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NECESSARY THICKNESS OF RANDOMLY ORIENTED ARMOR FOR METEOROID PROTECTION

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16. Abstract

Equations and data are presented for determination of armor thickness to provide a specified probability of no destructive impact by sporadic meteoroids during a space mission. Any preferred damage criterion may be used, in terms of powers of meteoroid mass and impact velocity, together with other constant parameters. Conditions in Earth orbit both near and far from Earth are treated. Corrections are made for photographic biases due to meteoroid velocity, mass, and spatial direction of true radiant. Frequency of destructive impacts is based on double integration of analytical expressions to find frequency of impacts with unacceptable combinations of mass and velocity.

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SUMMARY

Weighting factors developed earlier were used to obtain a histogram of velocity and a revised influx rate for meteoroids relative to a gravity-free Earth, fully corrected for photographic biasing effects of meteoroid mass, real velocity relative to Earth's atmosphere, gravitational focusing, and direction in space of approach of meteor toward Earth. Similar results were derived for conditions near the real Earth.

Bimodal log-normal equations were derived to match the histograms of velocity for both gravity-free Earth and real Earth conditions. A double integration yielded an equation for general use to determine the necessary magnitude of a damage criterion, such as armor thickness, for a desired probability of a successful mission from the meteoroid damage standpoint. The double integration involved the bimodal log-normal velocity distributions and a well confirmed equation for influx rate of meteoroids of various masses. As an example of the method of use of the equation for necessary magnitude of a damage criterion, use was also made of an expression for penetration by hypervelocity impact provided in a personal communication by Richard H. Fish and James L. Summers of Ames Research Center. However, if preferred by the reader, other criteria could readily be used in the same manner.

INTRODUCTION

Meteoroids, small particles within the solar system and in orbit about the sun, will continually impinge on radiators and other components of space vehicles. For space missions of long duration, meteoroids having masses within the so-called photographic range may represent a serious hazard. It is therefore desirable to develop accurate analytical expressions for calculation of probability of meteoroid impact damage.

Until recently, necessary refinements in analysis of meteoroid influx data have been lacking. Some recent efforts directed toward the necessary refinements will be summarized here, and the manner in which practical use may be made of the results will be illustrated.

Photographic meteoroids, on impact with the upper atmosphere, produce luminous trails, or meteors, of sufficient brilliance that they may be photographed. In the photography of such meteors, two important biases exist. Particles having greater velocity relative to Earth's atmosphere produce trails more easily photographed than those moving more slowly. Also, heavier particles produce trails more easily photographed than lighter particles. Unless properly corrected, these photographic biases strongly affect counts of photographic meteors of any particular type or class.

For the reasons described, a weighting factor was developed in reference 1 for correction of both the velocity and the mass bias. In references 2 and 3 that weighting factor was confirmed and refined. A corrected histogram was obtained for velocities of meteoroids relative to a hypothetical gravity-free Earth on the basis of data for more than 2000 sporadic meteors photographed and reduced by the Smithsonian Astrophysical Observatory (ref. 4). A bimodal log-normal equation was derived to express the velocity distribution shown by the histogram. An older equation for influx rates of particles of various masses was confirmed on a higher confidence level than had been possible before.

The velocity histogram and consequently also the bimodal log-normal equation were not corrected for spacewise bias. Such a bias exists because particles approaching Earth from some directions in space are more likely to enter the atmosphere over New Mexico (the camera site) than particles approaching from other directions. Correction of spacewise bias would have involved unnecessary complication and the effect of the correction would not even have been desirable relative to the primary purposes of the analyses. In the analysis reported here, use was made of factors for correction of spacewise bias for each sporadic meteor, which were calculated as part of the work recently reported in reference 5.

In addition to the more refined results for a gravity-free Earth, similar results were obtained for velocity distribution and influx rate relative to the atmosphere of the real Earth. The method that was developed for utilization of the results for meteoroid damage prediction was also applied to the real Earth condition.

METHOD OF PROCEDURE

This analysis covered the following four broad aspects:

(1) A repetition of part of the effort reported in reference 3, but with a weighting factor correcting for spacewise bias in the photography of meteors. This weighting factor was in addition to the correction that was made in reference 3 for meteoroid velocity,

mass, and other minor parameters. The results were a fully corrected histogram and bimodal log-normal equation for particle velocities relative to a hypothetical gravity-free Earth (approximately the same as for the real Earth at a distance equal to the radius of Earth's sphere of influence).

(2) A correction for spacewise bias on the constant factor α, which was determined in reference 3, for use in the equation

$$F_{>} = \alpha m^{-\beta} \tag{1}$$

which, with β approximately equal to 1.34, expresses the influx rate $F_{>}$ for meteoroids of mass greater than m in grams. (Symbols are defined in appendix A.)

(3) Derivation of a histogram and a bimodal log-normal equation for velocities of meteoroids relative to the atmosphere of the real Earth. This result was obtained by theoretical modification of the histogram for gravity-free Earth conditions, which was obtained in the first aspect of this analysis.

(4) Integration of the bimodal log-normal equations obtained in the first and third aspects of the analysis to obtain average values of velocity to an n^{th} power, and an illustration of how the average value of the n^{th} power may be used to calculate necessary armor thickness for a space mission.

Velocity Distribution for Gravity-Free Earth

The points plotted in figure 1 represent the distribution of v_G, which is the velocity of sporadic meteoroids relative to a gravity-free Earth, or the impact velocity on a space vehicle moving in Earth orbit but at a great distance from Earth. The data for the figure were obtained with the same computer program as described in reference 2 and as used in both references 2 and 3. Essentially, that computer program simply counts meteors belonging within each of a large number of intervals of velocity, but with application of a weighting factor to the count of each meteor; that is, if the weighting factor for a given meteor has a value φ_w, then such a meteor is counted not as one meteor but as φ_w meteors. For the data of figure 1, the weighting factor was more refined than as used in reference 2 or 3. It is defined by the following equation

$$\varphi_w = (\cos Z_R)^{-0.22} F(Z_R)_{av}^{0.712} v_\infty^{-3.87} \varphi_s \tag{2}$$

in which Z_R is zenith angle, or angle of the meteor path to the zenith, $F(Z_R)_{av}$ is a statistical function of zenith angle, position of the meteor within the field of view of the cameras, and azimuth of the meteor path, v_∞ is the velocity of the meteor particle relative to Earth's atmosphere just before the first deceleration of the particle by atmospheric resistance, and φ_s is a spatial weighting factor the inverse value of which was computed in the effort described in reference 5. The function $F(Z_R)_{av}$ was computed for each meteor as reported and described in reference 1.

The factor φ_w differs from the most refined form that was developed in reference 3 only in the substitution of φ_s for an approximate correction of the gravity focusing effect. A separate adjustment for the effect of gravity focusing was not used (as in refs. 2 and 3), because the factor φ_s included an accurate adjustment for that effect. The assumptions made and the principles followed in calculating the inverse value of φ_s for each meteor as part of the effort reported in reference 5 are described in appendix B.

From more than 2000 sporadic meteors reported in reference 4, only 1282 were chosen for use in obtaining the data of figure 1. The meteors selected for use all had values of $\cos Z_R$ at least as high as 0.2, to avoid unrealistically high values of φ_w. They all arrived at Earth's surface from directions in space lying within quadrant 1 as illustrated in appendix B. As discussed in that appendix, the velocity distribution for the meteors from quadrant 1 may reasonably be assumed to be the same as for all quadrants until contrary evidence is available. Moreover, the velocity distribution for quadrant 1 should more nearly represent the distribution for all quadrants than would a distribution obtained with indiscriminate use of all sporadic meteors reported in reference 4. This opinion is held because the camera sites in New Mexico could not completely cover any quadrant except quadrant 1.

The curve in figure 1 was fitted to the plotted data. It represents a bimodal log-normal equation of the form

$$f(v) = C_1 \exp\left\{-\frac{1}{2}\left[\frac{\log_e(v + \delta) - \mu_1}{\sigma_1}\right]^2\right\} + C_2 \exp\left[-\frac{1}{2}\left(\frac{\log_e v - \mu_2}{\sigma_2}\right)^2\right] \tag{3}$$

with values of the constants as follows:

$$\left.\begin{array}{llll} C_1 = 0.07594 & \mu_1 = 2.30 & \sigma_1 = 0.47 & \delta = 0 \\ C_2 = 0.0002317 & \mu_2 = 4.10 & \sigma_2 = 0.097 & \end{array}\right\} \tag{4}$$

(Equation form (3) contains the constant δ because that constant will have a nonzero value in later use for another purpose. A more usual form of, for example, the first mode of eq. (3) would substitute $C_1/(v + \delta)$ for C_1. The numerical values of C_1 and μ_1 would be changed and the physical significance of μ_1 would be different, but the same offset log-normal distribution would be represented. The form used in eq. (3) is more convenient for the present purposes.) In this case, $f(v)$ of equation (3) is $f(v_G)$. The function $f(v)$ should be read as "frequency of v." It may be defined by the equation

$$f(v) = \frac{p\left[\left(v - \frac{1}{2}\,dv\right) < v_r < \left(v + \frac{1}{2}\,dv\right)\right]}{dv} \tag{5}$$

in which $p\left[\left(v - \frac{1}{2}\,dv\right) < v_r < \left(v + \frac{1}{2}\,dv\right)\right]$ is the probability that an unknown real value v_r in a specific case lies within a randomly assumed differential range from $\left(v - \frac{1}{2}\,dv\right)$ to $\left(v + \frac{1}{2}\,dv\right)$. Thus, approximately, if $\Delta v = 1$ kilometer per second is substituted for dv and if v is given successive integral values (in km/sec), equation (3) for each successive integral value will give the fraction of all meteors with velocities within 1/2 kilometer per second of that integral value.

Revision of Total Unaccelerated Influx Rate

In reference 3 a value of $2.98 \times 10^{-15}\ g^{\beta} m^{-2} sec^{-1}$ was determined for α of equation (1). In appendix C, that value is increased to 3.287×10^{-15} because of a correction for spatial bias. In consequence, with $\beta = 1.34$ as shown in reference 3, it appears that the most accurate expression for cumulative mass rate of influx determinable under the assumptions and by the methods described in the appendix is

$$F_{>} = 3.287 \times 10^{-15} m^{-1.34} \tag{6}$$

relative to a hypothetical gravity-free Earth.

Velocity Distribution and Influx Rate Near Earth

The plotted points in figure 2 represent the distribution of v_{∞}, the impact velocity that would exist for a space vehicle in orbit about the Earth at an altitude just sufficient to avoid appreciable atmospheric resistance. The data for the plotted points were derived from those of figure 1 in the manner described in appendix D. The abscissas of figure 1 were adjusted for acceleration by Earth's gravity and the ordinates were adjusted for the effect of gravitational focusing. Also, increases of ordinates by gravitational focusing were integrated to provide a ratio F_c of influx rate near Earth to influx rate far from Earth. The effect of the orbital velocity about Earth for the space vehicle was neglected. The statistical effect of the orbital velocity should be small because it would subtract from the impact velocity nearly as often as it would add to it and also because it would cause a large percentage difference of impact velocity only at the lower impact velocities, which are not those that represent the principal hazard. In general, the orbital velocity of the space vehicle should cause some reduction of the levels of the two peaks of the distribution shown in figure 2, some increase in the level of the valley, a decrease in the minimum and an increase in the maximum abscissa for which a frequency of velocity is shown, and some increase in average velocity.

