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# LIBRATION POINT RENDEZVOUS

T. N. Edelbaum

Analytical Mechanics Associates, Incorporated
Cambridge, Massachusetts

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Final Report

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ANALYTICAL MECHANICS ASSOCIATES, INC.
a subsidiary of
scientific resources corporation

179 Fifth Street
Cambridge, Mass. 02141
617/868-1456

## ABSTRACT

A study of the flight mechanics and mass requirements for one-shot lunar missions has been made utilizing rendezvous at the libration points in front of, and behind, the Moon. The flight mechanics studies were carried out using the restricted problem of three-bodies. It was found that the lowest delta-V requirement to reach either libration point was to go to the $L_2$ point behind the Moon via a powered lunar swingby. It was also found that every point on the lunar surface is directly accessible from the $L_2$ point, for transfer times greater than about 59 hours.

The mass calculations were carried out assuming advanced cryogenic propellants in all stages. For such propellants, the least mass in Earth orbit for rendezvous at the $L_2$ point was determined. The mass requirement was found to be smaller than that for the standard lunar orbit rendezvous mode. Rendezvous at either the $L_2$ point, or in a Halo Orbit about the $L_2$ point, would also have operational advantages, including access to all points on the Moon and an infinite rendezvous launch window.

## ACKNOWLEDGMENT

This study was conducted for the Office of Control Theory and Applications under contract NAS 12-2118 with NASA Electronics Research Center. The author would like to thank the Contract Monitor, Robert W. Farquhar, for inspiring, supporting, and assisting with the study. He would also like to thank Saul Serben for programming the three-body problem and Johan Synnestvedt for running the programs and assisting with the computations.

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## INTRODUCTION

Reference 1 treats the libration points of the Earth-Moon system and their possible utilization in various space missions. Reference 2 provides a good review of the previous literature on the subject and summarizes the author's work in this area. As discussed in Reference 1, relatively little investigation has been made into the advantages of using the libration points of the Earth-Moon system as rendezvous points for lunar missions. The present study is intended to treat in more detail the flight mechanics of transfer trajectories to the libration points and the mass requirements of several of the more desirable mission modes.

## FLIGHT MECHANICS

In choosing a methematical model for the study, it is important to use a model which is sufficiently realistic to include the physical phenomena of interest. At the same time, it is not desirable to use too complex a model because one then spends considerable time calculating small effects of no particular consequence. The present study uses the model of the restricted problem of three bodies in three dimensions. This model is a reasonable compromise for the conflicting requirements mentioned earlier, and is somewhat more complex than models usually used for preliminary mission analyses. The restricted three-body model is necessary in the present study because the two-body and patched-conic models are not sufficiently accurate for these missions.

The equations of motion for the restricted problem are taken from Moulton (Reference 3). These equations are written in a rotating coordinate system whose center is at the barycenter of the Earth-Moon system. The potential in this rotating coordinate system is given by

$$U = \frac{1}{2} \left(x^2 + y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{1}$$

In this equation, $x$, $y$, and $z$ are Cartesian coordinates with the $x-y$ plane as the plane of motion of the Earth and the Moon; $r_1$ and $r_2$ are the distances from the Earth and Moon, respectively. The position of the Earth is at a distance $\mu$ from the origin on the negative $x$ axis, and the Moon is located at a distance $1-\mu$ from the origin on the positive $x$ axis.

The equations of motion are given by

$$\frac{d^2x}{dt^2} - 2\frac{dy}{dt} = \frac{\partial u}{\partial x}$$

$$\frac{d^2y}{dt^2} + 2\frac{dx}{dt} = \frac{\partial u}{\partial y} \tag{2}$$

$$\frac{d^2z}{dt^2} = \frac{\partial u}{\partial z}$$

The known integral of this system of equations is the Jacobi integral which is

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = V^2 = 2U - C \tag{3}$$

Various methods can be used to numerically integrate these equations of motion along a coasting arc. A straightforward integration of the equations as they stand, with constant step size, tends to run into problems, owing to the very large changes in velocity that occur near the singularities. Various methods have been suggested to circumvent this problem, including regularization of the variables, variation of parameters, and the Encke method. A different technique was utilized in this study. The equations of motion, Equations (2), were integrated by a fourth-order Runge-Kutta method using a step size that is inversely proportional to the potential given in Equation (1). This is essentially a numerical approach to regularizing the equations of motion rather than an analytical one. For very small time steps in a two-body problem, this method would be equivalent to integrating the equations of motion using constant increments of eccentric anomaly. A recent study (Ref. 4) has shown that utilization of the method employed in the analysis now being reported is highly advantageous as compared to the utilization of the methods of analytical regularization and constant time steps.

In order to solve the two-point boundary value problem having specified initial and final conditions, and fixed time, a Newton-Raphson iterator with numerically-determined partial derivatives was employed. The computer program was written in double precision and outputs position, velocity, and the value of the Jacobi constant in both rotating coordinates and in Moon-centered and Earth-centered inertial coordinates. To allow for terminal conditions at either fixed positions or fixed periapsis radii, several options were written into the program.

## TRAJECTORIES BETWEEN $L_1$ AND THE MOON

Various conventions have been employed for numbering the libration points. In this study, the libration point between the Earth and the Moon will be referred to as $L_1$, and the libration point beyond the Moon will be referred to as $L_2$. A series of trajectories which leave $L_1$ and just graze the surface of the Moon are illustrated in Fig. 1 in the rotating coordinate system. This figure illustrates the changes in the characteristics of the trajectory as the flight time increases. For short flight times, the velocities are fairly large and the trajectories approach straight lines. As the flight time increases, the velocity decreases, the curvature of the trajectory increases, and these trajectories approximate two-body trajectories in the rotating coordinate system. However, the perturbations due to the Earth on these trajectories are significant, particularly for the longer flight times. The corresponding trajectories from the Moon to $L_1$ may be found by reversing the direction of motion and reflecting the trajectory in the Earth-Moon line so that the positive and negative $y$ axes are interchanged.

There is another family of trajectories which has a larger curvature and which goes around the opposite side of the Moon. One member of this family is illustrated in Fig. 2 along with the corresponding trajectory from the first family for the same flight time of 48 hours. It should be noted that both of these trajectories graze the Moon at almost the same points, although they approach these points from different directions. The two trajectories shown in Fig. 2 may be looked upon as the trace, in the $x$-$y$ plane, of a family of three-dimensional grazing trajectories which completely envelops the Moon. This three-dimensional family of trajectories, for a fixed flight time, resemble similar families of trajectories which were described by Hoelker and Braud in Reference 5 for Earth-Moon trajectories. For this particular flight time, there is a small circular area on the back side of the Moon which cannot be reached by any direct transfer trajectory from $L_1$. For very short flight times, this circular area will approximate the back hemisphere of the Moon. As the flight time increases, this circular area becomes offset from the Earth-Moon line and shrinks in size until, at a flight time of 51 hours, the inaccessible region completely disappears. For flight times of 51 hours and above, all points on the Moon can be reached by direct two-impulse transfers to and from $L_1$. In order to reach a specific point on the Moon from $L_1$, it may be desirable to first enter a low parking orbit around the Moon and then descend from this parking orbit so that direct transfers are generally not necessary. The transfer trajectories used to enter the low parking orbit would be similar to those illustrated, except that they would have a slightly higher perilune radius.

The $\Delta V$ requirements for direct transfers between the Moon and $L_1$ are illustrated in Fig. 3 for the family of trajectories illustrated in Fig. 1. These $\Delta V$ requirements are theoretical impulsive $\Delta V$'s and do not make any allowance for gravity losses due to finite burning times. The additional $\Delta V$'s due to gravity losses would be negligible at $L_1$ but would be significant for the lunar landing maneuver. For the range of times shown, the $\Delta V$ required at $L_1$ decreases monotonically with increasing transfer time, as does the $\Delta V$ required at the Moon. The $\Delta V$ required at the Moon approaches a limiting value which is slightly below escape velocity from the Moon. The velocities at $L_1$ and at the Moon are related by Jacobi's integral, Equation (3). If the velocity is known at either point, the value of Jacobi's constant can be calculated as can the value of the velocity at the other point. If the transfer time is increased above 60 hours, the $\Delta V$ required would pass through a minimum and then increase. This phenomenon is illustrated in Fig. 4 which shows a plot of $\Delta V$ versus transfer time for the second family of trajectories in the Earth-Moon plane. For this family of trajectories the minimum $\Delta V$ occurs at a transfer time of about 58 hours and is more than twice the minimum at $L_1$ for the first family of trajectories. However, $\Delta V$ requirements at the Moon are very similar for the two families. The higher velocities for the second family of trajectories can be explained by looking at Fig. 2. Here it is shown that the path traversed by the trajectory which does not cross the Earth-Moon line is somewhat longer, and traverses a region of lower potential, so that the average velocity will tend to be reduced somewhat. As a result, the members of the second family of trajectories require higher initial velocities in order to reach the Moon within the same time of flight.

