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A NUMERICAL EVALUATION OF PRELIMINARY ORBIT DETERMINATION METHODS

by William F. Huseonica

John F. Kennedy Space Center
Kennedy Space Center, Fla.

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16. Abstract

This Technical Note presents a general FORTRAN Code and computer program flowcharts for twelve different Preliminary Orbit Determination Methods (PODM). A number of solutions were obtained from each PODM using input data from a predetermined reference orbit. A comparison of these PODMs in their ability to converge, error propogation, computation time, and total computer core requirements is presented.

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John F. Kennedy Space Center

SUMMARY

Solutions from twelve different Preliminary Orbit Determination Methods using data from two well defined orbits are presented. A number of different solutions were obtained from each method when the angular difference (true anomaly) between observation data was varied from several degrees to one complete revolution. The failure to converge and the numerical error propagation are indicated. The computation time and total computer core required for each PODM is tabulated. A computational algorithm was used to adapt inertial position, velocity, and time input data to angular, range, range rate, and time input data from several different observation stations. A general FORTRAN code and a computer program flowchart are documented and can be utilized with computers other than the Scientific Data Systems 930 used in these solutions.

INTRODUCTION

In preliminary orbit determination (the first approximation of the orbit) it is difficult to select a method which could be considered the best Preliminary Orbit Determination Method (PODM). The best method can be determined by considering several factors of interest to the particular analyst selecting an orbit determination method. These factors are:

Which method is the fastest from a computational point of view?

Which method has the least numerical error propagation?

Which method experiences the least convergence difficulties?

Which method will function most effectively with the observation data available (position, angles, range, range rate, and time)?

Which method can give the best numerical results from orbits of varying eccentricity and semimajor axis?

Which method gives the best results from observation data having small and large true anomaly angular differences?

Data presented in this report form the solutions of twelve different PODMs and will help in determining the best method for a given application. The twelve different PODMs encompass classical methods used in determining the motion of heavenly bodies and present day methods used in artificial satellite PODMs. These PODMs are found in computational algorithm form (Escobal, reference 1). The algorithms were programmed in a FORTRAN II code and the calculations were accomplished on a Scientific Data Systems (SDS) 930 computer.

The PODM input data were derived from two well defined orbits (with perturbations and differential corrections) of common occurrence for artificial earth satellites. One orbit has low eccentricity with a small semimajor axis; the second orbit has a higher eccentricity and a larger semimajor axis.

DISCUSSION

Symbols and Abbreviations

Because the nomenclature used within the field of PODM is so extensive and nonuniform from text to text, a list of symbols and abbreviations is included (appendix A). In addition, the unit vectors and orientation angles of the orbital plane are illustrated in appendix A, figure 1.

PODM Computational Algorithms

The twelve PODMs computed in this evaluation use various types of observation data necessary for a solution or preliminary determination of the orbit. Lambert-Euler, F and G series, Iteration of Semiparameter, Gaussian (time and position), and Iteration of the True Anomaly PODMs use inertial position vectors (x_1, y_1, z_1, and x_2, y_2, z_2) and their corresponding universal times (t_1 and t_2) as the input data. Method of Gauss (angles), Laplace, and Double R-Iteration PODMs require right ascension (α) and declination (δ) from three different stations and their corresponding universal times. Observation station data such as longitude, latitude, and elevation are also required. The remaining PODMs (Modified Laplacian, R-Iteration, Trilateration, and Herrick-Gibbs) require mixed data inputs. The mixed data inputs are selected from right ascension, declination, range and range rate along with the observation station data. Further discussion of these PODMs can be found in references 1 and 3. The computational algorithms for these PODMs are given in equations (1) through (439) in appendixes B through M.

Special considerations that must be given in the computational algorithms for retrograde orbits have been deleted. All orbits to be determined in this evaluation are those involving direct motion.

In nine of the PODMs an iteration of equations is involved which produces an iterative function that must be driven to zero or a lesser specified tolerance, i.e., epsilon.

For this evaluation, a number of 10^{-10} was selected and is in line with the significant figures involved with the input data as well as the PODM solutions. This value for epsilon eliminated the need for extended range accuracy in the computer solutions.

Input data for these nine PODMs were derived from two National Aeronautics and Space Administration (NASA) earth-orbited satellites, OSO-III and Relay-II. These satellite orbits will be used as the bases for evaluation of the PODMs. The OSO-III orbit has an eccentricity of 0.00216 and a semimajor axis of 4,306.81 miles; Relay-II orbit eccentricity is 0.24115 and semimajor axis is 6,915.52 miles. The inclination angles are 32.863 degrees and 46.323 degrees for OSO-III and Relay-II respectively. Additional orbital elements for these satellites are specified in appendixes N and O. Orbital data were furnished by the NASA Goddard Space Flight Center (GSFC), Greenbelt, Maryland. Observation data were received from the various NASA tracking stations (references 5 and 6), and the resultant inertial position and velocity vector data for each minute of two complete revolutions for both orbits were generated from GSFC R083 Orbit Generator Routine-3 (references 7 and 8). The tracking stations and coordinates are listed in appendix P.

The inertial position vector data and corresponding universal time obtained from OSO-III and Relay-II orbits can be used as input data for the five PODMs using position and time inputs. However, these data must be modified to define range, range rate, and angular data to be used as an input for the remaining seven PODMs and to maintain a well defined orbit on which to base an evaluation of all PODMs. A computational algorithm developed to find ρ , $\dot{\rho}$, α , and δ is detailed in appendix Q, equations (440) through (459). Results from this computational algorithm can be selected and applied to the seven PODMs requiring angles only and mixed data.

The PODM computational algorithms terminate when the inertial position and velocity vector for a corresponding observation point is determined; the orbit is then considered determined. In many cases, the classical orbital elements may serve to better illustrate the significant changes in the evaluation of the PODM. Therefore, a computational algorithm that solves for the classical elements (semimajor axis, a; eccentricity, e; inclination, i; longitude of the ascending node, Ω; argument of perigee, ω; and time of perifocal passage, T) from the position and velocity vector is detailed in appendix R, equations (460) through (480). This algorithm is computed subsequent to the determination of the inertial position and velocity vector of each PODM.

Computer Program Language

To facilitate this evaluation, the most obvious tool is the digital computer. The computational algorithms discussed in the previous paragraphs are readily translatable into a program language for communicating with digital computers. The FORTRAN II language was used because it is not really a single computer language. Rather, it is a family of similar languages, or dialects, with one or more being developed for each class of digital computer. A later generation of FORTRAN (FORTRAN IV) will further minimize the difference in this language for each class of computer (reference 2). The FORTRAN language provides engineers and scientists with an efficient and easily understood means of writing programs for computers.

Computer Program Flowcharts

In preparation for the programming of each computational algorithm, a program flowchart was constructed. The flowchart describes the code sequences that accomplish the processing of information to obtain the desirable result. In programs involving a great number of statements, it becomes cumbersome to follow the sequence of written statements. Since written statements can be stated or can proceed in a variety of ways, flowcharts are excellent for conveying procedural concepts.

The value of flowcharts is futher enchanced by consistency in the graphical conventions used. The conventions used in this paper are found in appendix S and were primarily adopted from reference 4.

Flowcharts describe the code sequences as written from the computational algorithms (appendixes B through M). The information within the flowchart symbols is the FORTRAN II code description of the expressions in the algorithm and in the program listings. Only statements conveying procedural concepts are presented in the flowcharts.

Computer Program Listing

For each PODM computed there is a computer program listing (appendixes B through M). The program listing is a sequence of FORTRAN language statements used in computation of the PODM. The program listing is a copy of the source language translated to machine code by the computer processor. The program listing serves as an indicator for the diagnostic report from the computer during the program debugging procedure. The algorithms are programmed in FORTRAN II for use with SDS Series 930 computer (references 9 and 10), but the output of the millisecond (run-time) clock on the SDS 930 was programmed in SDS Meta-Symbol language. The run-time clock tallied and obtained the total time necessary to compute the PODM programs by a program subroutine identified as ITIME. This subroutine used the programmed statements indicated on the program listing by S (SDS Meta-Symbol language). The millisecond clock was initialized by ITIME = 0 and incremented each millisecond by the ITIME subroutine and would subsequently be printed out upon command at the conclusion of a block of computed programmed statements. This procedure was accomplished several times during the computation of each PODM program in order to obtain only computation time and not time required for READ and PRINT statements.

Discussion Summary

The PODMs used for evaluation were found basically in reference 1, Escobal. They were programmed in FORTRAN II and SDS Meta-Symbol for use in the SDS 930 computer. Prior to programming, the procedural concept was established with flowcharts. The two reference orbit data were obtained from GSFC. The data were adapted to input data for angles only and mixed data PODM by a computational algorithm that was programmed and computed prior to the PODM computations. All PODM computations were accomplished on the SDS 930 computer. However, selected programs were successfully compiled and computed on an IBM 1800 and an IBM 360 with only slight modifications. The compilation of algorithms, flowcharts, and computer program listings used to conduct this evaluation of twelve PODMs are detailed in appendixes B through M.

RESULTS AND CONCLUSIONS

The inertial position and velocity orbit data with their corresponding times from epoch used in this PODM evaluation are listed in tables 1 and 2 for OSO-III and Relay-II satellites, respectively. Also contained within these tables is the change in true anomaly angle of each data point referenced to data point 1. Data points contained in these tables are the data points used for the inertial position and time PODM inputs. The same data points were used in the generation of data inputs by the computational algorithm for range, range rate, and angular data for the angles only and mixed data PODMs (appendix Q). The evaluation will consider the inertial position and time PODMs separately from the angles only and mixed data PODMs because sufficient differences exist in the computational algorithms and the practical usage of these PODMs.

Position and Time PODMs

The PODMs which use inertial position vectors and their corresponding times are found in appendixes B through F. These algorithms were applied using all data points referenced from data point 1 in tables 1 and 2. The computational algorithms for inertial position and time PODMs conclude by computing an inertial velocity vector corresponding to one of the times for which an input of inertial position is known. This inertial position and velocity vector and the corresponding time are sufficient to consider the orbit determined.

Subsequent to determination of the inertial velocity vector, the classical orbital elements are computed by using the computational algorithm contained in appendix R. The results of these computations are detailed in figures 2 through 11 and tables 3 through 17.

Figures 2 through 11 are detailed plots of the computed inertial velocity vectors in the $\underline{\dot{x}}$, $\underline{\dot{y}}$, $\underline{\dot{z}}$ components versus the true anomaly angular difference between input data components from tables 1 and 2. The true anomaly angular difference, of position and time PODM, is the angular difference between two inertial position vectors (figure 12). The true anomaly angular difference was varied from 3.8 to 360 degrees for OSO-III orbit and from 2.5 to 360 degrees for Relay-II orbit for convenience in adapting the same data to the angles only and mixed data PODM with consideration to station locations. A plot of the number of iterations required for the iteration loop within the PODM compuational algorithm for each set of data input used is also contained in figures 2 through 11. Tables 3 through 12 are the tabulated results which are plotted in figures 2 through 11.

For example, in figure 2, results of Lambert-Euler PODM for OSO-III, at 10 degrees difference in true anomaly the inertial velocity vectors are as follows: $\dot{x}$ is -0.67100 CUL/CUT; $\dot{y}$ is 0.45242 CUL/CUT; and $\dot{z}$ is -0.51970 CUL/CUT and the predicted number of iterations is seven. The nominal values are indicated for each component. Also denoted is the true anomaly angular difference beyond which the program fails to compute and yield satisfactory results.

A comparison in each case of the computed resultant classical orbital elements, with respect to the nominal values obtained from appendixes N and O, is listed in tables 13 through 17. Both the computed results and the nominal values from the reference orbit are referenced to the same time of epoch as denoted in tables 1 and 2.

Each PODM program listing as found in appendixes B through F requires a definite number of words available in the computer core before a successful computation can be accomplished. Table 18 lists the number of 24-bit words required in the computer core of the SDS 930 computer for variables, statements, and subprograms necessary for compuation of each PODM. The number of core words required can vary and may depend on the programming efficiency of the programmer. One programmer may be able to accomplish the same task with fewer core words than another programmer.

Another factor which can vary the computer core requirements is the efficiency of the computer manufacturer's library of translations of FORTRAN to machine language. In comparing the position and time PODMs, the core requirements for each PODM vary little except for the F and G Series (4649 words) requirement.

The time necessary to compute the computer coded program listing of each PODM was evaluated by printing time from the computer clock (ITIME) at the conclusion of a block of computations, ignoring the time necessary for READ and PRINT statements. The method used can be found in the computer program listing. The computation time required for each PODM is listed in table 19. The total time required for computation of each program with only one iteration ranges from 16 to 21 milliseconds, with F and G series being slowest and Lambert-Euler being fastest. The F and G series is slowest and Lambert-Euler and Gaussian PODMs fastest when comparing the time required for each additional iteration computation loop. However, the total time for computation during practical application of these PODMs is a function also of the rate of convergence. The average number of iterations required for the PODM iterative loop to converge is listed in table 20. Although the F and G series is slowest when computing for all portions of the algorithm, it is fastest in its ability to converge. The averages in table 20 considered only the data points for which the PODM yielded satisfactory results; i.e., the averages were computed from results of the PODM over true anomaly angular ranges which yielded acceptable solutions. The radius vector spread of the data input must be considered when choosing a PODM for a minimum computation time for a particular orbit because the convergence of the iteration loop is a function of the true anomaly difference.

Ease of convergence. - The ease of convergence of each PODM is indicated in table 20. The shape of the orbit appears to have some effect on the ability of the PODM to converge. Lambert-Euler, F and G series, and Iteration of True Anomaly PODMs decrease in ability to converge for an orbit with a larger semimajor axis and higher eccentricity while Gaussian and Iteration of Semiparameter PODMs increase.

The radius vector spread (true anomaly angular difference) over which these PODMs are likely to yield best results is concluded in table 21. The best result is a function of ease of convergence and accuracy.

Error propagation. - The position and time PODM that has the least error propagation is not readily distinguishable. There are relatively small differences in the propagation of error as indicated by the graph of inertial velocity versus true anomaly angular difference in figures 2 through 11. The profile of error in computing the inertial velocity in all PODMs appears the same until the radius vector spread becomes excessive for acceptable PODM results. The data also indicate that an optimum in radius vector spread for the most accurate computed velocity vector for these PODMs is 20 to 30 degrees.

Discussion of results. - In comparing the five PODMs using position and time input data, the results indicate that the optimum PODM is the Lambert-Euler followed by Iteration of Semiparameter, Iteration of True Anomaly, Gaussian, and F and G series. The optimum was a compromise between computation time, ease of convergence, and best overall accuracy considering radius vector spreads up to 360 degrees. These comparisons were made from the results of two different orbits; OSO-III and Relay-II. Table 22 indicates the standing of each PODM for consideration for determining the optimum.

Angles Only and Mixed Data PODMs

The PODMs using angles only and mixed data are found in appendixes G through M. These algorithms require a combination of three station observations of right ascension, declination, range or range rate, and their corresponding times from epoch in a topocentric coordinate system for a solution. The station location data is also required and is found in appendix P. From each data point in tables 1 and 2, values for range, range rate, declination, and right ascension were computed for several different stations using the computational algorithm found in appendix Q. These data are detailed in tables 23 and 24 for OSO-III and Relay-II, respectively. Tables 23 and 24 constitute the required input data to the angles only and mixed data PODMs being evaluated.

These PODMs require three observation data inputs for a solution and the observation station location data. There is also a requirement that the station observation data be from either three separate stations at three different times, or one station at three different times from epoch, or three stations with data input resolved to a common time from epoch. The number of stations required is determined in the computation algorithm by the input data necessary before a solution can be obtained from the PODM. The data points and observation stations combination used in computing results for evaluation of these PODMs are specified in tables 25 and 26.

The inertial velocity component results of these computations are specified in tables 27 through 39. These tables present the inertial velocity vector components $\dot{\underline{x}}$, $\dot{\underline{y}}$, and $\dot{\underline{z}}$ with reference to inertial velocity vector of the nominal orbit from tables 1 and 2. A comparison in each case of the resultant classical orbital elements, with respect to the nominal values of the elements from appendixes N and O, is specified in tables 40 through 44.

Both the computed results and the nominal values from the reference orbit are referenced to the same time of epoch as denoted in tables 1 and 2.

Table 18 indicates the computer core requirements for the program listings contained in appendixes G through M and Q. The requirements range from 3525 words for Herrick-Gibbs to 5254 words for Method of Gauss.

Computation time. - The computation time required for each PODM is specified in table 19. Two of the PODMs in this table, one under mixed data and the other under angles only, differ from the others. Herrick-Gibbs PODM has no iteration loop and is fastest from the computation time; Gauss PODM has two iteration loops and is the slowest. The total computing time required ranges from 13 to 26 milliseconds when only one pass through the iteration loop is present. Time for each additional pass through the iteration loop ranges from 5 to 9 milliseconds.

The average number of iterations of each PODM, using both OSO-III and Relay-II orbits, is specified in table 45. Herrick-Gibbs and Trilateration PODM do not have an iteration loop. However, Trilateration does have a branch which is computed twice to determine best approximation for the inertial position vector. Neither has an iteration loop computation time which can be compared with the other PODMs. Of the remaining PODMs which have iteration loops, Laplace and Modified Laplacian are the fastest at 5 milliseconds for each iteration loop while the Double R-Iteration PODM is slowest at 9 milliseconds.

Ease of convergence. - The radius vector spread between $\underline{r}_1$ to $\underline{r}_2$ and $\underline{r}_3$ for data inputs to the PODM was 3.8 to 360 degrees for OSO-III and 2.5 to 360 degrees for Relay-II. Considering the data points which yielded satisfactory results to define the orbit, table 45 indicates the difficulty in convergence. Double R-Iteration and Laplace (angles only) iteration loops did not converge in the allotted number indexed in the program (maximum number of iterations allowable is 25). It becomes apparent that changes are required in refining the iteration loop from either a mathematical or programming viewpoint or that observation station geometry is critical. From these two PODMs (Double R-Iteration and Laplace) only one set of results from each came close to resembling OSO-III or Relay-II orbits. As presented, these PODMs have difficulty in converging and require additional information.

The three remaining PODMs which have iteration loops (Method of Gauss, Modified Laplacian, and R-Iteration) have a greater ease of convergence with data from OSO-III orbit, having a lower eccentricity and semimajor axis, than with the data from Relay-II orbit.

The convergence question does not arise in Herrick-Gibbs or Trilateration PODMs since no iteration loops exist.

Error propagation. - Error propagation in the angles only and mixed data PODMs have no characteristic profile as in the case of the position and time PODMs. Many factors may contribute to the inconsistency of error propagation and overall accuracy of results.

One factor is that station observation data was generated by a scheme from inertial position and velocity data and not by direct station observations. The geometry established between the observing station and the orbiting body may also be a critical factor. The limited number of data points available and used may yield results not completely representative of the PODM error propagation. However, after such considerations, all PODMs used the same input data for the results being discussed. If an error propagation profile can be established sufficiently it would appear to be similar in the Herrick-Gibbs, Method of Gauss, Modified Laplacian, and R-Iteration PODMs. The Double R-Iteration and Laplace PODMs have no distinguishable error profile.

A more accurate and complete set of results exist from the Relay-II orbit input data to PODM than exists from the inputs used from the OSO-III orbit. It appears that an orbit with larger semimajor axis and eccentricity is more readily computable for acceptable results over a greater radius vector spread than an orbit of lesser semimajor axis and eccentricity (Relay-II versus OSO-III). The PODM with the best overall accuracy with a radius vector spread (υ) to 360 degrees is specified in table 46.

Discussion of results. - In comparing each PODM using angles only and mixed data, the optimum PODM was determined to be Herrick-Gibbs followed by Modified Laplacian, Method of Gauss, R-Iteration, Double R-Iteration, and Laplace. The optimum was a compromise between the computing time, ease of convergence, and best overall accuracy considering radius vector spreads up to 360 degrees. These comparisons were made using the results of OSO-III and Relay-II orbits. Table 47 indicates the rank of each PODM under several classifications.

A contrasting difference is apparent when comparing the angles only and mixed data PODMs in that the schemes converge more easily with an OSO-III type of orbit. However, acceptable results are more readily attainable over a greater radius vector spread with the Relay-II type orbit.

Trilateration

Trilateration PODM is unique in that it requires three different station observations at the same time. The geometry of the three stations is very critical for obtaining accurate results. A computed set of results for OSO-III and Relay-II orbits are detailed in table 39. The results of Relay-II are more accurate than those of OSO-III. This follows the same trend as the other PODMs using angles only or mixed data. Also, Trilateration does not have an iteration loop and, with the requirement of simultaneous observations, it makes this PODM sufficiently different to refrain from comparing it directly with other PODMs. Total computation time for Trilateration PODM was 17 milliseconds.

Conclusion

Solutions from twelve different PODMs using data from two well defined orbits are presented. A number of solutions were obtained from each PODM when the angular difference (true anomaly difference) between observation data was varied from several degrees to one complete revolution. The PODMs evaluated use combinations of inertial position, angels, range and range rate, and corresponding universal times as input data. The computation time required for each PODM is tabulated for a nearly circular orbit with a small semimajor axis and one of higher eccentricity and a larger semimajor axis.

In comparing the five PODMs using position and time imput data, the results indicate that the optimum PODM is the Lambert-Euler. Herrick-Gibbs is the optimum of the seven PODMs using angles only and mixed data.

A computational algorithm was used to adapt inertial position, velocity, and time input data to angular, range, range rate, and time input data from several different observation stations. A general FORTRAN code with program listings and computer program flowcharts is documented and can be utilized with computers other than the SDS 930 used in these solutions with only slight modifications. The computer core requirements for each program listing presented is tabulated.

The PODMs using inertial position and universal time input data yield solutions to the intercept, rendezvous, and interplanetary transfer problems of trajectory analysis. The angles only PODMs are the more classical PODMs which solve for fundamental orbital elements using the observer as main participant. Standing on a given location on the central planet of the orbiting body, an observer can measure the angular coordinates and determine the orbit. With the introduction of radar, the mixed data techniques are attractive to the trajectory analyst. The slant range from the observer to the satellite is obtainable as well as the rate at which this range is changing. The modern trajectory analyst uses the mixed data PODMs more frequently because of the excellent range and range rate data available.

The twelve PODMs may be used in any number of different problems confronting the trajectory analyst. The data presented can be used to predetermine a set of conditions which must exist in order to use the PODM which will yield the best determination of the orbit. Various combinations of observation stations and satellite observation data can be used effectively for orbit determination. With the computer programs available to each PODM, they may be used as computer program options which can be called on command to yield the best orbital results. This would be an efficient and accurate method for determining orbits of unknown space objects. The PODM results can be used to determine look angles for observation stations at later dates.

APPENDIX A
SYMBOLS AND ABBREVIATIONS

English Symbols

A	Azimuth angle. Miscellaneous constants. Area.
$\underline{A}$	Auxiliary vector used in the method of Gauss. Unit vector pointing due east.
a	Semimajor axis of a conic section. Matrix coefficient.
a_e	Equatorial radius of Earth.
B	Miscellaneous constants.
$\underline{B}$	Auxiliary vector used in the method of Gauss. Semiminor axis of a conic section.
C_ψ	The dot product of $(-\underline{R} \cdot \underline{L})$.
C_e	Element $(= e \cos E_0)$.
C_h	Element $(= e \cosh F_0)$.
C_ν	Element $(= e \cos \nu_0)$.
c	Ratio of sector to triangle in the method of Gauss.
E	Eccentric anomaly. Miscellaneous constants.
e	Orbital eccentricity. Mathematical constant.
f	Geometrical flattening of reference spheroid adopted for central planet. Functional notation. Coefficient of f and g series.
G	Station location and shape coefficients. Universal gravitational constant. Miscellaneous constants.
g	Coefficient of f and g series. Gravitational acceleration.
H	Station elevation measured normal to adopted ellipsoid.

h	Elevation angle.
$\underline{h}$	Angular momentum vector.
$\underline{I}$	Unit vector along the principal axis of a given coordinate system.
i	Orbital inclination. The imaginary $(= \sqrt{-1})$.
J	Harmonic coefficients of the Earth's potential function.
$\underline{J}$	Unit vector advanced to I by a right angle in the fundamental plane.
K	A constant.
$\underline{K}$	Unit vector defined by $\underline{I} \times \underline{J} = \underline{K}$.
k_e	Gravitational constant.
$\underline{L}$	Unit vector from observational station to satellite.
M	Mean anomaly $\left[= n(t - T)\right]$.
m	General symbol for mass. Meters.
N	Number of revolutions.
n	Mean motion $(= k\sqrt{\mu}/a^{\frac{3}{2}})$. Number of revolutions.
P	Orbital period (time from perigee crossing to perigee crossing).
P_h	Perifocus.
$\underline{P}$	Unit vector pointing toward perifocus.
p	Orbital semiparameter $\left[= a(1 - e^2)\right]$.
Q	Unit vector advanced to $\underline{P}$ by a right angle in the direction and plane of motion.
q	Generalized element. Perifocal distance $\left[= a(1 - e)\right]$. Parameter of f and g series expansions.
R	Perturbative function $(= \Phi - V)$. Magnitude of station coordinate vector.

$\underline{R}$	Station coordinate vector. Alternate notation for $\underline{U}$.
r	Magnitude of satellite radius vector.
$\underline{r}$	Satellite radius vector.
S	Satellite symbol.
S_e	Element ($= e \sin E_0$).
S_h	Element ($= e \sinh F_0$).
S_ν	Element ($= e \sin \nu_0$).
s	A parameter taking the value 1 or -1.
T	Time of perifocal passage.
t	Universal or ephemeris time.
$\underline{U}$	Unit vector pointing toward given satellite.
u	Argument of latitude. Parameter of f and g series expansions.
V	General symbol for velocity vector magnitude. Spherical potential of planet.
$\underline{V}$	Unit vector advanced to $\underline{U}$ by a right angle in the direction and plane of motion.
$\underline{W}$	Unit vector perpendicular to orbit plane.
X, Y, Z	Rectangular coordinates of station coordinate vector.
x, y, z	Rectangular coordinates of an object.
$\underline{Z}$	Unit vector in the zenith direction.

Special Symbols

$\equiv$	Identically equal to. Equal to by definition.
$\doteq$	Replace left side of equation with right side of equation.
$\simeq$	Approximately equal to.
♈	Vernal equinox (sign of the Ram's Horns).
∞	Infinity.
$\angle x, y$	Angle between x and y.
$\rightarrow$	Yields.
$\lvert x \rvert$	Absolute value of x.

Superscript Symbols

$\cdot$	Relating to modified time differentiation. Also ($\ddot{}$).
$'$	Relating to general differentiation. Relating to geocentric latitude. Minutes of arc.
$''$	Seconds of arc.
*	Particular parameter or special form of an analytical expression.
~	Particular parameter or special form of an analytical expression.
-	Used to denote average or special form of an analytical expression or parameter.
^{o}	Degrees.
hr	Hours.
min	Minutes.
sec	Seconds.

Greek Alphabet

A	α	Alpha.	N	ν	Nu.
B	β	Beta.	Ξ	ξ	Xi.
Γ	γ	Gamma.	O	o	Omicron.
Δ	δ	Delta.	Π	π	Pi.
E	ϵ	Epsilon.	P	ρ	Rho.
Z	ζ	Zeta.	Σ	σ	Sigma.
H	η	Eta.	T	τ	Tau.
Θ	θ	Theta.	Υ	υ	Upsilon.
I	ι	Iota.	Φ	ϕ	Phi.
K	κ	Kappa.	X	χ	Chi.
Λ	λ	Lambda.	Ψ	ψ	Psi.
M	μ	Mu.	Ω	ω	Omega.

Greek Symbols

α Right ascension.

Δ Increment or difference.

∇ Gradient operator.

$$\left[\nabla(\cdot) = \frac{\partial(\cdot)}{\partial x}\,\underline{I} + \frac{\partial(\cdot)}{\partial y}\,\underline{J} + \frac{\partial(\cdot)}{\partial z}\,\underline{K}\right]$$

δ Declination.
Variation.

ϵ Obliquity of the ecliptic.
Specified tolerance.

ζ Coefficient.

θ Sidereal time.

λ Longitude.

μ Sum of masses or mass.

ν True anomaly.

$\underline{\rho}$ Slant range vector.

Greek Symbols (Cont'd)

ϕ	Geodetic latitude.
ϕ	Geocentric latitude.
ϕ_a	Astronomical latitude.
Ω	Longitude of ascending node.
☋	Longitude of descending node.
ω	Argument of perigee.

Abbreviations

a.u.	Astronomical units.
cm	Centimeters.
c.m.	Central masses.
c.s.u.	Circular satellite units (also g.c.s.u.; geocentric circular satellite units)
c.u.	Characteristic units.
CUL	Canonical unit of length.
CUT	Canonical unit of time.
deg	Degrees.
e.m.	Earth masses.
e.r.	Earth radii.
ft	Feet.
gm	Grams.
hr	Hours.
h.c.s.u.	Heliocentric circular satellite units.
J.D.	Julian date.
km	Kilometers.
m	Meters.
min	Minutes.
sec	Seconds.
s.m.	Solar masses.

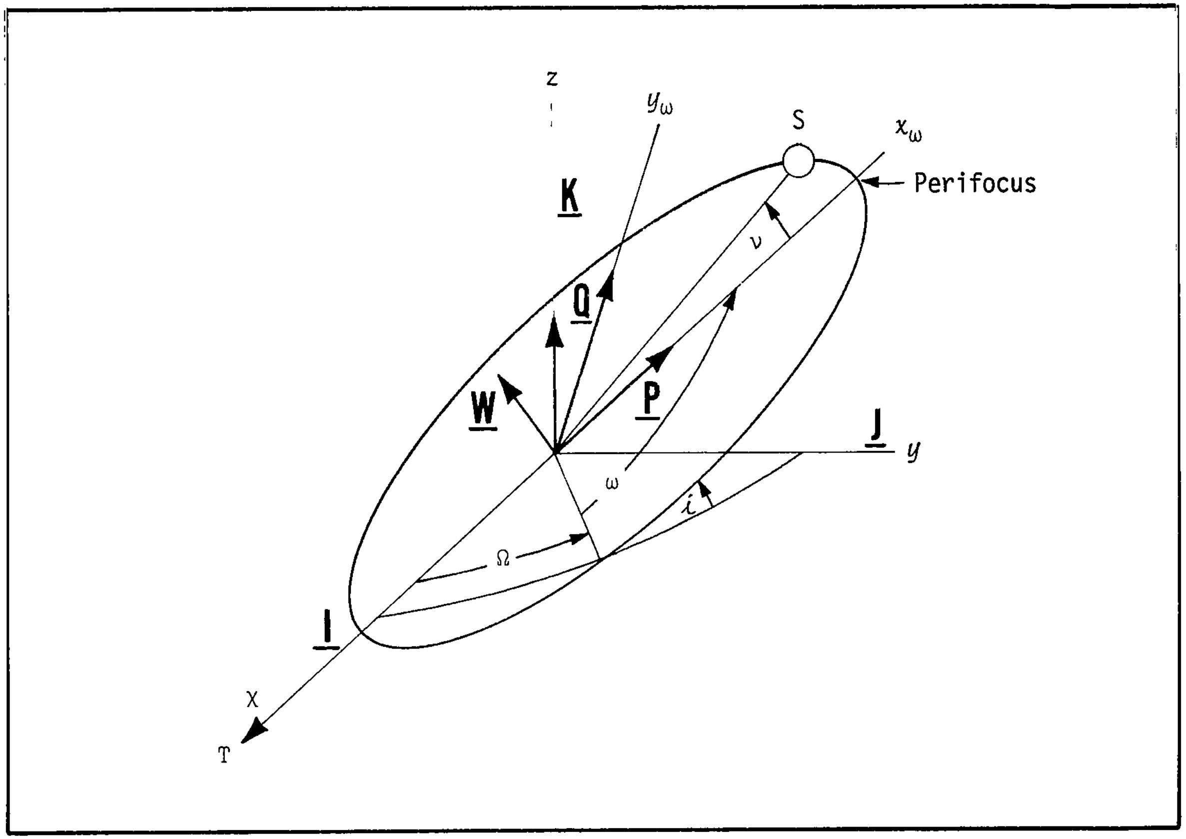


Figure 1. Orbit Plane Coordinate System Showing Unit Vectors and Orientation Angles

APPENDIX B
LAMBERT-EULER PODM, POSITION AND TIME

Given $\underline{r}_1$ (x_1, y_1, z_1), $\underline{r}_2$ (x_2, y_2, z_2) and their corresponding universal times, t_1 and t_2, proceed as follows:

$$\tau = k_e (t_2 - t_1) \tag{1}$$

$$r_1 = +\sqrt{\underline{r}_1 \cdot \underline{r}_1} \tag{2}$$

$$r_2 = +\sqrt{\underline{r}_2 \cdot \underline{r}_2} \tag{3}$$

$$\underline{U}_1 = \frac{\underline{r}_1}{r_1} \tag{4}$$

$$\underline{U}_2 = \frac{\underline{r}_2}{r_2} \tag{5}$$

$$\cos(\nu_2 - \nu_1) = \underline{U}_1 \cdot \underline{U}_2 \tag{6}$$

$$\sin(\nu_2 - \nu_1) = \frac{x_1 y_2 - x_2 y_1}{|x_1 y_2 - x_2 y_1|} \sqrt{1 - \cos^2(\nu_2 - \nu_1)} \tag{7}$$

As a first approximation, if no better estimate is available, set

$$a = \frac{(r_1 + r_2)}{2} \tag{8}$$

and continue calculating with

$$c = + \left[r_2{}^2 + r_1{}^2 - 2(x_1x_2 + y_1y_2 + z_1z_2) \right]^{\frac{1}{2}} \tag{9}$$

$$\sin \frac{1}{2} \varepsilon = + \sqrt{\frac{1}{4a} (r_2 + r_1 + c)} \tag{10}$$

$$\sin \frac{1}{2} \delta = + \frac{\sqrt{r_2r_1} \cos \left(\frac{\nu_2 - \nu_1}{2} \right)}{2a \sin \frac{1}{2} \varepsilon} \tag{11}$$

$$\cos \frac{1}{2} \delta = + \sqrt{1 - \frac{1}{4a} (r_2 + r_1 - c)} \tag{12}$$

Set

$$s = 1 \tag{13}$$

Later the analysis will be repeated for

$$s = -1 \tag{14}$$

Continue with

$$\cos \frac{1}{2} \varepsilon = s \sqrt{1 - \sin^2 \frac{1}{2} \varepsilon} \tag{15}$$

$$F = \tau - \frac{a^{\frac{3}{2}}}{\sqrt{\mu}} \left[(\varepsilon - \sin \varepsilon) - (\delta - \sin \delta) \right] \tag{16}$$

If

$$|F| < \Delta \tag{17}$$

where Δ is a given tolerance, i.e., 10^{-10}, proceed to equation (22); if it is not, save F(a) and increment a, by 5 percent, that is, Δa, to obtain:

$$a + \Delta a \tag{18}$$

Repeat equational loop (10) through (16), obtaining $F(a + \Delta a)$, and form

$$F'(a) \simeq \frac{F(a + \Delta a) - F(a)}{\Delta a} \tag{19}$$

Improve the value of a by

$$a_{j+1} = a_j - \frac{F(a_j)}{F'(a_j)}, \quad j = 1, 2, 3, \ldots, q \tag{20}$$

If

$$|a_{j+1} - a_j| < \Delta \tag{21}$$

Proceed to equation (22); if not return to equation (10), replacing a_j with a_{j+1}.

$$E_2 - E_1 = \varepsilon - \delta \tag{22}$$

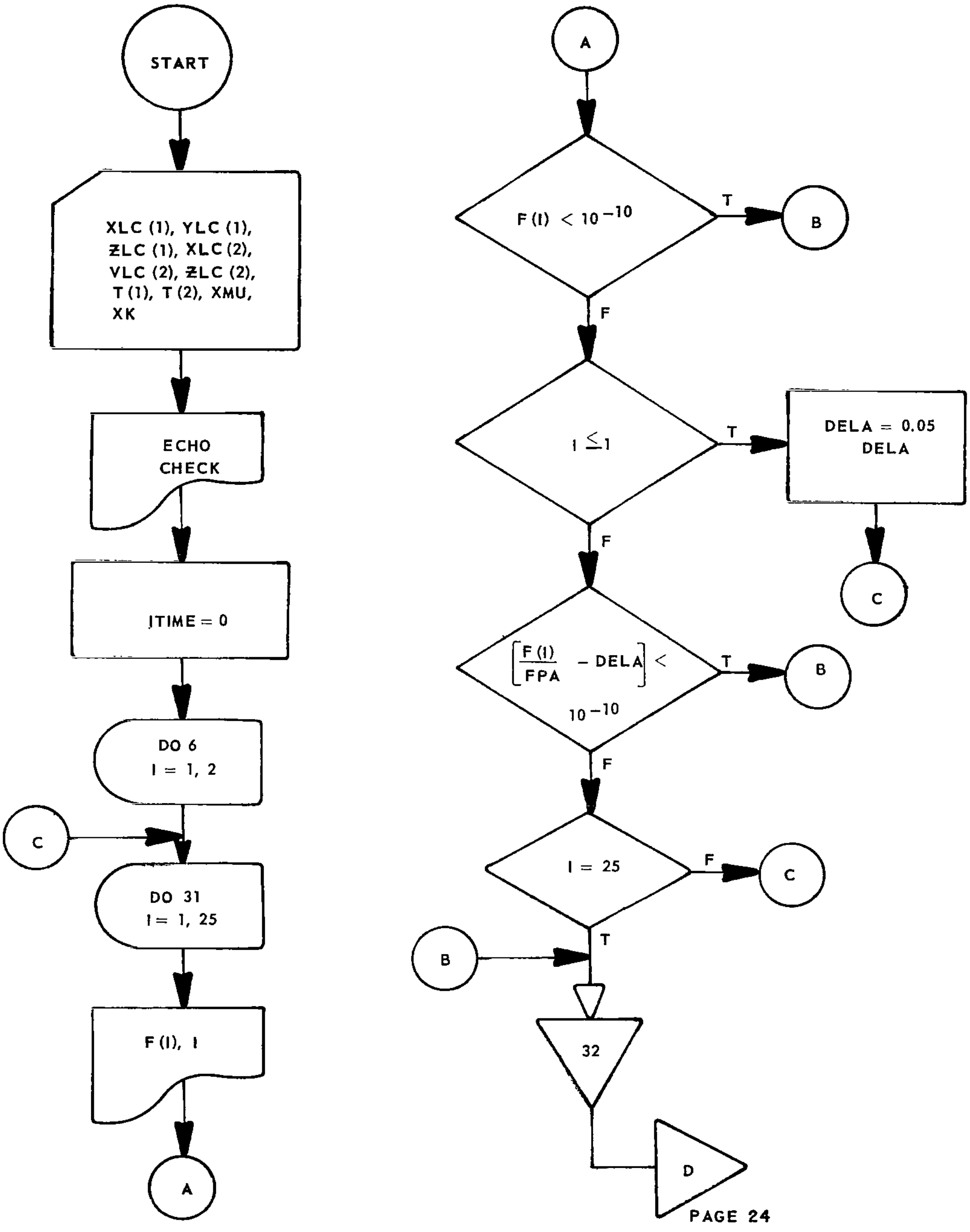
$$f = 1 - \frac{a}{r_1}\left[1 - \cos(E_2 - E_1)\right] \tag{23}$$

$$g = \tau - \frac{a^{\frac{3}{2}}}{\mu}\left[E_2 - E_1 - \sin(E_2 - E_1)\right] \tag{24}$$

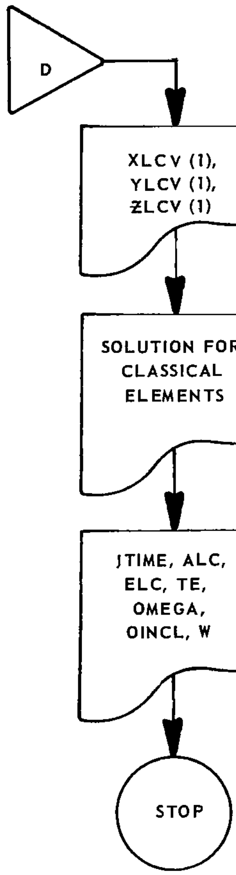
$$\dot{\underline{r}}_1 = \frac{\underline{r}_2 - f\,\underline{r}_1}{g} \tag{25}$$

Continue by calculating for the classical elements.

LAMBERT-EULER FLOWCHART



LAMBERT-EULER FLOWCHART (CONT'D)



```
C       LAMBERT-EULER PRELIMINARY ORBIT DETERMINATION
C       POSITION AND TIME (ESCOBAL,PAGE 205)
C
        DIMENSION F(25),UX(2),UY(2),UZ(2),RLC(2),XLC(2),
       CYLC(2),ZLC(2),T(2),XLCV(1),YLCV(1),ZLCV(1),RLCV(1)
        DO 40 N=1,6
C
C       READ TWO INERTIAL POSITION VECTORS AND THEIR CORRESPONDING TIMES
C
        READ 101, XLC(1), YLC(1), ZLC(1), T(1), XLC(2)
        READ 101, YLC(2), ZLC(2), T(2), XMU, XK
101     FORMAT(5E16.8)
C
C       ECHO CHECK
C
        PRINT 104, XLC(1),YLC(1),ZLC(1),T(1),XLC(2),YLC(2),ZLC(2), T(2),
       CXMU,XK
104     FORMAT(1H0,$XLC(1)=$E16.8,///,$YLC(1)=$E16.8,///,$ZLC(1)= [illegible],///,
       1$T(1)=$E16.8,///,$XLC(2)=$E16.8,///,$YLC(2)=$E16.8,///,$ZLC(2)=$E16.
       1///,$T(2)=$E16.8,///,$XMU=$E16.8,///,$XK=$E16.8)
C
C       BEGIN COMPUTATIONS
C
C       ALL METASYMBOL IS ITIME SUBROUTINE
C
        ITIME=0
S       LDA        205S
S       STA        0205
S       BRU        200S
S205    BRM        2050S
S200    COM        020020
S       PAT  =  00202000
S       EIR
        TAU=XK*(T(2)-T(1))
  2     DO 6 I=1,2
        RLC(I)=SQRT(XLC(I)**2+YLC(I)**2+ZLC(I)**2)
        UX(I)=XLC(I)/RLC(I)
        UY(I)=YLC(I)/RLC(I)
  6     UZ(I)=ZLC(I)/RLC(I)
        VCOS=UX(1)*UX(2)+UY(1)*UY(2)+UZ(1)*UZ(2)
        COM=XLC(1)*YLC(2)-XLC(2)*YLC(1)
        VSIN=(COM/ABS(COM))*SQRT(1.0-VCOS**2)
        C=SQRT(RLC(2)**2+RLC(1)**2-2.0*(XLC(1)*XLC(2)+YLC(1)*YLC(2)
       C+ZLC(1)*ZLC(2)))
        S=1.0
 14     A=(RLC(1)+RLC(2))/2.0
C
C       BEGIN LAMBERT EULER ITERATION
C
 15     DO 31 I=1,25
        SHEPS=SQRT((RLC(2)+RLC(1)+C)/(4.0*A))
        ANGV=ATAN(VSIN,VCOS)
        SHDEL=SQRT(RLC(1)*RLC(2))*COS(ANGV/2.0)/(2.0*A*SHEPS)
```

```
      CHDEL=SQRT((1.0)-(RLC(2)+RLC(1)-C)/(4.0*A))
      CHEPS=S*SQRT(1.0-SHEPS**2)
      EPSLN=2.0*ATAN(SHEPS,CHEPS)
      DELTA=2.0*ATAN(SHDEL,CHDEL)
      F(I)=TAU-SQRT(A**3/XMU)*((EPSLN-SIN(EPSLN))-(DELTA-SIN(DELTA)))
      CT1=ITIME
      PRINT 100,CT1
      PRINT 102, F(I),I
102   FORMAT(1H0,*F(I)=*E16.8*****I=*I2)
      ITIME=0
 24   IF(ABS(F(I))-0.0000000001) 32,25,25
 25   IF(I-1) 30,30,26
 26   FPA=(F(I)-F(I-1))/DELA
 27   IF(ABS(F(I)/FPA-DELA)-0.0000000001) 32,28,28
 28   DELA=-F(I)/FPA
 29   GO TO 31
 30   DELA=0.05*A
 31   A=ABS(A+DELA)
C
C     SOLVE FOR INERTIAL VELOCITY VECTORS XDOT,YDOT,ZDOT.
C
 32   DIFF=EPSLN-DELTA
 33   FLC=1.0-(A/RLC(1))*(1.0-COS(DIFF))
 34   GLC=TAU-SQRT(A**3/XMU)*(DIFF-SIN(DIFF))
      XLCV(1)=(XLC(2)-FLC*XLC(1))/GLC
      YLCV(1)=(YLC(2)-FLC*YLC(1))/GLC
      ZLCV(1)=(ZLC(2)-FLC*ZLC(1))/GLC
      CT2=ITIME
      PRINT 100,CT2
      PRINT 103, XLCV(1),YLCV(1),ZLCV(1)
103   FORMAT(1H0,*XLCV(1)=*F16.8,//,*YLCV(1)=*F16.8,//,*ZLCV(1)=*F16.8)
C
C     SOLUTION FOR CLASSICAL ELEMENTS
C
      ITIME=0
      RLC(1)=SQRT(XLC(1)**2+YLC(1)**2+ZLC(1)**2)
      RRDOT=XLC(1)*XLCV(1)+YLC(1)*YLCV(1)+ZLC(1)*ZLCV(1)
      RLCV(1)=RRDOT/RLC(1)
      V=SQRT(XLCV(1)**2+YLCV(1)**2+ZLCV(1)**2)
      ALC=(RLC(1)*XMU)/(2.0*XMU-V**2*RLC(1))
      CSUBE=(1.0-RLC(1)/ALC)
      SSUBE=(RLCV(1)*RLC(1))/SQRT(XMU*ALC)
      ELC=SQRT(SSUBE**2+CSUBE**2)
      COSE=(ALC-RLC(1))/(ALC*ELC)
      XSUBW=ALC*(COSE-ELC)
      COSV=XSUBW/RLC(1)
      SINV=SQRT(RLC(1)**2-XSUBW**2)/RLC(1)
      SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
      E=ATAN(SINE,COSE)
      TF=T(1)-((E-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(1)*ZLCV(1)-ZLC(1)*YLCV(1)
      HY=-(XLC(1)*ZLCV(1)-ZLC(1)*XLCV(1))
      HZ=XLC(1)*YLCV(1)-YLC(1)*XLCV(1)
      VANGE=ATAN(SINV,COSV)
```

```
      SINHX=HX
      COSHY=-HY
      OMEGA=ATAN(SINHX,COSHY)
      EXP=SQRT(HX**2+HY**2)
      OINCL=ATAN(EXP,HZ)
      UNUM=-XLC(1)*SIN(OMEGA)*COS(OINCL)+YLC(1)*COS(OMEGA)*COS(OINCL)+
     CZLC(1)*SIN(OINCL)
      DEM=XLC(1)*COS(OMEGA)+YLC(1)*SIN(OMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANGE
      CT3=ITIME
      PRINT 100,CT3
100   FORMAT($MILLISEC=$I8)
      PRINT 107, ALC,ELC,TE,OMEGA,OINCL,W
107   FORMAT(1H0,$ALC=$F16.8,//,$ELC=$F16.8,//,$TE=$F16.8,//,
     1$OMEGA=$F16.8,//,$OINCL=$F16.8,//,$W=$F16.8,//)
 40   CONTINUE
      GO TO  41
S2050 PZE
S     MIN        ITIME
S     BRU        *2050S
 41   END
```

APPENDIX C
F AND G SERIES PODM, POSITION AND TIME

Given $\underline{r}_1$ (x_1, y_1, z_1), $\underline{r}_2$ (x_2, y_2, z_2) and their corresponding universal times, t_1 and t_2, proceed as follows:

$$r_1 = +\sqrt{\underline{r}_1 \cdot \underline{r}_1} \tag{26}$$

$$r_2 = +\sqrt{\underline{r}_2 \cdot \underline{r}_2} \tag{27}$$

$$\underline{U}_1 = \frac{\underline{r}_1}{r_1} \tag{28}$$

$$\underline{U}_2 = \frac{\underline{r}_2}{r_2} \tag{29}$$

$$\cos(\nu_2 - \nu_1) = \underline{U}_1 \cdot \underline{U}_2 \tag{30}$$

$$\sin(\nu_2 - \nu_1) = \frac{x_1 y_2 - x_2 y_1}{|x_1 y_2 - x_2 y_1|} \sqrt{1 - \cos^2(\nu_2 - \nu_1)} \tag{31}$$

$$t_0 = \frac{t_2 + t_1}{2} \tag{32}$$

$$\tau_1 = k_e (t_1 - t_0) \tag{33}$$

$$\tau_2 = k_e (t_2 - t_0) \tag{34}$$

$$r_0 = \frac{r_2 + r_1}{2} \tag{35}$$

$$A = 1 - \frac{\mu \tau_1^2}{2r_0^3} \tag{36}$$

$$B = 1 - \frac{\mu \tau_2^2}{2r_0^3} \tag{37}$$

$$\Delta = A \tau_2 - B \tau_1 \tag{38}$$

$$\underline{r}_0 = \left(\frac{\tau_2}{\Delta} \right) \underline{r}_1 - \left(\frac{\tau_1}{\Delta} \right) \underline{r}_2 \tag{39}$$

$$\dot{\underline{r}}_0 = \left(\frac{A}{\Delta} \right) \underline{r}_2 - \left(\frac{B}{\Delta} \right) \underline{r}_1 \tag{40}$$

$$r_0 = \sqrt{\underline{r}_0 \cdot \underline{r}_0} \tag{41}$$

$$V_0 = \sqrt{\dot{\underline{r}}_0 \cdot \dot{\underline{r}}_0} \tag{42}$$

$$\dot{r}_0 = \frac{\underline{r}_0 \cdot \dot{\underline{r}}_0}{r_0} \tag{43}$$

$$\frac{1}{a} = \frac{2}{r_0} - \frac{V_0^2}{\mu} \tag{44}$$

$$U_0 = \frac{\mu}{r_0^3} \tag{45}$$

$$P_0 = \frac{r_0 \dot{r}_0}{r_0^2} \tag{46}$$

$$q_0 = \frac{V_0^2 - r_0^2 U_0}{r_0^2} \tag{47}$$

Utilize the f and g functions:

$$f_1 = f\ (V_0,\ r_0,\ \dot{r}_0,\ \tau_1) \tag{48}$$

$$f_2 = f\ (V_0,\ r_0,\ \dot{r}_0,\ \tau_2) \tag{49}$$

$$g_1 = g\ (V_0,\ r_0,\ \dot{r}_0,\ \tau_1) \tag{50}$$

$$g_2 = g\ (V_0,\ r_0,\ \dot{r}_0,\ \tau_2) \tag{51}$$

and form

$$D = f_1 g_2 - f_2 g_1 \tag{52}$$

$$C_1 = \frac{g_2}{D} \tag{53}$$

$$C_2 = \frac{-g_1}{D} \tag{54}$$

$$\dot{C}_1 = \frac{-f_2}{D} \tag{55}$$

$$\dot{C}_2 = \frac{f_1}{D} \tag{56}$$

Hence, a better approximation to $\underline{r}_0$, $\dot{\underline{r}}_0$ is given by

$$\underline{r}_0 = C_1\, \underline{r}_1 + C_2\, \underline{r}_2 \tag{57}$$

$$\dot{\underline{r}}_0 = \dot{C}_1\, \underline{r}_1 + \dot{C}_2\, \underline{r}_2 \tag{58}$$

Return to equation (41) and repeat the equational loop to equation (58); continue until r_0, $\dot{r}_0$, V_0 from equations (41), (42),and (43) do not vary, that is,

$$|(r_0)_{n+1} - (r_0)_n| < \varepsilon_1 \tag{59}$$

$$|(\dot{r}_0)_{n+1} - (\dot{r}_0)_n| < \varepsilon_2 \tag{60}$$

$$|(V_0)_{n+1} - (V_0)_n| < \varepsilon_3,\ n = 1, 2, \ldots, q \tag{61}$$

Where ε_1, ε_2, and ε_3 are tolerances, i.e., 10^{-10}. Having r , $\dot{r}$, and V , utilize the derivatives of the f and g functions, that is,

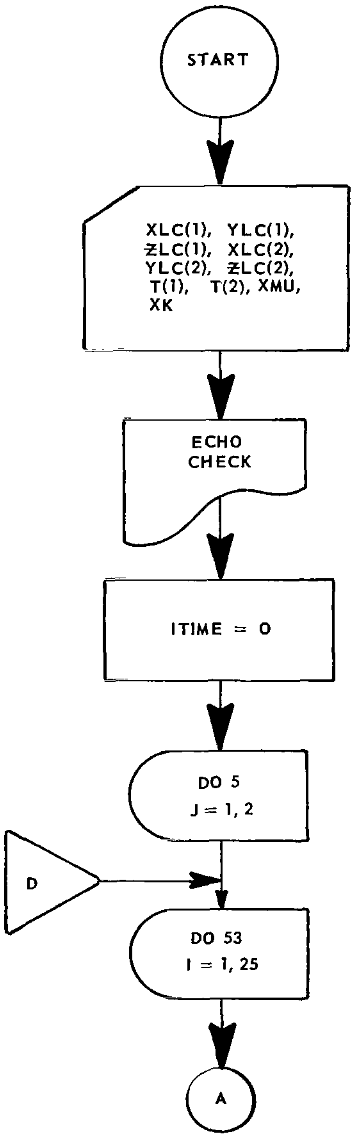
$$\dot{f}_1 = \dot{f}\ (V_0,\ r_0,\ \dot{r}_0,\ \tau_1) \tag{62}$$

