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ANALYTIC CALCULATION OF LAUNCH VEHICLE RESPONSE TO WINDS AND CALCULATION OF OPTIMAL BIASED PITCH PROGRAMS

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16. Abstract
A simplified method is described for computing trajectory response of rocket vehicles to wind disturbances. It is useful in determining aerodynamic loading, launch availability, and thrust-vector-control-system requirements. In the method presented, the equations of motion are linearized so they can be solved closed-form using certain simplifying assumptions. Closed-form equations are derived to determine angle of attack, flight attitude, and thrust deflection angle as functions of the wind disturbance. The analysis assumes a unity autopilot; however, any linearized autopilot model can be included. A method for obtaining optimum biased pitch and yaw programs using the simple linearized equations is also shown.

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SUMMARY

A set of simple, closed-form equations is derived to evaluate the trajectory response to a wind disturbance; that is, to compute

(1) Angle of attack caused by a wind disturbance for a trimmed vehicle (The vehicle is trimmed if nominal attitude is maintained in the presence of a wind disturbance.)

(2) Attitude bias caused by a wind disturbance if a load-relief control system is assumed (i.e., the nominal angle-of-attack profile is maintained in the presence of wind)

(3) Thrust vector deflection required to maintain nominal flight attitude in the presence of a wind disturbance

In deriving the closed-form solutions to the equations of motion, some simplifying assumptions were made. This introduces errors in the resulting angle-of-attack or attitude bias. However, the results obtained using the closed-form equations agree well with the detailed six-degree-of-freedom computer results.

The simplified equations are also used to derive optimum biased pitch and yaw programs based on a sample of actual wind measurements. The results presented show a significant improvement in launch availability (based on a constant angle-of-attack-times-dynamic-pressure capability of the vehicle) compared to the nominal (no bias) trajectory and to a biased pitch program based on a statistical (synthetic) wind profile considered herein.

INTRODUCTION

One of the many launch vehicle studies required prior to a flight is to determine the launch availability (the probability that the structural capability of the vehicle will not be exceeded during flight) which can be expected for a vehicle with a given structural strength or allowable aerodynamic loading. Alternately, in preliminary design studies it is necessary to determine the structural strength required for a specific launch availability. Since maximum structural loading of launch vehicles is often caused by the winds aloft, both of these studies require the calculation of aerodynamic loads using synthetic wind profiles (refs. 1 and 2) or a large sample of actual wind measurements.

Further studies involve the possible reduction of aerodynamic loads (or increasing launch availability) without structural modification of the vehicle. Aerodynamic loads can be reduced by using a load-relief autopilot or biased pitch and yaw programs. The basic principle of a load-relief autopilot is a continuous in-flight calculation of aerodynamic loads and turning the vehicle in such a way as to minimize these loads. The second method of reducing aerodynamic loads is by using a biased pitch program. The reference (nominal) pitch program is usually defined by a zero-angle-of-attack, no-wind trajectory simulation. The initial pitchover is selected to maximize payload or to satisfy an ascent aerodynamic heating constraint. To obtain a biased pitch program, the nominal flight attitude is adjusted to minimize the expected aerodynamic loads based on a sample of actual wind measurements. Biased pitch programs make use of the seasonal correlation of wind velocities and directions in reducing loads. This seasonal correlation of winds is observed in references 1 and 2. Derivation of a biased pitch program involves the calculation of aerodynamic loads for a large sample of actual wind measurements.

Biased pitch programs were developed for the Atlas-Centaur vehicle using a detailed computer program and a sample of actual wind measurements. One of the procedures used is discussed in reference 3. This procedure gave good biased programs. However, it was too costly in terms of computer time to be of practical use. To reduce the computer time required, biased pitch programs for the actual flights were derived based on artificial wind profiles (ref. 4) which were representative of the seasonal winds. The procedure gave good launch availability.

In the past, all these studies were accomplished by using detailed six-degree-of-freedom computer programs. This is time consuming and expensive. To reduce the cost and to simplify these studies, a set of simple, closed-form equations is derived in this report for calculating the trajectory response to a wind disturbance. Furthermore, these equations are used to derive biased pitch programs using a large sample of actual wind measurements.