These results illustrate the characteristics of the simplest families of trajectories between $L_1$ and the Moon. There are other, more complex, families of trajectories which involve repeated passes by the Moon or which wander around in Earth-Moon space, that are not considered in this study. The families illustrated constitute probably the most important class of trajectories from an astrodynamic viewpoint.

## TRAJECTORIES BETWEEN THE EARTH AND $L_1$

Fig. 5 illustrates two different trajectories between the Earth and the $L_1$ point. The first of these trajectories is a direct two-impulse transfer from a 100-nautical mile parking orbit around the Earth to $L_1$. The second trajectory is an indirect transfer which leaves Earth from the same 100-nautical mile parking orbit, passes within 50-nautical miles of the Moon, then has a second braking impulse about one hour after perilune, and finally brakes to a stop at $L_1$ with a third impulse.

One of the results of this study is that, if the perilune altitude is constrained, the optimum location of the intermediate impulse does not occur at perilune and is not a tangential impulse. The perilune altitude was specified in this study due to interest in a particular mission mode in which the lunar module separates from the command module and descends to the surface while the vehicle is flying by the Moon. Previous work at General Electric has shown that somewhat lower $\Delta V$'s required to reach $L_1$ can be achieved if the perilune altitude is allowed to increase. The GE work uses tangential impulses and does not represent truly optimum trajectories for a given flight time. However, their results are probably close to the true optimum for this class of trajectories.

Fig. 6 shows a plot of the $\Delta V$ requirements for the direct transfer as a function of flight time. The minimum $\Delta V$ to brake at $L_1$ occurs at a flight time of 92 hours and a braking velocity of about 2, 330 feet/second. Because the launch and braking velocities are related by Jacobi's integral, the minimum total $\Delta V$ will also occur at the same flight time. The launch $\Delta V$ at Earth for this trajectory is about 200 feet/second smaller than the minimum launch $\Delta V$

for a direct transfer to the Moon. Fig. 7 shows the total $\Delta V$ required for the indirect transfer to $L_1$ as a function of the time after perilune. This plot is drawn for the total time and also for the time of the Earth-Moon leg that minimizes total $\Delta V$. This plot demonstrates that the minimum $\Delta V$ does not occur at perilune. For the particular case considered, with a constraint on the perilune radius, the total $\Delta V$ for the indirect transfer is somewhat greater than that for the direct transfer to $L_1$. The GE results, without the perilune radius constraints, obtained somewhat lower $\Delta V$'s for the indirect transfer to $L_1$ than for the direct transfer; however, the most important conclusion derived is that the total $\Delta V$'s required to arrive at $L_1$ by either the direct or indirect transfers are in the same ballpark.

## TRAJECTORIES FROM EARTH TO THE $L_2$ POINT OR TO THE HALO ORBIT

Fig. 8 illustrates a direct transfer from Earth into a Halo Orbit. The Halo Orbit is sufficiently close to $L_2$, and its velocities relative to $L_2$ are so low, that it is possible to discuss transfer trajectories to either $L_2$ or the Halo Orbit interchangeably. The $\Delta V$ requirements for both missions are very similar. In addition, it makes very little difference at what point the Halo Orbit is entered. Fig. 9 shows $\Delta V$ versus time for direct transfers from the Earth to $L_2$. It should be noted that the $\Delta V$ required to enter $L_2$ by a direct transfer from Earth is significantly higher than the $\Delta V$ required to enter $L_1$. On the other hand, a previous study by GE (Ref. 6) has shown that indirect three-impulse transfers to $L_2$ significantly reduce the $\Delta V$ requirements, and even require less total $\Delta V$, than either a direct or indirect transfer to $L_1$. The particular trajectory utilized in this study requires a transfer time of 5.36 days between Earth and Moon and an additional 3.23 days between the Moon and $L_2$. It has a tangential braking impulse of 620 feet/second at perilune and a third braking impulse of 472 feet/second at $L_2$. The total $\Delta V$ required to enter $L_2$ from the translunar trajectory is only 1092 feet/second.

## TRAJECTORIES BETWEEN THE HALO ORBIT AND THE MOON

Fig. 10 shows two typical trajectories from the Halo Orbit to the Moon. The trajectories from either the Halo Orbit or the $L_2$ point to the Moon have very similar $\Delta V$ requirements and are generally similar in character to the trajectories from $L_1$ to the Moon. It is possible to reach any point on the Moon by a direct transfer from either the Halo Orbit or the $L_2$ point, for transfer times greater than about 59 hours. Figs. 11 and 12 show the $\Delta V$ requirements as a function of flight time for transfers from the Halo Orbit to the surface of the Moon. The figures are drawn for the two classes of grazing trajectories which pass around both sides of the Moon. The $\Delta V$ required to reach any other latitude on the Moon will be intermediate between the $\Delta V$'s shown for these two classes of trajectories. As was discussed in the section on trajectories from $L_1$ to the Moon, these two trajectories represent the limiting members of a family of three-dimensional grazing trajectories which completely envelop the Moon. They may be regarded as having inclinations to the lunar equator at perilunes of $0^{\circ}$ and of $180^{\circ}$. The trajectories having inclinations at values other than those specified will have intermediate $\Delta V$ requirements.

## MISSION MODES

A number of potential mission modes were considered in the course of the study. From these potential ways of performing lunar missions, four modes were selected for detailed $\Delta V$ and mass comparisons. The $\Delta V$ requirements for the four modes selected are listed in Table 1. The first of these modes is the standard lunar orbit rendezvous mode. In this mode, the lunar module descends from a low altitude circular parking orbit to the lunar surface, and then accomplishes rendezvous with the command module in the same lunar orbit. The allowances for gravity losses and hovering fuel have been made consistent with those in Reference 7. For the lunar orbit rendezvous mode, no allowance has been made for the additional $\Delta V$'s necessary for any required plane changes. Landing sites at high lunar latitudes may require either additional $\Delta V$ or long waiting time for mission completion. The total $\Delta V$'s shown are the $\Delta V$'s that the command module payload and the lunar module payload must be accelerated through. These total $\Delta V$'s give a rough idea of the efficiency of each mission mode. However, actual vehicle staging must be considered to obtain any realistic assessment of mass requirements.

The second mission mode to be considered is very similar to lunar orbit rendezvous, except that rendezvous occurs at the $L_1$ point instead of in lunar orbit. Both vehicles make a direct two-burn transfer from the initial parking orbit to the $L_1$ point. The LEM then descends to the lunar surface. It should be noted that there is no restriction on the latitude of the landing point. The LEM then takes off from the lunar surface and achieves rendezvous with the command module back at the $L_1$ point. It should be noted that there is an infinitely wide launch window for this rendezvous from any location on the Moon. The CSM payload is then placed on an Earth-return trajectory. Table 1 shows that the total $\Delta V$ for the LEM payload is somewhat higher than for lunar orbit rendezvous, while the total $\Delta V$ for the command module payload is somewhat lower than for lunar orbit rendezvous. What is, perhaps, more significant is that the total $\Delta V$ requirement for the lunar landing and ascent is larger than that for lunar orbit rendezvous. This is probably less desirable for one-shot missions such as those considered in this study, but may be more desirable for lunar shuttle missions.

The third mission mode considered is where the lunar module descends at perilune of a close lunar flyby and the command module continues on to the $L_1$ point, using the indirect multiple-impulse transfer. The return to Earth is a mirror image of the outbound trajectory. The command module starts towards the Moon and has a second burn to place it on an Earth-return trajectory about an hour before perilune. The LEM ascends from the lunar surface and accomplishes rendezvous with the command module at hyperbolic energy after perilune. While hyperbolic rendezvous is not a standard maneuver, it should present no particular difficulty, as there is plenty of time for the vehicles to link up on the way back to Earth. This mission mode is not quite as flexible as rendezvous at $L_1$, because landing sites far from the lunar equator may cause operational difficulties. For this mission mode, the LEM payload has somewhat lower $\Delta V$ requirements than those for libration point rendezvous, while the command module has higher $\Delta V$ requirements.

The fourth mode to be considered is where both the LEM and the command module use an indirect, three-burn transfer from Earth parking orbit to the $L_2$ point. The LEM descends to an arbitrary point on the lunar surface. After an arbitrary time has elapsed, the LEM ascends and rendezvous with the command module at the $L_2$ point is achieved. Then the command module returns to Earth along a trajectory which is a mirror image of the outbound trajectory. If it is desired to use the command module for lunar farside communications, it can be placed into a Halo Orbit using about the same $\Delta V$ requirements. This mission mode requires only slightly more $\Delta V$ for the LEM payload than does lunar orbit rendezvous, but has substantially lower $\Delta V$ requirements for the command module payload. It is not only attractive from a performance standpoint, but also offers the flexibility of arbitrary landing sites and an infinite rendezvous launch window, while allowing full communication on both sides of the Moon.

