUALS TEN TO MINUS SIXTH POWER

ZONAL HARMONICS* $\bar{C}$(N,0)

$\bar{C}$(2,0)=	-484.1733
$\bar{C}$(3,0)=	0.9623
$\bar{C}$(4,0)=	0.5497
$\bar{C}$(5,0)=	0.0633
$\bar{C}$(6,0)=	-0.1792
$\bar{C}$(7,0)=	0.0860
$\bar{C}$(8,0)=	0.0655
$\bar{C}$(9,0)=	0.0122
$\bar{C}$(10,0)=	0.0118
$\bar{C}$(11,0)=	0.0630
$\bar{C}$(12,0)=	0.0714
$\bar{C}$(13,0)=	0.0219
$\bar{C}$(14,0)=	-0.0332
$\bar{C}$(15,0)=	0.0

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TESTS AND EVALUATION OF THE COMBINED FIELD

Several tests of this new model have been made. The first pair of these were concerned with the fit to six day arcs of precision reduced optical data for GEOS I and GEOS II. The fits were made without and then with special values for resonant coefficients for the two satellites. For GEOS I the resonant coefficients were those obtained by Gaposchkin and Veis, Reference 13; and for GEOS II the resonant coefficients were those obtained by APL, Reference 14, and Goddard, Reference 15. Characteristics of these two satellite orbits are given in Table 3. In Table 4 there appears a list of resonant coefficients obtained previously for these two satellites. The combined field was then tested in orbit computations with and without resonant terms against some of the fields that were used in deriving it. After these tests were completed, the combined field was truncated in the following manner. All the coefficients of degree greater than eight with the exception of those of order twelve, thirteen, fourteen, and fifteen were deleted from the field. The same orbit determination runs were then repeated for this adjusted field, GSFC 1.70-T.

Table 3

GEOS I and GEOS II Orbit Characteristics

	GEOS I	GEOS II
semi major axis	8077.9 km	7701.1 km
eccentricity	.070	.033
inclination	$59^\circ.4$	$105^\circ.8$
argument of perigee	$312^\circ.0$	$194^\circ.8$
right ascension of ascending node	$266^\circ.1$	$353^\circ.7$
mean anomaly	$224^\circ.1$	$121^\circ.0$
epoch	July 11, 1966 $0^h 0^m$	April 28, 1968 $17^h 56^m$
resonant harmonics	12th order	13th order
resonant period	7.3 days	6.5 days
perigee height	1130.6 km	1071.8 km
apogee height	2268.9 km	1574.1 km

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Table 4

Resonant Coefficients for GEOS Satellites

GEOS I*	GEOS II**
$\bar{C}_{12,12} = -3.1 \times 10^{-8}$	$\bar{C}_{13,13} = -6.53 \times 10^{-8}$
$\bar{S}_{12,12} = .08 \times 10^{-8}$	$\bar{S}_{13,13} = 5.79 \times 10^{-8}$
$\bar{C}_{13,12} = -6.769 \times 10^{-8}$	$\bar{C}_{15,13} = -3.82 \times 10^{-8}$
$\bar{S}_{13,12} = 6.245 \times 10^{-8}$	$\bar{S}_{15,13} = -1.85 \times 10^{-8}$
$\bar{C}_{14,12} = .261 \times 10^{-8}$	$\bar{C}_{17,13} = 6.36 \times 10^{-9}$
$\bar{S}_{14,12} = -2.457 \times 10^{-8}$	$\bar{S}_{17,13} = 1.11 \times 10^{-8}$
$\bar{C}_{15,12} = -7.473 \times 10^{-8}$	$\bar{C}_{14,13} = 7.81 \times 10^{-9}$
$\bar{S}_{15,12} = -1.026 \times 10^{-8}$	$\bar{S}_{14,13} = 8.91 \times 10^{-8}$

*SAO

**APL & GSFC

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The fits to the data for the six day GEOS I and GEOS II arcs with the GSFC 1.70 C and GSFC 1.70T geopotential fields appear in Table 5. Also presented in Table 5 are fits obtained from previous studies, Reference 16, for the same orbital arcs using some of the complete fields used to derive the combined field. For the orbit computations involving the SAO B13.1 field, the recommended set of zonals, Reference 17, adopted for the 1969 SAO Standard Earth was used. Two points can be made concerning this table. First, after the resonant coefficients for these two satellites are inserted into the field, the fit to the data with the combined field is better than that of any other field for GEOS I, and for GEOS II the fit using the combined field was second only to the SAO B13.1 field. Secondly, after the field is adjusted so that it has only about forty percent of the terms that the original field had, the fit to the data remains about the same.