The basic equations of this report are obtained by linearizing the equations of motion about a reference trajectory. The simplified equations can be used to determine angle-of-attack change caused by a wind disturbance if the vehicle is assumed to be trimmed (i. e., the nominal attitude is maintained in the presence of a wind disturbance). If a load-relief autopilot is assumed (i. e., the angle-of-attack profile is minimized in the presence of a wind disturbance), the equations can be used to determine the change in flight attitude caused by a wind disturbance.

Maximum loading of launch vehicles occurs in the atmosphere near the maximum dynamic pressure region. Loads on the vehicle are made up of axial loads and bending moments. For a particular vehicle, axial loads cannot be changed, since they depend on axial acceleration and aerodynamic drag. The bending moments, on the other hand, depend on the angle-of-attack history encountered during flight. Bending moments are proportional to the product of angle of attack and dynamic pressure. Dynamic pressure does not change appreciably because of the wind disturbance. Therefore, in order to reduce bending moments, it is necessary to reduce the angle of attack. The simplified equations can be used to compute the angle-of-attack change resulting from the wind disturbance. This angle-of-attack change, in turn, can be used to compute the attitude bias (using the simplified equations) necessary to maintain a nominal (no wind) angle-of-attack history, thus giving the best possible bias. Since the equations are simple, it is possible to analyze a sample of actual wind measurements in a minimum of computer time.

Results obtained by using the simplified equations are presented for two vehicle configurations: a 260-inch solid-rocket booster with an SIV-B upper stage (configuration I) and the Atlas-Centaur (AC-15) vehicle (configuration II). The results presented include comparisons of angle-of-attack and attitude bias for three wind profiles using a detailed six-degree-of-freedom computer program and the simplified closed-form equations. Furthermore, the simplified equations were used to derive biased pitch and yaw programs for the two vehicle configurations. The biased programs are based on a sample of 100 March wind soundings.

To evaluate the biased pitch and yaw programs, launch availability is derived by using the simplified equations and the sample of 100 winds (ref. 2). A comparison of launch availability is made for the nominal and biased pitch and yaw programs. (The launch availability described is based on maximum angle of attack times dynamic pressure. This assumes the vehicle has a constant angle-of-attack-times-dynamic-pressure capability. Of course, if this capability is given as a function of flight time, the launch availability can be derived by using the given vehicle capability.)

ASSUMPTIONS

To obtain a closed-form solution to the equations of motion, the following assumptions are made:

(1) The vehicle is assumed to be a rigid body; no attempt is made to include or compensate for elastic effects.

(2) The inertial effect of the vehicle is assumed to be negligible; that is, the vehicle can be instantaneously turned to the desired attitude.

(3) The basic equations of motion are linearized about a nominal operating point, which is generally chosen to be a no-wind, zero-angle-of-attack trajectory. Since the analysis gives the changes from this nominal flight caused by wind disturbances, it requires a detailed nominal simulation to evaluate the perturbed flight.

(4) The change in velocity from the nominal caused by the wind disturbance is assumed to be small, and its effect on the flight path angle negligible. If greater accuracy is required, this effect can be included. In this case, however, two first-order linear differential equations (rather than one as will be discussed) must be solved simultaneously to obtain the change in angle of attack.

(5) The basic vehicle parameters (thrust, weight, drag, etc.) are assumed to remain constant in some small time interval. This allows a closed-form solution (in this interval) of the linear first-order variable-coefficient differential equations.

(6) The wind velocity is approximated by a straight line in some subinterval of the one chosen in assumption 5. Since both these intervals are arbitrary, this does not limit the accuracy of the results.

ANALYSIS

The variables and other notations used in the following discussion are defined in appendix A. The basic equations of motion have been derived in reference 5, and the equations for the flight path angle, velocity, attitude, and angle of attack are reproduced in appendix B for convenience and completeness.

