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MINIMUM ΔV, THREE-IMPULSE TRANSFER ONTO A TRANS-MARS ASYMPTOTIC VELOCITY VECTOR

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1970

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1. REPORT NO. NASA TN D-5757	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
4. TITLE AND SUBTITLE MINIMUM ΔV, THREE-IMPULSE TRANSFER ONTO A TRANS-MARS ASYMPTOTIC VELOCITY VECTOR		5. REPORT DATE April 1970 6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) William C. Bean, MSC		8. PERFORMING ORGANIZATION REPORT NO. MSC S-231
9. PERFORMING ORGANIZATION NAME AND ADDRESS Manned Spacecraft Center Houston, Texas 77058		10. WORK UNIT NO. 924-22-20-00-72 11. CONTRACT OR GRANT NO.
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D.C. 20546		13. REPORT TYPE AND PERIOD COVERED Technical Note 14. SPONSORING AGENCY CODE
15. SUPPLEMENTARY NOTES		

16. ABSTRACT

For the numerical study that is presented, the primary objective was to obtain a set of minimum ΔV, three-impulse maneuvers from a specified circular Earth parking orbit onto a constrained asymptotic velocity vector that is characteristic of Mars. A computer program based on an accelerated gradient method for function minimization was used. Twenty cases were examined in which the four parametric values (0°, 2°, 4°, and 6°) for the out-of-plane angle of the asymptotic velocity vector were chosen with the idea that launch window considerations dictate a small maximum tolerable error in parking-orbit orientation at departure, and in which the five parametric values (8, 12, 24, 48, and 72 hours) for the period of the elliptical first transfer conic were chosen arbitrarily to result in realistically small total flight times for a maneuver.

17. KEY WORDS (SUPPLIED BY AUTHOR) · Trajectory Optimization · Earth Escapes · Interplanetary Studies · Impulsive Trajectory · Trans-Mars Trajectories · Multiple Impulse Orbital Transfer · Noncoplanar Earth Escape	18. DISTRIBUTION STATEMENT Unclassified - Unlimited

19. SECURITY CLASSIFICATION (THIS REPORT)	20. SECURITY CLASSIFICATION (THIS PAGE)	21. NO. OF PAGES	22. PRICE*
None	None	25	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

MINIMUM ΔV, THREE-IMPULSE TRANSFER ONTO A TRANS-MARS ASYMPTOTIC VELOCITY VECTOR

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SUMMARY

For the numerical study that is presented, the primary objective was to obtain a set of minimum ΔV, three-impulse maneuvers from a specified circular Earth parking orbit onto a constrained asymptotic velocity vector characteristic of Mars. A computer program based on an accelerated gradient method for function minimization was used. Twenty cases were examined in which the four parametric values (0°, 2°, 4°, and 6°) for the out-of-plane angle of the asymptotic velocity vector and the five parametric values (8, 12, 24, 48, and 72 hours) for the period of the elliptical first transfer conic were used. It is believed that the numerical study makes the following quantitative contributions.

1. The study verified numerically the existence of three-impulse maneuvers which provide measurable improvement in 18 of 20 cases studied, even for realistically small launch windows, over corresponding one-impulse maneuvers. For the cases examined, the improvement is from -0.23 percent to +7.40 percent lower velocity penalty, that is, total ΔV.

2. For the cases examined, the study obtains the optimum relative distribution (among the impulses) of energy-changing and plane-changing duties. Previous studies have obtained minimum ΔV, three-impulse trajectories from sets of maneuvers in which all of the plane changing occurs at the high-altitude middle impulse. Imposition of this characteristic, which is valid for the minimum ΔV, infinite-time maneuver, would have been unrealistic for the (finite-time) cases examined, for which the low-altitude third impulse performed as much as 29.3 percent of the plane changing.

3. The study determines that the velocity penalty for the small-launch-window three-impulse cases examined is only 0.38 to 5.2 percent higher than for corresponding coplanar cases.

4. The analysis discerns that the three-impulse maneuvers involve a 0.13 to 5.8 percent greater velocity penalty than the corresponding absolute minimum ΔV, time-open three-impulse maneuvers involve.

INTRODUCTION

Studies frequently have emphasized energy considerations in the determination of a launch window for transfer from an Earth orbit onto a trans-Mars trajectory. These studies usually noted that a parking orbit could be established with an appropriate inclination containing the required asymptotic direction. Therefore, the supposition was made that launch would occur with an instantaneous burn at the perigee of an escape hyperbola, with the plane of the departure trajectory coincident with the parking-orbit plane.

For a specific mission, both the required asymptotic direction and the parking-orbit inclination vary in inertial space with time passage. Therefore, it is realistic to anticipate that the required asymptotic velocity vector may not lie in the parking-orbit plane. Consequently, the need to regulate fuel expenditure according to the dual requirements of performing plane changes and imparting energy indicates a further reduction in launch windows for Earth-to-Mars missions.

With regard to examination of the dual requirements, certain one- and two-burn requirements were studied by Deerwester (ref. 1). The following concepts were among the conclusions.

1. The actual departure window is much smaller than the departure window dictated by energy requirements alone.

2. The relationship between the declination of the required departure asymptote and the inclination of the parking orbit profoundly affects the size of the launch window.

3. A quantitative comparison of out-of-plane escape maneuvers from circular orbit and, in particular, selection of a set of maneuvers yielding a minimum velocity penalty for a particular asymptotic velocity vector suggest extensive use of a digital computer.

More recently, optimum three-impulse transfer onto an escape asymptote has been studied (ref. 2). For such a transfer, several additional degrees of freedom are permitted, including consideration of elliptical parking orbits.

Finally, an analysis presented by Edelbaum (ref. 3) indicated the likelihood that a class of three-impulse maneuvers from a circle to a prescribed asymptotic velocity vector exists, each maneuver consuming less fuel than the corresponding minimum ΔV, one- and two-impulse transfers. The assumption is made that hyperbolic perigee can occur at an altitude lower than the circular parking-orbit radius. Also, it is suggested that, although unrealizably long flight times result, the lower bound for velocity expenditure is attained by allowing the altitude at which the second impulse is applied to increase without limit and by allowing the altitude at which the third impulse is applied to decrease to zero with respect to the origin of attraction.