Table 5

Rms's (Sec's of Arc) of Fitted Orbits for Six Day Arcs of Optical Data from GEOS I and GEOS II when Different Gravity Models are used with and without Resonant Coefficients*

Geopotential Model	GEOS I		GEOS II		Number of Coefficients
	without	with	without	with	
SAO M 1	19.04	2.52	17.36	3.08	108
SAO B13.1	2.56	3.27	3.39	2.29	294
Köhnlein	14.65	2.89	9.41	3.12	236
Rapp 1967	7.81	6.91	11.30	5.48	206
NWL 5E-6	11.82	3.33	27.99	8.08	58
APL 3.5	13.51	6.64	59.41	5.79	74
Kaula 1967	5.80	5.95	16.67	9.32	88
GSFC 1.70C	6.48	2.30	6.67	2.95	236
GSFC 1.70T	6.67	2.43	6.87	3.23	88

*NOTE 1" equals approximately 7 m.

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Certain coefficients in the combined field were adjusted using optical data in the six day arcs referred to in Table 5. Other coefficients were adjusted using accelerations from seven twenty-four hour satellites in the Syncom, Intelsat and ATS (Applications Technology Satellite) series of spacecraft. These accelerations had been prepared by Mr. C. Wagner for use in some of his deep resonance studies (see Reference 12, for example) and made available for this work by him. Values of the original and adjusted geopotential coefficients using these data appears in Table 6. The second and third degree sectorial harmonics were improved with the sectorial harmonic of degree four and the tesseral harmonics of degree three order one and degree four order two which are also sensitive for the accelerations held fixed to the values for the coefficients appearing in the combined field.

Table 6

Adjusted Coefficients*

Term	A Priori	Improved	Satellite (S)
$\bar{C}_{2,2}$	2.402	2.434	ATS 1,3,5, INTELSAT 1,2-F3, Syncom 2, 3
$\bar{S}_{2,2}$	-1.370	-1.398	
$\bar{C}_{3,3}$	0.618	0.726	
$\bar{S}_{3,3}$	1.365	1.371	
$\bar{C}_{13,12}$	-0.0675	-0.0703	GEOS I
$\bar{S}_{13,12}$	0.0622	0.0681	
$\bar{C}_{13,13}$	-0.0653	-0.0629	GEOS II
$\bar{S}_{13,13}$	0.0579	0.0586	

*Multiply all coefficients by 10^{-6}

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When the twelfth order coefficient was adjusted, a two second of arc fit was obtained for the GEOS I arc. The corresponding improvement to the fit for the GEOS II arc was only a few percent. The RMS error to the twenty-four hour accelerations for the low degree coefficients that are sensitive to these accelerations appears in Table 7 for the SAO 1966 Standard Earth, SAO B13.1, SAO 1969 Standard Earth, GSFC 1.70-C and GSFC 1.70-C with the adjusted sectorial harmonics.

It may be concluded that based upon the results so far, the GSFC 1.70-C field modified with the SAO, APL, and Goddard GEOS I and II coefficients, Table 4, further modified with the adjusted coefficients of Table 7 might be the best field to be presented here for satellite orbit computations. If speed of computation becomes a factor, the analogue of this field based upon GSFC 1.70-T might be used with some small sacrifice in accuracy of computations instead of GSFC 1.70-C.

Table 7

Fits to 24 hr Satellite Accelerations

Geopotential Field	RMS, rad./(sid.da.)^2
1966 SAO Standard Earth	8.5×10^{-7}
SAO B13.1	3.9×10^{-7}
1969 SAO Standard Earth	2.5×10^{-7}
GSFC 1.70-C	3.8×10^{-7}
GSFC 1.70-C + Adjusted Sectorials	1.9×10^{-7}

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As a means of further testing the orbital solutions obtained with the GSFC 1.70-C and the modified GSFC 1.70-C gravity models, satellite position differences were computed using the orbits based upon the SAO M-1 (1966) model (modified by GEOS-I and GEOS-II resonant terms) and the more recent SAO B13.1 model as standards.