The basic equation of this analysis relating the attitude change $\Delta\theta$, change in angle of attack $\Delta\alpha$, and superimposed wind angle of attack α_W is derived in appendix B and is given by

$$\Delta\dot{\theta} + a_1 \Delta\theta = \Delta\dot{\alpha} + c_1 \Delta\alpha + \dot{\alpha}_W + b_1\alpha_W \qquad \text{(B5a)}$$

The superimposed wind angle of attack at any particular flight time depends on the wind velocity, the wind azimuth, and the vehicle's relative velocity and flight attitude. The change in angle of attack, on the other hand, depends on the vehicle and trajectory response to the wind disturbance. In the analysis, the coefficients a_1, c_1, and b_1 are assumed to be constant in some time interval. This assumption is made in order that a closed-form solution may be obtained for this equation.

Attitude Bias

In the following sections, simple closed-form solutions of equation (B5a) will be derived to compute the required change in flight attitude (attitude bias) that will maintain a nominal angle-of-attack profile in the presence of a wind disturbance. Simplified equations will also be derived to compute the attitude bias for a desired change in nominal angle of attack. These equations are given for a particular wind or change in angle of attack. Their usefulness in deriving biased pitch programs is discussed in the section Statistical Wind Analysis and Biased Pitch Programs.

Pitch and yaw plane attitude bias. - It is desired to maintain the nominal angle-of-attack profile (in general, near zero angle of attack) in the presence of winds. This assures that the aerodynamic loads will be minimized. Since

$$\Delta\alpha = \alpha - \alpha_n = 0 \tag{1}$$

equation (B5a) gives the desired relation of wind-induced angle of attack and the change in vehicle attitude

$$\Delta\dot{\theta} + a_1\,\Delta\theta = \dot{\alpha}_w + b_1\alpha_w \tag{2a}$$

The general solution of equation (2a) is given by

$$\Delta\theta = \alpha_w + (\Delta\theta_o - \alpha_{w,o})\exp\left[-\int_{t_o}^{t} a_1(\tau)d\tau\right] - \int_{t_o}^{t} \alpha_w(\xi)\exp\left[-\int_{\xi}^{t} a_1(\tau)d\tau\right] \times \left[a_1(\xi) - b_1(\xi)\right]d\xi \tag{2b}$$

Assume that coefficients a_1 and b_1 remain constant on the interval F_j; $F_j\left[t_j \leq t \leq (t_j + \Delta t_j)\right]$. With this assumption equation (2b) becomes

$$\Delta\theta_{j+1} = \alpha_{w,j+1} + (\Delta\theta_j - \alpha_{w,j})\exp(-a_1\,\Delta t_j) - (a_1 - b_1)\int_{t_j}^{t_j+\Delta t_j} \alpha_{w,j+1}(\xi)\exp\left[-a_1(t_{j+1} - \xi)\right]d\xi \tag{2c}$$

If the superimposed wind angle of attack is assumed to be linear on the interval E_i; $E_i\left[t_i \leq t \leq (t_i + \Delta t_i)\right]$ where $E_i \in F_j$, then

$$\alpha_{w,i+1} = \dot{\alpha}_{w,i}(t - t_i) + \alpha_{w,i} \tag{3a}$$

When equation (B1e) is used, the following expressions for $\alpha_{w,i}$ and $\dot{\alpha}_{w,i}$ are obtained:

$$\alpha_{w,i} = \left[\frac{V_w \sin(\theta - \alpha)_n}{V_r}\right]_{t=t_i} \tag{3b}$$

and

$$\dot{\alpha}_{w,i} = \frac{1}{\Delta t_i}\left\{\left[\frac{V_w \sin(\theta - \alpha)_n}{V_r}\right]_{t=t_i+\Delta t_i} - \alpha_{w,i}\right\} \tag{3c}$$

With these assumptions, the solution of equation (2c) becomes

$$\Delta\theta_{i+1} = \alpha_{w,i+1} + (\Delta\theta_i - \alpha_{w,i})\exp(-\,a_1\,\Delta t_i) - \left(1 - \frac{b_1}{a_1}\right)$$
$$\times\left\{\left(\alpha_{w,i} - \frac{\dot{\alpha}_{w,i}}{a_1}\right)\left[1 - \exp(-a_1\,\Delta t_i)\right] + \dot{\alpha}_{w,i}\,\Delta t_i\right\} \tag{4}$$