The conclusions of previous studies led to a numerical study of 20 cases in which the duration of the first transfer coasting arc and the target asymptotic velocity vector were constrained. In each case, the same target hyperbolic energy and the same lower bound for the perigee altitude of the escape hyperbola were fixed. The pair of parameters for the study were prescribed values for the period T_1 of the coasting arc between the first and second impulses and for the out-of-plane angle D of the asymptotic velocity vector.

For each case, the objectives were to determine the following facts.

1. A minimum ΔV, three-impulse maneuver from a specified circular Earth parking orbit onto the appropriate target asymptotic velocity vector

2. The decrease in velocity expenditure relative to the appropriate reference one-impulse transfer

3. The increase in velocity expenditure relative to the corresponding minimum ΔV, three-impulse coplanar maneuver which imparts the desired energy

4. The relative distribution, among the three impulses, of both energy-imparting and plane-changing responsibilities

5. The excess fuel expenditure over the corresponding absolute minimum ΔV, time-open, three-impulse transfer

In the numerical study, a 262-nautical-mile Earth parking orbit was assumed. The hyperbolic energy was fixed by the requirement that the magnitude of the asymptotic velocity vector $\vec{V}_\infty$ be 12 000 ft/sec. Hyperbolic perigee altitude was constrained in each case to be no lower than 100 nautical miles above the surface of the Earth. The set of values 8, 12, 24, 48, and 72 hours were considered for the period T_1 of the first coasting arc; and the set of values 0°, 2°, 4°, and 6° were considered for the out-of-plane angle D of the vector $\vec{V}_\infty$.

Finding a minimum ΔV maneuver involved a search in a class of pairs of transfer trajectories which could be ellipses, parabolas, or hyperbolas. However, specification of a period T_1 of motion for the first transfer coasting path guaranteed ellipticity of this trajectory. Also, the numerical study avoided the following presumptions.

1. That a given velocity increment was devoted entirely either to adding energy or to imparting plane changes

2. That the three velocity increments for a three-impulse maneuver were mutually parallel or antiparallel

3. That impulses must occur only at perigee or apogee of transfer conics

4. That insertion occurred at perigee of the escape hyperbola

5. That insertion must occur at or above circular parking-orbit altitude

For each case, a minimum ΔV, three-impulse maneuver subject to the specified constraints was found by using a modified program of an accelerated gradient method for function minimization (ref. 4). This report summarizes and discusses the results of such a numerical study, with relation to a class of possible Earth-to-Mars missions.

The author wishes to express gratitude to Ivan L. Johnson of the Mission Planning and Analysis Division, NASA Manned Spacecraft Center, for continued guidance in the adaptation of the program of the accelerated gradient method and for assistance in theoretical development. Also, the author wishes to thank T. N. Edelbaum and H. J. Kelley of Analytical Mechanics Associates for verbal communications and assistance.

SYMBOLS

[A] orthogonal 3 × 3 matrix relating $(\vec{i}, \vec{j}, \vec{k})^T$ to $(I, J, K)^T$

a semimajor axis of the terminal hyperbola, Earth radii (e. r.)

a_1 semimajor axis of the elliptical coast between the first and second impulses, e. r.

[B] orthogonal 2 × 2 matrix relating $\left(\vec{i}_p, \vec{j}_p\right)^T$ to $(\vec{i}, \vec{j})^T$

b semiminor axis of the terminal hyperbola, e. r.

D declination of $\vec{V}_\infty$, deg

d $\vec{r} \cdot \vec{V}$, $(\text{e. r.})^2/\text{hr}$

e eccentricity of the terminal hyperbola

F performance index (the sum of the magnitudes of the velocity increments for a given maneuver), e. r. /hr

F' augmented performance index, equation (41)

g column vector of the original set of constraints with the components g_1, g_2, g_3, and g_4

g' column vector of amended set of constraints with the components g_1, g_2', g_3, and g_4

g_1 intermediate equality constraint on a semimajor axis of the first coast

g_2, g_3	terminal equality constraints on $\vec{V}_\infty$ declination and semimajor axis, respectively
g_2'	amended $\vec{V}_\infty$ declination constraint
g_4	terminal inequality constraint on the radius of perigee
h	scalar angular momentum of the terminal hyperbola, $(\text{e.r.})^2/\text{hr}$
I	unit vector from the center of the Earth to the position of the vehicle at initial time
$\vec{i}, \vec{j}, \vec{k}$	orthonormal right-handed set of vectors
$\vec{i}_p, \vec{j}_p$	unit orthogonal vectors in the plane of the hyperbola
i_∞	unit vector from the center of the hyperbola along the asymptote coparallel to $\vec{V}_\infty$
J	unit vector in the direction of motion of the vehicle at initial time
K	unit vector such that $K = I \times J$
L	$y(yw - zv) - x(zw - xw)$
M	$x(xv - yu) - z(yw - zv)$
m	slope of the hyperbolic asymptote with respect to the $\left(\vec{i}_p, \vec{j}_p\right)$ system
N	$z(zu - xw) - y(xv - yu)$
p	semilatus rectum of the terminal hyperbola, e.r.
r	scalar magnitude of $\vec{r}$, e.r.
$\vec{r}$	terminal position vector (at insertion)
r_p	scalar magnitude of $\vec{r}_p$, e.r.
$\vec{r}_p$	radius-of-perigee vector of the terminal hyperbola
RA	$\vec{V}_\infty$ right ascension, deg

T_1 period of the elliptical coast between the first and second impulses for a given three-impulse maneuver, hr

u, v, w velocity components in the (I, J, K) coordinate system of the target hyperbola at r

V_p scalar magnitude of $\vec{V}_p$, e.r./hr

$\vec{V}_p$ velocity vector at perigee for the terminal hyperbola

$\vec{V}_\infty$ asymptotic velocity vector of the terminal hyperbola, e.r./hr

x, y, z position components of $\vec{r}$ in the (I, J, K) coordinate system

α control vector with the components $\alpha_1, \alpha_2, \ldots, \alpha_{11}$

$\hat{\alpha}$ control vector α which minimizes $F(\alpha)$

β_1, β_2 change in generalized eccentric anomaly between $\Delta\vec{V}_1$ and $\Delta\vec{V}_2$ and between $\Delta\vec{V}_2$ and $\Delta\vec{V}_3$, respectively, $(\text{e.r.})^{1/2}$