The position differences were computed at five minute intervals and were resolved into radial, along track, and cross track components. The differences were computed using the unit vectors $\underline{H}$, $\underline{L}$ and $\underline{C}$, respectively which were calculated from the following relationships:

$$\underline{H} = \frac{\underline{R}}{R}$$

$$\underline{L} = \left[\underline{V} - \left(\frac{\underline{R} \cdot \underline{V}}{R^2} \right) \underline{R} \right] \Big/ \sqrt{V^2 - \frac{(\underline{R} \cdot \underline{V})^2}{R}}$$

$$\underline{C} = \underline{H} \times \underline{L}$$

where $\underline{R}$ is the vector from the geocenter to the satellite,

R is the distance from the geocenter to the satellite, and

$\underline{V}$ is the velocity vector of the satellite.

The RMS of satellite position differences for the respective GEOS-I and GEOS-II orbital arcs are presented in Table 8. The largest position differences were along track for both the GEOS-I and GEOS-II orbits which was not unexpected due to poorly modeled resonance. These along track position differences were reduced significantly when the GSFC 1.70-C model was modified by the Gaposchkin-Veis 12th order coefficients for GEOS-I and the APL and Goddard 13th order coefficients for GEOS-II. As indicated in Table 8, the RMS of the along track position differences between the GEOS I and GEOS II SAO M-1 standard orbits and the orbits computed with the modified GSFC 1.70-C model were on the order of 25 meters. The RMS of the radial and crosstrack position differences were on the order of 10 meters or less for all cases. The comparisons between the orbits computed using the GSFC 1.70-C modified models with the coefficients (13,12) adjusted for GEOS-I and (13,13) adjusted for GEOS-II showed an increase in the along track component of 8 meters for GEOS-I and 1 meter for GEOS-II. This is consistent with the RMS of fits presented in Table 5 and the fact that adjustment of (13,13) for Geos II resulted in very little change to the orbital fit.

Although the RMS of fits for the GEOS-I orbital arc were within .3 arc seconds for the SAO B13.1 orbit and the GSFC 1.70-C orbit (2.56 arc seconds vs. 2.30 arc seconds as shown in Table 5) the trajectory comparison presented in Table 8 shows that the RMS of position differences were on the order of 67 meters.

Table 8
Satellite Position Comparisons
GEOS I July 11-16, 1966
GEOS II April 28-May 4, 1968

Satellite	SAO M 1 (Modified)* vs	RMS Position Difference (meters)				Maximum
		Radial	Crosstrack	Alongtrack	Total	Difference
GEOS I	GSFC 1.70-C	7.0	7.8	82.8	83.4	189.7
GEOS I	GSFC 1.70-C + Gaposchkin and Veis 12th order terms	9.0	6.4	25.8	28.1	71.5
GEOS II	GSFC 1.70-C	10.3	9.3	91.8	92.9	210.8
GEOS II	GSFC 1.70-C + APL (1968) and GSFC (1969) 13th order terms	7.8	8.3	26.9	29.2	75.1
	SAO B13.1 vs					
GEOS I	GSFC 1.70-C + Gaposchkin and Veis 12th order terms	19.9	7.3	59.0	62.7	159.3
GEOS II	GSFC 1.70-C + APL (1968) and GSFC (1969) 13th order terms	20.3	14.3	75.6	78.6	188.2

*Modified with Gaposchkin and Veis 12th order terms for Geos I Comparisons.
Modified with APL (1968) 13th order terms for Geos II Comparisons.

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To this point the GSFC 1.70C field has been tested against other fields using several satellites in distant 24hr orbits and the GEOS satellites in their close, resonant and drag free orbits. The next test of this model will involve orbit computations using tracking data from the Orbiting Geophysical Observatory (OGO-4) satellite. Fits to six two day arcs of Minitrack and Goddard Range and Range Rate data using NWL 5E-6, SAO M1, SAO B6.1, and GSFC 1.70C are presented in Table 9.