Equation (B5) can also be used to obtain the attitude bias required for a desired change in nominal angle-of-attack profile. To do this, the wind-induced angle of attack is assumed to be zero ($\dot{\alpha}_w = \alpha_w = 0$). Furthermore, if the angle-of-attack change is approximated in the same way as previously for the wind-induced angle of attack (by a straight line segment), the solution to equation (B5) becomes

$$\Delta\theta_{i+1} = \Delta\alpha_{i+1} + (\Delta\theta_i - \Delta\alpha_i)\exp(-a_1\,\Delta t_i) - \left(1 - \frac{c_1}{a_1}\right)$$
$$\times\left\{\left(\Delta\alpha_i - \frac{\Delta\dot{\alpha}_i}{c_1}\right)\left[1 - \exp(-a_1\,\Delta t_i)\right] + \Delta\dot{\alpha}_i\,\Delta t_i\right\} \tag{5}$$

where

$$\Delta\dot{\alpha}_i = \frac{1}{\Delta t_i}(\Delta\alpha_{i+1} - \Delta\alpha_i)$$

If a particular angle-of-attack history is desired in the presence of a wind disturbance, equations (4) and (5) can be used to compute the attitude biases required for the wind alone and the angle-of-attack bias alone, respectively. Since equation (B5) is a linear differential equation, the total attitude bias is the sum of the attitude biases due to the wind and to the angle-of-attack bias. The same equations can be used in computing attitude bias in the yaw plane. Of course, it must be remembered that a_1, b_1, and c_1 of appendix B are computed based on the yaw plane trajectory parameters.

Final conditions. - In the previous section, equations were presented to give attitude bias. This bias is required to maintain a given angle-of-attack profile in the presence of a wind disturbance. If the nominal trajectory is assumed to be zero angle of attack, the bias given by these equations is the same as would be obtained by using a perfect load-relief autopilot. However, this bias gives large dispersions in altitude and flight path angle at booster separation. This can result in payload loss or excessive aerodynamic heating. When biased pitch programs are derived, the dispersions can be minimized by varying the initial attitude bias to maintain nominal flight path angle at booster separation. This is equivalent to changing the initial pitchover in the trajectory. Linearizing equation (B1d) gives

$$\Delta\alpha = \Delta\theta - \Delta\gamma - \alpha_w \tag{6}$$

Since in deriving equation (4) it was assumed that the nominal angle-of-attack profile will be maintained (i.e., $\Delta\alpha = 0$), equation (6) reduces to

$$\Delta\theta = \Delta\gamma + \alpha_w$$

To reduce the dispersions, the flight path angle is chosen to be the same as the nominal flight path angle at booster cutoff (i.e., $\Delta\gamma_f = 0$). Then

$$\Delta\theta_f = \alpha_{w,f} \tag{7}$$

The initial attitude change is varied to satisfy the given final condition (eq. (7)). Since the relative velocity is large at booster engine cutoff, the superimposed wind angle of attack becomes small. This assures an essentially nominal burnout condition; that is, the attitude, angle of attack, and flight path angle are all close to their nominal values. This, of course, does not preclude small deviations in altitude and downrange position.

Simplified Angle-of-Attack Computation

Most present-day launch vehicle autopilots are designed to maintain nominal flight attitude (trimmed) in the presence of aerodynamic disturbances. If the vehicle is assumed to be trimmed, and a unity autopilot is assumed, equation (B5) can be used to give a functional relation between the change in angle of attack and superimposed wind-induced angle of attack; that is,

$$\Delta\dot{\alpha} + c_1 \Delta\alpha = -(\dot{\alpha}_w + b_1 \alpha_w) \tag{8}$$

The general solution to equation (8) is given by

$$\Delta\alpha = -\alpha_w + (\Delta\alpha_o + \alpha_{w,o})\exp\left[-\int_{t_o}^{t} c_1(\tau)d\tau\right] + \int_{t_o}^{t} \alpha_w(c_1 - b_1)\exp\left[-\int_{\xi}^{t} c_1(\tau)d\tau\right]d\xi \tag{9}$$