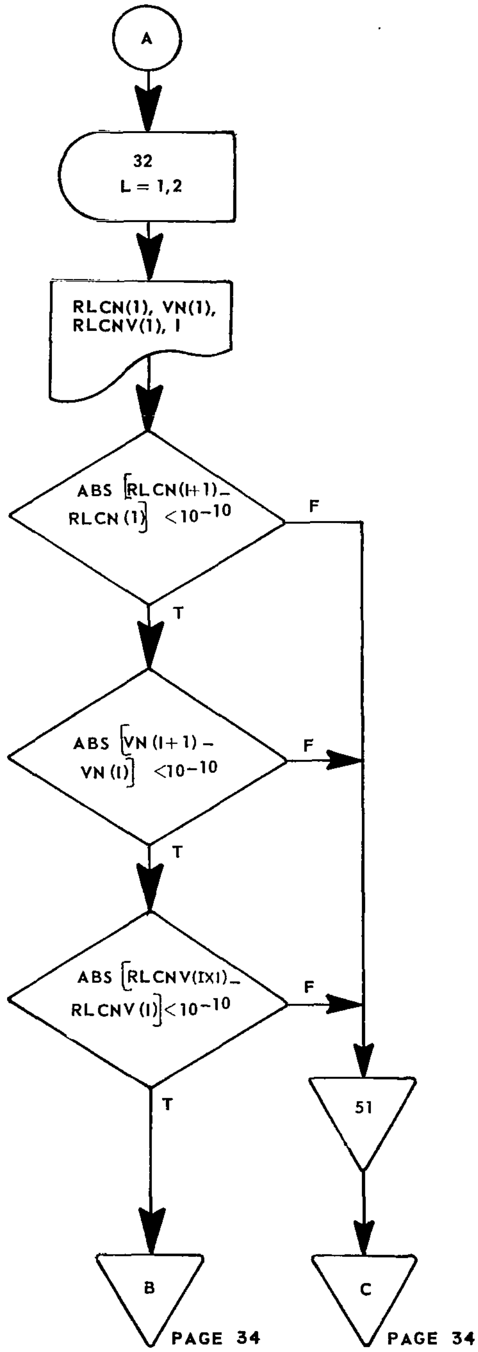
$$\dot{g}_1 = \dot{g}\ (V_0,\ r_0,\ \dot{r}_0,\ \tau_1) \tag{63}$$

to obtain

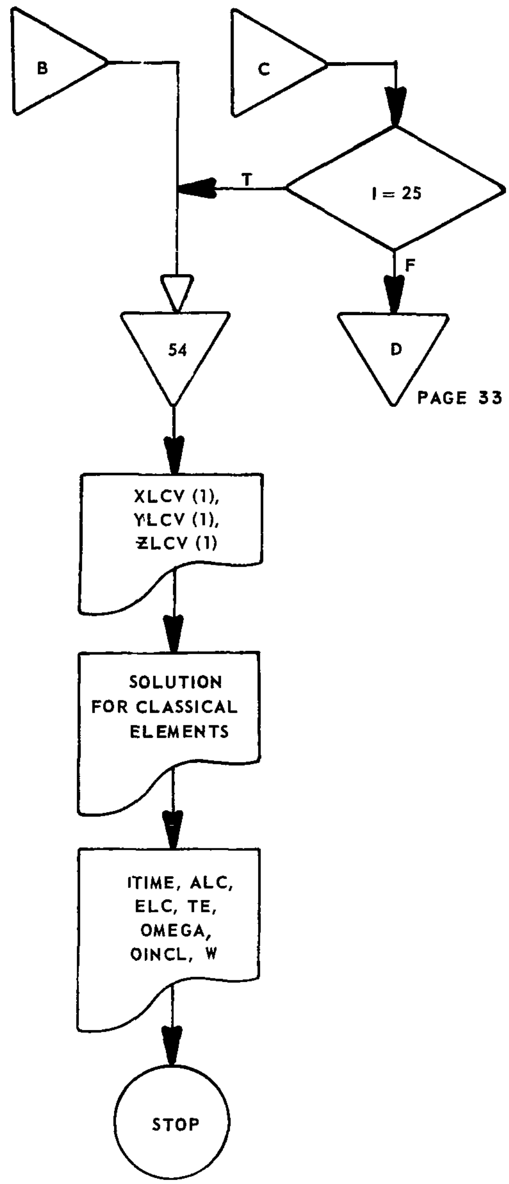
$$\dot{\underline{r}}_1 = \dot{f}_1\underline{r}_0 + \dot{g}_1\dot{\underline{r}}_0 \tag{64}$$

Continue by calculating for classical elements


START
XLC(1), YLC(1),
ZLC(1), XLC(2),
YLC(2), ZLC(2),
T(1), T(2), XMU,
XK
ECHO
CHECK
ITIME = 0
DO 5
J = 1, 2
D
DO 53
I = 1, 25
A


A
32
L = 1,2
RLCN(1), VN(1),
RLCNV(1), I
ABS [RLCN(I+1) - RLCN(1)] < 10-10
F
T
ABS [VN (I+1) - VN (I)] < 10-10
F
T
ABS [RLCNV(IXI) - RLCNV (I)] < 10-10
F
T
51
B
PAGE 34
C
PAGE 34

F AND G SERIES FLOWCHART (CONT'D)



```
C       F AND G SERIES PRELIMINARY ORBIT DETERMINATION METHOD
C       POSITION AND TIME (ESCOBAL,PAGE 221)
C
        DIMENSION RLC(3),UX(2),UY(2),UZ(2),XLC(2),YLC(2),T(3),
       CTAU(2),XLCV(1),YLCV(1),ZLCV(1),RLCN(25),C(2),CV(2),G(2),F(2),
       CVN(25),RLCNV(25),XLCN(25),YLCN(25),ZLCN(25),XLCNV(25),
       CYLCNV(25),ZLCNV(25),FV(1),GV(1),RLCV(1),ZLC(2)
        DO 90 K=1,6
C
C       READ TWO INERTIAL POSITION VECTORS AND THEIR CORRESPONDING TIMES
C
        READ 101, XLC(1), YLC(1), ZLC(1), T(1), XLC(2)
        READ 101, YLC(2), ZLC(2), T(2), XMU, XK
101     FORMAT(5F16.8)
C
C       ECHO CHECK
C
        PRINT 104, XLC(1),YLC(1),ZLC(1),T(1),XLC(2),YLC(2),ZLC(2),T(2),
       CXMU,XK
104     FORMAT(1H0,$XLC(1)=$F16.8,//,$YLC(1)=$F16.8,//,$ZLC(1)=$F16.8,//,
       1$T(1)=$F16.8,//,$XLC(2)=$F16.8,//,$YLC(2)=$F16.8,//,$ZLC(2)=$F16.
       1//,$T(2)=$F16.8,//,$XMU=$F16.8,//,$XK=$F16.8)
C
C       BEGIN COMPUTATIONS
C
C       ALL META'SYMBOL IS ITIME SUBROUTINE
C
        ITIME=0
S       LDA        205S
S       STA        0205
S       BRU        200S
S205    BRM        2050S
S200    EOM        020020
S       POT = 00202000
S       EIR
 1      DO 5 J=1,2
        RLC(J)=SQRT(XLC(J)**2+YLC(J)**2+ZLC(J)**2)
        UX(J)=XLC(J)/RLC(J)
        UY(J)=YLC(J)/RLC(J)
 5      UZ(J)=ZLC(J)/RLC(J)
        VCOS=UX(1)*UX(2)+UY(1)*UY(2)+UZ(1)*UZ(2)
        COM=XLC(1)*YLC(2)-XLC(2)*YLC(1)
        VSIN=COM/ABS(COM)*SQRT(1.0-VCOS**2)
        T(3)=(T(2)+T(1))/2.0
        TAU(1)=XK*(T(1)-T(3))
        TAU(2)=XK*(T(2)-T(3))
        RN=(RLC(1)+RLC(2))/2.0
        A=1.0-XMU*TAU(1)**2/(2.0*RN**3)
        B=1.0-XMU*TAU(2)**2/(2.0*RN**3)
        DELTA=A*TAU(2)-B*TAU(1)
        XLCN(1)=(TAU(2)/DELTA)*XLC(1)-(TAU(1)/DELTA)*XLC(2)
        YLCN(1)=(TAU(2)/DELTA)*YLC(1)-(TAU(1)/DELTA)*YLC(2)
        ZLCN(1)=(TAU(2)/DELTA)*ZLC(1)-(TAU(1)/DELTA)*ZLC(2)
```

```
      XLCNV(1)=(A/DELTA)*XLC(2)-(B/DELTA)*XLC(1)
      YLCNV(1)=(A/DELTA)*YLC(2)-(B/DELTA)*YLC(1)
      ZLCNV(1)=(A/DELTA)*ZLC(2)-(B/DELTA)*ZLC(1)
      RLCN(1)=SQRT(XLCN(1)**2+YLCN(1)**2+ZLCN(1)**2)
      VN(1)=SQRT(XLCNV(1)**2+YLCNV(1)**2+ZLCNV(1)**2)
      RLCNV(1)=(XLCN(1)*XLCNV(1)+YLCN(1)*YLCNV(1)+ZLCN(1)*ZLC
     ?/RLCN(1)
C
C     BEGIN F AND G SERIES ITERATION
C
      DO 53 I=1,25
      AINV=2.0/RLCN(I)-VN(I)**2/XMU
      UN=XMU/RLCN(I)**3
      PN=(RLCN(I)*RLCNV(I))/RLCN(I)**2
      QN=(VN(I)**2-RLCN(I)**2*UN)/RLCN(I)**2
 30   DO 32 L=1,2
      F(L)=1.0-0.5*UN*TAU(L)**2+0.5*UN*PN*TAU(L)**3+1.0/24.0*[illegible]
     CQN-15.0*UN*PN**2+UN**2)*TAU(L)**4+1.0/8.0*(7.0*UN*PN**3-3.0[illegible]
     C*QN-UN**2*PN)*TAU(L)**5+1.0/720.0*(630.0*UN*PN**2*QN-[illegible]
     C-UN**3-45.0*UN*QN**2-945.0*UN*PN**4+210.0*UN**2*PN**2)*T[illegible]
 32   G(L)=TAU(L)-1.0/6.0*UN*TAU(L)**3+1.0/4.0*UN*PN*TAU(L)**4+[illegible]/12.
     C*(9.0*UN*QN-45.0*UN*PN**2+UN**2)*TAU(L)**5+1.0/360.0*(210.0[illegible]*
     CPN**3-90.0*UN*PN*QN-15.0*UN**2*PN)*TAU(L)**6
      D=F(1)*G(2)-F(2)*G(1)
      C(1)=G(2)/D
      C(2)=-G(1)/D
      CV(1)=-F(2)/D
      CV(2)=F(1)/D
      XLCN(I+1)=C(1)*XLC(1)+C(2)*XLC(2)
      YLCN(I+1)=C(1)*YLC(1)+C(2)*YLC(2)
      ZLCN(I+1)=C(1)*ZLC(1)+C(2)*ZLC(2)
      XLCNV(I+1)=CV(1)*XLC(1)+CV(2)*XLC(2)
      YLCNV(I+1)=CV(1)*YLC(1)+CV(2)*YLC(2)
      ZLCNV(I+1)=CV(1)*ZLC(1)+CV(2)*ZLC(2)
      RLCN(I+1)=SQRT(XLCN(I+1)**2+YLCN(I+1)**2+ZLCN(I+1)**2)
      VN(I+1)=SQRT(XLCNV(I+1)**2+YLCNV(I+1)**2+ZLCNV(I+1)**[illegible])
      RLCNV(I+1)=(XLCN(I+1)*XLCNV(I+1)+YLCN(I+1)*YLCNV(I+1)+
     CZLCN(I+1)*ZLCNV(I+1))/RLCN(I+1)
      CT1=ITIME
      PRINT 100,CT1
      PRINT 102,RLCN(I),I,VN(I),I,RLCNV(I),I
 102  FORMAT(1H0,$RLCN(I)=$E16.8$*****I=$I2,//,$VN(I)=$E16.8$*****I=$I2,
     1//,$RLCNV(I)=$E16.8$*****I=$I2)
      ITIME=0
 47   IF(ABS(RLCN(I+1)-RLCN(I))-0.0000000001) 48,53,53
 48   IF(ABS(VN(I+1)-VN(I))-0.0000000001) 49,53,53
 49   IF(ABS(RLCNV(I+1)-RLCNV(I))-0.0000000001) 50,53,53
 53   CONTINUE
C
C     SOLVE FOR INERTIAL VELOCITY VECTORS XDOT,YDOT,ZDOT.
C
 50   XLCF=XLCN(I+1)
      YLCF=YLCN(I+1)
      ZLCF=ZLCN(I+1)
```

```
      XLCVF=XLCNV(I+1)
      YLCVF=YLCNV(I+1)
      ZLCVF=ZLCNV(I+1)
 54   FV(1)=-UN*TAU(1)+3.0/2.0*UN*PN*TAU(1)**2+1.0/6.0*(3.0*[illegible]
     CUN*PN**2+UN**2)*TAU(1)**3+5.0/8.0*(7.0*UN*PN**3-3.0*[illegible]
     C*PN)*TAU(1)**4+6.0/720.0*(630.0*UN*PN**2*QN-24.0*[illegible]**3-
     C45.0*UN*QN**2-945.0*UN*PN**4+210.0*UN**2*PN**2)*TAU(1)**[illegible]
      GV(1)=1.0-0.5*UN*TAU(1)**2+UN*PN*TAU(1)**3+1.0/24.0*([illegible]-
     C45.0*UN*PN**2+UN**2)*TAU(1)**4+1.0/60.0*(210.0*UN*PN**3-[illegible]
     CPN*QN-15.0*UN**2*PN)*TAU(1)**5
      XLCV(1)=FV(1)*XLCF+GV(1)*XLCVF
      YLCV(1)=FV(1)*YLCF+GV(1)*YLCVF
      ZLCV(1)=FV(1)*ZLCF+GV(1)*ZLCVF
      CT2=ITIME
      PRINT 100,CT2
      PRINT 103, XLCV(1),YLCV(1),ZLCV(1)
103   FØRMAT(1H0,$YLCV(1)=$E16.8,///,$YLCV(1)=$E16.8,///,$ZLCV(1)=$E16.8)
C
C     SØLUTIØN FØR CLASSICAL ELEMENTS
C
      ITIME=0
      RLC(1)=SQRT(XLC(1)**2+YLC(1)**2+ZLC(1)**2)
      RFDØT=XLC(1)*XLCV(1)+YLC(1)*YLCV(1)+ZLC(1)*ZLCV(1)
      RLCV(1)=RRDØT/RLC(1)
      V=SQRT(XLCV(1)**2+YLCV(1)**2+ZLCV(1)**2)
      ALC=(RLC(1)*XMU)/(2.0*XMU-V**2*RLC(1))
      CSUBE=(1.0-RLC(1)/ALC)
      SSUBE=(RLCV(1)*RLC(1))/SQRT(XMU*ALC)
      ELC=SQRT(SSUBE**2+CSUBE**2)
      CØSE=(ALC-RLC(1))/(ALC*ELC)
      XSUBV=ALC*(CØSE-ELC)
      CØSV=XSUBV/RLC(1)
      SINV=SQRT(RLC(1)**2-XSUBV**2)/RLC(1)
      SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
      E=ATAN(SINE,CØSE)
      TE=T(1)-((E-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(1)*ZLCV(1)-ZLC(1)*YLCV(1)
      HY=-(XLC(1)*ZLCV(1)-ZLC(1)*XLCV(1))
      HZ=XLC(1)*YLCV(1)-YLC(1)*XLCV(1)
      VANGE=ATAN(SINV,CØSV)
      SINHX=HX
      CØSHY=-HY
      ØMEGA=ATAN(SINHX,CØSHY)
      EXP=SQRT(HX**2+HY**2)
      ØINCL=ATAN(EXP,HZ)
      UNUM=-XLC(1)*SIN(ØMEGA)*CØS(ØINCL)+YLC(1)*CØS(ØMEGA)*CØS(ØINCL)+
     CZLC(1)*SIN(ØINCL)
      DEM=XLC(1)*CØS(ØMEGA)+YLC(1)*SIN(ØMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANGE
      CT3=ITIME
      PRINT 100,CT3
100   FØRMAT(*MILLISEC=*I3)
      PRINT 107, ALC,ELC,TE,ØMEGA,ØINCL,W
```

```
107    FØRMAT(1H0,$ALC=$F16.8,//,$CLC=$F16.8,//,$TF=$E16.8,//,
      1$ØMEGA=$E16.8,//,$ØINCL=$F16.8,//,$W=$E16.8,//)
 90    CØNTINUE
       GØ TØ 61
S2050  PZE
S      MIN        ITIME
S      BRU        *2050S
 61       END
```

APPENDIX D
ITERATION OF SEMIPARAMETER PODM, POSITION AND TIME

Given $\underline{r}_1$ (x_1, y_1, z_1), $\underline{r}_2$ (x_2, y_2, z_2) and their corresponding universal times, t_1 and t_2, proceed as follows:

$$\tau = k_e (t_2 - t_1) \tag{65}$$

$$r_1 = +\sqrt{\underline{r}_1 \cdot \underline{r}_1} \tag{66}$$

$$r_2 = +\sqrt{\underline{r}_2 \cdot \underline{r}_2} \tag{67}$$

$$\underline{U}_1 = \frac{\underline{r}_1}{r_1} \tag{68}$$

$$\underline{U}_2 = \frac{\underline{r}_2}{r_2} \tag{69}$$

$$\cos (\nu_2 - \nu_1) = \underline{U}_1 \cdot \underline{U}_2 \tag{70}$$

$$\sin (\nu_2 - \nu_1) = \frac{x_1 y_2 - x_2 y_1}{|x_1 y_2 - x_2 y_1|} \sqrt{1 - \cos^2 (\nu_2 - \nu_1)} \tag{71}$$

As a first estimate, let

$$p_g = 0.4 (r_1 + r_2) \tag{72}$$

and

$$p = p_g \tag{73}$$

and continue calculating with

$$e \cos \nu_1 = \frac{p}{r_1} - 1 \tag{74}$$

$$e \cos \nu_2 = \frac{p}{r_2} - 1 \tag{75}$$

$$e \sin \nu_1 = \frac{\cos (\nu_2 - \nu_1)(e \cos \nu_1) - (e \cos \nu_2)}{\sin (\nu_2 - \nu_1)} \tag{76}$$

$$e \sin \nu_2 = \frac{- \cos (\nu_2 - \nu_1)(e \cos \nu_2) - (e \cos \nu_1)}{\sin (\nu_2 - \nu_1)} \tag{77}$$

$$e = \sqrt{(e \cos \nu_1)^2 + (e \sin \nu_1)^2} \tag{78}$$

$$a = \frac{p}{1 - e^2} \tag{79}$$

$$n = k_e \sqrt{\frac{\mu}{a^3}} \tag{80}$$

If $e \neq o$, proceed with equation (81); if $e = o$ within a given tolerance, continue with equation (83).

$$\cos E_i = \frac{r_i}{p} (\cos \nu_i + e) \ , \quad i = 1, 2 \tag{81}$$

$$\sin E_i = \frac{r_i}{p} \sqrt{1 - e^2} \sin \nu_i \ , \quad i = 1, 2 \tag{82}$$

Continue calculating with equation (88).

$$e = 0 \ , \ \nu_1 = 0 \tag{83}$$

$$\cos E_1 = 1 \tag{84}$$

$$\cos E_2 = \cos (\nu_2 - \nu_1) \tag{85}$$

$$\sin E_1 = 0 \tag{86}$$

$$\sin E_2 = \sin (\nu_2 - \nu_1) \tag{87}$$

$$M_i = E_i - e \sin E_i \ , \quad i = 1, 2 \tag{88}$$

$$F = \tau - \left(\frac{M_2 - M_1}{n} \right) k_e \tag{89}$$

If F = 0, proceed to equation (92); if not, increment p by 5 percent and, by repeating equational loop (74) through (89), obtain

$$F'(p) \simeq \frac{F\ (p + \Delta p) - F\ (p)}{\Delta p} \tag{90}$$

Hence, a better approximation to the semiparameter is

$$p_{j+1} = p_j - \frac{F\ (p_j)}{F'(p_j)} \quad , \qquad j = 1, 2, \ldots, q \tag{91}$$

Repeat the above loop q times until p is constant within a given tolerance, i.e., 10^{-10}. Finally, continue calculating with equation (92).

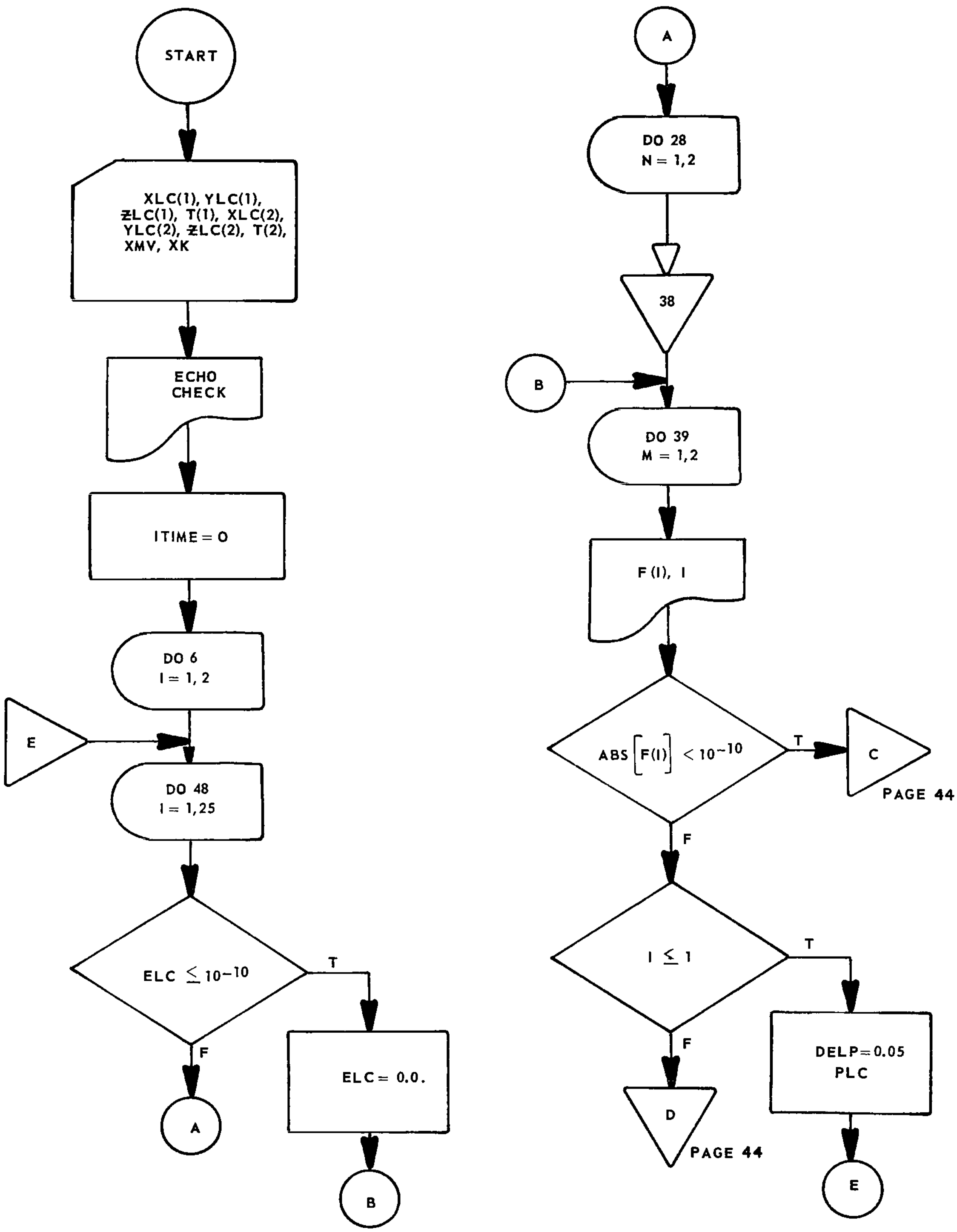
$$f = 1 - \frac{a}{r_1} \left[1 - \cos\ (E_2 - E_1)\right] \tag{92}$$

$$g = \tau - \sqrt{\frac{a^3}{\mu}} \left[E_2 - E_1 - \sin\ (E_2 - E_1)\right] \tag{93}$$

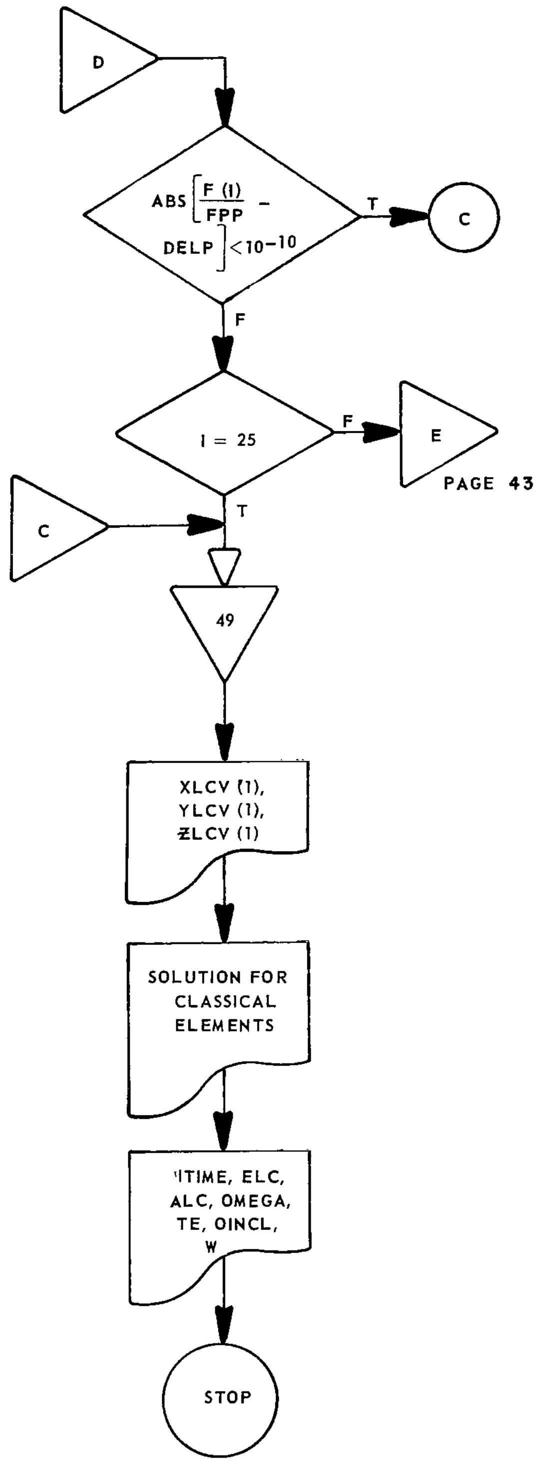
$$\dot{\underline{r}}_1 = \frac{\underline{r}_2 - f\ \underline{r}_1}{g} \tag{94}$$

Continue by calculating for classical elements.

ITERATION OF SEMIPARAMETER



ITERATION OF SEMIPARAMETER (CONT'D)



```
C       ITERATION OF SEMIPARAMETER PRELIMINARY OR IT GETS MI...
C       POSITION AND TIME (ESCOBAL,PAGE 211)
C
        DIMENSION F(25),RLC(2),UX(2),UY(2),UZ(2),FCOSV(2),ES...
       CCOSV(2),SINV(2),COSE(2),SINE(2),ANGE(2),XMEAN(2),XLCV(1),YLCV(1),
       CZLCV(1),RLCV(1),VLC(1),XLC(2),YLC(2),ZLC(2),T(2)
        DO 30 K=1,6
C
C       READ TWO INERTIAL POSITION VECTORS AND THEIR CORRESPONDING TIME
C
        READ 101, XLC(1), YLC(1), ZLC(1), T(1), XLC(2)
        READ 101, YLC(2), ZLC(2), T(2), XMU, XK
101     FORMAT(5F16.8)
C
C       ECHO CHECK
C
        PRINT 104, XLC(1),YLC(1),ZLC(1),T(1),XLC(2),YLC(2),ZLC(2),T(2),
       CXMU,XK
104     FORMAT(1H0,$XLC(1)=$F16.8,///,$YLC(1)=$F16.8,///,$ZLC(1)=$F16.8,///,
       1$T(1)=$F16.8,///,$XLC(2)=$F16.8,///,$YLC(2)=$F16.8,///,$ZLC(2)=$F16.8,
       1///,$T(2)=$F16.8,///,$XMU=$F16.8,///,$XK=$F16.8)
C
C       BEGIN COMPUTATIONS
C
C       ALL ...TA..SYMBOL TO ITIME SUBROUTINE
C
        ITIME=0
S       LDA         2058
S       STA         0206
S       BRU         2008
S205    BRU         2050S
S200    FSM         020020
S       PRT = 00000000
S       END
        TAU=XK*(T(2)-T(1))
        DO 6 J=1,2
        RLC(J)=SQRT(XLC(J)**2+YLC(J)**2+ZLC(J)**2)
        UX(J)=XLC(J)/RLC(J)
        UY(J)=YLC(J)/RLC(J)
  6     UZ(J)=ZLC(J)/RLC(J)
        VCOS=UX(1)*UX(2)+UY(1)*UY(2)+UZ(1)*UZ(2)
        COM=XLC(1)*YLC(2)-XLC(2)*YLC(1)
        VSIN=COM/ABS(COM)*SQRT(1.0-VCOS**2)
        PC=0.5*(RLC(1)+RLC(2))
        PLC=PC
C
C       BEGIN ITERATION OF SEMIPARAMETER
C
 11     DO 48 I=1,25
        ECOSV(1)=PLC/RLC(1)-1.0
        ECOSV(2)=PLC/RLC(2)-1.0
        ESINV(1)=(VCOS*ECOSV(1)-ECOSV(2))/VSIN
        ESINV(2)=(-VCOS*ECOSV(2)+ECOSV(1))/VSIN
```

```
      ELC=SQRT(ABS(ECOSV(1)**2+ESINV(1)**2))
      ALC=PLC/(1.0-ELC**2)
      ETA=XK*SQRT(ABS(XMU/ALC**3))
      COSV(1)=PLC/(RLC(1)*ELC)-1.0/ELC
      COSV(2)=PLC/(RLC(2)*ELC)-1.0/ELC
      SINV(1)=(VCOS*ECOSV(1)-ECOSV(2))/(VSIN*ELC)
      SINV(2)=(-VCOS*ECOSV(2)+ECOSV(1))/(VSIN*ELC)
 24   IF(ELC-0.0000000001) 30,30,25
 25   DO 28 N=1,2
      COSE(N)=RLC(N)/PLC*(COSV(N)+ELC)
      SINE(N)=RLC(N)/PLC*SQRT(1.0-ELC**2)*SINV(N)
 28   ANGE(N)=ATAN(SINE(N),COSE(N))
 29   GO TO 38
 30   ELC=0.0
 31   VLC(1)=0.0
      COSE(1)=1.0
      COSE(2)=VCOS
      SINE(1)=0.0
      SINE(2)=VSIN
      ANGE(1)=0.0
      ANGE(2)=ATAN(SINE(2),COSE(2))
 38   DO 39 M=1,2
 39   XMEAN(M)=ANGE(M)-ELC*SINE(M)
 40   F(I)=TAU-((XMEAN(2)-XMEAN(1))/ETA)*XK
      CT1=ITIME
      PRINT 100,CT1
      PRINT 102, F(I),I
102   FORMAT(1H0,$F(I)=$E16.8$*****I=$I2)
      ITIME=0
 41   IF(ABS(F(I))-0.0000000001) 49,42,42
 42   IF(I-1) 47,47,43
 43   FPP=(F(I)-F(I-1))/DELP
 44   IF(ABS(F(I)/FPP-DELP)-0.0000000001) 49,45,45
 45   DELP=-F(I)/FPP
 46   GO TO 48
 47   DELP=0.05*PLC
 48   PLC=ABS(PLC+DELP)
C
C     SOLVE FOR INERTIAL VELOCITY VECTORS XDOT,YDOT,ZDOT.
C
 49   FLC=1.0-(ALC/RLC(1))*(1.0-COS(ANGE(2)-ANGE(1)))
 50   GLC=TAU-SQRT(ALC**3/XMU)*(ANGE(2)-ANGE(1)-SIN(ANGE(2)-ANGE(1)))
      XLCV(1)=(XLC(2)-FLC*XLC(1))/GLC
      YLCV(1)=(YLC(2)-FLC*YLC(1))/GLC
      ZLCV(1)=(ZLC(2)-FLC*ZLC(1))/GLC
      CT2=ITIME
      PRINT 100,CT2
      PRINT 103, XLCV(1),YLCV(1),ZLCV(1)
103   FORMAT(1H0,$XLCV(1)=$E16.8,//,$YLCV(1)=$E16.8,//, ZL[illegible])
C
C     SOLUTION FOR CLASSICAL ELEMENTS
C
      ITIME=0
      RLC(1)=SQRT(XLC(1)**2+YLC(1)**2+ZLC(1)**2)
```

```
      RRDØT=XLC(1)*XLCV(1)+YLC(1)*YLCV(1)+ZLC(1)*ZLCV(1)
      RLCV(1)=RRDØT/RLC(1)
      V=SQRT(XLCV(1)**2+YLCV(1)**2+ZLCV(1)**2)
      ALC=(RLC(1)*XMU)/(2.0*XMU-V**2*RLC(1))
      CSUBE=(1.0-RLC(1)/ALC)
      SSUBE=(RLCV(1)*RLC(1))/SQRT(XMU*ALC)
      ELC=SQRT(SSUBE**2+CSUBE**2)
      CØSEC=(ALC-RLC(1))/(ALC*ELC)
      XSUBW=ALC*(CØSEC-ELC)
      CØSVC=XSUBW/RLC(1)
      SINVC=SQRT(RLC(1)**2-XSUBW**2)/RLC(1)
      SINEC=SQRT(1.0-ELC**2)*SINVC/(1.0+ELC*SINVC)
      E=ATAN(SINEC,CØSEC)
      TE=T(1)-((E-ELC*SINEC)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(1)*ZLCV(1)-ZLC(1)*YLCV(1)
      HY=-(XLC(1)*ZLCV(1)-ZLC(1)*XLCV(1))
      HZ=XLC(1)*YLCV(1)-YLC(1)*XLCV(1)
      VANGE=ATAN(SINVC,CØSVC)
      SINHX=HX
      CØSHY=-HY
      ØMEGA=ATAN(SINHX,CØSHY)
      EXP=SQRT(HX**2+HY**2)
      ØINCL=ATAN(EXP,HZ)
      UNUM=-XLC(1)*SIN(ØMEGA)*CØS(ØINCL)+YLC(1)*CØS(ØMEGA)*CØS(ØINCL)+
     CZLC(1)*SIN(ØINCL)
      DEM=XLC(1)*CØS(ØMEGA)+YLC(1)*SIN(ØMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANGE
      CT3=ITIME
      PRINT 100,CT3
100   FØRMAT(*MILLISEC=*I8)
      PRINT 107, ALC,ELC,TE,ØMEGA,ØINCL,W
107   FØRMAT(1H0,*ALC=*F16.8,//,*ELC=*F16.8,//,*TE=*F16.8,//,
     1*ØMEGA=*F16.8,//,*ØINCL=*F16.8,//,*W=*F16.8,//)
 90   CØNTINUE
      GØ TØ 51
S2050 PZE
S     MIN         ITIME
S     BRU         *2050S
 51      END
```

APPENDIX E
GAUSSIAN PODM, POSITION AND TIME

Given $\underline{r}_1$ (x_1, y_1, z_1), $\underline{r}_2$ (x_2, y_2, z_2) and their corresponding universal times, t_1 and t_2, proceed as follows:

$$\tau = k_e\,(t_2 - t_1) \tag{95}$$

$$r_1 = +\sqrt{\underline{r}_1 \cdot \underline{r}_1} \tag{96}$$

$$r_2 = +\sqrt{\underline{r}_2 \cdot \underline{r}_2} \tag{97}$$

$$\cos\,(\nu_2 - \nu_1) = \frac{\underline{r}_1 \cdot \underline{r}_2}{r_1 r_2} \tag{98}$$

$$\sin\,(\nu_2 - \nu_1) = \frac{x_1y_2 - x_2y_1}{|x_1y_2 - x_2y_1|}\sqrt{1 - \cos^2\,(\nu_2 - \nu_1)} \tag{99}$$

Obtain the constants

$$l = \frac{r_1 + r_2}{4\sqrt{r_1 r_2}\,\cos\left(\frac{\nu_2 - \nu_1}{2}\right)} - \frac{1}{2} \tag{100}$$

$$m = \frac{\mu\,\tau^2}{\left[2\sqrt{r_1 r_2}\,\cos\left(\frac{\nu_2 - \nu_1}{2}\right)\right]^3} \tag{101}$$

As a first approximation, set

$$y = 1 \tag{102}$$

and continue calculating with

$$x = \frac{m}{y^2} - 1 \tag{103}$$

$$\cos\left(\frac{E_2 - E_1}{2}\right) = 1 - 2x \tag{104}$$

$$\sin\left(\frac{E_2 - E_1}{2}\right) = \sqrt{4x\ (1 - x)} \tag{105}$$

$$X = \frac{(E_2 - E_1) - \sin\ (E_2 - E_1)}{\sin^3\left(\frac{E_2 - E_1}{2}\right)} \tag{106}$$

$$y = 1 + X\ (1 + x) \tag{107}$$

If y is now equal to the assumed value within some tolerance, continue with equation (108); if it is not, place the value of y from equation (107) into equation (103) and repeat equational loop (103) through (107). Continue calculating with

$$a = \left[\frac{\tau\sqrt{\mu}}{2y\sqrt{r_2 r_1}\ \cos\left(\frac{\nu 2 - \nu 1}{2}\right)\ \sin\left(\frac{E2 - E1}{2}\right)}\right]^2 \tag{108}$$

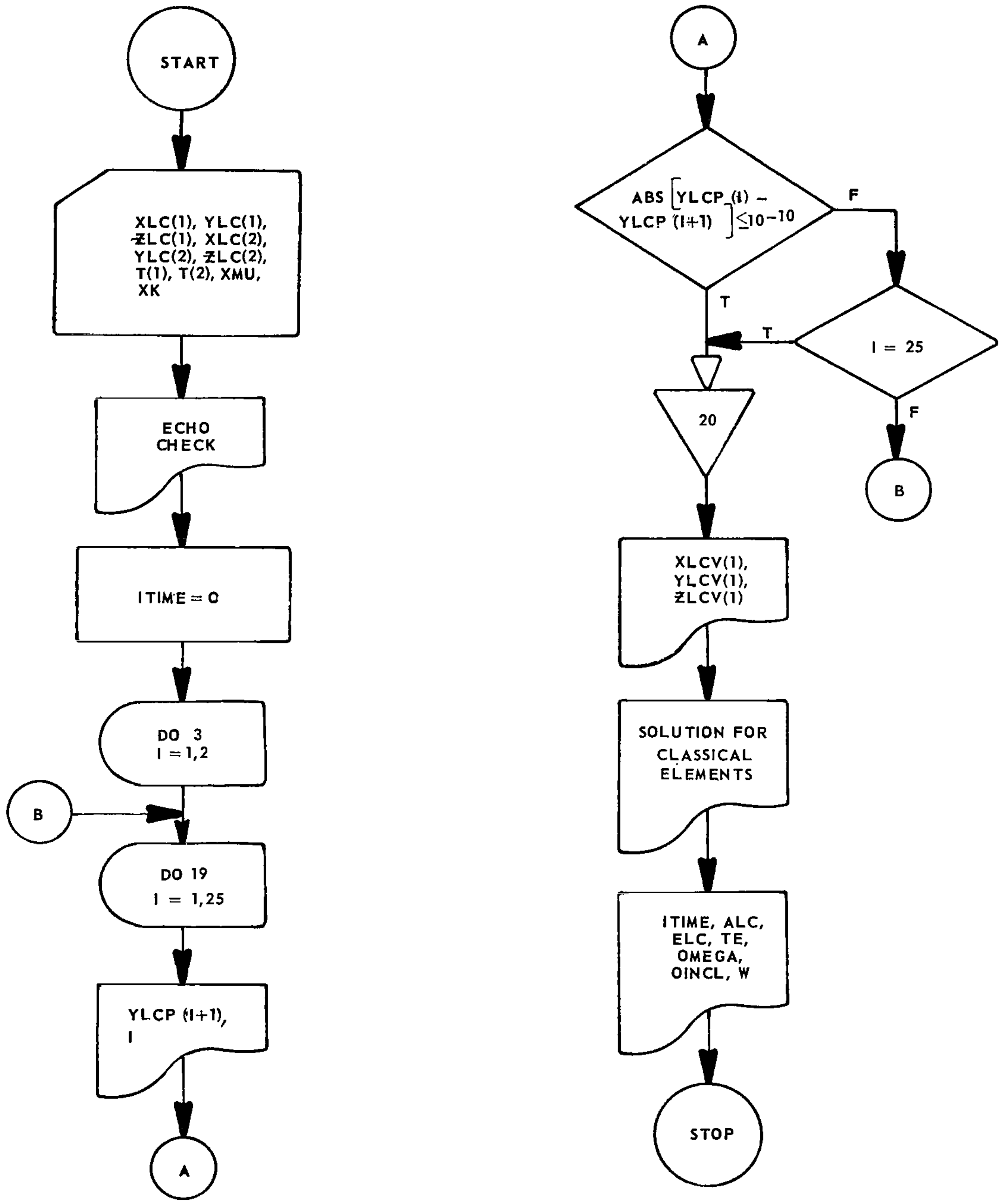
$$f = 1 - \frac{a}{r_1}\left[1 - \cos\left(E_2 - E_1\right)\right] \tag{109}$$

$$g = \tau - \sqrt{\frac{a^3}{\mu}}\left[\left(E_2 - E_1\right) - \sin\left(E_2 - E_1\right)\right] \tag{110}$$

$$\dot{\underline{r}}_1 = \frac{\underline{r}_2 - f\,\underline{r}_1}{g} \tag{111}$$

Continue to calculate for classical elements.

GAUSSIAN FLOWCHART



```
C      GAUSSIAN PRELIMINARY ORBIT DETERMINATION METHOD
C      POSITION AND TIME (ESCOBAL, PAGE 196)
C
       DIMENSION XLC(2),YLC(2),ZLC(2),RLC(2),YLCP(26),XLCV(1),
      CYLCV(1),ZLCV(1),T(2),RLCV(1)
       DO 70 N=1,6
C
C      READ TWO INERTIAL POSITION VECTORS AND THEIR CORRESPONDING TIME
C
       READ 101, XLC(1), YLC(1), ZLC(1), T(1), XLC(2)
       READ 101, YLC(2), ZLC(2), T(2), XMU, XK
101    FORMAT(5E16.8)
C
C      ECHO CHECK
C
       PRINT 104, XLC(1),YLC(1),ZLC(1),T(1),XLC(2),YLC(2),ZLC(2),T(2),
      CXMU,XK
104    FORMAT(1H0,$XLC(1)=$E16.8,//,$YLC(1)=$E16.8,//,$ZLC(1)=$E16.8,//,
      1$T(1)=$E16.8,//,$XLC(2)=$E16.8,//,$YLC(2)=$E16.8,//,$ZLC(2)=$E16.8,
      1//,$T(2)=$E16.8,//,$XMU=$E16.8,//,$XK=$E16.8)
C
C      BEGIN COMPUTATIONS
C
C      ALL METASYMBOL IS ITIME SUBROUTINE
C
       ITIME=0
S      LDA       205S
S      STA       0205
S      BRU       200S
S205   BRM       0205S
S200   FBM       020020
S      PRT  =  00000000
S      FIR
       TAU=XK*(T(2)-T(1))
  2    DO 3 I=1,2
  3    RLC(I)=SQRT(XLC(I)**2+YLC(I)**2+ZLC(I)**2)
       VCOS=(XLC(1)*XLC(2)+YLC(1)*YLC(2)+ZLC(1)*ZLC(2))/(RLC(1)*RLC(2))
       COM=XLC(1)*YLC(2)-XLC(2)*YLC(1)
       VSIN=COM/ABS(COM)*SQRT(1.0-VCOS**2)
       ANGV=ATAN(VSIN,VCOS)
       DL=(RLC(1)+RLC(2))/(4.0*SQRT(RLC(1)*RLC(2))*COS(ANGV/2.))-.5
       DM=(XMU*TAU**2)/(2.0*SQRT(RLC(1)*RLC(2))*COS(ANGV/2.))**3
       YLCP(1)=1.0
C
C      BEGIN GAUSSIAN ITERATION
C
 10    DO 19 I=1,25
       XLCP=DM/YLCP(I)**2-DL
       FCOS=1.0-2.0*XLCP
       FSIN=SQRT(4.0*XLCP*(1.0-XLCP))
       ANGE=ATAN(FSIN,FCOS)
       X=((2.0*ANGE)-SIN(2.0*ANGE))/(SIN(ANGE)**3)
       YLCP(I+1)=1.0+X*(DL+XLCP)
```

```
      CT1=ITIME
      PRINT 100,CT1
      PRINT 102, YLCP(I+1),I
102   FØRMAT(1H0,$YLCP(I+1)=$E16.8$*****I=$I2)
      ITIME=0
 18   IF(ABS(YLCP(I)-YLCP(I+1))-0.0000000001) 20,20,19
 19   CØNTINUE
C
C     SØLVE FØR INERTIAL VELØCITY VECTØRS XDØT,YDØT,ZDØT.
C
 20   A=((TAU*SQRT(XMU))/(2.0*YLCP(I+1)*SQRT(RLC(2)*RLC(1))*
     CCØS(ANGV/2.0)*SIN(ANGE)))**2
      FLC=1.0-(A/RLC(1))*(1.0-CØS(2.0*ANGE))
      GLC=TAU-SQRT(A**3/XMU)*(2.0*ANGE-SIN(2.0*ANGE))
      XLCV(1)=(XLC(2)-FLC*XLC(1))/GLC
      YLCV(1)=(YLC(2)-FLC*YLC(1))/GLC
      ZLCV(1)=(ZLC(2)-FLC*ZLC(1))/GLC
      CT2=ITIME
      PRINT 100,CT2
      PRINT 103, XLCV(1),YLCV(1),ZLCV(1)
103   FØRMAT(1H0,$XLCV(1)=$F16.8,//,$YLCV(1)=$F16.8,//,$ZLCV(1)=$F16.8)
C
C     SØLUTIØN FØR CLASSICAL ELEMENTS
C
      ITIME=0
      RLC(1)=SQRT(XLC(1)**2+YLC(1)**2+ZLC(1)**2)
      RDØT=XLC(1)*XLCV(1)+YLC(1)*YLCV(1)+ZLC(1)*ZLCV(1)
      RLCV(1)=RDØT/RLC(1)
      V=SQRT(XLCV(1)**2+YLCV(1)**2+ZLCV(1)**2)
      ALC=(RLC(1)*XMU)/(2.0*XMU-V**2*RLC(1))
      CSUBE=(1.0-RLC(1)/ALC)
      SSUBE=(RLCV(1)*RLC(1))/SQRT(XMU*ALC)
      ELC=SQRT(SSUBE**2+CSUBE**2)
      CØSE=(ALC-RLC(1))/(ALC*ELC)
      XSUBX=ALC*(CØSE-ELC)
      CØSV=XSUBX/RLC(1)
      SINV=SQRT(RLC(1)**2-XSUBX**2)/RLC(1)
      SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
      E=ATAN(SINE,CØSE)
      TE=T(1)-((E-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(1)*ZLCV(1)-ZLC(1)*YLCV(1)
      HY=-(XLC(1)*ZLCV(1)-ZLC(1)*XLCV(1))
      HZ=XLC(1)*YLCV(1)-YLC(1)*XLCV(1)
      VANGE=ATAN(SINV,CØSV)
      SINHX=HX
      CØSHY=-HY
      ØMEGA=ATAN(SINHX,CØSHY)
      EXP=SQRT(HX**2+HY**2)
      ØINCL=ATAN(EXP,HZ)
      UNUM=-XLC(1)*SIN(ØMEGA)*CØS(ØINCL)+YLC(1)*CØS(ØMEGA)*CØS(ØINCL)+
     CZLC(1)*SIN(ØINCL)
      DEM=XLC(1)*CØS(ØMEGA)+YLC(1)*SIN(ØMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANGE
```

```
      CT3=ITIME
      PRINT 100,CT3
100   FORMAT(*MILLISEC=*I8)
      PRINT 107, ALC,ELC,TE,OMEGA,OINCL,W
107   FORMAT(1H0,$ALC=*E16.8,//,*ELC=*E16.8,//,*TE=*E16.8,//,
     1$OMEGA=*E16.8,//,*OINCL=*E16.8,//,*W=*E16.8,//)
 70   CONTINUE
      GO TO 41
S2050 PZE
S     MIN       ITIME
S     BRU       *2050S
 41      END
```

APPENDIX F
ITERATION OF TRUE ANOMALY PODM, POSITION AND TIME

Given $\underline{r}_1$ (x_1, y_1, z_1), $\underline{r}_2$ (x_2, y_2, z_2) and their corresponding universal times, t_1 and t_2, proceed as follows:

$$\tau = k_e (t_2 - t_1) \tag{112}$$

$$r_1 = +\sqrt{\underline{r}_1 \cdot \underline{r}_1} \tag{113}$$

$$r_2 = +\sqrt{\underline{r}_2 \cdot \underline{r}_2} \tag{114}$$

$$\underline{U}_1 = \frac{\underline{r}_1}{r_1} \tag{115}$$

$$\underline{U}_2 = \frac{\underline{r}_2}{r_2} \tag{116}$$

$$\cos (\nu_2 - \nu_1) = \underline{U}_1 \cdot \underline{U}_2 \tag{117}$$

$$\sin (\nu_2 - \nu_1) = \frac{x_1 y_2 - x_2 y_1}{|x_1 y_2 - x_2 y_1|} \sqrt{1 - \cos^2 (\nu_2 - \nu_1)} \tag{118}$$

As a first approximation, set

$$\nu_1 = 0° \tag{119}$$

$$\nu_2 = \nu_1 + (\nu_2 - \nu_1) \quad (120)$$

$$e = \frac{(r_2 - r_1)}{r_1 \cos \nu_1 - r_2 \cos \nu_2} \quad (121)$$

If $e < 0$, return to equation (119) and increment ν_1 by $\Delta\nu_1$, 10 degrees; if $e > 0$, proceed with equation (122).

$$a = \frac{r_1 (1 + e \cos \nu_1)}{(1 - e^2)} \quad (122)$$

If $a < 0$, return to equation (119) and increment ν_1 by $\Delta\nu_1$, again 10 degrees; if $a > 0$, proceed with equation (123).

$$\sin E_1 = \frac{\sqrt{1 - e^2} \sin \nu_1}{1 + e \cos \nu_1} \quad (123)$$

$$\cos E_1 = \frac{\cos \nu_1 + e}{1 + e \cos \nu_1} \quad (124)$$

$$\sin E_2 = \frac{\sqrt{1 - e^2} \sin \nu_2}{1 + e \cos \nu_2} \quad (125)$$

$$\cos E_2 = \frac{\cos \nu_2 + e}{1 + e \cos \nu_2} \quad (126)$$

$$M_2 - M_1 = E_2 - E_1 + e (\sin E_1 - \sin E_2) \quad (127)$$

$$n = k_e \sqrt{\frac{\mu}{a^3}} \quad (128)$$

$$F = \tau - \left(\frac{M_2 - M_1}{n} \right) k_e \tag{129}$$

If the iterative function is less than a specified tolerance ε_1, that is, 10^{-10},

$$|F| < \varepsilon_1 \tag{130}$$

proceed to equation (135); if not, save the numerical value of F and increment ν_1 by a small amount, $\Delta\nu$, to obtain

$$\nu_1 + \Delta\nu \tag{131}$$

Repeat equational loop (120) to (129) obtain $F(\nu_1 + \Delta\nu)$ and form

$$F'(\nu_1) \simeq \frac{F\ (\nu_1 + \Delta\nu) - F\ (\nu_1)}{\Delta\nu} \tag{132}$$

Improve the value of ν_1 by

$$(\nu_1)_{j+1} = (\nu_1)_j - \frac{F\left[(\nu_1)_j\right]}{F'\left[(\nu_1)_j\right]} \quad , \qquad j = 1,\ 2,\ 3, \ldots, q \tag{133}$$

If

$$|(\nu_1)_{j+1} - (\nu_1)_j| < \varepsilon_2 \tag{134}$$

where ε_2 is another specified tolerance, i.e., 10^{-10}, proceed to equation (135); if not, return to equation (120) with the improved value of ν_1.