$\Delta\vec{V}_1$ column vector of the initial velocity increment with the components Δu_1, Δv_1, and $\Delta w_1 = \alpha_9$, α_{10}, and α_{11}, respectively, e.r./hr

$\Delta\vec{V}_2$ column vector of the intermediate velocity increment with the components Δu_2, Δv_2, and $\Delta w_2 = \alpha_5$, α_6, and α_7, respectively, e.r./hr

$\Delta\vec{V}_3$ column vector of the final velocity increment with the components Δu_3, Δv_3, and $\Delta w_3 = \alpha_1$, α_2, and α_3, respectively, e.r./hr

θ polar angle referenced from a specific axis, deg

θ_1 true anomaly of Q_1 (fig. 1) with respect to the perigee of the first transfer conic, deg

θ_2 true anomaly of Q_2 (fig. 1) with respect to the perigee of the first transfer conic, deg

θ_2' true anomaly of Q_2 (fig. 1) with respect to the perigee of the second transfer conic, deg

θ_3' true anomaly of Q_3 (fig. 1) with respect to the perigee of the second transfer conic, deg

θ_3'' true anomaly of Q_3 (fig. 1) with respect to the perigee of the terminal hyperbola, deg

λ column vector of constant Lagrange multipliers with the components λ_1, λ_2, λ_3, and λ_4

μ gravitational constant of the Earth, $(\text{e.r.})^3\ \text{hr}^2$

ϕ argument of perigee with respect to the terminal position vector, deg

Subscript:

p conditions at the hyperbolic perigee

Superscripts:

* desired value at convergence

T matrix transpose

-1 matrix inverse

METHOD

Geometric symmetry permitted simplification of the mathematical model for minimum ΔV orbit transfer from a circle to a $\vec{V}_\infty$ vector by substitution of an equivalent problem, namely, that of finding a minimum ΔV transfer, with the first impulse at a fixed point on the circular parking orbit, to a fixed $\vec{V}_\infty$ magnitude and out-of-plane angle with respect to the parking-orbit plane. To solve the new problem, the in-plane angle for the $\vec{V}_\infty$ vector was regarded as a parameter to be optimized in the solution. Then, in the first problem, any desired prespecified value for the in-plane angle could be regarded as having been obtained simply by a rotation through the appropriate angle of the initial point on the circular orbit.

The solution was expressed in a right-handed, orthonormal inertial coordinate system (I, J, K) based on the parking-orbit plane. In this system, I is a unit vector directed from the center of the Earth to the position of the vehicle at initial time, J is a unit vector in the direction of motion of the vehicle at initial time, and K is a unit vector given by the cross product $K = I \times J$. Because there was no loss of generality, it was assumed that the parking orbit was in the equatorial plane with I directed toward Aries, K directed toward the North Pole, and J completing the right-handed system (I, J, K). Subsequently, the out-of-plane angle of the $\vec{V}_\infty$ vector was referred to as declination and the in-plane angle as right ascension.

For each of the 20 computer runs generated in the numerical study, the transfer maneuver obtained was required to minimize the performance index

$$F(\alpha) = \left|\Delta\vec{V}_1\right| + \left|\Delta\vec{V}_2\right| + \left|\Delta\vec{V}_3\right| \tag{1}$$

subject to the intermediate constraint $g_1 = 0$, the terminal equality constraints $g_2 = g_3 = 0$, and the terminal inequality constraint $g_4 \leq 0$, where the column vector g was defined as

$$g = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix} = \begin{bmatrix} a_1 - a_1^* \\ \sin D - (\sin D)^* \\ a - a^* \\ r_p^* - r_p \end{bmatrix} \tag{2}$$

Fixing $(\sin D)^*$ sufficed to fix the declination angle D^*, because $-90° \leq D^* \leq 90°$ by definition. Furthermore, a_1^* was defined so that a corresponding desired value for the period T_1^* was fixed according to the equation

$$T_1^* = 2\pi\left[\frac{(a_1^*)^3}{\mu}\right]^{1/2} \tag{3}$$

The value a_1^* was selected so that $\left|V_\infty\right|^* = 12\ 000$ ft/sec, where

$$\left|V_\infty\right|^* = \left(\frac{-\mu}{a^*}\right)^{1/2} \tag{4}$$

The value r_p^* was fixed at 1.0290365646 e.r. (at a 100-nautical-mile altitude above the surface of the Earth).

In using the program of the accelerated gradient method, it was useful to express g as a vector function of intermediate and terminal position and velocity variables. The resulting formula for g_2 involved fractions; therefore, subsequent hand calculations of certain partial derivatives used in the program were simplified by the substitution of g_2' for g_2, where

$$g_2' = -erhg_2 \tag{5}$$

It was preferable to use a formula for r_p, namely

$$r_p = \frac{h^2(1 + e)}{\mu} \tag{6}$$

which is valid for all conics, including parabolas, because near-parabolic conics occurred as intermediate iteratives in the optimization process.

For cases in which $D^* = 0°$, it was found that inclusion of the second constraint introduced a redundancy with associated difficulties in obtaining convergence. The redundancy occurred because the optimization program naturally selects an in-plane solution in the absence of contrary constraints. Accordingly, for such cases, the constraint $g_2' = 0$ was deleted, leaving

$$g = \begin{bmatrix} a_1 - a_1^* \\ a - a^* \\ r_p^* - r_p \end{bmatrix} \tag{7}$$

with $g_1 = g_2 = 0$ and $g_3 = r_p^* - r_p \leq 0$.