The curve in figure 2 was fitted to the trend of the plotted points by the same method as with figure 1. It represents equation (3) with values of the constants as follows:

$$\left.\begin{array}{llll} C_1 = 0.1308 & \mu_1 = 1.40 & \sigma_1 = 0.62 & \delta = -8.0 \\ C_2 = 8.966\times10^{-5} & \mu_2 = 4.14 & \sigma_2 = 0.055 & \end{array}\right\} \tag{7}$$

Calculation of Necessary Armor Thickness to Avoid Critical Damage

Equations (1) and (3), with values of constants given in equations (4) or (7), may be used in conjunction with an equation expressing the extent of damage by a meteoroid impact to determine necessary armor thickness to avoid an unacceptable probability of critical damage. The general problem will be analyzed; then a sample calculation will be performed for a particular case to illustrate use of the method.

General expression for critical damage. - A general expression for a critical value of a damage criterion may be derived as follows (under the condition that the criterion consist essentially of a constant multiplied by powers of mass m and impact velocity v):

Let $F_{>Z_{cr}}$ be the number of impacts per unit area per unit time by particles having a combination of velocity and mass such as would cause a damage criterion Z to exceed a critical value Z_{cr}. Examples of the damage criterion Z might be necessary armor thickness to avoid penetration, or depth of crater formed in a very thick plate.

In the case of thickness of armor to avoid penetration, the value of Z may be (ref. 6)

$$Z = c_1 m^{1/3} v \tag{8}$$

and, in the case of depth of crater formed in a thick plate, the following equation may apply (ref. 6).

$$Z = c_2 m^{1/3} v^{2/3} \tag{9}$$

For the general case, under the condition to which this derivation is limited,

$$Z = c_3 m^{\lambda} v^{\epsilon} \tag{10}$$

where both λ and ϵ are positive constants. In equations (8) to (10), the symbols c_1,

c_2, and c_3 may be constants or functions of variable parameters other than m and v.

Now, for the general case (eq. (10)) the problem is to consider all meteoroids impinging on the vulnerable area and to determine which of those meteoroids will produce a value of Z greater than Z_{cr}. Actually, no equation is yet available to determine the total impingement. However, equation (1) will provide the total impingement for meteoroid masses above a value m_x, which may be designated as the minimum value of m for which equation (1) applies. For such total impingement, above the mass m_x, equation (1) indicates a value

$$F_{>m_x} = \alpha m_x^{-\beta} \tag{11}$$

For any value m_r above m_x, equation (1) is equivalent to

$$F_{>m_r} = \alpha\beta \int_{m_r}^{\infty} m^{-(\beta+1)}\, dm \tag{12}$$

where, for m of equation (1), a value m_r is substituted.

Now, from equations (11) and (12), the frequency of m in terms of the total impingement of meteoroids with mass above m_x must be

$$f(m) = \frac{\alpha\beta}{F_{>m_x}} m^{-(\beta+1)} \tag{13}$$

because then the fraction of the total population under consideration that have mass above any given value m_r will be

$$\frac{F_{>m_r}}{F_{>m_x}} = \int_{m_r}^{\infty} f(m)dm \tag{14}$$

(Here f(m) is the frequency of m, which may be defined by equation (5) on substitution of m for v and m_r for v_r.)

From the definitions of f(v) and f(m), the product f(v)f(m) dm dv must be the fraction of the total population under consideration that have the mass m ± 1/2 dm and the velocity v ± 1/2 dv. So the fraction of the total population that will produce a value of Z greater than Z_{cr} must be

$$\frac{F_{>Z_{cr}}}{F_{>m_x}} = \int_0^{\infty} \int_{m_{cr}}^{\infty} f(v)f(m)\, dm\, dv \tag{15}$$

where m_{cr}, from equation (10), is

$$m_{cr} = \left(\frac{Z_{cr}}{c_3 v^{\epsilon}}\right)^{1/\lambda} \tag{16}$$

So, from equations (13), (15), and (16)

$$\frac{F_{>Z_{cr}}}{F_{>m_x}} = \frac{\alpha\beta}{F_{>m_x}} \int_0^{\infty} \int_{\left(Z_{cr}/c_3 v^{\epsilon}\right)^{1/\lambda}}^{\infty} m^{-(\beta+1)} f(v)\, dm\, dv \tag{17}$$

with the reservation that the lower limit of the integration relative to m must never be less than m_x for any practical value of v, that is, for any value of v for which f(v) is significantly different from zero. So

$$F_{>Z_{cr}} = \alpha c_3^{\beta/\lambda} Z_{cr}^{-\beta/\lambda} \int_0^{\infty} v^{\epsilon\beta/\lambda} f(v) dv \tag{18}$$

or, as the velocity v is always measured positively in the direction of meteoroid motion and negative values of v are therefore not real,

$$F_{>Z_{cr}} = \alpha c_3^{\beta/\lambda} Z_{cr}^{-\beta/\lambda}\ \overline{v^{\epsilon\beta/\lambda}} \tag{19}$$

(See ref. 8 or other textbook on mathematical statistics.) Equation (19) may now be converted to

$$Z_{cr} = c_3 \left(\frac{\alpha \overline{v^{\epsilon\beta/\lambda}}}{F_{>Z_{cr}}}\right)^{\lambda/\beta} \tag{20}$$

Now, if exposure of a vulnerable area A is required for a time interval T, with an expected mean value p for number of impacts involving a value of the criterion Z greater than Z_{cr}, then the following equation must apply,

$$F_{>Z_{cr}} = \frac{p}{TA} \tag{21}$$

From equations (20) and (21), with $F_{>Z_{cr}}$ expressed in the same units as α,

$$Z_{cr} = c_3 \left(\frac{\alpha T A \overline{v^{\epsilon\beta/\lambda}}}{p}\right)^{\lambda/\beta} \tag{22}$$

Equation (22) permits direct determination of Z_{cr}, such as necessary thickness of armor, to assure a desired mean number of destructive impacts per mission.

Negligible error results from using a value of p in equation (22) equal to the complement of the desired probability p(0) of no destructive impact in a mission, if that desired probability is 0.99 or greater. For substantially smaller probabilities of no destructive impact, the following equation based on the Poisson distribution should be used (ref. 8 or other textbook on mathematical statistics):

$$p = -\log_e [p(0)] \tag{23}$$

Effect of change of influx rate on expression for critical damage. - Equation (22) includes as factors a value α concerning meteoroid influx rate in Earth's orbit but far from Earth and an average value of a power of impact velocity $\overline{v^{\epsilon\beta/\lambda}}$. In appendix D, F_c of equation (D11) is shown to be a factor that would approximately convert the influx rate of meteoroids far from Earth to the influx rate near Earth.

Now equation (22) can be applied to the near-Earth condition by substituting for α the product of α and the factor F_c and using the average value of the power of impact velocity that exists near Earth. However, it will be more convenient for use with the velocity distribution near Earth (as shown in fig. 2) and with other velocity distributions that might later be found to modify equation (22) to a form in which the constant α may always be used. This objective may be accomplished by combining the factor F_c (or its equivalent in later cases) with the average value of the power of impact velocity. For generality, a function φ_g will be used instead of F_c, which applies specifically to the near-Earth condition without regard to direction of impact. The product of α and φ_g may be substituted for α in equation (22) with the result

$$Z_{cr} = c_3 \left[\frac{\alpha TA\left(\overline{\varphi_g v^{\epsilon\beta/\lambda}}\right)}{p}\right]^{\lambda/\beta} \tag{24}$$

For general use, the function $\overline{\varphi_g v^n}$ may be plotted over a wide range of values of n. The parameter v in such function could be any one of an infinite variety of impact velocities. For example, instead of the full velocity on a randomly oriented plane, it could be the normal component of impact velocity on such a plane, the normal component of impact velocity on a plane with a particular orientation, the normal component on a cylinder with a particular orientation, and so on. In use of the plot, the exponent n in the function $\overline{\varphi_g v^n}$ would be given the value $\epsilon\beta/\lambda$ as in equation (24), the value of the function would be read from the plot for that value of n, and the result would be substituted into equation (24).

For impact velocities without regard to direction of impact, far from Earth, φ_g is equal to unity, as in equation (22), and the impact velocity v of equation (24) is the velocity v_G. For the near-Earth condition, without regard to direction of impact, φ_g is equal to the factor F_c of equation (D11) and the impact velocity v of equation (24) is the velocity v_∞.

Sample calculation of critical damage criterion. - A sample calculation will be described here for the purpose of illustration of the method. It is not intended as a definitive indication of necessary armor thickness, particularly because it involves no consideration of shower meteors. Because the calculation is intended only to illustrate the method, no support will be offered for the values of constants used. For this example, the necessary thickness of armor will be calculated for a randomly oriented space radiator of 2000 square feet vulnerable area, with an assumption of a 99 percent probability of no destructive impact in a period of 1000 days.

Use will be made of an equation by Fish and Summers of Ames Research Center (personal communication):

$$\frac{t}{d} = 0.65 \left(\frac{1}{\epsilon_t}\right)^{1/18} \left(\frac{\rho_p}{\rho_t}\right)^{1/2} v_{norm}^{7/8}\ d^{1/18} \tag{25}$$

where t is minimum armor thickness (cm) at which penetration would occur, d is diameter of particle (cm), ϵ_t is percent elongation of armor material, ρ_t is density of armor material (g/cm^3), ρ_p is density of particle (g/cm^3), and v_{norm} is the component of impact velocity normal to the surface of the armor at the point of impact (km/sec). With a simplifying approximation that all particles are spherical, equation (25) is equivalent to

$$t = 0.8162\ \epsilon_t^{-1/18} \rho_p^{8/54} \rho_t^{-1/2} m^{19/54}\ v_{norm}^{0.875} \tag{26}$$

(A procedure analogous to the following would apply with any damage criterion that might be chosen in preference to equations (25) or (26) so long as a power of v_{norm} and a power of m appeared as a factor within that criterion.)