If the same linear wind approximation as in the previous section on attitude bias is used and it is assumed that the variables c_1 and b_1 remain constant in the interval, the solution to equation (9) is given by

$$\Delta\alpha_{i+1} = -\alpha_{w,i+1} + (\Delta\alpha_i + \alpha_{w,i})\exp(-c_1 \Delta t_i) + \left(1 - \frac{b_1}{c_1}\right)$$

$$\times \left\{\left(\alpha_{w,i} - \frac{\dot{\alpha}_{w,i}}{c_1}\right)\left[1 - \exp(-c_1 \Delta t_i)\right] + \dot{\alpha}_{w,i} \Delta t_i\right\} \tag{10}$$

This equation can be used in both pitch and yaw planes with the appropriate coefficients.

Thrust Vector Deflection Requirements

Thrust vector deflection requirements caused by the wind, for a trimmed vehicle, may easily be computed from the linearized form of equation (B1c); that is,

$$\Delta\delta = \frac{l_a N_1}{l_c T} \Delta\alpha \tag{11}$$

This equation does not involve any characteristics of a real autopilot. However, because most control systems are designed to maintain near-trim conditions and their response time is small, equation (11) gives a good approximation to the thrust vector deflection requirement.

Statistical Wind Analysis and Biased Pitch Programs

A set of biased pitch programs has been used on some of the Atlas-Centaur flights with significant improvement in launch availability. However, it is difficult to obtain an optimum biased pitch program. There have been two different procedures used to derive the necessary bias; namely, biasing the nominal (zero angle of attack) flight to a statistical (synthetic) wind, or running a set of real wind soundings on a detailed computer program and adjusting the attitude (more or less empirically) until the resulting trajectories are acceptable in terms of aerodynamic loads. The first procedure has the disadvantage of giving a bias which may not be the best available. The second procedure gives good results. However, it requires prohibitively large computer times to derive a biased pitch program.

This section presents a simple, straightforward procedure for deriving a biased pitch program. Since structural loading is proportional to the angle of attack at a particular flight time, these loads can be minimized by minimizing angle of attack. The desired angle-of-attack bias can be derived from a statistical analysis of a set of real wind soundings. Equation (10) gives the change in angle of attack caused by a wind along the trajectory. Use of this equation does not require a trajectory integration to determine the change in angle of attack caused by the wind disturbance. It requires the integration of a single, first-order differential equation. In fact, if the trajectory parameters are assumed to be constant, a closed-form solution is available for each interval. By using this closed-form solution, the angle-of-attack profiles of a large sample of winds can be derived in a minimum of computer time. To obtain the best angle-of-attack bias, a statistical average of the angle-of-attack biases (for the sample of winds) is taken at every time point along the trajectory. Once this angle-of-attack bias is known, equation (5) can be used to compute the optimum attitude bias.

Generally, there will be large angles of attack in the early part of the flight. This is caused by the low relative velocity of the vehicle. The large angles of attack may give large angle-of-attack bias. The large angle-of-attack bias, in turn, would give undesirably large attitude bias. To eliminate the large attitude bias, the angle-of-attack bias is set equal to zero in the early portion of flight. The angle-of-attack bias can be set equal to zero in this flight region without adverse effects on loads since the dynamic pressure is very low.

RESULTS AND DISCUSSION

Figure 1 presents the wind profiles used in evaluating the accuracy of the simplified equations. The winds were chosen to give large perturbations: that is, high angles of attack and wind shear. The two winds given in figures 1(a) and (b) are designed to give high wind velocities and high wind shear near the maximum dynamic pressure region. They are given in terms of wind velocity and direction as a function of flight time. This choice of independent variable was made for convenience. A wind azimuth of 285 degrees for winds 1 and 2 was selected to eliminate any wind component in the yaw plane. Both vehicle configurations were launched at a 105-degree azimuth. The wind in figure 1(c) was selected from a sample of actual wind measurements (ref. 2) based on its high wind velocity, high wind shear, and a broad wind plateau near maximum dynamic pressure. The effect of wind direction is also included in the analysis of this wind. However, only the pitch plane results are presented.