## MASS COMPARISONS

The mass comparisons of this study were done for a somewhat advanced state-of-the-art, and for one-shot missions rather than shuttle missions. The state-of-the-art assumed is consistent with that used in Reference 7 for a study of lunar shuttle missions. The LEM payload was assumed to be 3, 500 pounds, while the command module payload was assumed to be 13, 000 pounds. The Earth-departure stage and the CSM stage were assumed to have 88% propellant and 12% inert mass, while the LEM ascent and descent stages were assumed to have 82% propellant and 18% inert mass. The specific impulse was assumed to be 460 seconds, representing an advanced cryogenic rocket engine. The staging analysis was done in the same fashion as Reference 8. The mass comparisons of the four different mission modes are shown in Fig. 13, which illustrates the masses of each stage and the payload for each mission mode. This figure shows that the two different modes for rendezvous at $L_1$ require somewhat larger masses in Earth orbit than does the standard lunar orbit rendezvous mode. This is partially due to the fact that the LEM, which has a greater mass fraction, must be used for more of the mission in these modes. On the other hand, the mission using rendezvous at $L_2$ requires a smaller mass in Earth orbit than any other mode, even though it suffers from the same disadvantage. The reason for this is that the $\Delta V$ required by the command module is so much smaller that it more than makes up for the inefficiency of using the LEM to accelerate its payload through a larger $\Delta V$ change.

Reference 7, in considering lunar shuttle missions, found that it was desirable to rendezvous in an elliptic lunar orbit instead of the standard circular orbit. The staging and other characteristics of the shuttle mission were such that it was advantageous to trade off command module $\Delta V$ for lunar module $\Delta V$. The net result for elliptic orbit rendezvous is that the LEM payload is accelerated through essentially the same total $\Delta V$, while the command module is accelerated through a smaller total $\Delta V$. In Reference 7, elliptic orbit rendezvous was found to be desirable in spite of the additional $\Delta V$'s required for rendezvous and plane changes. It is to be expected that rendezvous at the $L_2$ point would also be advantageous for lunar shuttle missions, and might even be superior to either elliptic or circular orbit rendezvous.

## CONCLUSIONS

1. Any point on the surface of the Moon can be reached by direct transfers to and from the $L_1$ point, for flight times greater than about 51 hours. The same is true for the $L_2$ point, or for the Halo Orbit, for flight times greater than about 59 hours.

2. The smallest mass requirements of any mission mode considered were for rendezvous at the $L_2$ point. Essentially the same performance could be obtained for rendezvous in a Halo Orbit around the $L_2$ point and this latter mode would allow full-time communication with landing sites on either the near or the far side of the Moon.

3. Rendezvous at $L_2$ also appears highly desirable for lunar shuttle missions, and should be investigated further for such missions.

## SUGGESTIONS FOR FUTURE RESEARCH

Relatively little work has been done on minimum-impulse three-body trajectories. It appears desirable to extend the results of the present analysis by determining whether there are additional multiple-impulse trajectories which require smaller total impulse than those considered herein. One way of doing this is by calculating the primer vector along the trajectories considered to determine whether they are locally optimal. The possible existence of trajectories to $L_1$ that pass in front of the Moon should be investigated as should the use of additional impulses.

Another important area for future research is the use of Halo Orbit rendezvous for lunar shuttle missions. Halo Orbit rendezvous has greater operational flexibility than elliptic orbit rendezvous, and may well have lower mass requirements as well. The lunar shuttle is a promising candidate for the application of Halo Orbit or libration-point rendezvous.

## Table I

## ΔV Requirements (fps)

### Lunar Orbit Rendezvous

| | | | |
|---|---|---|---|
| Translunar injection | | 10,300 | |
| Braking into lunar orbit | | 3,400 | |
| Lunar landing | | | 6,600 |
| Lunar ascent | | | 6,200 |
| Transearth injection | 3,400 | | |
| | CSM | | LEM |
| Total | 17,100 | | 26,500 |

### Direct Transfer to $L_1$, Rendezvous at $L_1$

| | | | |
|---|---|---|---|
| Translunar injection | | 10,100 | |
| Braking at $L_1$ | | 2,300 | |
| Lunar Landing | | | 9,200 |
| Lunar ascent | | | 8,800 |
| Transearth injection | 2,300 | | |
| | CSM | | LEM |
| Total | 14,700 | | 30,400 |

### Indirect Transfer to $L_1$, Hyperbolic Rendezvous at Moon

| | | | |
|---|---|---|---|
| Translunar injection | | 10,300 | |
| Braking to $L_1$ | 2,400 | | |
| Lunar Landing | | | 10,000 |
| Lunar ascent | | | 9,600 |
| Transearth injection | 2,400 | | |
| | CSM | | LEM |
| Total | 15,100 | | 29,900 |

### Indirect Transfer to $L_2$, Rendezvous at $L_2$

| | | | |
|---|---|---|---|
| Translunar injection | | 10,300 | |
| Braking to $L_2$ | | 1,100 | |
| Lunar Landing | | | 9,100 |
| Lunar Ascent | | | 8,700 |
| Transearth injection | 1,100 | | |
| | CSM | | LEM |
| Total | 12,500 | | 29,200 |

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FIGURE 1 - $L_1$ TO MOON FLYBYS