Continue calculating with:

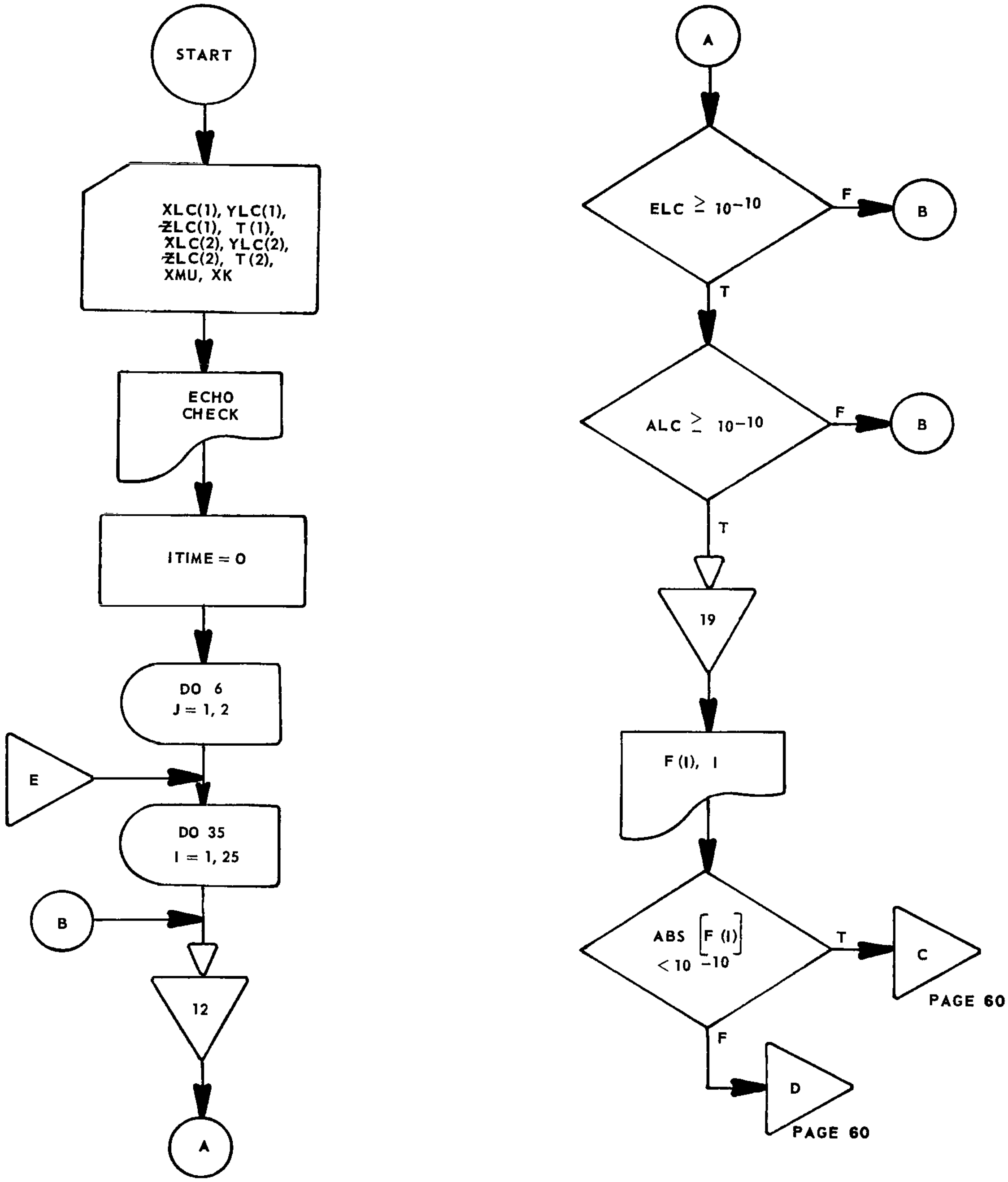
$$f = 1 - \frac{a}{r_1} \left[1 - \cos (E_2 - E_1)\right] \qquad (135)$$

$$g = \tau - \sqrt{\frac{a^3}{\mu}} \left[E_2 - E_1 - \sin (E_2 - E_1)\right] \qquad (136)$$

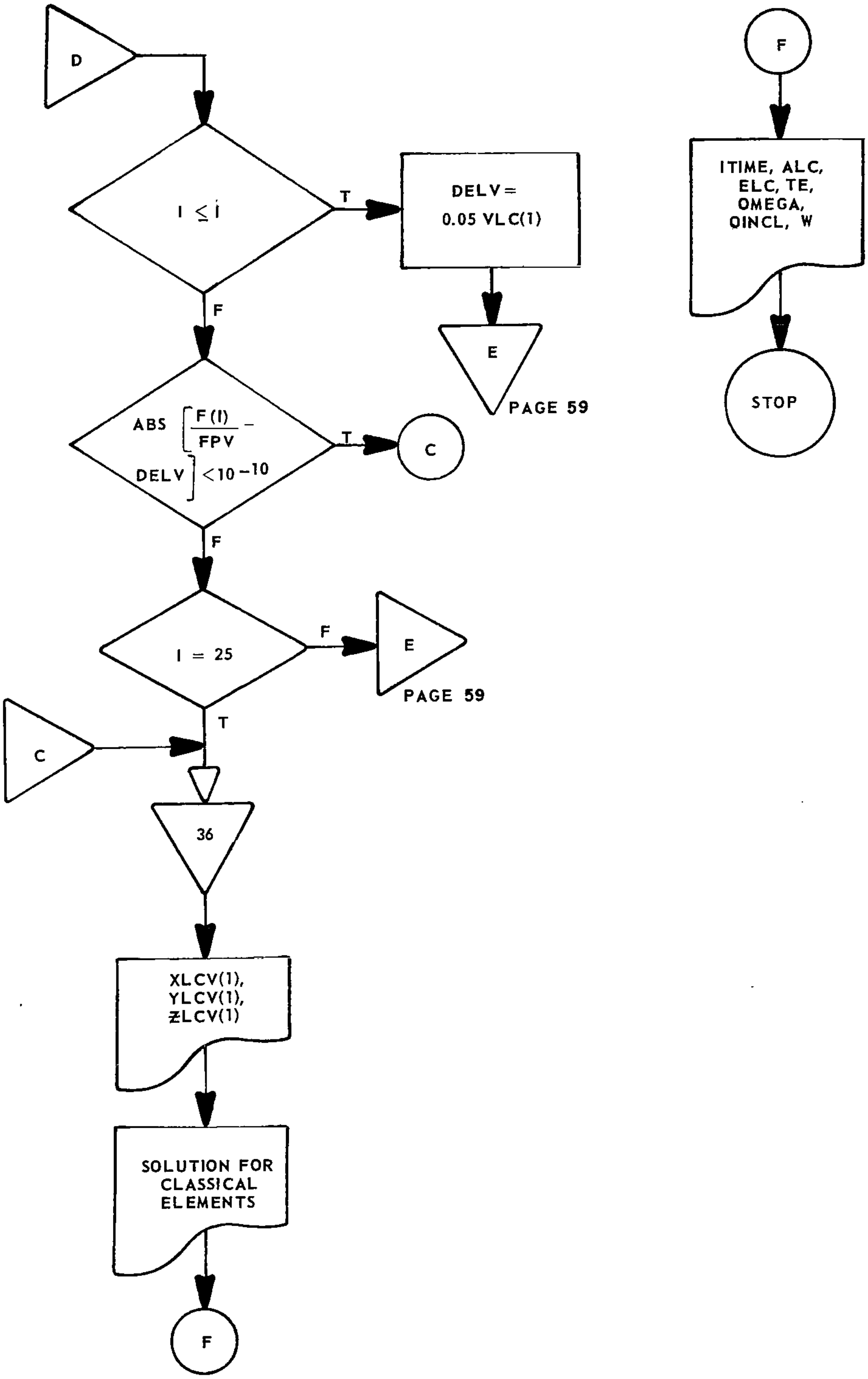
$$\dot{\underline{r}}_1 = \frac{\underline{r}_2 - f\,\underline{r}_1}{g} \qquad (137)$$

Continue by calculating for classical elements.

ITERATION OF TRUE ANOMALY
FLOWCHART



ITERATION OF TRUE ANOMALY
FLOWCHART
(CONT'D)



```
C       ITERATIØN ØF THE TRUE ANØMALY PRELIMINARY ØRBIT DETER[illegible]
C       PØSITIØN AND TIME (ESCØBAL,PAGE 215)
C
        DIMENSIØN F(25),RLC(2),UX(2),UY(2),UZ(2),VLC(2),ESIN([illegible])
       CECØS(2),ANGE(2),XLC(2),YLC(2),ZLC(2),XLCV(1),YLCV(1),Z[illegible](1),T(2)
        DØ 90 N=1,6
C
C       READ TWØ INERTIAL PØSITIØN VECTØRS AND THEIR CØRRESPØNDI[illegible] TIMES
C
        READ 101, XLC(1), YLC(1), ZLC(1), T(1), XLC(2)
        READ 101, YLC(2), ZLC(2), T(2), XMU, XK
101     FØRMAT(5E16.8)
C
C       ECHØ CHECK
C
        PRINT 104, XLC(1),YLC(1),ZLC(1),T(1),XLC(2),YLC(2),ZLC(2),T(2),
       CXMU,XK
104     FØRMAT(1H0,$XLC(1)=$E16.8,//,$YLC(1)=$E16.8,//,$ZLC(1)=$E16.8,//,
       1$T(1)=$E16.8,//,$XLC(2)=$E16.8,//,$YLC(2)=$E16.8,//,$ZLC(2)=$E16.8,
       1//,$T(2)=$E16.8,//,$XMU=$E16.8,//,$XK=$E16.8)
C
C       BEGIN CØMPUTATIØNS
C
C       ALL META-SYMBØL IS ITIME SUBRØUTINE
C
        ITIME=0
S       LDA         205S
S       STA         0205
S       BRU         200S
S205    BRM         2050S
S200    EØM         020020
S       PØT  =  00202000
S       FIR
        TAU=XK*(T(2)-T(1))
        DØ 6 J=1,2
        RLC(J)=SQRT(XLC(J)**2+YLC(J)**2+ZLC(J)**2)
        UX(J)=XLC(J)/RLC(J)
        UY(J)=YLC(J)/RLC(J)
   6    UZ(J)=ZLC(J)/RLC(J)
        VCØS=UX(1)*UX(2)+UY(1)*UY(2)+UZ(1)*UZ(2)
        CØM=XLC(1)*YLC(2)-XLC(2)*YLC(1)
        VSIN=CØM/ABS(CØM)*SQRT(1.0-VCØS**2)
        ANGV=ATAN(VSIN,VCØS)
        VLC(1)=0.05
C
C       BEGIN ITERATIØN ØF TRUE ANØMØLY
C
 11     DØ 35 I=1,25
 12     VLC(2)=VLC(1)+ANGV
        ELC=(RLC(2)-RLC(1))/(RLC(1)*CØS(VLC(1))-RLC(2)*CØS(VLC(2)))
        IF(ELC-0.0000000001) 17,17,15
 15     ALC=(RLC(1)*(1.0+ELC*CØS(VLC(1))))/(1.0-ELC**2)
 16     IF(ALC-0.0000000001) 17,17,19
```

```
 17     VLC(1)=VLC(1)+0.174532925
 18     GØ TØ 12
 19     ESIN(1)=SQRT(1.0-ELC**2)*SIN(VLC(1))/(1.0+ELC*CØS(VLC(1)))
        ECØS(1)=(CØS(VLC(1))+ELC)/(1.0+ELC*CØS(VLC(1)))
        ESIN(2)=(SQRT(1.0-ELC**2)*SIN(VLC(2)))/(1.0+ELC*CØS(VLC(2)))
        ECØS(2)=(CØS(VLC(2))+ELC)/(1.0+ELC*CØS(VLC(2)))
        ANGE(1)=ATAN(ESIN(1),ECØS(1))
        ANGE(2)=ATAN(ESIN(2),ECØS(2))
        DIFM=ANGE(2)-ANGE(1)+ELC*(ESIN(1)-ESIN(2))
        ETA=XK*SQRT(XMU/ALC**3)
        F(I)=TAU-(DIFM/ETA)*XK
        CT1=ITIME
        PRINT 100,CT1
        PRINT 102, F(I),I
102     FØRMAT(1H0,$F(I)=$E16.8$*****I=$I2)
        ITIME=0
        IF(ABS(F(I))-0.0000000001) 36,29,29
 29     IF(I-1) 34,34,30
 30     FPV=(F(I)-F(I-1))/DELV
 31     IF(ABS(F(I)/FPV-DELV)-0.0000000001) 36,32,32
 32     DELV=-F(I)/FPV
 33     GØ TØ 35
 34     DELV=0.05*VLC(1)
 35     VLC(1)=VLC(1)+DELV
C
C       SØLVE FØR INERTIAL VELØCITY VECTØRS XDØT,YDØT,ZDØT.
C
 36     FLC=1.0-(ALC/RLC(1))*(1.0-CØS(ANGE(2)-ANGE(1)))
        GLC=TAU-SQRT(ALC**3/XMU)*(ANGE(2)-ANGE(1)-SIN(ANGE(2)-ANGE(1)))
        XLCV(1)=(XLC(2)-FLC*XLC(1))/GLC
        YLCV(1)=(YLC(2)-FLC*YLC(1))/GLC
        ZLCV(1)=(ZLC(2)-FLC*ZLC(1))/GLC
        CT2=ITIME
        PRINT 100,CT2
        PRINT 103, XLCV(1),YLCV(1),ZLCV(1)
103     FØRMAT(1H0,$XLCV(1)=$E16.8,//,$YLCV(1)=$E16.8,//, ZLCV(1)=$E16.8)
C
C       SØLUTIØN FØR CLASSICAL ELEMENTS
C
        ITIME=0
        RLC(1)=SQRT(XLC(1)**2+YLC(1)**2+ZLC(1)**2)
        RDØT=XLC(1)*XLCV(1)+YLC(1)*YLCV(1)+ZLC(1)*ZLCV(1)
        RLCV(1)=RDØT/RLC(1)
        V=SQRT(XLCV(1)**2+YLCV(1)**2+ZLCV(1)**2)
        ALC=(RLC(1)*XMU)/(2.0*XMU-V**2*RLC(1))
        CSUBE=(1.0-RLC(1)/ALC)
        SSUBE=(RLCV(1)*RLC(1))/SQRT(XMU*ALC)
        ELC=SQRT(SSUBE**2+CSUBE**2)
        CØSE=(ALC-RLC(1))/(ALC*ELC)
        XSUBV=ALC*(CØSE-ELC)
        CØSV=XSUBV/RLC(1)
        SINV=SQRT(RLC(1)**2-XSUBV**2)/RLC(1)
        SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
        E=ATAN(SINE,CØSE)
```

```
      TE=T(1)-((E-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(1)*ZLCV(1)-ZLC(1)*YLCV(1)
      HY=-(XLC(1)*ZLCV(1)-ZLC(1)*XLCV(1))
      HZ=XLC(1)*YLCV(1)-YLC(1)*XLCV(1)
      VANGE=ATAN(SINV,CØSV)
      SINHX=HX
      CØSHY=-HY
      ØMEGA=ATAN(SINHX,CØSHY)
      EXP=SQRT(HX**2+HY**2)
      ØINCL=ATAN(EXP,HZ)
      UNUM=-XLC(1)*SIN(ØMEGA)*CØS(ØINCL)+YLC(1)*CØS(ØMEGA)*CØS(ØINCL)+
     CZLC(1)*SIN(ØINCL)
      DEM=XLC(1)*CØS(ØMEGA)+YLC(1)*SIN(ØMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANGE
      CT3=ITIME
      PRINT 100,CT3
100   FØRMAT(‡MILLISEC=‡I8)
      PRINT 107, ALC,ELC,TE,ØMEGA,ØINCL,W
107   FØRMAT(1H0,‡ALC=‡E16.8,//,‡ELC=‡E16.8,//,‡TE=‡E16.8,//,
     1‡ØMEGA=‡E16.8,//,‡ØINCL=‡E16.8,//,‡W=‡E16.8,//)
 90   CØNTINUE
      GØ TØ 41
S2050 PZE
S     MIN       ITIME
S     BRU       *2050S
 41       END
```

APPENDIX G
METHOD OF GAUSS PODM, ANGLES ONLY

Given α_i, δ_i, ϕ_i, λ_{Ei}, H_i, t_i for i = 1, 2, 3, and the constants $d\phi/dt$, f, a_e, μ, k_e, compute the following:

$$\tau_1 = k_e\ (t_1 - t_2) \tag{138}$$

$$\tau_3 = k_e\ (t_3 - t_2) \tag{139}$$

$$\tau_{13} = \tau_3 - \tau_1 \tag{140}$$

$$A_1 = \frac{\tau_3}{\tau_{13}} \tag{141}$$

$$B_1 = \left(\tau_{13}^{\ 2} - \tau_3^{\ 2}\right) \frac{A_1}{6} \tag{142}$$

$$A_3 = -\frac{\tau_1}{\tau_{13}} \tag{142.1}$$

$$B_3 = \left(\tau_{13}^{\ 2} - \tau_1^{\ 2}\right) \frac{A_3}{6} \tag{142.2}$$

$$Tu = \frac{J.D. - 2415020}{36525} \tag{143}$$

$$\theta_{g0} = 99\overset{\circ}{.}6909833 + 36000\overset{\circ}{.}7689\ Tu + 0\overset{\circ}{.}00038708\ Tu^2 \tag{144}$$

For i = 1, 2, 3, compute

$$L_{xi} = \cos\delta_i \cos\alpha_i \tag{145}$$

$$L_{yi} = \cos\delta_i \sin\alpha_i \tag{146}$$

$$L_{zi} = \sin \delta_i \tag{147}$$

$$\theta_i = \theta_g 0 + \frac{d\theta}{dt} (t_i - t_0) + \lambda_{Ei} \tag{148}$$

$$G_{1i} = \frac{a^e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \tag{149}$$

$$G_{2i} = \frac{(1 - f)^2 a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \tag{150}$$

$$X_i = - G_{1i} \cos \phi_i \cos \theta_i \tag{151}$$

$$Y_i = G_{1i} \cos \phi_i \sin \theta_i \tag{152}$$

$$Z_i = - G_{2i} \sin \phi_i \tag{153}$$

Compute the following:

$$D = L_{x1} (L_{y2}L_{z3} - L_{z2}L_{y3}) - L_{x2} (L_{y1}L_{z3} - L_{z1}L_{y3}) + L_{x3} (L_{y1}L_{z2} - L_{z1}L_{y2}) \tag{154}$$

$$a_{11} = \frac{L_{y2}L_{z3} - L_{y3}L_{z2}}{D} \tag{155}$$

$$a_{12} = - \frac{(L_{x2}L_{z3} - L_{x3}L_{z2})}{D} \tag{156}$$

$$a_{13} = \frac{L_{x2}L_{y3} - L_{x3}L_{y2}}{D} \tag{157}$$

$$a_{21} = - \frac{(L_{y1}L_{z3} - L_{y3}L_{z1})}{D} \tag{158}$$

$$a_{22} = \frac{L_{x1}L_{z3} - L_{x3}L_{z1}}{D} \tag{159}$$

$$a_{23} = - \frac{(L_{x1}L_{y3} - L_{x3}L_{y1})}{D} \tag{160}$$

$$a_{31} = \frac{L_{y1}L_{z2} - L_{y2}L_{z1}}{D} \tag{161}$$

$$a_{32} = - \frac{(L_{x1}L_{z2} - L_{x2}L_{z1})}{D} \tag{162}$$

$$a_{33} = \frac{L_{x1}L_{y2} - L_{x2}L_{y1}}{D} \tag{163}$$

and form the vectors

$$\underline{A} = \left[A_1, -1, A_3\right] \tag{164}$$

$$\underline{B} = \left[B_1, 0, B_3\right] \tag{165}$$

$$\underline{X} = \left[X_1, X_2, X_3\right] \tag{166}$$

$$\underline{Y} = \left[Y_1, Y_2, Y_3\right] \tag{167}$$

$$\underline{Z} = \left[Z_1, Z_2, Z_3\right] \tag{168}$$

Evaluate the coefficients:

$$A_2^* = -\,(a_{21}\,\underline{A}\cdot\underline{X} + a_{22}\,\underline{A}\cdot\underline{Y} + a_{23}\,\underline{A}\cdot\underline{Z}) \tag{169}$$

$$B_2^* = -\,(a_{21}\,\underline{B}\cdot\underline{X} + a_{22}\,\underline{B}\cdot\underline{Y} + a_{23}\,\underline{B}\cdot\underline{Z}) \tag{170}$$

$$C_\psi = -\,2\,(X_2L_{x2} + Y_2L_{y2} + Z_2L_{z2}) \tag{171}$$

$$R_2^2 = X_2^2 + Y_2^2 + Z_2^2 \tag{172}$$

$$a = - (C_\psi A_2^* + A_2^{*2} + R_2^2) \tag{173}$$

$$b = - \mu (C_\psi B_2^* + 2A_2^* B_2^*) \tag{174}$$

$$c = - \mu^2 B_2^{*2} \tag{175}$$

Solve

$$r_2^8 + ar_2^6 + br_2^3 + c = 0 \tag{176}$$

to obtain the applicable real root r_2, and continue calculating with

$$u_2 = \frac{\mu}{r_2^3} \tag{177}$$

$$D_1 = A_1 + B_1 u_2 \tag{178}$$

$$D_3 = A_3 + B_3 u_2 \tag{179}$$

$$A_1^* = a_{11} \underline{A} \cdot \underline{X} + a_{12} \underline{A} \cdot \underline{Y} + a_{13} \underline{A} \cdot \underline{Z} \tag{180}$$

$$B_1^* = a_{11}\, \underline{B} \cdot \underline{X} + a_{12}\, \underline{B} \cdot \underline{Y} + a_{13}\, \underline{B} \cdot \underline{Z} \tag{181}$$

$$A_3^* = a_{31}\, \underline{A} \cdot \underline{X} + a_{32}\, \underline{A} \cdot \underline{Y} + a_{33}\, \underline{A} \cdot \underline{Z} \tag{182}$$

$$B_3^* = a_{31}\, \underline{B} \cdot \underline{X} + a_{32}\, \underline{B} \cdot \underline{Y} + a_{33}\, \underline{B} \cdot \underline{Z} \tag{183}$$

$$\rho_1 = \frac{A_1^* + B_1^* u_2}{D_1} \tag{184}$$

$$\rho_2 = A_2^* + B_2^* u_2 \tag{185}$$

$$\rho_3 = \frac{A_3^* + B_3^* u_2}{D_3} \tag{186}$$

$$\underline{r}_i = \rho_i \underline{L}_i - \underline{R}_i \qquad \text{for } i = 1, 2, 3 \tag{187}$$

Then, utilizing the Herrick-Gibbs formulas, calculate

$$d_1 = \tau_3 \left(\frac{\mu}{12 r_1^{\,3}} - \frac{1}{\tau_1 \tau_{13}} \right) \tag{188}$$

$$d_2 = (\tau_1 + \tau_3) \left(\frac{\mu}{12 r_2^{\,3}} - \frac{1}{\tau_1 \tau_3} \right) \tag{189}$$

$$d_3 = -\, \tau_1 \left(\frac{\mu}{12 r_3^{\,3}} + \frac{1}{\tau_3 \tau_{13}} \right) \tag{190}$$

$$\dot{\underline{r}}_2 = - d_1\underline{r}_1 + d_2\underline{r}_2 + d_3\underline{r}_3 \tag{191}$$

$$r_2 = \sqrt{\underline{r}_2 \cdot \underline{r}_2} \tag{192}$$

$$\dot{r}_2 = \frac{\dot{\underline{r}}_2 \cdot \underline{r}_2}{r_2} \tag{193}$$

$$V_2 = \sqrt{\dot{\underline{r}}_2 \cdot \dot{\underline{r}}_2} \tag{194}$$

$$\frac{1}{a} = \frac{2}{r_2} - \frac{V_2{}^2}{\mu} \tag{195}$$

From the f and g functions, calculate

$$f_1 = f\ (V_2, r_2, \dot{r}_2, \tau_1) \tag{196}$$

$$f_3 = f\ (V_2, r_2, \dot{r}_2, \tau_3) \tag{197}$$

$$g_1 = g\ (V_2, r_2, \dot{r}_2, \tau_1) \tag{198}$$

$$g_3 = g\ (V_2, r_2, \dot{r}_2, \tau_3) \tag{199}$$

Continue calculating with

$$D^* = f_1g_3 - f_3g_1 \tag{200}$$

$$c_1 = \frac{g_3}{D^*} \tag{201}$$

$$c_2 = -1.0 \tag{202}$$

$$c_3 = -\frac{g_1}{D^*} \tag{203}$$

$$\underline{G} = c_1\underline{R}_1 + c_2\underline{R}_2 + c_3\underline{R}_3 \tag{204}$$

$$(\rho_1)_n = \frac{1}{c_1}(a_{11}G_x + a_{12}G_y + a_{13}G_z) \tag{205}$$

$$(\rho_2)_n = -(a_{21}G_x + a_{22}G_y + a_{23}G_z) \tag{206}$$

$$(\rho_3)_n = \frac{1}{c_3}(a_{31}G_x + a_{32}G_y + a_{33}G_z) \tag{207}$$

The first time through, test to see if

$$|(\rho_1)_n - \rho_1| < \varepsilon_1 \tag{208}$$

$$|(\rho_2)_n - \rho_2| < \varepsilon_2 \tag{209}$$

$$|(\rho_3)_n - \rho_3| < \varepsilon_3 \tag{210}$$

where ε_1, ε_2, ε_3 are tolerances, i.e., 10^{-10}. If so, proceed to equation (214); if not, return to equation (187) using $(\rho_i)_n$ and repeat equational loop (188) to (207); however, from this point on, test to see if

$$|(\rho_1)_{n+1} - (\rho_1)_n| < \varepsilon_1 \tag{211}$$

$$|(\rho_2)_{n+1} - (\rho_2)_n| < \varepsilon_2 \tag{212}$$

$$|(\rho_3)_{n+1} - (\rho_3)_n| < \varepsilon_3 \tag{213}$$

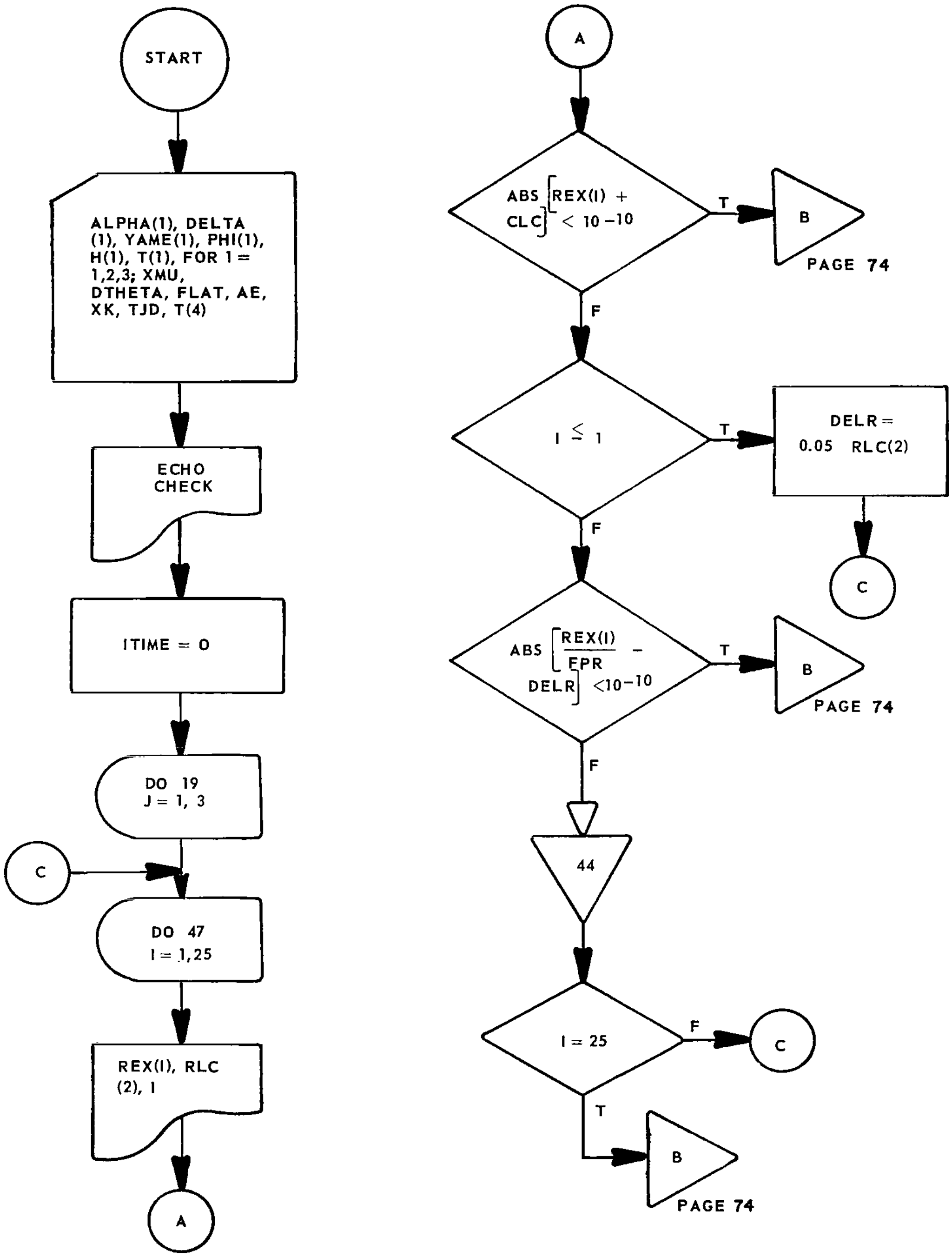
And repeat equational loop (188) to (207) until test is successful. Continue by calculating

$$\underline{r}_2 = \rho_2 \underline{L}_2 - \underline{R}_2 \tag{214}$$

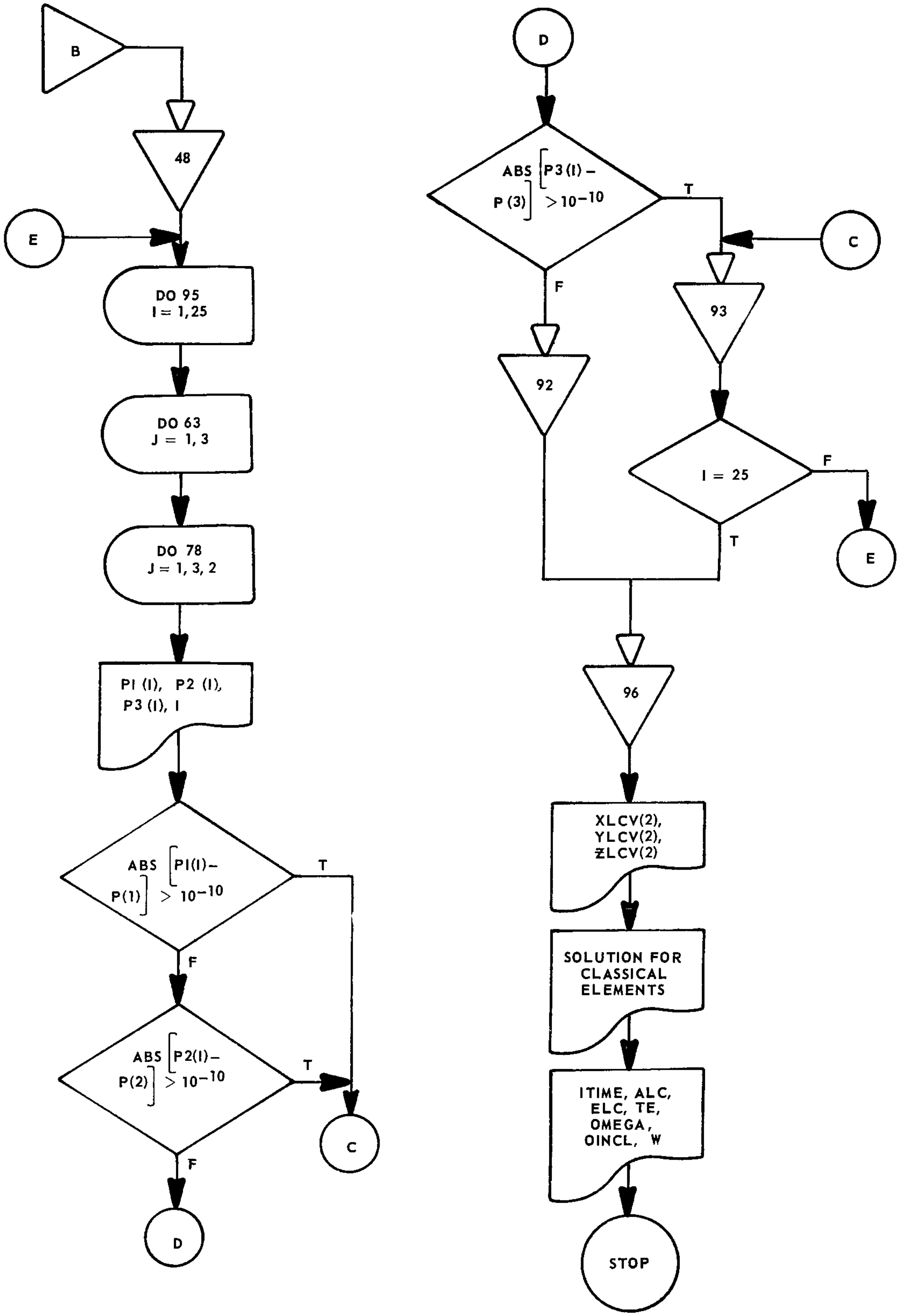
$$\dot{\underline{r}}_2 = - d_1 \underline{r}_1 + d_2 \underline{r}_2 + d_3 \underline{r}_3 \tag{215}$$

Continue by calculating the classical elements.

METHOD OF GAUSS FLOWCHART



METHOD OF GAUSS FLOWCHART (CONT'D)



```
C       METHOD OF GAUSS PRELIMINARY ORBIT DETERMINATION METHOD
C       ANGLES ONLY (ESCOBAL,PAGE 258)
C
        DO 97 K=1,25
C
        DIMENSION TAU(3),A(3),B(3),XL(3),YL(3),ZL(3),THETA(3),DEMG(3),
       CREX(25),YLC(3),YLC(3),ZLC(3),RLC(3),ULC(3),D(3),P(3),DLC(3),
       CG1(3),G2(3),X(3),Y(3),Z(3),R(3),A1(3),A2(3),A3(3),AS(3),BS(3),
       CXLCV(3),YLCV(3),ZLCV(3),RLCV(3),V(3),PLC(3),QLC(3),FO(3),FT(3)
        DIMENSION GO(3),GT(3),F(3),G(3),C(3),P1(25),P2(25),P3(25),
       CT(4),ALPHA(3),DELTA(3),PHI(3),YAME(3),H(3)
C
C       READ ANGLE INPUT DATA
C
        READ 108,FLAT,AE,YK,XMU,DTHETA
        READ 108,T(4),T(1),T(2),T(3),TJD
        READ 108,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2)
        READ 108,DELTA(3),YAME(1),YAME(2),YAME(3),PHI(1)
        READ 108,PHI(2),PHI(3),H(1),H(2),H(3)
108     FORMAT(5E16.8)
C
C       ECHO CHECK
C
        PRINT 110,FLAT,AE,XK,XMU,DTHETA,T(4),TJD,T(1),T(2),T(3)
110     FORMAT(1HO,$FLAT=$E16.8$**AE=$E16.8$**XK=$E16.8$**XMU=$E16.8,//,
       1$DTHETA=$E16.8$**T(4)=$E16.8$**TJD=$E16.8,//,
       1$T(1)=$E16.8$**T(2)=$E16.8$**T(3)=$E16.8)
        PRINT 111,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2),DELTA(3),
       CYAME(1),YAME(2),YAME(3)
111     FORMAT(1HO,$ALPHA(1)=$E16.8$**ALPHA(2)=$E16.8$**ALPHA(3)=$E16.8,
       1//,$DELTA(1)=$E16.8$**DELTA(2)=$E16.8$**DELTA(3)=$E16.8,//,
       1$YAME(1)=$E16.8$**YAME(2)=$E16.8$**YAME(3)=$E16.8)
        PRINT 112,PHI(1),PHI(2),PHI(3),H(1),H(2),H(3)
112     FORMAT(1HO,$PHI(1)=$E16.8$**PHI(2)=$E16.8$**PHI(3)=$E16.8,//,
       1$H(1)=$E16.8$**H(2)=$E16.8$**H(3)=$E16.8)
C
C       BEGIN COMPUTATIONS
C
C       ALL META-SYMBOL IS ITIME SUBROUTINE
C
        ITIME=0
S       LDA       2059
S       STA       0205
S       BRU       2008
S205    BRM       20505
S200    EOM       020020
S       POT  =  0020200O
S       EIR
        TAU(1)=XK*(T(1)-T(2))
        TAU(3)=XK*(T(3)-T(2))
        DTAU=TAU(3)-TAU(1)
        A(1)=TAU(3)/DTAU
        B(1)=(DTAU**2-TAU(3)**2)*A(1)/6.0
```

```
      A(3)=-TAU(1)/DTAU
      B(3)=(DTAU**2-TAU(1)**2)*A(3)/6.0
      TU=(TJD-2415020.0)/36525.0
      GTHETA=(99.6909833+36000.7689*TU+0.00038708*TU**2)/57.2957795131
    8 DO 19 J=1,3
      XL(J)=COS(DELTA(J))*COS(ALPHA(J))
      YL(J)=COS(DELTA(J))*SIN(ALPHA(J))
      ZL(J)=SIN(DELTA(J))
      THETA(J)=GTHETA+DTHETA*(T(J)-T(4))+YAME(J)
      DEMG(J)=SQRT(1.0-(2.0*FLAT-FLAT**2)*(SIN(PHI(J)))**2)
      G1(J)=AE/DEMG(J)+H(J)
      G2(J)=((1.0-FLAT)**2*AE)/DEMG(J)+H(J)
      X(J)=-G1(J)*COS(PHI(J))*COS(THETA(J))
      Y(J)=-G1(J)*COS(PHI(J))*SIN(THETA(J))
   19 Z(J)=-G2(J)*SIN(PHI(J))
      D1=XL(1)*(YL(2)*ZL(3)-ZL(2)*YL(3))-XL(2)*(YL(1)*ZL(3)-ZL(1)*YL(3))
     C+XL(3)*(YL(1)*ZL(2)-ZL(1)*YL(2))
      A1(1)=(YL(2)*ZL(3)-YL(3)*ZL(2))/D1
      A1(2)=-(XL(2)*ZL(3)-XL(3)*ZL(2))/D1
      A1(3)=(XL(2)*YL(3)-XL(3)*YL(2))/D1
      A2(1)=-(YL(1)*ZL(3)-YL(3)*ZL(1))/D1
      A2(2)=(XL(1)*ZL(3)-XL(3)*ZL(1))/D1
      A2(3)=-(XL(1)*YL(3)-XL(3)*YL(1))/D1
      A3(1)=(YL(1)*ZL(2)-YL(2)*ZL(1))/D1
      A3(2)=-(XL(1)*ZL(2)-XL(2)*ZL(1))/D1
      A3(3)=(XL(1)*YL(2)-XL(2)*YL(1))/D1
      AX=A(1)*X(1)-X(2)+A(3)*X(3)
      AY=A(1)*Y(1)-Y(2)+A(3)*Y(3)
      AZ=A(1)*Z(1)-Z(2)+A(3)*Z(3)
      BX=B(1)*X(1)+B(3)*X(3)
      BY=B(1)*Y(1)+B(3)*Y(3)
      BZ=B(1)*Z(1)+B(3)*Z(3)
      AS(2)=-(A2(1)*AX+A2(2)*AY+A2(3)*AZ)
      BS(2)=-(A2(1)*BX+A2(2)*BY+A2(3)*BZ)
      CHI=-2.0*(X(2)*XL(2)+Y(2)*YL(2)+Z(2)*ZL(2))
      R(2)=SQRT(X(2)**2+Y(2)**2+Z(2)**2)
      ALC=-(CHI*AS(2)+AS(2)**2+R(2)**2)
      BLC=-XMU*(CHI*BS(2)+2.0*AS(2)*BS(2))
      CLC=-XMU**2*BS(2)**2
      RLC(2)=1.0
C
C     ITERATIVE LOOP FOR DETERMINING APPLICABLE REAL ROOT OF RLC(2)
C
   37 DO 47 I=1,25
      REX(I)=RLC(2)**8+ALC*RLC(2)**6+BLC*RLC(2)**3+CLC
      CT1=ITIME
      PRINT 100,CT1
      PRINT 102,REX(I),RLC(2),I
  102 FORMAT(1H0,$REX(I)=$E16.8$***RLC(2)=$E16.8$***I=$I2)
      ITIME=0
      IF(ABS(REX(I)-REX(I-1))-0.0000000001) 48,48,49
   49 IF(ABS(REX(I)-0.0000000001)) 48,48,43
   43 IF(I-1) 46,46,44
   44 RPR=(REX(I)-REX(I-1))/DELR
```

```
      IF(ABS(REX(I)/RPR-DELR)-0.0000000001) 48,45,45
   45 DELR=-REX(I)/RPR
      GO TO 47
   46 DELR=0.05*RLC(2)
   47 RLC(2)=ABS(RLC(2)+DELR)
C
   48 ULC(2)=XMU/RLC(2)**3
      D(1)=A(1)+B(1)*ULC(2)
      D(3)=A(3)+B(3)*ULC(2)
      AS(1)=A1(1)*AX+A1(2)*AY+A1(3)*AZ
      BS(1)=A1(1)*BX+A1(2)*BY+A1(3)*BZ
      AS(3)=A3(1)*AX+A3(2)*AY+A3(3)*AZ
      BS(3)=A3(1)*BX+A3(2)*BY+A3(3)*BZ
      P(1)=(AS(1)+BS(1)*ULC(2))/D(1)
      P(2)=AS(2)+BS(2)*ULC(2)
      P(3)=(AS(3)+BS(3)*ULC(2))/D(3)
C
C     ITERATIVE LOOP FOR DETERMINING SCALAR OF THE RANGE VECTOR
C
   58 DO 95 I=1,25
   59 DO 63 J=1,3
      XLC(J)=P(J)*XL(J)-X(J)
      YLC(J)=P(J)*YL(J)-Y(J)
      ZLC(J)=P(J)*ZL(J)-Z(J)
   63 RLC(J)=SQRT(XLC(J)**2+YLC(J)**2+ZLC(J)**2)
      DLC(1)=TAU(3)*(XMU/(12.0*RLC(1)**3)-1.0/(TAU(1)*DTAU))
      DLC(2)=(TAU(1)+TAU(3))*(XMU/(12.0*RLC(2)**3)-1.0/(TAU(1)*TAU(3)))
      DLC(3)=-TAU(1)*(XMU/(12.0*RLC(3)**3)+1.0/(TAU(3)*DTAU))
      XLCV(2)=-DLC(1)*XLC(1)+DLC(2)*XLC(2)+DLC(3)*XLC(3)
      YLCV(2)=-DLC(1)*YLC(1)+DLC(2)*YLC(2)+DLC(3)*YLC(3)
      ZLCV(2)=-DLC(1)*ZLC(1)+DLC(2)*ZLC(2)+DLC(3)*ZLC(3)
      RLCV(2)=(XLCV(2)*XLC(2)+YLCV(2)*YLC(2)+ZLCV(2)*ZLC(2))/RLC(2)
      V(2)=SQRT(XLCV(2)**2+YLCV(2)**2+ZLCV(2)**2)
      AI=2.0/RLC(2)-V(2)**2/XMU
      ULC(2)=XMU/RLC(2)**3
      PLC(2)=RLC(2)*RLCV(2)/RLC(2)**2
      QLC(2)=(V(2)**2-RLC(2)**2*ULC(2))/RLC(2)**2
   76 DO 78 J=1,3,2
      F0(J)=1.0-0.5*ULC(2)*TAU(J)**2+0.5*ULC(2)*PLC(2)*TAU(J)**3+
     C1.0/24.0*(3.0*ULC(2)*QLC(2)-15.0*ULC(2)*PLC(2)**2+ULC(2)**2)*
     CTAU(J)**4+1.0/8.0*(7.0*ULC(2)*PLC(2)**3-3.0*ULC(2)*PLC(2)*QLC(2)-
     CULC(2)**2*PLC(2))*TAU(J)**5
      FT(J)=1.0/720.0*(630.0*ULC(2)*PLC(2)**2*QLC(2)-24.0*ULC(2)**2*
     CQLC(2)-ULC(2)**3-45.0*ULC(2)*QLC(2)**2-945.0*ULC(2)*PLC(2)**4+
     C210.0*ULC(2)**2*PLC(2)**2)*TAU(J)**6
      F(J)=F0(J)+FT(J)
      G0(J)=TAU(J)-1.0/6.0*ULC(2)*TAU(J)**3+1.0/4.0*ULC(2)*PLC(2)*
     CTAU(J)**4+1.0/120.0*(9.0*ULC(2)*QLC(2)-45.0*ULC(2)*PLC(2)**2+
     CULC(2)**2)*TAU(J)**5
      GT(J)=1.0/360.0*(210.0*ULC(2)*PLC(2)**3-90.0*ULC(2)*PLC(2)*QLC(2)
     C-15.0*ULC(2)**2*PLC(2))*TAU(J)**6
   78 G(J)=G0(J)+GT(J)
      DS=F(1)*G(3)-F(3)*G(1)
      C(1)=G(3)/DS
```

```
      C(2)=-1.0
      C(3)=-G(1)/DS
      GX=C(1)*X(1)+C(2)*X(2)+C(3)*X(3)
      GY=C(1)*Y(1)+C(2)*Y(2)+C(3)*Y(3)
      GZ=C(1)*Z(1)+C(2)*Z(2)+C(3)*Z(3)
      P1(I)=(1.0/C(1))*(A1(1)*GX+A1(2)*GY+A1(3)*GZ)
      P2(I)=-(A2(1)*GX+A2(2)*GY+A2(3)*GZ)
      P3(I)=(1.0/C(3))*(A3(1)*GX+A3(2)*GY+A3(3)*GZ)
      CT2=ITIME
      PRINT 100,CT2
      PRINT 103,P1(I),I,P2(I),I,P3(I),I
103   FORMAT(1H0,$P1(I)=$E16.8$***I=$I2,///,$P2(I)=$E16.8$***I=$I2,///,
     1$P3(I)=$E16.8$***I=$I2,//)
      ITIME=0
      IF(ABS(P1(I)-P(1))-0.0000000001) 90,93,93
90    IF(ABS(P2(I)-P(2))-0.0000000001) 91,93,93
91    IF(ABS(P3(I)-P(3))-0.0000000001) 92,93,93
92    GO TO 96
93    P(1)=P1(I)
      P(2)=P2(I)
95    P(3)=P3(I)
C
C     SOLVE FOR INERTIAL POSITION AND VELOCITY VECTORS
C
96    XLC(2)=P(2)*XL(2)-X(2)
      YLC(2)=P(2)*YL(2)-Y(2)
      ZLC(2)=P(2)*ZL(2)-Z(2)
      XLCV(2)=-DLC(1)*XLC(1)+DLC(2)*XLC(2)+DLC(3)*XLC(3)
      YLCV(2)=-DLC(1)*YLC(1)+DLC(2)*YLC(2)+DLC(3)*YLC(3)
      ZLCV(2)=-DLC(1)*ZLC(1)+DLC(2)*ZLC(2)+DLC(3)*ZLC(3)
      CT3=ITIME
      PRINT 100,CT3
      PRINT 104,XLCV(2),YLCV(2),ZLCV(2)
104   FORMAT(1H0,$XLCV(2)=$E16.8,///,$YLCV(2)=$E16.8,///,$ZLCV(2)=$E16.8,
     1//)
C
C
C     SOLUTION FOR CLASSICAL ELEMENTS
      RLC(2)=SQRT(XLC(2)**2+YLC(2)**2+ZLC(2)**2)
      RRDOT=XLC(2)*XLCV(2)+YLC(2)*YLCV(2)+ZLC(2)*ZLCV(2)
      RLCV(2)=RRDOT/RLC(2)
      VE=SQRT(XLCV(2)**2+YLCV(2)**2+ZLCV(2)**2)
      ALC=(RLC(2)*XMU)/(2.0*XMU-VE**2*RLC(2))
      CSUBE=(1.0-RLC(2)/ALC)
      SSUBE=(RLCV(2)*RLC(2))/SQRT(XMU*ALC)
      ELC=SQRT(SSUBE**2+CSUBE**2)
      COSE=(ALC-RLC(2))/(ALC*ELC)
      XSUBW=ALC*(COSE-ELC)
      COSV=XSUBW/RLC(2)
      SINV=SQRT(RLC(2)**2-XSUBW**2)/RLC(2)
      SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
      E=ATAN(SINE,COSE)
      TE=T(2)-((E-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(2)*ZLCV(2)-ZLC(2)*YLCV(2)
```

```
      HY=-(XLC(2)*ZLCV(2)-ZLC(2)*XLCV(2))
      HZ=XLC(2)*YLCV(2)-YLC(2)*XLCV(2)
      VANGE=ATAN(SINV,COSV)
      SINHX=HX
      COSHY=-HY
      OMEGA=ATAN(SINHX,COSHY)
      EXP=SQRT(HX**2+HY**2)
      OINCL=ATAN(EXP,HZ)
      UNUM=-XLC(2)*SIN(OMEGA)*COS(OINCL)+YLC(2)*COS(OMEGA)*COS(OINCL)+
     CZLC(2)*SIN(OINCL)
      DEM=XLC(2)*COS(OMEGA)+YLC(2)*SIN(OMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANGE
      CT4=ITIME
      PRINT 100,CT4
      PRINT 107, ALC,ELC,TE,OMEGA,OINCL,W
107   FORMAT(1H0,$ALC=$E16.8,//,$ELC=$E16.8,//,$TE=$E16.8,//,
     1$OMEGA=$E16.8,//,$OINCL=$E16.8,//,$W=$E16.8,//)
100   FORMAT($MILLISEC=$I8)
 97   CONTINUE
      GO TO 98
S2050 PZE
S     MIN         ITIME
S     BRU         *2050S
 98   END
```

APPENDIX H
LAPLACE PODM, ANGLES ONLY

Given α_{ti}, δ_{ti}, t_i, ϕ_i, λ_{Ei}, H_i for i = 1, 2, 3 and the constants $d\theta/dt$ f, a_e, μ, k, compute the following:

$$\tau_1 = k_e \, (t_1 - t_2) \tag{216}$$

$$\tau_3 = k_e \, (t_3 - t_2) \tag{217}$$

$$S_1 = \frac{-\tau_3}{\tau_1 \, (\tau_1 - \tau_3)} \tag{218}$$

$$S_2 = \frac{-(\tau_3 + \tau_1)}{\tau_1 \tau_3} \tag{219}$$

$$S_3 = \frac{-\tau_1}{\tau_3 \, (\tau_3 - \tau_1)} \tag{220}$$

$$S_4 = \frac{2}{\tau_1 \, (\tau_1 - \tau_3)} \tag{221}$$

$$S_5 = \frac{2}{\tau_1 \tau_3} \tag{222}$$

$$S_6 = \frac{2}{\tau_3 \, (\tau_3 - \tau_1)} \tag{223}$$

For i = 1, 2, 3, calculate:

$$L_{xi} = \cos \delta_{ti} \cos \alpha_{ti} \tag{224}$$

$$L_{yi} = \cos \delta_{ti} \sin \alpha_{ti} \tag{225}$$

$$L_{zi} = \sin \delta_{ti} \tag{226}$$

and determine

$$\dot{\underline{L}}_2 = S_1\underline{L}_1 + S_2\underline{L}_2 + S_3\underline{L}_3 \tag{227}$$

$$\ddot{\underline{L}}_2 = S_4\underline{L}_1 + S_5\underline{L}_2 + S_6\underline{L}_3 \tag{228}$$

For i = 1, 2, 3, proceed as follows:

$$Tu = \frac{J.D. - 2415020}{36525} \tag{229}$$

$$\theta_{g0} = 99^\circ.6909833 + 36000^\circ.7689\ Tu + 0^\circ.00038708\ Tu^2 \tag{230}$$

$$G_{1i} = \frac{a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \tag{231}$$

$$G_{2i} = \frac{(1 - f)^2 a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \tag{232}$$

Continue calculating with

$$\theta_i = \theta_{g0} + \frac{d\theta}{dt} (t_i - t_0) + \lambda_{Ei} \tag{233}$$

$$X_i = - G_{1i} \cos \phi_i \sin \theta_i \tag{234}$$

$$Y_i = - G_{1i} \cos \phi_i \sin \theta_i \tag{235}$$

$$Z_i = - G_{2i} \sin \phi_i \tag{236}$$

If the observations are not from a single station, that is, $\phi_1 \neq \phi_2 \neq \phi_3 \neq \phi_1$ and $\lambda_{E1} \neq \lambda_{E2} \neq \lambda_{E3} \neq \lambda_{E1}$, continue calculating with equation (237); if the observations are from a single station, proceed to equation (239).

$$\dot{\underline{R}}_2 = S_1 \underline{R}_1 + S_2 \underline{R}_2 + S_3 \underline{R}_3 \tag{237}$$

$$\ddot{\underline{R}}_2 = S_4 \underline{R}_1 + S_5 \underline{R}_2 + S_6 \underline{R}_3 \tag{238}$$

Proceed to equation (241)

$$\dot{\underline{R}}_2 = \begin{bmatrix} - Y_2 \\ X_2 \\ 0 \end{bmatrix} \frac{1}{k_e} \left(\frac{d\theta}{dt} \right) \tag{239}$$

$$\ddot{\underline{R}}_2 = \begin{bmatrix} -X_2 \\ -Y_2 \\ 0 \end{bmatrix} \frac{1}{k_e^2} \left(\frac{d\theta}{dt}\right)^2 \tag{240}$$

Numerically evaluate the following determinants:

$$\Delta = 2 \begin{vmatrix} L_{x2} & \dot{L}_{x2} & \ddot{L}_{x2} \\ L_{y2} & \dot{L}_{y2} & \ddot{L}_{y2} \\ L_{z2} & \dot{L}_{z2} & \ddot{L}_{z2} \end{vmatrix} \tag{241}$$

$$D_a = \begin{vmatrix} L_{x2} & \dot{L}_{x2} & \ddot{X}_2 \\ L_{y2} & \dot{L}_{y2} & \ddot{Y}_2 \\ L_{z2} & \dot{L}_{z2} & \ddot{Z}_2 \end{vmatrix} \tag{242}$$

$$D_b = \begin{vmatrix} L_{x2} & \dot{L}_{x2} & X_2 \\ L_{y2} & \dot{L}_{y2} & Y_2 \\ L_{z2} & \dot{L}_{z2} & Z_2 \end{vmatrix} \tag{243}$$

$$D_c = \begin{vmatrix} L_{x2} & \ddot{X}_2 & \ddot{L}_{x2} \\ L_{y2} & \ddot{Y}_2 & \ddot{L}_{y2} \\ L_{z2} & \ddot{Z}_2 & \ddot{L}_{z2} \end{vmatrix} \tag{244}$$

$$D_d = \begin{vmatrix} L_{x2} & X_2 & \ddot{L}_{x2} \\ L_{y2} & Y_2 & \ddot{L}_{y2} \\ L_{z2} & Z_2 & \ddot{L}_{z2} \end{vmatrix} \tag{245}$$

and form:

$$A_2^* = \frac{2D_a}{\Delta} \tag{246}$$

$$B_2^* = \frac{2D_b}{\Delta} \tag{247}$$

$$C_2^* = \frac{D_c}{\Delta} \tag{247}$$

$$D_2^* = \frac{D_d}{\Delta} \tag{249}$$

$$C_\psi = - 2 \, (\underline{L}_2 \cdot \underline{R}_2) \tag{250}$$

$$a = - \, (C_\psi A_2^* + A_2^{*2} + R_2^2) \tag{251}$$

$$b = - \, \mu \, (C_\psi B_2^* + 2A_2^* B_2^*) \tag{252}$$

$$c = - \, \mu^2 B_2^{*2} \tag{253}$$

Solve

$$r_2^8 + ar_2^6 + br_2^3 + c = 0 \tag{254}$$

to obtain the applicable real root r_2, and continue calculating with

$$\rho_2 = A_2^* + \frac{\mu B_2^*}{r_2^3} \tag{255}$$

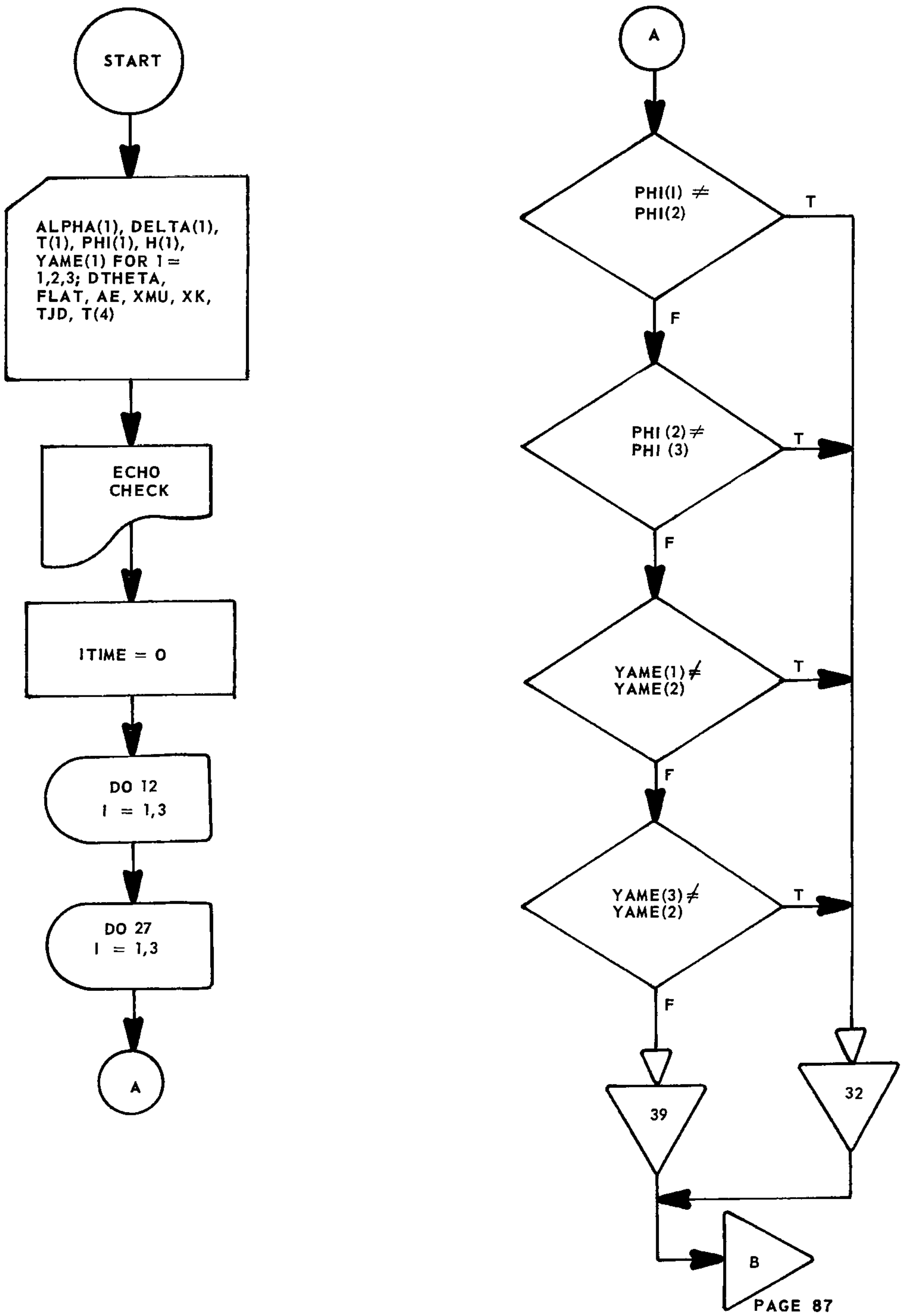
$$\dot{\rho}_2 = C_2^* + \frac{\mu D_2^*}{r_2^3} \tag{256}$$

$$\underline{r}_2 = \rho_2 \underline{L}_2 - \underline{R}_2 \tag{257}$$

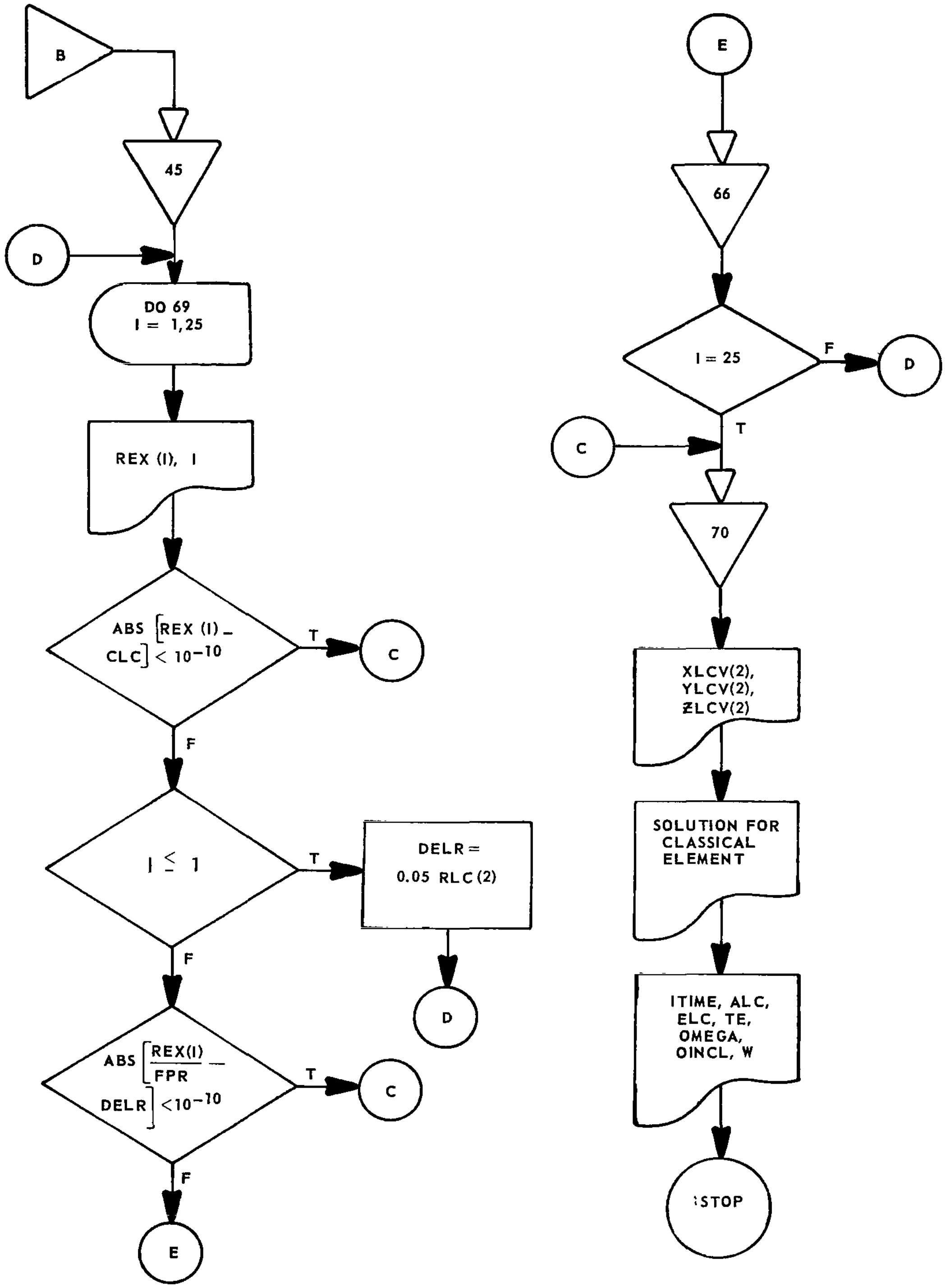
$$\dot{\underline{r}}_2 = \dot{\rho}_2 \underline{L}_2 + \rho_2 \dot{\underline{L}}_2 - \dot{\underline{R}}_2 \tag{258}$$

Continue by calculating for classical elements.