The analysis of OGO-4 data is of particular interest for a number of reasons. In the first place OGO-4 was not used in deriving any of the models discussed in this paper. This satellite in its near polar close earth orbit (a=7023.4km, e=.03492, I=86°.00) with only modest drag and slight resonance (150 m) is ideal for sampling the total gravity field.

Orbital comparisons similar to the ones made for the GEOS orbits have been also made using the OGO-4 arcs discussed above. In this case we were interested in the overlap region of successive pairs of orbital arcs. The results of these orbital comparisons appear in Table 10 for the NWL 5E-6, SAO M1, SAO B6.1 and GSFC 1.70C models.

Table 9

OGO-4 RMS of Orbital Solutions

Data Arc and Number of Observations	Model	Range (m)	Range-Rate (cm/sec)	Direction Cosines 1 (mils)	m(mils)
8/16/67 and 8/17/67	NWL 5E-6	84	67	1.7	3.4
R=59, RR=59, ℓ =m=27	SAO M1	63	47	.3	3.5
	SAO B6.1	60	45	.3	3.4
	GSFC 1.70-C	54	33	.4	3.3
8/17/67 and 8/18/67	NWL 5E-6	150	109	2.0	4.0
R=55, RR=54, ℓ=m=33	SAO M1	90	57	1.4	3.7
	SAO B6.1	72	49	1.3	3.8
	GSFC 1.70-C	60	28	.9	3.6
8/21/67 and 8/22/67	NWL 5E-6	126	89	1.7	3.4
R=85, RR=82, ℓ=m=38	SAO M1	69	51	1.0	3.3
	SAO B6.1	60	22	.2	1.9
	GSFC 1.70-C	48	27	.6	2.4
8/22/67 and 8/23/67	NWL 5E-6	102	65	.7	3.4
R=101, RR=98, ℓ=m=36	SAO M1	54	51	.7	3.0
	SAO B6.1	66	39	.7	3.0
	GSFC 1.70-C	45	32	.3	2.3
8/26/67 and 8/27/67	SAO M1	81	57	.8	3.1
R=78, RR=78, ℓ=m=34	GSFC 1.70-C	72	50	.8	2.9
8/27/67 and 8/28/67	SAO M1	81	41	.7	3.0
R=98, RR=97, ℓ =m=36	GSFC 1.70-C	75	50	.8	3.0

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Table 10

OGO 4 Orbital Comparisons

Overlap Region	Model	RMS Position Difference (meters) Radial	Crosstrack	Alongtrack	Total	Maximum Difference
8/17/67	NWL 5E-6	99.4	407.8	263.2	495.4	785.1
	SAO M1	31.1	298.4	105.7	318.1	484.8
	SAO B6.1	20.4	441.2	131.1	460.7	764.9
	GSFC 1.70C	19.9	79.5	84.7	117.9	254.7
8/22/67	NWL 5E-6	26.4	151.1	151.7	215.8	479.0
	SAO M1	17.7	69.5	247.4	257.6	770.6
	SAO B6.1	30.9	26.7	296.2	299.0	916.9
	GSFC 1.70C	19.1	21.8	166.7	169.2	479.0
8/27/67	SAO M1	20.2	25.5	62.9	70.8	144.5
	GSFC 1.70C	4.4	8.5	28.9	30.5	64.9

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The best orbital fits for practically every two day arc were obtained when the GSFC 1.70C model was used. In several instances the RMS fits for the Range data were a factor of two lower than those obtained with the NWL 5E-6 model and the fits to the range rate data were as much a factor of three lower in some cases.

Similar results were obtained from the orbital overlap comparisons. The results presented in Table 10 indicate that the smallest total RMS position differences were obtained for every case when the GSFC 1.70-C model was used. Although in general smaller position differences were obtained for all components, the largest reduction was observed in the crosstrack component with the GSFC 1.70C cross track differences on 8/22/67 a factor of six smaller than those obtained using the NWL 5E-6 model.

