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A SURVEY OF GENERAL COVERAGE NAVAIDS FOR V/STOL AIRCRAFT— A VOR/DME ERROR MODEL

by Henry Johansen

Prepared by
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge, Mass.
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ABSTRACT

A promising navigation concept for the V/STOL aircraft is to make a hybrid system comprising a low cost inertial navigation system which is updated by a radio navigation aid. This report gives a short description of suitable en route and terminal radio navaids which are available or may be available during the next decade. A statistical model for the VOR/DME errors is derived together with other information required by a Kalman filter approach to estimate the hybrid navigation system errors.

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TABLE OF CONTENTS

ABBREVIATIONS USED

ADF	- Automatic Direction Finder
ATC	- Air Traffic Control
CAS	- Collision Avoidance System
CTOL	- Conventional Take Off and Landing
CDI	- Course Deviation Indicator
DME	- Distance Measuring Equipment
DR	- Dead Reckoning
DVOR	- Doppler VOR
E[ ]	- Expected Value
FAA	- Federal Aviation Agency
HARCO	- Hyperbolic Area Coverage, also known as Decca
IFR	- Instrument Flight Rules
INS	- Inertial Navigation System
OBI	- OmniBearing Indicator
OBS	- OmniBearing Selector
PSD	- Power Spectral Density
PVOR	- Precision VOR
Rho	- Refers to distance information
RNAV	- Area Navigation (airways)
RVR	- Runway Visual Range
TACAN	- TACtical Air Navigation
Theta	- Refers to bearing information
VFR	- Visual Flight Rules
VOR	- VHF OmniRange
VORTAC	- Station comprising VOR ,and DME from TACAN
V/STOL	- Vertical/Short Take Off and Landing

1. INTRODUCTION

The navigation problem encountered by the V/STOL aircraft can be broken into three parts: en route navigation, terminal area navigation, and approach and landing. The requirements for the en route navigation will not differ significantly from those for the CTOL aircraft. In the terminal area V/STOL aircraft may be required to follow certain curved flight paths to reduce noise and to keep clear of CTOL airways.

A high degree of accuracy will be necessary to provide closely spaced tracks in high density terminal areas. For the final approach and landing phase all weather capability is desired. No single navigation aid exists today which can fulfill all these requirements. One promising approach to the problem is to use already developed radio navigation systems combined with inertial navigation systems, applying statistical filtering techniques to minimize the errors.

Many books and reports are available giving a detailed description of how the most common radio navaids function (See references [1], [2], [3], [4], [5], [41]). Thus these descriptions will not be repeated here. Only working principles which are necessary in order to describe certain error sources will be mentioned. The same navigation system can probably be used for the en route phase and the terminal phase. Low visibility landing requires a special precision navaid which most likely cannot be used during the en route and terminal area flight. This report will cover only the en route and terminal phase and give a survey of systems which most likely will be available and suitable for use with an I.N.S. for V/STOL navigation. A more detailed description of performance and a statistical error model for the only generally available continental navigation system, the VOR/DME system, is given.

2. SURVEY OF EN ROUTE AND TERMINAL AREA NAVAIDS SUITABLE FOR V/STOL APPLICATION

Some of the best known radio navigation systems designed for short to medium range navigation are listed in Table 2.1. Many other systems have been proposed such as:

Land, Litton	Theta-Theta
Halop, Hastings-Raydist	Hyperbolic
Radio Mesh, French	Hyperbolic
Satellites	

Table 2.1. En Route and Terminal Area Radio Navigation Systems

System	Status	Saturability	Range n.m.	Information and Rate
ADF	Operational, beacons in the 200-600 khtz band	None	50-200 (useful down to the surface)	Theta, continuous
CONSOL CONSOLAN	Operational, three stations in U.S.	None	500-700 (useful down to the surface	Theta, 1 fix per 60 sec.
VOR/DME VORTAC	Nationwide Coverage Recommended by ICAO	VOR 6-12	$R=1.27\sqrt{h}$, h - altitude in ft. Line of sight	VOR gives Theta continuous. DME gives Rho at 15 samples/sec.
TACAN	Nationwide coverage Bearing information for military users only		Line of sight	Theta and Rho. (Same Rho as DME)
Loran A	Fully implemented along coastal regions	None	500	Hyperbolic Continuous
Loran C	Good coverage along coastal regions of U.S.	None	1500	Hyperbolic 10 samples/sec.
Loran D	Designed for tactical use. Not operational	None	250	AS Loran C
DECCA Harco	One chain in New York	None	300	Hyperbolic Continuous
Omega	Still in the experimental stage	None	Goal is whole Earth	Hyperbolic 1 per 10 sec.
Doppler	Operational			Velocity Continuous

Accuracy	Price (A/C Equip. in $1,000)	Remarks
±2° (1σ). Very low frequency variation due to random fluctuation of ionosphere. High noise level during thunderstorms. Long distance reception difficult at night due to sky wave interference.	4.4	Bearings from two stations provide a fix. Two bearings at 90° from stations 50 n.m. away give a 1.5 n.m. (1σ) error.
<6-24 n.m. 95% of the time	0.5	Direction only.
VOR: ±1.2° (1σ) (Low frequency random) DME: ±0.14 n.m. (1σ)	VOR: 6-12 DME 7.8	See Chapter 3
Theta: 0.4-1.0° Rho: as VORTAC		Co-located with VOR stations. Distance measurement unit also used for VORTAC.
1,000-4,000 ft. groundwave 6,000-8,000 ft. skywave	3.0	
±1,500 ft. (2σ) at extreme range	80	
As Loran C		Transportable ground equipment
350 ft. (1σ) day at 100 n.m.; 3,000 ft. (1σ) night at 100 n.m.	6.4 w/automatic lane ident.	
±1n.m. (2σ) day ±2n.m. (2σ) night Low frequency random due to variations of ionosphere.	15-65	Lane ambiguity is a problem.
±0.5% (2σ) of distance traveled or 1 n.m., very low frequency random error	40	

They are not treated here, either because of lack of data or because no production models exist. None of these systems seem to offer a significant advantage over the systems in Table 2.1 A number of systems, often with a high degree of accuracy, have also been left out because they are designed only for one or a few simultaneous users. None of these systems have been recommended by the air traffic authorities or by international navigation committees.

Radio navaids which could be suitable for V/STOL have been discussed in the open literature. In references [6] and [7] it is stated that the V/STOL requires high flexibility in the terminal area and low instrument landing minima. This requires that the navigation system give volumetric coverage down to the earth's surface, free of shadows caused by intervening buildings and high ground. In order to reduce the noise and to avoid the CTOL airways, curved paths will be required. In the high-density terminal areas parallel lanes closer than 5,000 ft. have been proposed. These requirements may be fulfilled by Decca or Loran C/D. The insurmountable limitation of the VOR/DME system in the terminal area is its line of sight characteristics which restrict its low altitude coverage. Its ability to provide the required accuracy in the high-density areas when used without any optimal data processing has also been questioned. Reference [10] suggests that proper use of existing VORTAC facilities will give accuracies on the order of ± 0.3 n.m. in certain metropolitan areas. Where reflections preclude the use of VHF systems, Loran C could provide good navigational accuracy in the terminal areas. A study made at the University of Ohio [8] of low altitude flights (below 300 ft.) concludes that no existing system provides for satisfactory safe, low-altitude instrument flights.

The VOR/DME system is the only short range navigation system which has nationwide coverage. Reference [9] claims that no foreseeable system can replace VOR/DME from a practical and an economical point of view for the remainder of the twentieth century. The F.A.A. is now introducing area navigation [11] by authorizing the use of airways which do not overfly the VOR/DME stations but where VOR/DME signals can be used for primary navigation. This increases the VOR/DME system's capability to serve as an adequate en route navigation aid for V/STOL in the next decade. The present coverage of Decca and

Loran C is very limited. There are no definite plans for full implementation of these systems in the U.S. to give nationwide coverage or to give coverage in all the high-density terminal areas. This would indicate that V/STOL operations would be forced to rely upon VOR/DME for en route and terminal navigation.

Until now only limited experience has been gained with navigation systems for V/STOL. New York Airways has operated helicopter routes in the New York area for some years. Since the operating altitude has been about 1,000 feet, below line-of-sight, the VOR signals are useless due to reflections. Thus the use of Decca was authorized for use as a primary navigation system [12] with the following restrictions upon minimum ceiling and visibility: 400 ft.-1 mile, 500 ft. -3/4 mile, or 600 ft.-1/2 mile.

Eastern Airlines used for their STOL test program the Decca Omnitrac computer with a moving map display. The computer could process navigation signals from VOR/DME, Loran C or Decca (also called Harco for Hyperbolic Area Coverage) [13].

American Airlines is now conducting a STOL test program [14]. They are testing the following terminal and en route navigation systems:

- Vector Analog Computer with inputs from VOR/DME.
- Decca Omnitrac= System which takes inputs from VOR/DME, Loran C, Doppler, or Decca.
- Litton Systems LTN-51 I.N.S. with vertical navigation. VOR/DME used for updating I.N.S.

Other airlines have also tested area navigation systems where inputs have been VOR/DME, Decca, Doppler, or I.N.S. This brief survey gives an indication of navaids which will play a significant role in the en route and terminal area navigation of V/STOL in the near future.

Published reports show that the navigation accuracy using existing radio navaids can be substantially improved using hybrid systems and optimal filtering techniques. In reference [15] a simulation is performed showing that a substantial reduction in the position fix error can be obtained by using the existing VOR/DME system. By receiving signals from two VORTAC stations simultaneously, a maximum likelihood estimate of the position is performed. For a favorable choice of VORTAC stations, accuracy is primarily given by the accuracy of the DME signal. Reference [16] gives simulation results from a hybrid system using DR (Dead Reckoning) together with radio navaids such as

ADF, VOR/DME, and Loran C. Both I.N.S., Doppler and airdata DR were used. The results indicated that the combination of ADF-DR gives accuracies as good as raw VOR/DME data. Reference [17] gives an analysis of an optimal implementation of VOR/DME information with I.N.S. data. The results obtained indicate a factor of 4 to 5 improvement of position accuracy, a "cleaning up" of VOR signals without introducing unacceptable lag, and a filtering of beam bands which is a real part of the ATC airway structure. Butler National Corporation [18] has introduced geometric filtering of the VOR signals by using information about the maximum angular velocity of the aircraft about the VOR station at a specific distance to significantly reduce the position errors. Methods for improving Loran accuracy and coverage, demonstrated in [19] and consisting of a software remechanization of the receiver, can provide a passive closed-loop, one-way direct range measurement to the individual Loran stations.

The combination of I.N.S.-Doppler for V/STOL navigation has been given little attention in the recent literature. Although Doppler augmented I.N.S. systems will give improved performance, the system will give a growing position error with time, resulting in the largest inaccuracies entering the terminal area where the accuracy requirement is of most importance. Improving I.N.S.-Doppler performance to reflect terminal area requirements tends to be more expensive than using available position updates from radio navaids. Many military projects and some commercial aircraft use or are scheduled to use Doppler-I.N.S. [21]. Also, Doppler-I.N.S.-Loran C/D have been proposed. It is more likely that Doppler-radar will compete with I.N.S. for continental V/STOL flights as a dead reckoning system with position updates from radio navaids. For example, the F.A.A. has given authorization to TWA to use pure Doppler radar navigation as a primary en route navaid in the U.S. [20]. The combination ADF-I.N.S. is not expected to result in a satisfactory system because of large angular errors long distances from the station.