LAPLACE FLOWCHART



LAPLACE FLOWCHART (CONT'D)



```
C      LAPLACE PRELIMINARY ØRBIT DETERMINATIØN METHØD
C      ANGLES ØNLY (ESCØBAL,PAGE 267)
C
       DØ 74 K=1,25
C
       DIMENSIØN TAU(3),S(6),XL(3),YL(3),ZL(3),XLV(3),YLV(3),ZLV(3),
      CXA(3),YA(3),ZA(3),XV(3),YV(3),ZV(3),XLA(3),YLA(3),ZLA(3),
      CDEMG(3),G1(3),G2(3),THETA(3),X(3),Y(3),Z(3),R(3),AS(3),BS(3)
       DIMENSIØN CS(3),DS(3),REX(25),RLC(3),P(3),PV(3),XLC(3),YLC(3),
      CZLC(3),XLCV(3),YLCV(3),ZLCV(3),RLCV(3),T(4),ALPHA(3),DELTA(3),
      CYAME(3),PHI(3),H(3)
C
C      READ ANGLE INPUT DATA
C
       READ 108,FLAT,AE,XK,XMU,DTHETA
       READ 108,T(4),T(1),T(2),T(3),TJD
       READ 108,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2)
       READ 108,DELTA(3),YAME(1),YAME(2),YAME(3),PHI(1)
       READ 108,PHI(2),PHI(3),H(1),H(2),H(3)
108    FØRMAT(5E16.8)
C
C      ECHØ CHECK
C
       PRINT 110,FLAT,AE,XK,XMU,DTHETA,T(4),TJD,T(1),T(2),T(3)
110    FØRMAT(1H0,$FLAT=$E16.8$**AE=$E16.8$**XK=$E16.8$**XMU=$E16.8,///,
      1$DTHETA=$E16.8$**T(4)=$E16.8$**TJD=$E16.8,///,
      1$T(1)=$E16.8$**T(2)=$E16.8$**T(3)=$E16.8)
       PRINT 111,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2),DELTA(3),
      CYAME(1),YAME(2),YAME(3)
111    FØRMAT(1H0,$ALPHA(1)=$E16.8$**ALPHA(2)=$E16.8$**ALPHA(3)=$E16.8,
      1///,$DELTA(1)=$E16.8$**DELTA(2)=$E16.8$**DELTA(3)=$E16.8,///,
      1$YAME(1)=$E16.8$**YAME(2)=$E16.8$**YAME(3)=$E16.8)
       PRINT 112,PHI(1),PHI(2),PHI(3),H(1),H(2),H(3)
112    FØRMAT(1H0,$PHI(1)=$E16.8$**PHI(2)=$E16.8$**PHI(3)=$E16.8,///,
      1$H(1)=$E16.8$**H(2)=$E16.8$**H(3)=$E16.8)
C
C      BEGIN CØMPUTATIØNS
C
C      ALL META-SYMBØL IS ITIME SUBRØUTINE
C
       ITIME=0
S      LDA        205S
S      STA        0205
S      BRU        200S
S205   BRM        0505S
S200   EØM        020020
S      PØT  =  00202000
S      MIR
       TAU(1)=XK*(T(1)-T(2))
       TAU(3)=XK*(T(3)-T(2))
       S(1)=-TAU(3)/(TAU(1)*(TAU(1)-TAU(3)))
       S(2)=-(TAU(3)+TAU(1))/(TAU(1)*TAU(3))
       S(3)=-TAU(1)/(TAU(3)*(TAU(3)-TAU(1)))
```

```
      S(4)=2.0/(TAU(1)*(TAU(1)-TAU(3)))
      S(5)=2.0/(TAU(1)*TAU(3))
      S(6)=2.0/(TAU(3)*(TAU(3)-TAU(1)))
9     DO 12 I=1,3
      XL(I)=COS(DELTA(I))*COS(ALPHA(I))
      YL(I)=COS(DELTA(I))*SIN(ALPHA(I))
12    ZL(I)=SIN(DELTA(I))
      XLV(2)=S(1)*XL(1)+S(2)*XL(2)+S(3)*XL(3)
      YLV(2)=S(1)*YL(1)+S(2)*YL(2)+S(3)*YL(3)
      ZLV(2)=S(1)*ZL(1)+S(2)*ZL(2)+S(3)*ZL(3)
      XLA(2)=S(4)*XL(1)+S(5)*XL(2)+S(6)*XL(3)
      YLA(2)=S(4)*YL(1)+S(5)*YL(2)+S(6)*YL(3)
      ZLA(2)=S(4)*ZL(1)+S(5)*ZL(2)+S(6)*ZL(3)
      TU=(TJD-2415020.0)/36525.0
      GTHETA=(99.6909833+36000.7689*TU+0.00038708*TU**2)/57.2957795131
21    DO 27 I=1,3
      DENG(I)=SQRT(1.0-(2.0*FLAT-FLAT**2)*(SIN(PHI(I)))**2)
      G1(I)=AE/DENG(I)+H(I)
      G2(I)=((1.0-FLAT)**2*AE)/DENG(I)+H(I)
      THETA(I)=GTHETA+DTHETA*(T(I)-T(4))+YAME(I)
      X(I)=-G1(I)*COS(PHI(I))*COS(THETA(I))
      Y(I)=-G1(I)*COS(PHI(I))*SIN(THETA(I))
27    Z(I)=-G2(I)*SIN(PHI(I))
C
C     DETERMINE IF OBSERVATIONS ARE FROM SINGLE STATION
C
      IF(PHI(1)-PHI(2)) 32,29,32
29    IF(PHI(2)-PHI(3)) 32,30,32
30    IF(YAME(1)-YAME(2)) 32,31,32
31    IF(YAME(2)-YAME(3)) 32,39,32
C
32    XV(2)=S(1)*X(1)+S(2)*X(2)+S(3)*X(3)
      YV(2)=S(1)*Y(1)+S(2)*Y(2)+S(3)*Y(3)
      ZV(2)=S(1)*Z(1)+S(2)*Z(2)+S(3)*Z(3)
      XA(2)=S(4)*X(1)+S(5)*X(2)+S(6)*X(3)
      YA(2)=S(4)*Y(1)+S(5)*Y(2)+S(6)*Y(3)
      ZA(2)=S(4)*Z(1)+S(5)*Z(2)+S(6)*Z(3)
      GO TO 45
39    XV(2)=-(Y(2)*DTHETA)/XK
      YV(2)=(X(2)*DTHETA)/XK
      ZV(2)=0.0
      XA(2)=-(X(2)*DTHETA**2)/XK**2
      YA(2)=-(Y(2)*DTHETA**2)/XK**2
      ZA(2)=0.0
45    DEL=2.0*(XL(2)*YLV(2)*ZLA(2)+XLV(2)*YLA(2)*ZL(2)+XLA(2)*ZLV(2)*
     CYL(2)-ZL(2)*YLV(2)*XLA(2)-ZLV(2)*YLA(2)*XL(2)-ZLA(2)*XLV(2)*YL(2))
      DA=XL(2)*YLV(2)*ZA(2)+XLV(2)*YA(2)*ZL(2)+XA(2)*ZLV(2)*YL(2)-ZL(2)*
     CYLV(2)*XA(2)-YL(2)*XLV(2)*ZA(2)-XL(2)*YA(2)*ZLV(2)
      DB=XL(2)*YLV(2)*Z(2)+XLV(2)*Y(2)*ZL(2)+X(2)*ZLV(2)*YL(2)-ZL(2)*
     CYLV(2)*X(2)-ZLV(2)*Y(2)*XL(2)-Z(2)*XLV(2)*YL(2)
      DC=XL(2)*YA(2)*ZLA(2)+XA(2)*YLA(2)*ZL(2)+XLA(2)*ZA(2)*YL(2)-ZL(2)*
     CYA(2)*XLA(2)-ZA(2)*YLA(2)*XL(2)-ZLA(2)*XA(2)*YL(2)
      DD=XL(2)*Y(2)*ZLA(2)+X(2)*YLA(2)*ZL(2)+XLA(2)*Z(2)*YL(2)-ZL(2)*
     CY(2)*XLA(2)-Z(2)*YLA(2)*XL(2)-ZLA(2)*X(2)*YL(2)
```

```
      AS(2)=(2.0*DA)/DEL
      BS(2)=(2.0*DB)/DEL
      CS(2)=DC/DEL
      DS(2)=DD/DEL
      CHI=-2.0*(XL(2)*X(2)+YL(2)*Y(2)+ZL(2)*Z(2))
      R(2)=SQRT(X(2)**2+Y(2)**2+Z(2)**2)
      ALC=-(CHI*AS(2)+AS(2)**2+R(2)**2)
      BLC=-XMU*(CHI*BS(2)+2.0*AS(2)*BS(2))
      CLC=-XMU**2*BS(2)**2
      RLC(2)=1.0
C
C     ITERATIVE LOOP FOR DETERMINING APPLICABLE REAL ROOT OF RLC(2)
C
 59   DO 69 I=1,25
      REX(I)=RLC(2)**8+ALC*RLC(2)**6+BLC*RLC(2)**3+CLC
      CT1=ITIME
      PRINT 100,CT1
      PRINT 103, REX(I),RLC(2),I
103   FORMAT(1H0,$REX(I)=$E16.8*****$RLC(2)=$E16.8$*****,I=$I2)
      ITIME=0
      IF(ABS(REX(I)-REX(I-1))-0.000000001) 70,70,61
 61   IF(ABS(REX(I))-0.000000001) 70,70,63
 63   IF(I-1) 68,68,64
 64   RPR=(REX(I)-REX(I-1))/DELR
      IF(ABS(REX(I)/RPR-DELR)-0.000000001) 70,65,65
 65   DELR=-REX(I)/RPR
      GO TO 69
 68   DELR=0.05*RLC(2)
 69   RLC(2)=ABS(RLC(2)+DELR)
C
C     SOLVE FOR INERTIAL POSITION AND VELOCITY VECTORS
C
 70   P(2)=AS(2)+(XMU*BS(2)/RLC(2)**3)
      PV(2)=CS(2)+(XMU*DS(2)/RLC(2)**3)
      XLC(2)=P(2)*XL(2)-X(2)
      YLC(2)=P(2)*YL(2)-Y(2)
      ZLC(2)=P(2)*ZL(2)-Z(2)
      XLCV(2)=PV(2)*XL(2)+P(2)*XLV(2)-XV(2)
      YLCV(2)=PV(2)*YL(2)+P(2)*YLV(2)-YV(2)
      ZLCV(2)=PV(2)*ZL(2)+P(2)*ZLV(2)-ZV(2)
      CT2=ITIME
      PRINT 100,CT2
      PRINT 104, XLCV(2),YLCV(2),ZLCV(2)
104   FORMAT(1H0,$XLCV(2)=$E16.8,//,$YLCV(2)=$E16.8,//,$ZLCV(2)=$E16.8)
C
C     SOLUTION FOR CLASSICAL ELEMENTS
C
      ITIME=0
      RLC(2)=SQRT(XLC(2)**2+YLC(2)**2+ZLC(2)**2)
      RRDOT=XLC(2)*XLCV(2)+YLC(2)*YLCV(2)+ZLC(2)*ZLCV(2)
      RLCV(2)=RRDOT/RLC(2)
      V=SQRT(XLCV(2)**2+YLCV(2)**2+ZLCV(2)**2)
      ALC=(RLC(2)*XMU)/(2.0*XMU-V**2*RLC(2))
      CSUBE=(1.0-RLC(2)/ALC)
```

```
      HX=YLC(2)*ZLCV(2)-ZLC(2)*YLCV(2)
      HZ=XLC(2)*YLCV(2)-YLC(2)*XLCV(2)
      VANGE=ATAN(SINV,COSV)
      SINHX=HX
      COSHY=-HY
      OMEGA=ATAN(SINHX,COSHY)
      EXP=SQRT(HX**2+HY**2)
      OINCL=ATAN(EXP,HZ)
      UNUM=-XLC(2)*SIN(OMEGA)*COS(OINCL)+YLC(2)*COS(OMEGA)*COS(OINCL)+
     CZLC(2)*SIN(OINCL)
      DEM=XLC(2)*COS(OMEGA)+YLC(2)*SIN(OMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANGE
      CT3=ITIME
      PRINT 100,CT3
      PRINT 107, ALC,ELC,TE,OMEGA,OINCL,W
107   FORMAT(1H0,$ALC=$E16.8,//,$ELC=$E16.8,//,$TE=$E16.8,//,
     1$OMEGA=$E16.8,//,$OINCL=$E16.8,//,$W=$E16.8,//)
100   FORMAT($MILLISEC=$I5)
 74   CONTINUE
      GO TO 75
S2050 PZE
S     MIN         ITIME
S     BCI         *POPOS
 75       END
```

APPENDIX I
DOUBLE R-ITERATION PODM, ANGLES ONLY

Given α_{ti}, δ_{ti}, t_i, ϕ_i, λ_{Ei}, H_i, for i = 1, 2, 3, and the constants $d\theta/dt$, f, a_e, μ, and k_e, proceed as follows:

$$\tau_1 = k_e (t_1 - t_2) \tag{259}$$

$$\tau_3 = k_e (t_3 - t_2) \tag{260}$$

$$Tu = \frac{J.D. - 2415020}{36525} \tag{261}$$

$$\theta_{g0} = 99^\circ.6909833 + 36000^\circ.7689\ Tu + 0^\circ.00038708\ Tu^2 \tag{262}$$

For i = 1, 2, 3, compute:

$$L_{xi} = \cos \delta_{ti} \cos \alpha_{ti} \tag{263}$$

$$L_{yi} = \cos \delta_{ti} \sin \alpha_{ti} \tag{264}$$

$$L_{zi} = \sin \delta_{ti} \tag{265}$$

$$G_{1i} = \frac{a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \tag{266}$$

$$G_{2i} = \frac{(1 - f)^2 a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \qquad (267)$$

$$\theta_i = \theta_{g0} + \frac{d\theta}{dt} (t_i - t_0) + \lambda_{Ei} \qquad (268)$$

$$X_i = - G_{1i} \cos \phi_i \cos \theta_i \qquad (269)$$

$$Y_i = - G_{1i} \cos \phi_i \sin \theta_i \qquad (270)$$

$$Z_i = - G_{2i} \sin \phi_i \qquad (271)$$

$$C_{\psi i} = 2\underline{L}_i \cdot \underline{R}_i \quad , \qquad i = 1, 2, 3 \qquad (272)$$

As a first approximation, set

$$r_1 = r_{1g} \quad , \qquad r_2 = r_{2g} \qquad (273)$$

For near-Earth orbits, set

$$r_{1g} = r_{2g} = 1.1 \text{ e.r.} \qquad (274)$$

and compute ρ_i from

$$\rho_i = \frac{1}{2}\left[- C_{\psi i} + \sqrt{C_{\psi i}^{\;2} - 4\left(R_i^{\;2} - r_i^{\;2}\right)}\;\right] \tag{275}$$

Continue calculating with

$$\underline{r}_i = \rho_i \underline{L}_i - \underline{R}_i \quad , \qquad i = 1, 2 \tag{276}$$

Compute $\tilde{\underline{W}}$ as

$$\tilde{W}_x = \frac{y_1 z_2 - y_2 z_1}{r_1 r_2} \tag{277}$$

$$\tilde{W}_y = \frac{x_2 z_1 - x_1 z_2}{r_1 r_2} \tag{278}$$

$$\tilde{W}_z = \frac{x_1 y_2 - x_2 y_1}{r_1 r_2} \tag{279}$$

Continue calculating with

$$\rho_3 = \frac{\underline{R}_3 \cdot \tilde{\underline{W}}}{\underline{L}_3 \cdot \tilde{\underline{W}}} \tag{280}$$

$$\underline{r}_3 = \rho_3 \underline{L}_3 - \underline{R}_3 \tag{281}$$

$$r_3 = \sqrt{\underline{r}_3 \cdot \underline{r}_3} \tag{282}$$

$$\cos\,(\nu_j - \nu_k) = \frac{\underline{r}_j \cdot \underline{r}_k}{r_j r_k} \qquad j = 2,\ 3,\ k = 1,\ 2 \tag{283}$$

If $W_z \geq 0$, calculate

$$\sin\,(\nu_j - \nu_k) = \frac{x_k y_j - x_j y_k}{|x_k y_j - x_j y_k|}\sqrt{1 - \cos^2\,(\nu_j - \nu_k)} \tag{284}$$

If $W_z < 0$, calculate

$$\sin\,(\nu_j - \nu_k) = -\,\frac{x_k y_j - x_j y_k}{x_k y_j - x_j y_k}\sqrt{1 - \cos^2\,(\nu_j - \nu_k)} \tag{285}$$

If $\nu_3 - \nu_1 > \pi$, determine p from

$$c_1 = \left(\frac{r_2}{r_1}\right)\frac{\sin\,(\nu_3 - \nu_2)}{\sin\,(\nu_3 - \nu_1)} \tag{286}$$

$$c_3 = \left(\frac{r_2}{r_3}\right)\frac{\sin\,(\nu_2 - \nu_1)}{\sin\,(\nu_3 - \nu_1)} \tag{287}$$

$$p = \frac{c_1 r_1 + c_3 r_3 - r_2}{c_1 + c_3 - 1} \tag{288}$$

If $\nu_3 - \nu_1 \leq \pi$, determine p from

$$c_1 = \left(\frac{r_1}{r_2}\right)\frac{\sin(\nu_3 - \nu_1)}{\sin(\nu_3 - \nu_2)} \tag{289}$$

$$c_3 = \left(\frac{r_1}{r_3}\right)\frac{\sin(\nu_2 - \nu_1)}{\sin(\nu_3 - \nu_2)} \tag{290}$$

$$p = \frac{r_1 + c_3 r_3 - c_1 r_2}{1 + c_3 - c_1} \tag{291}$$

Continue calculating with

$$e \cos \nu_i = \frac{p}{r_i} - 1 \quad , \qquad i = 1, 2, 3 \tag{292}$$

and for $\nu_2 - \nu_1 \neq \pi$, obtain

$$e \sin \nu_2 = - \frac{\cos(\nu_2 - \nu_1)(e \cos \nu_2) + (e \cos \nu_1)}{\sin(\nu_2 - \nu_1)} \tag{293}$$

or, if $\nu_2 - \nu_1 = \pi$, obtain

$$e \sin \nu_2 = \frac{\cos(\nu_3 - \nu_2)(e \cos \nu_2) - (e \cos \nu_3)}{\sin(\nu_3 - \nu_1)} \tag{294}$$

Evaluate

$$e = \sqrt{(e \cos \nu_2)^2 + (e \sin \nu_2)^2} \tag{295a}$$

$$a = \frac{p}{1 - e^2} \tag{295b}$$

For orbit determination in this paper, $e^2 < 1$, therefore continue calculating with

$$n = k_e\sqrt{\frac{\mu}{a^3}} \tag{296}$$

$$S_e = \frac{r_2}{p}\sqrt{1 - e^2}\ e \sin \nu_2 \tag{297}$$

$$C_e = \frac{r_2}{p}\ (e^2 + e^2 \cos \nu_2) \tag{298}$$

$$\sin (E_3 - E_2) = \frac{r_3}{\sqrt{ap}} \sin (\nu_3 - \nu_2) - \frac{r_3}{p} \left[1 - \cos (\nu_3 - \nu_2)\right] S_e \tag{299}$$

$$\cos (E_3 - E_2) = 1 - \frac{r_3 r_2}{ap} \left[1 - \cos (\nu_3 - \nu_2)\right] \tag{300}$$

$$\sin (E_2 - E_1) = \frac{r_1}{\sqrt{ap}} \sin (\nu_2 - \nu_1) + \frac{r_1}{p} \left[1 - \cos (\nu_2 - \nu_1)\right] S_e \tag{301}$$

$$\cos (E_2 - E_1) = 1 - \frac{r_2 r_1}{ap} \left[1 - \cos (\nu_2 - \nu_1)\right] \tag{302}$$

$$M_3 - M_2 = E_3 - E_2 + 2S_e \sin^2 \left(\frac{E_3 - E_2}{2}\right) - C_e \sin (E_3 - E_2) \tag{303}$$

$$M_1 - M_2 = -(E_2 - E_1) + 2S_e \sin^2\left(\frac{E_2 - E_1}{2}\right) + C_e \sin(E_2 - E_1) \tag{304}$$

$$F_1 = \tau_1 - k_e\left(\frac{M_1 - M_2}{n}\right) \tag{305}$$

$$F_2 = \tau_3 - k_e\left(\frac{M_3 - M_2}{n}\right) \tag{306}$$

Save F_1, F_2, r_1; increment r_1 by Δr_1 (about 4 percent); and return to equation (275). The end result of this calculation will be $F_1\ (r_1 + \Delta r_1,\ r_2)$, $F_2\ (r_1 + \Delta r_1,\ r_2)$, so that

$$\frac{\partial F_1}{\partial r_1} \simeq \frac{F_1\ (r_1 + \Delta r_1, r_2) - F_1\ (r_1, r_2)}{\Delta r_1} \tag{307}$$

$$\frac{\partial F_2}{\partial r_1} \simeq \frac{F_2\ (r_1 + \Delta r_1, r_2) - F_2\ (r_1, r_2)}{\Delta r_1} \tag{308}$$

Save $\partial F_1/\partial r_1$, $\partial F_2/\partial r_1$; set r_1 back to the original value; increment r_2 by Δr_2 (about 4 percent); and return to equation (275). The end result of this calculation will be $F_1\ (r_1,\ r_2 + \Delta r_2)$, $F_2\ (r_1,\ r_2 + \Delta r_2)$, so that

$$\frac{\partial F_1}{\partial r_2} \simeq \frac{F_1\ (r_1, r_2 + \Delta r_2) - F_1\ (r_1, r_2)}{\Delta r_2} \tag{309}$$

$$\frac{\partial F_2}{\partial r_2} \simeq \frac{F_2\ (r_1, r_2 + \Delta r_2) - F_2\ (r_1, r_2)}{\Delta r_2} \tag{310}$$

Continue calculating with

$$\Delta = \left(\frac{\partial F_1}{\partial r_1}\right) \left(\frac{\partial F_2}{\partial r_2}\right) - \left(\frac{\partial F_2}{\partial r_1}\right) \left(\frac{\partial F_1}{\partial r_2}\right) \tag{311}$$

$$\Delta_1 = \left(\frac{\partial F_2}{\partial r_2}\right) F_1 - \left(\frac{\partial F_1}{\partial r_2}\right) F_2 \tag{312}$$

$$\Delta_2 = \left(\frac{\partial F_1}{\partial r_1}\right) F_2 - \left(\frac{\partial F_2}{\partial r_1}\right) F_1 \tag{313}$$

$$\Delta r_1 = - \frac{\Delta_1}{\Delta} \tag{314}$$

$$\Delta r_2 = - \frac{\Delta_2}{\Delta} \tag{315}$$

Check to see if

$$|\Delta r_1| < \varepsilon \tag{316a}$$

$$|\Delta r_2| < \varepsilon \tag{316b}$$

where ε is a tolerance, i.e. 10^{-10}. If the test is not satisfied, let

$$(r_1)_{n+1} = (r_1)_n + \Delta r_1 \tag{317a}$$

$$(r_2)_{n+1} = (r_2)_n + \Delta r_2 \tag{317b}$$

and return to equation (275); if it is satisfied, continue calculating with

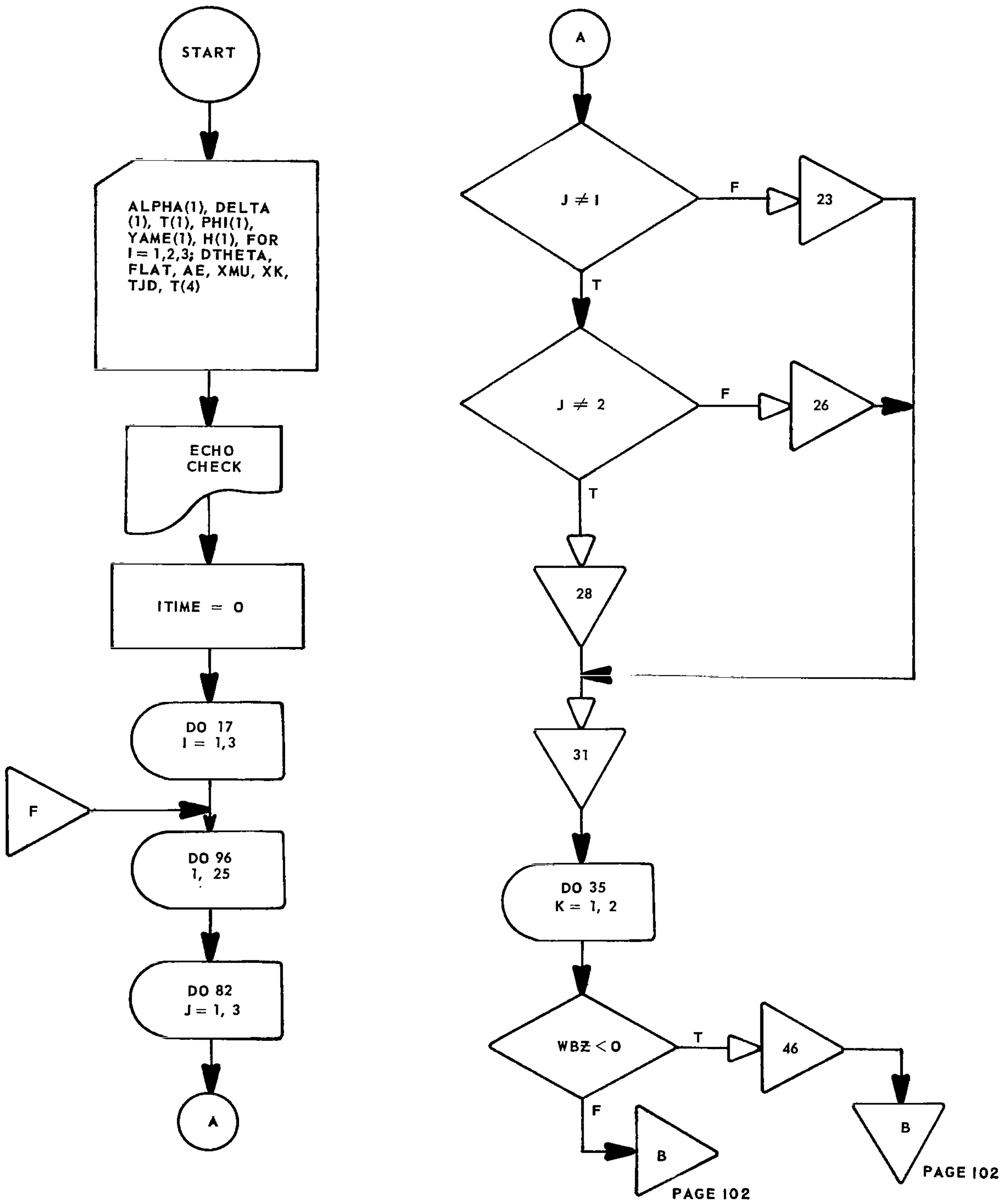
$$f = 1 - \frac{a}{r_2}\left[1 - \cos\left(E_3 - E_2\right)\right] \tag{318}$$

$$g = \tau_3 - \sqrt{\frac{a^3}{\mu}}\left[\left(E_3 - E_2\right) - \sin\left(E_3 - E_2\right)\right] \tag{319}$$

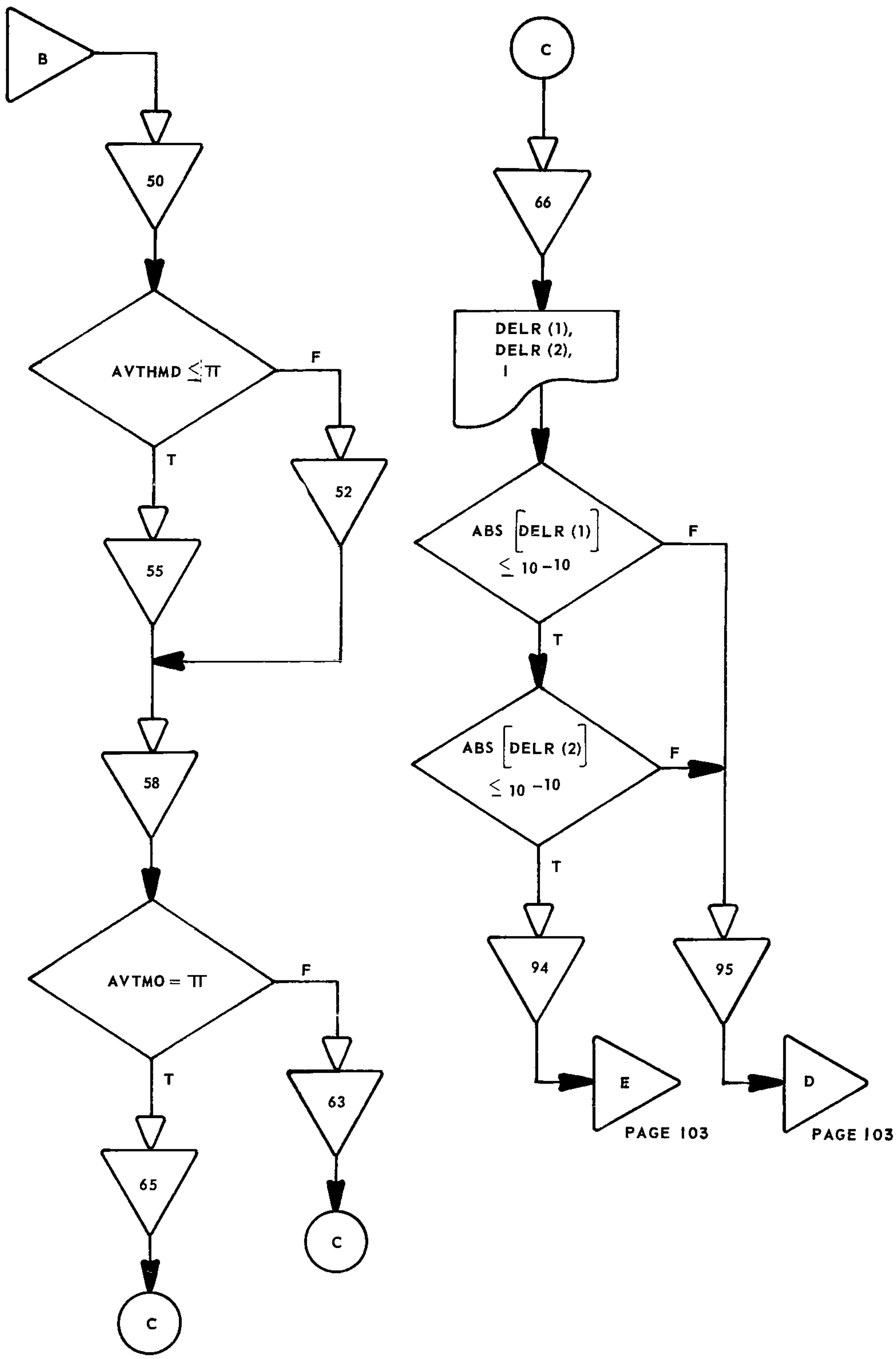
$$\dot{\underline{r}}_2 = \frac{\underline{r}_3 - f\underline{r}_2}{g} \tag{320}$$

Continue by calculating for the classical elements.

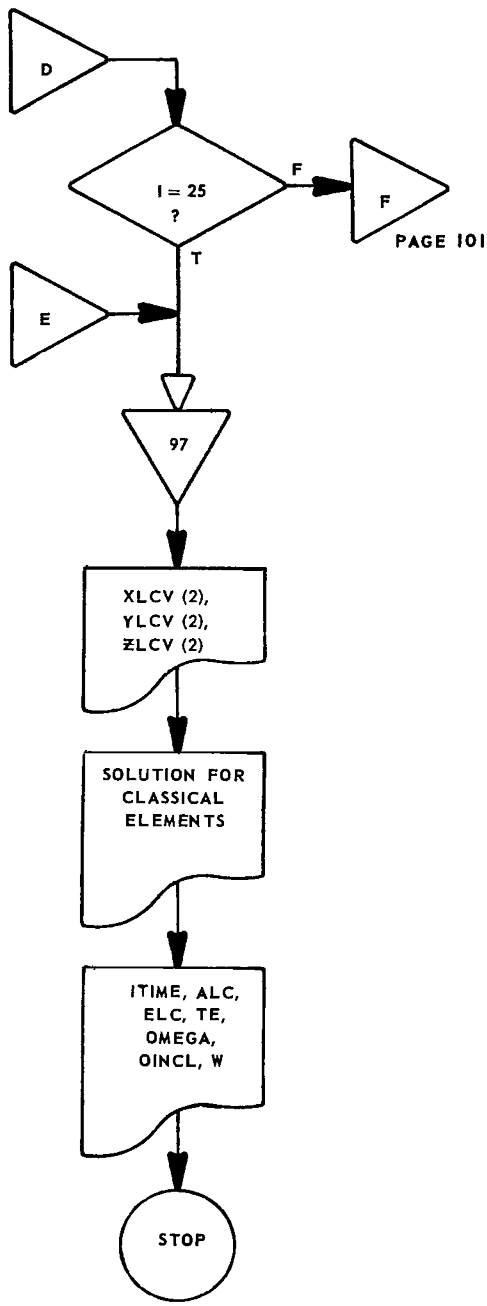
DOUBLE R-ITERATION FLOWCHART



DOUBLE R-ITERATION FLOWCHART (CONT'D)



DOUBLE R-ITERATION FLOWCHART (CONT'D)



```
C       DOUBLE-R ITERATION PRELIMINARY ORBIT DETERMINATION METHOD
C       ANGLES ONLY (ESCOBAL,PAGE 293)
C
        DO 119 N=1,25
C
        DIMENSION TAU(3),XL(3),YL(3),ZL(3),G1(3),G2(3),X(3),Y(3),Z(3),
       CTHETA(3),DEMG(3),RLC1(25),RLC2(25),RLC(3,3),CHI(3),R(3),P(3),
       CXLC(3),YLC(3),ZLC(3),C(3),F(3,3),DEL(3,3),PDEL(3),DELR(3),
       CXLCV(3),YLCV(3),ZLCV(3),RLCV(3),RLS(3)
        DIMENSION T(4),ALPHA(3),DELTA(3),YAME(3),PHI(3),H(3)
C
C       READ ANGLE INPUT DATA
C
        READ 108,FLAT,AE,XK,XMU,DTHETA
        READ 108,T(4),T(1),T(2),T(3),TJD
        READ 108,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2)
        READ 108,DELTA(3),YAME(1),YAME(2),YAME(3),PHI(1)
        READ 108,PHI(2),PHI(3),H(1),H(2),H(3)
108     FORMAT(5E16.8)
C
C       ECHO CHECK
C
        PRINT 110,FLAT,AE,XK,XMU,DTHETA,T(4),TJD,T(1),T(2),T(3)
110     FORMAT(1H0,$FLAT=$E16.8$**AE=$E16.8$**XK=$E16.8$**XMU=$E16.8,//,
       1$DTHETA=$E16.8$**T(4)=$E16.8$**TJD=$E16.8,//,
       1$T(1)=$E16.8$**T(2)=$E16.8$**T(3)=$E16.8)
        PRINT 111,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2),DELTA(3),
       CYAME(1),YAME(2),YAME(3)
111     FORMAT(1H0,$ALPHA(1)=$E16.8$**ALPHA(2)=$E16.8$**ALPHA(3)=$E16.8,
       1//,$DELTA(1)=$E16.8$**DELTA(2)=$E16.8$**DELTA(3)=$E16.8,//,
       1$YAME(1)=$E16.8$**YAME(2)=$E16.8$**YAME(3)=$E16.8)
        PRINT 112,PHI(1),PHI(2),PHI(3),H(1),H(2),H(3)
112     FORMAT(1H0,$PHI(1)=$E16.8$**PHI(2)=$E16.8$**PHI(3)=$E16.8,//,
       1$H(1)=$E16.8$**H(2)=$E16.8$**H(3)=$E16.8)
C
C       BEGIN COMPUTATIONS
C
C       ALL META-SYMBOL IS ITIME SUBROUTINE
C
        ITIME=0
S       LDA        2058
S       STA        0205
S       BRU        2008
S205    BRM        2050S
S200    EOM        020020
S       POT  =  01202000
S       EIR
  1     TAU(1)=XK*(T(1)-T(2))
        TAU(3)=XK*(T(3)-T(2))
        TU=(TJD-2415020.0)/36525.0
        GTHETA=(99.6909833+36000.7689*TU+.00038708*TU**2)/57.29577951
  5     DO 17 I=1,3
        XL(I)=COS(DELTA(I))*COS(ALPHA(I))
```

```
      YL(I)=COS(DELTA(I))*SIN(ALPHA(I))
      ZL(I)=SIN(DELTA(I))
      DENG(I)=SQRT(1.0-(2.0*FLAT-FLAT**2)*(SIN(PHI(I)))**2)
      G1(I)=AE/DENG(I)+H(I)
      G2(I)=((1.0-FLAT)**2*AE)/DENG(I)+H(I)
      THETA(I)=GTHETA+DTHETA*(T(I)-T(4))+YAME(I)
      X(I)=-G1(I)*COS(PHI(I))*COS(THETA(I))
      Y(I)=-G1(I)*COS(PHI(I))*SIN(THETA(I))
      Z(I)=-G2(I)*SIN(PHI(I))
      CHI(I)=-2.0*(XL(I)*X(I)+YL(I)*Y(I)+ZL(I)*Z(I))
 17   R(I)=SQRT(X(I)**2+Y(I)**2+Z(I)**2)
      RLC1(1)=1.0
      RLC2(1)=1.0
C
C     ITERATION FOR DETERMINING THE SCALAR OF THE INERTIAL POSITIO
C
 18   DO 36 I=1,25
 21   DO 32 J=1,3
 22   IF(J-1) 23,23,25
 23   RLC(1,1)=RLC1(I)
 24   RLC(2,1)=RLC2(I)
      GO TO 31
 25   IF(J-2) 26,26,28
 26   RLC(1,2)=RLC1(I)*1.04
      RLC(2,2)=RLC2(I)
      GO TO 31
 28   IF(J-3) 29,29,31
 29   RLC(1,3)=RLC1(I)
      RLC(2,3)=RLC2(I)*1.04
 31   DO 35 K=1,2
 32   P(K)=0.5*(-CHI(K)+SQRT(ABS(CHI(K)**2-4.0*(R(K)**2-RLC(K,J)**2))))
      XLC(K)=P(K)*XL(K)-X(K)
      YLC(K)=P(K)*YL(K)-Y(K)
 35   ZLC(K)=P(K)*ZL(K)-Z(K)
      WEX=(YLC(1)*ZLC(2)-YLC(2)*ZLC(1))/(RLC(1,J)*RLC(2,J))
      WEY=(XLC(2)*ZLC(1)-XLC(1)*ZLC(2))/(RLC(1,J)*RLC(2,J))
      WEZ=(XLC(1)*YLC(2)-XLC(2)*YLC(1))/(RLC(1,J)*RLC(2,J))
      P(3)=(X(3)*WEX+Y(3)*WEY+Z(3)*WEZ)/(XL(3)*WEX+YL(3)*WEY+ZL(3)*WEZ)
      XLC(3)=P(3)*XL(3)-X(3)
      YLC(3)=P(3)*YL(3)-Y(3)
      ZLC(3)=P(3)*ZL(3)-Z(3)
      RLS(3)=SQRT(XLC(3)**2+YLC(3)**2+ZLC(3)**2)
      CVTMP=(XLC(2)*XLC(1)+YLC(2)*YLC(1)+ZLC(2)*ZLC(1))/(RLC(1,J)*
     CRLC(2,J))
      CVTHMT=(XLC(3)*XLC(2)+YLC(3)*YLC(2)+ZLC(3)*ZLC(2))/(RLS(3)*
     CRLC(2,J))
      CVTHMP=(XLC(3)*XLC(1)+YLC(3)*YLC(1)+ZLC(3)*ZLC(1))/(RLC(1,J)*
     CRLS(3))
      SVTMP=(XLC(1)*YLC(2)-XLC(2)*YLC(1))/ABS(XLC(1)*YLC(2)-XLC(2)*
     CYLC(1))*SQRT(ABS(1.0-CVTMP**2))
      SVTHMT=(XLC(2)*YLC(3)-XLC(3)*YLC(2))/ABS(XLC(2)*YLC(3)-XLC(3)*
     CYLC(2))*SQRT(ABS(1.0-CVTHMT**2))
      SVTHMP=(XLC(1)*YLC(3)-XLC(3)*YLC(1))/ABS(XLC(1)*YLC(3)-XLC(3)*
     CYLC(1))*SQRT(ABS(1.0-CVTHMP**2))
```

```
      IF(ABZ-0.0000000000) 46,50,50
46    SVTMB=-SVTMB
      SVTHMT=-SVTHMT
      SVTHMB=-SVTHMB
50    AVTHMB=ATAN(SVTHMB,CVTHMB)
51    IF(AVTHMB-3.1415926536) 55,55,52
52    C(1)=RLC(2,J)/RLC(1,J)*SVTHMT/SVTHMB
      C(3)=RLC(2,J)/RLS(3)*SVTMB/SVTHMT
      PLC=(C(1)*RLC(1,J)+C(3)*RLS(3)-RLC(2,J))/(C(1)+C(3)-1.0)
      GO TO 58
55    C(1)=RLC(1,J)/RLC(2,J)*SVTHMB/SVTHMT
      C(3)=RLC(1,J)/RLS(3)*SVTMB/SVTHMT
      PLC=(RLC(1,J)+C(3)*RLS(3)-C(1)*RLC(2,J))/(1.0+C(3)-C(1))
58    ELCVB=PLC/RLC(1,J)-1.0
      ELCVT=PLC/RLC(2,J)-1.0
      ELCVTH=PLC/RLS(3)-1.0
      AVTMB=ATAN(SVTMB,CVTMB)
62    IF(AVTMB-3.1415926536) 63,65,63
63    ELSVT=(-CVTMB*ELCVT+ELCVB)/SVTMB
64    GO TO 66
65    ELSVT=(CVTHMT*ELCVT-ELCVTH)/SVTHMT
66    ELC=SQRT(ELCVT**2+ELSVT**2)
      ALC=PLC/(1.0-ELC**2)
      ETA=XK*SQRT(ABS(XMU/ALC**3))
      SSUBE=(RLC(2,J)/PLC)*SQRT(ABS(1.0-ELC**2))*ELSVT
      CSUBE=(RLC(2,J)/PLC)*(ELC**2+ELCVT)
      SETHMT=(RLS(3)/SQRT(ABS(ALC*PLC)))*SVTHMT-(RLS(3)/PLC)*
     C(1.0-CVTHMT)*SSUBE
      CETHMT=1.0-(RLS(3)*RLC(2,J)/(ALC*PLC))*(1.0-CVTHMT)
      SETMB=(RLC(1,J)/SQRT(ABS(ALC*PLC)))*SVTMB+RLC(1,J)/PLC*
     C(1.0-CVTMB)*SSUBE
      CETMB=1.0-((RLC(2,J)*RLC(1,J)/(ALC*PLC))*(1.0-CVTMB))
      ETHMT=ATAN(SETHMT,CETHMT)
      ETMB=ATAN(SETMB,CETMB)
      ATHMT=ETHMT+(2.0*SSUBE*(SIN(ETHMT/2.0))**2)-CSUBE*SETHMT
      ABMT=-ETMB+2.0*SSUBE*(SIN(ETMB/2.0))**2+CSUBE*SETMB
      F(1,J)=TAU(1)-XK*(ABMT/ETA)
82    F(2,J)=TAU(3)-XK*(ATHMT/ETA)
      DEL(1,1)=(F(1,2)-F(1,1))/(RLC(1,2)-RLC(1,1))
      DEL(2,1)=(F(2,2)-F(2,1))/(RLC(1,2)-RLC(1,1))
      DEL(1,2)=(F(1,3)-F(1,1))/(RLC(2,3)-RLC(2,1))
      DEL(2,2)=(F(2,3)-F(2,1))/(RLC(2,3)-RLC(2,1))
      PADEL=DEL(1,1)*DEL(2,2)-DEL(2,1)*DEL(1,2)
      PDEL(1)=DEL(2,2)*F(1,3)-DEL(1,2)*F(2,3)
      PDEL(2)=DEL(1,1)*F(2,2)-DEL(2,1)*F(1,2)
      DELR(1)=-PDEL(1)/PADEL
      DELR(2)=-PDEL(2)/PADEL
      CT1=ITIME
      PRINT 100,CT1
      PRINT 106, DELR(1),I,DELR(2),I
106   FORMAT(1H0,$DELR(1)=$F16.8$*****I=$I2,//,$DELR(2)=$F16.8$*****I=
     1$I2)
      ITIME=0
92    IF(ABS(DELR(1))-0.0000000001) 93,93,95
```

```
 93     IF(ABS(DELR(2))-0.0000000001) 94,94,95
 94     GØ TØ 97
 95     RLC1(I+1)=ABS(RLC1(I)+DELR(1))
        RLC2(I+1)=ABS(RLC2(I)+DELR(2))
 96     CØNTINUE
C
C       SØLVE FØR INERTIAL VELØCITY VECTØR
C
 97     RLCF=RLC2(I)
        FLC=1.0-(ALC/RLCF)*(1.0-CØS(ETHMT))
        GLC=TAU(3)-SQRT(ALC**3/XMU)*(ETHMT-SETHMT)
        XLCV(2)=(XLC(3)-FLC*XLC(2))/GLC
        YLCV(2)=(YLC(3)-FLC*YLC(2))/GLC
        ZLCV(2)=(ZLC(3)-FLC*ZLC(2))/GLC
        CT2=ITIME
        PRINT 100,CT2
        PRINT 107, XLCV(2),YLCV(2),ZLCV(2)
107     FØRMAT(1H0,*XLCV(2)=*E16.8,//,*YLCV(2)=*E16.8,//,*ZLCV(2)=*E16.8)
C
C
        ITIME=0
C       SØLUTIØN FØR CLASSICAL ELEMENTS
        RLS(2)=SQRT(XLC(2)**2+YLC(2)**2+ZLC(2)**2)
        RRDØT=XLC(2)*XLCV(2)+YLC(2)*YLCV(2)+ZLC(2)*ZLCV(2)
        RLCV(2)=RRDØT/RLS(2)
        V=SQRT(XLCV(2)**2+YLCV(2)**2+ZLCV(2)**2)
        ALC=(RLS(2)*XMU)/(2.0*XMU-V**2*RLS(2))
        CSUBE=(1.0-RLS(2)/ALC)
        SSUBE=(RLCV(2)*RLS(2))/SQRT(XMU*ALC)
        ELC=SQRT(SSUBE**2+CSUBE**2)
        CØSE=(ALC-RLS(2))/(ALC*ELC)
        XSUBV=ALC*(CØSE-ELC)
        CØSV=XSUBV/RLS(2)
        SINV=SQRT(RLS(2)**2-XSUBV**2)/RLS(2)
        SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
        E=ATAN(SINE,CØSE)
        TE=T(2)-((E-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
        HX=YLC(2)*ZLCV(2)-ZLC(2)*YLCV(2)
        HY=-(XLC(2)*ZLCV(2)-ZLC(2)*XLCV(2))
        HZ=XLC(2)*YLCV(2)-YLC(2)*XLCV(2)
        VANGE=ATAN(SINV,CØSV)
        SINHX=HX
        CØSHY=-HY
        ØMEGA=ATAN(SINHX,CØSHY)
        EXP=SQRT(HX**2+HY**2)
        ØINCL=ATAN(EXP,HZ)
        UNUM=-XLC(2)*SIN(ØMEGA)*CØS(ØINCL)+YLC(2)*CØS(ØMEGA)*CØS(ØINCL)+
       CZLC(2)*SIN(ØINCL)
        DEM=XLC(2)*CØS(ØMEGA)+YLC(2)*SIN(ØMEGA)
        U=ATAN(UNUM,DEM)
        W=U-VANGE
        CT3=ITIME
        PRINT 100,CT3
100     FØRMAT(*MILLISEC=*I8)
```

```
      PRINT 109, ALC,ELC,TE,ØMEGA,ØINCL,W
109   FØRMAT(1H0,$ALC=$E16.8,///,$ELC=$E16.8,///,$TE=$E16.8,///,
     1$ØMEGA=$E16.8,///,$ØINCL=$E16.8,///,$W=$E16.8,///)
119   CØNTINUE
      GØ TØ 120
S2050 PZE
S     MIN        ITIME
S     BRU        *2050S
120   END
```

APPENDIX J
MODIFIED LAPLACIAN PODM, MIXED DATA

Given the mixed data $\dot{\rho}_i$, α_{ti}, t_i, δ_{ti} for i = 1, 2, 3. along with ϕ_i, λ_{Ei}, H_i and the constants, a_e, k_e, μ, f, $d\theta/dt$, proceed as follows:

$$\tau_1 = k_e \, (t_1 - t_2) \tag{321}$$

$$\tau_3 = k_e \, (t_3 - t_2) \tag{322}$$

$$S_1 = \frac{-\tau_3}{\tau_1 \, (\tau_1 - \tau_3)} \tag{323}$$

$$S_2 = -\frac{(\tau_3 + \tau_1)}{\tau_1 \tau_3} \tag{324}$$

$$S_3 = \frac{-\tau_1}{\tau_3 \, (\tau_3 - \tau_1)} \tag{325}$$

$$Tu = \frac{J.D. - 2415020}{36525} \tag{326}$$

$$\theta_{g0} = 99^{\circ}\!.6909833 + 36000^{\circ}\!.7689 \; Tu + 0^{\circ}\!.00038708 \; Tu^2 \tag{327}$$

For i = 1, 2, 3, compute

$$\theta_i = \theta_{g0} + \frac{d\theta}{dt} (t_i - t_o) + \lambda_{Ei} \tag{328}$$

$$G_{1i} = \frac{a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \tag{329}$$

$$G_{2i} = \frac{(1 - f)^2 \, a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \tag{330}$$

$$L_{xi} = \cos \delta_{ti} \cos \alpha_{ti} \tag{331}$$

$$L_{yi} = \cos \delta_{ti} \sin \alpha_{ti} \tag{332}$$

$$L_{zi} = \sin \delta_{ti} \tag{333}$$

Continue calculating with

$$\ddot{\rho}_2 = S_1 \dot{\rho}_1 + S_2 \dot{\rho}_2 + S_3 \dot{\rho}_3 \tag{334}$$

$$\dot{\underline{L}}_2 = S_1 \underline{L}_1 + S_2 \underline{L}_2 + S_3 \underline{L}_3 \tag{335}$$

$$X_2 = - G_{12} \cos \phi_2 \cos \theta_2 \tag{336}$$

$$Y_2 = - G_{12} \cos \phi_2 \sin \theta_2 \tag{337}$$

$$Z_2 = - G_{22} \sin \phi_2 \tag{338}$$

$$\dot{\underline{R}}_2 = \frac{1}{k_e} \begin{bmatrix} -Y_2 \\ X_2 \\ 0 \end{bmatrix} \frac{d\theta}{dt} \tag{339}$$

$$\ddot{\underline{R}}_2 = \frac{1}{k_e^2} \begin{bmatrix} -X_2 \\ -Y_2 \\ 0 \end{bmatrix} \left(\frac{d\theta}{dt}\right)^2 \tag{340}$$

$$A = \ddot{\rho}_2 - (\underline{L}_2 \cdot \ddot{\underline{R}}_2) \tag{341}$$

$$B = -\mu\ (\underline{L}_2 \cdot \underline{R}_2) \tag{342}$$

$$C = \dot{\underline{L}}_2 \cdot \dot{\underline{L}}_2 \tag{343}$$

$$D = -\mu \tag{344}$$

$$C_\psi = -2\ (\underline{L}_2 \cdot \underline{R}_2) \tag{345}$$

As a first approximation, set $r_2 = r_{2G}$, where r_{2G} is an assumed value of r_2, i.e., 1.1 e.r., and initiate the following iterative scheme:

$$\rho_2 = \frac{A + \left(\dfrac{B}{r_2^3}\right)}{C + \left(\dfrac{D}{r_2^3}\right)} \tag{346}$$

$$F(r_2) = \rho_2^{\,2} + \rho_2 C_\psi + R_2^{\,2} - r_2^{\,2} \tag{347}$$

$$F'(r_2) = \left(\frac{3}{r_2^{\,4}}\right) \frac{(2\rho_2 + C_\psi)(D\rho_2 - B)}{C + \left(\frac{D}{r_2^{\,3}}\right)} - 2r_2 \tag{348}$$

and obtain a better value of r_2, that is,

$$(r_2)_{n+1} = (r_2)_n - \frac{F\left[(r_2)n\right]}{F'\left[(r_2)_n\right]} \quad , \qquad n = 1, 2, \ldots, q \tag{349}$$

If the improved value of r_2 does not vary, that is,

$$\left|(r_2)_{n+1} - (r_2)_n\right| < \varepsilon \tag{350}$$

where ε is a specified tolerance, i.e., 10^{-10}, proceed to equation (351); if not, return to equation (346) and using the latest value of r_2, repeat equational loop (347) to (349).