Figures 2 and 3 give the angle-of-attack change from the nominal obtained by using both the simplified procedure and the six-degree-of-freedom computer simulation for two vehicle configurations, respectively. The two vehicle configurations are the 260-inch solid-rocket booster with an SIV-B upper stage (configuration I) and the Atlas-Centaur (AC-15) vehicle (configuration II). The results obtained by using the simplified equations for the three wind profiles show good agreement with those obtained from a detailed computer simulation.

Attitude biases were also derived for the winds presented in figure 1. In deriving the attitude biases, the simplified closed-form equation was used, and a nominal angle-of-attack history is assumed in the presence of the wind.

Figures 4 and 5 represent the attitude biases for the two vehicle configurations. The curves show the bias with and without iterated end conditions. Both biases were obtained by changing the attitude along the trajectory to cancel the wind-induced angle of attack. For the noniterated bias, the initial attitude bias was set equal to zero. However, for the iterated bias, the initial attitude bias was adjusted to achieve the nominal flight path angle at booster cutoff. These figures show a significant reduction in maximum bias requirement when the iterated end conditions are used. This reduction is caused by an increased attitude bias in the low-wind-velocity region. Because of the inertial and aerodynamic properties of configuration II, the large wind velocities encountered in the early part of the flight with wind 3 caused the attitude to diverge in the case of noniterated bias. Therefore, the data for configuration II with wind 3 are not presented in figure 5 and later figures.

Since the biases are designed to maintain nominal angle of attack in the presence of winds, the accuracy of the simplified equations can be evaluated by simulating a detailed trajectory with a wind and its corresponding attitude bias. The resulting angle of attack is compared with the nominal; the difference in angle of attack is caused by the approximations made in deriving the equations. The difference between the nominal angle-of-attack profile and the angle-of-attack profile obtained by using the attitude bias from figures 4 and 5 is shown in figures 6 and 7. These results were obtained from a detailed six-degree-of-freedom computer simulation. The angle-of-attack errors are quite small compared to the derived angle-of-attack biases of figures 2 and 3. This indicates that the simplified analytic attitude-bias equation is quite accurate. Note that in figures 6 and 7 the angle-of-attack error using noniterated attitude bias is larger than that for the iterated attitude bias. This is caused by the large drift velocities encountered with a noniterated bias. The larger the drift velocity, the greater the deviation from the nominal trajectory, and the more the accuracy of the equations is reduced. The iterated bias anticipates the wind profile. Therefore, the attitude is changed in advance, which, in turn, changes the direction of the relative velocity vector. The velocity vector is changed in such a way that adding the wind velocity and sideslip effect to it results in the nominal relative velocity vector. This minimizes sideslip, which, in turn, improves the accuracy of the linearized attitude-bias equation. The noniterated bias does not anticipate the wind, and the wind-induced angle of attack must be cancelled by the attitude change. Because the velocity vector is not biased, there is no reduction in sideslip, and a larger angle-of-attack error results.

Figures 8 and 9 give a comparison of altitude dispersions using iterated and noniterated attitude biases. The results were obtained from a six-degree-of-freedom computer simulation by using the attitude biases derived by the simplified procedure and the corresponding wind profile. In all cases, the altitude dispersions obtained by using iterated end conditions were much smaller than the altitude dispersions for noniterated attitude bias. Figure 8(c) shows a maximum altitude error at 110 seconds of 12 450 meters for the noniterated bias compared with 150 meters using the iterated bias. This, of course, is very significant, since the vehicle is only 36 570 meters in altitude at this flight time and a 12 450-meter dispersion can cause severe aerodynamic heating problems and payload loss.

Figures 8 and 9 also give a comparison between a perfect load-relief control system and biased pitch programs. The noniterated attitude bias gives the same flight attitude that a perfect load-relief system would follow. Because of the large dispersions introduced, actual load-relief systems are designed to compromise between load reduction and flight dispersions. Because biased pitch programs are derived by using the iterated end conditions, the attitude, flight path angle, and attitude dispersions are minimized.