Stainless-steel armor will be assumed with values of ρ_t equal to 8 grams per cubic centimeter and ϵ_t equal to 10 percent. Meteoroid density ρ_p will be taken as 0.2 gram per cubic centimeter. On substitution of these values into equation (26), and combining of constants, that equation becomes equivalent to equation (10) with use of v_{norm} for v and t for Z, with

$$\left.\begin{aligned} c_3 &= 0.2000 \\ \lambda &= 0.3519 \\ \epsilon &= 0.875 \end{aligned}\right\} \tag{27}$$

Now equation (24) has been shown to be an expression for a critical value Z_{cr} of the damage criterion Z (t in this case) if that damage criterion is expressed by equation (10). For the present case, for solution of equation (24), use will be made of a value of β equal to 1.34, as was well confirmed in reference 3. Values of c_3, λ, and ϵ from equations (27) will be used. Use will be made of a value of α equal to $\alpha_{1(s)}$ of equation (C8), a value of T equal to the time of the mission (in sec), a value of A equal to the vulnerable area (in m^2), and a value of p approximately equal to the complement of the desired probability of no destructive impact 0.01. The velocity v in equation (24) will be taken as v_{norm}, as required for application of equation (26). The result for both near-Earth and far-from-Earth conditions and for a critical thickness t_{cr} is

$$t_{cr} = 0.05046\left(\varphi_g \overline{v_{norm}^{3.332}}\right)^{0.2626} \tag{28}$$

In appendix E, equation (E11) is developed to express the value of $\overline{v_{norm}^{n}}$ in terms of $\overline{v^n}$, for any unshadowed randomly oriented surface. From equations (28) and (E11),

$$t_{cr} = 0.039005\left(\varphi_g \overline{v^{3.332}}\right)^{0.2626} \tag{29}$$

For far-from-Earth conditions in equation (29) φ_g equals one and v equals v_G. For near-Earth conditions φ_g equals F_c of equation (D11) and v equals v_∞.

For both conditions, values of $\overline{v^n}$ have been determined in manners described in appendix F for values of n from 0 to 6, and the results are plotted in figure 3. Also plotted in figure 3 are values of $F_c\overline{v_\infty^n}$ or, for this case, $\varphi_g\overline{v_\infty^n}$. In the calculations of the plotted values, equation (3) was used with values of the constants as shown in equations (4) and (7).

Values of the function $\varphi_g\overline{v^{3.332}}$ for far-from-Earth and near-Earth conditions, somewhat more accurate than can be read from figure 3, are

$$\left.\begin{aligned} \overline{v_G^{3.332}} &= 18\,270 \\ 2.447\,\overline{v_\infty^{3.332}} &= 35\,030 \end{aligned}\right\} \tag{30}$$

From equations (29) and (30), for far-from-Earth conditions

$$t_{cr} = 0.513 \text{ cm } (0.202 \text{ in.}) \tag{31}$$

and for near-Earth conditions

$$t_{cr} = 0.609 \text{ cm } (0.240 \text{ in.}) \tag{32}$$

In the transformation from equation (17) to (18) a reservation was necessary that the expression for m_{cr} in equation (16) must always exceed the value of m_x, the lower limit of mass for which equation (1) applies. From figures 1 and 2, the highest practical value of impact velocity is certainly lower than 80 kilometers per second. The lowest practical value of m_{cr} will therefore be not lower than

$$m_{cr(min)} = \left(\frac{0.513}{0.2\times 80^{0.875}}\right)^{1/0.3519} = 2.7\times 10^{-4} \text{ g} \tag{33}$$

for the condition far from Earth, and

$$m_{cr(min)} = \left(\frac{0.609}{0.2\times 80^{0.875}}\right)^{1/0.3519} = 4.4\times 10^{-4} \text{ g} \tag{34}$$

for the condition near Earth. By reference to figure 7 of reference 3 and the pertinent textual discussion, m_x appears to be at least as low as 6.3×10^{-4}. Moreover, a straight-line extrapolation to the value 2.7×10^{-4} appears to be quite reasonable. Hence, the reservation appears to be satisfied.

For comparison, the same problem will now be solved by a method heretofore in common use with an average impact velocity of 30 kilometers per second that has also been in common use for substitution into equation (10). Then, with the values shown in equation (27) and with substitution of a critical thickness t_{cr} for Z, equation (10) yields a value of critical mass

$$m_{cr} = \left(\frac{t_{cr}}{0.2\times 30^{0.875}}\right)^{1/0.3519} = 0.02058\, t_{cr}^{2.841} \tag{35}$$

Next, with substitution of m_{cr} for m and p for $F_>$ in equation (6) and multiplication of the result by TA,

$$\begin{aligned} p &= 3.287\times 10^{-15} \times \left(0.02058\, t_{cr}^{2.841}\right)^{-1.34} \times 8.64\times 10^{7} \times 185.8 \\ &= 9.6016\times 10^{-3}\, t_{cr}^{-3.807} \end{aligned} \tag{36}$$

Finally, for $p = 0.01$,

$$t_{cr} = 0.9894 \text{ cm } (0.3895 \text{ in.}) \tag{37}$$

The critical thickness of 0.3895 is nearly twice the value shown by equation (31) for the unaccelerated condition. However, if the older method of calculation in common use is given the benefit of using only the normal components of velocity, the contrast of results is far less striking. Thus, by equation (E11), the assumed average value $\overline{v_G} = 30$ yields an average of the normal component $\overline{v_{G(norm)}} = 20$. Use of this value in equation (35) produces a value of t_{cr} equal to 0.2732 instead of the value shown in equation (37). This value is only 35 percent greater than the value shown in equation (31).

A much greater reduction of critical thickness might superficially be expected because the average impact velocity is now known to be less than one half the value of 30 kilometers per second previously assumed and used. The much greater reduction did not materialize because the older method erred not only in the use of an average velocity much too high, but also in its assumption that the critical thickness would even depend on the simple average velocity. As may be seen from equation (29), for the damage criterion used, the critical thickness really depends on the function $\left(\overline{v^{3.332}}\right)^{0.2626}$. Unfortunately, the function $\overline{v^n}$ grows more rapidly than does $(\overline{v})^n$ as n increases above unity. From figure 3, the value of $\left(\overline{v^{3.332}}\right)^{0.2626}$ is approximately 12.9. By the older method, with use of equation (26), the corresponding value would be $30^{0.875}$, or 19.6. This value is only 52 percent greater than the value of $\left(\overline{v_G^{3.332}}\right)^{0.2626}$. This comparison would call for 52 percent greater critical thickness by the old method. However, with the older method, $\overline{v^n_{norm}}$ according to equation (E11) is a smaller fraction of $\overline{v^n}$ than with the method presented here, with the result that the older method calls for only the 35 percent greater thickness noted.

DISCUSSION OF RESULTS

Equation (3), with values of constants as in equations (4), appears to be the most accurate that can be derived, by the methods used, for the distribution of velocity v_G. Such distribution is without regard to direction of impact; that is, it is the expected distribution of impact velocity on a randomly oriented surface. It differs from the final equation developed in reference 3 only because of correction for spatial bias. The histogram is but little different than that obtained in reference 3, but it permitted a more exact fit of a bimodal log-normal equation. Because the photographs were taken at all feasible hours of the night and on dates well distributed throughout the year, a degree of statistical masking of the effect of the spacewise bias was expected. The slight unmasked bias shown by the results may exist for the following reasons: (1) photographs were exposed predominantly during later hours of the night, a condition that favored meteors in retrograde orbits, (2) the photographic site was north of the equator, a condition that favored meteoric particles approaching Earth from directions north of the ecliptic plane, and (3) the photographs were necessarily exposed entirely at night, a condition that favored meteoric particles approaching Earth from the direction opposite the sun.

Equation (3), with values of constants as in equations (7), seems to be the most accurate that can be derived, by the methods used, for the distribution of velocity v_∞. It also is without regard to direction of impact.

A value of α for use in equation (1) equal to $3.287\times10^{-15}\ g^{\beta}m^{-2}sec^{-1}$ ($\alpha_{1(s)}$ of eq. (C8)) appears to be the most accurate determination that can be made by these methods. That value applies for statistically expected impacts on a randomly oriented surface moving in Earth's orbit, but not near enough to Earth that Earth's gravitation can appreciably increase the frequency of impacts. At Earth's atmosphere, the value of α should apparently be increased by the factor F_c of equation (D11).

The method of calculation of the critical value of a damage criterion, with use of armor thickness to avoid penetration as a specific example, is a culmination of all the effort reported in references 1 to 3 and in the present analysis. It appears to rest on a good foundation, but it is incomplete in that shower-associated meteoroids are neglected. Ultimate modification to include the effect of shower-associated meteoroids would need to include at least one additive term on the right side of equation (12) and a repetition of all work on which equation (12) has an effect. Moreover, in equation (3) and all work affected by it changes would have to be made to account for the frequency $f(v)$ applying to shower-associated meteoroids.

The additional hazard due to shower-associated meteoroids may be great or small, depending on the directionality pattern of sporadic meteoroids. Considering as a simple example a single radiator tube, later study may show that a much different hazard due to sporadic meteoroids exists when the axis of that tube points in the direction of the apex of Earth's motion than when it points toward the sun, or in a direction normal to the ecliptic plane. In that event, the optimum result might involve one of two recourses: (1) use of the preferred alinement with additional thickening of tube walls to protect against the additional hazard from shower-associated meteoroids and (2) alinement of the tube axis parallel to the paths of shower-associated meteoroids while passing through showers, with additional thickening of tube walls to protect against the additional hazard from sporadic meteoroids while the tube axis is so alined.

On the other hand, if later study should show little effect of directionality on the hazard from sporadics, then the axis of the tube would always be alined parallel to the paths of shower-associated meteors. The only additional hazard to the cylindrical part of the tube wall would exist if the tube passed through two showers simultaneously.

Even aside from any uncertainty regarding the effects of meteor showers, no argument will be presented here for the exactness of the results as shown by equations (31) and (32). The calculation is an illustrative example only. If the reader prefers any other criterion than that shown by equation (25) or if he prefers other values of constants than those used, he may quickly arrive, by the same methods used herein, at values other than those shown by equations (31) and (32). A part of the purpose of this paper is to enable him to do so.

The more exact results that have been developed in references 1 to 3, and in the work reported here, have greatly reduced the average velocity of meteoroids below earlier estimates. The consequent decrease in necessary armor thickness is not great because of compensating inaccuracies in the earlier treatment. But the new, slightly lower, calculated thickness is believed to be far more firmly grounded. The compensating inaccuracies might have proved serious if a much lower average velocity had not been found.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, July 14, 1969,
120-27.