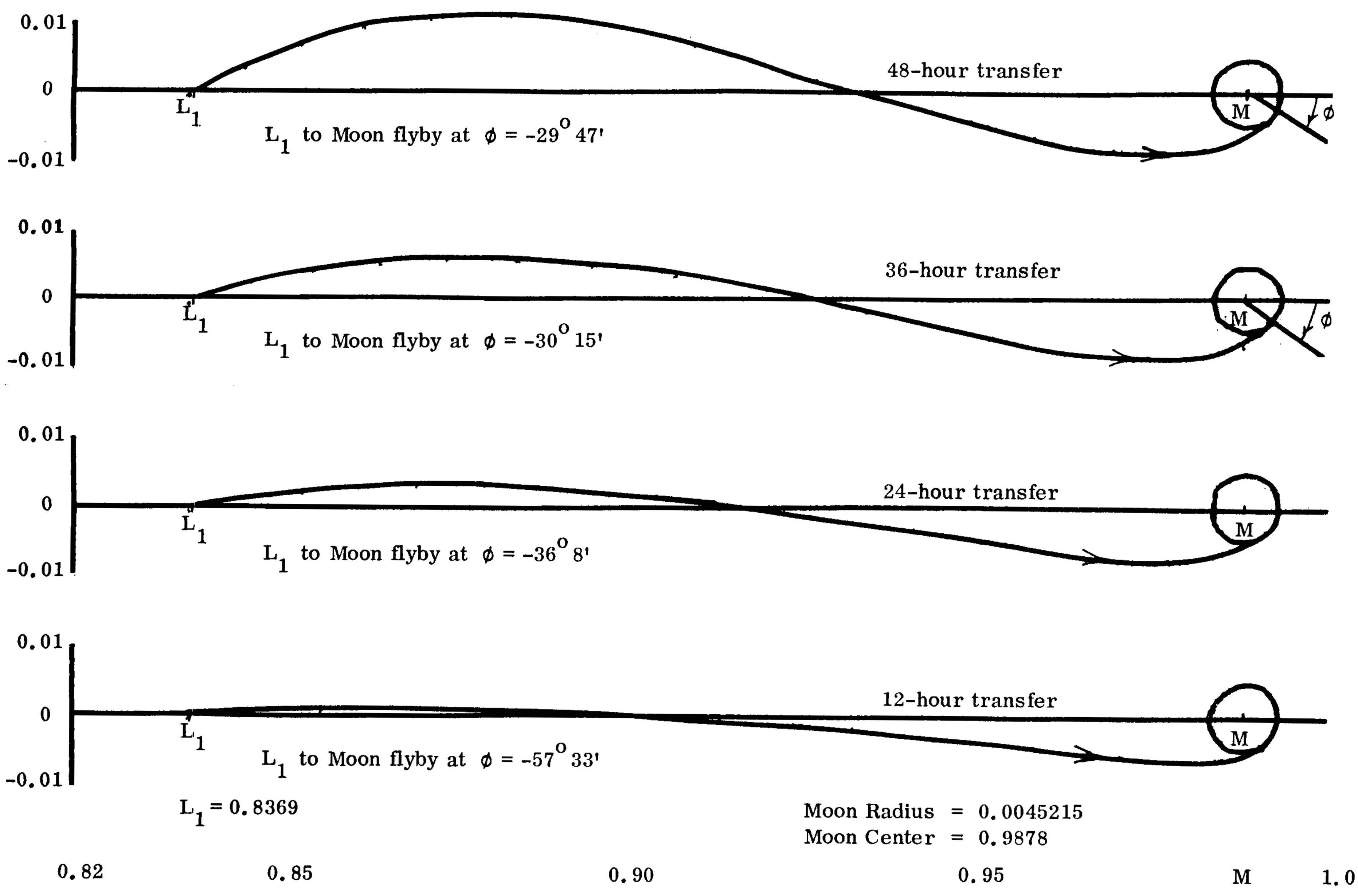


FIGURE 2 - 48-HOUR TRANSFERS

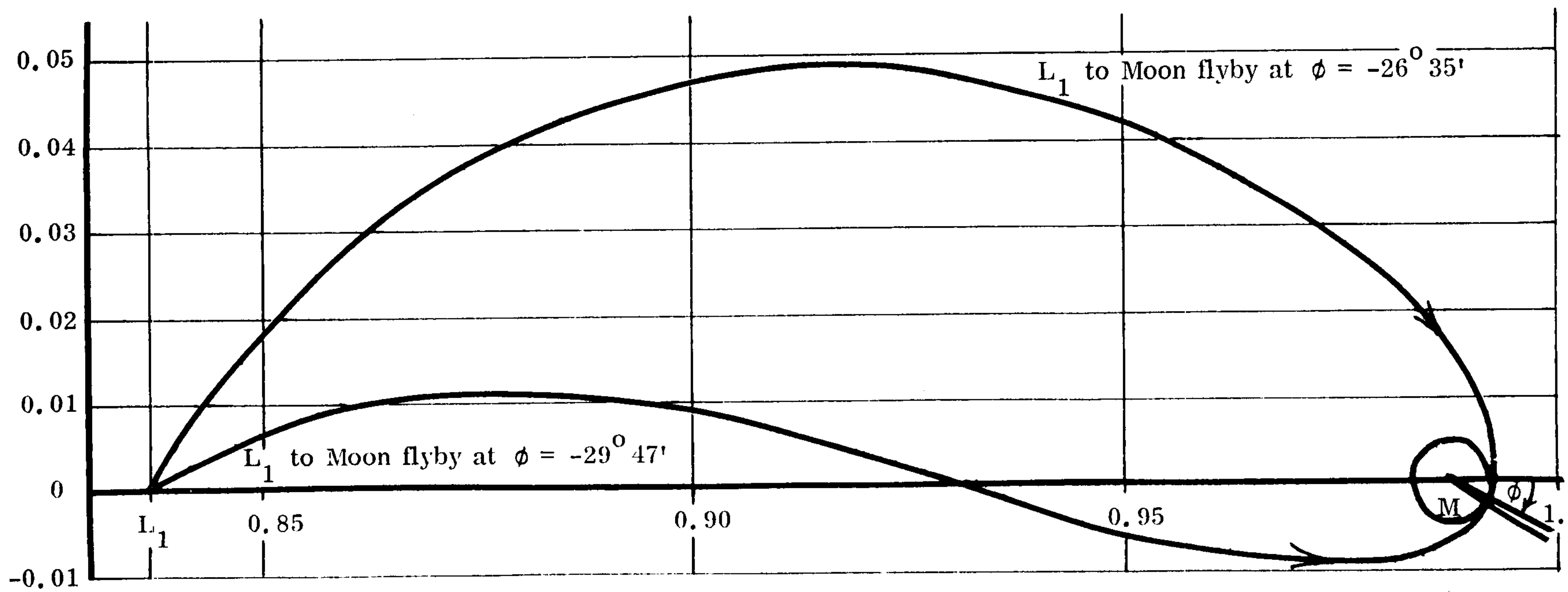


FIGURE 3

$L_1$ TO MOON FLYBYS - AROUND TRAILING SIDE OF MOON

| Transfer Time | ΔV at $L_1$ | ΔV at Moon | Total ΔV |
|---|---|---|---|
| 12 | 4,075 | 8,632 | 12,707 |
| 24 | 1,743 | 7,806 | 9,549 |
| 36 | 968 | 7,670 | 8,638 |
| 48 | 611 | 7,634 | 8,245 |
| 64 | 448 | 7,622 | 8,070 |

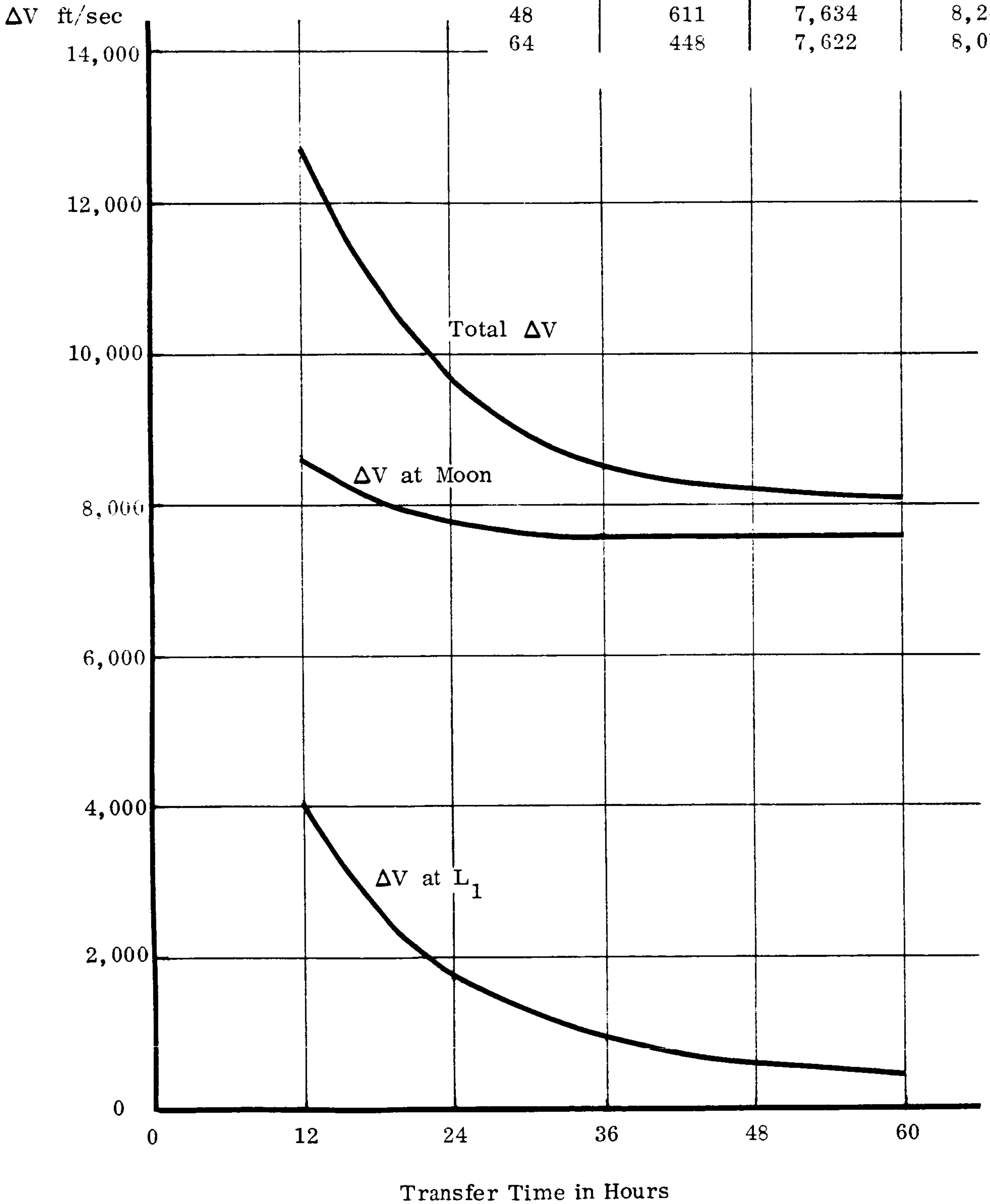


FIGURE 4

$L_1$ TO MOON FLYBYS - AROUND LEADING SIDE OF MOON

| Transfer Time | ΔV at $L_1$ | ΔV at Moon | Total ΔV |
|---|---|---|---|
| 46 | 1014.4 | 7676.3 | 8690.7 |
| 48 | 992.9 | 7674.6 | 8667.5 |
| 50 | 977.7 | 7671.5 | 8649.2 |
| 52 | 966.6 | 7670.1 | 8636.7 |
| 54 | 959.7 | 7669.2 | 8628.9 |
| 56 | 955.9 | 7668.7 | 8624.6 |
| 58 | 954.9 | 7668.5 | 8623.4 |
| 60 | 956.4 | 7668.8 | 8625.2 |
| 62 | 960.1 | 7669.3 | 8629.4 |