Finally, consider the second constraint, $g_2 = 0$ initially, where

$$g_2 = \sin D - (\sin D)^* \tag{8}$$

In spherical coordinates, the velocity vector at infinity $\vec{V}_\infty$ is

$$\vec{V}_\infty = \left|\vec{V}_\infty\right|(I \cos RA \cos D + J \sin RA \cos D + K \sin D) \tag{9}$$

Therefore

$$\sin D = \frac{\left(\vec{V}_\infty\right)_z}{\left|\vec{V}_\infty\right|} \tag{10}$$

By analysis, for a hyperbola

$$\vec{i}_\infty = -\left[\frac{\left(\vec{i}_p + \overrightarrow{m j}_p\right)}{\left(1 + m^2\right)}\right] \tag{11}$$

But for all conics

$$m = \frac{b}{a} = \frac{-a\left(e^2 - 1\right)^{1/2}}{a} = -\left(e^2 - 1\right)^{1/2} \tag{12}$$

Therefore

$$e = \left(1 + m^2\right)^{1/2} \tag{13}$$

Equation (11) becomes

$$\vec{i}_\infty = \frac{-\left[\vec{i}_p - \left(e^2 - 1\right)^{1/2} \vec{j}_p\right]}{e} \tag{14}$$

However, by definition

$$\left.\begin{aligned} \vec{i}_\infty &= \frac{\vec{V}_\infty}{\left|\vec{V}_\infty\right|} \\ \vec{i}_p &= \frac{\vec{r}_p}{\vec{r}_p} \\ \vec{j}_p &= \frac{\vec{V}_p}{V_p} \end{aligned}\right\} \tag{15}$$

Therefore, equation (14) becomes

$$\vec{V}_\infty = \frac{-\left|\vec{V}_\infty\right| \left[\frac{\vec{r}_p}{r_p} - \left(e^2 - 1\right)^{1/2} \frac{\vec{V}_p}{V_p}\right]}{e} \tag{16}$$

Using the fact that

$$\left(\vec{V}_\infty\right)_z = \frac{-\left|\vec{V}_\infty\right| \left[\left(\frac{z_p}{r_p}\right) - \left(e^2 - 1\right)^{1/2} \frac{w_p}{V_p}\right]}{e} \tag{17}$$

converts equation (10) into

$$\sin D = \frac{-\left[\left(\frac{z_p}{r_p}\right) - \left(e^2 - 1\right)^{1/2} \frac{w_p}{V_p}\right]}{e} \tag{18}$$

As an intermediate step toward obtaining the necessary conditions for the problem, it was important to express the constraint g_2 as a function of the components with respect to the initial rectangular coordinate system of the position and velocity vectors immediately after application of the third velocity increment. In other words, it was useful to change the right member of equation (18) from a function of state variables at hyperbolic perigee to a function of state variables immediately after insertion into the hyperbola. This change can be made by using the following equations.

$$\left.\begin{aligned} \vec{r} &= xI + yJ + zK \\ \vec{V} &= uI + vJ + wK \end{aligned}\right\} \tag{19}$$

The unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$ are defined as follows, where $\vec{i}$ and $\vec{j}$ lie in the hyperbolic plane of motion.

$$\left.\begin{aligned} \vec{k} &= \vec{r} \times \frac{\vec{V}}{h} \\ \vec{i} &= \frac{\vec{r}}{r} \\ \vec{j} &= \vec{k} \times \vec{i} = \left(r \times \vec{V}\right) \times \frac{\vec{r}}{hr} \end{aligned}\right\} \tag{20}$$

where $h = \left|\vec{r} \times \vec{V}\right|$ is the scalar angular momentum.

Using equation (20), the following coordinate transformation can be written in matrix form.

$$\begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix} = [A] \begin{bmatrix} I \\ J \\ K \end{bmatrix} \tag{21}$$

where $[A]$ is the 3×3 square matrix given by

$$[A] = \begin{bmatrix} \dfrac{x}{r} & \dfrac{y}{r} & \dfrac{z}{r} \\ \dfrac{z(zu - xw) - y(xv - yu)}{rh} & \dfrac{x(xv - yu) - z(yw - zv)}{rh} & \dfrac{y(yw - zv) - x(zu - xw)}{rh} \\ \dfrac{(yw - zv)}{h} & \dfrac{(zu - xw)}{h} & \dfrac{(xv - yu)}{h} \end{bmatrix} \tag{22}$$

The orthogonality of $[A]$ implies that $[A]^{-1} = [A]^{T}$; therefore

$$\begin{bmatrix} I \\ J \\ K \end{bmatrix} = [A]^{T} \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix} \tag{23}$$

Accordingly, equations (19) become, after simplification

$$\left.\begin{aligned} \vec{r} &= r\vec{i} \\ \vec{V} &= \frac{d\vec{i}}{r} + \frac{h\vec{j}}{r} \end{aligned}\right\} \tag{24}$$

where

$$d = \vec{r} \cdot \vec{V} = xu + yv + zw \tag{25}$$

The symbols N, M, and L are defined as

$$\left.\begin{aligned} N &= z(zu - xw) - y(xv - yu) \\ M &= x(xv - yu) - z(yw - zv) \\ L &= y(yw - zv) - x(zw - xw) \end{aligned}\right\} \tag{26}$$

To convert equation (18) into an expression involving only quantities related to the initial rectangular reference frame at the instant of hyperbolic insertion, a pair of rectangular coordinate axis systems $(\vec{i}, \vec{j})$ and $(\vec{i}_p, \vec{j}_p)$ in the plane of the hyperbola was defined. Therefore, it is possible to define an argument-of-perigee angle ϕ in such a manner that the $(\vec{i}_p, \vec{j}_p)$ system is related to the $(\vec{i}, \vec{j})$ system by

$$\begin{bmatrix} \vec{i}_p \\ \vec{j}_p \end{bmatrix} = [B] \begin{bmatrix} \vec{i} \\ \vec{j} \end{bmatrix} \tag{27}$$

where $[B]$ is the 2×2 square matrix

$$[B] = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \tag{28}$$

Therefore

$$\begin{bmatrix} \vec{i}_p \\ \vec{j}_p \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \dfrac{(xI + yJ + zK)}{r} \\ \dfrac{(NI + MJ + NK)}{rh} \end{bmatrix} \tag{29}$$

that is

$$\begin{bmatrix} \vec{i}_p \\ \vec{j}_p \end{bmatrix} = \begin{bmatrix} \dfrac{(xI + yJ + zK)\cos\phi}{r} + \dfrac{(NI + MJ + LK)\sin\phi}{rh} \\ \dfrac{-(xI + yJ + zK)\sin\phi}{r} + \dfrac{(NI + MJ + LK)\cos\phi}{rh} \end{bmatrix} \tag{30}$$

Equations (30) are equivalent to the pair of equations

$$\left.\begin{aligned} \vec{i}_p &= \left(\frac{x\cos\phi}{r} + \frac{N\sin\phi}{rh}\right)I + \left(\frac{y\cos\phi}{r} + \frac{M\sin\phi}{rh}\right)J \\ &\quad + \left(\frac{z\cos\phi}{r} + \frac{h\sin\phi}{rh}\right)K \\ \vec{j}_p &= \left(-\frac{x\sin\phi}{r} + \frac{N\cos\phi}{rh}\right)I + \left(-\frac{y\sin\phi}{r} + \frac{M\cos\phi}{rh}\right)J \\ &\quad + \left(-\frac{z\sin\phi}{r} + \frac{L\cos\phi}{rh}\right)K \end{aligned}\right\} \tag{31}$$