In conclusion, it is realistic to assume that the only general available en route and terminal radio navigation for V/STOL for the next decade will be the VOR/DME (VORTAC) system. Limited coverage of Loran C/D and Decca is also expected. The ATC Advisory Committee will soon release a report on traffic control systems adequate for the 1980's and beyond. They recommend a prompt improvement of the VORTAC system to meet the demand for higher capacity (The saturation of the DME is a major problem). This system is then predicted to handle traffic loads projected for the 1995 period [22]. Implementation of time frequency techniques similar to the CAS has been proposed as a replacement for the VORTAC system, but it is expected to be more than 15 or 20 years before this system will be in service, assuming that the system were agreed upon and funded today. This enormous time lag involved in implementing a new system with nationwide coverage emphasizes the importance of the phrase "in operational use" when choosing a navaid for the next decade. Thus the most likely radio navaids which can be integrated with I.N.S. for en route and terminal area navigation are:

- VOR/DME
- Decca
- Loran C/D

The performance of the VOR/DME system will be treated in detail in the next chapter.

3. *THE VOR/DME SYSTEM*

The VOR/DME system in the U.S. makes use of colocated VOR and TACAN transmitters. Civil aviation uses the VOR and the DME parts of the TACAN station while the bearing information from TACAN is for military users only. This combination is referred to as a VORTAC station.

In the literature, a variety of different VOR systems are described. They can be summarized as in [23]:

VOR	Conventional VOR. In general use in the U.S.
DVOR(SSB)	Doppler VOR with single sideband transmission. This is in general use in the U.S. and, for some receivers, fully compatible with VOR.
DVOR(DSB), DVOR(ASB)	Doppler VOR with double respective alternating sideband. Proposed to reduce errors in low cost VOR receivers.
PVOR-PDVOR	VOR and DVOR where increased precision is obtained by various means; for instance, by use of FM modulation instead of AM modulation.
PDVOR(H), PDVOR(M)	The first uses an additional FM carrier; the second uses multilobe techniques to increase accuracy.

In this report only the VOR and DVOR(SSB) system will be treated because they are in general use and the future use of the other systems is uncertain. The Airlines Electrical Engineering Committee recommended in 1966 that only VOR and DVOR should be used and that poor performing VOR sites should be replaced by DVOR, which is less dependent upon the terrain. Two speed systems such as PDVOR(M) and VOR systems with special FM subcarrier systems were not recommended because a similar accuracy improvement could be obtained by improving the VOR receiver design and by more accurate adjustments of the standard VOR and DVOR stations.

3.1 VOR/DME Navigation

3.1.1 The VOR/DME Coverage

The range of the VOR transmitter depends upon aircraft altitude and the class of VOR. The line of sight characteristics of the VOR ($\approx 115 MH_z$) and DME signal ($\approx 1 GH_z$) makes the coverage depend upon altitude, see Figure 3.1, [12]. (The region with 5μV signal strength implies a barely receivable signal.) An approximate formula for the range as a function of altitude is:

$$R[n.m.] = 1.27\sqrt{h[ft]} \qquad (3.1)$$

Because of the large number of VOR stations, some will be transmitting on the same frequency, and especially at high altitudes, interference can take place. The following minimum requirements for coverage have been allocated for the three VOR categories [24]:

Category	Frequency Protected Volume
H - high altitude, 18,000 to 45,000 ft	130 n.m. radius to 45,000 ft
	100 n.m. radius to 45,000 ft
L - low altitude, 18,000 ft	40 n.m. radius to 18,000 ft
T - terminal	25 n.m. radius to 12,000 ft

The Flight Inspection Handbook [25] states that the minimum range should be greater than 40 miles at 1,000 feet above antenna or terrain for H and L category VOR stations and a minimum of 25 miles for the T category.

The VOR/DME system gives full coverage in the continental U.S. at most cruising altitudes. Above 20,000 feet, the 250 H category VORTAC facilities also give almost full coverage. Above the VORTAC station

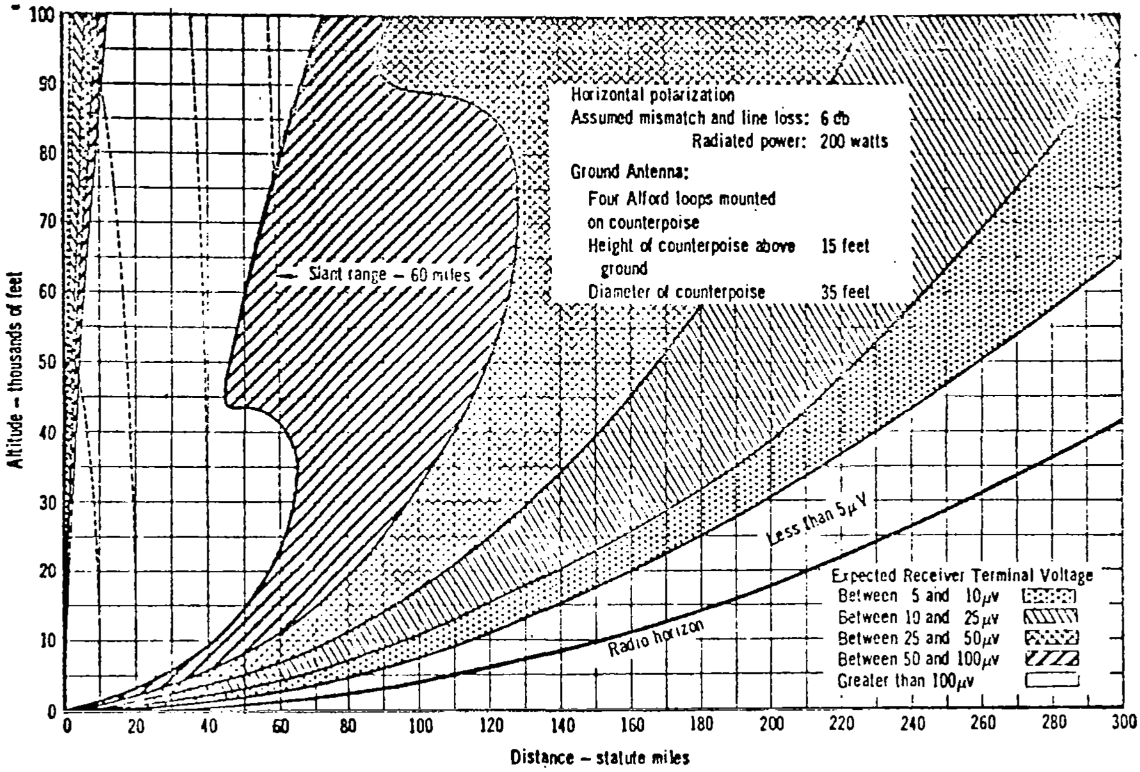


FIG. 3.1 TYPICAL COVERAGE DIAGRAM FOR A COMMON TYPE OF VOR INSTALLATION

there is a cone of confusion where the vertical polarization results in unusable signals. This cone should be less than 45°, but in most cases it is closer to 30°.

3.1.2 The Use of the VOR/DME Information

The information received from a VORTAC station is the bearing referred to magnetic north and the slant range to the station. This is often denoted as theta and rho, respectively. The use of this information can be categorized as follows:

a) Theta - radials ∿ Gives only a line of position

b) Theta - theta ∿ The bearing to two VOR stations are measured simultaneously. This rather cumbersome triangulation was used before the widespread implementation of DME. Because of the extensive computation involved and the poor accuracy obtained, this method will not be given further consideration.

c) Rho - theta ∿ Using VOR/DME signals from only one station, a position fix relative to that station is obtained without any computation. This is the simplest way of using the VOR/DME system and the method used by the commercial airlines today.

d) Multiple rho ∿ Because the range determined is much more accurate than the bearing at some distance from the VORTAC station, various multiple rho systems have been proposed:

Rho-rho ∿ The range information from two VORTAC stations will give ambiguity in the position fix which can be resolved by using the bearing to one of the VORTAC stations. One concept referred to as "cross range position determination" has been simulated in [15]. Here one receiver determines a position fix using a VORTAC station in the flight direction, while a second set tuned to an "off airway" VORTAC station determines another fix which, by use of a maximum likelihood filter, enables computation of the best position estimate. It is further claimed that by using only H-category VORTAC stations, almost complete coverage is obtained in the U.S. when crossing angles from 60° to 120° are employed.

Rho-rho-rho ∿ Here three simultaneous DME signals are used to determine a fix. This method can give a very accurate fix, but it results in frequent switching between VORTAC stations. The coverage, especially at low altitudes, will also in effect be reduced, a range limitation determined by the most distant VORTAC station. Another problem which can develop in the high traffic density areas, if multiple rho systems become common, is the possibility of saturation of the DME transmitters. The weakest interrogation will then be disregarded.

3.1.3 The Airways

Under IFR conditions an aircraft is not allowed to fly an arbitrary path between take-off and destination but has to follow airways defined by the F.A.A. These airways are now primarily based on radials to and from VORTAC stations. The desire to fly straight lines between airports together with the congestion along TH more traveled airways and terminal areas has led to the development of area navigation equipment. The F.A.A. has now followed up with a proposal of special area navigation (RNAV) airways [11]. The suggested total errors, which include errors in RNAV equipment, airborne VOR/DME equipment, VORTAC ground station, and pilot imprecision, are:

Less than	51 n.m. from VORTAC station:	Total error less than 4 n.m.
At	25 n.m. from VORTAC station:	Total error less than 2 n.m.
At	10 n.m. from VORTAC station:	Total error less than 0.85 n.m.

To reduce the effect of slant-range error, the F.A.A. has proposed that at 18,000 feet altitude the centerline of the RNAV route should not be closer than 8 n.m. from the VORTAC station, and at 18,000 to 45,000 feet, no closer than 15 n.m. The extension of the airway structure with the RNAV airways for en route and terminal area navigation (final approach also proposed), increases the usefulness of the VOR/DME system for V/STOL aircraft.

3.2 Some Control Parameters of the VOR/DME System

3.2.1 Data Rate

The bearing information is obtained by measuring the relative phase between two 30 H_z sinewaves, yielding a continuous measurement of bearing to the VOR station.

The range is found by measuring the round trip travel time of a pulse from aircraft to ground station and return. The data rate for the DME system is 15 pulse pairs per second corresponding to a sampling rate of the slant distance to the VORTAC station of 15 times per second.

The time required for an initial VOR fix is determined by the time it takes to tune in a new VOR station, the response time, plus a possible delay due to a holding mechanism. Reference [5] claims typical times of 2 minutes not including tuning for the VOR. This figure, however, will depend upon receiver design. The initial DME fix time is about 3 sec.