APPENDIX A

SYMBOLS

A vulnerable area of armor, m^2

C with subscripts 1 or 2, constant factor for mode 1 or 2 of bimodal log-normal velocity distribution

c unsubscripted, constant factor for a unimodal log-normal velocity distribution or for unspecified mode of a bimodal log-normal distribution; with subscript 0, 1, 2, or 3, factor containing constants and variable parameters other than meteoroid mass and impact velocity within various damage criterions Z (c_3 for general case)

d diameter of meteoroid, cm

F_c value of φ_g for influx at Earth's atmosphere without regard to direction

$F(Z_R)_{av}$ average statistical function of zenith angle Z_R, position of meteor within camera field of view, and azimuth of meteor path (ref. 1)

$F_>$ number of meteoroid impacts from all directions of mass greater than a given lower limit at distance from Earth equal to Earth's sphere of influence, $m^{-2}\,sec^{-1}$

$F_{>Z_{cr}}$ number of impacts for which damage criterion Z will exceed the value Z_{cr}, $m^{-2}\,sec^{-1}$

f( ) with variable argument within parentheses, frequency of a given value of argument, defined for argument v by equation (5)

m mass of meteoroid before any ablation by atmosphere, g

m_{cr} meteoroid mass at a given impact velocity just sufficient to cause damage criterion Z to exceed the critical value Z_{cr}, g

p probability of destructive impact during a mission

p( ) with argument within parentheses, probability of the argument

T duration of mission, sec

t maximum thickness of armor at which penetration would occur on impact by a given meteoroid, cm

t1, t2, t3 (subscripts) first tentative, second tentative, third tentative

t_{cr} critical thickness of armor that will provide a specified probability of penetration by a meteoroid, cm

v	meteoroid velocity of nature to be specified wherever used, km sec^{-1}
v_e	velocity of escape from Earth's gravitational field at position 90 km above Earth's surface, km sec^{-1}
v_G	velocity of meteoroid relative to Earth at distance equal to radius of Earth's sphere of influence, km sec^{-1}
v_{norm}	component of impact velocity normal to surface at point of impact, km sec^{-1}
v_∞	velocity of meteoroid relative to Earth just before deceleration by Earth's atmosphere begins, km sec^{-1}
$v_{\infty(\ )}$	with number as argument within parentheses, same as $v_{\infty(v_G)}$ with v_G equal to argument, km sec^{-1}
$v_{\infty(v_G)}$	velocity of meteoroid relative to Earth at entry into Earth's atmosphere after acceleration from velocity v_G relative to Earth at a distance equal to radius of Earth's sphere of influence, km sec^{-1}
Z	damage criterion relative to meteor impact, such as minimum thickness of armor to avoid penetration or depth of crater formed
Z_{cr}	critical value of damage criterion Z at which probability of destructive damage is just tolerable
Z_R	angle of meteor path to zenith, or zenith angle
α	constant factor governing rate of meteoroid impact in eq. (1), $g^{\beta}m^{-2}sec^{-1}$
α_1	same as α but restricted to meteoroids arriving from quadrant 1 as described in appendix B, $g^{\beta}m^{-2}sec^{-1}$
α_p	flux rate for impacts of meteoroids on unit surface that is arbitrarily turned to orientation perpendicular to path of each approaching meteoroid, unspecified units
α_r	expected flux rate for impacts of meteoroids on randomly oriented unit surface, unspecified units
β	negative value of exponent of mass in equation for cumulative influx rate relative to mass
δ	in log-normal distribution equation for function $f(x)$, offset from argument x to be used in measuring distances along logarithmic scale from modal position
ϵ	exponent of impact velocity in damage criterion Z
ϵ_t	percent elongation of armor material
λ	exponent of meteoroid mass in damage criterion Z

μ	unsubscripted or with subscript 1 or 2, modal value in unimodal log-normal equation or value of mode 1 or 2 of bimodal log-normal equation
ρ_p	density of meteoroid, g cm^{-3}
ρ_t	density of armor material, g cm^{-3}
σ	unsubscripted or with subscripts 1 or 2, standard deviation in unimodal log-normal equation or in mode 1 or 2 of bimodal log-normal equation
φ_g	factor by which α may be multiplied to provide analogous influx rate under specified condition other than that to which α applies
φ_s	weighting factor for use in count of meteors to correct for biasing effect of direction in space from which meteoroid arrives
φ_w	weighting factor applied in count of meteors to correct for biasing effects of zenith angle Z_R, function $F(Z_R)_{av}$, meteoroid mass m, meteoroid velocity relative to Earth's atmosphere v_∞, and direction in space from which meteoroid arrives
$\varphi_{w(G)}$	weighting factor to apply to count of meteors to correct approximately for effect of gravity focusing

APPENDIX B

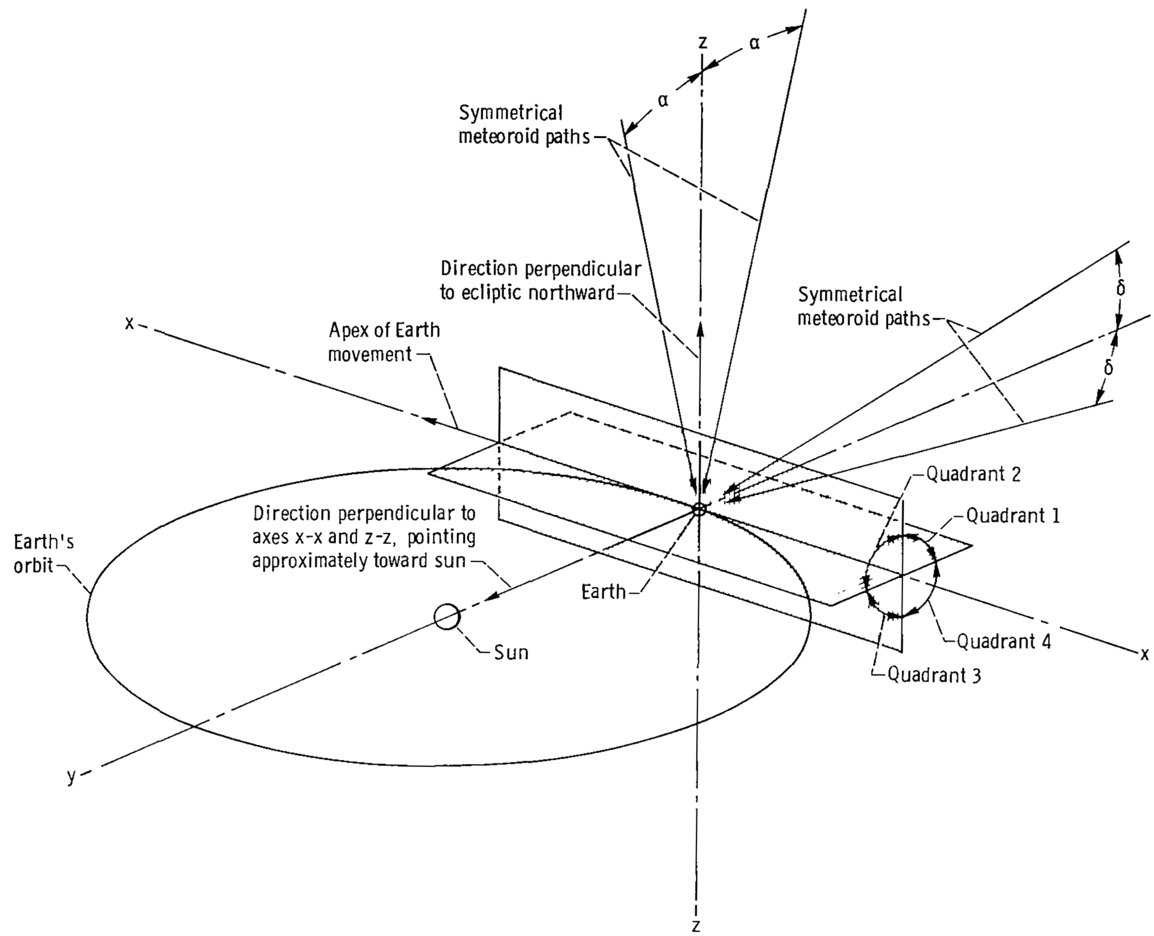
WEIGHTING FACTOR FOR CORRECTION OF SPACEWISE BIAS IN METEOR PHOTOGRAPHY

The values of φ_s as used in equation (2) are based on work by Albers and Diedrich (ref. 5). With the exception of meteors having a cosine of zenith angle less than 0.2, they computed the inverse value of a weighting factor for each sporadic meteor reported in reference 4 to correct for the biasing effect of the direction in space from which such meteor came. Certain preliminary assumptions made possible the definition of a set of coordinate axes about which the meteoroid flux should be symmetrical. The symmetry allows a logical correction for spacewise bias.

Preliminary Assumptions and Resulting Set of Coordinate Axes

The following two assumptions were made: (1) For each meteoroid arriving at Earth from a direction north of the ecliptic plane, an equal probability exists that a meteoroid might arrive in a symmetrically oriented orbit from a direction south of that plane, and (2) for each meteoroid arriving at Earth from the spatial hemisphere opposite the sun, an equal probability exists that a meteoroid might arrive in a symmetrically oriented orbit from the spatial hemisphere toward the sun. These assumptions give full scope to (1) the tendency of meteor orbits to lie within planes that make only small dihedral angles with the ecliptic plane, (2) the tendency for some values of eccentricity of meteoroid orbits to be more frequent than others, and (3) the tendency for some values of aphelion or perihelion to be more frequent than others. Equivalent assumptions would be that (1) for a given inclination of meteoroid orbit to the ecliptic, a meteoroid is as likely to intersect Earth's orbit at the descending node as at the ascending node, (2) equal probability exists for a meteoroid to pass through Earth's orbit during its transit from aphelion to perihelion as for it to pass through Earth's orbit during its transit from perihelion to aphelion.

The two assumptions may be discussed with reference to sketch (a), which illustrates a coordinate system based on the symmetry involved in those assumptions. The origin of the set of axes x-x, y-y, and z-z, as seen in the sketch, should be understood as always moving along with the Earth, and the x-x and y-y axes should be understood as always rotating about the origin in such manner that the positive direction of the x-x axis always points in the direction of the apex of the Earth movement and the positive direction of the y-y axis always extends in the direction within the plane of the ecliptic,



(a)

perpendicular to axis x-x, and approximately in the direction toward the sun.

All four quadrants illustrated in the sketch are associated with the two assumptions mentioned. Those assumptions, with reference to the sketch, are equivalent to the assumption of two planes of symmetry: (1) the ecliptic plane, that is, the plane of the x-x and y-y axes, and (2) the plane tangent to Earth's orbit and perpendicular to the ecliptic plane, that is, the plane of axes x-x and z-z. More specifically, for every meteoric particle intersecting Earth's orbit at any angle α to the x-z plane and within quadrants 1 or 4, an equal probability exists for a particle from quadrants 2 or 3, with path at the same angle α to the x-z plane and with the same projection of path on the x-z plane. Also, for every particle intersecting Earth's orbit at any angle δ to the x-y plane and within quadrant 1 or 2, an equal probability exists for a particle from quadrant 4 or 3, with path at the same angle δ to the x-y plane, and with the same projection of path on the x-y plane.

The advantages of the use of the coordinate axes shown in the sketch, with the two planes of symmetry, will now be discussed. The degree of reasonableness of the assumptions will be considered somewhat later.

Advantages of Use of Four Symmetrical Quadrants of Space

A meteoric particle arriving at the Earth orbit from any direction whatever within quadrant 1 could possibly be photographed by the cameras in New Mexico at least at some hour during the total time throughout which they were operated in obtaining the pho-photographs reported in reference 4. But particles arriving from any one of many directions within each of quadrants 2, 3, and 4 would have no possibility of being photographed at any time. With a suitable spatial weighting factor, and with a large enough sample, determination of a complete directional distribution would be possible for quadrant 1, though not for quadrant 2, 3, or 4. A directional distribution was not sought in this analysis. However, the velocity distribution that was sought is so closely associated with direction of meteoroid travel that complete directional coverage is necessary in order to allow a complete determination of the velocity distribution.

One of the advantages of dividing the space surrounding Earth into the four quadrants as shown in the sketch is now apparent. Meteors arriving from quadrants 2, 3, and 4 may be eliminated from consideration for statistical purposes. Then, with a proper spatial weighting factor for quadrant 1, an accurate velocity distribution may be obtained for that quadrant including particles arriving from any possible direction within the quadrant. Because of symmetry, the result should also apply to quadrants 2, 3, and 4, and, hence, for all space surrounding the Earth.