The k-components of $\vec{i}_p$ and $\vec{j}_p$ are

$$\left.\begin{aligned} \frac{z_p}{r_p} &= \left(\frac{z\cos\phi}{r}\right) + \left(\frac{L\sin\phi}{rh}\right) \\ \frac{w_p}{V_p} &= -\left(\frac{z\cos\phi}{r}\right) + \left(\frac{L\sin\phi}{rh}\right) \end{aligned}\right\} \tag{32}$$

Therefore, equation (18) becomes

$$\sin D = -\frac{1}{e}\left[\frac{z \cos \phi}{r} + \frac{L \sin \phi}{rh} - \left(e^2 - 1\right)^{1/2}\left(-\frac{z \sin \phi}{r} + \frac{L \cos \phi}{rh}\right)\right] \tag{33}$$

It is useful to express e, h, $\cos \phi$, $\sin \phi$, and L in terms of x, y, z, u, v, and w. According to the laws of orbital mechanics

$$h = \left[r^2V^2 - \left(\vec{r} \cdot \vec{V}\right)^2\right]^{1/2} \tag{34}$$

Similarly

$$e = \left[1 - \left(\frac{h^2}{\mu_a}\right)\right]^{1/2} \tag{35}$$

The value of L was determined in equation (26). The remaining steps are to express $\cos \phi$ and $\sin \phi$ in terms of x, y, z, u, v, and w.

The polar equation for a conic is

$$r = \frac{h^2}{\mu[1 + e \cos (-\phi)]} \tag{36}$$

Therefore

$$\cos \phi = \frac{\left[\left(\frac{h^2}{r\mu}\right) - 1\right]}{e} \tag{37}$$

Because $\cos^2 \phi + \sin^2 \phi = 1$ and because $d > 0$ on the upper half of a conic and $d < 0$ on the lower half of a conic

$$\sin \phi = -\left(\frac{hd}{r\mu e}\right) \tag{38}$$

Accordingly, it is helpful to replace the constraint g_2 by $g_2' = -erhg_2$. It is possible to consider $g_2' = 0$, because $g_2' = 0$ if and only if $g_2 = 0$. From equation (35)

$$g_2' = \left\{ erh(\sin D)^* + zh \cos \phi + L \sin \phi - \left[\left(e^2 - 1\right)^{1/2} (-zh \sin \phi + L \cos \phi)\right]\right\} \tag{39}$$

In all cases, the control vector α was defined as

$$\begin{aligned} \alpha &= \left(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}\right)^T \\ &= \left(\Delta u_3, \Delta v_3, \Delta w_3, \beta_2, \Delta u_2, \Delta v_2, \Delta w_2, \beta_1, \Delta u_1, \Delta v_1, \Delta w_1\right)^T \end{aligned} \tag{40}$$

In equation (40), β_1 and β_2 were the changes in generalized eccentric anomaly between $\Delta\vec{V}_1$ and $\Delta\vec{V}_2$ and between $\Delta\vec{V}_2$ and $\Delta\vec{V}_3$, respectively.

The initial position on the 262-nautical-mile parking orbit in each case was selected on the positive X-axis. Prior to the first transfer maneuver, the characteristic speed in the circular parking orbit of radius 1.07607599156 e.r. was 25 003 ft/sec.

Verification of a relative minimum for F was obtained by comparing the values found for the components of g' and $\partial F'/\partial\alpha$ with zero, where

$$F' = F + \lambda^T g' \tag{41}$$

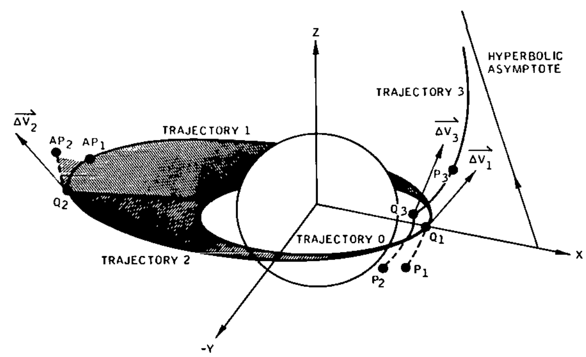
Therefore

$$\left[\frac{\partial F'}{\partial \alpha}\right] = \left(\frac{\partial F'}{\partial \alpha_1}, \frac{\partial F'}{\partial \alpha_2}, \ldots, \frac{\partial F'}{\partial \alpha_{11}}\right)^T = (0, 0, \ldots, 0)^T \tag{42}$$

is a necessary condition for a minimum of F subject to the g-constraints.

It was not possible to resolve a velocity impulse into components such that one component did only plane changing and did not alter the energy state. Nevertheless, it was possible to resolve an impulse into two components, one normal to the current orbital plane and the other lying in the orbital plane. Because only the normal component of an impulse resulted in orbital plane change, that component was called the "component of the impulse associated with plane changing." The in-plane component was called the "component of the impulse associated with energy changing." The "amount of plane changing accomplished by a three-impulse maneuver" was the sum of the magnitudes of the components associated with plane changing over all the impulses for the maneuver. The "percent of plane changing accomplished by a certain impulse" was defined as the percentage equivalent of the ratio of the magnitude of the "component of this plane-changing impulse" with respect to "the amount of plane changing associated with the entire maneuver." A similar definition was adopted for "percent of energy changing accomplished by a certain impulse."

Each of the 20 computer runs with the accelerated gradient method program assumed an impulse-coast-impulse-coast-impulse type of maneuver. For each case, initial position and velocity components on a circular Earth parking orbit were input to the program. First guesses for α were made to start the program converging to find the value of $\alpha*$ resulting in a minimum total velocity expenditure. The geometry for a typical minimum ΔV, three-impulse transfer sequence is shown in figure 1. The first impulse $\Delta\vec{V}_1$ for the maneuver was made on the parking orbit at a true anomaly θ_1 beyond the perigee of the first transfer conic. The second impulse $\Delta\vec{V}_2$ was made at a true anomaly θ_2 beyond the perigee of the first transfer conic, causing injection into a second transfer conic. The position of the second impulse occurred at a true anomaly θ_2' beyond the perigee of the second transfer conic. Injection into a hyperbolic escape trajectory then occurred at a true anomaly θ_3' beyond the perigee of the second transfer conic, that is, at a true anomaly θ_3'' beyond the perigee of the target hyperbola.