3.2.2 Maneuver Induced Effects

When orbiting a VOR station in the same direction as the rotating antenna, the frequency of the signal from the rotating antenna changes from $30H_z$ to $(30-V/2\pi R)H_z$, where V is the aircraft velocity and R is the distance to the station. Flying at 300 kt one n.m. from the VOR station gives a frequency shift of 0.013 H_z. Using an OBI servo to measure the changing bearing, the phase of the received reference $30H_z$ signal will be changed such that the phase difference between the shifted reference and the signal from the rotating antenna is zero, i.e., $_{ref} - A \sin (2\pi 30t \pm \phi) = A \sin (2\pi 30 \pm V/R)t$, since $\theta = \frac{V}{R}\,t$, where θ is the instantaneous VOR bearing. Errors will therefore not be created as long as the bandwidth of the receiver and the servo is sufficient.

Traveling along the radials the Doppler effect will give identical frequency shift on both reference and signal and therefore no phase errors develop. The frequency shift for this flight path is negligible.

The transit time of the DME pulses at maximum distance is about 2 msec, corresponding to a traveled distance of 1 ft at 300 kt aircraft speed, i.e., a negligible effect. The maximum traveled distance between each sample at 300 kt is 34 ft, which is at least one order of magnitude less than rated accuracy.

3.2.3 Transfer Function of the Instruments

Most VOR receivers today have an interval RC filter to reduce the effects of scalloping and roughness of the course. The time constant of this filter could be on the order of a few seconds. In a new digital VOR receiver [26], two phase-lock loops have been used having a second order transfer function with a natural frequency chosen equal to 0.3 H_z and a relative damping of 1 which gives adequate damping of erroneous information. This receiver type will be used by the F.A.A. to check ground stations.

The present VOR and DME receivers do not possess bandwidth limitations which will degrade an external optimum filter.

3.2.4 Some VOR/DME Features

A "To-From" indicator on the VOR display simplifies the navigation procedure for the pilot. When the OBS (the desired VOR radial selected by the pilot) agrees (within ±90°) with the measured VOR radial, the indication is "From." When the indicator shows "To," the phase difference between VOR bearing and selected heading is 180° (±90°). When the pilot flies over the VOR station on a manually selected heading, the indicator flips from "To" to "From."

The edge of the cone of confusion above the VOR station, where the VOR signal becomes unusable, can be found by sensing the rate of change of the VOR signal. A heading memory mode of operation can then be initiated.

When signal loss occurs because of station malfunction or at long ranges, a warning indicator is activated and the CDI output goes to zero.

3.3 Desription of VOR Errors

3.3.1 System Errors

Some of the most important VOR system errors can be categorized as follows:

- Calibration errors
- Station errors
- Site and terrain effects
- Propagation errors
- Receiver processing errors

These errors result partly from current practice and regulations, nonideal equipment and inherent limitations of the VOR system. Because the largest errors are caused by the currently used airborne equipment [27], [28], [29], a quantitative description of the errors are difficult. Greatly improved airborne equipment has been demonstrated, so the total error experienced can improve considerably. This development is also expected to have an influence on the station calibration regulations.

To get a background for the numbers given later, a short description of the above mentioned error sources will be given:

a) System Calibration

Station specification: as specified in the Flight Inspection Manual [25], the alignment of the radials, each identified by its nominal magnetic bearing from the station, should be within 2.5° of the correct magnetic bearing, measured out to a maximum of 40 miles from the station. Normal practice is to adjust the radials such that this error is less than 1° (1σ) [30]. The mean error of the network of VOR stations is essentially zero. A $\pm 1.0°$ station shift is typically allowed.

Receiver specifications: Part 91.25 of Federal Air Regulations specifies the airborne receiver during a flight test to be within 6° of a geographic reference, within 4° in a ground check, while a bench check should give an error less than 3.0°. Reference [30] claims that 0.6° (1σ) under operating conditions is a more realistic error value. ICAO, Annex 10 [2] specifies a maximum of ±2° error in the airborne equipment.

b) Station Errors

Polarization effect: this is caused by the radiation of undesired vertical polarized signals and can give errors during aircraft banking. Reference [25] specifies less than 2° at 30° bank angle.

Reference frequency: since $60H_z$ power line frequency is used as a reference for the VOR station, significant frequency variations have been noticed. Step changes can occur, especially when switching from main power to auxiliary power takes place [29].

Nonideal transmitters and antenna design: the transmitters are to some extent dependent upon line voltage changes and temperature changes. Step changes can be experienced when the transmission is switched from one transmitter to another. Of some importance is the rotating $30H_z$ parasitic pattern which cannot be removed by the receiver.

c) Site and Terrain Effects

The reflection caused by fixed obstables in the VOR coverage region can cause deviation of the beam. These deviations are normally categorized as follows:

Bends: slow, flyable excursions of the ideal course. Reference [25] specifies that they should not exceed 3.5° of the correct course.

Roughness: rapid, irregular, non-flyable excursions of the course which have to be averaged out.

Scalloping: rhythmic, non-flyable excursions of the course. Reference [25] says that the deviation caused by the combination of roughness and scalloping must be less than 3° from the average course. The maximum momentary displacement which can be tolerated is 6.5°.

The Airlines Electronic Engineering Committee claimed in 1966 that in most cases the course perturbations have an almost zero mean value and that true course shifts are rare.

The course perturbations can vary from VOR station to VOR station and also differ from one sector to another. The most difficult sites have now been equipped with DVOR which is much less sensitive to site location and reflections from the terrain.

d) Propagation Errors

VHF reception is affected very little by ionospheric and atmospheric conditions. This should enable day, night, all weather, static-free bearing measurements. The stability of the VOR radials has been studied extensively at Ohio University [27], [28], [29], using fixed position receivers. Their findings can be summarized as follows:

• Precipitation has probably no deleterious effect on VOR accuracy. A realistic bound for the worst meterological model appears to be about 0.2°. A maximum of 0.2° offset for approximately 10-15 minutes has been noted.

• Reflection from other aircraft. Aircraft passing near a VOR station can give rise to an impulse-like course deflection as large as 1° at an aircraft in a far field line of sight position. Below the line of sight to the VOR station, errors up to 3° have been measured. (The actual magnitudes of the errors depend upon the receiver used). The magnitude and number of impulses was a direct function of the ratio of the magnitude of the reflected signal to that received by direct transmission. Most deflections lasted for less than 3 seconds, although durations of 20 seconds were experienced. Up to 40 impulses on a single day were reported for that particular VOR station. The effect of reflections from aircraft is expected to be reduced for DVOR stations.

e) Receiver Processing Errors

In the above mentioned stability study [27], [28], [29], it was found that the major contribution to the total VOR error is due to inadequate receiver processing. Some of the findings were:

- Sensitivity to frequency changes in the 30 Hz VOR signal. From 0.6° to 3° per 1 Hz was found for different types of receivers. This effect normally appeared as a steplike error. By proper design this error could be made negligibly small for normally occurring frequency changes.

- Sensitivity to signal strength variations. This is of significance at very low signal levels only. The resulting errors have the shape of short-period variations (0.01 Hz found on one recording), or diural drift. This sensitivity could cause an error up to 1°, depending upon the receiver used.

- Sensitivity to low frequency amplitude modulation caused by multipath. Spurious signals caused by reflections from other aircraft could in one "quality" receiver give rise to an error 5 times as large as in a similar receiver from another company.

- Noise level at low signal strength. This receiver-produced noise could be up to 0.5° peak to peak in one receiver while barely recognizable in another.

- Nonlinear receiver elements can introduce distortion in the detection process such that the detector system does not give the correct average value when exposed to roughness and scalloping.

- Interchannel modulation which can also be reduced by proper receiver design [31]. By use of a split channel receiver, this effect was reduced and the scalloping errors were thereby reduced by a factor of 4-10.

From the discussion of the points a) to f) it is clear that the inherent capability of the VOR system is only limited by the propagation errors which are found to be 0.2° (3σ) using ideal equipment. It is expected that the VOR station equipment will improve gradually, but a significant performance improvement of the network as a whole for the next decade is not very likely, although the worst sites probably will be improved or replaced by DVOR. The airborne equipment for V/STOL, with highly improved performance is or will soon be available. Most of the current improvements have been obtained by using standard receivers with additional equipment to filter the data. Development of improved receivers have also been reported [31], [26]. It is believed that a total error of less than 0.5° max to 1.5° (95% of the time) with the existing VOR system can be achieved.

Transitions between VOR stations can cause severe steplike changes in the indicated course. Flying between two stations separated by 100 n.m., a step in the indicated lateral position of 1 3/4 n.m. can occur when switching from the outbound radial of one VOR to the inbound radial of the next (when both stations have a one degree error).

3.3.2 Measurements Taken on the Existing VOR System

Most measurements have been performed by flying radials out to about 40 miles from the VOR station or by orbiting the station at 5, 20, or 40 miles radius.

A large amount of measurements have been made and the published results [2], [32], [33] normally specify the errors as maximum errors, 95% probability of occurrence, or as r.m.s. errors. Any computations of the PSD functions for the errors registered during these measurement series have not been performed. It is somewhat difficult to evaluate the older results because the many receiver deficiencies found recently [29] could be the cause of some of the errors found. Some of the more interesting results found in the literature are listed below.

From reference [2]:

- 6,355 observations in 1955 of overall VOR system errors gave 1.6°, 1σ.

- Measurements of 276 stations (made before 1958) gave 1°, 1σ, with an error distribution very close to a Gaussian distribution.

- 196 stations measured in 1960 revealed 0.76°, 1σ.

- Large numbers of tests in 1960 with quality receivers gave 0.7°, 1σ.

Reference [33] reports a standard deviation, including the pilot errors, found by measuring aircraft position along certain airways to be 3.3°, with a mean value of 1.6°, 1σ.

Reference [32] measured the stability of the VOR stations during a two-year interval. The discrepancies found were generally less than 0.2°, with a maximum of 0.5° for some radials.

Many references quote numbers varying from 1°, 1σ, to 1.7°, 1σ for the overall system errors.

An inquiry to the F.A.A. recently resulted in the following numbers:

- Ground station error: 0.9°, 1σ
- Airborne receiver error: 1.0°, 1σ

Measurements made on DVOR stations indicate smaller errors, from 0.3°-0.7° maximum.

These results are typical examples of errors quoted in the literature. Interesting results are expected to be revealed by the F.A.A. as a result of a test program where high quality, specially designed equipment [26] is used to measure the accuracy of the VOR/DME system.

3.4 VOR Statistics

A statistical description of the VOR system based on the data found in the literature cannot be too detailed or accurate. When looking at the data received from one station only, it should be possible to split the VOR bearing error up into a mean value and a random varying component fitted to a lowpass filtered white noise model. Some of the known features of the VOR system cannot be included in the statistics but can hopefully be used in a practical data processor.

The parameters of the model are expected to depend upon whether the aircraft flies along the VOR radial or is crossing the radials, as would be common when using area navigation equipment. Error data for the latter case can be obtained from orbital flight recordings.

It is assumed that the angular errors are independent of the distance from the VOR station. No dependency has been observed by inspection of available recordings or has been discussed in the literature except for flights below the line of sight or at extreme distances from the station where the signal strength is very low (See section 3.3.1, items d and e), and for high altitude operations where interference can occur.