An additional advantage is concerned with determination of the total influx rate of meteoroids, that is, the value of α in equation (1). The site of the cameras in New Mexico cannot be assumed to encounter the same influx rate of particles as the statistically expected rate for a randomly oriented surface. The fact that the cameras were operated mostly during the later hours of the night favors impact by the comparatively rare particles in retrograde orbits, but probably disfavors impact by the average particle in a direct orbit. Location of the cameras nearer the equator than the pole tends to favor impacts by particles in orbits having only small inclinations to the ecliptic plane. It is well known that orbits having small inclinations to the ecliptic are more numerous than those having larger inclinations. These uncertainties can be eliminated by restricting the analysis to meteors arriving from quadrant 1 and with use of a spatial weighting factor that relates the probability of an impact on the atmosphere over New Mexico to the probability of impact on an equivalent area randomly oriented. The properly corrected influx rate for quadrant 1 alone needs only be multiplied by four to obtain a corrected influx rate for all quadrants.

Justification of Assumptions

The assumptions of symmetry that have been used are a necessary corollary of a concept of randomness of orientation of meteoroid orbital planes, namely, that the orientation is subject to a distribution law favoring small inclinations to the ecliptic plane but that the orientation is otherwise random. Such a condition is not known to exist but has also not been disproved. Because of the impossibility of photographing meteors in daylight, symmetry between quadrants 1 and 2 or between quadrants 3 and 4 could be checked eventually only by methods other than photographic. Symmetry between quadrants 1 and 4 could be checked by repetition of the work of references 1 to 4 and by repetition of the analysis reported here, with use of cameras located within the southern hemisphere. Until such checks become possible, the assumptions of symmetry appear to represent the simpler condition and should be used until discredited.

A definite possibility exists that meteoroids not identified with known showers are all nevertheless members of unknown minor showers. These minor showers might possibly be roughly grouped in a manner such that the sporadic meteors could be regarded as members of a few very diffuse showers. "Diffuse shower," here, is a shower in which the true radiants of the different meteoroids are not identical but are more or less approximately the same. A possible consequence would be that the sporadic meteors would tend to vary in the distribution of their directions from one part of the year to another as expressed relative to the coordinate axes that have been illustrated. The correct integrated effect throughout the year, then, would differ from that obtained in reference 5 because of the tilt of the Earth's axis relative to the ecliptic north. It is not believed that the error would be great.

Spatial Weighting Factor

In addition to the determination and recording of the inverse value of a spatial weighting factor for sporadic meteors reported in reference 4, Albers and Diedrich (ref. 5) determined and recorded the quadrant from which each meteor arrived. For their use, the authors of reference 4 supplied data as to the number of minutes of each hour of each day that the cameras were operated throughout the period covered by the data reported in reference 4.

For each meteor, Albers and Diedrich performed the following operations: (1) For each hour of camera operation, a ratio was determined equal to a unit cross-sectional area perpendicular to the meteor path at a great distance from Earth, through which the meteor might have passed, divided by the corresponding area on the surface of the Earth at the camera sites in New Mexico within which the meteor might have arrived. The determination of this ratio included a determination of the hyperbolic orbit of the particle about Earth center and an accurate determination of the effect of gravitational focusing. (2) The ratio so determined for each hour was multiplied by the number of minutes of camera operation within that hour. (3) All results of item (2) were added and divided by the total number of minutes of operation of the cameras. The reciprocal of the result is the spatial weighting factor φ_s for the meteor as used in equation (2).

APPENDIX C

REVISION OF INFLUX RATE FOR GRAVITY-FREE EARTH FOR CORRECTION OF SPATIAL BIAS

The value of 2.98×10^{-15} $g^{\beta}m^{-2}sec^{-1}$ that was found in reference 3 for α of equation (1) will be modified to correct for the spacewise bias caused by the fact that an area in the sky above the camera location would not necessarily receive the statistically expected frequency of impacts for a similar area located at random over Earth's surface.

Determination of the factor 2.98×10^{-15} involved use of a basic reference value from reference 9 and an actual count of sporadic meteors of mass greater than 0.705 gram. The count was corrected for an estimated fractional failure of observation because of faintness of meteor traces and approximately corrected for gravitational focusing by application of the factor

$$\varphi_{w(G)} = 1 + \left(\frac{v_e}{v_G}\right)^2 \tag{C1}$$

where v_e is the velocity of escape from Earth's gravitation (ref. 10). The lower mass limit of 0.705 gram was an arbitrary value above which observational failure caused by meteor faintness was believed to be small. Because it was believed to be small and because the failure should apply to all spatial quadrants shown in appendix B about equally, correction of this observational failure will be ignored in the following treatment. Accordingly, the value of α, 2.98×10^{-15}, may be reduced to a value applying to quadrant 1 alone by the equation

$$\alpha_1 = \frac{K_1}{K_{1-4}}\alpha \tag{C2}$$

where K_1 is a count of sporadic meteors within quadrant 1 and K_{1-4} is a count of sporadic meteors for all quadrants, in both cases with $\cos Z_R$ not less than 0.2, with mass greater than 0.705 gram, and with correction for gravitational focusing by application of the factor $\varphi_{w(G)}$ of equation (C1). The corrected counts K_1 and K_{1-4} and the consequent value of the ratio K_1/K_{1-4} were determined by a minor addition to the computer program used for the results described in the preceding section. The result was

$$\frac{K_1}{K_{1-4}} = 0.608 \tag{C3}$$

or

$$\alpha_1 = 1.812\times10^{-15} \tag{C4}$$

Now the adjusted count K_1 of equation (C2) may be expressed as

$$K_1 = \sum \varphi_{w(G)} \tag{C5}$$

where the summation is performed for all sporadic meteors within the classification described, with each meteor counted as $\varphi_{w(G)}$ meteors. This adjusted count may be further adjusted for spacewise bias as

$$K_{1(s)} = \sum \varphi_s \tag{C6}$$

and an adjusted value of α may be obtained as

$$\alpha_{1(s)} = \frac{K_{1(s)}}{K_1} \alpha_1 \tag{C7}$$

a determination of which was included in the computer program, with the result

$$\alpha_{1(s)} = 1.814\ \alpha_1 = 3.287\times10^{-15} \tag{C8}$$

The value $\alpha_{1(s)}$ is applicable to sporadic meteors arriving from quadrant 1 only. The factor α_1, of course, refers to impacts occurring on the atmospheric surface as it existed over the camera sites. But, as the factor φ_s is referred to an area arbitrarily alined normal to the path of each meteor, the factor $\alpha_{1(s)}$ also applies under the assumption that the vulnerable area would always be so oriented as to be normal to the path of any approaching particle. As shown in appendix E (eq. (E1)), a randomly oriented surface should receive only one-quarter as many impacts. However, $\alpha_{1(s)}$ applies to one quadrant only. By the assumption of symmetry, the total flux from all quadrants should be four times as great. The two adjustments cancel, so that the value of $\alpha_{1(s)}$ shown in equation (C8) represents the statistically expected total influx from all directions on one side only of a randomly oriented surface. Hence, this value is used in equation (6) for the cumulative mass distribution.

APPENDIX D

DERIVATION OF VELOCITY HISTOGRAM AND INFLUX RATE FOR NEAR-EARTH CONDITIONS

The histogram represented by the circular points in figure 2 was derived theoretically from that of figure 1. Use was made of the relation, for change of variable,

$$f(v_\infty) = f(v_G)\, dv_G/dv_\infty \tag{D1}$$

(see ref. 8 or other textbook on mathematical statistics). For application to a histogram, equation (D1) must be converted to an approximate form

$$f(v_\infty) = f(v_G)\, \Delta v_G/\Delta v_\infty \tag{D2}$$

Equation (D2) converts from frequency of v_G to frequency of v_∞, but only with regard to the relative values of the two variables. Account must still be taken of the effect of Earth's gravity in focusing the influx of particles in such manner as to increase the rate of influx for all particles, and to increase the rates of influx for slower particles relative to those for faster particles. Thus, equation (D2) becomes

$$f(v_\infty) = C_g f(v_G)\, \Delta v_G/\Delta v_\infty \tag{D3}$$

where C_g is the concentration factor for the given value of v_G due to the gravitational focusing effect. Finally, the values of $f(v_\infty)$ must be normalized to provide a total value of unity.

Details of the method of deriving the histogram shown in figure 2 in accordance with equation (D3) will be described by considering an illustrative example, as follows:

In the data for figure 1, four values of $f(v_G)$ are

$$\left.\begin{aligned} f(1.5) &= 0.002704 \\ f(2.5) &= 0.01030 \\ f(3.5) &= 0.005130 \\ f(4.5) &= 0.02960 \end{aligned}\right\} \tag{D4}$$

Now, for each of the frequencies of v_G shown in equations (D4), an augmented velocity

$v_{\infty}(v_G)$ was determined by use of the equation

$$v_{\infty}(v_G) = \sqrt{v_G^2 + v_e^2} \tag{D5}$$

where v_e^2 is the square of the escape velocity, taken as

$$v_e^2 = 123.25\ \mathrm{km}^2/\mathrm{sec}^2 \tag{D6}$$

The left side of equation (D5) may be regarded as an argument of the frequency function at the left side of equation (D3). The augmentation of velocity from v_G to $v_{\infty}(v_G)$ with use of equation (D5) was based on the obvious fact that the kinetic energy of a particle must be increased by the energy of escape when a particle moves from an infinite distance to the Earth's surface. Very little error is involved, even at low values of v_G because of the incorrect treatment of particles as being accelerated from an infinite distance. The results of the augmentation of velocity for the four examples were

$$\left.\begin{aligned} v_{\infty}(1.5) &= 11.203 \\ v_{\infty}(2.5) &= 11.380 \\ v_{\infty}(3.5) &= 11.640 \\ v_{\infty}(4.5) &= 11.979 \end{aligned}\right\} \tag{D7}$$

The frequencies of v_G, that is, $f(v_G)$ as given by equations (D4), represent influx rates at the values of v_G given (±0.5 km/sec), divided by the total influx rate far from Earth for all velocities. Tentative values of the same frequencies augmented by gravitational focusing $f\left[v_{\infty}(v_G)\right]_{t1}$ were obtained with use of the following equation (refs. 2 and 10)

$$f\left[v_{\infty}(v_G)\right]_{t1} = \left[1 + \left(\frac{v_e}{v_G}\right)^2\right] f(v_G) \tag{D8}$$

The left side of equation (D8) corresponds to the left side of equation (D3). The right side of equation (D8) corresponds to the first two factors of the right side of equation (D3), but not yet in their final form. The results from equation (D8) are normalized relative to the total influx far from Earth rather than relative to the total influx near Earth. Also, $f\left[v_{\infty}(v_G)\right]_{t1}$ represents a fractional influx rate within an interval of 1 kilometer per second in the value of v_G, not the value of $v_{\infty(v_G)}$.