Although a study of this nature has not been performed for the OGO-6 satellite data (OGO-6 orbital paramaters are quite similar to those for OGO-4) it is anticipated that the results of such a study would be comparable to those presented in Tables 9 and 10. That is, we would expect the overlap errors to be reduced significantly.

The combined field was compared to terrestrial free-air anomalies and to other geopotential fields through consideration of degree variances. The degree variances computed from the terrestrial data were obtained from Reference 2. The degree variances, σ_ℓ^2 , for the various geopotential fields were obtained from equation (4) which is based upon an analysis of gravity appearing in Reference 18. Thus,

$$\sigma_\ell^2 = \gamma^2 (\ell - 1)^2 \sum_m (\overline{C}_{\ell m}^2 + \overline{S}_{\ell m}^2) \qquad (4)$$

where γ is the mean acceleration of gravity. These degree variances for the input fields, GSFC 1.70-C gravimetric, and for the SAO 1969 standard earth, Reference 20, appear in Table 11.

In Table 12 there appears an RMS coefficient difference between GSFC 1.70-C and the various other geopotential fields. By both of these methods of comparison GSFC 1.70-C compares very well with Köhnlein's field. Good agreement between GSFC 1.70-C and Kaula CA, SAO B13.1, Rapp 67, and Rapp 68 on the two means of comparison are obtained.

The mean degree variance, $\overline{\sigma}_\ell^2$, is given by equation (5) for a set of non-zonal harmonics. Thus,

$$\overline{\sigma}_\ell^2 = \sum_\ell (\overline{C}_{\ell m}^2 + \overline{S}_{\ell m}^2)/2\, a_\ell \qquad (5)$$

where a_ℓ is the number of pairs of non-zonal harmonics of degree ℓ. In Figure 1 a plot of the log of 10^{17} times the mean degree variance for 1.70-C versus the log of the degree is shown.

The rule of thumb that the size of a normalized gravity coefficient of degree ℓ is $10^{-5}/\ell^2$ is adhered to closely by the data points in this figure. The straight line so obtained is in good agreement with one presented in Reference 19 for NWL-8 D by Anderle and Smith.

Finally, a geoid map based upon GSFC 1.70-C and the other constants in Table 2 is given in Figure 2.

Table 11

Degree Variances σ_ℓ^2 (m gal^2)

Degree	NWL 5E6	APL 3.5	SAO M1	Köhnlein	Kaula 656	Kaula CA	Kaula K8	SAO B6.1	SAO B13.1	Rapp 67	Rapp 68	GSFC 1.70C	Gravimetric	SAO 69 S.E.
2	7.9	6.8	7.2	7.2	7.5	7.4	7.5	7.6	7.4	6.8	7.1	7.3	6.3	7.4
3	36.2	25.9	29.0	24.6	26.6	25.8	25.5	28.7	28.1	25.1	25.3	26.8	31.8	40.1
4	21.6	14.4	16.4	14.8	20.6	16.9	17.8	18.3	19.1	14.8	14.9	16.3	18.6	19.2
5	21.7	15.4	18.2	13.5	19.3	9.7	21.1	18.7	15.4	9.0	10.5	12.0	8.4	17.1
6	30.8	24.9	17.9	14.9	49.6	18.8	19.9	15.7	15.1	14.9	12.9	17.4	22.2	15.7
7	32.8	44.9	13.3	10.5	6.1		21.0	13.6	14.3	9.2	7.2	11.9	11.0	17.1
8		28.4	11.7	8.2	2.7		10.4	7.3	8.4	7.6	5.1	5.5	9.2	6.9
9				3.3	3.7			30.9	12.2	7.3	3.5	5.6	10.1	13.0
10				4.6				25.7	11.2	8.3	4.3	6.3		13.8
11				3.4				51.8	25.5	4.2	1.8	7.8		16.0
12				3.2				59.1	15.3	7.3	2.3	5.9		11.4
13				3.3				80.5	18.5	7.0	2.2	8.8		15.9
14				4.9				67.9	27.7	9.9	10.8	8.8		16.1
15				5.3				18.8	20.5			10.8		30.3

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Table 12
RMS Coefficient Difference
Between Fields