3.4.1 The Mean Value of the Angular Errors

a) The VOR Network

The mean value of the angular errors of the VOR network as a whole is zero.

b) The Receiver

The receiver can have a bias caused by calibration inaccuracy. In a letter from the Butler National Corporation it is stated that a properly calibrated airline receiver can be trimmed to an offset less than 0.25°. This requirement is much lower than that required by regulations (Section 3.3.1, point a). Assuming that high quality, regularly calibrated receivers will be required for V/STOL area navigation, a realistic assumption is:

$$E[\gamma_{RCV}] = \beta_{RCV} = 0.3° \qquad (3.2)$$

c) The mean value of a VOR radial error must be less than 2.5°,(3.3.1a). In the above mentioned letter from Butler, VOR stations having alignment errors as large as 1° were found. It is expected that the station misalignments will be reduced due to improved measurement instrumentation now available. The reported stability of the VOR stations (Section 3.3.2) is good, and an alarm is activated when a 1° shift occurs. A probable mean value valid for the VOR radials is then:

$$E[\gamma_{STra}] = 0.7°, \quad 1\sigma \qquad (3.3)$$

d) VOR Station Alignment Error Determined from an Orbital Flight

The mean value of the angular errors obtained by orbiting a VOR station is expected to be smaller than that found from flying radials since the deviations of the radials caused by reflections are likely to give positive as well as negative contributions when going around the VOR. On the other hand, the specifications given for the VOR station could allow a maximum 2.5° turn of the entire station. In an actual flight only a part of an orbit will be flown, such that bends in one sector could give rise to a mean value. Errors with periods up to 360° can be seen on some orbital plots. It is probably not worth while to distinguish between the mean value expected for radial flights and flights along parts of an orbit so that the mean value suggested is:

$$E[\gamma_{STor}] = E[\gamma_{STra}] = \beta_{ST} = 0.7°, \ 1\sigma \qquad (3.4)$$

3.4.2 The Standard Deviation of the Random Angular Error in the Signal from a VOR Station

The standard deviation of the random perturbations of the VOR radials is of course not dependent upon the flight direction. The major angular errors are caused by scalloping, beam roughness and beam bends. Instabilities in the propagation seem to be negligibly small. Because the magnitude of the beam reflection effects are highly dependent upon the receiver design, no attempt will be made to relate the errors to the receiver or to the station. Using data measured with high quality receivers (See Section 3.3.2), together with the specifications given for the station (Section 3.3.1, point a) the following value seems reasonable for the standard deviation of the random angular error component:

$$(E[\gamma(t)^2])^{1/2} = \sigma_\gamma = 0.9° \tag{3.5}$$

Equations (3.2), (3.4), and (3.5) yield the following standard deviation for the overall VOR system error to be expected when quality receivers are used and a large number of stations are measured.

$$\sigma_{\gamma t} = (\beta_{RCV}^2 + \beta_{ST}^2 + \sigma_\gamma^2)^{1/2} \approx 1.2° \tag{3.6}$$

The individual error sources are independent. It can also be assumed that the angular error distribution function is Gaussian (Section 3.3.2).

3.4.3 The P.S.D. Error Function for a VOR Station

Disregarding any range limitation offsets in the random angular error, there is reason to believe that the statistics can be described by the following exponentially correlated function:

$$E[\gamma(t)\gamma(t+\tau)] = \sigma_\gamma{}^2 e^{-\omega_2|\tau|} = \phi_{\gamma\gamma}(\tau) \tag{3.7}$$

This corresponds to the following P.S.D. function:

$$\Phi_{\gamma\gamma}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_{\gamma\gamma}(\tau)e^{j\omega\tau}\,d\tau = \frac{\sigma_\gamma^2}{\pi\omega_2}\,\frac{1}{1+(\omega/\omega_2)^2} \tag{3.8}$$

which can be looked upon as low pass filtered white noise. A P.S.D. function which is flat below ω_2 assumes that the reflections from the terrain give rise to scalloping and bends with no preferred frequency due to the irregularity of the terrain. Objects such as a fence, buildings, power lines, and trees which cause much of the perturbations, are rather regular in shape, but their distance and direction to the

VOR station can be assumed random. Inspection of actual data taken from radial and orbital flights strengthen the assumption that no special frequency band is dominating below a maximum occurring frequency denoted ω_2.

Because most of the VOR errors are geometrically fixed, ω_2 will be proportional to the aircraft velocity.

a) Flying Along a Radial

By inspection of a limited number of recordings, the scalloping frequency seems to be independent of the distance from the VOR station. The maximum scalloping frequency of importance for a recording said to be typical [17] for a conventional VOR station, is $0.17\text{-}0.34 \times 10^{-3}$ r/s/kt. Reference [15] indicates the maximum dominant scalloping frequency to be approximately 0.12×10^{-3} r/s/kt. Reference [18] has found that during their flight tests that the maximum scalloping frequency is higher than $0.7\text{-}2.5 \times 10^{-3}$ r/s/kt. A measurement of a DVOR radial [31] gave 4.1×10^{-3} r/s/kt. This higher frequency can be ascribed to the DVOR design, references [34] and [35]. In the Flight Inspection Manual [25], a typical plot for bends, scalloping, and roughness is shown, indicating the scalloping frequencies to be 16-20 times higher than the bends. By restricting the bank angles to be less than 10° for en route flights at 145 kt (DC-3), one can compute the maximum flyable bend frequency to be 0.1×10^{-3} r/s/kt 20 miles from the VOR. This indicates possible scalloping frequencies to be on the order of $1.6\text{-}2 \times 10^{-3}$ r/s/kt. The discrepancies in the measured values could be due to the "natural" spread in the VOR network or perhaps reflect the different sampling rates or smoothing filters used in the receivers. (A 20 second time constant in the low pass filter would give a 0.35×10^{-3} r/s/kt frequency bandwidth when a 145 kt aircraft is used to collect the data.) Considering only conventional VOR stations (DVOR amounts to less than 3% of the total number of stations), a realistic value along the radials could be:

$$\omega_{2ra} = 0.7 \times 10^{-3} \text{ r/s/kt} \tag{3.9}$$

b) Orbital Flights

The scalloping frequency experienced is expected to depend upon the radius of the orbit. For example, misaligned radials or bends caused by reflections near the VOR stations, will give an indicated frequency equal to:

$$\omega = \frac{0.1}{R\phi_p} \text{ [r/s/kt]} \tag{3.10}$$

where

R - radius of orbit in n.m.

ϕ_p - period of scallops as seen on a 360° error plot, degrees

As the radius is increased, new objects may cause reflection phenomema and thereby weaken this inverse proportionality law to some extent. Reference [36] shows orbital measurements made at different radii. The predominant scalloping period remains almost unaltered at 20 ft, 6 n.m., 12 n.m., and 20 n.m., confirming the above equation. Some higher frequencies of lower amplitudes show up at the greatest distance. The result of a theoretical description of the scalloping frequency valid for orbital flights is also shown for scallops caused by nondirectional reflection from a single source, as:

$$\omega_{or} = 1.2 \frac{d}{R}|\sin \psi| \quad [r/s/kt] \tag{3.11}$$

where

d - distance from VOR to reflector

R - distance from VOR to aircraft

ψ - angle between aircraft and reflecting object seen from VOR station

This equation also gives a frequency inversely proportional to the radius of the orbit. A directional reflecting source seems to give a frequency which can be calculated from (3.10) with ϕ_p = 360° according to orbital measurements shown in the same reference. Many examples of measurements taken while orbiting the VOR station are shown in the literature. Most of them are made with a "sampling rate" of 10° along the orbit and thus leave out the fine structure of the course deviation. (Scallops with 20° period give 0.25×10^{-3} r/s/kt at a radius of 20 n.m. and 1×10^{-3} r/s/kt at 5 n.m. radius.)

A high resolution chart for a 20 mile orbital flight [34] revealed maximum frequencies at a particular site of

$$5.7 \times 10^{-3} \text{ r/s/kt}$$

This site was characterized as useless with conventional VOR due to some nearby towers causing this high frequency reflection (The use of (2.11) gives five times the measured frequency). A high resolution plot in reference [32] gave a shortest scalloping period of approximately 10° for repeated measurements of two different VOR stations.

There is reason to believe that a 10° period can be used as a practical upper limit, which by use of equation (3.10), gives the maximum frequency of the scallops during an orbital flight. The best estimate then becomes:

$$\omega_{2or} = 10/R(\text{n.m.}) \times 10^{-3} \text{ r/s/kt} \tag{3.12}$$

Bounds on R can now be set. Minimum values of interest for parts of an orbital flight are given by the specifications on the RNAV airways (Section 3.1.3), which yields:

$$\omega_{2or\ max}\ R_{min} \begin{cases} 1.25 \times 10^{-3} \text{ r/s/kt below 18,000 ft altitude} \\ 0.67 \times 10^{-3} \text{ r/s/kt above 18,000 ft altitude} \end{cases}$$

The maximum value of R is given by the separation of the VOR stations. In terminal areas and along high density airways, values of R greater than 40 n.m. would be very rare. For en route flights close lane specifications for RNAV airways is given up to 51 n.m. Thus:

$$\omega_{2or\ max}\ R_{max} \begin{cases} 0.25 \times 10^{-3} \text{ r/s/kt terminal} \\ 0.2 \times 10^{-3} \text{ r/s/kt en route} \end{cases} \tag{3.12b}$$

As was remarked in connection with equation (3.10), the maximum scalloping frequencies which could be expected are higher than those given by (3.10) at longer distances.

3.4.4 Suggested Statistical Description of the Angular Error Signal from a VOR Station

The statistical model derived in the preceeding paragraphs is valid for flights within the coverage of a VOR station. Comparison of the values estimated for flying along the radials with the values for flying across the radials shows only a moderate difference in the estimated maximum perturbation frequencies. This result encourages the approximation of using only one model valid for all flight directions with respect to the VOR station. This will, to a great extent, simplify the mechanization of the optimal filter. Because of the spread and incompleteness in the data used to derive the statistical parameters, such a simplified model will probably give as good a result as that obtained using a more complicated model.

The proposed statistical description of the overall VOR system error, valid within the coverage of one station is:

Mean value:

$$E[\gamma_t] = \beta_t = (\beta^2_{RCV} + \beta^2_{ST})^{1/2} = (0.3^2 + 0.7^2) = 0.75° \ (1\sigma) \tag{3.13}$$

The parameters of the random component, equation (3.7):

$$\sigma_\gamma = 0.9° \qquad (3.5),\ (3.14)$$

$$\omega_2 = 0.7 \times 10^{-3} \text{ r/s/kt} \tag{3.15}$$

This model can be depicted as follows:

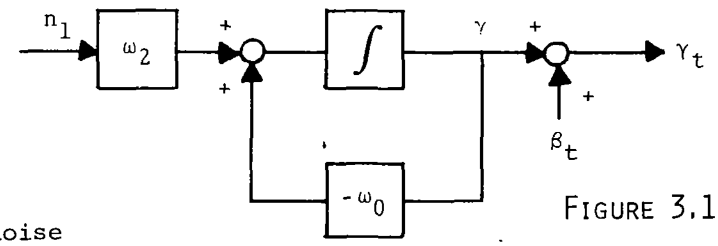


FIGURE 3.1

n_1 - white noise

γ_t - the total angular error in the VOR signal

Using state space notation, this can be written as:

$$\begin{bmatrix} \dot{\gamma} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\omega_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma \\ \beta_t \end{bmatrix} + \begin{bmatrix} \omega_2 \\ 0 \end{bmatrix} n_1 \tag{3.16}$$

where

$$\gamma_t = [1 \quad 1] \begin{bmatrix} \gamma \\ \beta_t \end{bmatrix} \tag{3.17}$$

and

$$E[n_1(t)n_1(t+\tau)] = Q_2\ \delta(\tau) \tag{3.18}$$

The P.S.D. function for this white noise is:

$$\Phi_{n_1 n_1}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Q_2 \delta(\tau) e^{j\omega\tau}\, d\tau = Q/2\pi$$

Passing this white noise through a low-pass filter yields the following P.S.D. function:

$$\Phi_{\gamma\gamma}(\omega) = |H_1(j\omega)|^2\ \Phi_{n_1 n_1}(\omega) = \frac{Q_2}{2\pi} \frac{1}{1 + (\omega/\omega_2)^2}$$

Comparing this expression with (3.8) finally gives:

$$Q_2 = 2\sigma^2_\gamma/\omega_2 \tag{3.19}$$

Using the above proposed figures, (3.19) becomes:

$$Q_2 = 2.3 \times 10^{-3} (\text{degr})^2/(\text{r/s/kt}) \tag{3.20}$$

3.4.5 Statistical Model for the VOR Network Angular Error

The total time an aircraft at 300 kt uses a particular VOR station will vary from 7 to 12 minutes for proposed RNAV airways. Flying any particular radial will take half that time. It is therefore of interest to describe the VOR network as a whole.