Equation (D8) involves substantial error at low values of v_G because its derivation assumes that the meteoroid is accelerated by Earth gravity from an infinite distance, while, in fact, because the particle was originally in orbit about the sun, its acceleration by Earth gravity does not occur from an infinite distance. An estimate of this inaccuracy was presented in figure 8 of reference 2. From that figure, for each value of v_G used in equation (D8), a correction factor φ_{er} was read and used as follows to obtain a second tentative frequency value

$$f\left[v_{\infty(v_G)}\right]_{t2} = f\left[v_{\infty(v_G)}\right]_{t1} \varphi_{er} \tag{D9}$$

The results, for the four cases of equations (D4) and (D7) were

$$\left.\begin{aligned} f\left[v_{\infty(1.5)}\right]_{t2} &= 0.09426 \\ f\left[v_{\infty(2.5)}\right]_{t2} &= 0.1905 \\ f\left[v_{\infty(3.5)}\right]_{t2} &= 0.05328 \\ f\left[v_{\infty(4.5)}\right]_{t2} &= 0.2021 \end{aligned}\right\} \tag{D10}$$

The left side of each of the equations (D10) again corresponds to the left side of equation (D3), while the right side of these equations corresponds to the first two factors in the right side of equation (D3) in their final form. At this stage, the values of $f\left[v_{\infty(v_G)}\right]_{t2}$ were still in units of fraction of total flux at great distance from Earth per unit value of Δv_G. The conclusion follows that the total of all values of $f\left[v_{\infty(v_G)}\right]_{t2}$ obtained with use of equation (D9) is a concentration factor, equal to the ratio of total flux rates per unit area at Earth's atmosphere and at a great distance from Earth but within the Earth orbit. The value of such concentration factor proved to be

$$F_c = 2.447 \tag{D11}$$

The results of the use of equation (D9), as shown in equations (D10), were next converted to fractions per unit value of $\Delta v_{\infty(v_G)}$, but still normalized relative to the total influx rate far from Earth, with the equation

$$f\left[v_\infty(v_G)\right]_{t3} = f\left[v_\infty(v_G)\right]_{t2} \times \frac{1}{\Delta v_\infty(v_G)} \tag{D12}$$

in which

$$\Delta v_\infty(v_G) = \frac{1}{2}\left[v_\infty(v_G+1) - v_\infty(v_G-1)\right] \tag{D13}$$

except in end cases, in which

$$\left.\begin{aligned} \Delta v_\infty(v_G) &= v_\infty(v_G+1) - v_\infty(v_G) \\ &\text{or} \\ \Delta v_\infty(v_G) &= v_\infty(v_G) - v_\infty(v_G-1) \end{aligned}\right\} \tag{D14}$$

Equation (D12) yields a final equivalent result to equation (D3). Equations (D13) and (D14) yield approximate averages of the intervals between one value of $v_{\infty(v_G)}$ and the next. Hence, $f\left[v_{\infty(v_G)}\right]_{t3}$ given by equation (D12) is a fraction of total influx far from Earth at the near Earth velocity $v_{\infty(v_G)}$ ±1/2 kilometer per second.

The results of application of equations (D12) to (D14) to the values shown by equations (D7) and (D10) are

$$\left.\begin{aligned} \Delta v_\infty(1.5) &= v_\infty(2.5) - v_\infty(1.5) = 11.380 - 11.203 = 0.177 \\ \Delta v_\infty(2.5) &= \frac{1}{2}\left[v_\infty(3.5) - v_\infty(1.5)\right] = \frac{1}{2}(11.640 - 11.203) = 0.2185 \\ \Delta v_\infty(3.5) &= \frac{1}{2}\left[v_\infty(4.5) - v_\infty(2.5)\right] = \frac{1}{2}(11.979 - 11.380) = 0.2995 \end{aligned}\right\} \tag{D15}$$

and

$$\left.\begin{aligned}
f\left[v_{\infty(1.5)}\right]_{t3} &= f\left[v_{\infty(1.5)}\right]_{t2}\left[\frac{1}{\Delta v_{\infty(1.5)}}\right] = \frac{0.09426}{0.177} = 0.533 \\
f\left[v_{\infty(2.5)}\right]_{t3} &= f\left[v_{\infty(2.5)}\right]_{t2}\left[\frac{1}{\Delta v_{\infty(2.5)}}\right] = \frac{0.1905}{0.2185} = 0.872 \\
f\left[v_{\infty(3.5)}\right]_{t3} &= f\left[v_{\infty(3.5)}\right]_{t2}\left[\frac{1}{\Delta v_{\infty(3.5)}}\right] = \frac{0.05328}{0.2995} = 0.178
\end{aligned}\right\} \qquad \text{(D16)}$$

The frequency values shown by equations (D16) and the other similar frequency values not shown were still not normalized relative to the total near-Earth flux.

Finally, for values of v_∞ equal to 1.5, 2.5, . . . 79.5, frequencies $f(v_\infty)$ were obtained by linear interpolation relative to the values of $v_{\infty(v_G)}$ and the values of $f\left[v_{\infty(v_G)}\right]_{t3}$ of equations (D7) and (D16). The results for values of v_∞ from 1.5 to 10.5 were taken as zero because these values of v_∞ were less than $v_{\infty(1.5)}$. For $v_\infty = 11.5$, interpolation was executed between the values of $v_{\infty(2.5)}$ and $v_{\infty(3.5)}$ of equations (D7), relative to the corresponding values of $f\left[v_{\infty(2.5)}\right]_{t3}$ and $f\left[v_{\infty(3.5)}\right]_{t3}$ of equations (D16). The unnormalized result of this interpolation is

$$f(11.5)_{un} = 0.552 \qquad \text{(D17)}$$

The result as shown by equation (D17) and the similar results for all values of v_∞ from 12.5 to 79.5 were normalized by dividing each value by the total of all, which was 2.571. Thus, for the case of v_∞ equal to 11.5 kilometers per second, the normalized result was

$$f(11.5)_{norm} = 0.2147 \qquad \text{(D18)}$$

The normalized results for all velocities from 11.5 to 79.5 kilometers per second, converted to percentages, are plotted as the circular points in figure 2.

APPENDIX E

AVERAGE n^{th} POWER OF NORMAL COMPONENT OF IMPACT VELOCITY

As part of the problem of necessary armor thickness for meteoroid protection, detailed consideration must be given to the statistical relation between normal components of impact velocity and the full impact velocity for a plane surface exposed on one side to an isotropic flux of meteoroids. The problem is identical to that involving the statistically expected flux arriving from one direction only, or from any distribution of directions, on one side of a randomly oriented surface. In one case, it is the direction of movement of particles that varies randomly while the surface remains stationary. In the other case, the surface orientation varies randomly while the flux remains fixed.

This problem as stated is independent of the nature of the velocity distribution, whether log-normal or of some entirely different nature. For the case of the plane with fixed orientation, the only assumption is that the meteoroid flux is isotropic. For the expected relation between normal component of velocity and the full velocity, in the case of the randomly oriented plane, even the assumption of isotropicity is not needed.

At the outset, all meteoroids will be treated as having the same velocity v, relative to the point of impact on the surface. Later, a velocity distribution of unspecified nature will be considered. A value $\overline{v^n_{norm}}$ will be sought, an expected arithmetic average of v^n_{norm}, where v_{norm} is the component of the impact velocity normal to the surface. Impacts of any obliquity whatever will be included in the treatment.

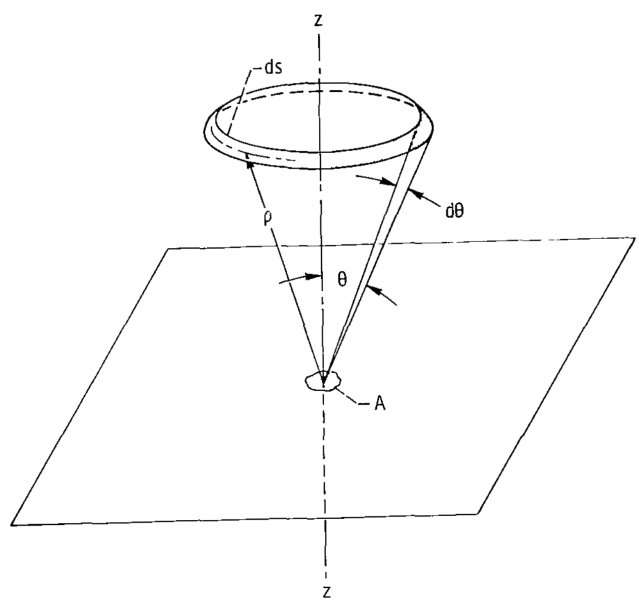
The following proposition will prove useful: given an influx rate of meteoroids α_p measured as impacts per unit area per unit time on one side of a plane surface that is always arbitrarily oriented to face any approaching particle, prove that one side of a randomly oriented surface will receive an influx $\alpha_r = (1/4)\alpha_p$.

Now, α_p, as defined to represent the given condition, is the same as the average number of impacts of any obliquity whatever per unit time on the total surface of a sphere per unit area of a cross section constructed through the center of the sphere. This fact may be seen by the consideration that a sphere offers exactly its cross sectional area for interception of a meteoroid approaching from any direction. But α_r as defined (the influx on one side of a randomly oriented surface) is the same as the average number of impacts per unit time on the total surface of a sphere per unit area of that surface. This fact is apparent in view of the consideration that a sphere has infinitesimal elements of surface facing impartially in all possible directions. Hence, over an infinite span of time, the statistical effect of the infinitesimal elements would be the same as that of a plane area that changes its orientation randomly from time to time. (Every impact will involve two punctures of a sphere, one on entering and one on leaving. The two punctures, for the purpose of this discussion, constitute one impact.)

From the foregoing considerations

$$\alpha_r = \frac{1}{4}\alpha_p \tag{E1}$$

Now detailed consideration will be given to isotropic impacts on one side of an infinitesimal plane area A as shown in sketch (b):



(b)

For this purpose, a hypothetical hemispherical surface of radius ρ will be considered, with the infinitesimal area A at its center. Equal influx rates should exist through all parts of that hemispherical surface, passing through the elementary surface A, if that elementary surface is always oriented to face any approaching particle. A differential element ds of the hemispherical surface will be considered, bounded by two circles centered about the axis z-z and separated by an arc $d\theta$ (where θ is the angle from the z axis).

Now an expression $d\varphi$ will be sought for the flux-area product (for the infinitesimal area A) passing through the surface ds and impacting on the surface element A. The flux-area product for impingement on the area A from any direction whatever, if the area A were always oriented normally to the path of any approaching particle, would be

$$d\varphi_1 = \alpha_p A = 4\alpha_r A \tag{E2}$$

Under the assumption of isotropicity, the part of that flux-area product that would pass through the annular surface ds would be

$$d\varphi_2 = d\varphi_1 \frac{2\pi\rho^2 \sin\theta \, d\theta}{4\pi\rho^2} = 2\alpha_r A \sin\theta \, d\theta \tag{E3}$$

But the area A is always oriented at an angle θ to the flux passing through the annular surface, so that the flux-area product becomes

$$d\varphi = d\varphi_2 \cos\theta = 2\alpha_r A \sin\theta \cos\theta \, d\theta \tag{E4}$$

The factor $A \cos\theta$ in equation (E4) represents the projection of area A on a plane normal to the meteoroid path, that is, the effective area presented by A for interception of meteoroids approaching at the angle θ.