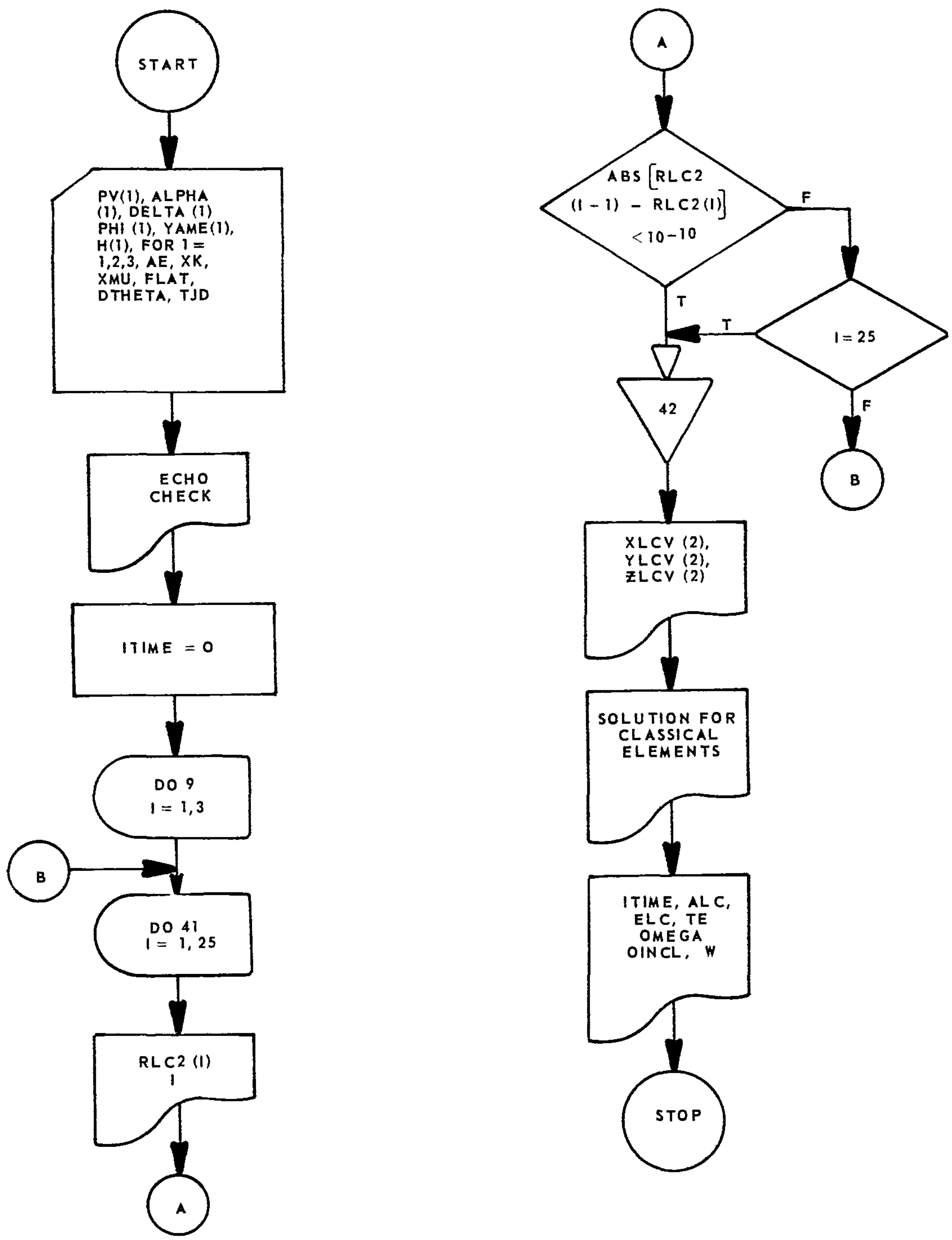
Continue calculating with

$$\underline{r}_2 = \rho_2 \, \underline{L}_2 - \underline{R}_2 \tag{351}$$

$$\dot{\underline{r}}_2 = \dot{\rho}_2 \underline{L}_2 + \rho_2 \dot{\underline{L}}_2 - \dot{\underline{R}}_2 \tag{352}$$

Continue by calculating for classical elements.

MODIFIED LAPLACIAN FLOWCHART



```
C      MODIFIED LAPLACIAN PRELIMINARY ORBIT DETERMINATION METHOD
C      RANGE RATE AND ANGLES (ESCOBAL,PAGE 297)
C
       DO 59 N=1,25
C
       DIMENSION TAU(3),S(3),XL(3),YL(3),ZL(3),PA(3),XLV(3),YLV( )
      CZLV(3),S1(3),X(3),Y(3),Z(3),R(3),XV(3),YV(3),ZV(3),  (3),  (3),
      CYA(3),ZA(3),RLCR(25),P(3),XLC(3),YLC(3),ZLC(3),XLCV(3),YLCV(3),
      CZLCV(3),RLC(3),RLCV(3),G2(3),SEMS(3),THETA(3)
       DIMENSION T(4),ALPHA(3),DELTA(3),YAME(3),PHI(3),H(3),PV(3)
C
C      READ RANGE RATE AND ANGULAR INPUT DATA
C
       READ 108,FLAT,AE,XK,XMU,DTHETA
       READ 108,T(4),T(1),T(2),T(3),TJD
       READ 108,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2)
       READ 108,DELTA(3),YAME(1),YAME(2),YAME(3),PHI(1)
       READ 108,PHI(2),PHI(3),H(1),H(2),H(3)
       READ 109,PV(1),PV(2),PV(3)
108    FORMAT(5E16.8)
109    FORMAT(3E16.8)
C
C      ECHO CHECK
C
       PRINT 110,FLAT,AE,XK,XMU,DTHETA,T(4),TJD,T(1),T(2),T(3)
110    FORMAT(1H0,$FLAT=$E16.8$**AE=$E16.8$**XK=$E16.8$**XMU=$E16.8,//,
      1$DTHETA=$E16.8$**T(4)=$E16.8$**TJD=$E16.8,//,
      1$T(1)=$E16.8$**T(2)=$E16.8$**T(3)=$E16.8)
       PRINT 111,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2),DELTA(3),
      CYAME(1),YAME(2),YAME(3)
111    FORMAT(1H0,$ALPHA(1)=$E16.8$**ALPHA(2)=$E16.8$**ALPHA(3)=$E16.8,
      1//,$DELTA(1)=$E16.8$**DELTA(2)=$E16.8$**DELTA(3)=$E16.8,//,
      1$YAME(1)=$E16.8$**YAME(2)=$E16.8$**YAME(3)=$E16.8)
       PRINT 112,PHI(1),PHI(2),PHI(3),H(1),H(2),H(3),PV(1),PV(2),PV(3)
112    FORMAT(1H0,$PHI(1)=$E16.8$**PHI(2)=$E16.8$**PHI(3)=$E16.8,//,
      1$H(1)=$E16.8$**H(2)=$E16.8$**H(3)=$E16.8,//,
      1$PV(1)=$E16.8$**PV(2)=$E16.8$**PV(3)=$E16.8)
C
C      BEGIN COMPUTATIONS
C
C      ALL META-SYMBOL IS ITIME SUBROUTINE
C
       ITIME=0
S      LDA        205S
S      STA        0205
S      BRU        200S
S205   BRM        2050S
S200   EOM        030020
S      POT  =  0002020  0
S      EIR
       TAU(1)=XK*(T(1)-T(2))
       TAU(3)=XK*(T(3)-T(2))
       S(1)=-TAU(3)/(TAU(1)*(TAU(1)-TAU(3)))
```

```
      S(2)=-(TAU(3)+TAU(1))/(TAU(1)*TAU(3))
      S(3)=-TAU(1)/(TAU(3)*(TAU(3)-TAU(1)))
      TU=(TJD-2415020.0)/36525.0
      GTHETA=(99.6909833+36000.7689*TU+0.00038708*TU**2)/57.2957795131
 6    DO 9 I=1,3
      XL(I)=COS(DELTA(I))*COS(ALPHA(I))
      YL(I)=COS(DELTA(I))*SIN(ALPHA(I))
      ZL(I)=SIN(DELTA(I))
      DENS(I)=SQRT(1.0-(2.0*FLAT-FLAT**2)*(SIN(PHI(I)))**2)
      G1(I)=AE/DENS(I)+H(I)
      G2(I)=((1.0-FLAT)**2*AE)/DENS(I)+H(I)
      THETA(I)=GTHETA+DTHETA*(T(I)-T(4))+YANE(I)
      X(I)=-G1(I)*COS(PHI(I))*COS(THETA(I))
      Y(I)=-G1(I)*COS(PHI(I))*SIN(THETA(I))
      Z(I)=-G2(I)*SIN(PHI(I))
 9    R(I)=SQRT(X(I)**2+Y(I)**2+Z(I)**2)
      PA(2)=S(1)*PV(1)+S(2)*PV(2)+S(3)*PV(3)
      XLV(2)=S(1)*XL(1)+S(2)*XL(2)+S(3)*XL(3)
      YLV(2)=S(1)*YL(1)+S(2)*YL(2)+S(3)*YL(3)
      ZLV(2)=S(1)*ZL(1)+S(2)*ZL(2)+S(3)*ZL(3)
      XV(2)=-Y(2)*DTHETA/XK
      YV(2)=X(2)*DTHETA/XK
      ZV(2)=0.0
      XA(2)=-X(2)*DTHETA**2/XK**2
      YA(2)=-Y(2)*DTHETA**2/XK**2
      ZA(2)=0.0
      A=PA(2)-(XL(2)*XA(2)+YL(2)*YA(2)+ZL(2)*ZA(2))
      B=-XMU*(XL(2)*X(2)+YL(2)*Y(2)+ZL(2)*Z(2))
      C=XLV(2)**2+YLV(2)**2+ZLV(2)**2
      D=-XMU
      CHI=-2.0*(XL(2)*X(2)+YL(2)*Y(2)+ZL(2)*Z(2))
      RLC2(1)=1.0
C
C     ITERATION FOR RANGE VECTOR FOR CENTRAL DATE
C
 34   DO 41 I=1,25
      P(2)=(A+(B/RLC2(I)**3))/(C+(D/RLC2(I)**3))
      FRLC2=P(2)**2+P(2)*CHI+R(2)**2-RLC2(I)**2
      FPRLC2=(3.0/RLC2(I)**4)*(2.0*P(2)+CHI)*(D*P(2)-B)/(C+D/RLC2(I)**3)
     C-(2.0*RLC2(I))
      RLC2(I+1)=RLC2(I)-FRLC2/FPRLC2
      CT1=ITIME
      PRINT 100,CT1
      PRINT 103,RLC2(I),I
 103  FORMAT(1H0,$RLC2(I)=$E16.8,5X*****I=$I2)
      ITIME=0
      IF(ABS(RLC2(I+1)-RLC2(I))-0.0000000001) 42,42,41
 41   CONTINUE
C
C     COMPUTE INERTIAL POSITION AND VELOCITY VECTORS
C
 42   XLC(2)=P(2)*XL(2)-X(2)
      YLC(2)=P(2)*YL(2)-Y(2)
      ZLC(2)=P(2)*ZL(2)-Z(2)
```

```
      XLCV(2)=QV(2)*XL(2)+P(2)*YL (2)-XV(2)
      YLCV(2)=QV(2)*YL(2)+P(2)*YL (2)-YV(2)
      ZLCV(2)=QV(2)*ZL(2)+P(2)*ZL (2)-ZV(2)
      CT2=ITIME
      PRINT 100,CT2
      PRINT 104, XLCV(2),YLCV(2),ZLCV(2)
104   FORMAT(1H0,$XLCV(2)=$F16.8,//,$YLCV(2)=$F16.8,//,$ZLCV(2)=$F16.8)
C
C     SOLUTION FOR CLASSICAL ELEMENTS
C
      ITIME=0
      RLC(2)=SQRT(XLC(2)**2+YLC(2)**2+ZLC(2)**2)
      RDOT=XLC(2)*XLCV(2)+YLC(2)*YLCV(2)+ZLC(2)*ZLCV(2)
      RLCV(2)=RDOT/RLC(2)
      V=SQRT(XLCV(2)**2+YLCV(2)**2+ZLCV(2)**2)
      ALC=(RLC(2)*XMU)/(2.0*XMU-V**2*RLC(2))
      CSUBE=(1.0-RLC(2)/ALC)
      SSUBE=(RLCV(2)*RLC(2))/SQRT(XMU*ALC)
      ELC=SQRT(SSUBE**2+CSUBE**2)
      COSE=(ALC-RLC(2))/(ALC*ELC)
      XSUBV=ALC*(COSE-ELC)
      COSV=XSUBV/RLC(2)
      SINV=SQRT(RLC(2)**2-XSUBV**2)/RLC(2)
      SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
      E=ATAN(SINE,COSE)
      TF=T(2)-((E-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(2)*ZLCV(2)-ZLC(2)*YLCV(2)
      HY=-(XLC(2)*ZLCV(2)-ZLC(2)*XLCV(2))
      HZ=XLC(2)*YLCV(2)-YLC(2)*XLCV(2)
      VANGE=ATAN(SINV,COSV)
      SINHX=HX
      COSHY=-HY
      OMEGA=ATAN(SINHX,COSHY)
      EXP=SQRT(HX**2+HY**2)
      OINCL=ATAN(EXP,HZ)
      UNUM=-XLC(2)*SIN(OMEGA)*COS(OINCL)+YLC(2)*COS(OMEGA)*COS(OINCL)+
     CZLC(2)*SIN(OINCL)
      DEM=XLC(2)*COS(OMEGA)+YLC(2)*SIN(OMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANGE
      CT3=ITIME
      PRINT 100,CT3
      PRINT 107, ALC,ELC,TF,OMEGA,OINCL,W
107   FORMAT(1H0,$ALC=$E16.8,//,$ELC=$E16.8,//,$TF=$E16.8,//,
     1$OMEGA=$E16.8,//,$OINCL=$E16.8,//,$W=$E16.8,//)
100   FORMAT(1H MILLISEC=I8)
 59   CONTINUE
      GO TO 60
S2050 PZE
S     MIN        ITIME
S     BSH        *2050S
 60        END
```

APPENDIX K
R-ITERATION PODM, MIXED DATA

Given the mixed data ρ_i, α_{ti}, δ_{ti}, t_i, for i = 1, 2, 3, along with ϕ_i, λ_{Ei}, H_i and the constants a_e, k_e, μ, f, $d\theta/dt$, proceed as follows:

$$\tau_1 = k_e\,(t_1 - t_2) \tag{353}$$

$$\tau_3 = k_e\,(t_3 - t_2) \tag{354}$$

$$S_1 = \frac{-\tau_3}{\tau_1\,(\tau_1 - \tau_3)} \tag{355}$$

$$S_2 = -\left(\frac{\tau_3 + \tau_1}{\tau_1 \tau_3}\right) \tag{356}$$

$$S_3 = \frac{-\tau_1}{\tau_3\,(\tau_3 - \tau_1)} \tag{357}$$

$$Tu = \frac{J.D. - 2415020}{36525} \tag{358}$$

$$\theta_{g0} = 99^\circ.6909833 + 36000^\circ.7689\ Tu + 0^\circ.00038708\ Tu^2 \tag{359}$$

For i = 1, 2, 3, compute

$$L_{xi} = \cos\delta_{ti}\cos\alpha_{ti} \tag{360}$$

$$L_{yi} = \cos \delta_{ti} \sin \alpha_{ti} \tag{361}$$

$$L_{zi} = \sin \delta_{ti} \tag{362}$$

$$\theta_i = \theta_{g0} + \frac{d\theta}{dt}(t_i - t_0) + \lambda_{Ei} \tag{363}$$

$$G_{1i} = \frac{a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \tag{364}$$

$$G_{2i} = \frac{(1 - f)^2 a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi_i}} + H_i \tag{365}$$

$$X_i = - G_{1i} \cos \phi_i \cos \theta_i \tag{366}$$

$$Y_i = - G_{1i} \cos \phi_i \sin \theta_i \tag{367}$$

$$Z_i = - G_{2i} \sin \phi_i \tag{368}$$

$$\dot{\underline{R}}_i = \frac{1}{k_e} \begin{bmatrix} -Y_i \\ X_i \\ 0 \end{bmatrix} \frac{d\theta}{dt} \tag{369}$$

$$C_\psi = -2(L_{x2}X_2 + L_{y2}Y_2 + L_{z2}Z_2) \tag{370}$$

As a first approximation, set $r_2 = r_g$. For near-Earth orbits, set $r_g = 1.1$ and obtain

$$\rho_2 = \frac{1}{2}\left\{-C_\psi + \left[C_\psi^{\,2} - 4(R_2^{\,2} - r_2^{\,2})\right]^{\frac{1}{2}}\right\} \tag{371}$$

Compute the radius vector at the central date from

$$\underline{r}_2 = \rho_2\underline{L}_2 - \underline{R}_2 \tag{372}$$

Obtain the numerical derivative

$$\dot{\underline{L}}_2 = S_1\underline{L}_1 + S_2\underline{L}_2 + S_3\underline{L}_3 \tag{373}$$

Continue calculating with

$$\dot{\underline{r}}_2 = \dot{\rho}_2\underline{L}_2 + \rho_2\dot{\underline{L}}_2 - \dot{\underline{R}}_2 \tag{374}$$

$$\dot{r}_2 = \frac{\underline{r}_2 \cdot \dot{\underline{r}}_2}{r_2} \tag{375}$$

$$V_2 = \sqrt{\dot{\underline{r}}_2 \cdot \dot{\underline{r}}_2} \tag{376}$$

Utilize the derivatives of the f and g series to compute

$$\dot{f}_i = \dot{f}(V_2\ ,\ r_2\ ,\ \dot{r}_2\ ,\ \tau_i)\quad ,\qquad i = 1,\ 3 \tag{377}$$

$$\dot{g}_i = \dot{g}(V_2 , r_2 , \dot{r}_2 , \tau_i) , \qquad i = 1, 3 \tag{378}$$

Continue calculating with:

$$E = \dot{f}_1\dot{g}_3\underline{L}_1 \cdot \underline{L}_2 - \dot{f}_3\dot{g}_1\underline{L}_3 \cdot \underline{L}_2$$

$$+ \dot{g}_1\dot{g}_3\underline{L}_2 \cdot (\underline{L}_1 - \underline{L}_3) \tag{379}$$

$$A = \{\dot{f}_1\dot{g}_3\underline{L}_1 \cdot \underline{R}_2 - \dot{f}_3\dot{g}_1\underline{L}_3 \cdot \underline{R}_2$$

$$+ \dot{g}_1\dot{g}_3 (\underline{L}_1 - \underline{L}_3) \cdot \underline{\dot{R}}_2 - \dot{g}_3\underline{L}_1 \cdot \underline{\dot{R}}_1 \tag{380}$$

$$+ \dot{g}_1\underline{L}_3 \cdot \underline{\dot{R}}_3\}/E$$

$$B = \frac{\dot{g}_3}{E} \tag{381}$$

$$C = - \frac{\dot{g}_1\dot{g}_3\underline{L}_2 \cdot (\underline{L}_1 - \underline{L}_3)}{E} \tag{382}$$

$$D = - \frac{\dot{g}_1}{E} \tag{383}$$

$$\rho_2 = A + \dot{\rho}_1 B + \dot{\rho}_2 C + \dot{\rho}_3 D \tag{384}$$

If

$$|(\rho_2)_{n+1} - (\rho_2)_n| < \varepsilon \tag{385}$$

where ε is a specified tolerance, i.e., 10^{-10}, proceed to equation (386); if not, return to equation (372) with the latest value of ρ_2 obtained from equation (384) and repeat equational loop (372) to (385).

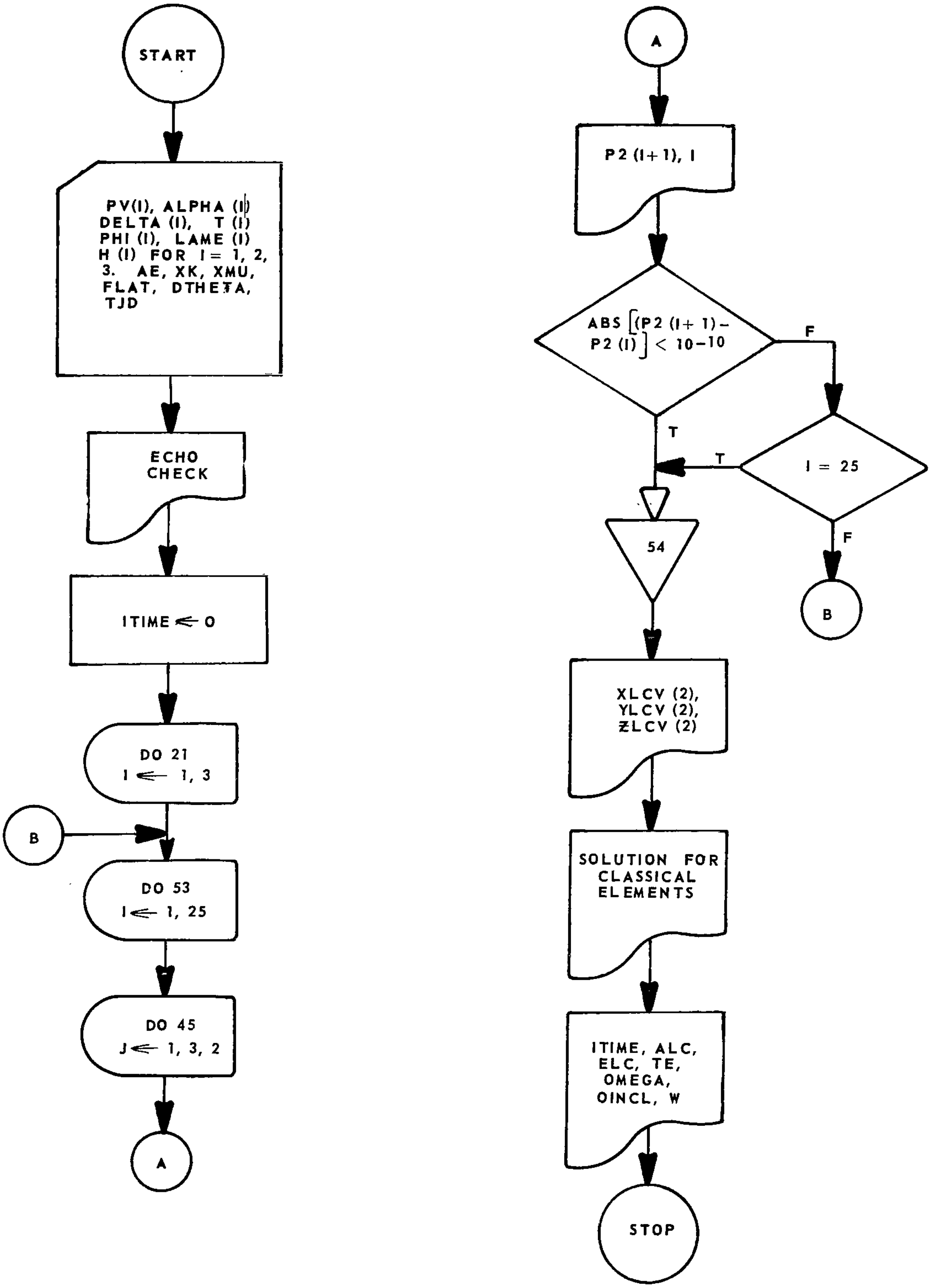
Continue calculating with

$$\underline{r}_2 = \rho_2 \underline{L}_2 - \underline{R}_2 \tag{386}$$

$$\dot{\underline{r}}_2 = \dot{\rho}_2 \underline{L}_2 + \rho_2 \dot{\underline{L}}_2 - \dot{\underline{R}}_2 \tag{387}$$

Continue by calculating for classical elements.

R-ITERATION FLOWCHART



```
C       R-ITERATION PRELIMINARY ORBIT DETERMINATION METHOD
C       RANGE RATE AND ANGLES (ESCOBAL,PAGE 302)
C
        DO 59 N=1,25
C
        DIMENSION TAU(3),S(3),G1(3),XL(3),YL(3),ZL(3),THETA(3),X(3),Y(3),
       CZ(3),XV(3),YV(3),ZV(3),RLC(3),R(3),P2(25),XLC(3),YLC(3),ZLC(3),
       CXLV(3),YLV(3),ZLV(3),XLCV(3),YLCV(3),ZLCV(3),FV(3),GV(3),RLCV(3),
       CV(3),G2(3),DEMG(3),T(4),ALPHA(3),DELTA(3),YAME(3),PHI(3), [illegible](3)
C
C       READ RANGE RATE AND ANGULAR INPUT DATA
C
        READ 108,FLAT,AE,XK,XMU,DTHETA
        READ 108,T(4),T(1),T(2),T(3),TJD
        READ 109,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2)
        READ 109,DELTA(3),YAME(1),YAME(2),YAME(3),PHI(1)
        READ 109,PHI(2),PHI(3),H(1),H(2),H(3)
        READ 109,P1V,P2V,P3V
108     FORMAT(5F16.8)
109     FORMAT(3F16.8)
C
C       ECHO CHECK
C
        PRINT 110,FLAT,AE,XK,XMU,DTHETA,T(4),TJD,T(1),T(2),T(3)
110     FORMAT(1H0,$FLAT=$E16.8$**AE=$F16.8$**XK=$E16.8$**XMU=$F1[illegible],//,
       1$DTHETA=$E16.8$**T(4)=$F16.8$**TJD=$F16.8,//,
       1$T(1)=$F16.8$**T(2)=$F16.8$**T(3)=$F16.8)
        PRINT 111,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2),DELTA(3),
       CYAME(1),YAME(2),YAME(3)
111     FORMAT(1H0,$ALPHA(1)=$F16.8$**ALPHA(2)=$F16.8$**ALPHA(3)=$F16.8,
       1//,$DELTA(1)=$F16.8$**DELTA(2)=$F16.8$**DELTA(3)=$F16.8,//,
       1$YAME(1)=$F16.8$**YAME(2)=$F16.8$**YAME(3)=$E16.8)
        PRINT 112,PHI(1),PHI(2),PHI(3),H(1),H(2),H(3),P1V,P2V,P3V
112     FORMAT(1H0,$PHI(1)=$F16.8$**PHI(2)=$E16.8$**PHI(3)=$F16.8,//,
       1$H(1)=$F16.8$**H(2)=$F16.8$**H(3)=$F16.8,//,
       1$P1V=$F16.8$**P2V=$F16.8$**P3V=$F16.8)
C
C       BEGIN COMPUTATIONS
C
C       ALL META-SYMBOL IS ITIME SUBROUTINE
C
        ITIME=0
S       LDA         [illegible]
S       STA         [illegible]
S       BRU         [illegible]
S205    BRM         [illegible]
S200    EXM         [illegible]
S       PAT =  [illegible]
S       END
        TAU(1)=XK*(T(1)-T(2))
        TAU(3)=XK*(T(3)-T(2))
        TU=(TJD-2415020.0)/36525.0
        GTHETA=(99.6909833+36000.7689*TU+.00038708*TU**2)/57.29578 [illegible]
```

```
      S(1)=-TAU(3)/(TAU(1)*(TAU(1)-TAU(3)))
      S(2)=-(TAU(3)+TAU(1))/(TAU(1)*TAU(3))
      S(3)=-TAU(1)/(TAU(3)*(TAU(3)-TAU(1)))
 11   DO 21 I=1,3
      DENS(I)=SQRT(1.0-(2.0*FLAT-FLAT**2)*(SIN(PHI(I)))**2)
      G1(I)=AE/DENS(I)+H(I)
      G2(I)=(1.0-FLAT)**2*AE/DENS(I)+H(I)
      XL(I)=COS(DELTA(I))*COS(ALPHA(I))
      YL(I)=COS(DELTA(I))*SIN(ALPHA(I))
      ZL(I)=SIN(DELTA(I))
      THETA(I)=STHETA+DTHETA*(T(I)-T(4))+YAME(I)
      X(I)=-G1(I)*COS(PHI(I))*COS(THETA(I))
      Y(I)=-G1(I)*COS(PHI(I))*SIN(THETA(I))
      Z(I)=-G2(I)*SIN(PHI(I))
      XV(I)=-(Y(I)*DTHETA)/XK
      YV(I)=(X(I)*DTHETA)/XK
 21   ZV(I)=0.0
      CHI=-2.0*(XL(2)*X(2)+YL(2)*Y(2)+ZL(2)*Z(2))
      RLC(2)=1.0
      R(2)=SQRT(X(2)**2+Y(2)**2+Z(2)**2)
      P2(1)=0.5*(-CHI+SQRT(CHI**2-4.0*(R(2)**2-RLC(2)**2)))
      XLV(2)=S(1)*XL(1)+S(2)*XL(2)+S(3)*XL(3)
      YLV(2)=S(1)*YL(1)+S(2)*YL(2)+S(3)*YL(3)
      ZLV(2)=S(1)*ZL(1)+S(2)*ZL(2)+S(3)*ZL(3)
C
C     ITERATION FOR DETERMINING STATION RANGE VECTORS FROM INITIAL DATA
C
 26   DO 53 I=1,25
      XLC(2)=P2(I)*XL(2)-X(2)
      YLC(2)=P2(I)*YL(2)-Y(2)
      ZLC(2)=P2(I)*ZL(2)-Z(2)
      RLC(2)=SQRT(XLC(2)**2+YLC(2)**2+ZLC(2)**2)
      XLCV(2)=P2V*XL(2)+P2(I)*XLV(2)-XV(2)
      YLCV(2)=P2V*YL(2)+P2(I)*YLV(2)-YV(2)
      ZLCV(2)=P2V*ZL(2)+P2(I)*ZLV(2)-ZV(2)
      RLCV(2)=(XLC(2)*XLCV(2)+YLC(2)*YLCV(2)+ZLC(2)*ZLCV(2))/RLC(2)
      V(2)=SQRT(XLCV(2)**2+YLCV(2)**2+ZLCV(2)**2)
      U=XMU/RLC(2)**3
      P=(RLC(2)*RLCV(2))/RLC(2)**2
      Q=(V(2)**2-RLC(2)**2*U)/RLC(2)**2
 43   DO 45 J=1,3,2
      FV(J)=-U*TAU(J)+3.0/2.0*U*P*TAU(J)**2+1.0/6.0*(3.0*U*Q-15.0*U*
     CP**2+U**2)*TAU(J)**3+5.0/8.0*(7.0*U*P**3-3.0*U*P*Q-U**2*P)*TAU(J)
     C**4+1.0/120.0*(630.0*U*P**2*Q-24.0*U**2*Q-U**3-45.0*U*Q**2-945.0*U
     C*P**4+210.0*U**2*P**2)*TAU(J)**5
 45   GV(J)=1.0-0.5*U*TAU(J)**2+U*P*TAU(J)**3+1.0/24.0*(3.0*U*Q-45.0*U*
     CP**2+U**2)*TAU(J)**4+1.0/60.0*(210.0*U*P**3-90.0*U*P*Q-15.0*U**2*
     CP)*TAU(J)**5
      E=FV(1)*GV(3)*(XL(1)*XL(2)+YL(1)*YL(2)+ZL(1)*ZL(2))-FV(3)*GV(1)*
     C(XL(3)*XL(2)+YL(3)*YL(2)+ZL(3)*ZL(2))+GV(1)*GV(3)*(XLV(2)*(XL(1)-
     CXL(3))+YLV(2)*(YL(1)-YL(3))+ZLV(2)*(ZL(1)-ZL(3)))
      A=(FV(1)*GV(3)*(XL(1)*X(2)+YL(1)*Y(2)+ZL(1)*Z(2))-FV(3)*GV(1)*(XL
     C(3)*X(2)+YL(3)*Y(2)+ZL(3)*Z(2))+GV(1)*GV(3)*((XL(1)-XL(3))*XV(2)+
     C(YL(1)-YL(3))*YV(2)+(ZL(1)-ZL(3))*ZV(2))-GV(3)*(XL(1)*XV(1)+YL(1)*
```

```
     CYV(1)+ZL(1)*ZV(1))+ (1)*(XL(3)*X (3)+YL(3)*YV(3)+ZL( )*Z ( )))/E
      B=GV(3)/E
      C=-GV(1)*GV(3)*(XL( )*(XL(1)-XL(3))+YL(2)*(YL(1)-YL( ))+Z ( )*(
     CZL(1)-ZL(3)))/E
      D=-GV(1)/E
      P2(I+1)=A+P1V*B+P2V*C+P3V*D
      CT1=ITIME
      PRINT 100,CT1
      PRINT 103, P2(I+1),I
103   FORMAT(1H0,$P2(I+1)=*E16.8$*****I=$I2)
      ITIME=0
 52   IF(ABS(P2(I+1)-P2(I))-0.0000000001) 54,53,53
 53   CONTINUE
C
C     COMPUTE INERTIAL POSITION AND VELOCITY VECTORS
C
 54   P2F=P2(I)
      XLC(2)=P2F*XL(2)-X(2)
      YLC(2)=P2F*YL(2)-Y(2)
      ZLC(2)=P2F*ZL(2)-Z(2)
      XLCV(2)=P2 *XL(2)+P2F*XLV(2)-XV(2)
      YLCV(2)=P2 *YL(2)+P2F*YLV(2)-YV(2)
      ZLCV(2)=P2 *ZL(2)+P2F*ZLV(2)-ZV(2)
      CT2=ITIME
      PRINT 100,ITIME
      PRINT 104, XLCV(2),YLCV(2),ZLCV(2)
104   FORMAT(1H0,$XLCV(2)=*E16.8,//,$YLCV(2)=$E16.8,//, ZLC (2)=  1 .  )
C
C     SOLUTION FOR CLASSICAL ELEMENTS
C
      ITIME=0
      RLC(2)=SQRT(XLC(2)**2+YLC(2)**2+ZLC(2)**2)
      RDOT=XLC(2)*XLCV(2)+YLC(2)*YLCV( )+ZLC(2)*ZLCV(2)
      RLCV(2)=RDOT/RLC(2)
      VC=SQRT(XLCV(2)**2+YLCV(2)**2+ZLCV(2)**2)
      ALC=(RLC(2)*XMU)/(2.0*XMU-VC**2*RLC(2))
      COSBE=(1.0-RLC(2)/ALC)
      SSUBE=(RLCV(2)*RLC(2))/SQRT(XMU*ALC)
      ELC=SQRT(SSUBE**2+COSBE**2)
      COSE=(ALC-RLC(2))/(ALC*ELC)
      XSUB =ALC*(COSE-ELC)
      COSV=XSUB /RLC(2)
      SINV=SQRT(RLC(2)**2-XSUBW**2)/RLC(2)
      SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
      E=ATAN(SINE,COSE)
      TE=T(2)-((E-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(2)*ZLCV(2)-ZLC(2)*YLCV(2)
      HY=-(XLC(2)*ZLCV(2)-ZLC(2)*XLCV(2))
      HZ=XLC(2)*YLCV(2)-YLC(2)*XLCV(2)
      VANGE=ATAN(SINV,COSV)
      SINHY=HX
      COSHY=-HY
      OMEGA=ATAN(SINHX,COSHY)
      EXP=SQRT(HX**2+HY**2)
```

```
       OINCL=ATAN(EYP,HZ)
       UNUM=-XLC(2)*SIN(OMEGA)*COS(OINCL)+YLC(2)*COS(OMEGA)*COS(OINCL)+
      CZLC(2)*SIN(OINCL)
       DEM=XLC(2)*COS(OMEGA)+YLC(2)*SIN(OMEGA)
       U=ATAN(UNUM,DEM)
       W=U-VANGE
       CT3=ITIME
       PRINT 100,CT3
       PRINT 107, ALC,BLC,TE,OMEGA,OINCL,W
107    FORMAT(1H0,*ALC=*E16.8,//,*BLC=*E16.8,//,*TE=*E16.8,//,
      1*OMEGA=*E16.8,//,*OINCL=*E16.8,//,*W=*E16.8,//)
100    FORMAT(*MILLISEC=*I8)
 59    CONTINUE
       GO TO 60
S2050  BZE
S      NI            ITIME
S      ARG           *20F0S
 60        END
```

APPENDIX L
TRILATERATION PODM, MIXED DATA

Given the mixed data ρ_j, $\dot{\rho}_j$, t_j, $j = 1, 2, \ldots, q$, for a set of observing stations with coordinates ϕ_i, λ_{Ei}, H_i, $i = 1, 2, 3$, and constants a_e, f, $d\theta/dt$, proceed as follows. Reduce the range and range-rate data to a common simultaneous time such that ρ_i, $\dot{\rho}_i$, $i = 1, 2, 3$, are available for an arbitrary modified time τ_0 and compute

$$Tu = \frac{J.D. - 2415020}{36525} \tag{388}$$

$$\theta_{g0} = 99.^{\circ}6909833 + 36000.^{\circ}7689\ Tu + 0.^{\circ}00038708\ Tu^2 \tag{389}$$

For i = 1, 2, 3, compute

$$G_{1i} = \frac{a_e}{\sqrt{1 - (2f - f^2)\sin^2\phi_i}} + H_i \tag{390}$$

$$G_{2i} = \frac{(1 - f)^2 a_e}{\sqrt{1 - (2f - f^2)\sin^2\phi_i}} + H_i \tag{391}$$

$$\theta_i = \theta_{g0} + \frac{d\theta}{dt}(t_i - t_0) + \lambda_{Ei} \tag{392}$$

$$X_i = -G_{1i}\cos\phi_i\cos\theta_i \tag{393}$$

$$Y_i = -G_{1i}\cos\phi_i\sin\theta_i \tag{394}$$

$$Z_i = - G_{2i} \sin \phi_i \tag{395}$$

$$R_i^2 = \underline{R}_i \cdot \underline{R}_i \tag{396}$$

$$\zeta_{21} = \frac{1}{2}\left[\rho_2^2 - \rho_1^2 - (R_2^2 - R_1^2)\right] \tag{397}$$

$$\zeta_{31} = \frac{1}{2}\left[\rho_3^2 - \rho_1^2 - (R_3^2 - R_1^2)\right] \tag{398}$$

$$\Delta_1 = (Z_3 - Z_1)(Y_2 - Y_1) - (Z_2 - Z_1)(Y_3 - Y_1) \tag{399}$$

$$A = \frac{(X_2 - X_1)(Y_3 - Y_1) - (X_3 - X_1)(Y_2 - Y_1)}{\Delta_1} \tag{400}$$

$$B = \frac{\zeta_{31} (Y_2 - Y_1) - \zeta_{21} (Y_3 - Y_1)}{\Delta_1} \tag{401}$$

$$\Delta_2 = (Y_3 - Y_1)(Z_2 - Z_1) - (Y_2 - Y_1)(Z_3 - Z_1) \tag{402}$$

$$C = \frac{(X_2 - X_1)(Z_3 - Z_1) - (X_3 - X_1)(Z_2 - Z_1)}{\Delta_2} \tag{403}$$

$$D = \frac{\zeta_{31} (Z_2 - Z_1) - \zeta_{21} (Z_3 - Z_1)}{\Delta_2} \tag{404}$$

$$\varepsilon_1 = A^2 + C^2 + 1 \tag{405}$$

$$\varepsilon_2 = 2(AB + CD + X_1 + CY_1 + AZ_1) \tag{406}$$

$$\varepsilon_3 = B^2 + D^2 + 2DY_1 + 2BZ_1 + R_1^2 - \rho_1^2 \tag{407}$$

$$x_{0j} = -\frac{\varepsilon_2 \pm \sqrt{\varepsilon_2^2 - 4\varepsilon_1\varepsilon_3}}{2\varepsilon_1} \tag{408}$$

$$y_{0j} = Cx_{0j} + D \tag{409}$$

$$z_{0j} = Ax_{0j} + B \tag{410}$$

$$r_{0j}^2 = \underline{r}_{0j} \cdot \underline{r}_{0j} \tag{411}$$

Reject the $\underline{r}_{0j}$ that does not satisfy

$$\rho_1^2 = r_{0j}^2 + 2\underline{r}_{0j} \cdot \underline{R}_1 + R_1^2 \tag{412}$$

and continue calculating for i = 1, 2, 3, with

$$\dot{\underline{R}}_i = \frac{1}{k_e} \begin{bmatrix} -Y_i \\ X_i \\ Z_i \end{bmatrix} \frac{d\theta}{dt} \tag{413}$$

$$\underline{\rho}_i = \underline{r}_0 + \underline{R}_i \tag{414}$$

$$E_i = \rho_i \dot{\rho}_i - \underline{\dot{R}}_i \cdot \underline{\rho}_i \tag{415}$$

Invert the matrix

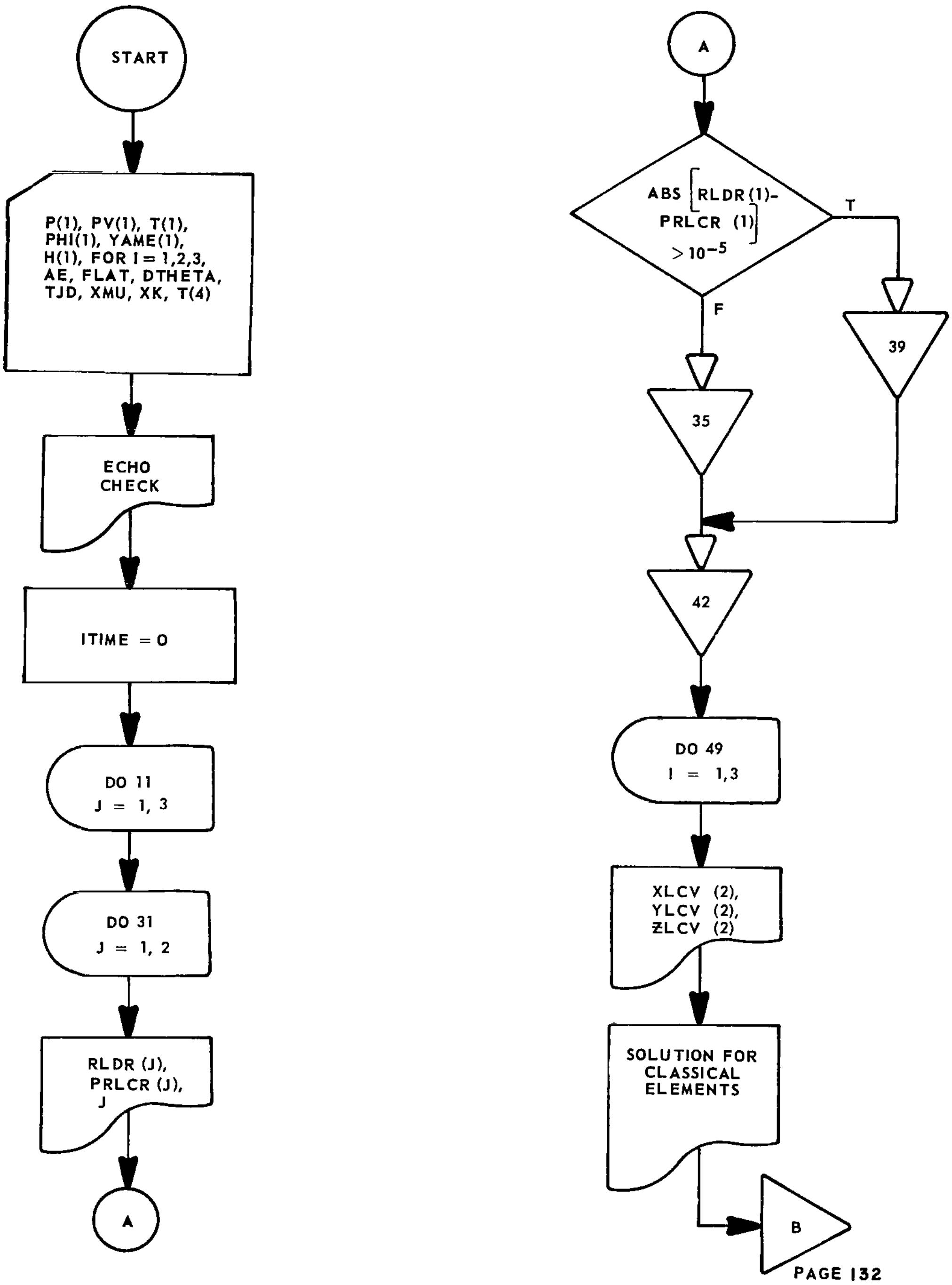
$$M_s = \begin{bmatrix} \rho_{x1} & \rho_{y1} & \rho_{z1} \\ \rho_{x2} & \rho_{y2} & \rho_{z2} \\ \rho_{x3} & \rho_{y3} & \rho_{z3} \end{bmatrix} \tag{416}$$

and obtain

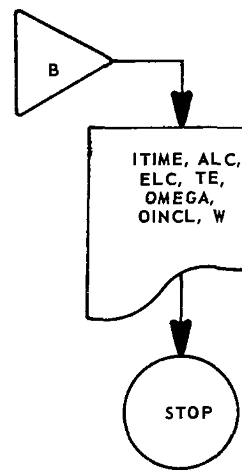
$$\begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{z}_0 \end{bmatrix} = \left[M_s \right]^{-1} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{417}$$

Continue by calculating for classical elements.