Denoting the average time an aircraft uses a VOR station as T_O, we have that:

$$\frac{T_O}{R_O} = 3.6 \text{ sec/kt} \tag{3.21}$$

where R_O is the average distance travelled in n.m. using one VOR station. Examining the angular offsets of the VOR stations, it is seen that they will be sensed as a pulse train with an average pulse width of T_O and an amplitude described by the Gaussian distribution given in Section 3.4.1. The mean of the VOR stations can be regarded as uncorrelated. Assuming for the moment that the time each station is used is equal, the expectation becomes:

$$E[\beta_{ST}(t)\beta_{ST}(t+\tau)] = \sigma_\beta^2 \left(1 - \frac{|\tau|}{T_O}\right) \tag{3.22}$$

where σ_β^2 is the variance of the station offsets (3.4). Comparing this result with an exponentially correlated function:

$$\sigma_\beta^2 e^{-|\tau|/T_O} = \sigma_\beta^2 (1 - |\tau|/T_O + . . .), \tag{3.23}$$

equation (3.22) can be said to be an approximation of (3.23). Because the actual flight times using a particular VOR have a distribution with T_O as a mean value, the approximation is improved. The expectation of the angular error of the VOR network can then be described as:

$$E[\beta_{ST}(t)\beta_{ST}(t-\tau)] \times \sigma_\beta^2 e^{-\omega_O|\tau|} \tag{3.24}$$

where

$$\omega_O = 1/T_O = \frac{0.28}{R_O} \times 10^{-3} \text{ r/s/kt}$$

A value of R_O found from the proposed RNAV airways between New York and Boston and between New York and Washington, D.C. [13] is:

$$R_O = 45 \text{ n.m.} \tag{3.25}$$

giving:

$$\omega_o = 0.006 \times 10^{-3} \text{ r/s/kt}$$

yielding T_o = 9 minutes for a 300 kt aircraft.

The limited flight time within the coverage of a VOR station imposes a lower limit on the possible frequency of the random varying angular error sensed from each station. Because the bends and scallops have a zero mean, the maximum nominal period of the bends is:

$$T_1 = 3.6\ R_o \times 10^3 \text{ s kt}$$

or: (3.26)

$$\omega_1 = \frac{1.74}{R_o} \times 10^{-3} \text{ r/s/kt}$$

Using the above found value of R_o yields:

$$\omega_1 = 0.039 \times 10^{-3} \text{ r/s/kt}$$

The simplest model for this case is obtained by approximating the random components as white noise filtered by a bandpass filter given by the equation:

$$H_1(s) = \frac{s/\omega_1}{(1 + s/\omega_1)(1 + s/\omega_2)}$$

Here ω_1 is given by (3.26) and ω_2 by (3.15). The P.S.D. function of the filtered noise then becomes:

$$\Phi_{11}(\omega) = \frac{(\omega/\omega_1)^2 \qquad N_1}{(1 + (\omega/\omega_1)^2][1 + (\omega/\omega_2)^2]}$$

where N_1 is the P.S.D. of the unfiltered white noise.

The variance can now be found as:

$$E[\gamma(t)^2] = \frac{1}{j}\int_{-j\infty}^{j\infty} H_1(s)H_1(-s)\ N_1 ds = 2\pi N\ I$$

where I is a tabulated integral in [42]

This gives:

$$E[\gamma(t)^2] = \pi N_1 \frac{\omega_2}{1 + \omega_1/\omega_2} = \sigma_\gamma^2$$

where σ_γ is given by (3.5). The value of the N_1 then becomes:

$$N_1 = \frac{\sigma_\gamma^2}{\pi\omega_2}(1 + \omega_1/\omega_2) \approx \sigma_\gamma^2/\pi\omega_2 \tag{3.27}$$

since when ω_2 (3.15) is compared with ω_1 (3.26), a ratio of about

18:1 is found.

The effect of extending the frequency range of the P.S.D. function from ω_1 to zero for the random component will now be evaluated. Adding a lowpass filtered white noise component, n_2, which is assumed uncorrelated with n_1, to the bandpass filtered white noise, we get:

$$\Phi_{22}(\omega) = \frac{(\omega/\omega_1)^2 \quad N_1}{(1 + \omega^2/\omega_1^2)(1 + \omega^2/\omega_2^2)} + \frac{(\omega/\omega_3)\ N_2}{1 + \omega^2/\omega_3^2} \tag{3.28}$$

where

N_2 = P.S.D. of $n_2 = \sigma_2^2/\pi\omega_3$

ω_3 = breakpoint of lowpass filter

For $N_1 = N_2$ and $\omega_1 = \omega_3$ and assuming $\omega_2 >> \omega_1$, (3.28) becomes:

$$\Phi_{22}(\omega) = \frac{N_1}{1 + \omega^2/\omega_2^2} \tag{3.29}$$

The assumed standard deviation of the added lowpass filtered noise signal can be found from the assumption of $N_1 = N_2$ to be:

$$\sigma_2 = \sigma_\gamma(\omega_1/\omega_2)^{1/2} \tag{3.30}$$

Using the values quoted in equations (3.26), (3.14), and (3.15) gives:

$$\sigma_2 = 0.19°$$

Using R_O = 45 n.m. and a speed of 300 kt, the shortest fluctuation period becomes from (3.26):

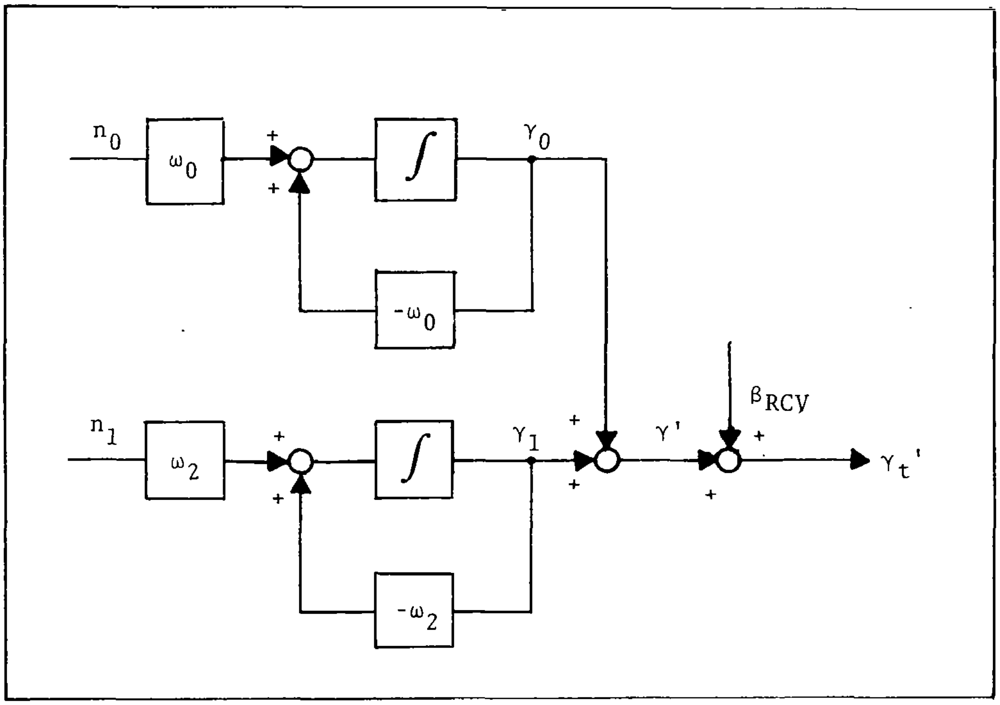
$$T_{P3} = 2\pi/\omega_3 = 9.3 \text{ minutes}$$

Such low frequency noise can enter the system from the following sources:

- Receiver bias changes due to temperature or supply voltage fluctuations
- Drift in the 30 Hz VOR frequency, which can give low frequency components when a group of VOR stations are connected to the same power station.

The values indicated above would be reasonable values for these error sources. The bandpass filtered white noise model can therefore be replaced by a lowpass filtered white noise model.

The proposed statistical description of the VOR network then becomes:



n_0
ω_0
+
+
$\int$
γ_0
$-\omega_0$
β_{RCV}
n_1
ω_2
+
+
$\int$
γ_1
+
+
γ'
+
+
γ_t'
$-\omega_2$

FIGURE 3.2

γ_t^1 - total angular error in the VOR network

γ_O, γ_1 - state variables

β_{RCV} - constant bias in VOR receiver (3.2)

ω_O - highest "bias chopping" frequency (3.24)

ω_2 - highest scalloping frequency (3.15)

Using state space notation:

$$\begin{bmatrix} \dot{\gamma}_O \\ \dot{\gamma}_1 \\ \dot{\beta}_{RCV} \end{bmatrix} = \begin{bmatrix} -\omega_O & 0 & 0 \\ 0 & -\omega_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma_O \\ \gamma_1 \\ \beta_{RCV} \end{bmatrix} + \begin{bmatrix} \omega_O & 0 \\ 0 & \omega_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} n_O \\ n_1 \end{bmatrix} \tag{3.31}$$

$$\gamma_t^1 = [1 \quad 1 \quad 1] \begin{bmatrix} \gamma_O \\ \gamma_1 \\ \beta_{RCV} \end{bmatrix}$$

$$\begin{aligned} E[n_O(t)n_1(t+\tau)] &= 0 \\ E[n_O(t)n_O(t+\tau)] &= Q_O\delta(\tau) \\ E[n_1(t)n_1(t+\tau)] &= Q_2\delta(\tau) \end{aligned} \tag{3.32}$$

where

$Q_2 = 2.3 \times 10^3 (\text{degr})^2/(\text{r/s/kt})$, see (3.20)

$Q_O = 2\sigma_\beta{}^2/\omega_O$, similar to equation (3.19)

Using (3.24) and (3.25) for ω_O, and (3.4) for σ_β yields:

$$\begin{aligned} Q_O &= 3{,}500\ R_O (\text{degr})^2/(\text{r/s/kt}) \\ Q_O &= 160 \times 10^3 (\text{degr})^2/(\text{r/s/kt}) \end{aligned} \tag{3.33}$$

These results for the VOR network can also be summarized as the sum of two uncorrelated random angular error signals having the following expectations:

$$E[\beta_{ST}(t)\beta_{ST}(t+\tau)] = \sigma_\beta{}^2 e^{-\omega_o|\tau|}, \text{ see (3.24)}$$

$$\sigma_\beta = 0.7°, \text{ see (3.4)} \tag{3.34}$$

$$\omega_o = 0.006° \times 10^{-3} \text{ r/s/kt nominal, (3.25)}$$

$$E[\gamma(t)\gamma(t+\tau)] = \sigma_\gamma{}^2 e^{-\omega_2|\tau|}$$

$$\sigma_\gamma = 0.9°, \text{ see (3.14)}$$

$$\omega_2 = 0.7 \times 10^{-3} \text{ r/s/kt, see (3.15)}$$

In addition, a constant receiver bias with the following standard deviation is added:

$$\beta_{RCV} = 0.3° \quad \text{See (3.2)}$$

The total r.m.s. error then becomes:

$$\sigma^1_{\gamma t} = (\sigma_\beta{}^2 + \sigma_\gamma{}^2 + \beta^2_{RCV})^{1/2} = 1.2°,$$

identical to (3.6) as presumed.