TRAJECTORY	0	CIRCULAR EARTH PARKING ORBIT
TRAJECTORY	1	FIRST TRANSFER ELLIPSE
TRAJECTORY	2	SECOND TRANSFER ELLIPSE
TRAJECTORY	3	TERMINAL HYPERBOLA
Q_1, Q_2, Q_3		POINTS OF APPLICATION OF FIRST, SECOND, AND THIRD VELOCITY INCREMENTS
$\Delta V_1, \Delta V_2, \Delta V_3$		FIRST, SECOND, AND THIRD VELOCITY INCREMENTS
AP_1, AP_2		APOGEE POINTS OF FIRST AND SECOND TRANSFER CONICS
P_1, P_2, P_3		PERIGEE POINTS OF FIRST TRANSFER CONIC, SECOND TRANSFER CONIC, AND TERMINAL HYPERBOLA

Figure 1. - Geometry for a typical minimum ΔV, three-impulse transfer sequence.

For each of the in-plane cases, where $D* = 0$, the minimum ΔV transfer sequence was found to be a "double Hohmann" type of maneuver, that is, a 180°-180° perigee-apogee-perigee type of maneuver. Specifically, for each in-plane case, $\theta_1 = \theta_3' = \theta_3'' = 0°$ and $\theta_2 = \theta_2' = 180°$. Or, for each in-plane case, the first impulse was at the perigee of the first transfer conic, the second impulse was at the apogee of both the first and the second transfer conics, and the third impulse was at the perigee of both the second transfer conic and the terminal hyperbolic trajectory.

For all in-plane and out-of-plane cases, except the two cases in which $T_1 = 12$ hours and $D = 2°$ and in which $T_1 = 8$ hours and $D = 2°$, the inequality constraint was satisfied as if it were an equality constraint; that is, the minimum permissible value for target hyperbolic perigee r_p^* was invariably attained. For the two exceptional cases, r_p exceeded r_p^* by 0.04697 and 0.03731 e.r., respectively. Furthermore, in all cases, a retrofire was accomplished at the intermediate impulse. (The vector $\Delta\vec{V}_2$ always had a negative component in the direction of motion.) The Hohmann features of the minimum ΔV transfer maneuvers apply only to in-plane sequences. Computer run 4 ($T_1 = 8$ hours and $D = 6°$) is a sufficiently typical transfer sequence for a full three-dimensional case and is illustrated in figure 1.

In figure 1, trajectory 0 is the circular parking orbit about the Earth. Trajectories 1, 2, and 3 are the first and second transfer ellipses and the terminal hyperbolic path, respectively. The actual transfer flight path is from Q_1 to Q_2 on trajectory 1, from Q_2 to Q_3 on trajectory 2, and thereafter on trajectory 3. The points Q_1, Q_2, and Q_3 are the points of application of the first, second, and third velocity increments, respectively. The points P_1, AP_1, P_2, and AP_2 are the perigee and apogee points of the first transfer ellipse and the perigee and apogee points of the second transfer ellipse, respectively. The point P_3 is the perigee of the terminal hyperbolic trajectory. Impulses do not occur at apsidal points. In fact, the points P_1, AP_2, and P_2 are not even reached in this illustrative minimum ΔV maneuver.

RESULTS

Illustrated in figure 2 is the fact that the three-impulse technique provides some improvement in launch window over the reference one-impulse method, but falls short of the absolute minimum ΔV maneuver (corresponding to infinite T_1) referred to in reference 3.

As an illustration of the advantage of the three-impulse technique, note that for $D = 6°$, the minimum ΔV, three-impulse maneuver for $T_1 = 72$ hours requires a velocity expenditure of 2.15203 e.r./hr, a 7.4 percent smaller velocity penalty than for the reference one-impulse case which requires a total velocity increment of 2.32409 e.r./hr.

The velocity expenditure reduction attainable by increasing the period T_1 of the first coast ellipse for a given target declination D is comparable. For $D = 6°$, the fuel expenditure for the case $T_1 = 72$ hours is 2.15203 e.r./hr, only 3.9 percent less than for the most expensive minimum ΔV case examined for $D = 6°$; namely, $T_1 = 8$ hours, for which the velocity expended is 2.23849 e.r./hr.

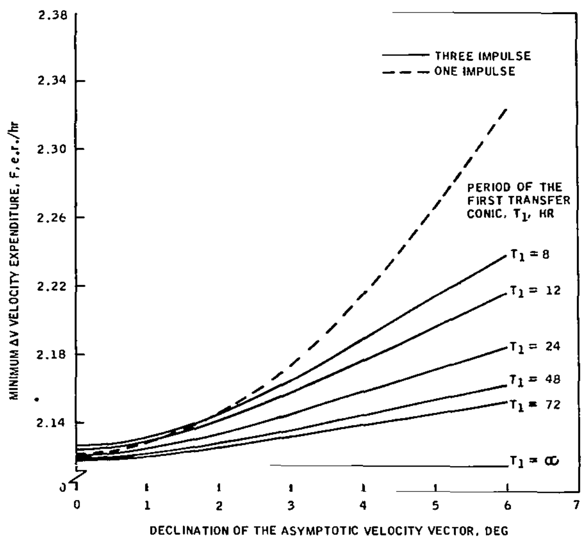


Figure 2. - Minimum ΔV velocity expenditure as a function of D (with a parameter T_1).

For the cases examined, the noncoplanarity velocity penalty is measurable, but not drastic (fig. 3). To illustrate the magnitude of the plane-change penalty, consider $T_1 = 12$ hours. The velocity expenditure for the $D = 6°$ out-of-plane case is is 2.21599 e.r./hr, or 4.3 percent higher than for the corresponding $D = 0°$ coplanar case, with a velocity expended of 2.12430 e.r./hr. For $T_1 = 72$ hours, the velocity requirements for the 6° case exceed those for the 0° case by only 1.6 percent.