TRILATERATION FLOWCHART



TRILATERATION FLOWCHART (CONT'D)



```
C       TRILATERATION PRELIMINARY ORBIT DETERMINATION METHOD
C       RANGE AND RATE (ESCOBAL,PAGE 312)
C
        DO 59 N=1,19
C
        DIMENSION DEMG(3),G1(3),G2(3),THETA(3),X(3),Y(3),Z(3),R(3),XLC(3),
       CDEL2(3),DEL3(3),DELTA(3),FPS(3),XLCN(3),YLCN(3),ZLCN(3),PLC (3),
       CRLDP(3),RPLCR(3),XV(3),YV(3),ZV(3),PX(3),PY(3),PZ(3),E(3),
       CYLC(3),ZLC(3),RLC(3),XLCV(3),YLCV(3),PLCV(3),COFP (3,3)
        DIMENSION T(4),PHI(3),YAME(3),P(3),PV(3),H(3),ZLCV(3)
C
C       READ RANGE AND RANGE RATE INPUT DATA
C
        READ 108,FLAT,AE,XK,XMU,DTHETA
        READ 108,T(4),T(1),T(2),T(3),TJD
        READ 108,PHI(1),PHI(2),PHI(3),YAME(1),YAME(2)
        READ 108,YAME(3),H(1),H(2),H(3),P(1)
        READ 108,P(2),P(3),PV(1),PV(2),PV(3)
108     FORMAT(5E16.8)
C
C       ECHO CHECK
C
        PRINT 110,FLAT,AE,XK,XMU,DTHETA,T(4),TJD,T(1),T(2),T(3)
110     FORMAT(1H0,$FLAT=$E16.8$**AE=$E16.8$**XK=$E16.8$**XMU=$E16.8,//,
       1$DTHETA=$E16.8$**T(4)=$E16.8$**TJD=$E16.8,//,
       1$T(1)=$E16.8$**T(2)=$E16.8$**T(3)=$E16.8)
        PRINT 111,PHI(1),PHI(2),PHI(3),YAME(1),YAME(2),YAME(3),H(1),H(2),
       CH(3)
111     FORMAT(1H0,$PHI(1)=$E16.8$**PHI(2)=$E16.8$**PHI(3)=$E16.8,//,
       1$YAME(1)=$E16.8$**YAME(2)=$E16.8$**YAME(3)=$E16.8,//,
       1$H(1)=$E16.8$**H(2)=$E16.8$**H(3)=$E16.8)
        PRINT 112,P(1),P(2),P(3),PV(1),PV(2),PV(3)
112     FORMAT(1H0,$P(1)=$E16.8$**P(2)=$E16.8$**P(3)=$E16.8,//,
       1$PV(1)=$E16.8$**PV(2)=$E16.8$**PV(3)=$E16.8)
C
C       BEGIN COMPUTATIONS
C
C       ALL META-SYMBOL IS ITIME SUBROUTINE
C
        ITIME=0
S       LDA         2259
S       STA         2205
S       BRU         2205
S205    BRM         22525
S200    EOM         020020
S       POT =  00202000
S       EIR
        TU=(TJD-2415020.0)/36525.0
        GTHETA=(99.6909833+36000.7689*TU+0.00038708*TU**2)/57.2957795131
   3    DO 11 J=1,3
        DEMG(J)=SQRT(1.-(2.0*FLAT-FLAT**2)*(SIN(PHI(J)))**2)
        G1(J)=AE/DEMG(J)+H(J)
        G2(J)=((1.0-FLAT)**2*AE)/DEMG(J)+H(J)
```

```
      THETA(J)=GTHETA+DTHETA*(T(2)-T(4))+YAME(J)
      X(J)=-G1(J)*COS(PHI(J))*COS(THETA(J))
      Y(J)=-G1(J)*COS(PHI(J))*SIN(THETA(J))
      Z(J)=-G2(J)*SIN(PHI(J))
   11 R(J)=SQRT(X(J)**2+Y(J)**2+Z(J)**2)
      DEL2(1)=0.5*(P(2)**2-P(1)**2-(R(2)**2-R(1)**2))
      DEL3(1)=0.5*(P(3)**2-P(1)**2-(R(3)**2-R(1)**2))
      DELTA(1)=(Z(3)-Z(1))*(Y(2)-Y(1))-(Z(2)-Z(1))*(Y(3)-Y(1))
      A=((X(2)-X(1))*(Y(3)-Y(1))-(X(3)-X(1))*(Y(2)-Y(1)))/DELTA(1)
      B=(DEL3(1)*(Y(2)-Y(1))-DEL2(1)*(Y(3)-Y(1)))/DELTA(1)
      DELTA(2)=(Y(3)-Y(1))*(Z(2)-Z(1))-(Y(2)-Y(1))*(Z(3)-Z(1))
      C=((X(2)-X(1))*(Z(3)-Z(1))-(X(3)-X(1))*(Z(2)-Z(1)))/DELTA(2)
      D=(DEL3(1)*(Z(2)-Z(1))-DEL2(1)*(Z(3)-Z(1)))/DELTA(2)
      EPS(1)=A**2+C**2+1.0
      EPS(2)=2.0*(A*B+C*D+X(1)+C*Y(1)+A*Z(1))
      EPS(3)=B**2+D**2+2.0*D*Y(1)+2.0*B*Z(1)+R(1)**2-P(1)**2
      XLCN(1)=(-EPS(2)+SQRT(ABS(EPS(2)**2-4.0*EPS(1)*EPS(3))))/
     C(2.0*EPS(1))
      XLCN(2)=(-EPS(2)-SQRT(ABS(EPS(2)**2-4.0*EPS(1)*EPS(3))))/
     C(2.0*EPS(1))
   26 DO 31 J=1,2
      YLCN(J)=C*XLCN(J)+D
      ZLCN(J)=A*XLCN(J)+B
      RLCN(J)=SQRT(XLCN(J)**2+YLCN(J)**2+ZLCN(J)**2)
      RLDR(J)=2.0*(XLCN(J)*X(1)+YLCN(J)*Y(1)+ZLCN(J)*Z(1))
      PRLCR(J)=P(1)**2-RLCN(J)**2-R(1)**2
      CT1=ITIME
      PRINT 100,CT1
      PRINT 101,RLDR(J),PRLCR(J),J
  101 FORMAT(1H0,$RLDR(J)=$E16.8$***PRLCR(J)=$E16.8$***J=$I2)
      ITIME=0
   31 CONTINUE
C
C     DETERMINE APPLICABLE REAL ROOT
C
   32 IF(ABS(RLDR(1)-PRLCR(1))-0.000000001) 39,39,35
   35 XLC(2)=XLCN(1)
      YLC(2)=YLCN(1)
      ZLC(2)=ZLCN(1)
   38 GO TO 42
   39 XLC(2)=XLCN(2)
      YLC(2)=YLCN(2)
      ZLC(2)=ZLCN(2)
   42 DO 49 I=1,3
      XV(I)=-Y(I)*DTHETA/XK
      YV(I)=X(I)*DTHETA/XK
      ZV(I)=Z(I)*DTHETA/XK
      PX(I)=XLC(2)+X(I)
      PY(I)=YLC(2)+Y(I)
      PZ(I)=ZLC(2)+Z(I)
   49 F(I)=F(I)*VV(I)-(XV(I)*PX(I)+YV(I)*PY(I)+ZV(I)*PZ(I))
C
C     MATRIX INVERSION
C
```

```
      PMAT=PX(1)*PY(2)*PZ(3)+PY(1)*PZ(2)*PX(3)+PZ(1)*PY(3)*PX(2)-PX(3)*
     CPY(2)*PZ(1)-PY(3)*PZ(2)*PX(1)-PZ(3)*PX(2)*PY(1)
      COFPM(1,1)=PY(2)*PZ(3)-PY(3)*PZ(2)
      COFPM(1,2)=-(PX(2)*PZ(3)-PX(3)*PZ(2))
      COFPM(1,3)=PX(2)*PY(3)-PX(3)*PY(2)
      COFPM(2,1)=-(PY(1)*PZ(3)-PY(3)*PZ(1))
      COFPM(2,2)=PX(1)*PZ(3)-PX(3)*PZ(1)
      COFPM(2,3)=-(PX(1)*PY(3)-PX(3)*PY(1))
      COFPM(3,1)=PY(1)*PZ(2)-PY(2)*PZ(1)
      COFPM(3,2)=-(PX(1)*PZ(2)-PX(2)*PZ(1))
      COFPM(3,3)=PX(1)*PY(2)-PX(2)*PY(1)
C
C     SOLVE FOR INERTIAL VELOCITY VECTORS
C
      XLCV(2)=COFPM(1,1)/PMAT*E(1)+COFPM(2,1)/PMAT*E(2)+COFPM(3,1)/PMAT*
     CE(3)
      YLCV(2)=COFPM(1,2)/PMAT*E(1)+COFPM(2,2)/PMAT*E(2)+COFPM(3,2)/PMAT*
     CE(3)
      ZLCV(2)=COFPM(1,3)/PMAT*E(1)+COFPM(2,3)/PMAT*E(2)+COFPM(3,3)/PMAT*
     CE(3)
      CT2=ITIME
      PRINT 100,CT2
      PRINT 103,XLCV(2),YLCV(2),ZLCV(2)
103   FORMAT(1H ),'XLCV(2)='F16.8,//,'YLCV(2)='F16.8,//,' ZLCV(2)='F16.8)
C
C     SOLUTION FOR CLASSICAL ELEMENTS
C
      ITIME=0
      RLC(2)=SQRT(XLC(2)**2+YLC(2)**2+ZLC(2)**2)
      RDDOT=XLC(2)*XLCV(2)+YLC(2)*YLCV(2)+ZLC(2)*ZLCV(2)
      RLCV(2)=RDDOT/RLC(2)
      V=SQRT(XLCV(2)**2+YLCV(2)**2+ZLCV(2)**2)
      ALC=(RLC(2)*XMU)/(2.0*XMU-V**2*RLC(2))
      CSUBE=(1.0-RLC(2)/ALC)
      SSUBE=(RLCV(2)*RLC(2))/SQRT(XMU*ALC)
      ELC=SQRT(SSUBE**2+CSUBE**2)
      COSE=(ALC-RLC(2))/(ALC*ELC)
      XSUBW=ALC*(COSE-ELC)
      COSV=XSUBW/RLC(2)
      SINV=SQRT(RLC(2)**2-XSUBW**2)/RLC(2)
      SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
      EA=ATAN2(SINE,COSE)
      TE=T(2)-((EA-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(2)*ZLCV(2)-ZLC(2)*YLCV(2)
      HY=-(XLC(2)*ZLCV(2)-ZLC(2)*XLCV(2))
      HZ=XLC(2)*YLCV(2)-YLC(2)*XLCV(2)
      VANGE=ATAN2(SINV,COSV)
      SINHX=HX
      COSHY=-HY
      OMEGA=ATAN2(SINHX,COSHY)
      EXP=SQRT(HX**2+HY**2)
      OINCL=ATAN2(EXP,HZ)
      UNUM=-XLC(2)*SIN(OMEGA)*COS(OINCL)+YLC(2)*COS(OMEGA)*COS(OINCL)+
     CZLC(2)*SIN(OINCL)
```

```
      DEM=XLC(2)*COS(OMEGA)+YLC(2)*SIN(OMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANOD
      CT3=ITIME
      PRINT 100,CT3
      PRINT 107, ALC,ELC,TE,OMEGA,OINCL,W
107   FORMAT(1H0,$ALC=$F16.8,//,$ELC=$F16.8,//,$TE=$F16.8,//,
     1$OMEGA=$F16.8,//,$OINCL=$F16.8,//,$W=$F16.8,//)
100   FORMAT(1H MILLISEC=I13)
 59   CONTINUE
      GO TO 60
S2050 PZE
S      MIT          ITIME
S      BRU          *2050S
 60       END
```

Appendix M
Herrick-Gibbs PODM, Mixed Data

Given the mixed data ρ_i, α_{ti}, δ_{ti}, for some t_i with $i = 1, 2, 3$ along with station data ϕ_i, λ_{Ei}, H_i and the constants a_e, K_e, μ, f, $\frac{d\theta}{dt}$, proceed as follows:

$$Tu = \frac{JD - 2415020}{36525} \tag{418}$$

$$\theta_g 0 = 99^\circ.6909833 + 36000^\circ.7689\ Tu + 0^\circ.00038708\ Tu^2 \tag{419}$$

For $i = 1, 2, 3$ compute

$$L_{xi} = \text{Cos}\ \delta_{ti}\ \text{Cos}\ \alpha_{ti} \tag{420}$$

$$L_{yi} = \text{Cos}\ \delta_{ti}\ \text{Sin}\ \alpha_{ti} \tag{421}$$

$$L_{zi} = \text{Sin}\ \alpha_{ti} \tag{422}$$

$$G_{1i} = \frac{a_e}{1 - (2f - f^2)\ \text{Sin}^2\ \phi_i} + H_i \tag{423}$$

$$G_{2i} = \frac{(1 - f)^2\ a_e}{1 - (2f - f^2)\ \text{Sin}^2\ \phi_i} + H_i \tag{424}$$

$$\theta_i = \theta_{go} + \frac{d\theta}{dt}(t_i - t_o) + \lambda_{Ei} \tag{425}$$

$$X_i = - G_{1i} \cos \phi_i \cos \theta_i \tag{426}$$

$$Y_i = - G_{1i} \cos \phi_i \sin \theta_i \tag{427}$$

$$Z_i = - G_{2i} \sin \phi_i \tag{428}$$

$$\underline{r}_i = \rho_i \underline{L}_i - \underline{R}_i \tag{429}$$

From the observation times, one may compute the respective modified times, that is

$$\tau_{ij} = K_e (t_j - t_i) \tag{430}$$

with $\quad j = 1, 2, 3$ and $i = 2$

$$G'_1 \equiv \frac{\tau_{23}}{\tau_{12}\,\tau_{13}} \tag{431}$$

$$G'_3 \equiv \frac{\tau_{12}}{\tau_{23}\,\tau_{13}} \tag{432}$$

$$G'_2 \equiv G'_1 - G'_3 \tag{433}$$

with $$\tau_{13} \equiv \tau_3 - \tau_1 \tag{434}$$

Continue by computing

$$H'_1 \equiv \frac{\mu \ \tau_{23}}{12} \tag{435}$$

$$H'_3 \equiv \frac{\mu \ \tau_{12}}{12} \tag{436}$$

$$H'_2 \equiv H'_1 - H'_3 \tag{437}$$

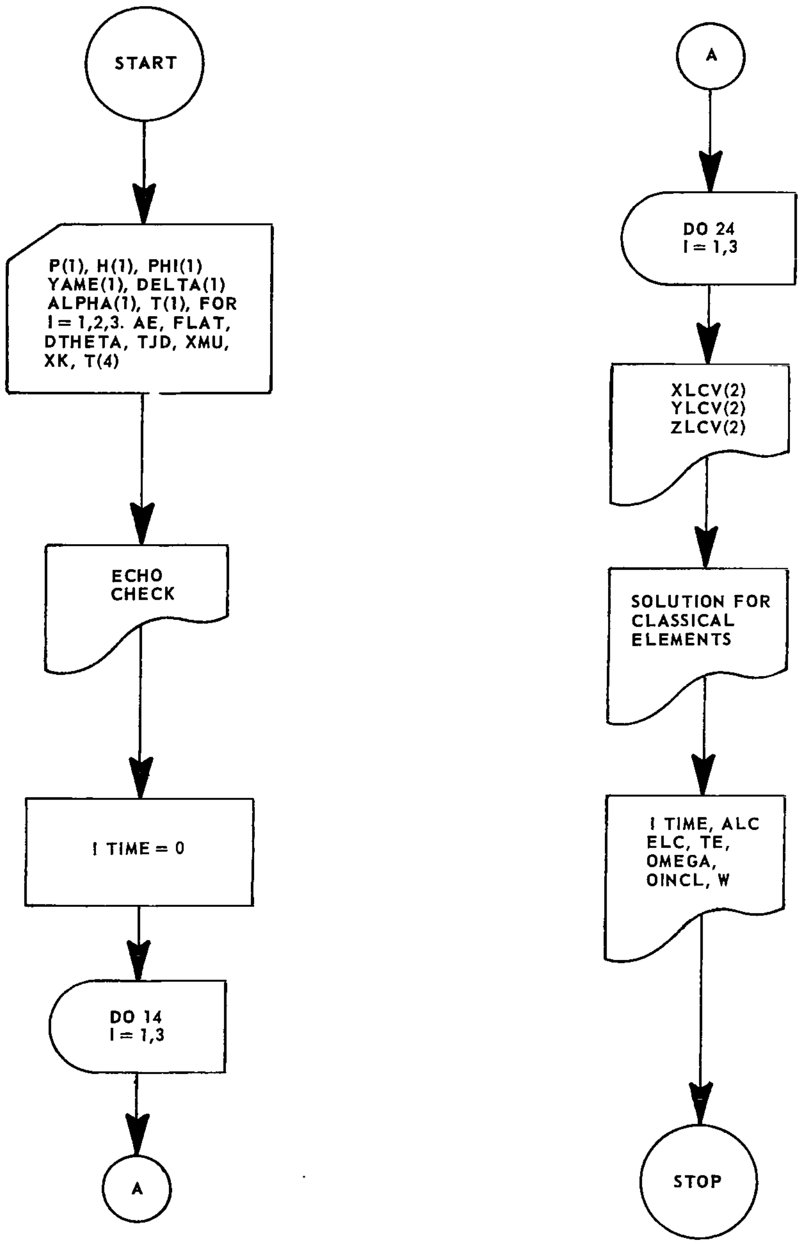
and form the coefficients

$$d_i = G'_i + \frac{H'_i}{r_i^3} \text{ for } i = 1 , 2 , 3 \tag{438}$$

$$\underline{\dot{r}}_2 = - d_1 \underline{r}_1 + d_2 \underline{r}_2 + d_3 \underline{r}_3 \tag{439}$$

Continue by calculating for the classical elements.

HERRICK-GIBBS FLOWCHART



```
C       HERRICK-GIBBS PRELIMINARY ORBIT DETERMINATION METHOD
C       RANGE AND ANGLES (ESCOBAL,PAGE 305)
C
        DO 59 N=1,25
C
        DIMENSION XL(3),YL(3),ZL(3),DEMO(3),G1(3),G2(3),THETA(3),X(3),
       CY(3),Z(3),XLC(3),YLC(3),ZLC(3),RLC(3),T(4),P(3),ALPHA(3),
       CDELTA(3),SR(3),HR(3),D(3),XLCV(3),YLCV(3),ZLCV(3),RLCV(3),
       CYAME(3),PHI(3),H(3)
C
C       READ RANGE AND ANGULAR INPUT DATA
C
        READ 108,FLAT,AE,XK,XMU,DTHETA
        READ 108,T(4),T(1),T(2),T(3),TJD
        READ 108,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2)
        READ 108,DELTA(3),YAME(1),YAME(2),YAME(3),PHI(1)
        READ 108,PHI(2),PHI(3),H(1),H(2),H(3)
        READ 109,P(1),P(2),P(3)
108     FORMAT(5E16.8)
109     FORMAT(3E16.8)
C
C       ECHO CHECK
C
        PRINT 110,FLAT,AE,XK,XMU,DTHETA,T(4),TJD,T(1),T(2),T(3)
110     FORMAT(1H0,$FLAT=$E16.8$**AE=$E16.8$**XK=$E16.8$**XMU=$E16.8,//
       1$DTHETA=$E16.8$**T(4)=$E16.8$**TJD=$E16.8,//,
       1$T(1)=$E16.8$**T(2)=$E16.8$**T(3)=$E16.8)
        PRINT 111,ALPHA(1),ALPHA(2),ALPHA(3),DELTA(1),DELTA(2),DELTA(3),
       CYAME(1),YAME(2),YAME(3)
111     FORMAT(1H0,$ALPHA(1)=$E16.8$**ALPHA(2)=$E16.8$**ALPHA(3)=$E16.8,
       1//,$DELTA(1)=$E16.8$**DELTA(2)=$E16.8$**DELTA(3)=$E16.8,//,
       1$YAME(1)=$E16.8$**YAME(2)=$E16.8$**YAME(3)=$E16.8)
        PRINT 112,PHI(1),PHI(2),PHI(3),H(1),H(2),H(3),P(1),P(2),P(3)
112     FORMAT(1H0,$PHI(1)=$E16.8$**PHI(2)=$E16.8$**PHI(3)=$E16.8,//,
       1$H(1)=$E16.8$**H(2)=$E16.8$**H(3)=$E16.8,//,$P(1)=$E16.8,
       1$**P(2)=$E16.8$**P(3)=$E16.8)
C
C       BEGIN COMPUTATIONS
C
C       ALL META-SYMBOL IS ITIME [illegible]
C
        ITIME=0
S       LDA     [illegible]
S       STA     [illegible]
S       BRU     [illegible]
S205    BRM     [illegible]
S200    ERM     [illegible]
S       PRT  =  [illegible]
S       FIR
        TU=(TJD-2415020.0)/36525.0
        GTHETA=(99.6909833+36000.7689*TU+0.00038708*TU**2)/57.2957795131
        DO 14 I=1,3
        XL(I)=COS(DELTA(I))*COS(ALPHA(I))
```

```
      YL(I)=CØS(DELTA(I))*SIN(ALPHA(I))
      ZL(I)=SIN(DELTA(I))
      DEMG(I)=SQRT(1.0-(2.0*FLAT-FLAT**2)*(SIN(PHI(I)))**2)
      G1(I)=AE/DEMG(I)+H(I)
      G2(I)=(1.0-FLAT)**2*AE/DEMG(I)+H(I)
      THETA(I)=GTHETA+DTHETA*(T(I)-T(4))+YAME(I)
      X(I)=-G1(I)*CØS(PHI(I))*CØS(THETA(I))
      Y(I)=-G1(I)*CØS(PHI(I))*SIN(THETA(I))
      Z(I)=-G2(I)*SIN(PHI(I))
      XLC(I)=P(I)*XL(I)-X(I)
      YLC(I)=P(I)*YL(I)-Y(I)
      ZLC(I)=P(I)*ZL(I)-Z(I)
 14   RLC(I)=SQRT(XLC(I)**2+YLC(I)**2+ZLC(I)**2)
      DT23=XK*(T(3)-T(2))
      DT12=XK*(T(2)-T(1))
      DT13=XK*(T(3)-T(1))
      GB(1)=DT23/(DT12*DT13)
      GB(3)=DT12/(DT23*DT13)
      GB(2)=GB(1)-GB(3)
      HB(1)=(XMU*DT23)/12.0
      HB(3)=(XMU*DT12)/12.0
      HB(2)=HB(1)-HB(3)
      DØ 24 I=1,3
 24   D(I)=GB(I)+HB(I)/RLC(I)**3
      XLCV(2)=-D(1)*XLC(1)+D(2)*XLC(2)+D(3)*XLC(3)
      YLCV(2)=-D(1)*YLC(1)+D(2)*YLC(2)+D(3)*YLC(3)
      ZLCV(2)=-D(1)*ZLC(1)+D(2)*ZLC(2)+D(3)*ZLC(3)
      CT1=ITIME
      PRINT 100,CT1
      PRINT 103,XLCV(2),YLCV(2),ZLCV(2)
103   FØRMAT(1H0,*XLCV(2)=*F16.8,//,*YLCV(2)=*F16.8,//,*ZLCV(2)=*F16.8)
C
C     SØLUTIØN FØR CLASSICAL ELEMENTS
C
      ITIME=0
      RLC(2)=SQRT(XLC(2)**2+YLC(2)**2+ZLC(2)**2)
      RRDØT=XLC(2)*XLCV(2)+YLC(2)*YLCV(2)+ZLC(2)*ZLCV(2)
      RLCV(2)=RRDØT/RLC(2)
      V=SQRT(XLCV(2)**2+YLCV(2)**2+ZLCV(2)**2)
      ALC=(RLC(2)*XMU)/(2.0*XMU-V**2*RLC(2))
      CSUBE=(1.0-RLC(2)/ALC)
      SSUBE=(RLCV(2)*RLC(2))/SQRT(XMU*ALC)
      ELC=SQRT(SSUBE**2+CSUBE**2)
      CØSE=(ALC-RLC(2))/(ALC*ELC)
      XSUBN=ALC*(CØSE-ELC)
      CØSV=XSUBN/RLC(2)
      SINV=SQRT(RLC(2)**2-XSUBN**2)/RLC(2)
      SINE=SQRT(1.0-ELC**2)*SINV/(1.0+ELC*SINV)
      EA=ATAN(SINE,CØSE)
      TE=T(2)-((EA-ELC*SINE)/(XK*SQRT(XMU)))*SQRT(ALC**3)
      HX=YLC(2)*ZLCV(2)-ZLC(2)*YLCV(2)
      HY=-(XLC(2)*ZLCV(2)-ZLC(2)*XLCV(2))
      HZ=XLC(2)*YLCV(2)-YLC(2)*XLCV(2)
      VANGE=ATAN(SINV,CØSV)
```

```
      SINNX=PX
      COSNY=-PY
      OMEGA=ATAN(SINNX,COSNY)
      EXP=SQRT(PX**2+PY**2)
      AINCL=ATAN(EXP,-Z)
      UNUM=-XLC(2)*SIN(OMEGA)*COS(AINCL)+YLC(2)*COS(OMEGA)*COS(AINCL)+
     CZLC(2)*SIN(AINCL)
      DEM=XLC(2)*COS(OMEGA)+YLC(2)*SIN(OMEGA)
      U=ATAN(UNUM,DEM)
      W=U-VANCE
      CT2=ITIME
      PRINT 100,CT2
      PRINT 107, ALC,ELC,TF,OMEGA,AINCL,W
107   FORMAT(1H ,$ALC=$E16.8,//,$ELC=$E16.8,//,$TF=$E16.8,//,
     1$OMEGA= E16.8,//,$AINCL=$E16.8,//,$W =$E16.8,//)
100   FORMAT($ TIME SEC=$I )
 59   CONTINUE
      GO TO 60
S2050 PZE
S     MIN        ITIME
S     BSS        ,PCEDS
 60      END
```

APPENDIX N
OSO-III ORBITAL PARAMETERS
Epoch 67Y 10M 27D 00H 00M

Parameter	Value
Semimajor Axis	006931.15 km or 004306.81 mi
Eccentricity	0.00216
Inclination	032.863°
Mean Anomaly	351.947°
Argument of Perigee	226.399°
RA of Ascending Node	187.347°
Anomalistic Period	0095.70901 min
Height of Perigee	000537.76 km or 000334.15 mi
Height of Apogee	000567.76 km or 000352.79 mi
Velocity at Perigee	027360 km/hr or 017001 mi/hr
Velocity at Apogee	027242 km/hr or 016928 mi/hr
Geocentric Latitude of Perigee	-23.138°

APPENDIX O
RELAY-II ORBITAL PARAMETERS
Epoch 67Y 10M 11D 20H 00M

Parameter	Value
Semimajor Axis	011129.48 km or 006915.5 mi
Eccentricity	0.24115°
Inclination	046.323°
Mean Anomaly	291.027°
Argument of Perigee	248.553°
RA of Ascending Node	161.988°
Anomalistic Period	0194.74113 min
Height of Perigee	002067.24 km or 001284.52 mi
Height of Apogee	007434.94 km or 004619.85 mi
Velocity at Perigee	027554 km/hr or 017121 mi/hr
Velocity at Apogee	016847 km/hr or 010468 mi/hr
Geocentric Latitude of Perigee	-42.311°

APPENDIX P
STATION COORDINATES

Station	Latitude (ϕ)		Longitude (λ_E)		Height (H)	
	Degrees	Radians	Degrees	Radians	Meters	e.r. (10^{-7})
Fort Myers	26° 32′53.78	0.46335476	278° 08′04.60	4.8543647	9	14.110639
Newfoundland	47° 44′28.94	0.83324413	307° 16′46.71	5.3630414	112	175.59907
Quito	00° 37′20.55	0.01086249	281° 25′15.62	4.9117231	3,578	5609.7632
Lima	-11° 46′34.86	-0.20553608	282° 50′59.14	4.9366596	516	809.00999
Santiago	-33° 08′56.23	-0.57855837	289° 19′52.88	5.0497847	681	1067.7050
Winkfield	51° 26′45.43	0.89790126	359° 18′13.57	6.2710337	87	136.40285
Johannesburg	-25° 53′00.98	-0.45175414	27° 42′28.49	0.48359432	1,565	2453.6834
Madagascar	-19° 00′25.21	-0.33173478	47° 18′00.46	0.82554296	1,361	2133.8422
Orroral	-35° 37′37.51	-0.62180996	148° 57′10.71	2.5997184	947	1484.7528

APPENDIX Q
RANGE, RANGE RATE, AND ANGULAR DATA COMPUTATIONAL ALGORITHM AND COMPUTER PROGRAM LISTING

Given $\underline{r}$ (x, y, z) and $\dot{\underline{r}}$ ($\dot{x}$, $\dot{y}$, $\dot{z}$) at a time t with constants ϕ, H, λ_E, $d\theta/dt$, k_e, μ, t_g, a_e, f, proceed as follows:

$$Tu = \frac{J.D. - 2415020}{36525} \tag{440}$$

$$\theta_g = 99^\circ.6909833 + 36000^\circ.7689\ Tu + 0^\circ.00038708\ Tu^2$$

$$\theta = \theta_g + \frac{d\theta}{dt}(t - t_g) - (2\pi - \lambda_E) \tag{441}$$

$$G_1 = \frac{a_e}{\sqrt{1 - (2f - f^2)\sin^2\phi}} + H \tag{442}$$

$$G_2 = \frac{(1 - f)^2 a_e}{\sqrt{1 - (2f - f^2)\sin^2\phi}} + H \tag{443}$$

$$X = - G_1 \cos\phi \cos\theta \tag{444}$$

$$Y = - G_1 \cos\phi \sin\theta \tag{445}$$

$$Z = - G_2 \sin\phi \tag{446}$$

$$\dot{X} = - \frac{d\theta}{dt} Y \tag{447}$$

$$\dot{Y} = \frac{d\theta}{dt} X \tag{448}$$

$$\dot{Z} = 0.0 \tag{449}$$

$$\underline{\rho} = \underline{r} + \underline{R} \tag{450}$$

$$\rho = \sqrt{\underline{\rho} \cdot \underline{\rho}} \tag{451}$$

$$\dot{\underline{\rho}} = \dot{\underline{r}} + \dot{\underline{R}} \tag{452}$$

$$\dot{\rho} = \frac{\underline{\rho} \cdot \dot{\underline{\rho}}}{\rho} \tag{453}$$

$$r_p = \sqrt{x^2 + y^2} \tag{454}$$

$$r = \sqrt{x^2 + y^2 + z^2} \tag{455}$$

$$\cos \delta = \frac{r_p}{r} \tag{456}$$

$$\sin \delta = \frac{z}{r} \tag{457}$$

$$\cos \alpha = \frac{x}{r_p} \tag{458}$$

$$\sin \alpha = \frac{y}{r_p} \tag{459}$$

```
C     COMPUTATION FOR RANGE,RANGE RATE,AND ANGLE DATA
C     TOPOCENTRIC COORDINATE SYSTEM
      DO 45 N=1,66
      READ 107,XLCV,YLCV,ZLCV
      READ 108,AE,TUD,FLAT,OTHETA,XK
      READ 108,XLC,YLC,ZLC,T,TG
      READ 107,PHI,YAWE,H
107   FORMAT(3F16.8)
108   FORMAT(5F16.8)
      PRINT 110,AE,FLAT,TUD,PHI,H,OTHETA
110   FORMAT(1H0,9HAE=[illegible]F16.[illegible]*FLAT=$F16.8$**TUD=$F16.8,//,
     1$PHI=$F16.8$**H=$F16.8$**OTHETA=$F16.8)
      PRINT 111,XLC,YLC,ZLC,T,TG
111   FORMAT(1H0,$XLC=$F16.8$**YLC=$F16.8$**ZLC=$E16.8,//,
     1$T=$F16.8$**TG=$F16.8)
      PRINT 112,XLCV,YLCV,ZLCV,YAWE,XK
112   FORMAT(1H0,$XLCV=$E16.8$**YLCV=$F16.8$**ZLCV=$E16.8,//,
     C$YAWE=$F16.8$**[illegible]=$F16.8,///)
      TU=(TUD-2415020.0)/36525.0
      GTHETA=([illegible]+[illegible].76[illegible]*TU+.000387[illegible]8*TU**2)/57.29577[illegible]5131
      THETA=GTHETA+OTHETA*(T-TG)+YAWE
      DEN=(1.-FLAT)**2
      DENO=SQRT(1.0-(2.0*FLAT-FLAT**2)*(SIN(PHI))**2)
      G1=AE/DENO+H
      G2=AE*DEN/DENO+H
      X=-G1*COS(PHI)*COS(THETA)
      Y=-G1*COS(PHI)*SIN(THETA)
      Z=-G2*SIN(PHI)
      XV=-OTHETA*Y/XK
      YV=OTHETA*X/XK
      ZV=0.0
      PX=XLC+X
      PY=YLC+Y
      PZ=ZLC+Z
      P=SQRT(PX**2+PY**2+PZ**2)
      PXV=XLCV+XV
      PYV=YLCV+YV
      PZV=ZLCV+ZV
      PV=(PX*PXV+PY*PYV+PZ*PZV)/P
      RPT=SQRT(PX**2+PY**2)
      COSDT=RPT/P
      SINDT=PZ/P
      COSAT=PX/RPT
      SINAT=PY/RPT
      DELTA=ATAN2(SINDT,COSDT)
      ALPHA=ATAN2(SINAT,COSAT)
      PRINT 102,P,PV,DELTA,ALPHA,N
102   FORMAT(1H0,$P=$F16.8,///,$PV=$E16.8,///,$DELTA=$F16.8,///,
     1$ALPHA=$F16.8,///,$N=$I2,//)
45    CONTINUE
      END
```

APPENDIX R
SOLUTION FOR CLASSICAL ELEMENTS

Given $\underline{r}_1$ (x_1, y_1, z_1) or $\underline{r}_2$ (x_2, y_2, z_2) and the velocity $\dot{\underline{r}}_1$ $(\dot{x}_1, \dot{y}_1, \dot{z}_1)$ or $\dot{\underline{r}}_2$ $(\dot{x}_2, \dot{y}_2, \dot{z}_2)$, proceed as follows:

$$r_1 = \sqrt{\underline{r}_1 \cdot \underline{r}_1} \tag{460}$$

$$r_1\dot{r}_1 = x_1\dot{x}_1 + y_1\dot{y}_1 + z_1\dot{z}_1 \tag{461}$$

$$\dot{r}_1 = \frac{\underline{r}_1 \cdot \dot{\underline{r}}_1}{r_1} \tag{462}$$

$$V = \sqrt{\dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1} \tag{463}$$

Semimajor axis, a,

$$a = \frac{r_1 \mu}{2\mu - V^2 r_1} \tag{464}$$

$$C_e = 1 - \frac{r_1}{a} \tag{465}$$

$$S_e = \frac{\dot{r}_1 r_1}{\sqrt{\mu a}} \tag{466}$$

Eccentricity, e,

$$e = \sqrt{S_e^{\,2} + C_e^{\,2}} \tag{467}$$

$$\cos E = \frac{a - r_1}{a_e} \tag{468}$$

$$x_w = a\,(\cos E - e) \tag{469}$$

$$\cos \nu = \frac{x_w}{r_1} \tag{470}$$

$$\sin \nu = \frac{\sqrt{r_1^2 - x_w^2}}{r_1} \tag{471}$$

$$\sin E = \sqrt{1 - e^2}\left(\frac{\sin \nu}{1 + e \cos \nu}\right) \tag{472}$$

Time of perifocal passage, T

$$T = t_1 - \frac{(E - e \sin E)}{k_e \sqrt{\mu a^3}} \tag{473}$$

$$h_x = y_1 \dot{z}_1 - z_1 \dot{y}_1 \tag{474}$$

$$h_y = -\,(x_1 \dot{z}_1 - z_1 \dot{x}_1) \tag{475}$$

$$h_z = x_1 \dot{y}_1 - y_1 \dot{x}_1 \tag{476}$$

Longitude of ascending node, Ω

$$\tan \Omega = \frac{h_x}{-h_y} \tag{477}$$

Orbit inclination, i

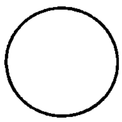
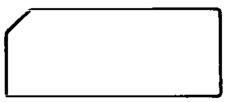
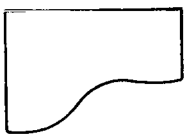
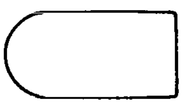
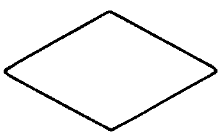
$$\tan i = \frac{\sqrt{h_x^2 + h_y^2}}{h_z} \tag{478}$$

$$\tan u = \frac{- x_1 \sin \Omega \cos i + y_1 \cos \Omega \cos i + z_1 \sin i}{x_1 \cos \Omega + y_1 \sin \Omega} \tag{479}$$

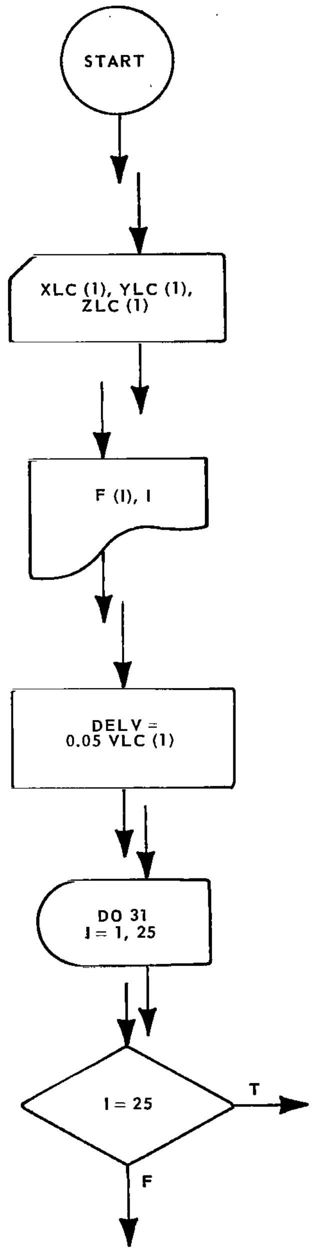
Augument of perigee, ω

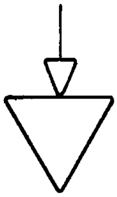
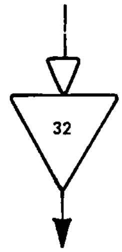
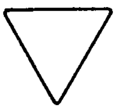
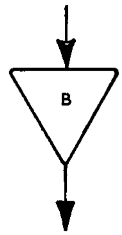
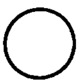
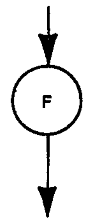
$$w = u - \nu \tag{480}$$

APPENDIX S
FLOWCHART SYMBOL DEFINITIONS

Symbol Shape	Definition	Information Inside Symbol
	Start/stop statement	Start or stop
	Card input statement	Input items
	Printer output statement	Output items
	Assignment statement	One or more statements
	DO statement	Repetition parameters
	Decision or IF statements	True and false conditions

Example



Symbol Shape	Definition	Information Inside Symbol	Example
	Unconditional transfer or GO TO statement	Numerical statement	
	Off-page connector label	Alphabetical letter	
	On-page connector label	Alphabetical letter	

APPENDIX T
ASSUMED VALUES OF GEOPHYSICAL CONSTANTS

Constant	Symbol	Assumed Value	FORTRAN Name
Flatness coefficient	f	$0.33528919 \times 10^{-2}$	FLAT
Canonical unit of length	CUL	0.63781660×10^{7} meters	-
Earth radius	e.r.	0.10000000 X 10 CUL	AE
Gravitational constant of Earth	k_e	$0.74366728 \times 10^{-1} \left(\frac{\text{e.r.}^{\frac{3}{2}}}{\text{min.}}\right)$	XK
Sum of masses	μ	0.100000000 X 10	XMU
Rotation of Earth	$\frac{d\theta}{dt}$	$0.43752691 \times 10^{-2} \left(\frac{\text{radians}}{\text{min.}}\right)$	DTHETA
Julian Date	J.D.		
OSO-III EPOCH		0.24397835×10^{7}	TJD
RELAY-II EPOCH		0.24398075×10^{7}	TJD
Canonical unit of time	CUT	0.13446874×10^{2} min.	-