3.4.6 Statistical Models for the VOR System Described in the Literature

In references [17] and [37] an exponentially correlated model was derived from a limited number of F.A.A. recordings. Their findings were:

$$E[\gamma(t)\gamma(t+\tau)] = \sigma^2 e^{\beta|\tau|}$$

with:

$$\sigma = 1.1°$$

$$\beta = 0.2 \times 10^{-3} \text{ r/s/kt}$$

The bias of the VOR station and the receiver were set to zero.

Simulations made at Boeing [16] made use of the following model:

Bias in the VOR system: 1°, 1σ.
The random part of the VOR signal: 1°, 1σ.
No frequency limits were mentioned.

Reference [15] made use of a very short average time when describing the angular errors used for their simulations:

Mean value: $1° + 5° \sin \omega t$, $\omega = 0.12 \times 10^{-3}$ r/s/kt.
Standard deviation: Varying from 0.5° to maximum 4° with the same frequency given above.

Other models have been found in the survey. The above models show a satisfactory similarity with the model proposed in this report.

3.5 Description of the DME Errors

The most important DME error sources are:

- Pulse rise time and pulse distortion
- Calibration of fixed delays
- Frequency stability
- Receiver processing errors

Because only the leading edge of the pulse is used for timing, the system is virtually immune to multipath caused by reflection.

Reference [32] claims that the potential accuracy of the current system is ±0.5μsec. corresponding to ±0.084 n.m.

The specification given by the Flight Inspection Manual [25] says that the accuracy of the DME shall be within 3% of the distance or 0.5 mi, whichever is greater. In the older manual [38], 0.2 mi. or 2% of the distance was specified.

3.5.1 Observed DME Errors

Some measurements of the DME system accuracy are reported in the literature.

The DME transmitter:

Reference [2] has found the error to be ±0.033 n.m. 1σ. Reference [30] concludes that the TACAN station error is 0.03 mi., 1 Reference [39] states a 0.27 n.m. maximum error which is said to correspond to a 1σ value of 0.29 x 0.27 = 0.077 n.m.

DME receivers:

Reference [2] quotes ±0.1 n.m. ±0.2% of range for the airborne equipment. Reference [40] states that the accuracy of the receivers have improved from about 5% of distance to about 0.2 mi. independent of distance. A comparison between the distance indicated by two DME receivers in the Eastern program [13] showed a discrepancy of up to 0.25 n.m., which was within the stated accuracy of ±0.2 n.m. for the receivers. The error seems to be caused by bias type errors in the receivers.

The overall DME system:

Reference [2] concludes that the system error is 0.1 n.m., 1σ.

Reference [30] says that typical distance error is ±0.1 mi. to ±0.2 mi. Other references use ±0.2 n.m. or 1% of range describing the errors as very low frequency random error.

3.5.2 DME Statistics

The main error source is probably uncertainties in the fixed delays in the receiver and the DME station, giving rise to a bias type error. It is expected that this bias will vary slowly with time due to component drift and temperature and power supply changes. There is reason to believe that the DME station bias exhibits only small changes in the time interval an aircraft is using the station. Distribution of the leading edge caused by noise can give rise to a higher frequency error component. Because of the 15 samples per sec, paragraph 3.2.1, the maximum information frequency would be about 30 rad/sec.

a) The r.m.s. Errors

The estimated standard deviation of the receiver based on the above mentioned references is:

$$(E[\alpha^2_{RCV}])^{1/2} = \sigma_{\alpha RCV} = 0.1 \text{ n.m., } 1\sigma \qquad (3.35)$$

A quality receiver is assumed so that the distance dependency can be disregarded.

The station error is estimated to be:

$$(E[\alpha^2_{ST}])^{1/2} = \sigma_{\alpha ST} = 0.1 \text{ n.m., } 1\sigma \qquad (3.36)$$

The difficulty in these estimations is due to the great diversity in the DME transmitter specifications and in the reported accuracies.

The overall r.m.s. error thus becomes:

$$\sigma_{\alpha t} = (\sigma^2_{\alpha RCV} + \sigma^2_{\alpha ST})^{1/2} = 0.14 \text{ n.m., } 1\sigma \tag{3.37}$$

b) <u>The Expectation of the DME Errors</u>

From the data available it is only possible to guess at the expectation of the DME errors. Provided that the station bias can be regarded as slowly varying, the same bias steps flying from one station to another, as described in paragraph 3.4.5, can be expected. This would give the following expectation for the DME station network:

$$E[\alpha_{ST}(t)\ \alpha_{ST}(t+\tau)] = \sigma^2_{\alpha ST}\ e^{-\omega_o|\tau|} \tag{3.38}$$

where

$\sigma_{\alpha ST} = 0.1$ n.m. See (3.36)

$\omega_o = 0.006° \times 10^{-3}$ r/s/kt nominal. See (3.25)

The fluctuations in the receiver bias can also be predicted to be in the same frequency range (but independent of aircraft speed). A rough estimate of the model for the total DME error is:

$$E[\alpha(t)\ \alpha(t+\tau)] = \sigma^2_{\alpha}\ e^{-\omega_o|\tau|} \tag{3.39}$$

$\sigma_\alpha = 1.4$ n.m. See (3.37)

Assuming V = 300 kt, $1/\omega_0$ becomes 530 seconds. The recording shown in reference [13] does not depict any high frequency noise content, indicating that the above model may have some relevance.

The DME error equations can then by written as:

$$\dot{\alpha} = -\omega_o\alpha + \omega_o n_D$$

$$E[n_D(t)\ n_D(t+\tau)] = Q_D\delta(\tau)$$

Similarly to equation (3.19), this yields:

$$Q_D = 2\sigma^2_\alpha/\omega_o = 7 \times 10^3 (\text{n.m.})^2/(\text{r/s/kt}) \tag{3.40}$$

where

n_D - white noise

c) Statistical Models for the DME Found in the Literature

Reference [37] has used the following model:

$$E[\alpha(t)\ \alpha(t+\tau)] = (0.2\ \text{n.m.})^2\ e^{-1|\tau|}$$

This autocorrelation function was based on a rather limited amount of data. An inquiry to the author has revealed that the assumed autocorrelation time for the DME station was 300 sec. Reference [16] uses a DME error of 0.1 n.m. 2σ in their simulations. In [15] a DME error varying from 0.02 n.m. to 0.11 n.m. with a nominal value of 0.07 n.m. was used for their simulation.

3.6 Optimum Use of the VOR/DME Data

3.6.1 Results from Programs Described in the Literature

In the Eastern STOL program [13] the VOR/DME information was fed to the Omnitrac Computer together with heading, airspeed data and altitude. The computer did the following data reduction:

- Correction of the slant range error in the DME signal.
- Mathematical filtering of VOR signal which significantly reduced the VOR bearing error.
- A dead reckoning mode obtained by use of true airspeed data and magnetic bearing. Wind speed was then estimated. If sensor signal was lost, the computer provided its normal outputs using airspeed, compass heading, and the most recent stored value of the wind speed.

The measured errors in the VOR/DME position fix in the terminal area was less than 0.2 n.m., 1σ. These results were probably achieved without use of Kalman filtering, but probably with use of the dead reckoning data.

Reference [15] has obtained in their simulations a considerable improvement in the position errors derived from the VOR/DME signals with use of a maximum likelihood filter but without a dead reckoning system. The error reduction is achieved by using an additional VORTAC station located "off-airway". For a transcontinental flight simulation, the following results were published:

Mean error:	±0.11 n.m. across track
	±0.083 n.m. along track
Standard deviation:	±0.072 n.m.

(Except for a few intentionally bad selections of VORTAC station pairs.)

The interesting simulations performed in reference [17] and [37] processed the VOR/DME data together with the data from an I.N.S. and an air speed-magnetic heading dead reckoner, respectively. It was assumed that the I.N.S. had a velocity error of 1.4 n.m. per hour and that the VOR/DME signal could be described by

$$E[\gamma(t)\ \gamma(t+\tau)] = (1.1°)^2\ e^{-0.1|\tau|}$$

for the VOR and 0.2 n.m. r.m.s. error in the DME. Using a six dimensional state vector and a sampling time interval of five seconds, an improvement in the position error from 2% of the distance to the VORTAC station to about 0.4% of the distance was achieved. Using the air-speed-magnetic heading dead reckoning, the r.m.s. accuracy in the position is better than that of the VOR/DME by a factor of 2.5.

The Vector Analog Computer developed by Bulter National Corporation [18] makes use of a distance proportional filter. In this filter an OBI servo in the VAC has a speed limitation which is inversely proportional with the distance to the VORTAC station. The speed limits selected corresponds to the maximum angular rotations of the VOR bearing an aircraft can experience at a given speed. Thus the servo speed does not depend upon the amplitude of the course deviations, but only upon the polarity of the CDI output from the VOR receiver. The filter equation is:

$$\frac{d\theta}{dt} = \frac{V\ \text{(Aircraft Speed)}}{D\ \text{(Distance)}}, \quad D > 0.5\ \text{n.m.}$$

$$\frac{d\theta}{dt} = \frac{V}{D = 0.5\ \text{n.m.}}, \quad D < 0.5\ \text{n.m.}$$

The receivers which are presently on the market have an internal filter. The effect of this filter is removed by a passive network in the VAC.

It is said that this simple filter technique makes an improvement in the VOR accuracy of an order of magnitude and enables the use of so-called unusable VOR signals.

3.6.2 Some Considerations When Designing an Optimal Filter

a) The VOR/DME Position Fix

The VOR/DME signal gives a position fix relative to the VORTAC station with the following r.m.s. errors:

Across the direction to the VORTAC station:

$$\pm R\sigma_{\gamma t} \approx \pm 0.021\ R(\text{n.m.})$$

Along the direction to the VORTAC station:

$$\pm\sigma_{\alpha} = 0.14\ \text{n.m.}$$

These errors are shown in Figure 3.3 as a function of the distance from the station.