Now the arithmetic average of the n^th power of normal component of impact velocity (v constant) may be expressed as

$$\overline{v^n_{norm(v=k)}} = \frac{\int_{\theta=0}^{\pi/2} v^n_{norm(v=k)} \, d\varphi}{\alpha_r A} \tag{E5}$$

The numerator of equation (E5) is the integrated value of $v^n_{norm(v=k)}$ multiplied by the differential flux-area product. The denominator, by previous definition, is the total flux-area product for the one side of the area element A.

From equations (E4) and (E5)

$$\overline{v^n_{norm(v=k)}} = 2\int_0^{\pi/2} v^n_{norm(v=k)} \sin\theta \cos\theta \, d\theta \tag{E6}$$

But

$$\left.\begin{aligned} v_{norm} &= v\cos\theta \\ &\text{or} \\ v^n_{norm} &= v^n \cos^n\theta \end{aligned}\right\} \tag{E7}$$

From equations (E6) and (E7),

$$\overline{v^n_{norm\,(v=k)}} = 2v^n \int_0^{\pi/2} \sin\theta \cos^{n+1}\theta \, d\theta$$

$$= 2v^n \left(\frac{-1}{n+2} \cos^{n+2}\theta \right)_0^{\pi/2}$$

$$= \frac{2}{n+2} v^n \tag{E8}$$

Now for any point on any solid body, with random orientation, the value of $\overline{v^n_{norm\,(v=k)}}$ will be given by equation (E8) so long as the surface at that point is not shadowed by other parts of the surface of the body. But equation (E8) applies only for a specific value of v.

In the writing of an expression for $\overline{v^n_{norm}}$ for a distribution of velocities, use will be made of the fact that equation (E8) must apply for each velocity. Use will also be made of the statistical frequency of velocity $f(v)$.

The arithmetic average of the n^{th} power of the normal component of impact velocity, then, may be expressed as

$$\overline{v^n_{norm}} = \frac{\int_0^{\infty} \int_{\theta=0}^{\pi/2} v^n_{norm} \, d\varphi \, f(v) \, dv}{\alpha_r A \int_0^{\infty} f(v) \, dv} = \frac{\int_0^{\infty} \overline{v^n_{norm\,(v=k)}} \, f(v) \, dv}{\int_0^{\infty} f(v) \, dv} \tag{E9}$$

in which the final denominator is the total fractional flux (unity).

From equations (E8) and (E9)

$$\overline{v^n_{norm}} = \frac{2}{n+2} \frac{\int_0^{\infty} v^n f(v) \, dv}{\int_0^{\infty} f(v) \, dv} \tag{E10}$$

or

$$\overline{v^n_{norm}} = \frac{2}{n+2} \overline{v^n} \tag{E11}$$

APPENDIX F

AVERAGE VALUE OF A POWER OF IMPACT VELOCITY BY INTEGRATION OF BIMODAL LOG-NORMAL EQUATION

In the main text, expressions were derived for critical values of damage criteria for meteoroid impact. Those expressions included an average value of a power of impact velocity $\overline{v^n}$, with n equal to $\epsilon\beta/\lambda$. As bimodal log-normal distribution equations have been found for velocities v_G and v_∞, it is therefore desirable to find a solution for the function $\overline{v^n}$ calculable from those equations.

The function $\overline{v^n}$, which involves no negative values of v, may be expressed as

$$\overline{v^n} = \int_0^\infty v^n f(v)\, dv \tag{F1}$$

For the bimodal log-normal case, equation (3),

$$\overline{v^n} = \sum_{i=1}^{2} c_i \int_0^\infty v^n \exp\left\{-\frac{1}{2}\left[\frac{\log_e(v + \delta_i) - \mu_i}{\sigma_i}\right]^2\right\} dv \tag{F2}$$

with δ_2 equal to zero.

Obviously, equation (F2) can be integrated if the following unimodal expression can be integrated

$$\overline{v_u^n} = c \int_0^\infty v^n \exp\left\{-\frac{1}{2}\left[\frac{\log_e(v + \delta) - \mu}{\sigma}\right]^2\right\} dv \tag{F3}$$

No simple closed solution or converging series solution has been found for the combination of (1) values of n that are not positive integers and (2) nonzero δ. For all other cases, however, solutions are found as follows:

Let

$$y = v + \delta \tag{F4}$$

Then,

$$\overline{v_u^n} = c \int_\delta^\infty (y - \delta)^n \exp\left\{-\frac{1}{2}\left[\frac{\log_e y - \mu}{\sigma}\right]^2\right\} dy \tag{F5}$$

or, for positive integral values of n,

$$\overline{v_u^n} = c \sum_{i=0}^{n} \frac{n!}{i!\,(n-i)!} (-\delta)^i \int_{\delta}^{\infty} y^{n-i} \exp\left\{-\frac{1}{2}\left[\frac{\log_e y - \mu}{\sigma}\right]^2\right\} dy \tag{F6}$$

with the conventions that both zero to the zero power and factorial zero equal unity. Now, let

$$x = \log_e y \tag{F7}$$

Then

$$\overline{v_u^n} = c \sum_{i=0}^{n} \frac{n!}{i!\,(n-i)!} (-\delta)^i \int_{\log_e \delta}^{\infty} \exp\left[(n-i+1)x - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx \tag{F8}$$

The squared part of the exponent of e may be expanded and terms in x may then be combined. The resulting terms in x^2 and x will then form the first two terms of an expanded square of a binomial. That square may be completed, so that the exponent of e will be a squared binomial and a constant term. The constant power of e may then be removed from the integral, with the following result:

$$\overline{v_u^n} = c \sum_{i=0}^{n} \frac{n!}{i!\,(n-i)!} (-\delta)^i \exp\left\{(n-i+1)\left[\mu + \frac{1}{2}\sigma^2(n-i+1)\right]\right\}$$

$$\times \int_{\log_e \delta}^{\infty} \exp\left(-\frac{1}{2}\left\{\frac{x - [\mu + (n-i+1)\sigma^2]}{\sigma}\right\}^2\right) dx \tag{F9}$$

or

$$\overline{v_u^n} = c\sigma\sqrt{2\pi} \sum_{i=0}^{n} \frac{n!}{i!\,(n-i)!} (-\delta)^i \exp\left\{(n-i+1)\left[\mu + \frac{1}{2}\sigma^2(n-i+1)\right]\right\}$$

$$\times \left[\frac{1}{\sigma\sqrt{2\pi}} \int_{\log_e \delta}^{\infty} \exp\left(-\frac{1}{2}\left\{\frac{x - [\mu + \sigma^2(n-i+1)]}{\sigma}\right\}^2\right) dx\right] \tag{F10}$$

Now the bracketed part of equation (F10) is an ordinary normal distribution integral and may be evaluated as follows:

Let

$$t = \frac{x - [\mu + \sigma^2(n - i + 1)]}{\sigma} \tag{F11}$$

and let

$$\tau = \frac{\log_e \delta - \left[\mu + \sigma^2(n - i + 1)\right]}{\sigma} \tag{F12}$$

Then (see ref. 8 or other textbook on mathematical statistics)

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{\log_e \delta}^{\infty} \exp\left(-\frac{1}{2}\left\{\frac{x - [\mu + \sigma^2(n - i + 1)]}{\sigma}\right\}^2\right)dx = \frac{1}{\sqrt{2\pi}}\int_{\tau}^{\infty} \exp\left(-\frac{1}{2}t^2\right)dt$$

$$= 0.5 - \frac{\tau}{\sqrt{2\pi}}\sum_{j=0}^{\infty}(-1)^j \frac{\tau^{2j}}{2^j\, j!\, (2j + 1)} \tag{F13}$$

So, from equations (F10) and (F13),

$$\overline{v_u^n} = c\sigma\sqrt{2\pi}\sum_{i=0}^{n}\frac{n!}{i!\,(n - i)!}(-\delta)^i \exp\left\{(n - i + 1)\left[\mu + \frac{1}{2}\sigma^2(n - i + 1)\right]\right\}$$

$$\times\left[\frac{1}{2} - \frac{\tau}{\sqrt{2\pi}}\sum_{j=0}^{\infty}(-1)^j\frac{\tau^{2j}}{2^j\, j!\, (2j + 1)}\right] \quad (\delta > 0,\ n \text{ a positive integer}) \tag{F14}$$

In use of equation (F14) the value of τ must be determined for each value of the index i with use of equation (F12). The value of the second term within the brackets of equation (F14) may then be found in a table of probability integrals (see ref. 8 or other textbook on mathematical statistics).

When $\delta = 0$, all parts of the compound series in equation (F14) reduce to zero for $i > 0$. In that case, the value of τ from equation (F12) becomes equal to $-\infty$, and the bracketed part of equation (F14) becomes equal to unity, so that

$$\overline{v_u^n} = c\sigma\sqrt{2\pi}\,\exp\left\{(n+1)\left[\mu + \frac{1}{2}\sigma^2(n+1)\right]\right\} \qquad (\delta = 0) \tag{F15}$$

Equation (F15) is valid for $\delta = 0$ and any real value of n.

If δ is negative, equation (F12) indicates an imaginary value for τ. But, in that case, equation (F3) implies nonexistence of values of v less than $-\delta$. Hence, the real part of the integral in equation (F3) extends from $v = -\delta$ to $v = \infty$. Accordingly, the real part of the integral in equation (F5) extends from zero to ∞, and the real parts of the integrals in equations (F8) to (F10) extend from $-\infty$ to $+\infty$. Hence, a value of $\tau = -\infty$ may be obtained from equation (F12) and, as in the case with δ equal to zero the bracketed part of equation (F10) becomes equal to unity. If n is a positive integer, equation (F10) becomes

$$\overline{v_u^n} = c\sigma\sqrt{2\pi}\sum_{i=0}^{n}\frac{n!}{i!\,(n-i)!}(-\delta)^i\exp\left\{(n-i+1)\left[\mu + \frac{1}{2}\sigma^2(n-i+1)\right]\right\} \tag{F16}$$

($\delta < 0$, n a positive integer).

If $\delta \neq 0$, and n is not a positive integer, none of the equations (F14), (F15), or (F16) may be used. In that case, equation (F3) must be subjected directly to numerical integration.