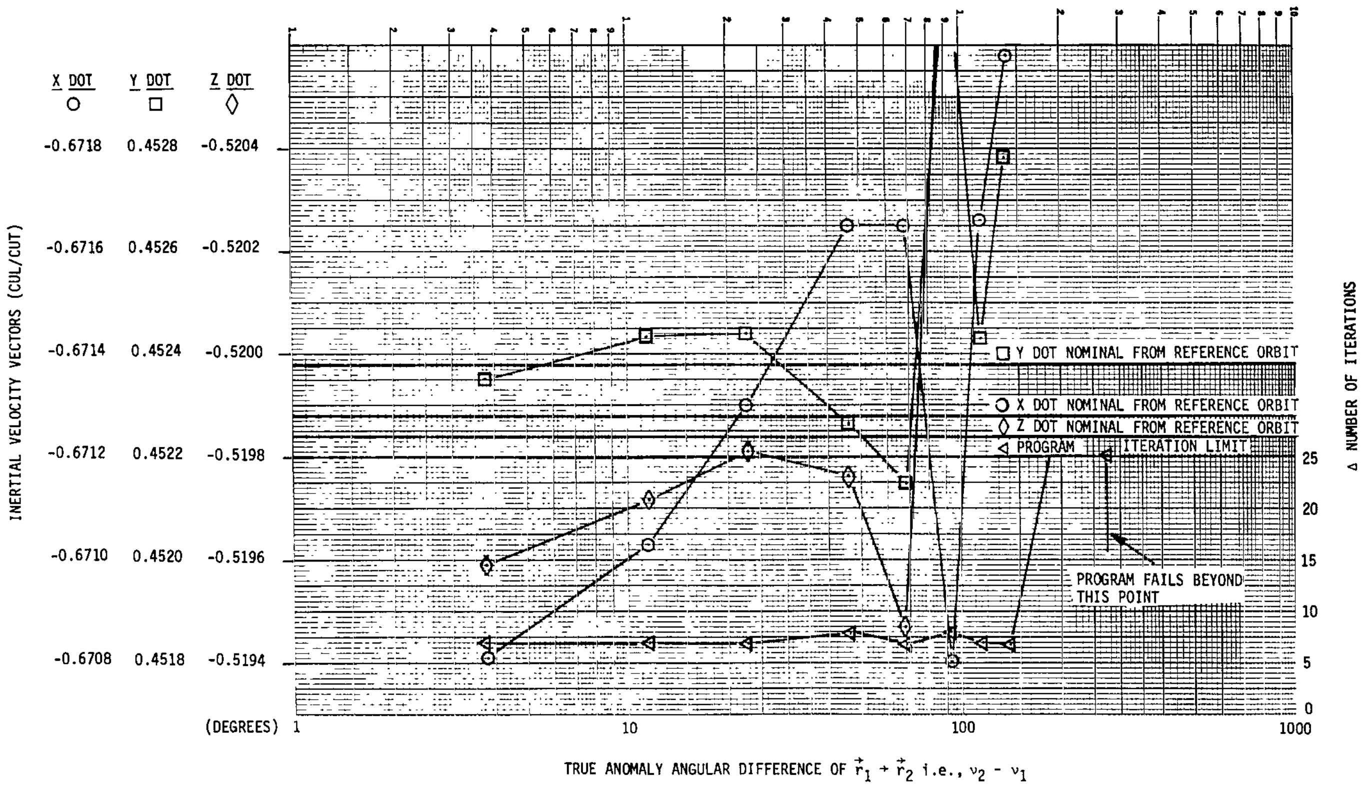


Figure 2. Results of Lambert-Euler PODM for OSO-III Orbit

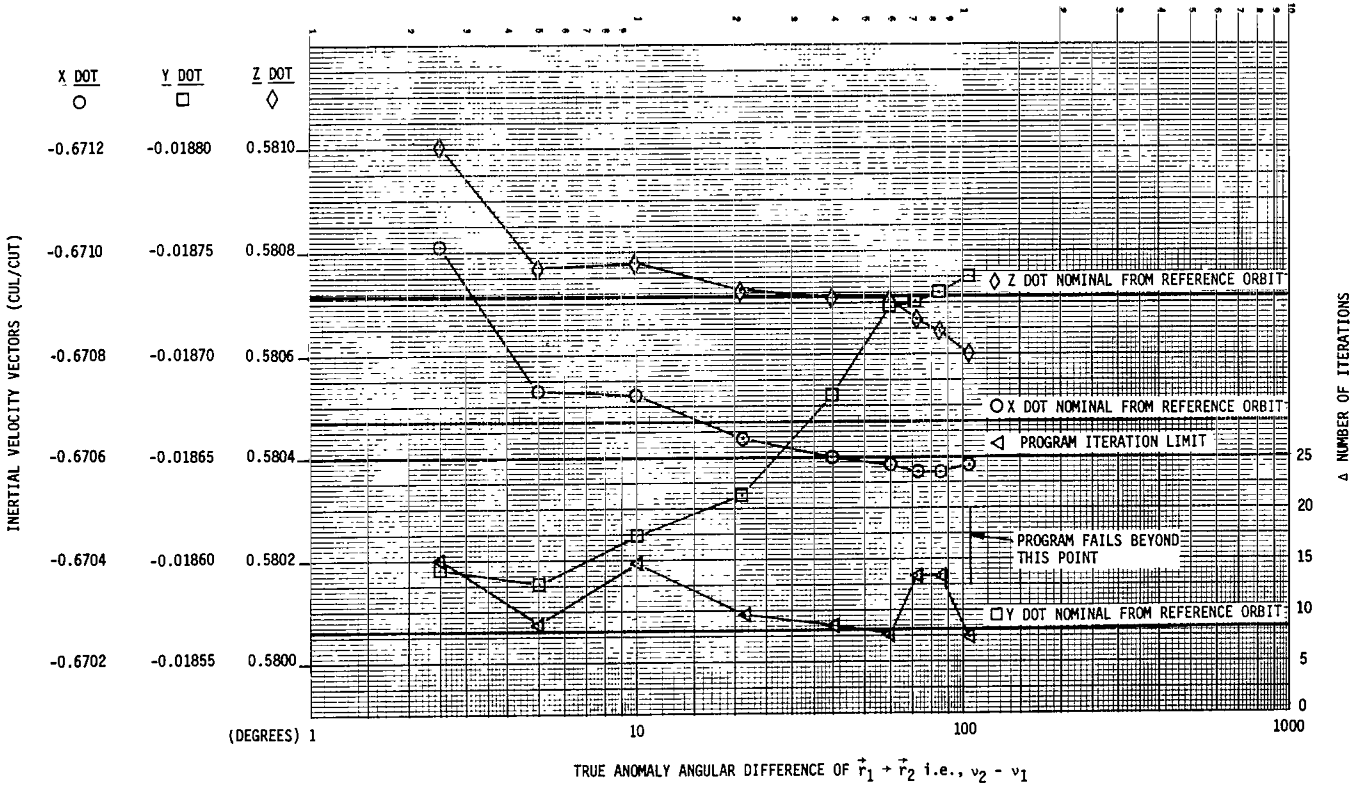


Figure 3. Results of Lambert-Euler PODM for Relay-II Orbit

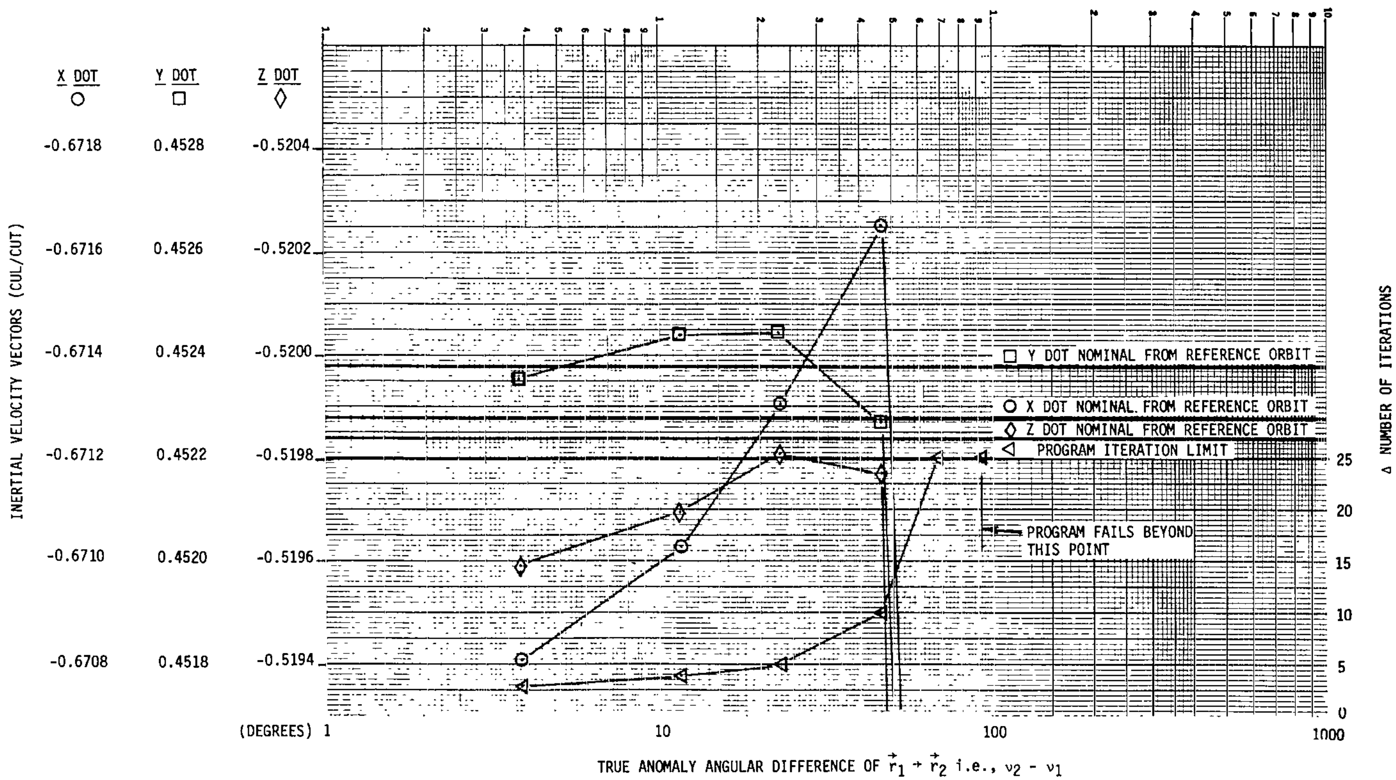


Figure 4. Results of F and G Series PODM for OSO-III Orbit

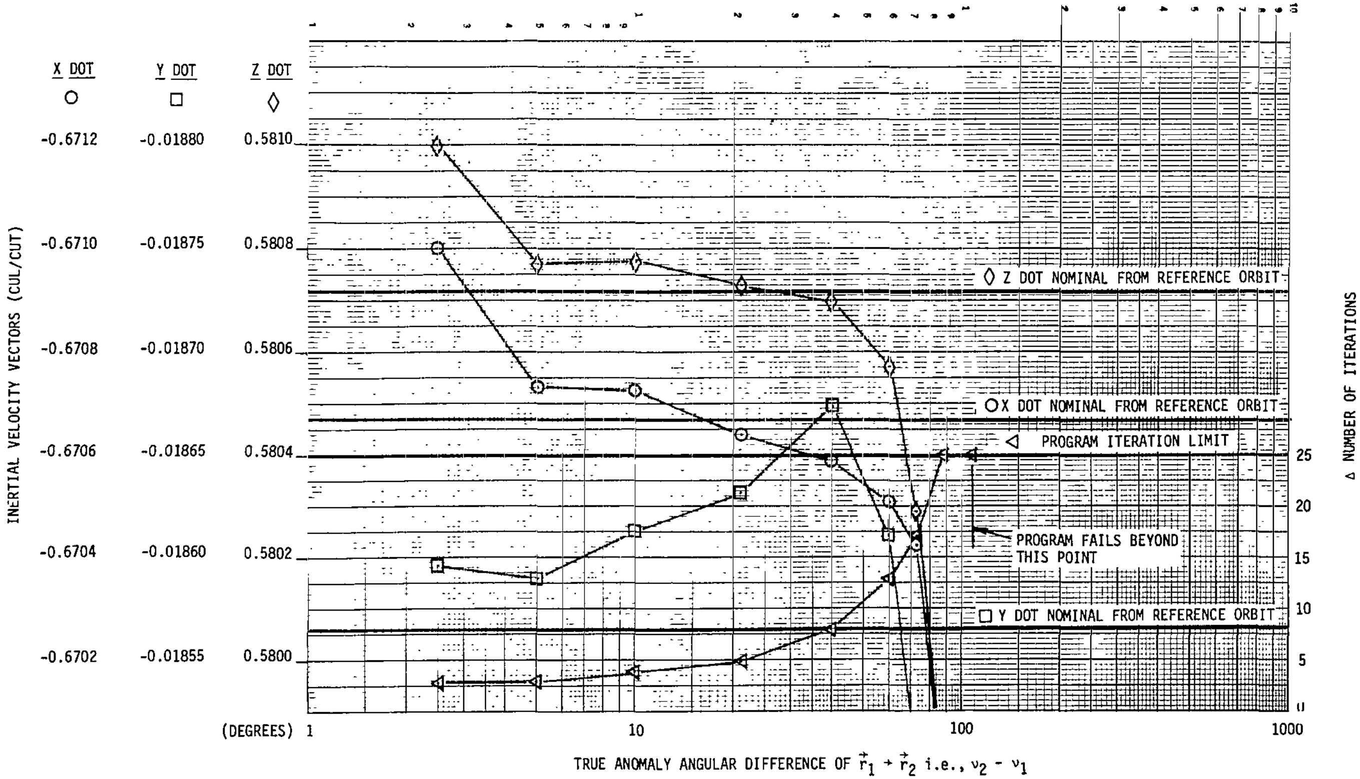


Figure 5. Results of F and G Series PODM for Relay-II Orbit

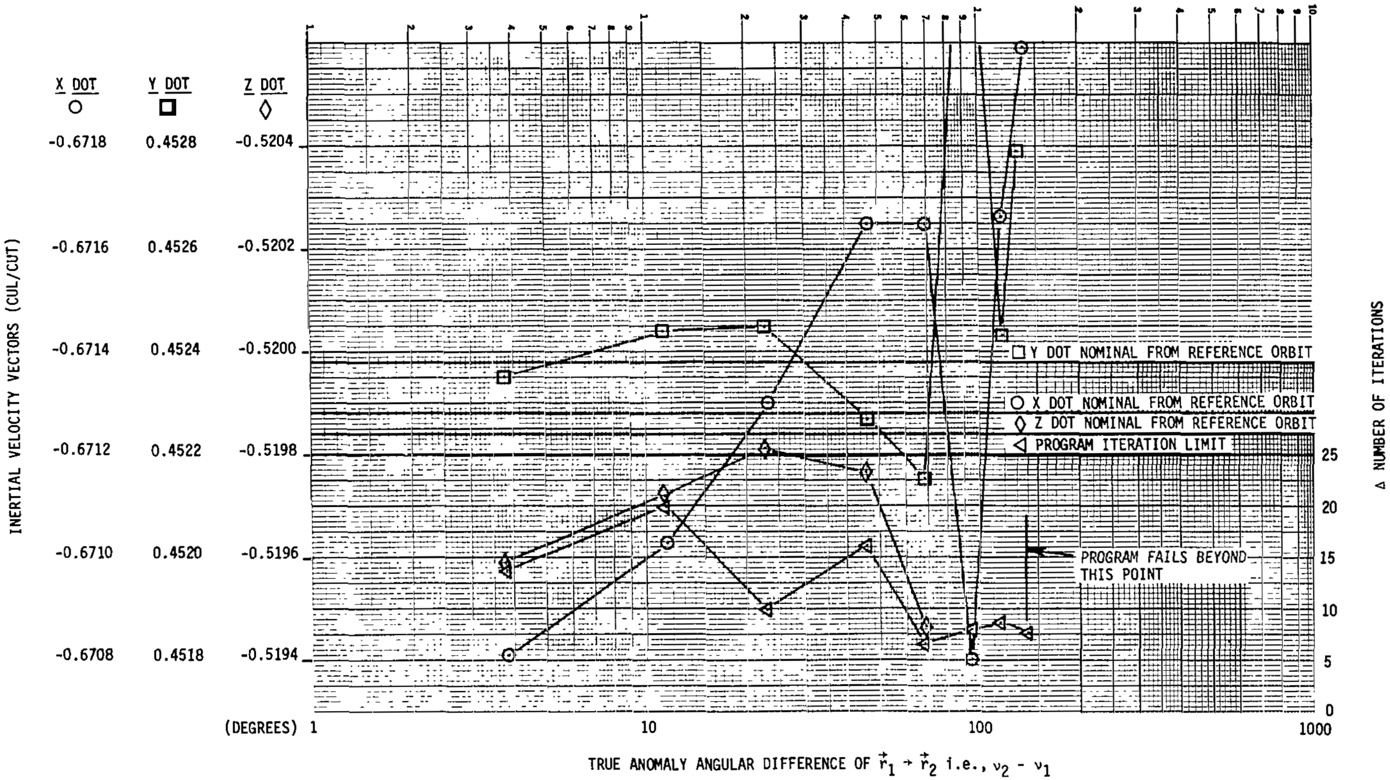


Figure 6. Results of Iteration of Semiparameter PODM for OSO-III Orbit

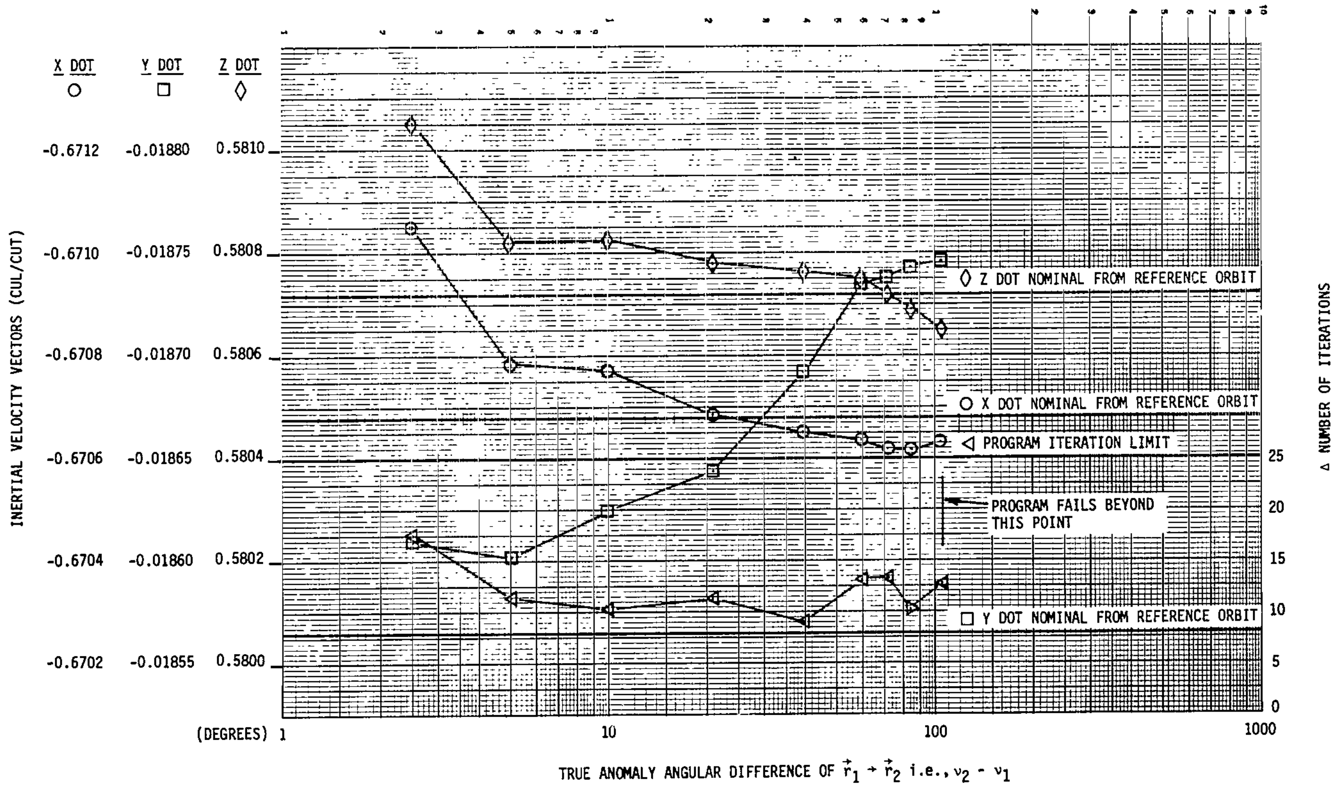


Figure 7. Results of Iteration of Semiparameter PODM for Relay-II Orbit

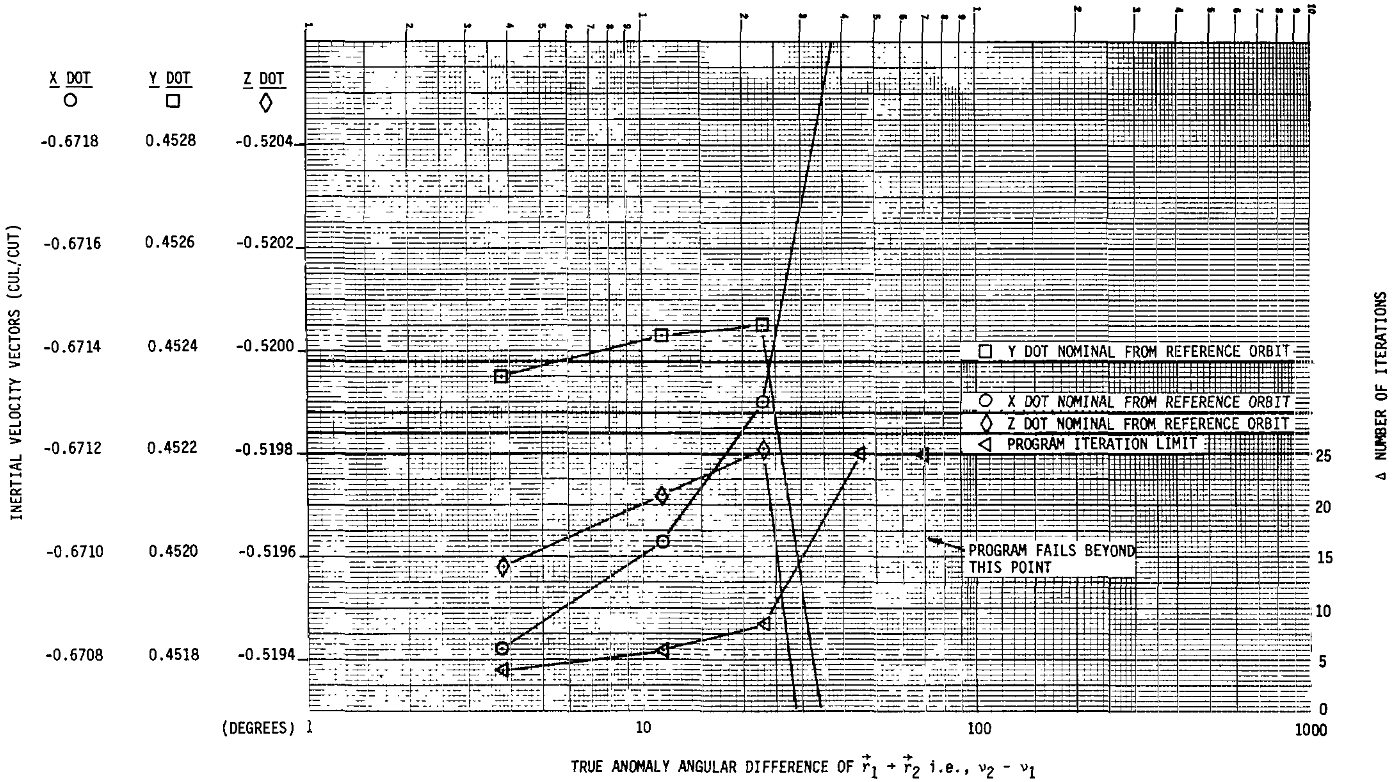


Figure 8. Results of Gaussian PODM for OSO-III Orbit

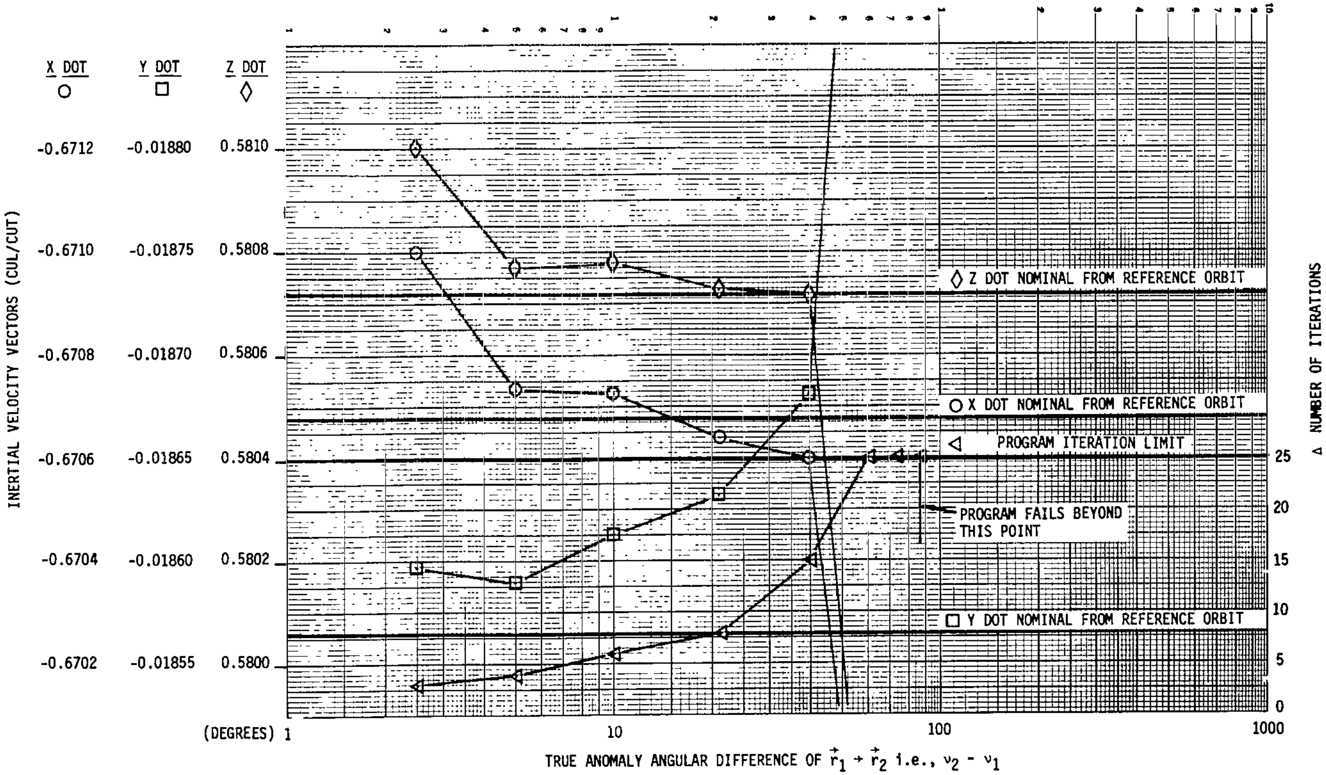


Figure 9. Results of Gaussian PODM for Relay-II Orbit

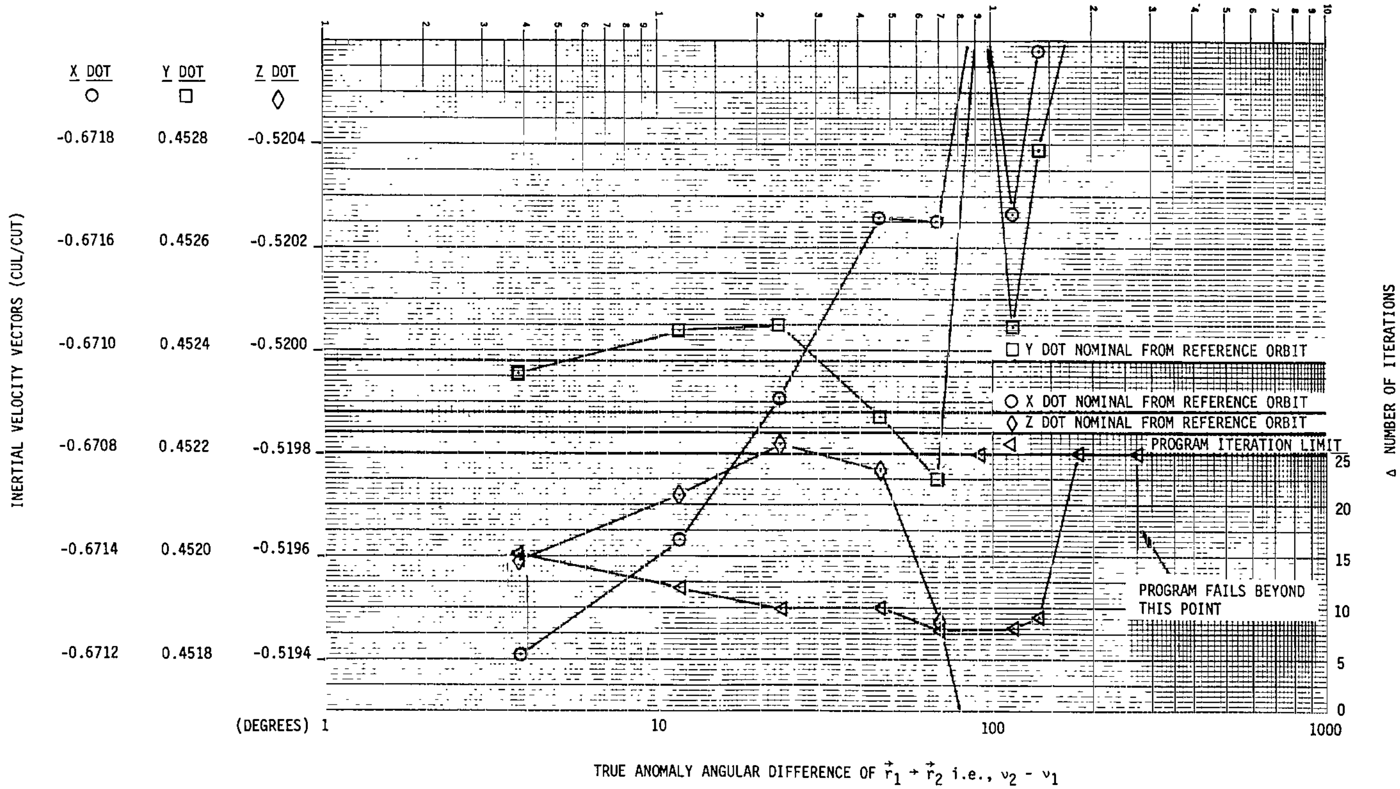


Figure 10. Results of Iteration of True Anomaly PODM for OSO-III Orbit

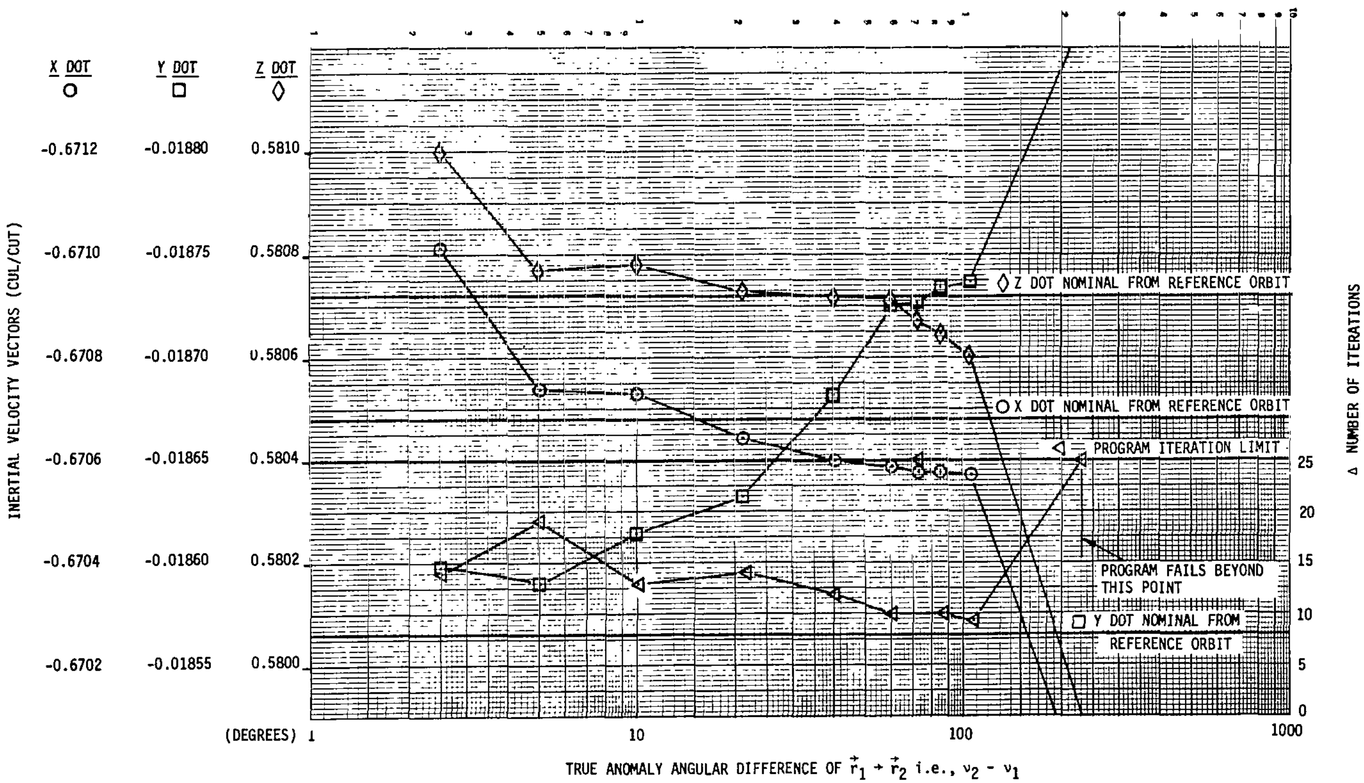


Figure 11. Results of Iteration of True Anomaly PODM for Relay-II Orbit

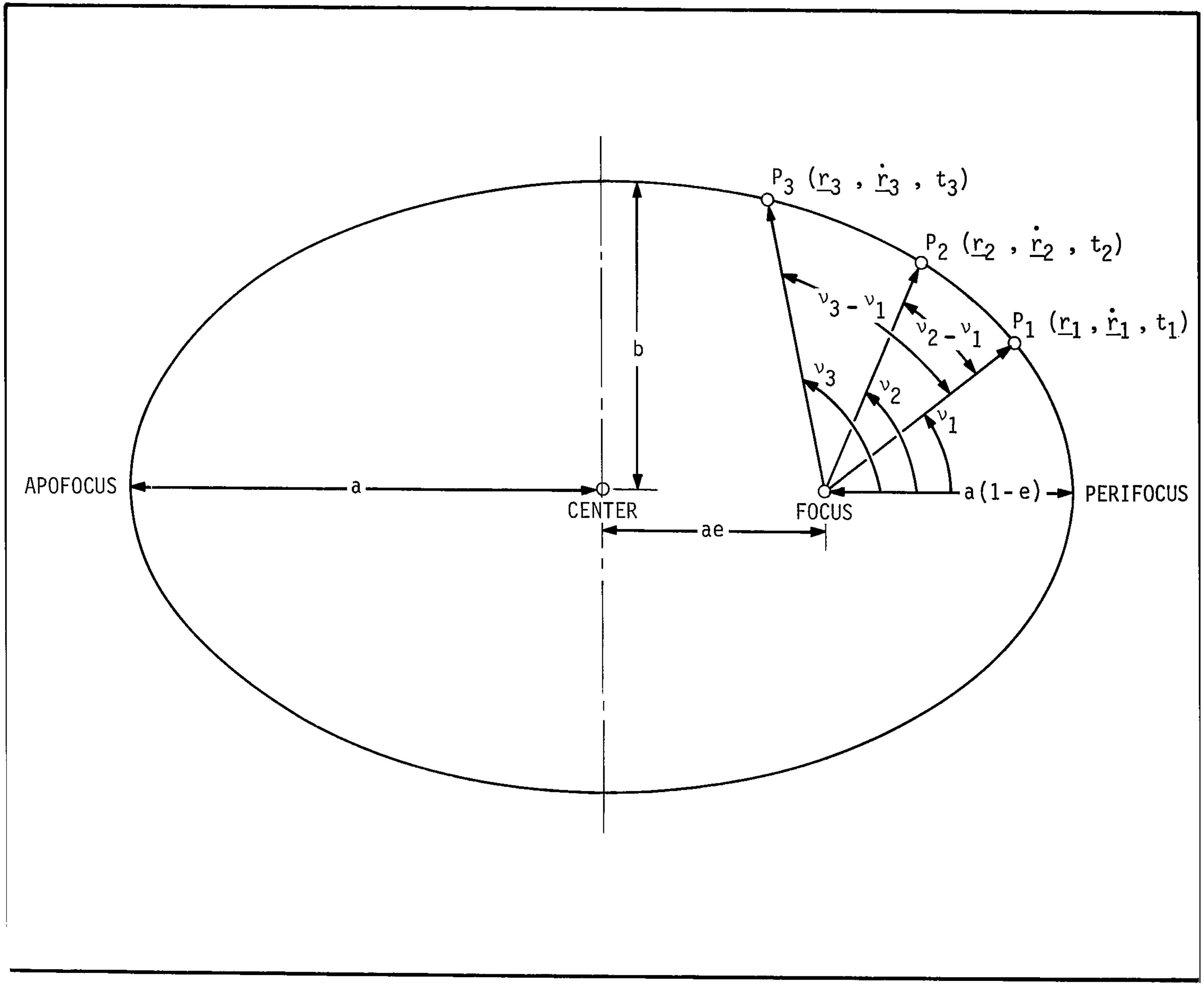


Figure 12. Elliptical Orbit

Table 1. OSO-III Position and Velocity Orbit Data*
Epoch 67Y 10M 20D 00H 00M 00S

Data Point	Position Vector (Canonical Units of Length)			Time from Epoch (Minutes)	Resultant Velocity Vector (Canonical Unit of Length Per Canonical Unit of Time)			Change in True Anomaly from Data Point 1 (Degrees)
	X	Y	Z		X DOT	Y DOT	Z DOT	
1	0.63397379 E00	0.87714911 E00	-0.57285980 E-01	0.42900000 E03	-0.67128213 E00	0.45237915 E00	-0.51983933 E00	0
2	0.58274812 E00	0.90885977 E00	-0.95773336 E-01	0.43000000 E03	-0.70685743 E00	0.40013314 E00	-0.51534094 E00	3.8
3	0.47289180 E00	0.96034300 E00	-0.17136390 E00	0.43200000 E03	-0.76862972 E00	0.29068616 E00	-0.49963709 E00	11.4
4	0.29327509 E00	0.10061443 E01	-0.27881096 E00	0.43500000 E03	-0.83592404 E00	0.11781151 E00	-0.45992297 E00	22.8
5	-0.92932753 E-01	0.97992039 E00	-0.45733638 E00	0.44100000 E03	-0.87135390 E00	-0.23408489 E00	-0.32909258 E00	45.6
6	-0.46473516 E00	0.80288180 E00	-0.56523331 E00	0.44700000 E03	-0.77289578 E00	-0.54884506 E00	-0.14805021 E00	68.4
7	-0.76519048 E00	0.50282255 E00	0.58621929 E00	0.45300000 E03	-0.55646495 E00	-0.77864062 E00	0.55149247 E-01	91.2
8	-0.94868622 E00	0.12595263 E00	-0.51737727 E00	0.45900000 E03	-0.25549497 E00	-0.88905191 E00	0.24948641 E00	114.0
9	-0.98742402 E00	-0.27017428 E00	-0.36935944 E00	0.46500000 E03	0.84194416 E-01	-0.86372645 E00	0.40548748 E00	136.8
10	-0.62955513 E00	-0.88498102 E00	0.65285980 E-01	0.47700000 E03	0.67560492 E00	-0.44112696 E00	0.51698187 E00	180.0
11	0.76766608 E00	-0.49528024 E00	0.58396071 E00	0.50100000 E03	0.55145966 E00	0.78482607 E00	-0.62454770 E-01	270.0
12	0.61361294 E00	0.89000851 E00	-0.77740021 E-01	0.52500000 E03	-0.68743522 E00	0.42989298 E00	-0.51773733 E00	360.0

*From reference 3.

Table 2. Relay-II Position and Velocity Orbit Data*
Epoch 67Y 11M 13D 00H 00M 00S

Data Point	Position Vector (Canonical Units of Length) X	Y	Z	Time from Epoch (Minutes)	Resultant Velocity Vector (Canonical Unit of Length Per Canonical Unit of Time) X DOT	Y DOT	Z DOT	Change in True Anomaly from Data Point 1 (Degrees)
1	-0.62706086 E00	0.13026303 E01	-0.26788816 E00	0.66500000 E03	-0.67069755 E00	-0.18565986 E-01	0.58071281 E00	0
2	-0.67640560 E00	0.13001240 E01	-0.22446287 E00	0.66600000 E03	-0.65562172 E00	-0.48674037 E-01	0.58641873 E00	2.5
3	-0.72456249 E00	0.12954130 E01	-0.18069118 E00	0.66700000 E03	-0.63983417 E00	-0.77927626 E-01	0.59099381 E00	5.0
4	-0.81727365 E00	0.12796195 E01	-0.92290918 E-01	0.66900000 E03	-0.60637538 E00	-0.13383906 E00	0.53694559 E00	10.0
5	-0.98699837 E00	0.12244558 E01	0.85668977 E-01	0.67300000 E03	-0.53391142 E00	-0.23461233 E00	0.59733711 E00	21.0
6	-0.12588173 E01	0.10352957 E01	0.43259412 E00	0.68100000 E03	-0.37896284 E00	-0.39161701 E00	0.56220952 E00	40.0
7	-0.14383867 E01	0.76921052 E00	0.74843089 E00	0.68900000 E03	-0.22604802 E00	-0.49460573 E00	0.49560149 E00	60.0
8	-0.15150151 E01	0.53705523 E00	0.95602583 E00	0.69500000 E03	-0.11873926 E00	-0.54226741 E00	0.43373466 E00	72.0
9	-0.15435262 E01	0.33061169 E00	0.11069735 E01	0.70000000 E03	-0.35627838 E-01	-0.56593395 E00	0.37767977 E00	85.0
10	-0.15282029 E01	-0.11262742 E-01	0.13038324 E01	0.70800000 E08	0.84472657 E-01	-0.57869376 E00	0.28350944 E00	105.0
11	0.89934644 E-01	-0.17919032 E01	0.10225903 E01	0.76800000 E03	0.49784070 E00	-0.75270923 E-01	-0.37661014 E00	237.0
12	0.10671941 E01	-0.13527369 E01	-0.76826469 E-01	0.79900000 E03	0.27460969 E00	0.48148531 E00	-0.52859756 E00	290.0
13	-0.64038080 E00	0.13030522 E01	-0.15192970 E00	0.86000000 E03	-0.66588429 E00	-0.27532078 E-01	0.58314165 E00	360.0

*From reference 3.

Table 3. Results of Lambert-Euler PODM for OSO-III Orbit

True Anomaly Angular Difference of $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67128213 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is 0.45237915 E0 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is -0.51983933 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\leq 10^{-10}$
3.8	-0.67081054 E0	0.45235122 E0	-0.51959007 E0	7
11.4	-0.67103078 E0	0.45243688 E0	-0.51971812 E0	7
22.8	-0.67130422 E0	0.45244467 E0	-0.51981342 E0	7
45.6	-0.67165405 E0	0.45226798 E0	-0.51976476 E0	8
68.4	-0.67164899 E0	0.45215526 E0	-0.51947102 E0	7
91.2	-0.67080666 E0	0.47883872 E0	-0.52650669 E0	8
114.0	-0.67166326 E0	0.45243605 E0	-0.51859662 E0	7
136.8	-0.67198271 E0	0.45278865 E0	-0.51775009 E0	7
180.0	Computer halted after second iteration.			
270.0	0.65513605 E0	-0.39298239 E0	0.48590209 E0	I=25*
360.0	Computer halted after six iterations.			

* Did not converge.

Table 4. Results of Lambert-Euler PODM for Relay-II Orbit

True Anomaly Angular Difference of $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67069755 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is -0.18565986 E-01 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is 0.58071281 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\leq 10^{-10}$
2.5	-0.67100717 E0	-0.18597544 E-01	0.58100110 E0	15
5.0	-0.67073406 E0	-0.18589597 E-01	0.58076722 E0	9
10.0	-0.67072314 E0	-0.18613539 E-01	0.58077790 E0	15
21.0	-0.67063993 E0	-0.18632342 E-01	0.58072454 E0	10
40.0	-0.67060216 E0	-0.18680951 E-01	0.58071562 E0	9
60.0	-0.67058860 E0	-0.18723884 E-01	0.58070555 E0	8
72.0	-0.67057670 E0	-0.18726358 E-01	0.58066965 E0	14
85.0	-0.67057675 E0	-0.18730622 E-01	0.58064458 E0	14
105.00	-0.67058715 E0	-0.18733006 E-01	0.58060167 E0	8
237.0	Computer halted after two iterations.			
290.0	Computer halted after two iterations.			
360.0	Computer halted after fifteen iterations.			

Table 5. Results of F and G Series PODM for OSO-III Orbit

True Anomaly Angular Difference of $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67128213 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is 0.45237915 E0 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is -0.51983933 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\leq 10^{-10}$
3.8	-0.67081054 E0	0.45235122 E0	-0.51959007 E0	3
11.4	-0.67103078 E0	0.45243689 E0	-0.51971812 E0	4
22.8	-0.67130428 E0	0.45244469 E0	-0.51981347 E0	5
45.6	-0.67165853 E0	0.45226850 E0	-0.51976718 E0	10
68.4	0.45123977 E0	-0.51913683 E0	0.33426901 E0	I=25*
91.2	0.23846019 E1	0.70582817 E0	-0.23591702 E1	I=25*
114.0	Computer halted after six iterations.			
136.8	Computer halted after three iterations.			
180.0	Computer halted after one iteration.			
270.0	Computer halted after one iteration.			
360.0	Computer halted after two iterations.			

*Did not converge.

Table 6. Results of F and G Series PODM for Relay-II Orbit

True Anomaly Angular Difference of $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67069755 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is -0.18565986 E-01 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is 0.58071281 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\leq 10^{-10}$
2.5	-0.67100717 E0	-0.18597544 E-01	0.58100110 E0	3
5.0	-0.67073406 E0	-0.18589597 E-01	0.58076722 E0	3
10.0	-0.67072313 E0	-0.18613540 E-01	0.58077789 E0	4
21.0	-0.67063956 E0	-0.18632358 E-01	0.58072423 E0	5
40.0	-0.67058782 E0	-0.18673756 E-01	0.58069903 E0	8
60.0	-0.67050862 E0	-0.18611443 E-01	0.58056860 E0	13
72.0	-0.67043325 E0	-0.18305457 E-01	0.58028936 E0	17
85.0	-0.19824139 E-01	0.57848394 E0	0.53834986 E0	I=25*
105.0	-0.24107564 E-01	0.57356778 E0	0.43500250 E0	I=25*
237.0	Computer halted after four iterations.			
290.0	Computer halted after one iteration.			
360.0	Computer halted after one iteration.			

* Did not converge.

Table 7. Results of Iteration of Semiparameter PODM for OSO-III Orbit

True Anomaly Angular Difference of $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67128213 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is 0.45237915 E0 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is -0.51983933 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\leq 10^{-10}$
3.8	-0.67081054 E0	0.45235122 E0	-0.51959007 E0	14
11.4	-0.67103078 E0	0.45243688 E0	-0.51971812 E0	20
22.8	-0.67130422 E0	0.45244466 E0	-0.51981342 E0	10
45.6	-0.67165405 E0	0.45226799 E0	-0.51976476 E0	16
68.4	-0.67164899 E0	0.45215526 E0	-0.51947102 E0	7
91.2	-0.67080666 E0	0.47883870 E0	0.52650669 E0	8
114.0	-0.67166326 E0	0.45243607 E0	-0.51859662 E0	9
136.8	-0.67198271 E0	0.45278865 E0	-0.51775009 E0	8
180.0	Computer halted after one iteration.			
270.0	Computer halted after two iterations.			
360.0	Computer halted after two iterations.			

Table 8. Results of Iteration of Semiparameter PODM for Relay-II Orbit

True Anomaly Angular Difference of $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67069755 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is -0.18565986 E-01 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is 0.58071281 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\le 10^{-10}$
2.5	-0.67100717 E0	-0.18597543 E-01	0.58100110 E0	15
5.0	-0.67073406 E0	-0.18589597 E-01	0.58076722 E0	9
10.0	-0.67072314 E0	-0.18613537 E-01	0.58077790 E0	8
21.0	-0.67063993 E0	-0.18632343 E-01	0.58072454 E0	9
40.0	-0.67060216 E0	-0.18680947 E-01	0.58071562 E0	7
60.0	-0.67058860 E0	-0.18723889 E-01	0.58070555 E0	11
72.0	-0.67057669 E0	-0.18726365 E-01	0.58066965 E0	11
85.0	-0.67057675 E0	-0.18730629 E-01	0.58064458 E0	8
105.0	-0.67058717 E0	-0.18732987 E-01	0.58060167 E0	10
237.0	Computer halted after five iterations.			
290.0	Computer halted after two iterations.			
360.0	Computer halted after two iterations.			

Table 9. Results of Gaussian PODM for OSO-III Orbit

True Anomaly Angular Difference of $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67128213 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is 0.45237915 E0 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is -0.51983933 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\leq 10^{-10}$
3.8	-0.67081054 E0	0.45235122 E0	-0.51959007 E0	4
11.4	-0.67103078 E0	0.45243688 E0	-0.51971812 E0	6
22.8	-0.67130423 E0	0.45244466 E0	-0.51981342 E0	8
45.6	-0.91948458 E0	-0.37633925 E01	0.11297649 E01	I=25*
68.4	-0.79344996 E0	-0.83165543 E-02	-0.38585390 E0	I=25*
91.2	Computer halted during first iteration.			
114.0	Computer halted during first iteration.			
136.8	Computer halted during first iteration.			
180.0	Computer halted during first iteration.			
270.0	Computer halted during first iteration.			
360.0	Computer halted during first iteration.			

* Did not converge.

Table 10. Results of Gaussian PODM for Relay-II Orbit

True Anomaly Angular Difference of $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67069755 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is -0.18565986 E-01 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is 0.58071281 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\le 10^{-10}$
2.5	-0.67100717 E0	-0.18597544 E-01	0.58100110 E0	3
5.0	-0.67073406 E0	-0.18589598 E-01	0.58076722 E0	4
10.0	-0.67072314 E0	-0.18613541 E-01	0.58077790 E0	6
21.0	-0.67063993 E0	-0.18632347 E-01	0.58072454 E0	8
40.0	-0.67060215 E0	-0.18680959 E-01	0.58071562 E0	15
60.0	0.18744650 E-01	-0.38893012 E-01	0.79794763 E-02	I=25*
72.0	0.29766430 E-01	-0.61606750 E-01	0.12576050 E-01	I=25*
85.0	0.38514860 E-01	-0.79439075 E-01	0.16103859 E-01	I=25*
105.0	Computer halted after first iteration.			
237.0	Computer halted during first iteration.			
290.0	Computer halted during first iteration.			
360.0	Computer halted during first iteration.			

* Did not converge.

Table 11. Results of Iteration of True Anomaly PODM for OSO-III Orbit

True Anomaly Angular Difference of $\vec{r}1 \to \vec{r}2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67128213 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is 0.45237915 E0 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is -0.51983933 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\leq 10^{-10}$
3.8	-0.67081054 E0	0.45235122 E0	-0.51959007 E0	15
11.4	-0.67103078 E0	0.45243688 E0	-0.51971812 E0	12
22.8	-0.67130422 E0	0.45244467 E0	-0.51981342 E0	10
45.6	-0.67165404 E0	0.45226800 E0	-0.51976476 E0	10
68.4	-0.67164899 E0	0.45215526 E0	-0.51947102 E0	8
91.2	-0.67744460 E0	0.50667361 E0	0.53936484 E0	I=25*
114.0	-0.67166326 E0	0.45243607 E0	-0.51859662 E0	8
136.8	-0.67198271 E0	0.45278862 E0	-0.51775008 E0	7
180.0	-0.17226110 E01	0.11138352 E01	-0.19506859 E01	I=25*
270.0	0.12672460 E0	-0.85052773 E-01	0.97780343 E-01	I=25*
360.0	Computer halted after six iterations.			

* Did not converge.

Table 12. Results of Iteration of True Anomaly PODM for Relay-II Orbit

True Anomaly Angular Difference of $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot is -0.67069755 E0 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot is -0.18565986 E-01 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot is 0.58071281 E0 (CUL/CUT)	Iterations Required to Obtain an Epsilon (ε) of $\leq 10^{-10}$
2.5	-0.67100717 E0	-0.18597543 E-01	0.58100110 E0	14
5.0	-0.67073406 E0	-0.18589597 E-01	0.58076722 E0	19
10.0	-0.67072314 E0	-0.18613537 E-01	0.58077790 E0	13
21.0	-0.67063993 E0	-0.18632342 E-01	0.58072454 E0	14
40.0	-0.67060216 E0	-0.18680947 E-01	0.58071562 E0	12
60.0	-0.67058860 E0	-0.18723889 E-01	0.58070555 E0	10
72.0	-0.67057669 E0	-0.18726361 E-01	0.58066965 E0	I=25*
85.0	-0.67057675 E0	-0.18730629 E-01	0.58064458 E0	10
105.0	-0.67058716 E0	-0.18732997 E-01	0.58060167 E0	9
237.0	-0.46843289 E-01	-0.32805744 E-02	0.41666293 E-01	I=25*
290.0	Computer halted after two iterations.			
360.0	Computer halted after four iterations.			

*Did not converge.

Table 13. Position and Time PODM Classical Orbital Element Comparisons - Semimajor Axis

True Anomaly Angular Difference of $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Nominal Semimajor Axis from Reference Orbit (Earth Radii)	Gaussian PODM	F and G Series PODM	Iteration of True Anomaly PODM	Iteration of Semiparameter PODM	Lambert-Euler PODM
OSO-III	1.0866609					
3.8		1.0860143	1.0860143	1.0860143	1.0860143	1.0860143
11.4		1.0866115	1.0866115	1.0866115	1.0866115	1.0866115
22.8		1.0871705	1.0871707	1.0871705	1.0871707	1.0871705
45.6		No data	1.0874878	1.0874771	1.0874771	1.0874771
68.4		0.93732551	0.79332067	1.0869877	1.0869877	1.0869877
91.2		No data	No data	1.1921556	1.1249820	1.1249820
114.0		No data	No data	1.0862386	1.0862386	1.0862385
136.8		No data	No data	1.0860864	1.0860865	1.0860865
180.0		No data	No data	No data	No data	No data
270.0		No data	No data	0.55171611	No data	0.97499981
360.0		No data	No data	No data	No data	No data
RELAY-II	1.7448736					
2.5		1.7479539	1.7479539	1.7479539	1.7479539	1.7479539
5.0		1.7460054	1.7460054	1.7460054	1.7460054	1.7460054
10.0		1.7460013	1.7460012	1.7460013	1.7460013	1.7460013
21.0		1.7454744	1.7454718	1.7454744	1.7454744	1.7454744
40.0		1.7452940	1.7451760	1.7452940	1.7452940	1.7452940
60.0		No data	1.7443844	1.7452079	1.7452079	1.7452079
72.0		0.73778052	1.7430571	1.7450325	1.7450325	1.7450326
85.0		0.73953397	1.3598060	1.7449446	1.7449446	1.7449446
105.0		No data	1.1883995	1.7448357	1.7448357	1.7448357
237.0		No data	No data	0.73729180	No data	No data
290.0		No data	No data	No data	No data	No data
360.0		No data	No data	No data	No data	No data

No data indicates program failed in computing these values.

Table 14. Position and Time PODM Classical Orbital Element Comparisons - Eccentricity

True Anomaly Angular Difference of $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Nominal Eccentricity from Reference Orbit	Gaussian PODM	F and G Series PODM	Iteration of True Anomaly PODM	Iteration of Semiparameter PODM	Lambert-Euler PODM
OSO-III	0.0021640595					
3.8		0.0023845575	0.0023845584	0.0023845588	0.0023845579	0.0023845587
11.4		0.0028469817	0.0028469864	0.0028469841	0.0028469843	0.0028469841
22.8		0.0032708105	0.0032709635	0.0032708124	0.0032708120	0.0032708124
45.6		No data	0.0034625667	0.0034533924	0.0034533867	0.0034533750
68.4		0.52792947	0.42286780	0.0029925489	0.0029925485	0.0029925490
91.2		No data	No data	0.092067645	0.049560352	0.049560354
114.0		No data	No data	0.0023672168	0.0023672094	0.0023671832
136.8		No data	No data	0.0022529044	0.0022529505	0.0022529528
180.0		No data	No data	No data	No data	No data
270.0		No data	No data	0.96439317	No data	0.11970044
360.0		No data	No data	No data	No data	No data
RELAY-II	0.24114781					
2.5		0.24171947	0.24171947	0.24171947	0.24171947	0.24171947
5.0		0.24112427	0.24112427	0.24112427	0.24112427	0.24112427
10.0		0.24109974	0.24109972	0.24109974	0.24109974	0.24109974
21.0		0.24091843	0.24091762	0.24091843	0.24091843	0.24091843
40.0		0.24082014	0.24079030	0.24082016	0.24082016	0.24082016
60.0		No data	0.24060980	0.24076101	0.24076101	0.24076102
72.0		0.99999997	0.2404882	0.24071194	0.24071194	0.24071196
85.0		0.99999982	0.53935368	0.24068833	0.24068833	0.24068834
105.0		No data	0.63807485	0.24066680	0.24066682	0.2406678
237.0		No data	No data	0.99433781	No data	No data
290.0		No data	No data	No data	No data	No data
360.0		No data	No data	No data	No data	No data

No data indicates program failed in computing these values.

Table 15. Position and Time PODM Classical Orbital Element Comparisons - Longitude of Ascending Node

True Anomaly Angular Difference of $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Nominal Longitude of Ascending Node from Reference Orbit (Radians)	Gaussian PODM	F and G Series PODM	Iteration of True Anomaly PODM	Iteration of Semiparameter PODM	Lambert-Euler PODM
OSO-III	-2.2460589					
3.8		-2.2786124	-2.2786124	-2.2786124	-2.2786124	-2.2786124
11.4		-2.2786149	-2.2786149	-2.2786149	-2.2786149	-2.2786149
22.8		-2.2786216	-2.2786216	-2.2786216	-2.2786216	-2.2786216
45.6		No data	-2.2786445	-2.2786445	-2.2786445	-2.2786445
68.4		-2.2786827	0.83662099	-2.2786827	-2.2786827	-2.2786827
91.2		No data	No data	1.0276607	1.0276607	1.0276607
114.0		No data	No data	-2.2788396	-2.2788396	-2.2788396
136.8		No data	No data	-2.2790210	-2.2790210	-2.2790210
180.0		No data	No data	No data	No data	No data
270.0		No data	No data	0.86280731	No data	0.86280731
360.0		No data	No data	No data	No data	No data
RELAY-II	2.2064792					
2.5		2.1972221	2.1972221	2.1972221	2.1972221	2.1972221
5.0		2.1972213	2.1972213	2.1972213	2.1972213	2.1972213
10.0		2.1972198	2.1972198	2.1972198	2.1972198	2.1972198
21.0		2.1972183	2.1972183	2.1972183	2.1972183	2.1972183
40.0		2.1972201	2.1972201	2.1972201	2.1972201	2.1972201
60.0		No data	2.1972270	2.1972270	2.1972270	2.1972270
72.0		-0.94435814	2.1972345	2.1972345	2.1972345	2.1972345
85.0		-0.94435049	1.9516982	2.1972422	2.1972422	2.1972422
105.0		No data	1.9406196	2.1972568	2.1972568	2.1972568
237.0		No data	No data	2.1976536	No data	No data
290.0		No data	No data	No data	No data	No data
360.0		No data	No data	No data	No data	No data

No data indicates program failed in computing these values.

Table 16. Position and Time PODM Classical Orbital Element Comparisons - Orbital Inclination

True Anomaly Angular Difference of $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Nominal Orbital Inclination from Reference Orbit (Radians)	Gaussian PODM	F and G Series PODM	Iteration of True Anomaly PODM	Iteration of Semiparameter PODM	Lambert-Euler PODM
OSO-III	0.57356194					
3.8		0.57386440	0.57386440	0.57386440	0.57386440	0.57386440
11.4		0.57385039	0.57385039	0.57385039	0.57385039	0.57385039
22.8		0.57381367	0.57381367	0.57381367	0.57381367	0.57381367
45.6		No data	0.57368666	0.57368666	0.57368666	0.57368666
68.4		0.57347473	2.6863359	0.57347473	0.57347473	0.57347473
91.2		No data	No data	0.56982104	0.56982104	0.56982104
114.0		No data	No data	0.57260595	0.57260595	0.57260595
136.8		No data	No data	0.57160489	0.57160489	0.57160489
180.0		No data	No data	No data	No data	No data
270.0		No data	No data	2.5686864	No data	2.5686864
360.0		No data	No data	No data	No data	No data
RELAY-II	0.80848228					
2.5		0.80872844	0.80872844	0.80872844	0.80872844	0.80872844
5.0		0.80873061	0.80873061	0.80873061	0.80873061	0.80873061
10.0		0.80873462	0.80873462	0.80873462	0.80873462	0.80873462
21.0		0.80873900	0.80873900	0.80873900	0.80873900	0.80873900
40.0		0.80873386	0.80873386	0.80873386	0.80873386	0.80873386
60.0		No data	0.80871476	0.80871476	0.80871476	0.80871476
72.0		2.3328988	0.80869383	0.80869383	0.80869383	0.80869383
85.0		2.3329201	1.9210250	0.80867257	0.80867257	0.80867257
105.0		No data	1.9725943	0.80863180	0.80863180	0.80863180
237.0		No data	No data	0.80753039	No data	No data
290.0		No data	No data	No data	No data	No data
360.0		No data	No data	No data	No data	No data

No data indicates program failed in computing these values.

Table 17. Position and Time PODM Classical Orbital Element Comparisons - Nominal Argument of Perigee

True Anomaly Angular Difference of $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Nominal Argument of Perigee from Reference Orbit (Radians)	Gaussian PODM	F and G Series PODM	Iteration of True Anomaly PODM	Iteration of Semiparameter PODM	Lambert-Euler PODM
OSO-III	-3.4856807					
3.8		-3.5809687	-3.5809693	-3.5809695	-3.5809689	-3.5809694
11.4		-3.4656506	-3.4656515	-3.4656518	-3.4656519	-3.4656518
22.8		-3.3580944	-3.3580766	-3.3580952	-3.3580950	-3.3580952
45.6		No data	-3.2308644	-3.2320094	-3.2320072	-3.2320031
68.4		-5.4079583	-2.9238213	-3.2244081	-3.2244080	-3.2244081
91.2		No data	No data	-2.7283392	-0.87159489	-0.87159441
114.0		No data	No data	-3.3514835	-3.3514834	-3.3514790
136.8		No data	No data	-3.3938025	-3.3938061	-3.3938062
180.0		No data	No data	No data	No data	No data
270.0		No data	No data	-3.2392317	No data	-2.9099143
360.0		No data	No data	No data	No data	No data
RELAY-II	-1.3234053					
2.5		-1.3088962	-1.3088962	-1.3088962	-1.3088962	-1.3088962
5.0		-1.3118151	-1.3118151	-1.3118151	-1.3118151	-1.3118151
10.0		-1.3117024	-1.3117024	-1.3117024	-1.3117024	-1.3117024
21.0		-1.3123907	-1.3123945	-1.3123907	-1.3123907	-1.3123907
40.0		-1.3124450	-1.3126543	-1.3124450	-1.3124450	-1.3124450
60.0		No data	-1.3141631	-1.3124132	-1.3124132	-1.3124132
72.0		-6.0285606	-1.3176087	-1.3127055	-1.3127055	-1.3127055
85.0		-6.0285203	-2.4581196	-1.3128599	-1.3128599	-1.3128599
105.0		No data	-2.7249224	-1.3130937	-1.3130937	-1.3130937
237.0		No data	No data	-3.3956230	No data	No data
290.0		No data	No data	No data	No data	No data
360.0		No data	No data	No data	No data	No data

No data indicates program failed in computing these values.

Table 18. Computer Core Requirements

PODM	No. of 24-Bit Words Required
Lambert-Euler	3352
F and G Series	4649
Iteration of Semiparameter	3479
Gaussian	3308
Iteration of True Anomaly	3406
Method of Gauss	5254
Laplace	4470
Double R-Iteration	4919
Modified Laplacian	3981
R-Iteration	4458
Trilateration	4231
Herrick-Gibbs	3525
Computation for Range, Range Rate, and Angle Data	2731

Table 19. PODM Computation Time

PODM	Total Time for Program with One Iteration (Milliseconds)	Total Time Without "Solution for Classical Elements" (Milliseconds)	Time for Each Additional Iteration (Milliseconds)
Position and Time			
F and G Series	21	15	8
Gaussian	17	11	5
Iteration of Semiparameter	16.5	10.5	6
Iteration of the True Anomaly	16.5	10.5	6
Lambert-Euler	16	10	5
Angles Only			
Laplace	19	13	5
Double R-Iteration	19	13	9
Method of Gauss (1)	26	16	5 & 8
Mixed Data			
Herrick-Gibbs	13	7	--
R-Iteration	20	14	8
Modified Laplacian	17	11	5
Triateration	17	11	5

(1) Method of Gauss has two iteration loops

Table 20. Ease of Convergence

PODM	Average Number of Iterations Required		
	OSO-III	Relay-II	Combined Average
Lambert-Euler	7	11	9
F and G Series	6	8	7
Gaussian	9	7	8
Iteration of Semiparameter	12	10	11
Iteration of True Anomaly	10	14	12

Table 21. Best Overall Results for Radius Vector Spread

Range of Radius Vector Spread	PODM
$0° < \nu < 45°$	F and G Series
$45° < \nu < 140°$	Gaussian
	Lambert-Euler
	Iteration of True Anomaly
	Iteration of Semiparameter

Table 22. Order of Selection for Optimum PODM

	Computation Time	Ease of Convergence	Best Overall Accuracy
Lambert-Euler	1	1-2	1-2
Iteration of Semiparameter	2-3	3-4	1-2
Iteration of True Anomaly	2-3	3-4	3
Gaussian	4	1-2	5
F and G Series	5	5	4

Table 23. OSO-III Range/Range Rate and Angular Data
(Topocentric Coordinate System)
Epoch 67Y 10M 20D 00H 00M 00S

Data Point	Range ρ (CUL)	Range Rate $\dot{\rho}$ (CUL/CUT)	Declination δ (Radians)	Right Ascension α (Radians)	Time from Epoch (Minutes)	Station Name
1	0.11634686 E0	-0.45635762 E-2	-0.62507848 E0	0.44485366 E0	0.42900000 E3	Quito
1	0.19238541 E0	-0.67367391 E0	0.85750708 E0	0.36812866 E0	0.42900000 E3	Lima
1	0.57151514 E0	-0.65374571 E0	0.10182674 E1	0.41054875 E0	0.42900000 E3	Santiago
2	0.13288929 E0	0.42086265 E0	-0.93053247 E0	0.10773335 E1	0.43000000 E3	Quito
2	0.14805040 E0	-0.49604487 E0	0.80771167 E0	0.81921376 E0	0.43000000 E3	Lima
2	0.52415134 E0	-0.61897421 E0	0.10249512 E1	0.58326064 E0	0.43000000 E3	Santiago
3	0.22583837 E0	0.74340715 E0	-0.93836480 E0	0.20783978 E1	0.43200000 E3	Quito
3	0.12994566 E0	0.30464432 E0	0.24410711 E0	0.18328840 E1	0.43200000 E3	Lima
3	0.43970803 E0	-0.50531992 E0	0.10101581 E1	0.10121787 E1	0.43200000 E3	Santiago
4	0.40313098 E0	0.81886282 E0	-0.80148994 E0	0.25539242 E1	0.43500000 E3	Quito
4	0.26871481 E0	0.77208888 E0	-0.28689805 E0	0.24667446 E1	0.43500000 E3	Lima
4	0.36009344 E0	-0.17300730 E0	0.82681412 E0	0.17591614 E1	0.43500000 E3	Santiago
5	0.76749498 E0	0.79928922 E0	-0.65599498 E0	0.29501521 E1	0.44100000 E3	Quito
5	0.63169893 E0	0.81632973 E0	-0.41477732 E0	0.29287723 E1	0.44100000 E3	Lima
5	0.46777177 E0	0.55520424 E0	0.18582274 E0	0.26801299 E1	0.44100000 E3	Santiago
6	0.76352988 E0	0.71594029 E0	-0.28127920 E-1	0.31159661 E1	0.44700000 E3	Santiago
6	0.74785942 E0	-0.80054013 E0	-0.17638387 E0	0.97278986 E0	0.44700000 E3	Johannesburg
6	0.10629221 E1	-0.74142968 E0	-0.22924942 E0	0.10951673 E1	0.44700000 E3	Madagascar
7	0.10793174 E1	-0.23126828 E0	0.12383610 E1	0.12266098 E1	0.45300000 E3	Johannesburg
7	0.11280901 E1	-0.45427354 E0	0.93836178 E0	0.13128296 E1	0.45300000 E3	Madagascar
8	0.10531220 E0	0.11922778 E0	-0.91352007 E0	-0.27920773 E1	0.45900000 E3	Johannesburg
8	0.35115898 E0	-0.81909318 E0	-0.58424762 E0	0.16303254 E1	0.45900000 E3	Madagascar

Table 23. OSO-III Range/Range Rate and Angular Data
(Topocentric Coordinate System)
Epoch 67Y 10M 20D 00H 00M 00S (Cont'd)

Data Point	Range ρ (CUL)	Range Rate $\dot{\rho}$ (CUL/CUT)	Declination δ (Radians)	Right Ascension α (Radians)	Time from Epoch (Minutes)	Station Name
9	0.41131892 E0	0.82112059 E0	0.15782305 E0	-0.18084010 E1	0.46500000 E3	Johannesburg
9	0.10993237 E0	0.37522557 E0	-0.42843954 E0	-0.22250441 E1	0.46500000 E3	Madagascar
10	0.11169323 E1	0.72873755 E0	0.46343195 E0	-0.12996834 E1	0.47700000 E3	Johannesburg
10	0.80621682 E0	0.79408998 E0	0.50347015 E0	-0.11542474 E1	0.47700000 E3	Madagascar
10	0.11916632 E1	-0.18574710 E0	0.57157446 E0	-0.29859676 E1	0.47700000 E3	Orroral
12	0.47239474 E0	-0.59420234 E0	-0.18853377 E0	-0.21196418 E0	0.52500000 E3	Quito
12	0.50359872 E0	-0.79805117 E0	0.25089801 E0	-0.16507695 E0	0.52500000 E3	Lima
12	0.75736078 E0	-0.79022343 E0	0.66279248 E0	0.87279491 E-1	0.52500000 E3	Santiago

Table 24. Relay-II Range/Range Rate and Angular Data
(Topocentric Coordinate System)
Epoch 67Y 11M 13D 00H 00M 00S

Data Point	Range ρ (CUL)	Range Rate $\dot{\rho}$ (CUL/CUT)	Declination δ (Radians)	Right Ascension α (Radians)	Time from Epoch (Minutes)	Station Name
1	0.89888122 E0	0.14301893 E0	0.31193977 E0	0.14776794 E1	0.66500000 E3	Santiago
1	0.70207264 E0	-0.14905871 E0	-0.92884152 E-1	0.13783888 E1	0.66500000 E3	Lima
1	0.72304980 E0	-0.31760081 E0	-0.39567188 E0	0.13681167 E1	0.66500000 E3	Quito
2	0.91079344 E0	0.17681771 E0	0.35817955 E0	0.15330759 E1	0.66600000 E3	Santiago
2	0.69280362 E0	-0.99906213 E-1	-0.31316161 E-1	0.14447979 E1	0.66600000 E3	Lima
2	0.70088515 E0	-0.27766471 E0	-0.34230491 E0	0.14374326 E1	0.66600000 E3	Quito
3	0.92511571 E0	0.20802940 E0	0.40330177 E0	0.15872940 E1	0.66700000 E3	Santiago
3	0.68722148 E0	-0.50294936 E-1	0.32133870 E-1	0.15102860 E1	0.66700000 E3	Lima
3	0.68181821 E0	-0.23487703 E0	-0.28467688 E0	0.15058797 E1	0.66700000 E3	Quito
4	0.96023887 E0	0.26243882 E0	0.48947537 E0	0.16924963 E1	0.66900000 E3	Santiago
4	0.68703725 E0	0.47002459 E-1	0.16150698 E0	0.16384893 E1	0.66900000 E3	Lima
4	0.65366838 E0	-0.14232829 E0	-0.15836583 E0	0.16400535 E1	0.66900000 E3	Quito
5	0.64019751 E0	0.50475988 E-1	0.11722174 E0	0.18964100 E1	0.67300000 E3	Quito
5	0.72696304 E0	0.21314890 E0	0.40799880 E0	0.18833043 E1	0.67300000 E3	Lima
5	0.79087014 E0	-0.22489191 E0	-0.47058734 E0	0.20403390 E1	0.67300000 E3	Ft. Myers
6	0.76057235 E0	0.31746370 E0	0.58785855 E0	0.23705836 E1	0.68100000 E3	Quito
6	0.91466433 E0	0.38498172 E0	0.76792261 E0	0.23382376 E1	0.68100000 E3	Lima
6	0.73482427 E0	0.37758981 E-1	-0.15872731 E-1	0.24594815 E1	0.68100000 E3	Ft. Myers
7	0.98056431 E0	0.40079706 E0	0.85147922 E0	0.28211775 E1	0.68900000 E3	Quito
7	0.11581804 E1	0.41989126 E0	0.96366759 E0	0.27765901 E1	0.68900000 E3	Lima
7	0.82155313 E0	0.23363976 E0	0.37926968 E0	0.28470387 E1	0.68900000 E3	Ft. Myers

Table 24. Relay-II Range/Range Rate and Angular Data
(Topocentric Coordinate System)
Epoch 67Y 11M 13D 00H 00M 00S (Cont'd)

Data Point	Range ρ (CUL)	Range Rate $\dot{\rho}$ (CUL/CUT)	Declination δ (Radians)	Right Ascension α (Radians)	Time from Epoch (Minutes)	Station Name
8	0.94366490 E0	0.30450685 E0	0.57319622 E0	0.31312779 E1	0.69500000 E3	Ft. Myers
8	0.98530497 E0	-0.37367764 E-1	0.22465771 E0	0.26491460 E1	0.69500000 E3	Newfoundland
8	0.14629012 E1	-0.18644757 E0	0.12172679 E0	0.24023389 E1	0.69500000 E3	Winkfield
9	0.10622621 E1	0.32951433 E0	0.67368809 E0	-0.29201484 E1	0.70000000 E3	Ft. Myers
9	0.98419844 E0	-0.29869375 E-1	0.38590132 E0	0.28497676 E1	0.70000000 E3	Newfoundland
9	0.13977209 E1	-0.16398231 E0	0.23730549 E0	0.25287457 E1	0.70000000 E3	Winkfield
10	0.12613358 E1	0.33464199 E0	0.74978306 E0	-0.25714658 E1	0.70800000 E3	Ft. Myers
10	0.10284237 E1	0.11327257 E0	0.58431282 E0	-0.30762094 E1	0.70800000 E3	Newfoundland
10	0.13111786 E1	-0.12686322 E0	0.41232413 E0	0.27536675 E1	0.70800000 E3	Winkfield
11	0.17091122 E1	-0.26426344 E0	0.10202773 E1	-0.14992882 E1	0.76800000 E3	Johannesburg
11	0.16446661 E1	-0.32404875 E0	0.95892110 E0	-0.18418508 E1	0.76800000 E3	Madagascar
12	0.20003376 E1	-0.48310473 E0	0.25394107 E0	-0.13362718 E1	0.79900000 E3	Orroral
13	0.15747209 E1	0.56274620 E-1	0.18575302 E0	0.14646764 E1	0.86000000 E3	Santiago
13	0.15069416 E1	-0.13631837 E0	-0.33291602 E-1	0.13537631 E1	0.86000000 E3	Lima
13	0.15144226 E1	-0.22774245 E0	-0.17503491 E0	0.13343712 E1	0.86000000 E3	Quito