GSFC 1.70-C and	RMS Difference (unit is 10^{-6})
NWL 5E-6	.163
APL 3.5	.192
SAO M1	.067
Köhnlein	.047
Kaula UCLA 656	.165
Kaula CA	.061
Kaula K8	.179
SAO B6.1	.098
SAO B13.1	.055
Rapp 67	.066
Rapp 68	.052
SAO 69 S.E.	.062

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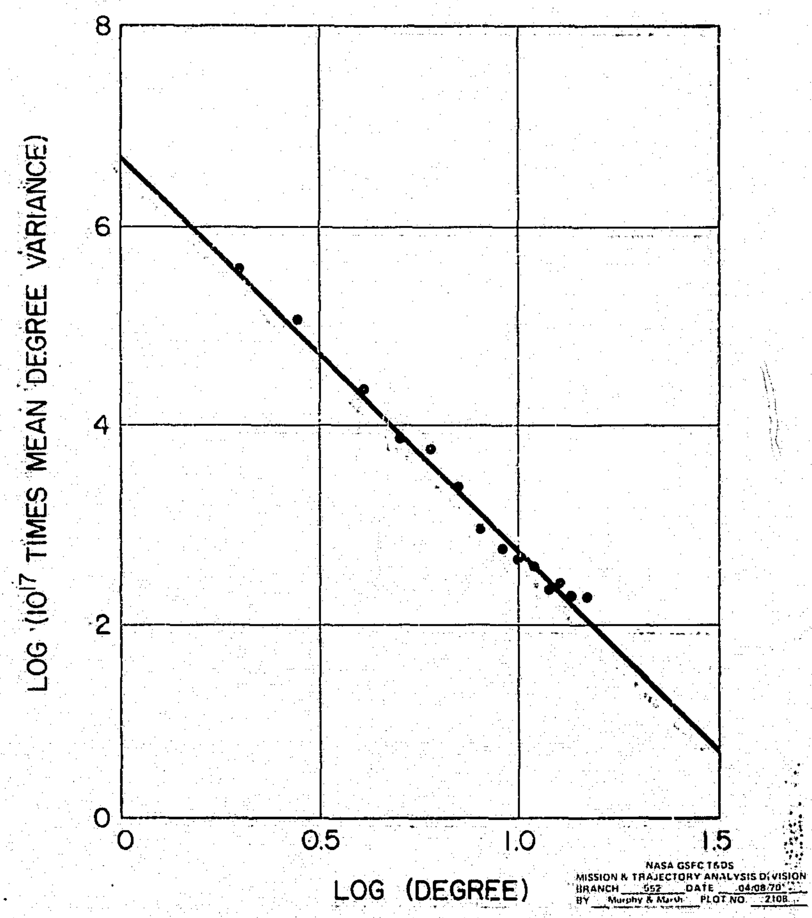

Figure 1. Mean Degree Variances, $\sigma_\ell^2 = \sum_m (\bar{C}_{\ell m}^2 + \bar{S}_{\ell m}^2)/2a_\ell$, for GSFC 1.70-C.