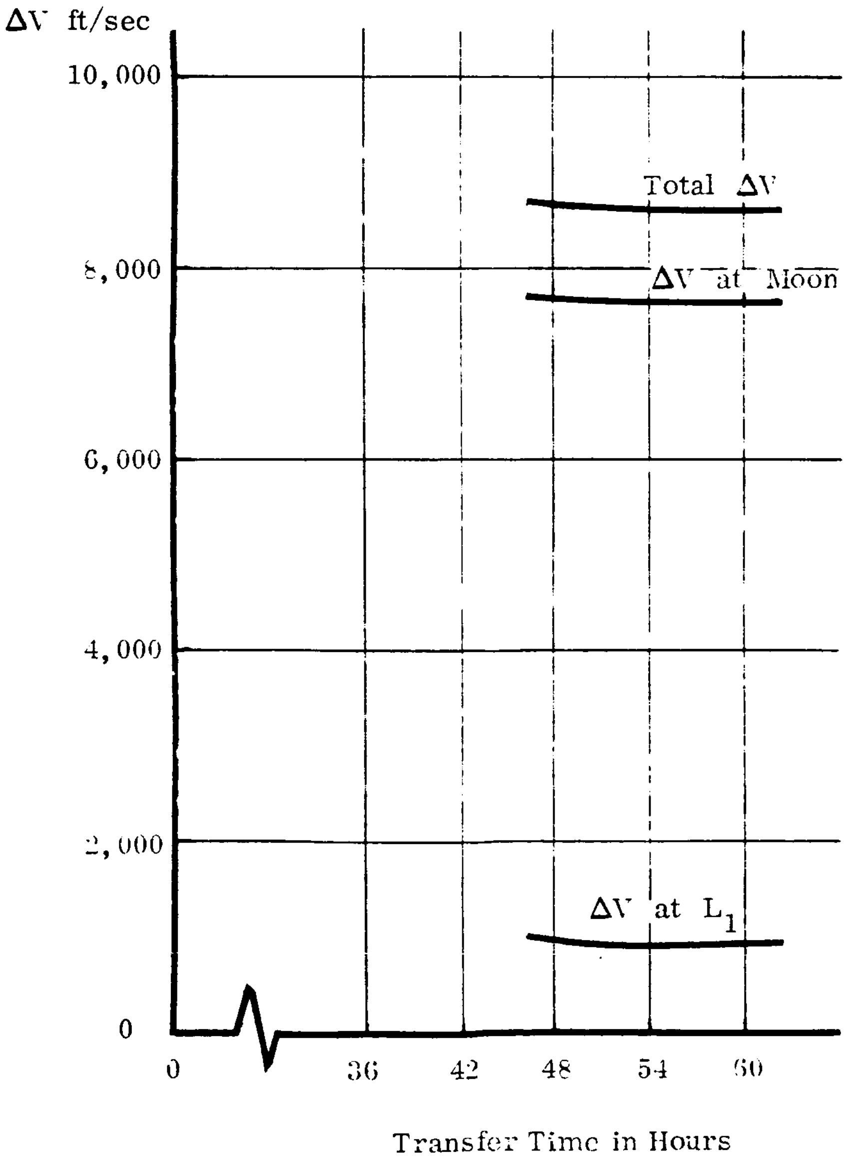


FIGURE 5 - TRANSFERS FROM EARTH TO $L_1$

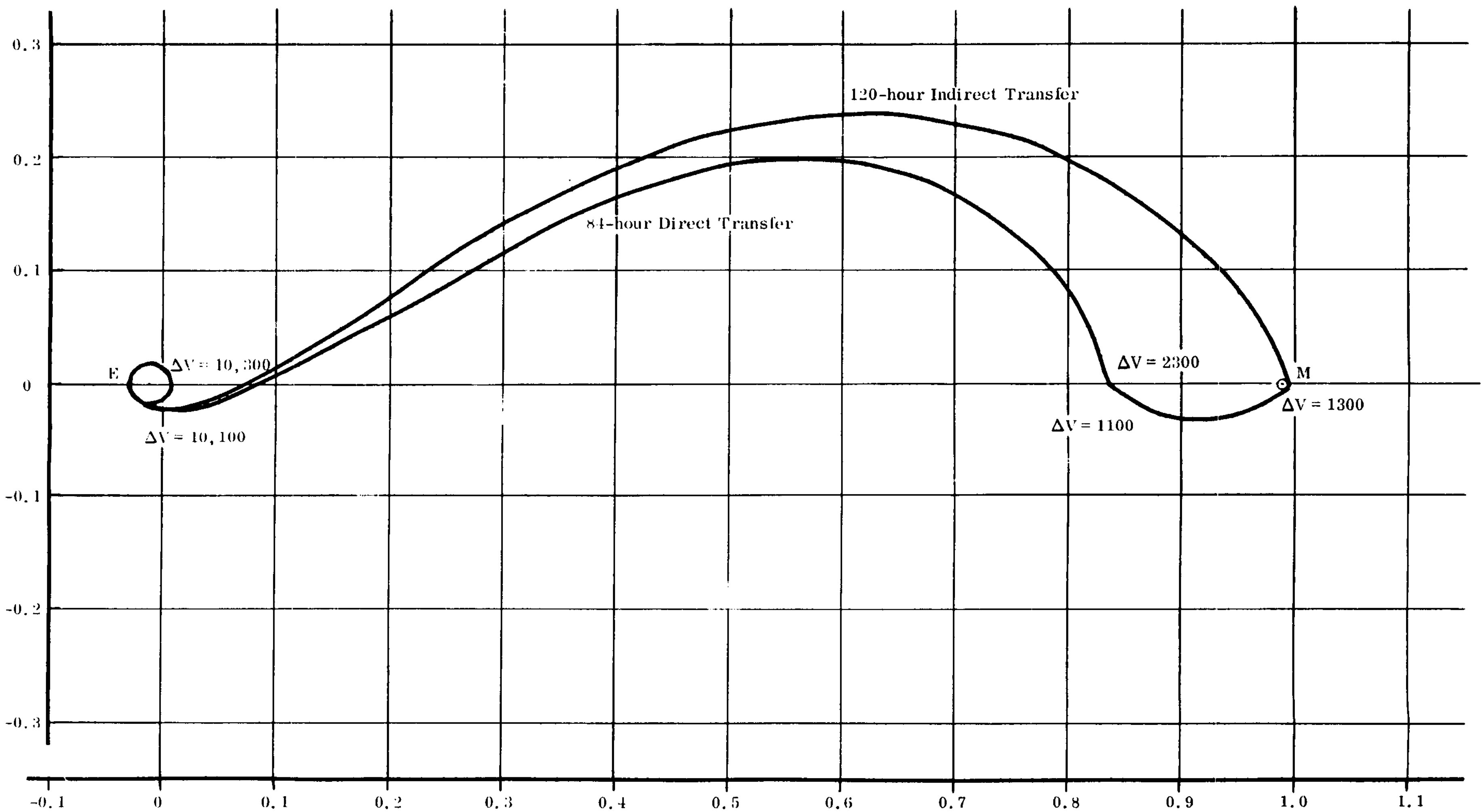


FIGURE 6

BRAKING $\Delta V$ REQUIREMENTS AT $L_1$

DIRECT TRANSFER


| Transfer Time | $\|V\|_{L_1}$ |
|---|---|
| 82 | 2386.1 |
| 84 | 2364.2 |
| 86 | 2348.1 |
| 88 | 2337.2 |
| 90 | 2330.9 |
| 92 | 2328.8 |
| 94 | 2330.5 |
| 96 | 2335.5 |
| 98 | 2343.5 |
| 100 | 2354.1 |
| 102 | 2366.9 |
| 104 | 2381.7 |