Table 25. OSO-III Data Points and Stations Used for PODMs Requiring Angular and Mixed Data Inputs

Data Points Used	Station for Three-Station Inputs	Station for Single-Station Input	Three Stations with Input Resolved to Single Time Input	
			Data Point	Station
1	Quito	Quito	1	Santiago
2	Lima	Quito	1	Lima
3	Santiago	Quito	1	Quito
1	Quito	Quito	2	Santiago
2	Lima	Quito	2	Lima
4	Santiago	Quito	2	Quito
1	Quito	Quito	3	Santiago
2	Lima	Quito	3	Lima
5	Santiago	Quito	3	Quito
1	Quito	Quito	4	Santiago
3	Lima	Quito	4	Lima
5	Santiago	Quito	4	Quito
1	Quito	Quito	5	Santiago
4	Lima	Quito	5	Lima
5	Santiago	Quito	5	Quito
6	Santiago	Johannesburg	6	Santiago
7	Johannesburg	Johannesburg	6	Johannesburg
8	Madagascar	Johannesburg	6	Madagascar
6	Santiago	Johannesburg	10	Johannesburg
7	Johannesburg	Johannesburg	10	Madagascar
9	Madagascar	Johannesburg	10	Orroral
6	Santiago	Johannesburg	12	Santiago
7	Johannesburg	Johannesburg	12	Lima
10	Madagascar	Johannesburg	12	Quito
6	Santiago	Johannesburg	N/A	N/A
8	Johannesburg	Johannesburg	N/A	N/A
9	Madagascar	Johannesburg	N/A	N/A

Table 25. OSO-III Data Points and Stations Used for PODMs Requiring Angular and Mixed Data Inputs (Cont'd)

Data Points Used	Station for Three-Station Inputs	Station for Single-Station Input	Three Stations with Input Resolved to Single Time Input	
			Data Point	Station
6	Santiago	Johannesburg	N/A	N/A
9	Johannesburg	Johannesburg	N/A	N/A
10	Madagascar	Johannesburg	N/A	N/A
1	Quito	Quito	N/A	N/A
2	Lima	Quito	N/A	N/A
12	Santiago	Quito	N/A	N/A
1	Quito	Quito	N/A	N/A
5	Lima	Quito	N/A	N/A
12	Santiago	Quito	N/A	N/A

Table 26. Relay-II Data Points and Stations Used for PODMs Requiring Angular and Mixed Data Inputs

Data Points Used	Station for Three-Station Inputs	Station for Single-Station Input	Three Stations with Input Resolved to Single Time Input	
			Data Point	Station
1	Santiago	Quito	1	Santiago
2	Lima	Quito	1	Lima
3	Quito	Quito	1	Quito
1	Santiago	Quito	2	Santiago
2	Lima	Quito	2	Lima
4	Quito	Quito	2	Quito
1	Santiago	Quito	3	Santiago
2	Lima	Quito	3	Lima
5	Quito	Quito	3	Quito
1	Santiago	Quito	4	Santiago
3	Quito	Quito	4	Lima
5	Lima	Quito	4	Quito
1	Santiago	Quito	5	Quito
4	Quito	Quito	5	Lima
5	Lima	Quito	5	Ft. Myers
6	Quito	Ft. Myers	6	Quito
7	Lima	Ft. Myers	6	Lima
8	Ft. Myers	Ft. Myers	6	Ft. Myers
6	Quito	Ft. Myers	7	Quito
7	Lima	Ft. Myers	7	Lima
9	Ft. Myers	Ft. Myers	7	Ft. Myers
6	Quito	Ft. Myers	8	Ft. Myers
7	Lima	Ft. Myers	8	Newfoundland
10	Ft. Myers	Ft. Myers	8	Winkfield
6	Quito	Ft. Myers	9	Ft. Myers
8	Lima	Ft. Myers	9	Newfoundland
9	Ft. Myers	Ft. Myers	9	Winkfield

Table 26. Relay-II Data Points and Stations Used for PODMs Requiring Angular and Mixed Data Inputs (Cont'd)

Data Points Used	Station for Three-Station Inputs	Station for Single-Station Input	Three Stations with Input Resolved to Single Time Input	
			Data Point	Station
6	Quito	Ft. Myers	10	Ft. Myers
9	Ft. Myers	Ft. Myers	10	Newfoundland
10	Newfoundland	Ft. Myers	10	Winkfield
1	Santiago	Quito	13	Santiago
2	Lima	Quito	13	Lima
13	Quito	Quito	13	Quito
1	Santiago	Quito	N/A	N/A
5	Lima	Quito	N/A	N/A
13	Quito	Quito	N/A	N/A
1	Santiago	Quito	N/A	N/A
7	Lima	Quito	N/A	N/A
13	Quito	Quito	N/A	N/A

Table 27. Results of Method of Gauss PODM for OSO-III

True Anomaly Angular $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	True Anomaly Difference $\vec{r}_3 \to \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	Computed X Dot Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations (1)
3.8	11.4	-0.70791722 E0 -0.70685743 E0	0.39743942 E0 0.40013314 E0	-0.52158271 E0 -0.51534094 E0	19/8
3.8	22.8	-0.70667326 E0 -0.70685743 E0	0.39969767 E0 0.40013314 E0	-0.51601203 E0 -0.51534094 E0	10/5
3.8	45.6	-0.70657644 E0 -0.70685743 E0	0.39983035 E0 0.40013314 E0	-0.51529424 E0 -0.51534094 E0	9/16
11.4	45.6	-0.76769882 E0 -0.76862972 E0	0.29034934 E0 0.29068616 E0	-0.49971037 E0 -0.49963709 E0	15/25
22.8	45.6	-0.83775843 E0 -0.83592404 E0	0.11826982 E0 0.11781151 E0	-0.45899419 E0 -0.45992297 E0	13/11
(2) 22.8	45.6	NO DATA -0.55646495 E0	NO DATA -0.77864062 E0	NO DATA 0.55149247 E-1	8/3
22.8	68.4	-0.11614366 E1 -0.55646495 E0	-0.16762643 E1 -0.77864062 E0	-0.35725661 E0 0.55149247 E-1	25/25
(3) 22.8	111.6	NO DATA -0.55646495 E0	NO DATA -0.77864062 E0	NO DATA 0.55149247 E-1	15/5
45.0	68.4	-0.25517320 -0.25549497 E0	-0.88735304 E0 -0.88905191 E0	0.24888084 E0 0.24948641 E0	9/20
68.4	111.6	0.65269272 E-1 0.84194416 E-1	-0.69242933 E0 -0.86372645 E0	0.32903333 E0 0.40548748 E0	8/25

Table 27. Results of Method of Gauss PODM for OSO-III (Cont'd)

True Anomaly		Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
(4) 3.8	360.0	NO DATA / -0.70685743 E0	NO DATA / 0.40013314 E0	NO DATA / -0.51534094 E0	14/3
(5) 45.6	360.0	NO DATA / -0.87135390 E0	NO DATA / -0.23408489 E0	NO DATA / -0.32909258 E0	25/6

(1) Method of Gauss has two iteration loops (1/2)
(2) Computer halted after third iteration of second loop
(3) Computer halted after fifth iteration of second loop
(4) Computer halted after third iteration of second loop
(5) Computer halted after sixth iteration of second loop

Table 28. Results of Method of Gauss PODM for Relay-II

True Anomaly		Computed X Dot Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations (1)
Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
2.5	5.0	-0.65573896 E0 -0.65562172 E0	-0.48529845 E-1 -0.48674037 E-1	0.58465493 E0 0.58641873 E0	25/6
2.5	10.0	-0.65577567 E0 -0.65562172 E0	-0.47736815 E-1 -0.48674037 E-1	0.58613153 E0 0.58641873 E0	25/7
2.5	21.0	-0.65584460 E0 -0.65562172 E0	-0.47958021 E-1 -0.48674037 E-1	0.58649359 E0 0.58641873 E0	24/7
5.0	21.0	-0.63987321 E0 -0.63983417 E0	-0.77623212 E-1 -0.77927626 E-1	0.59110496 E0 0.59099381 E0	15/7
10.0	21.0	-0.60642894 E0 -0.60637538 E0	-0.13341274 E0 -0.13383906 E0	0.59706499 E0 0.59694559 E0	25/5
20.0	32.0	-0.22620569 E0 -0.22604802 E0	-0.49453221 E0 -0.49460573 E0	0.49575304 E0 0.49560149 E0	25/8
20.0	45.0	-0.22630405 E0 -0.22604802 E0	-0.49457044 E0 -0.49460573 E0	0.49582764 E0 0.49560149 E0	25/9
20.0	65.0	-0.22658515 E0 -0.22604802 E0	-0.49480670 E0 -0.49460573 E0	0.49613260 E0 0.49560149 E0	25/25
32.0	45.0	-0.11872069 E0 -0.11873926 E0	-0.54240256 E0 -0.54226741 E0	0.43382895 E0 0.43373466 E0	22/9
(2) 45.0	65.0	NO DATA -0.35627838 E-1	NO DATA -0.56593395 E0	NO DATA 0.37767977 E0	25/5
(3) 2.5	360.0	NO DATA -0.65562172 E0	NO DATA -0.48674037 E-1	NO DATA 0.58641873 E0	6/3

Table 28. Results of Method of Gauss PODM for Relay-II (Cont'd)

True Anomaly		Computed X Dot	Computed Y Dot	Computed Z Dot	Number of Iterations
Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	Reference Orbit X Dot at T_2 (CUL/CUT)	Reference Orbit Y Dot at T_2 (CUL/CUT)	Reference Orbit Z Dot at T_2 (CUL/CUT)	
(4) 21.0	360.0	NO DATA -0.53391142 E0	NO DATA -0.23461233 E0	NO DATA 0.59733711 E0	14/3
(5) 60.0	360.0	NO DATA -0.22604802 E0	NO DATA -0.49460573 E0	NO DATA 0.49560149 E0	25/3

(1) Method of Gauss has two iteration loops (1/2)
(2) Computer halted after fifth iteration of second loop
(3) Computer halted after third iteration of second loop
(4) Computer halted after third iteration of second loop
(5) Computer halted after third iteration of second loop

Table 29. Results of Laplace PODM for OSO-III

True Anomaly: Angular $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	True Anomaly: Difference $\vec{r}_3 \to \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
3.8	11.4	-0.62214854 E0 -0.70685743 E0	-0.42083550 E1 0.40013314 E0	-0.18298844 E2 -0.51534094 E0	25
3.8	22.8	-0.12509150 E1 -0.70685743 E0	0.81876243 E0 0.40013314 E0	0.26324664 E1 -0.51534094 E0	25
3.8	45.6	0.62338167 E0 -0.70685743 E0	-0.97365651 E0 0.40013314 E0	-0.91009868 E1 -0.51534094 E0	24
11.4	45.6	-0.17521341 E1 -0.76862972 E0	0.10642148 E1 0.29068616 E0	0.36921444 E0 -0.49963709 E0	25
22.8	45.6	-0.11041127 E1 -0.83592404 E0	0.19952538 E0 0.11781151 E0	-0.47175310 E0 -0.45992297 E0	17
22.8	45.6	-0.44101836 E1 -0.55646495 E0	-0.17955487 E1 -0.77864062 E0	-0.11655648 E1 0.55149247 E-1	19
22.8	68.4	0.24578202 E0 -0.55646495 E0	-0.73672249 E0 -0.77864062 E0	0.23126402 E1 0.55149247 E-1	25
22.8	111.6	0.25079742 E1 -0.55646495 E0	-0.11870974 E0 -0.77864062 E0	0.45458931 E1 0.55149247 E-1	25
45.0	68.4	-0.19467675 E1 -0.25549497 E0	-0.27913786 E0 -0.88905191 E0	-0.18056729 E1 0.24948641 E0	25

Table 29. Results of Laplace PODM for OSO-III (Cont'd)

True Anomaly		Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
68.4	111.6	0.95421127 E-1 / 0.84194416 E-1	-0.44695439 E0 / -0.86372645 E0	0.27811717 E0 / 0.40548748 E0	25
3.8	360.0	-0.17796285 E1 / -0.70685743 E0	0.78874290 E0 / 0.40013314 E0	0.38617498 E1 / -0.51534094 E0	10
45.6	360.0	0.42930140 E1 / -0.87135390 E0	0.33948553 E0 / -0.23408489 E0	-0.56191323 E0 / -0.32909258 E0	18

Table 30. Results of Laplace PODM for Relay-II

True Anomaly: Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	True Anomaly: Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
2.5	5.0	-0.72714109 E0 -0.65562172 E0	-0.71504655 E-1 -0.48674037 E-1	0.59489353 E0 0.58641873 E0	25
2.5	10.0	-0.10267281 E1 -0.65562172 E0	-0.18628955 E1 -0.48674037 E-1	0.36820326 E1 0.58641873 E0	25
2.5	21.0	-0.53878453 E4 -0.65562172 E0	-0.23647739 E5 -0.48674037 E-1	0.36040611 E5 0.58641873 E0	25
5.0	21.0	-0.48696044 E0 -0.63983417 E0	-0.18666574 E1 -0.77927626 E-1	0.35635083 E1 0.59099381 E0	25
10.0	21.0	0.31209321 E1 -0.60637538 E0	-0.57152970 E2 -0.13383906 E0	0.99120958 E1 0.59694559 E0	25
20.0	32.0	-0.26693959 E0 -0.22604802 E0	-0.43433233 E0 -0.49460573 E0	0.69450731 E0 0.49560149 E0	25
20.0	45.0	-0.13796895 E0 -0.22604802 E0	-0.56147737 E0 -0.49460573 E0	0.45146978 E0 0.49560149 E0	25
20.0	65.0	-0.16272038 E0 -0.22604802 E0	0.33983267 E0 -0.49460573 E0	-0.12063448 E0 0.49560149 E0	10
32.0	45.0	-0.52113641 E0 -0.11873926 E0	-0.10880935 E1 -0.54226741 E0	0.13009035 E1 0.43373466 E0	16
45.0	65.0	-0.36805806 E-1 -0.35627838 E-1	-0.59926305 E0 -0.56593395 E0	0.54858426 E0 0.37767977 E0	25

Table 30. Results of Laplace PODM for Relay-II (Cont'd)

True Anomaly		Computed X Dot Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
2.5	360.0	-0.20583222 E1 -0.65562172 E0	0.42296244 E0 -0.48674037 E-1	0.16115016 E2 0.58641873 E0	25
21.0	360.0	0.77590448 E1 -0.53391142 E0	0.89489232 E0 -0.23461233 E0	-0.16625441 E1 0.59733711 E0	25
60.0	360.0	0.14735411 E1 -0.22604802 E0	0.15238060 E1 -0.49460573 E0	-0.12408895 E1 0.49560149 E0	25

Table 31. Results of Double R-Iteration PODM for OSO-III

True Anomaly		Computed X Dot / Reference Orbit X Dot at T_2	Computed Y Dot / Reference Orbit Y Dot at T_2	Computed Z Dot / Reference Orbit Z Dot at T_2	Number of Iterations
Angular $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \to \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	(CUL/CUT)	(CUL/CUT)	(CUL/CUT)	
3.8	11.4	0.10753446 E-1 / -0.70685743 E0	0.66555841 E-1 / 0.40013314 E0	0.51606808 E-1 / -0.51534094 E0	25
3.8	22.8	-0.14092275 E0 / -0.70685743 E0	-0.29962388 E-1 / 0.40013314 E0	-0.91042634 E0 / -0.51534094 E0	25
3.8	45.6	0.13710653 E-1 / -0.70685743 E0	-0.11286502 E0 / 0.40013314 E0	-0.16121196 E0 / -0.51534094 E0	25
11.4	45.6	0.26168193 E0 / -0.76862972 E0	-0.38736629 E1 / 0.29068616 E0	-0.16471906 E1 / -0.49963709 E0	25
22.8	45.6	-0.78886572 E0 / -0.83592404 E0	0.12043147 E0 / 0.11781151 E0	-0.48667308 E0 / -0.45992297 E0	25
(1) 22.8	45.6	NO DATA / -0.55646495 E0	NO DATA / -0.77864062 E0	NO DATA / 0.55149247 E-1	25
(2) 22.8	68.4	NO DATA / -0.55646495 E0	NO DATA / -0.77864062 E0	NO DATA / 0.55149247 E-1	25
22.8	111.6	-0.11258185 E0 / -0.55646495 E0	0.18068761 E0 / -0.77864062 E0	-0.19051456 E0 / 0.55149247 E-1	25
45.0	68.4	-0.26254772 E0 / -0.25549497 E0	-0.88822925 E0 / -0.88905191 E0	0.24562602 E0 / 0.24948641 E0	25
(3) 68.4	111.6	NO DATA / 0.84194416 E-1	NO DATA / -0.86372645 E0	NO DATA / 0.40548748 E0	25

Table 31. Results of Double R-Iteration PODM for OSO-III (Cont'd)

True Anomaly Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	True Anomaly Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
3.8	360.0	-0.15743479 E0 -0.70685743 E0	-0.13771405 E0 0.40013314 E0	-0.21949831 E0 -0.51534094 E0	25
45.6	360.0	NO DATA -0.87135390 E0	NO DATA -0.23408489 E0	NO DATA -0.32909258 E0	25

(1) Computer halted after twenty-fifth iteration
(2) Computer halted after twenty-fifth iteration
(3) Computer halted after twenty-fifth iteration

Table 32. Results of Double R-Iteration PODM for Relay-II

True Anomaly: Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	True Anomaly: Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
(1) 2.5	5.0	NO DATA / -0.65562172 E0	NO DATA / -0.48674037 E-1	NO DATA / 0.58641873 E0	25
2.5	10.0	-0.91421077 E0 / -0.65562172 E0	0.28383640 E1 / -0.48674037 E-1	-0.67260774 E-1 / 0.58641873 E0	25
2.5	21.0	-0.27019848 E0 / -0.65562172 E0	-0.45272405 E0 / -0.48674037 E-1	0.46024761 E0 / 0.58641873 E0	25
(2) 5.0	21.0	NO DATA / -0.63983417 E0	NO DATA / -0.77927626 E-1	NO DATA / 0.59099381 E0	25
(3) 10.0	21.0	NO DATA / -0.60637538 E0	NO DATA / -0.13383906 E0	NO DATA / 0.59694559 E0	25
(4) 20.0	32.0	NO DATA / -0.22604802 E0	NO DATA / -0.49460573 E0	NO DATA / 0.49560149 E0	25
20.0	45.0	0.66520513 E-1 / -0.22604802 E0	0.52704750 E-1 / -0.49460573 E0	0.58966469 E0 / 0.49560149 E0	25
20.0	65.0	0.37515994 E-1 / -0.22604802 E0	0.50194898 E-1 / -0.49460573 E0	0.43918827 E0 / 0.49560149 E0	25
(5) 32.0	45.0	NO DATA / -0.11873926 E0	NO DATA / -0.54226741 E0	NO DATA / 0.43373466 E0	25
(6) 45.0	65.0	NO DATA / -0.35627838 E-1	NO DATA / -0.56593395 E0	NO DATA / 0.37767977 E0	25

Table 32. Results of Double R-Iteration PODM for Relay-II (Cont'd)

True Anomaly: Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees) [1]	True Anomaly: Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees) [1]	Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
2.5	360.0	-0.52220508 E-1 / -0.65562172 E0	-0.34564548 E0 / -0.48674037 E-1	0.22778349 E-1 / 0.58641873 E0	25
(7) 21.0	360.0	NO DATA / -0.53391142 E0	NO DATA / -0.23461233 E0	NO DATA / 0.59733711 E0	25
60.0	360.0	0.41528271 E-2 / -0.22604802 E0	-0.38793737 E-1 / -0.49460573 E0	-0.12068895 E-1 / 0.49560149 E0	25

(1) Computer halted after twenty-fifth iteration
(2) Computer halted after twenty-fifth iteration
(3) Computer halted after twenty-fifth iteration
(4) Computer halted after twenty-fifth iteration
(5) Computer halted after twenty-fifth iteration
(6) Computer halted after twenty-fifth iteration
(7) Computer halted after twenty-fifth iteration

Table 33. Results of Modified Laplacian PODM for OSO-III

True Anomaly		Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \to \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
3.8	11.4	-0.71593645 E0 -0.70685743 E0	0.48563242 E0 0.40013314 E0	-0.62694617 E0 -0.51534094 E0	5
3.8	22.8	-0.69978116 E0 -0.70685743 E0	0.52655320 E0 0.40013314 E0	-0.67720686 E0 -0.51534094 E0	5
3.8	45.6	-0.69070303 E0 -0.70685743 E0	0.54224031 E0 0.40013314 E0	-0.68928764 E0 -0.51534094 E0	5
11.4	45.6	-0.10306731 E1 -0.76862972 E0	0.50739234 E0 0.29068616 E0	-0.71356305 E0 -0.49963709 E0	5
22.8	45.6	-0.11293956 E1 -0.83592404 E0	0.28689117 E0 0.11781151 E0	-0.47564288 E0 -0.45992297 E0	5
22.8	45.6	-0.13744573 E0 -0.55646495 E0	-0.23924188 E0 -0.77864062 E0	0.38397540 E-1 0.55149247 E-1	5
22.8	68.4	-0.22370122 E0 -0.55646495 E0	-0.41542275 E0 -0.77864062 E0	0.10946262 E0 0.55149247 E-1	5
22.8	111.6	-0.14220677 E0 -0.55646495 E0	-0.26481078 E0 -0.77864062 E0	0.47927197 E-1 0.55149247 E-1	6
45.0	68.4	-0.32805748 E-1 -0.25549497 E0	-0.82710003 E0 -0.88905191 E0	0.49787300 E0 0.24948641 E0	8
68.4	111.6	0.96889297 E-2 0.84194416 E-1	0.34904833 E0 -0.86372645 E0	-0.25621297 E0 0.40548748 E0	25

Table 33. Results of Modified Laplacian PODM for OSO-III (Cont'd)

True Anomaly Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	True Anomaly Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	$\frac{\text{Computed X Dot}}{\text{Reference Orbit X Dot at } T_2}$ (CUL/CUT)	$\frac{\text{Computed Y Dot}}{\text{Reference Orbit Y Dot at } T_2}$ (CUL/CUT)	$\frac{\text{Computed Z Dot}}{\text{Reference Orbit Z Dot at } T_2}$ (CUL/CUT)	Number of Iterations
3.8	360.0	$\frac{-0.68062580 \text{ E0}}{-0.70685743 \text{ E0}}$	$\frac{0.5495026 \text{ E0}}{0.40013314 \text{ E0}}$	$\frac{-0.69971598 \text{ E0}}{-0.51534094 \text{ E0}}$	5
45.6	360.0	$\frac{-0.13166452 \text{ E1}}{-0.87135390 \text{ E0}}$	$\frac{0.60437789 \text{ E-1}}{-0.23408489 \text{ E0}}$	$\frac{-0.49458454 \text{ E0}}{-0.32909258 \text{ E0}}$	5

Table 34. Results of Modified Laplacian PODM for Relay-II

True Anomaly		Computed X Dot Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \to \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
2.5	5.0	-0.65588544 E0 -0.65562172 E0	-0.49754409 E-1 -0.48674037 E-1	0.58705798 E0 0.58641873 E0	25
2.5	10.0	-0.65668485 E0 -0.65562172 E0	-0.51458662 E-1 -0.48674037 E-1	0.58841259 E0 0.58641873 E0	25
2.5	21.0	-0.65795561 E0 -0.65562172 E0	-0.52382219 E-1 -0.48674037 E-1	0.58982847 E0 0.58641873 E0	13
5.0	21.0	-0.64295328 E0 -0.63983417 E0	-0.84693810 E-1 -0.77927626 E-1	0.59620329 E0 0.59099381 E0	13
10.0	21.0	-0.60804220 E0 -0.60637538 E0	-0.14289316 E0 -0.13383906 E0	0.60159792 E0 0.59694559 E0	12
20.0	32.0	-0.20757021 E0 -0.22604802 E0	0.10461109 E1 -0.49460573 E0	-0.77340509 E0 0.49560149 E0	25
20.0	45.0	-0.16278096 E0 -0.22604802 E0	0.10200810 E1 -0.49460573 E0	-0.78809914 E0 0.49560149 E0	25
20.0	65.0	-0.10887273 E0 -0.22604802 E0	0.99482536 E0 -0.49460573 E0	-0.81125674 E0 0.49560149 E0	25
32.0	45.0	-0.53087633 E0 -0.11873926 E0	0.87105242 E0 -0.54226741 E0	-0.37080802 E0 0.43373466 E0	25
45.0	65.0	-0.42783549 E0 -0.35627838 E-1	0.25999349 E0 -0.56593395 E0	0.37270621 E-1 0.37767977 E0	6

Table 34. Results of Modified Laplacian PODM for Relay-II (Cont'd)

True Anomaly		$\frac{\text{Computed X Dot}}{\text{Reference Orbit X Dot at } T_2}$ (CUL/CUT)	$\frac{\text{Computed Y Dot}}{\text{Reference Orbit Y Dot at } T_2}$ (CUL/CUT)	$\frac{\text{Computed Z Dot}}{\text{Reference Orbit Z Dot at } T_2}$ (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
2.5	360.0	-0.65095335 E0 / -0.65562172 E0	-0.19624521 E-1 / -0.48674037 E-1	0.56898767 E0 / 0.58641873 E0	11
21.0	360.0	-0.63087748 E0 / -0.53391142 E0	0.44128080 E-1 / -0.23461233 E0	0.58437370 E0 / 0.59733711 E0	25
60.0	360.0	0.20119010 E0 / -0.22604802 E0	0.45091972 E0 / -0.49460573 E0	-0.38224743 E0 / 0.49560149 E0	25

Table 35. Results of R-Iteration PODM for OSO-III

True Anomaly		Computed X Dot Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \to \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
3.8	11.4	-0.67303769 E0 -0.70685743 E0	0.47295788 E0 0.40013314 E0	-0.61114236 E0 -0.51534094 E0	7
3.8	22.8	-0.68262750 E0 -0.70685743 E0	0.52046653 E0 0.40013314 E0	-0.66963444 E0 -0.51534094 E0	7
3.8	45.6	-0.72791534 E0 -0.70685743 E0	0.55636971 E0 0.40013314 E0	-0.70650216 E0 -0.51534094 E0	10
11.4	45.6	-0.78954637 E0 -0.76862972 E0	0.47857224 E0 0.29068616 E0	-0.67776446 E0 -0.49963709 E0	10
(1) 22.8	45.6	NO DATA -0.83592404 E0	NO DATA 0.11781151 E0	NO DATA -0.45992297 E0	NO DATA
22.8	45.6	-0.94933236 E0 -0.55646495 E0	-0.11649079 E1 -0.77864062 E0	0.20859521 E1 0.55149247 E-1	13
22.8	68.4	-0.50000178 E0 -0.55646495 E0	-0.84474399 E0 -0.77864062 E0	0.59120426 E0 0.55149247 E-1	17
22.8	111.6	-0.45529692 E0 -0.55646495 E0	-0.68089611 E0 -0.77864062 E0	0.83025921 E0 0.55149247 E-1	25
45.0	68.4	-0.30194235 E-1 -0.25549497 E0	-0.87126672 E0 -0.88905191 E0	0.53275537 E0 0.24948641 E0	6
68.4	111.6	-0.67218747 E-1 0.84194416 E-1	0.43304281 E-3 -0.86372645 E0	-0.13491177 E0 0.40548748 E0	5

Table 35. Results of R-Iteration PODM for OSO-III (Cont'd)

True Anomaly		Computed X Dot Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)[1]	Difference $\vec{r}_3 \to \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)[1]				
3.8	360.0	0.10841023 E1 -0.70685743 E0	-0.14663795 E0 0.40013314 E0	0.15214168 E0 -0.51534094 E0	25
45.6	360.0	0.26869148 E1 -0.87135390 E0	0.61834314 E0 -0.23408489 E0	-0.45069544 E0 -0.32909258 E0	25

(1) Computer halt prior to iteration loop

Table 36. Results of R-Iteration PODM for Relay-II

True Anomaly: Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	True Anomaly: Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
2.5	5.0	-0.65536606 E0 / -0.65562172 E0	-0.49981041 E-1 / -0.48674037 E-1	0.58661809 E0 / 0.58641873 E0	25
2.5	10.0	-0.65496732 E0 / -0.65562172 E0	-0.52202059 E-1 / -0.48674037 E-1	0.58695597 E0 / 0.58641873 E0	25
2.5	21.0	-0.65354322 E0 / -0.65562172 E0	-0.54280921 E-1 / -0.48674037 E-1	0.58608381 E0 / 0.58641873 E0	25
5.0	21.0	-0.63575963 E0 / -0.63983417 E0	-0.86952051 E-1 / -0.77927626 E-1	0.58974413 E0 / 0.59099381 E0	25
10.0	21.0	-0.60023973 E0 / -0.60637538 E0	-0.14346819 E0 / -0.13383906 E0	0.59381180 E0 / 0.59694559 E0	25
20.0	32.0	-0.24804671 E0 / -0.22604802 E0	-0.23651090 E0 / -0.49460573 E0	0.30288142 E0 / 0.49560149 E0	25
20.0	45.0	-0.25742519 E0 / -0.22604802 E0	-0.20546412 E0 / -0.49460573 E0	0.28512663 E0 / 0.49560149 E0	25
20.0	65.0	-0.14597404 E0 / -0.22604802 E0	0.71919810 E0 / -0.49460573 E0	-0.55706753 E0 / 0.49560149 E0	25
32.0	45.0	-0.42319203 E0 / -0.11873926 E0	0.46893844 E0 / -0.54226741 E0	-0.13421643 E0 / 0.43373466 E0	25
45.0	65.0	-0.52868635 E0 / -0.35627838 E-1	0.50775696 E0 / -0.56593395 E0	-0.78652931 E-1 / 0.37767977 E0	25

Table 36. Results of R-Iteration PODM for Relay-II (Cont'd)

True Anomaly		Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \to \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
2.5	360.0	-0.19164933 E-1 / -0.65562172 E0	-0.33057738 E0 / -0.48674037 E-1	0.49087828 E-1 / 0.58641873 E0	25
21.0	360.0	0.80376633 E0 / -0.53391142 E0	-0.61899776 E-1 / -0.23461233 E0	-0.84950024 E0 / 0.59733711 E0	25
60.0	360.0	-0.17054887 E1 / -0.22604802 E0	-0.11853038 E1 / -0.49460573 E0	0.23051794 E1 / 0.49560149 E0	11

Table 37. Results of Herrick-Gibbs PODM for OSO-III

True Anomaly Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	True Anomaly Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
3.8	11.4	-0.70645695 E0 -0.70685743 E0	0.40020864 E0 0.40013314 E0	-0.51517444 E0 -0.51534094 E0	N/A
3.8	22.8	-0.70643282 E0 -0.70685743 E0	0.40017858 E0 0.40013314 E0	-0.51514648 E0 -0.51534094 E0	N/A
3.8	45.6	-0.70629247 E0 -0.70685743 E0	0.40012606 E0 0.40013314 E0	-0.51504945 E0 -0.51534094 E0	N/A
11.4	45.6	-0.76818828 E0 -0.76862972 E0	0.29083187 E0 0.29068616 E0	-0.49945671 E0 -0.49963709 E0	N/A
22.8	45.6	-0.83577205 E0 -0.83592404 E0	0.11803920 E0 0.11781151 E0	-0.45991218 E0 -0.45992297 E0	N/A
22.8	45.6	-0.31078045 E-2 -0.55646495 E0	-0.17694896 E-1 -0.77864062 E0	0.73806815 E-1 0.55149247 E-1	N/A
22.8	68.4	-0.55558350 E0 -0.55646495 E0	-0.77687066 E0 -0.77864062 E0	0.14025474 E1 0.55149247 E-1	N/A
22.8	111.6	-0.55417017 E0 -0.55646495 E0	-0.76854725 E0 -0.77864062 E0	0.21244346 E1 0.55149247 E-1	N/A
45.0	68.4	-0.25473601 E0 -0.25549497 E0	-0.88721359 E0 -0.88905191 E0	0.24905497 E0 0.24948641 E0	N/A
68.4	111.6	0.85339904 E-1 0.84194416 E-1	-0.84943977 E0 -0.86372645 E0	0.39986042 E0 0.40548748 E0	N/A

Table 37. Results of Herrick-Gibbs PODM for OSO-III (Cont'd)

True Anomaly		Computed X Dot Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)				
3.8	360.0	-0.70521277 E0 -0.70685743 E0	0.43640725 E0 0.40013314 E0	-0.52981753 E0 -0.51534094 E0	N/A
45.6	360.0	-0.95443170 E0 -0.87135390 E0	0.13423173 E0 -0.23408489 E0	-0.52536724 E0 -0.32909258 E0	N/A

Table 38. Results of Herrick-Gibbs PODM for Relay-II

True Anomaly					
Angular $\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	Difference $\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees)	Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations (1)
2.5	5.0	-0.65566707 E0 / -0.65562172 E0	-0.48663300 E-1 / -0.48674037 E-1	0.58645073 E0 / 0.58641873 E0	N/A
2.5	10.0	-0.65584596 E0 / -0.65562172 E0	-0.48675139 E-1 / -0.48674037 E-1	0.58660992 E0 / 0.58641873 E0	N/A
2.5	21.0	-0.65589575 E0 / -0.65562172 E0	-0.48662218 E-1 / -0.48674037 E-1	0.58664349 E0 / 0.58641873 E0	N/A
5.0	21.0	-0.63986719 E0 / -0.63983417 E0	-0.77898726 E-1 / -0.77927626 E-1	0.59100122 E0 / 0.59099381 E0	N/A
10.0	21.0	-0.60637559 E0 / -0.60637538 E0	-0.13379122 E0 / -0.13383906 E0	0.59691250 E0 / 0.59694559 E0	N/A
20.0	32.0	-0.22612738 E0 / -0.22604802 E0	-0.49459717 E0 / -0.49460573 E0	0.49566263 E0 / 0.49560149 E0	N/A
20.0	45.0	-0.22626938 E0 / -0.22604802 E0	-0.49464458 E0 / -0.49460573 E0	0.49581165 E0 / 0.49560149 E0	N/A
20.0	65.0	-0.22663254 E0 / -0.22604802 E0	-0.49481561 E0 / -0.49460573 E0	0.49622461 E0 / 0.49560149 E0	N/A
32.0	45.0	-0.11888973 E0 / -0.11873926 E0	-0.54230026 E0 / -0.54226741 E0	0.43388195 E0 / 0.43373466 E0	N/A
45.0	65.0	-0.36077449 E0 / -0.35627838 E-1	-0.56616147 E0 / -0.56593395 E0	0.37820133 E0 / 0.37767977 E0	N/A

Table 38. Results of Herrick-Gibbs PODM for Relay-II (Cont'd)

True Anomaly Angular Difference		Computed X Dot / Reference Orbit X Dot at T_2 (CUL/CUT)	Computed Y Dot / Reference Orbit Y Dot at T_2 (CUL/CUT)	Computed Z Dot / Reference Orbit Z Dot at T_2 (CUL/CUT)	Number of Iterations
$\vec{r}_1 \rightarrow \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees) [1]	$\vec{r}_3 \rightarrow \vec{r}_1$ i.e., $\nu_3 - \nu_1$ (Degrees) [1]				
2.5	360.0	-0.67240405 E0 / -0.65562172 E0	-0.46594379 E-1 / -0.48674037 E-1	0.59939317 E0 / 0.58641873 E0	N/A
21.0	360.0	-0.64066348 E0 / -0.53391142 E0	-0.23290743 E0 / -0.23461233 E0	0.68697493 E0 / 0.59733711 E0	N/A
60.0	360.0	-0.43829390 E0 / -0.22604802 E0	-0.50943763 E0 / -0.49460573 E0	0.68517316 E0 / 0.49560149 E0	N/A

(1) No iteration loop exists

Table 39. Computation Results from Trilateration PODM

Parameter	OSO-III	RELAY-II
Computed X-Dot Reference Orbit X-Dot	-0.77396768 E0 -0.77289578 E0	-0.65232511 E-1 -0.35627838 E-1
Computed Y-Dot Reference Orbit Y-Dot	-0.53944807 E0 -0.54884506 E0	-0.58262553 E0 -0.56593395 E0
Computed Z-Dot Reference Orbit Z-Dot	-0.13339736 E0 -0.14805021 E0	0.36304436 E0 0.37767977 E0
Computed Semimajor Axis Reference Orbit Semimajor Axis	0.10715168 E1 0.10866609 E1	0.17798733 E1 0.17448736 E1
Computed Eccentricity Reference Orbit Eccentricity	0.13944822 E-1 0.21640595 E-2	0.24677798 E0 0.24114781 E0
Computed Longitude of Ascending Node Reference Orbit Longitude of Ascending Node	-0.23098294 E1 -0.22460589 E1	0.21387843 E1 0.22064792 E1
Computed Orbit Inclination Reference Orbit Orbit Inclination	0.56873906 E0 0.57356194 E0	0.77806829 E0 0.80848228 E0
Computed Argument of Perigee Reference Orbit Argument of Perigee	-0.48379221 E1 -0.34856807 E1	-0.11822875 E1 -0.13234053 E1

Table 40. Angles Only and Mixed Data PODM Classical Orbital Element Comparisons - Semimajor Axis

True Anamoly		Method of Gauss (Angles Only)	Laplace (Angles Only)	Double-R Iteration (Angles Only)	Modified Laplacian (Mixed Data)	R-Iteration (Mixed Data)	Herrick-Gibbs (Mixed Data)
$\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	$\vec{r}_1 \to \vec{r}_3$ $\nu_3 - \nu_1$ (Degrees)						
Nominal semimajor axis from reference orbit (Earth Radii) 1.0866609 for OSO-III							
3.8	11.4	1.0933545	No Data	0.56680724	1.4565199	1.2706987	1.0862822
3.8	22.8	1.0870821	No Data	0.76012870	1.6411056	1.5309293	1.0861796
3.8	45.6	1.0862247	No Data	0.77258918	1.6740630	2.0096861	1.0857783
11.4	45.6	1.0849989	No Data	No Data	3.4415947	1.6797845	1.0861159
22.8	45.6	1.0900871	2.9676586	1.0212752	3.2250214	No Data	1.0866567
22.8	45.6	No Data	No Data	No Data	0.49742429	No Data	0.54530960
22.8	68.4	No Data	No Data	No Data	0.51325886	1.0992819	No Data
22.8	111.6	No Data	No Data	0.51375022	0.49432114	1.1398184	No Data
45.0	68.4	1.0820681	No Data	1.0864170	2.1854150	3.1140459	1.0818763
68.4	111.6	0.89356833	0.74728463	No Data	0.34190386	0.21995489	1.0537560
3.8	360.0	No Data	No Data	1.7849994	1.6703803	1.0261115	1.1405874
45.6	360.0	No Data	No Data	No Data	1.6933086	No Data	1.5681016
Nominal semimajor axis from reference orbit (Earth Radii) 1.7448736 for RELAY-II							
2.5	5.0	1.7408033	2.6172861	No Data	1.7526733	1.7476601	1.7459612
2.5	10.0	1.7453902	No Data	No Data	1.7699487	1.7531379	1.7472501
2.5	21.0	1.7468151	No Data	0.88413473	1.8025055	1.7583358	1.7475660
5.0	21.0	1.7460754	No Data	No Data	1.8410649	1.7669713	1.7458116
10.0	21.0	1.7459635	No Data	No Data	1.8560241	1.7725920	1.7454941
20.0	32.0	1.7453169	1.7139800	No Data	1.6332744	0.83095475	1.7452667
20.0	45.0	1.7457094	1.4443873	0.56058119	1.5308440	0.79084692	1.7460556
20.0	65.0	1.7477173	0.52260853	0.51562444	1.4267193	0.27337010	1.7483249
32.0	45.0	1.7453685	No Data	No Data	1.3976667	0.30390446	1.7454417
45.0	65.0	No Data	0.69717653	No Data	0.36436660	0.62453156	1.7468162
2.5	360.0	No Data	No Data	3.7080780	1.6919533	0.51226546	1.8678605
21.0	360.0	No Data	No Data	No Data	2.1014806	0.50041346	3.0014965
60.0	360.0	No Data	No Data	0.60708685	0.60224454	No Data	5.1725237

Table 41. Angles Only and Mixed Data PODM Classical Orbital Element Comparisons - Eccentricity

True Anamoly $\vec{r} \rightarrow \vec{r}$ i.e., $\nu_2 - \nu_1$ (Degrees)	True Anamoly $\vec{r} \rightarrow \vec{r}$ $\nu_3 - \nu_1$ (Degrees)	Method of Gauss (Angles Only)	Laplace (Angles Only)	Double-R Iteration (Angles Only)	Modified Laplacian (Mixed Data)	R-Iteration (Mixed Data)	Herrick-Gibbs (Mixed Data)
Nominal eccentricity from reference orbit 0.0021640595 for OSO-III							
3.8	11.4	0.0090209644	No Data	0.99587324	0.26042608	0.16855448	0.0025816264
3.8	22.8	0.0031179903	No Data	0.32552456	0.35348243	0.31165306	0.0024932554
3.8	45.6	0.0023833076	No Data	0.96217539	0.37236123	0.47077550	0.0022031445
11.4	45.6	0.0016172480	No Data	No Data	0.96888166	0.38658497	0.0024264521
22.8	45.6	0.52162234	0.62995502	0.0059085738	0.67259723	No Data	0.0027043814
22.8	45.6	No Data	No Data	No Data	0.92938662	No Data	0.99497186
22.8	68.4	No Data	No Data	No Data	0.79069600	0.25253568	No Data
22.8	111.6	No Data	No Data	0.92855803	0.91547050	0.29056091	No Data
45.0	68.4	0.0054796699	No Data	0.0097580110	0.44034632	0.58784565	0.0056700754
68.4	111.6	0.33483581	0.65056730	No Data	0.91258559	0.99088686	0.032882344
3.8	360.0	No Data	No Data	0.97721692	0.37632616	0.45937567	0.060286596
45.6	360.0	No Data	No Data	No Data	0.94214231	No Data	0.47958596
Nominal eccentricity from reference orbit 0.24114781 for RELAY-II							
2.5	5.0	0.23998301	0.42608590	No Data	0.24135718	0.24000998	0.24112274
2.5	10.0	0.24170341	No Data	No Data	0.24305787	0.23847364	0.24150217
2.5	21.0	0.24191983	No Data	0.54264214	0.24721999	0.23499117	0.24161043
5.0	21.0	0.24134923	No Data	No Data	0.25052518	0.233033181	0.24107510
10.0	21.0	0.24138999	No Data	No Data	0.24819047	0.22726255	0.24096353
20.0	32.0	0.24083358	0.29548092	No Data	0.48116751	0.77573682	0.24069890
20.0	45.0	0.24088657	0.12926246	0.75155324	0.44973306	0.81512154	0.24080103
20.0	65.0	0.24095068	0.94594361	0.85163884	0.42517904	0.65480736	0.24102223
32.0	45.0	0.24043689	No Data	No Data	0.31354452	0.86390255	0.24071615
45.0	65.0	No Data	0.37710462	No Data	0.90172398	0.57811616	0.24076141
2.5	360.0	No Data	No Data	0.98480947	0.24716890	0.92494449	0.28042783
21.0	360.0	No Data	No Data	No Data	0.56096324	0.38796165	0.52988183
60.0	360.0	No Data	No Data	0.99870767	0.87779807	No Data	0.73185317

Table 42. Angles Only and Mixed Data PODM Classical Orbital Element Comparisons - Longitude of Ascending Node

True Anamoly		Method of Gauss (Angles Only)	Laplace (Angles Only)	Double-R Iteration (Angles Only)	Modified Laplacian (Mixed Data)	R-Iteration (Mixed Data)	Herrick-Gibbs (Mixed Data)
$\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	$\vec{r}_1 \to \vec{r}_3$ $\nu_3 - \nu_1$ (Degrees)						
Nominal longitude of ascending node from reference orbit -2.2460589 (radians) for OSO-III							
3.8	11.4	-2.2773522	No Data	1.0126258	-2.2767968	-2.2666136	-2.2786146
3.8	22.8	-2.2784461	No Data	-2.1376049	-2.2636157	-2.2599112	-2.2786155
3.8	45.6	-2.2785933	No Data	-2.3031889	-2.2554272	-2.2630652	-2.2786171
11.4	45.6	-2.2784670	No Data	No Data	-2.2697945	-2.2035252	-2.2786425
22.8	45.6	-2.2800338	-2.3766156	-2.2481640	-2.3692610	No Data	-2.2787244
22.8	45.6	No Data	No Data	No Data	-2.4006765	No Data	2.4262738
22.8	68.4	No Data	No Data	No Data	-2.7372935	2.5108421	No Data
22.8	111.6	No Data	No Data	-0.67519090	-2.4489075	2.6737071	No Data
45.0	68.4	-2.2814877	No Data	-2.2814843	-2.3155691	-2.3114001	-2.2814521
68.4	111.6	-2.2632779	-2.2944144	No Data	1.7764004	2.7874679	-2.2818564
3.8	360.0	No Data	No Data	-2.0361503	-2.2460081	2.1237891	-2.2786043
45.6	360.0	No Data	No Data	No Data	-2.3291864	No Data	-2.2781742
Nominal longitude of ascending node from reference orbit 2.2064792 (radians) for RELAY-II							
2.5	5.0	2.1974520	2.1681778	No Data	2.1969734	2.1971352	2.1972214
2.5	10.0	2.1971107	No Data	No Data	2.1962855	2.1968205	2.1972214
2.5	21.0	2.1971102	No Data	2.4010042	2.194417	2.1958196	2.1972216
5.0	21.0	2.1971711	No Data	No Data	2.1921265	2.1943008	2.1972206
10.0	21.0	2.1971844	No Data	No Data	2.1898849	2.1921711	2.1972193
20.0	32.0	2.1972313	2.3980710	No Data	2.9387537	2.0552814	2.1971611
20.0	45.0	2.1972194	2.2414262	2.4642598	2.9469973	2.0235991	2.1971607
20.0	65.0	2.1971614	1.8753231	2.4657669	2.9636265	-2.6702419	2.1971604
32.0	45.0	2.1971293	No Data	No Data	2.7068033	-2.6797294	2.1971086
45.0	65.0	No Data	2.1694441	No Data	2.2536846	2.5808903	2.1970559
2.5	360.0	No Data	No Data	-3.0040220	2.1933827	2.3263347	2.1972200
21.0	360.0	No Data	No Data	No Data	2.1855524	0.58183673	2.1972238
60.0	360.0	No Data	No Data	-0.19772600	0.97178151	No Data	2.1973668

Table 43. Angles Only and Mixed Data PODM Classical Orbital Element Comparisons - Argument of Perigee

True Anamoly		Method of Gauss (Angles Only)	Laplace (Angles Only)	Double-R Iteration (Angles Only)	Modified Laplacian (Mixed Data)	R-Iteration (Mixed Data)	Herrick-Gibbs (Mixed Data)
$\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	$\vec{r}_1 \to \vec{r}_3$ $\nu_3 - \nu_1$ (Degrees)						
Nominal argument of perigee from reference orbit -3.4856807 (radians) for OSO-III							
3.8	11.4	-3.0968797	No Data	-3.1814587	-3.3619857	-3.6361456	-3.5210052
3.8	22.8	-3.2650512	No Data	-5.2972215	-3.4415148	-3.5098716	-3.5352457
3.8	45.6	-3.3949683	No Data	-0.21050044	-3.4828409	-3.3734575	-3.6330051
11.4	45.6	-3.8062089	No Data	No Data	-3.0101077	-3.3281029	-3.5415507
22.8	45.6	-2.9632696	-2.6250891	-5.5696599	-2.6373267	No Data	-3.3808190
22.8	45.6	No Data	No Data	No Data	-4.1367965	No Data	-2.5557482
22.8	68.4	No Data	No Data	No Data	-3.8143743	-0.70287052	No Data
22.8	111.6	No Data	No Data	-0.41738087	-4.0967220	-0.84080418	No Data
45.0	68.4	-3.9403840	No Data	-2.7746583	-2.0454965	-1.8559031	-4.0303982
68.4	111.6	-3.4096793	-3.5488346	No Data	-5.8607012	-5.5550579	-3.7467118
3.8	360.0	No Data	No Data	-0.48393262	-3.5159384	-0.67183791	-3.6142215
45.6	360.0	No Data	No Data	No Data	-2.5321856	No Data	-3.5871972
Nominal argument of perigee from reference orbit -1.3234053 (radians) for RELAY-II							
2.5	5.0	-1.3233052	-0.77865676	No Data	-1.2977156	-1.3042309	-1.3119424
2.5	10.0	-1.3168437	No Data	No Data	-1.2662402	-1.2874691	-1.3099387
2.5	21.0	-1.3133931	No Data	-2.9812441	-1.2194290	-1.2728954	-1.3095217
5.0	21.0	-1.3125766	No Data	No Data	-1.1592059	-1.2493791	-1.3121522
10.0	21.0	-1.3129503	No Data	No Data	-1.1343231	-1.2468223	-1.3125518
20.0	32.0	-1.3120577	-1.0566229	No Data	-2.9431307	-2.3016439	-1.3121980
20.0	45.0	-1.3110767	-2.3886245	-3.4087926	-3.0704280	-2.2983913	-1.3104178
20.0	65.0	-1.3063930	-4.0643189	-3.5001147	-3.2495198	-6.0954398	-1.3052735
32.0	45.0	-1.3117088	No Data	No Data	-2.9471956	-5.2429044	-1.3116275
45.0	65.0	No Data	-2.4620371	No Data	-3.5935690	-4.2994821	-1.3078684
2.5	360.0	No Data	No Data	-4.6937989	-1.5240942	-2.9165604	-1.1712039
21.0	360.0	No Data	No Data	No Data	-1.7190749	-5.3109605	-0.72101312
60.0	360.0	No Data	No Data	-0.35315446	-4.2662515	No Data	-0.47568945

Table 44. Angles Only and Mixed Data PODM Classical Orbital Element Comparisons - Orbit Inclination

True Anamoly		Method of Gauss (Angles Only)	Laplace (Angles Only)	Double-R Iteration (Angles Only)	Modified Laplacian (Mixed Data)	R-Iteration (Mixed Data)	Herrick-Gibbs (Mixed Data)
$\vec{r}_1 \to \vec{r}_2$ i.e., $\nu_2 - \nu_1$ (Degrees)	$\vec{r}_1 \to \vec{r}_3$ $\nu_3 - \nu_1$ (Degrees)						
Nominal orbital inclination from reference orbit 0.57356194 (radians) for OSO-III							
3.8	11.4	0.57979070	No Data	1.1021145	0.62999778	0.64063228	0.57385747
3.8	22.8	0.57467945	No Data	1.4583617	0.65878711	0.66282570	0.57385448
3.8	45.6	0.57405038	No Data	2.0592190	0.66558984	0.65726740	0.57384928
11.4	45.6	0.57440879	No Data	No Data	0.56910681	0.63508759	0.57380134
22.8	45.6	0.57212004	0.47891522	0.61416430	0.44646964	No Data	0.57370990
22.8	45.6	No Data	No Data	No Data	0.42986477	No Data	1.3651483
22.8	68.4	No Data	No Data	No Data	0.35649591	0.54414129	No Data
22.8	111.6	No Data	No Data	1.9260220	0.40124284	0.79586700	No Data
45.0	68.4	0.57331131	No Data	0.57328656	0.74807940	0.75551440	0.57338118
68.4	111.6	0.58915829	0.64790733	No Data	1.2654971	1.7389577	0.57351640
3.8	360.0	No Data	No Data	1.2257388	0.67468083	2.9635559	0.57389136
45.6	360.0	No Data	No Data	No Data	0.46035030	No Data	0.57389401
Nominal orbital inclination from reference orbit 0.80848228 (radians) for RELAY-II							
2.5	5.0	0.80728742	0.74777617	No Data	0.80844311	0.80837493	0.80873040
2.5	10.0	0.80883062	No Data	No Data	0.80791993	0.80769499	0.80873045
2.5	21.0	0.80897325	No Data	0.72791660	0.80716883	0.80659368	0.80872986
5.0	21.0	0.80894600	No Data	No Data	0.80603485	0.80514310	0.80873331
10.0	21.0	0.80901589	No Data	No Data	0.80530187	0.80448760	0.80873922
20.0	32.0	0.80869180	0.94644204	No Data	0.66670269	0.74309483	0.80864734
20.0	45.0	0.80866257	0.77927170	1.7102357	0.68262804	0.72570899	0.80864684
20.0	65.0	0.80859972	1.9870953	1.7124440	0.70170114	0.66729803	0.80864661
32.0	45.0	0.80862534	No Data	No Data	0.57966024	0.21713257	0.80860271
45.0	65.0	No Data	0.97979288	No Data	0.21852801	0.48607085	0.80856940
2.5	360.0	No Data	No Data	0.067846464	0.81129006	0.20124354	0.80873516
21.0	360.0	No Data	No Data	No Data	0.87233394	0.10445771	0.80879221
60.0	360.0	No Data	No Data	0.31354446	1.7972711	No Data	0.80885823

No data indicates program failed in computing these values.

Table 45. Average Number of Iterations Using Both OSO-III and Relay-II Orbit Results

PODM	Number of Iterations
Method of Gauss	19/11*
Laplace	25
Double R-Iteration	25
Modified Laplacian	14
R-Iteration	18
Herrick-Gibbs	Not applicable
Trilateration	Not applicable

*Two Iteration loops

Table 46. Best Overall Results for Radius Vector Spread to 360°

Radius Vector Spread	PODM
$65^\circ < \nu < 360^\circ$	Herrick-Gibbs
$30^\circ < \nu < 65^\circ$	Method of Gauss
	Modified Laplacian
$\nu < 30^\circ$	R-Iteration
Undetermined	Double R-Iteration
	Laplace

Table 47. Considerations for Selecting Optimum PODM

PODM	Computation Time	Ease of Convergence	Best Overall Accuracy
Herrick-Gibbs	1	N/A	1
Modified Laplacian	2-3	1	3
Method of Gauss	7	3	2
R-Iteration	6	2	4
Double R-Iteration	4-5	4-5	5
Laplace	4-5	4-5	6
Trilaterations	2-3	N/A	7

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