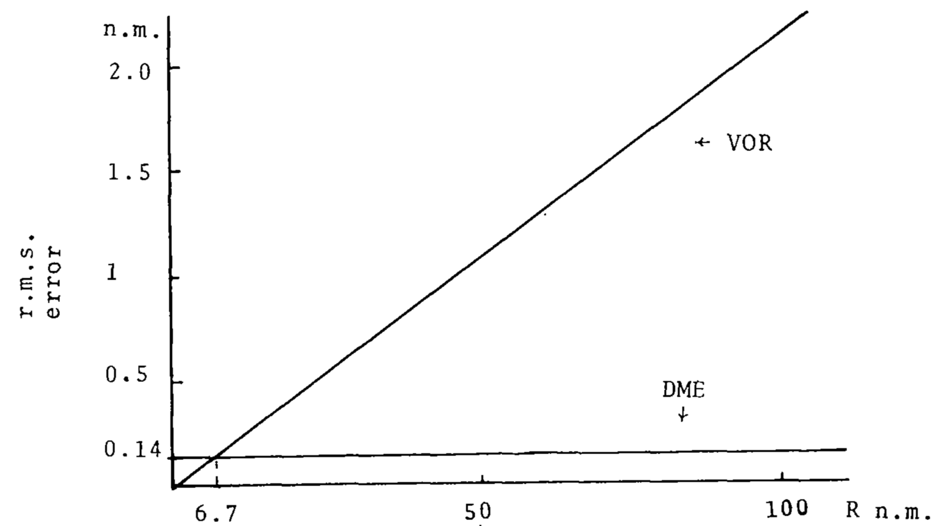


FIG. 3.3 VOR-DMS POSITION FIX ERRORS

Using the average range found for certain RNAV-airways, the mean VOR error is 0.95 n.m. At ranges greater than 10 n.m. the DME error is much less than the VOR error. A valuable property is that $E[R\gamma(t)\alpha(t+\tau)]=0$ and that the errors appear orthogonal to each other.

In the navigation computer the latitude, longitude, and angle between magnetic and geographic north for each VORTAC station have to be stored. Denoting the VOR bearing with respect to geographic north as θ, the VOR/DME position fix transformed to the geographic coordinate frame becomes:

$$\Delta r_x' = R'\cos\theta' \qquad (3.40)$$

$$\Delta r_y' = R' \sin\theta'$$

where

$\Delta r_x'$ = component of R' along the north axis

$\Delta r_y'$ = component of R' along the east axis

The "prime" indicates a measured value. Now the measured values can be written as the correct value plus an error term:

$$R' = R + \alpha$$

$$\theta' = \theta + \gamma_t$$

$$\Delta r_N' = \Delta r_N + \delta r_{VDN}, \qquad \Delta r_N = R \cos \theta$$

$$\Delta r_E' = \Delta r_E + \delta r_{VDE}, \qquad \Delta r_E = R \sin \theta$$

Using small angle approximation and neglecting products of uncertainties, Eq.(3.40) then gives:

$$\begin{bmatrix} \delta r_{VDN} \\ \delta r_{VDE} \end{bmatrix} = \begin{bmatrix} -R \sin \theta & \cos \theta \\ R \cos \theta & \sin \theta \end{bmatrix} \begin{bmatrix} \gamma_t \\ \alpha \end{bmatrix} \qquad (3.41)$$

b) Example of a Filter Design

Similar to the system described in [37], position is measured simultaneously by the INS and the VOR/DME. Denoting the position vectors indicated respectively as

$$\underline{r}_I = \underline{r} + \delta\underline{r}_I$$

$$\underline{r}_{VD} = \underline{r} + \delta\underline{r}_{VD}$$

Where $\underline{r}$ is the correct position vector, an error free comparison between the two measurements gives:

$$\underline{m}' = \underline{r}_I - \underline{r}_{VD} = \delta\underline{r}_I - \delta\underline{r}_{VD} \tag{3.42}$$

That is, $\underline{m}'$ depends only upon the errors in the INS and VOR/DME system.

As an example, a very simple model of the INS uncertainties is used, consisting of two uncoupled channels with white noise at the accelerometer level (random walk gyro drift). The noise is integrated twice to get the position uncertainty. The VOR model valid for the VOR network is used, Eq. (3.31). The state equation then becomes:

$$\begin{bmatrix} \dot{\gamma}_0 \\ \dot{\gamma}_1 \\ \dot{\beta}_{RCV} \\ \dot{\alpha} \\ \delta\dot{V}_N \\ \delta\dot{r}_{IN} \\ \delta\dot{V}_E \\ \delta\dot{r}_{IE} \end{bmatrix} = \begin{bmatrix} -\omega_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\omega_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \beta_{RCV} \\ \alpha \\ \delta V_N \\ \delta r_{IN} \\ \delta V_E \\ \delta r_{IE} \end{bmatrix} +$$

$$+ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ n_\alpha \\ n_{IN} \\ n_{IE} \end{bmatrix} \tag{3.43}$$

or $$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{G}\,\underline{n}$$

By subtracting at the discrete time intervals the VOR/DME signals from the INS signals and using equation (3.41), we get:

$$\begin{bmatrix} m_N \\ m_E \end{bmatrix} = \begin{bmatrix} R \sin\theta & R \sin\theta & R \sin\theta & -\cos\theta & 0 & 1 & 0 & 0 \\ -R\cos\theta & -R\cos\theta & -R\cos\theta & -\sin\theta & 0 & 0 & 0 & 1 \end{bmatrix} \underline{x}(t_m) + \underline{u} \tag{3.44}$$

or

$$\underline{m}(t_m) = \underline{H}(t_m)\,\underline{x}(t_m) + \underline{n}$$

where $\underline{u}$ is a gaussian distributed noise representing an erroneous comparison of the INS and VOR/DME data:

$$E[\underline{u}\ \underline{u}^T] = \underline{u} = \begin{bmatrix} \sigma_u^{\ 2} & 0 \\ 0 & \sigma_u^{\ 2} \end{bmatrix} \tag{3.45}$$

An estimate of σ_u is 0.05 n.m. Now

$$E[\underline{n}] = E[\underline{u}] = 0$$

and

$$E[\underline{n}(t)\underline{n}(t+\tau)^T] = \underset{\sim}{Q}\,\delta(\tau) \qquad (3.46)$$

where

$$\underset{\sim}{Q} = \begin{bmatrix} Q_0 & & & 0 & \\ & Q_2 & & & \\ & & Q_\alpha & & \\ & & & Q_I & \\ & 0 & & & Q_I \end{bmatrix}$$

and

$$Q_0 = 48.5(\mathrm{rad})^2/(\mathrm{r/s/kt}) \quad \text{from} \quad (3.30)$$

$$Q_2 = 0.7(\mathrm{rad})^2/(\mathrm{r/s/kt}) \quad \text{from} \quad (3.20)$$

$$Q_\alpha = 7 \times 10^3(\mathrm{n.m.})^2/(\mathrm{r/s/kt}) \quad \text{from} \quad (3.40)$$

$$E[n_{IN}(t)n_{IN}(t+\tau)] = Q_{IN}\delta(\tau) = Q_{IE}\delta(\tau) = Q_I\delta(\tau)$$

By choosing a sampling time interval, paragraph 3.6.2 c, the information needed to solve the discrete Kalman filter equation for estimation of the VOR/DME and INS errors is at hand. These estimates can be applied as corrections to the measured quantities.

The H matrix is time varying because some of the entries depend upon the measured VOR bearing, θ, and range R. Becasue R and θ are not known a priori as functions of time, the gain and covariance matrices in the Kalman filter equation can not be precomputed.

The introduction of the $\underline{u}$ noise source in the measurement equation is somewhat artificial, because the errors introduced in the electronics could be made negligibly small. In his paper [43], Kalman solved the estimation problem for a discrete-time system driven by white noise where all measurements were assumed noise free. The method described by Bryson and Johansen, [44], and Bryson and Henrikson, [45], should be considered.

c) The Sampling Time Interval

The simulations performed in [17] and [37] made use of a 5-second sampling time interval. Reference [16] used both 10 and 1 seconds. With a sampling time interval T_s = 5 seconds, the following theoretical maximum signal frequency ω_s, can be reproduced:

$$\omega_s = \frac{2\pi}{2T_s} = 0.63 \text{ r/s}$$

The highest scalloping frequency at 300 kt is:

$$\omega_2 = 0.7 \times 10^{-3} \times 300 = 0.21 \text{ r/s}$$

The highest course aberration frequencies will therefore be preserved in the sampled measurement. This means that prefiltering of the measured signals should not be necessary to prevent the frequency folding effect from arising when sampling signals with frequencies above ω_s.

The distance traveled in 5 seconds at 300 kt is 0.42 n.m. This would indicate that the signal should not be averaged between the samples because of the lag introduced. From the above considerations, 5 seconds seems to be a reasonable choice of sampling time interval.

d) Some Practical Considerations

The performance of the INS will probably not be based upon the sampling time interval only, but also upon the duration of possible loss of VOR/DME signals. When flying over a VOR-station, the cone of confusion may cause unusable signals for a time lasting for maximum: (Ref. Para. 3.1.1.)

$$T_{cone} = \frac{2h \tan 35}{V} = 0.6 \frac{h(ft)}{V(kt)} \text{ seconds}$$

for h = 10,000 ft and V = 300 kt this time becomes 2.0 seconds. The edge of the cone of confusion can be sensed, 3.2.4, and a program change signal can be derived. A signal loss warning can also occur, 3.2.4. This may happen occasionally, and the system has to be switched to a dead reckoning mode while another VOR station is selected. The availability figure of the VORTAC stations has been found to be 98.7% [33], giving an indication of the occurance of signal loss. Flying at low altitudes in metropolitan areas, shadows can cause temporary signal loss. Sporadic signal loss can also be expected in the DME signal due to saturation which occurs when more than two airplanes interrogate a DME station simultaneously.

A very important bit of information which should be used in an optimum filter design, is the knowledge of the maximum VOR bearing rate which can occur. This rate is given by:

$$\dot{\theta}_{max} = \frac{V}{R} |\sin \psi|$$

where

V = aircraft speed

R = range

ψ = 90° when flying an orbital flight, 0° flying a radial.

Butler National Corp. experienced a great improvement using this information with $\sin \psi = 1$ in their design of the VAC.

The fact that the VOR derived position error increases linearly with range will be taken into account when writing the equations for the Kalman filter and should result in a weighting function which depends upon the range.

When computing the slant error correction, a flat earth approximation can be used. At 100 n.m. this will give an error of 50 ft (increasing with the cube of the distance).

Two models were developed for the VOR errors, one valid within the coverage of one station, the other for the network. Using the first model, the filtering process has to be reinitiated every time a new station is used. In the other model the information about the shift of station is used only indirectly in the autocorrelation time $1/\omega_0$. A disadvantage of the second model is that it requires one extra state variable. No preferences will be made here.