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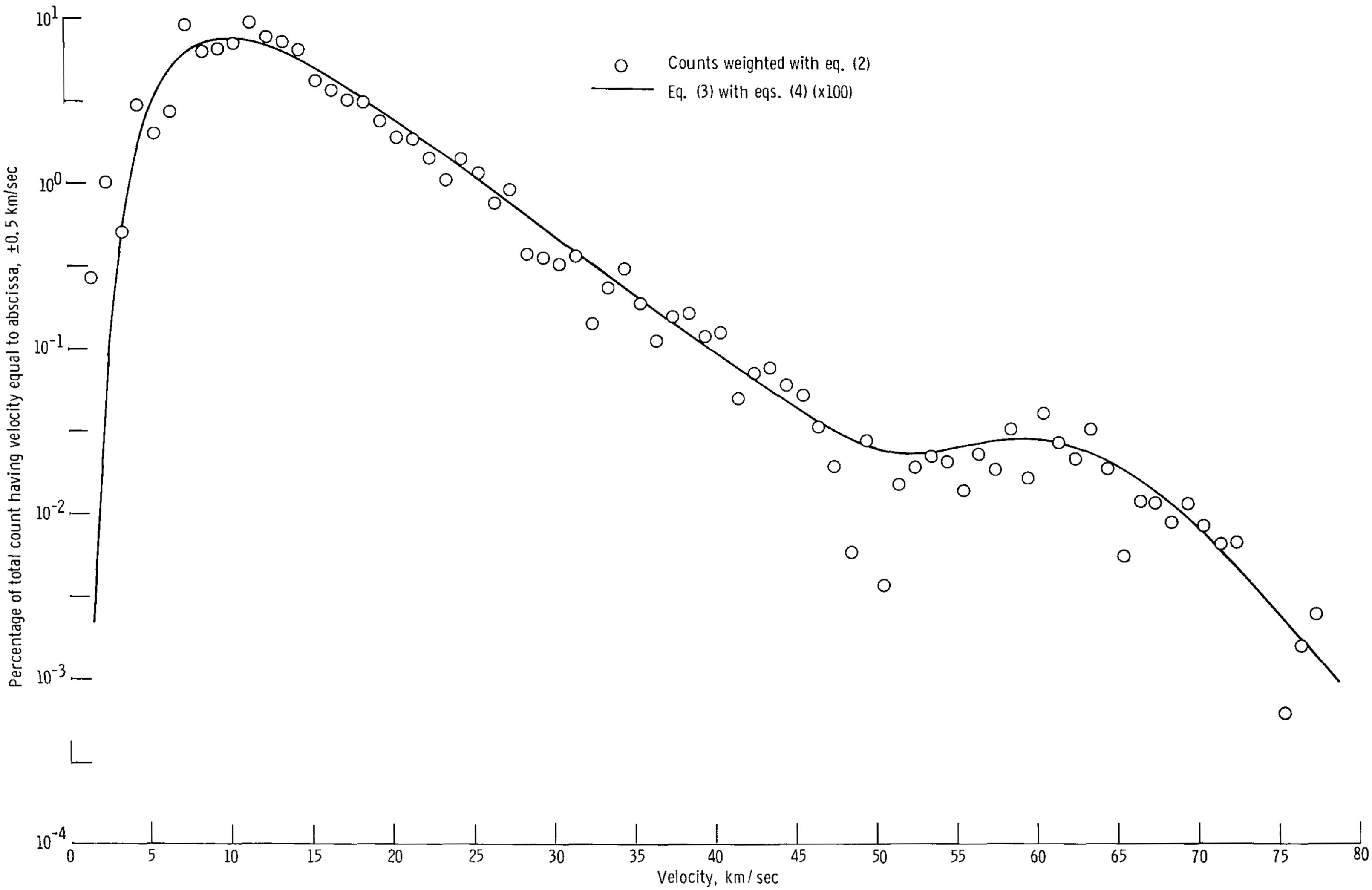

Figure 1. - Velocity distribution for impacts of meteoroids from all directions on a gravity-free Earth.

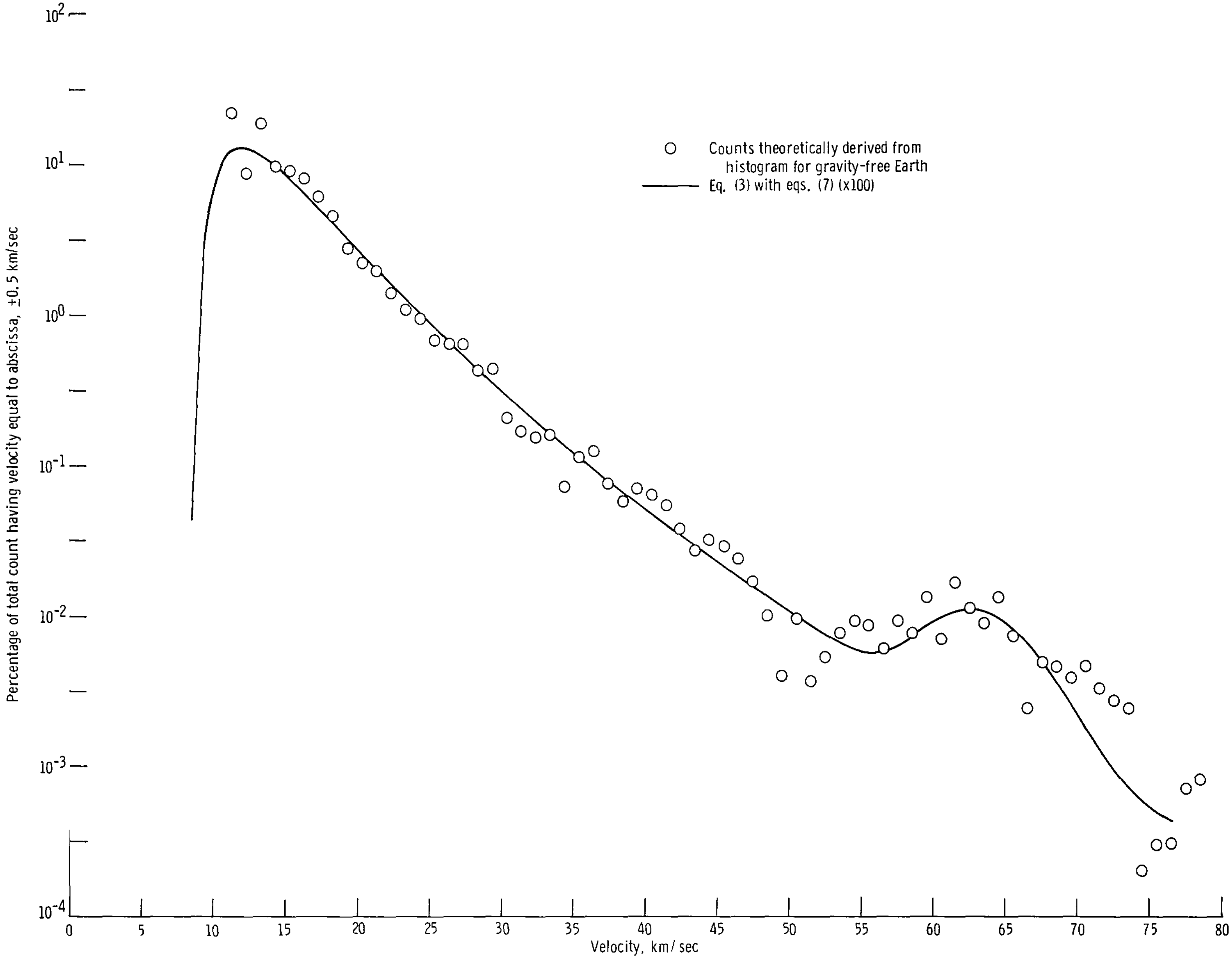

Figure 2. - Velocity distribution for meteoroids from all directions relative to atmosphere of real Earth.

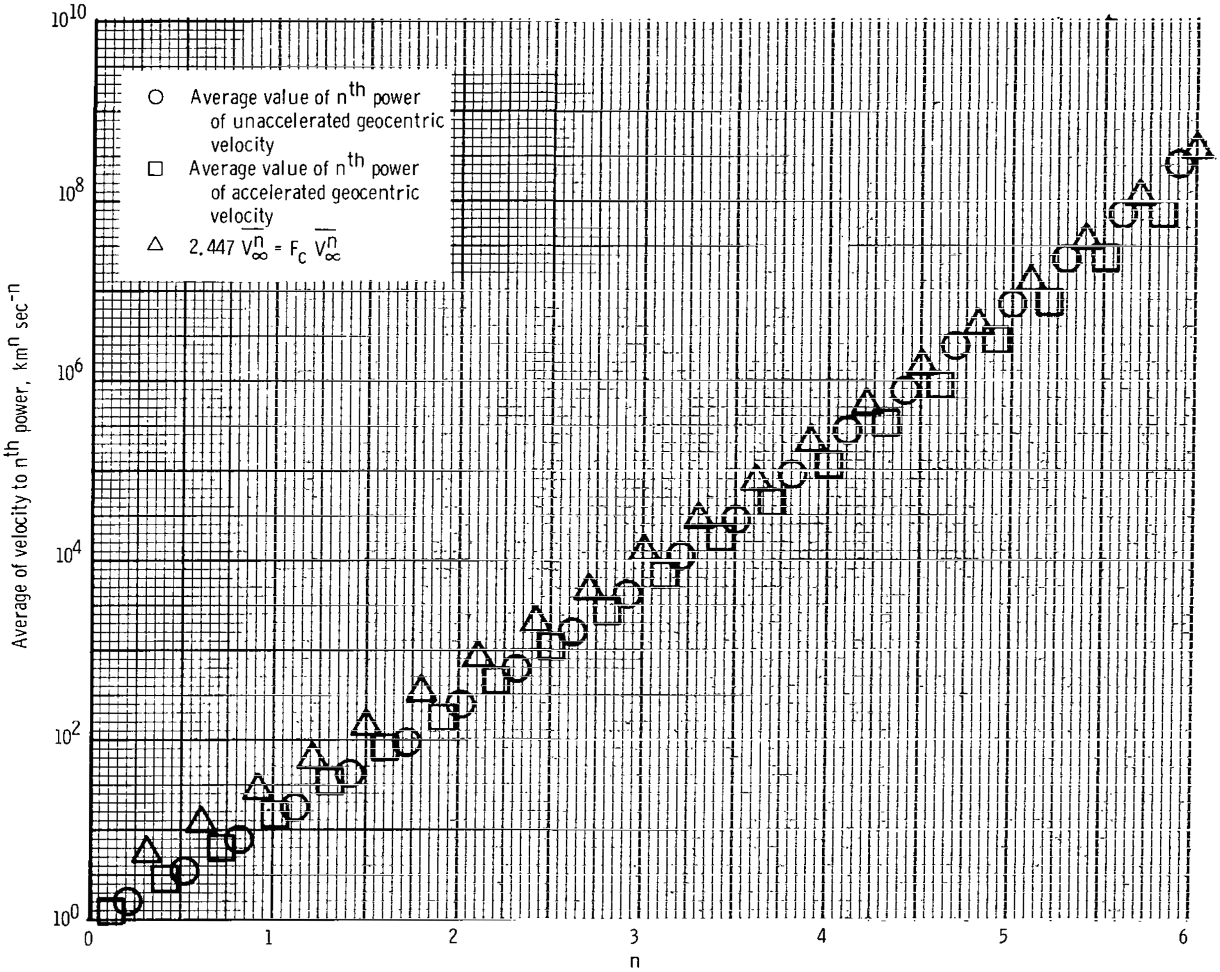

Figure 3. - Average of n^{th} power of meteoroid impact velocity times concentration factor far from Earth and near Earth.

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