The results presented indicate that the simplified analytic equations give good results. In the discussion that follows, the simplified equations are used to compute biased pitch and yaw programs. The biased pitch and yaw programs are derived for a sample of 100 March wind measurements (ref. 2). The measurements were taken at the Eastern Test Range between 1956 and 1959. The pitch and yaw wind components were computed based on wind direction and launch azimuth. These wind components were used to compute pitch and yaw attitude biases. The launch azimuth for both vehicle configurations was 105 degrees.

Figures 10 to 13 give the attitude-bias results for configuration I in the pitch plane. The simplified angle-of-attack bias equation was used to obtain angle-of-attack profiles for the wind sample. The statistical average of these angle-of-attack profiles is given by the dashed curve in figure 10. This angle-of-attack bias was used to derive the attitude bias of figure 11 by applying the simplified attitude-bias equation and the desired final condition. To check the accuracy of the bias equation, the attitude bias of figure 11 was simulated by the six-degree-of-freedom computer program. If a zero wind is used in the simulation, the angle-of-attack profile should be equal to the angle-of-attack bias of figure 10, except for the inaccuracies of the simplified equations. The resulting angle-of-attack bias is given by the solid curve in figure 10. It compares well with the statistical angle-of-attack bias. This indicates again that the simplified attitude-bias computation gives good accuracy. Since the nominal (unbiased) trajectory is near zero angle of attack, the angle-of-attack bias of figure 10 is essentially the same as the angle-of-attack history of the biased no-wind trajectory.

Figure 12 is a histogram of the wind sample analyzed based on maximum angle of attack times dynamic pressure. As was expected, the attitude bias drastically reduced maximum angle of attack times dynamic pressure, on the average. Thus, the probability of having a wind with smaller maximum angle of attack times dynamic pressure is increased. This is shown in figure 13. For example, if configuration I had an angle-of-attack-times-dynamic-pressure capability of 120 000 degrees times newtons per square meter (deg $\times$ N/m^2) (from fig. 13), 6 percent of the winds considered would not exceed this value if the nominal pitch program is used, while approximately 70 percent of the winds would not exceed the vehicle capability if the biased pitch program derived is used. As another example, if configuration I were designed with 85 percent launch availability, its design structural strength would have to provide for a 390 000-deg $\times$ N/m^2 capability without biased pitch programs. The requirement can be reduced to 180 000 deg $\times$ N/m^2 using the biased pitch program. From the preceding discussion, the advantages of using biased pitch and yaw programs are obvious.

Figures 14 to 18, with the exception of figure 16, correspond to figures 10 to 13 but for configuration II in the pitch plane. However, this configuration had a biased nominal pitch program. The biased nominal pitch program is given in reference 4, where it is designated as the one to be used for the month of March. This pitch program was derived based on a synthetic wind profile also given in reference 4. Figure 16 shows the angle-of-attack profiles for the two biased pitch programs, when the vehicle is flown on a no-wind trajectory. Note that the nominal pitch program has a maximum angle of attack of 3.4 degrees (fig. 16), while the biased pitch program derived herein has a 2.3 degree maximum angle-of-attack bias. Because of this overbias, the nominal pitch program gives a reduced launch availability. This is shown in figure 18. For example, if configuration II had an angle-of-attack-times-dynamic-pressure capability of 50 000 deg $\times$ N/m^2 (from fig. 18), 18 percent of the winds considered would not exceed this value if the nominal pitch program is used, while approximately 52 percent of winds would not exceed the vehicle capability if the biased pitch program derived is used.

To illustrate the applicability of the simplified equations to the yaw plane, a biased yaw program was derived for configuration I. The results are given on figures 19 to 22. Since the nominal trajectory is zero angle of attack, the angle-of-attack bias is identical to the total angle of attack. Therefore, the total-angle-of-attack profile is not plotted. From figure 22, if the vehicle capability is 100 000 deg $\times$ N/m^2, the launch availability can be improved from 47 percent without bias to 72 percent with the bias.