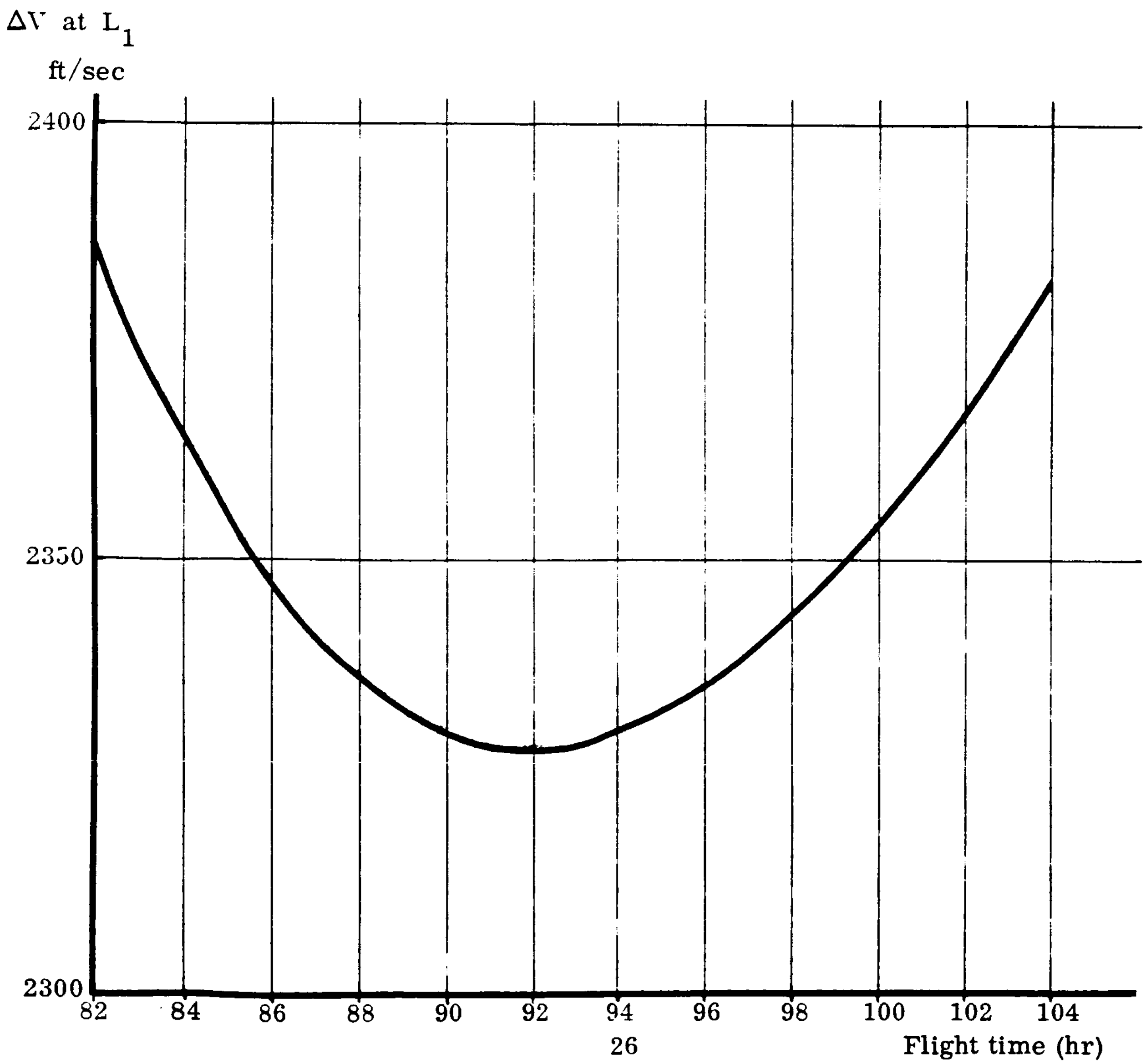


FIGURE 7 - 120-HOUR INDIRECT TRANSFERS TO $L_1$

Perilune Occurs at 82 Hours

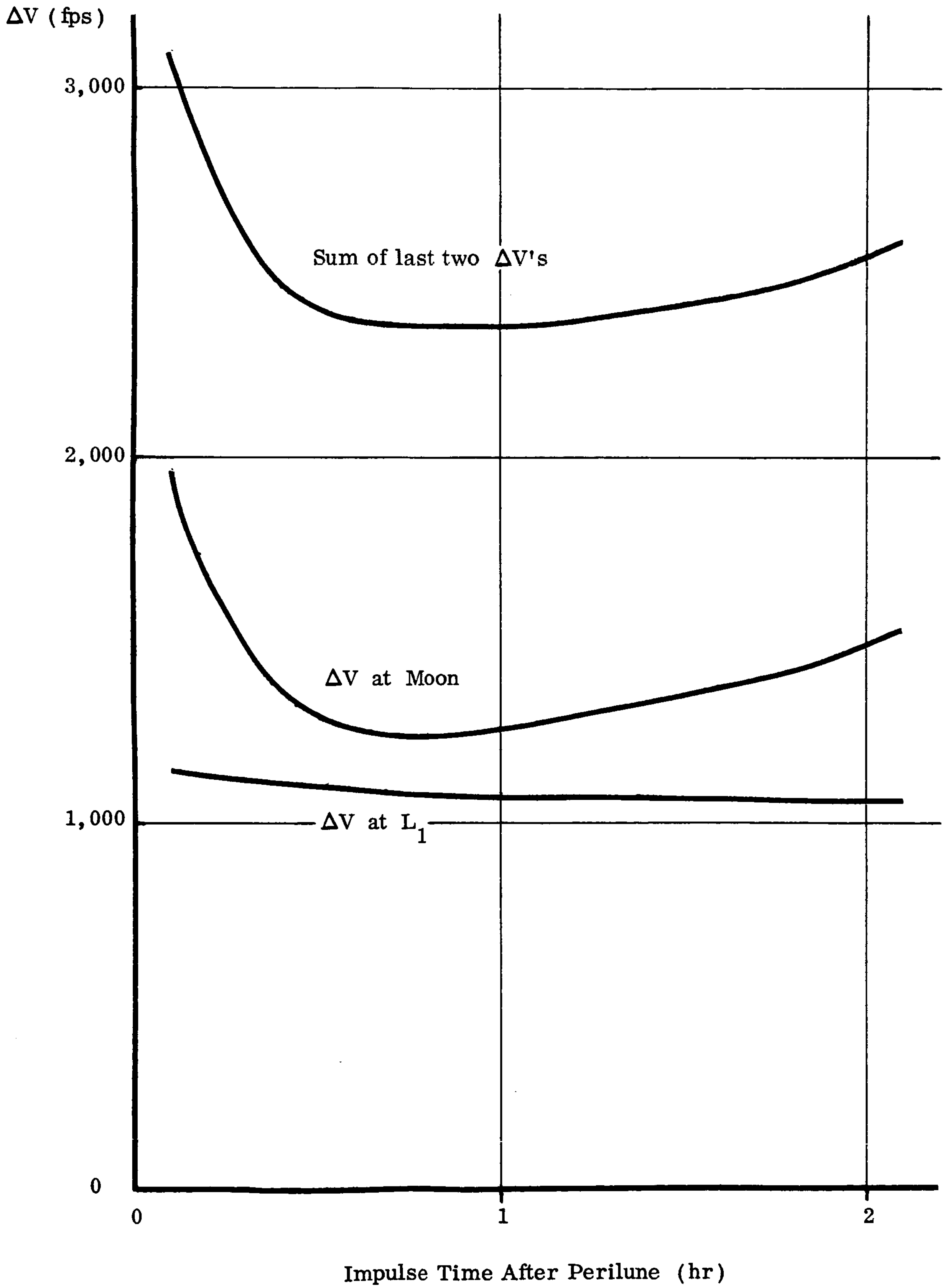


FIGURE 8 - HALO ORBIT TO EARTH TRANSFERS

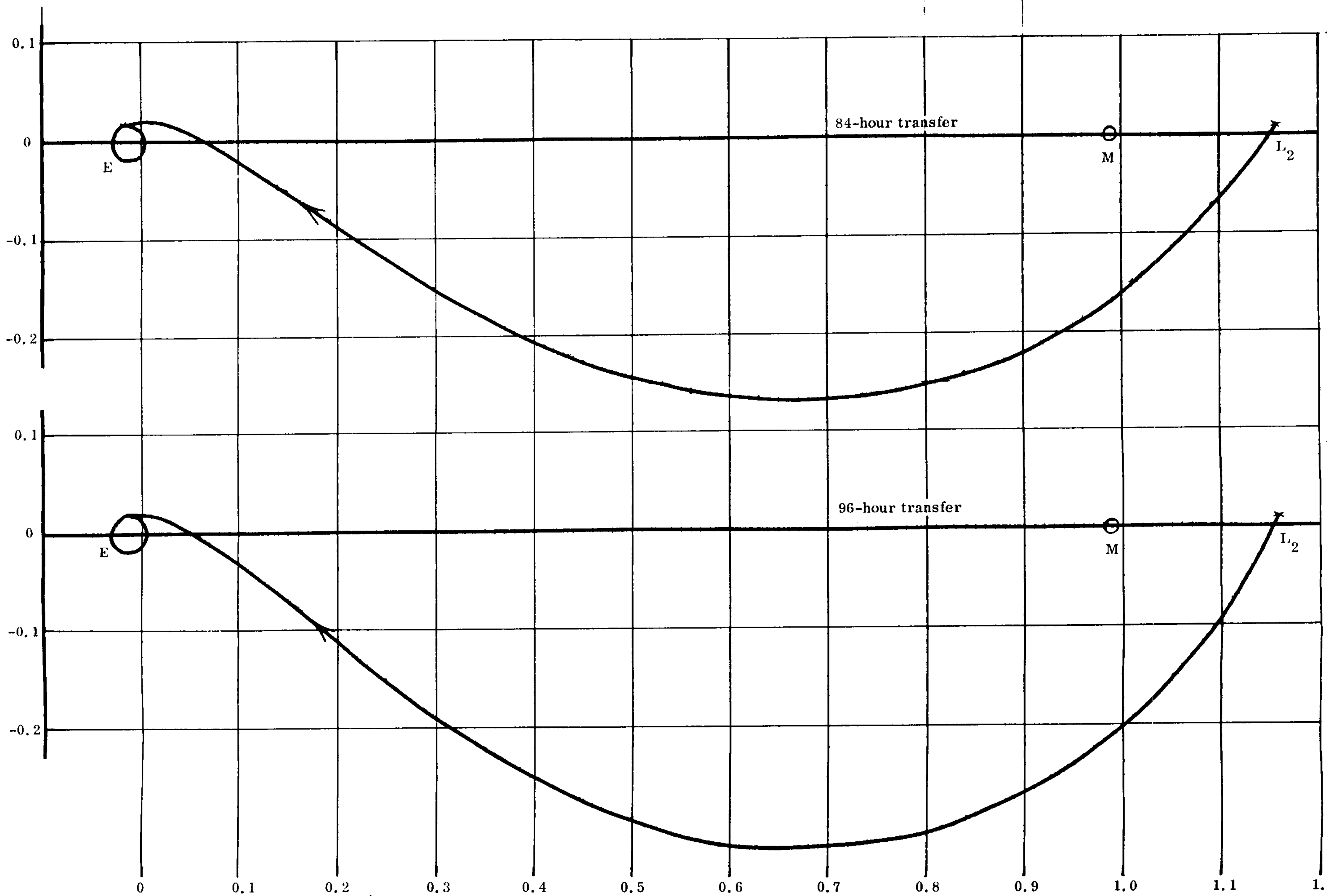
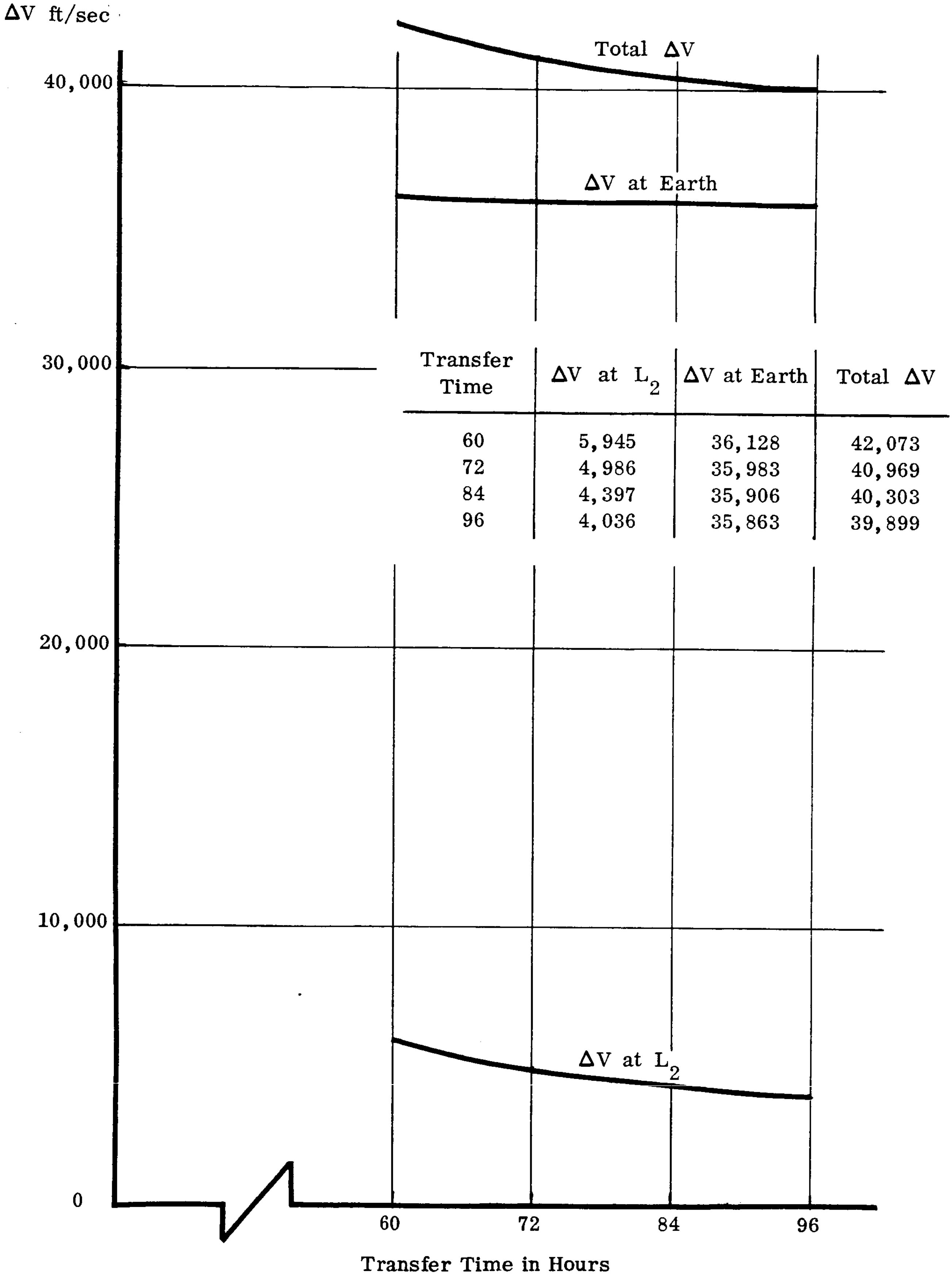


FIGURE 9 - $L_2$ TO EARTH FLYBYS AT 100 N. M.




| Transfer Time | $\Delta V$ at $L_2$ | $\Delta V$ at Earth | Total $\Delta V$ |
|---|---|---|---|
| 60 | 5,945 | 36,128 | 42,073 |
| 72 | 4,986 | 35,983 | 40,969 |
| 84 | 4,397 | 35,906 | 40,303 |
| 96 | 4,036 | 35,863 | 39,899 |