Another result is that for the cases considered, the typical minimum ΔV, three-impulse maneuver does not come close to completely uncoupling the energy-changing and plane-changing duties among the three impulses (table I). As a particular example, for computer run 4

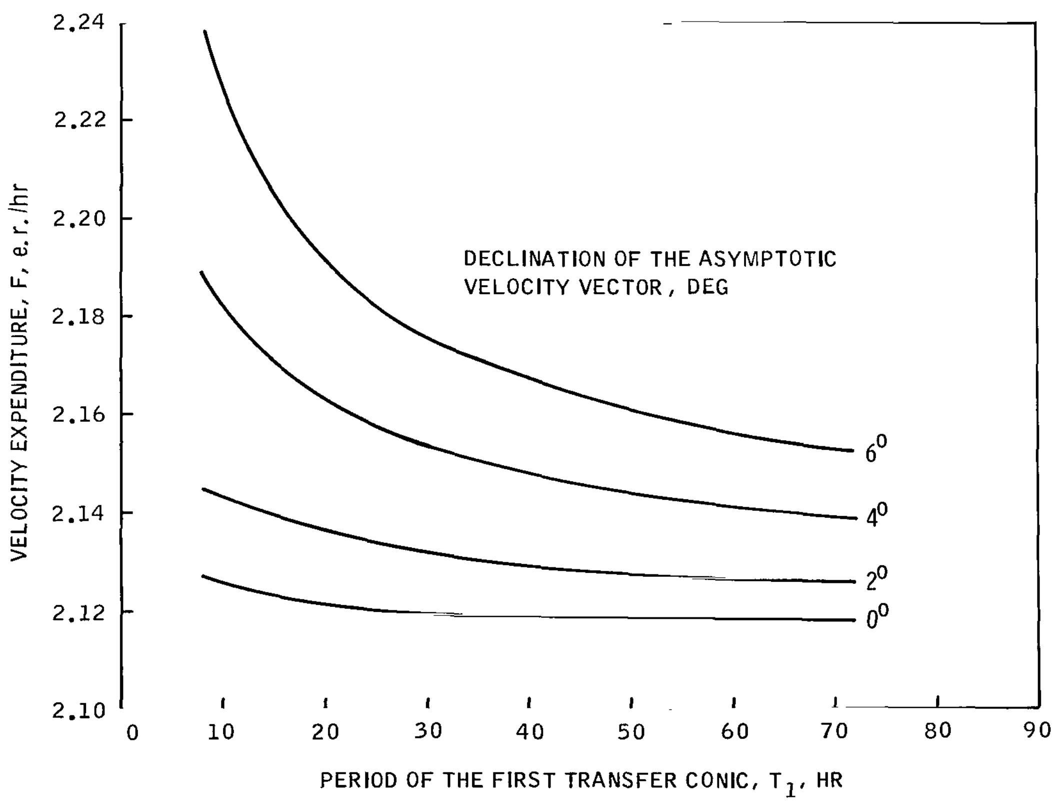


Figure 3. - Velocity expenditure as a function of T_1 (with constant D) for a three-impulse sequence.

TABLE I. - DISTRIBUTION OF PLANE-CHANGING AND ENERGY-CHANGING DUTIES FOR THE MINIMUM ΔV, THREE-IMPULSE MANEUVERS

Run	T_1, hr	D, deg	Plane changing associated with $\Delta\vec{V}_1$, percent	Energy changing associated with $\Delta\vec{V}_1$, percent	Plane changing associated with $\Delta\vec{V}_2$, percent	Energy changing associated with $\Delta\vec{V}_2$, percent	Plane changing associated with $\Delta\vec{V}_3$, percent	Energy changing associated with $\Delta\vec{V}_3$, percent
1	8	0	--	58.4	--	1.0	--	40.6
2	8	2	23.8	67.3	0.8	.1	75.4	32.6
3	8	4	69.2	52.2	4.4	.6	26.4	47.2
4	8	6	71.4	56.8	14.5	.9	14.1	42.3
5	12	0	--	63.5	--	.8	--	35.7
6	12	2	57.4	64.9	14.0	.7	28.6	34.4
7	12	4	22.0	65.3	25.9	.5	52.1	34.2
8	12	6	43.4	65.2	15.1	15.6	41.5	33.2
9	24	0	--	72.0	--	.5	--	27.5
10	24	2	61.3	72.2	4.6	.6	34.1	27.2
11	24	4	49.4	73.2	7.5	.8	43.1	26.0
12	24	6	64.4	72.8	6.8	.7	28.8	26.5
13	48	0	--	75.3	--	.3	--	24.4
14	48	2	46.2	76.9	5.8	.3	48.0	22.8
15	48	4	72.1	76.6	6.6	.4	21.3	23.0
16	48	6	64.2	76.7	12.0	.4	23.8	22.9
17	72	0	--	78.3	--	.2	--	21.5
18	72	2	57.1	78.5	7.8	.3	35.1	21.2
19	72	4	67.3	78.4	10.3	.3	22.4	21.3
20	72	6	57.4	78.4	17.4	.4	25.2	21.2

(T_1 = 8 hours and D = 6°), $\Delta\vec{V}$ does 32.1 percent of the plane changing and 40.8 percent of the energy changing; $\Delta\vec{V}_2$ does 56.0 percent of the plane changing and 28.2 percent of the energy changing; and $\Delta\vec{V}_3$ does 11.9 percent of the plane changing and 31.0 percent of the energy changing.

The ΔV components and generalized eccentric anomaly angles for the minimum ΔV, three-impulse maneuvers are given in table II in units of Earth radii and hours. It is observed that $\theta_2 - \theta_1 \geq 180°$ and $\theta_3' - \theta_2' \leq 180°$ for all cases, with θ_1 always small, positive, and acute. The flight time between first and third impulses for most maneuvers was within an hour of the stated period of the first transfer conic.

As a further illustration, consider again computer run 4 (T_1 = 8 hours and D = 6°) illustrated in figure 1. For this case, $\theta_1 = 1.05°$, $\theta_2 = 199.78°$, $\theta_2' = -160.50°$, $\theta_3' = -6.17°$, and $\theta_3'' = -3.90°$. The constraints for run 4 were well satisfied with converged values for the four components of the constraint vector and for the 11 components of the vector $\partial F'/\partial\alpha$ of the order of 10^{-13}. Approximately this degree of numerical accuracy was attained in all cases examined.