Prior to take-off, the initial velocity and position errors in the INS should be negligible, otherwise it can be difficult to distinguish between constant INS position error and VOR bias errors. The measurement of the VOR error at the airport does not give any information about the VOR bias, because when the aircraft is stationary, it is not possible to distinguish between bias, bend, or scalloping. The best initial estimate of the VOR bias is therefore zero. The DME signal, however, should give a value of the bias (slowly varying), but a distinction between receiver bias and transmitter bias can not be made if only one station can be received.

4. THE USE OF VOR/DME IN THE FINAL APPROACH PHASE

The VOR/DME system can be used together with altitude information in the final approach phase. In the Eastern STOL program [13] a curved approach path was followed down to a final way point at 200 ft altitude and 1400 ft from the runway. From there the approach was flown visually. A number of approaches using the Omnitrac computer with VOR/DME and altitude inputs were also flown on instruments down to touchdown. In a flight profile suggested for an improved STOL, the descent is initiated at about 22 n.m. from the airport. A speed of 250 n.m. is assumed down to where the final decent starts at 2000 ft altitude, and 2.3 n.m. from the airport. The final approach speed is 75 kt.

Reference [10] suggests that a PVOR colocated with a DME installed at the STOL or VTOL ports could be used for approach down to the point where visual contact can be made. Because of the line of sight characteristics of the VOR/DME system, some regions in the metropolitan area can have unusable VOR reception conditions. Some airports are equipped with terminal VOR/DME stations, and a few have installed PVOR. According to the Flight Inspection Manual [25], the terminal VOR/DME should have a minimum range of 25 n.m. The approach radial is evaluated from 15 n.m. inbound to a point where missed approach is executed. (The missed approach radial is also evaluated regularly.) The same tolerances on the terminal VOR and DME accuracies as for the enroute VOR/DME system are specified. Some airports suitable for V/STOL are not equipped with VOR/DME facilities, but will be covered by nearby VORTAC stations. FAA will propose RNAV instrument approach procedures [11] to permit use of additional runways under higher landings minima. Measurements will be made to assure good VOR/DME signal coverage from the final approach fix to the minimum descent altitude. The combined RNAV errors should be less than 0.85 n.m. at 10 n.m. range from the VORTAC station.

4.1 Results from Simulations and Demonstration

The approach phase has been simulated in reference [16] using different sensors with an INS as inputs to a Kalman filter. Assuming perfect velocity information and a VOR with 2° (2σ) bias and 2° (2σ) random component, it took 60 seconds to reduce the initial error of 2 n.m. to 0.13 n.m. with 1 sample per second. After 200 seconds the error was reduced to 80 ft. Perfect velocity information and DME with 0.1 n.m. (2σ) gave a reduction from 2 n.m. to 200 ft in 100 seconds. As a conclusion it was stated that the smoothing of VOR/DME data using a 10 second sampling time interval gives accuracies nearly equal to that required of a Category I localizer.

In the simulation of the cross-range position determination system, [15], (model mentioned in paragraph 3.4.6) the cross-track error during approach was found equal to 0.1 n.m. for the mean value and 0.07 n.m. for the standard deviation.

Evaluation of the approaches made by Eastern [13] shows that the VOR/DME errors during final approach were less than in the terminal area. This was probably due to the shorter distance to the VOR station. When using the VOR/DME as an input to the Omnitrac computer, the errors were comparable with the errors experienced when using Decca as an input. With a 90% confidence level the following approximate standard deviations of the cross range errors were found:

At 200 ft altitude:	350-750 ft
At 100 ft altitude:	300-900 ft
At touchdown:	190-600 ft
Along runway centerline:	470-660 ft

4.2 VOR/DME Error Model for the Approach Phase

Because the error tolerances specified for the VOR and DME are the same for the terminal VOR/DME station as for the rest of the network and because the sites and terrain effects should be expected to be simular or in some cases worse because of the presence of towers and hangars, the same error model as described in paragraphs 3.4 and 3.5.2 can be used. The terminal VOR/DME station can be regarded as part of the network, paragraph 3.4.5 or described as in paragraph 3.4.4. Using the figures for the proposed advanced STOL, [13], the terminal VOR/DME station will be used for 5 - 9 minutes, not significantly different from the time each VORTAC station is used while enroute. The obtained accuracy will be better, however, because the terminal VOR station will be used from a distance of about 15-25 n.m. to a final decision height at a range of about 0.23 n.m. The VOR r.m.s. error will then be reduced from about 0.5 n.m. down to 300 ft, while the DME error should be 840 ft r.m.s. independent of range. These figures indicate that theta-rho navigation would give less error than the more complicated rho-rho system in the approach phase where the accuracy requirements are greatest. If an adequate updating of the INS is possible during the relatively short final approach (2-3 minutes) the theta-rho system would also be preferred because of the improved accuracy.

REFERENCES

1. Henney, Radio Engineering Handbook, Chapters 25 and 26. Gives a brief description of almost every type of navigation in use prior to 1958.

2. Bauss (ed), Radio Navigation Systems for Aviation and Maritime Use, MacMillan 1963, product of a committee of AGARD. Good description of common radio navaids, error analysis and comparison of various systems.

3. Sandretto, Aviation Engineering, 1958. Detail description of common radio navaids.

4. Summary of Radio Navigation Systems, University of Michigan Summer Conference 1965. Brief description and accuracy of almost all known radio navaids.

5. A. Cameron, A Survey of Electronic Navigation Report ESD-TR-68-273, 1966. The MITRE Corp., Mass. Brief discussion of radio navaids, good bibliographical listings.

6. T. Vickers, Navigation of V/STOL Aircraft in High-Density Urban Areas

7. A.B. Winick, Air Navigation Trends, Navigation, Vol. 15, No.1, 1968.

8. D.W. Pidwell, Essential Criteria for Low Level Navigation, NACCON 1969.

9. G.A. Filbert, Navigation for General Aviation, Navigation Vol.14, No.4, 1968.

10. J.B. Barriage, STOL and VTOL Air Transportation from the Ground Up, Astronautics & Aeronautics, Sept. 1968.

11. FAA Asks Area Navigation View in July, AW. and ST., June 30, 1969.

12. G. B. Litchford, Air Traffic Control for VTOL Systems. Lecture Notes of a VTOL Seminar at Boeing, Vertol Division.

13. STOL Demonstration Program, Technical Report, Eastern, McDonnell Douglas, March 1969, EAL Report: Flight R&D 68-315, MDC Report: G984.

14. American Amasses STOL Navaid Data, AW and ST, May 26, 1969.

15. L.E. DeGroot, J. Larsen, Extended Capability from Existing Navaids, Navigation, Vol. 15, No.4, Winter 1968-69. The authors are at the Lear Siegler,Inc., Grand Rapids.

16. J.W. Burrows, Combined Radio - Dead Reckoning Navigation Systems, Proceedings of the 10N National Air Meeting on Supersonic Navigation, 15-16 Nov. 1967, Seattle Washington. The author is at the Boeing Corp., Seattle, Wash.

17. N.B. Hemesath, D.H. Meyer, H.M. Schweighofer, Complementing VOR/DME with INS - An Improved Navigation System, Society of Automotive Engineers, National Business Aircraft Meeting, March 26-28, 1969, Paper No. 690338.

18. Introduction to VAC Distance Proportional Filtering, Technical Memorandum 101, Butler National Corporation

19. L. Horowitz, Direct-Ranging Loran, NAECON, 1969, the author is at Teledyne Systems Co., Calif.

20. Doppler System Performance by TWA, IATA 16th Technical Conference, Miami, March 1965.

21. R.N. Brown, Doppler-Inertial Navigation and Where It's Going, Signal, Jan. 1969. Brown is Executive Vice Pres. of Kearfott, Singer General Prec.

22. FAA Studying New Navaid System, AW and ST, July 1969. The The Autonetic Incise system described.

23. B. Winick, An Analysis of Recent VOR Developments, FAA Washington, D.C., IATA 16th Conference, Miami, April 1965, Paper WP-100

24. W.B. Hawthorne, VOR/DME/TACAN Frequency Technology. IEEE Trans. on Aerospace and Navigational Electronics, Vol. ANE-12, No.7, March 1965.

25. Flight Inspection Manual, U.S. Standards, FAA Handbook DA P 8200

26. E.M. Dragan, Steering a Course to Safer Air Travel, Electronics, Nov. 27, 1967, Airborne Instrument Lab., Cutler-Hammer, Inc.

27. R.H. McFarland, Experiments in the Determination of VOR Radial Stability IEEE Trans. on Aerospace and Navigational Electronics, Vol. ANE-12, No.1, March 1965. Completed by VOR Error Tolerance, Stability, and Accuracy, Correspondence in IEEE Trans. in Aerospace and Electronic Systems, May 1967. The author is a professor at Ohio University.

28. R.H. McFarland, J.T. Gorman, T. A. Gutwein, VOR Propagation and Stability Study, Report RD-66-92, November 1966.

29. R.H. McFarland, F.J. Kiko, Results of VOR Path Stability Investigations, Eascon'68 Record.

30. J.S. Prichard, The VOR/DME/TACAN System, Its Present State and Its Potential, IEEE Trans. on Aerospace and Navigational Electronics, March 1965, Vol. ANE-12, No.1.

31. A.C. Marshall et al, A Compatible Precision Doppler VOR System, WP-95, IATA 16th Technical Conference, Miami, April 1965.

32. McKeel, Craymer; VOR/DME/VORTAC Facilities, IEEE Trans. on Aerospace and Navigational Electronics, ANE-12, No.1, March 1965.

33. Measurements of Air Navigation Accuracy Along Certain VOR Airways, FAA report PB-161 652.

34. Flint, Hollin, VOR Improvements in the U.S., IEEE Trans. on Aerospace and Navigational Electronics, ANE-12, No.1, March 1965.

35. Crone, Kramar, Doppler VOR in Europe, IEEE Trans. on Aerospace and Navigational Electronics, ANE-12, No.1, March 1965.

36. Anderson, VHF Omnirange Accuracy, IEEE Trans. on Aerospace and Navigational Electronics, ANE-12, No.2, March 1965.

37. N.B. Hemesath, The Optimum Complementation of VOR/DME with Air Data, AIAA Guidance, Control and Flight Mechanics Conference, Princeton, N.J., Aug. 18-20, 1969.

38. Flight Inspection Manual, U.S. Standard, Dec. 1956

39. Hirsch, TACAN - Compatible DME, IEEE Trans. on Aerospace and Navigational Electronics,ANE-12, No.1, March 1965.

40. S.H. Dodington, Recent Improvements in Today's DME, IEEE Trans. on Aerospace and Navigational Electronics, ANE-12, No.1, March 1965.

41. M. Kayton, W. Fried, Avionics Navigation Systems, Book from J. Wiley and Sons, Inc., 1969.

42. Newton, Gould, Kaiser, Analytical Design of Linear Feedback Controls, John Wiley and Sons, Inc.

43. Kalman, A New Approach to Linear Filtering and Prediction Problems, Trans. ASME, J. of Basic Eng. March, 1960.

44. Bryson, Johansen, Linear Filtering for Time-Varying Systems Using Measurements Containing Colored Noise, IEEE Vol. AC-10 Jan. 1965.

45. Bryson, Henrikson, Estimation Using Sampled-Data Containing Sequentially Correlated Noise, AIAA 1967, Paper No. 67-541

NASA-Langley, 1970 —— 21 CR-1588