FIGURE 10 - HALO ORBIT TO MOON FLYBY IN 48 HOURS

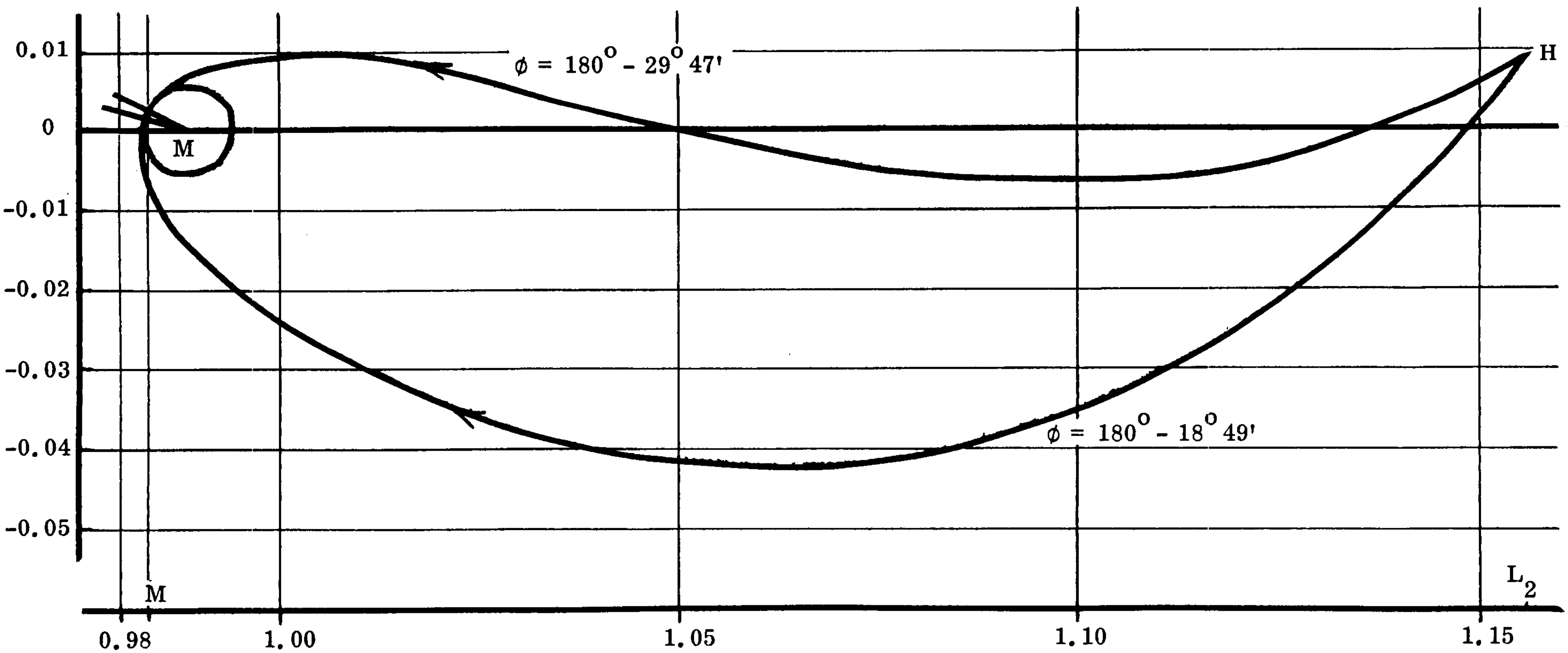


FIGURE 11 - HALO ORBIT TO MOON FLYBYS
AROUND TRAILING SIDE OF MOON

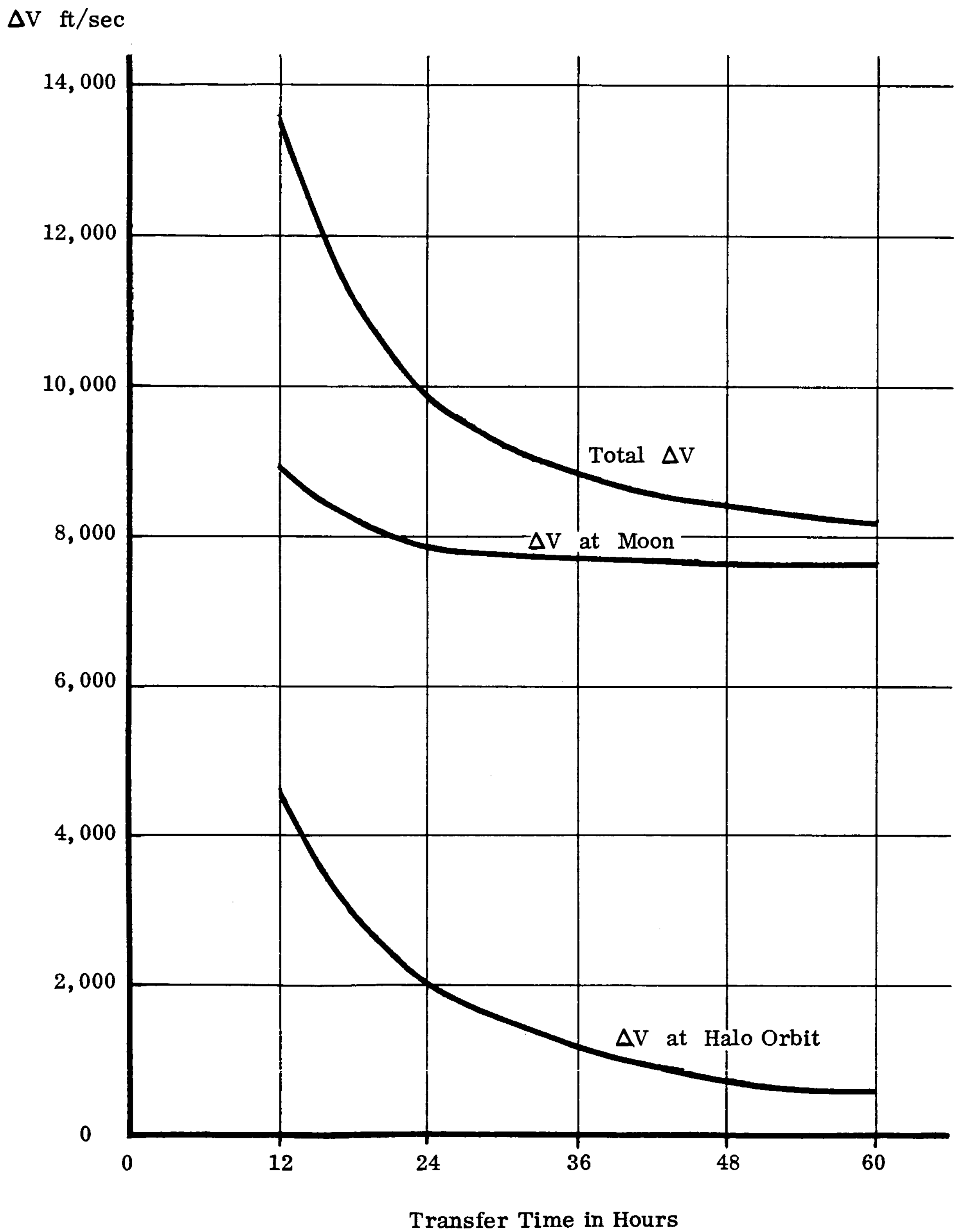


FIGURE 12

HALO ORBIT TO MOON FLYBYS - AROUND LEADING SIDE OF MOON

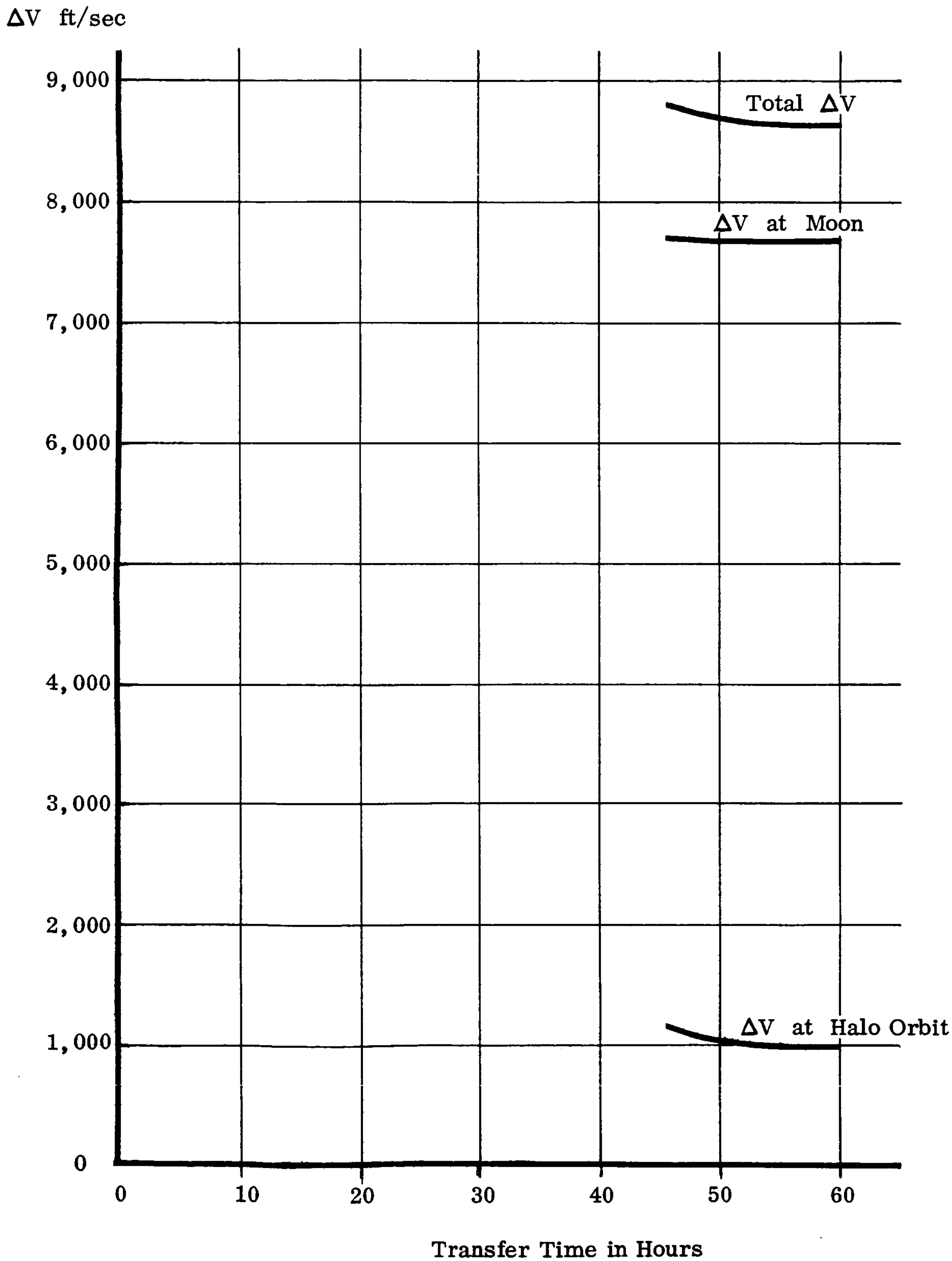


FIGURE 13 - VEHICLE WEIGHTS IN EARTH ORBIT (LB)