TABLE II. - CONTROL-VECTOR ΔV COMPONENTS, GENERALIZED ECCENTRIC ANOMALY ANGLES,

TOTAL ΔV, AND FLIGHT TIMES FOR THE MINIMUM ΔV, THREE-IMPULSE MANEUVERS

Run	Δu_3	Δv_3	Δw_3	β_2	Δu_2	Δv_2	Δw_2	β_1	Δu_1	Δv_1	Δw_1	F
1	0.00000	0.86246	0.00000	5.5850	0.00000	0.02083	0.00000	5.6057	0.00000	1.2439	0.00000	2.1272
2	.02204	.86894	.14545	4.7932	-.00011	.00467	-.00478	6.4110	.02733	1.2401	.20415	2.1451
3	.06396	.84661	.18669	4.3831	.00250	.02781	-.05038	6.7632	.03710	1.2386	.24068	2.1892
4	.11027	.83462	.21315	4.1818	.01057	.03707	-.09788	6.9075	.04022	1.2378	.25691	2.2385
5	.00000	.73213	.00000	6.3988	.00000	.01625	.00000	6.4169	.00000	1.3759	.00000	2.1243
6	.01595	.72800	.09653	5.6231	-.00047	.01747	-.01565	7.1844	.01591	1.3736	.16148	2.1412
7	.04728	.72066	.13974	5.0302	.00264	.02223	-.04812	7.7397	.02573	1.3720	.20975	2.1769
8	.08001	.71204	.16470	4.8726	.00833	.02892	-.08644	7.8485	.02713	1.3716	.21894	2.2160
9	.00000	.58103	.00000	8.0704	.00000	.01050	.00000	8.0848	.00000	1.5294	.00000	2.1209
10	.01036	.57840	.06403	6.9593	-.00011	.01157	-.01458	9.1860	.00933	1.5279	.13060	2.1341
11	.02855	.57386	.09425	6.4962	.00193	.01440	-.03881	9.6176	.01257	1.5273	.15331	2.1587
12	.04716	.56813	.11839	6.3827	.00476	.01833	-.06451	9.6955	.01300	1.5272	.15708	2.1848
13	.00000	.48810	.00000	10.175	.00000	.00671	.00000	10.186	.00000	1.6240	.00000	2.1188
14	.00675	.48631	.04551	8.8195	.00004	.00745	-.01189	11.532	.00466	1.6232	.09516	2.1285
15	.01745	.48305	.07113	8.4604	.00109	.00910	-.02844	11.868	.00572	1.6230	.10598	2.1449
16	.02833	.47859	.09443	8.3720	.00240	.01154	-.04533	11.931	.00588	1.6230	.10769	2.1621
17	.00000	.45110	.00000	11.650	.00000	.00515	.00000	11.660	.00000	1.6617	.00000	2.1180
18	.00523	.44962	.03872	10.191	.00006	.00571	-.01008	13.110	.00297	1.6612	.07681	2.1259
19	.01314	.44677	.06295	9.8755	.00073	.00694	-.02307	13.407	.00352	1.6611	.08402	2.1387
20	.02118	.44271	.08574	9.7969	.00154	.00883	-.03621	13.465	.00362	1.6611	.08514	2.1520

CONCLUDING REMARKS

It is believed that the numerical study makes several quantitative contributions.

1. The study verifies numerically the existence of three-impulse maneuvers which afford measurable improvement in 18 of 20 cases studied, even for realistically small launch windows, over corresponding one-impulse maneuvers. For the cases examined, the improvement is from -0.23 percent to +7.40 percent lower velocity penalty, that is, total ΔV.

2. For the cases examined, the study obtains the optimum relative distribution (among the impulses) of energy-changing and plane-changing duties. Previous studies have obtained minimum ΔV, three-impulse trajectories from sets of maneuvers in each of which all of the plane changing occurs at the high-altitude middle impulse. Imposition of this characteristic — which is valid for the minimum ΔV, infinite-time maneuver — would have been unrealistic for the (finite-time) cases examined, for which the low-altitude third impulse performed as much as 29.3 percent of the plane changing.

3. The study determines that the velocity penalty for the small-launch-window three-impulse cases examined is only 0.38 to 5.2 percent higher than for corresponding coplanar cases.

4. The analysis discerns that the three-impulse maneuvers involve a 0.13 to 5.8 percent greater velocity penalty than the corresponding absolute minimum ΔV, time-open three-impulse maneuvers.

The program of the accelerated gradient method has a high degree of flexibility and can be adapted to a variety of parameter optimization problems. The current study was conducted on both IBM 7094 and Univac 1108 computers, and a typical case consumed 10 to 15 minutes of machine time. A large percentage of the core storage of the 7094 was used. After the current problem was incorporated into the program, the only usual remaining difficulty was to select appropriate penalty constants for the penalty function phase of the convergence process and a first guess for an optimum control vector. A second computational phase inherent in the accelerated gradient program, using a Newton method for functional minimization subject to nonlinear constraints, encountered numerical convergence problems only infrequently. The accelerated gradient program is not usually strongly dependent on the first guess for the control vector in problems with a payoff function with reasonably uncomplicated contours. However, the level of complexity of the three-impulse study usually required correct guessing of at least the algebraic sign and order of magnitude of each component of the control vector before convergence would ensue. An occasional further difficulty was the obtaining of a degenerate solution, that is, one in which an impulse or coast duration vanished, even though a nondegenerate solution existed. Degenerate solutions were rejected, and alteration of the first guess for the optimum control vector eventually led to attainment of a true three-impulse maneuver.

A further related study under consideration deals with minimum ΔV, two-impulse and three-impulse transfer onto a trans-Mars asymptotic velocity vector from a regressing oblate Earth-assembly parking ellipse, with fixed period of the first transfer conic. Another study underway seeks minimum ΔV, four-impulse transfer from an Earth parking ellipse onto a conjunction-class trans-Mars asymptotic velocity vector, with fixed time of flight between first and fourth impulses.

Recommended for further study are (1) solution of specific fixed-time four-impulse problems, (2) construction of a good first-guess generator to use as an adjunct to the optimization method for fixed-time four-impulse problems, (3) determination of absolute minimum ΔV, three-impulse or four-impulse maneuvers for fixed-time problems, and (4) modification of an optimization method to eliminate degeneracies of the type in which an impulse or a coast vanishes.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, February 11, 1970
924-22-20-00-72

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