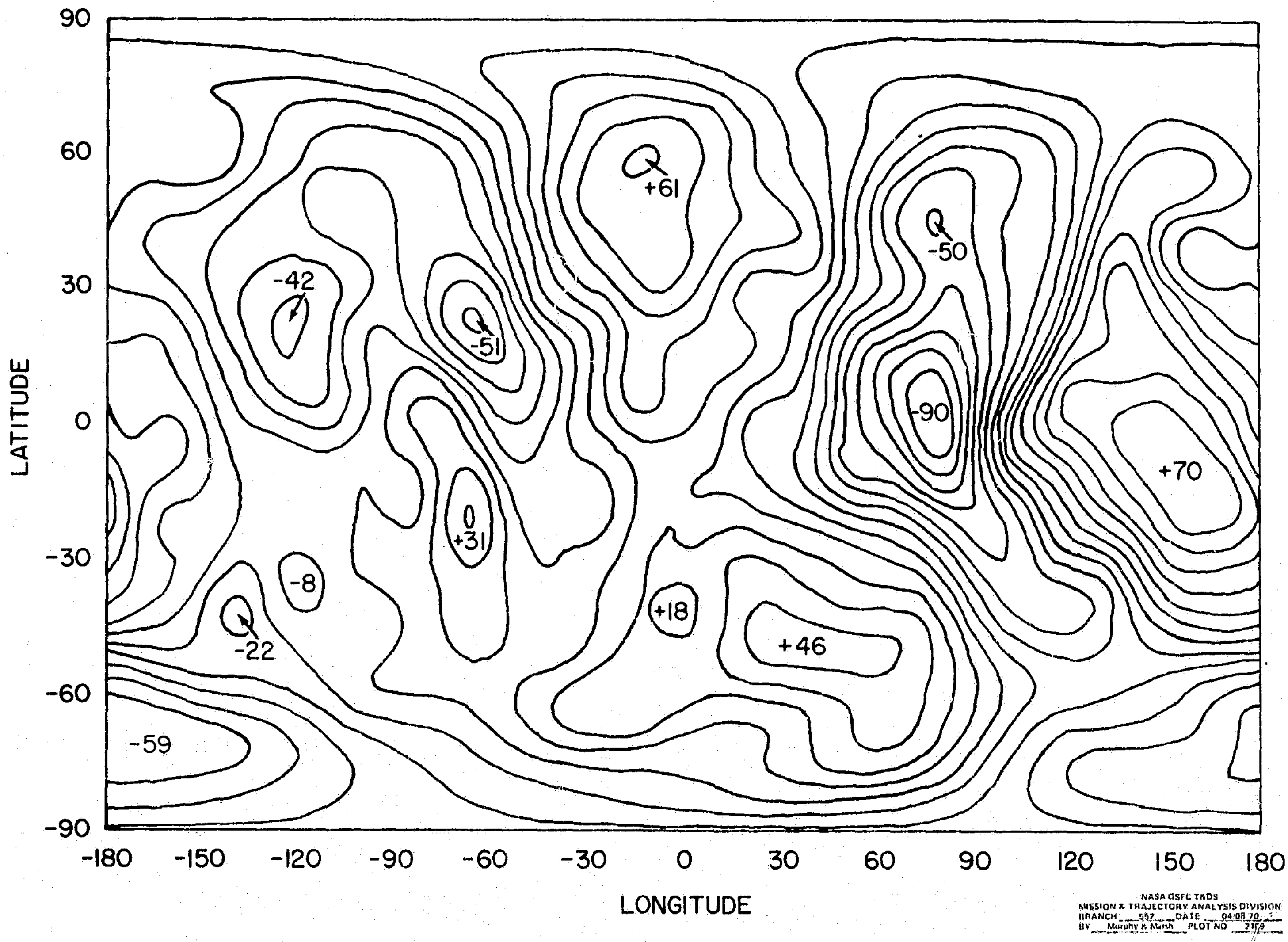

Figure 2. Geoid undulations from GSFC 1.70-C solution in meters.

COMMENT AND CONCLUSIONS

Several possible conclusions with regard to the performance of the combined gravity field for orbit computations can be made.

1. After the proper resonant terms are inserted in the combined field, the fit to the tracking data is as good or better than the fits obtained with any of the fields used in composing it.

2. The truncated field with 60% fewer coefficients fit the data almost as well as any of the fields tested. This implies that a 60% savings in computer time for trajectory integration could be realized over a (15×15) field with GSFC 1.70-T for some satellite prediction problems.

3. Adjustments to the resonant coefficients were small for both GEOS-I and GEOS-II. This seems to indicate that the complement of the field with respect to these resonant coefficients represents the gravity field adequately.

4. Comparisons of 1.70-C with other fields from the point of view of satellite position errors, degree variances, and geoid maps are favorable.

5. The tracking data processing carried out in this study was accomplished using the NONAME (orbit and geodetic parameter estimation) system, Reference 21, which includes all satellite perturbations such as lunar-solar gravitation, solar radiation pressure, etc. to the required degree of accuracy.

6. Professor Kaula's conclusion concerning the superiority of the arithmetic mean of different solutions to any single solution has been reexamined using several additional models.

ACKNOWLEDGEMENT

We thank Mr. Carl A. Wagner for making accelerations on several twenty-four hour satellites available to us. We also thank him for help in the analysis of those accelerations with regards to the adjustment of low degree coefficients.

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