To determine the launch availability of a particular vehicle using biased pitch programs, the total angle of attack times dynamic pressure must be computed for the wind sample. Once the angle-of-attack-times-dynamic-pressure histories are known, the same statistical procedure can be used as previously to derive launch availability. The total angle of attack can be calculated from the pitch and yaw angles of attack by using spherical geometry.

SUMMARY OF RESULTS

The results of this analytic calculation of launch vehicle response to winds are twofold:

1. First, a set of analytic equations was derived to evaluate
   a. The change in angle of attack caused by a wind disturbance for a trimmed vehicle (eq. (10)) (A vehicle is trimmed if nominal attitude is maintained in the presence of aerodynamic disturbances.)
   b. The change in attitude caused by a wind disturbance if the nominal angle-of-attack profile is maintained (eq. (4))
   c. The change in attitude necessary to obtain a desired angle-of-attack profile (This equation (eq. (5)) was derived by assuming a no-wind nominal trajectory.)

d. Deflection requirements for a particular angle-of-attack change (eq. (11)) The analytic equations can be used in preliminary design studies to determine the vehicle's structural strength requirement. This determination can be based on a large sample of actual wind soundings instead of a synthetic wind profile. Since the results are based on a large number of wind measurements, they will give a more accurate representation of load requirements.

2. Second, the simplified equations were used to derive biased pitch and yaw programs. It was shown that substantial improvement in launch availability is obtained by using the procedure described herein compared to a biased pitch program derived based on an artificial wind profile. The launch availability is computed based on maximum angle of attack times dynamic pressure, which assumes the vehicle has a constant angle-of-attack-times-dynamic-pressure capability. If the variation of the vehicle capability with flight time is known, the launch availability can be computed based on the given vehicle capability. Also, the present procedure requires a minimum of computer time to derive such a bias. To derive a biased pitch or yaw program using a sample of 100 winds requires approximately 7 minutes of (IBM 7094) computer time compared with approximately 500 minutes using the same wind sample and a six-degree-of-freedom computer program.

The simplified equations may make it feasible to derive a biased pitch and yaw program at the time of launch for the existing launch wind. This has not been evaluated in the present report.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, March 25, 1970,
731-25.

APPENDIX A

SYMBOLS

a_1	constant defined by eq. (B5b) or (B9b), sec^{-1}
a_2	constant defined by eq. (B2c) or (B7b), N
b_1	constant defined by eq. (B5c) or (B9c), sec^{-1}
C_N	normal force coefficient
$C_{N,0}$	normal force coefficient for zero angle of attack
c_1	constant defined by eq. (B5d) or (B9d), sec^{-1}
E_i	interval defined by $t_i \leq t \leq t_{i+1}$
F_A	axial force, N
F_j	interval defined by $t_j \leq t \leq t_{j+1}$
$F_{\parallel V}$	force parallel to velocity vector, N
$F_{\perp V}$	force perpendicular to velocity vector, N
g	gravitational acceleration, m/sec^2
I	moments of inertia, $N\text{-}m\text{-}sec^2$
l_a	aerodynamic moment arm, m
l_c	control moment arm, m
M_{CG}	moment about center of gravity, N-m
m	gross mass, kg
N	total normal force, N
N_1	normal force per angle of attack, N/rad
N_2	zero-angle-of-attack normal force, N
Q	dynamic pressure, N/m^2
R_1, R_2	defined by eqs. (B1a) and (B1b), N
S_{ref}	aerodynamic reference area, m^2
T	total thrust, N
t	time, sec
V	velocity, m/sec
W_e	effective gravity force, N

$\dot{x}$ time derivative of x, sec^{-1}

α angle of attack, rad

γ flight path angle, rad

Δ linearized variable (i. e., $\Delta X = X - X_n$)

δ thrust deflection angle, rad

θ flight attitude, rad

θ_p reference pitch attitude, rad

ξ dummy variable

τ dummy variable

Subscripts:

e effective

f final

i, j time intervals

n nominal

o starting value

r relative

w wind

APPENDIX B

DERIVATION OF BASIC EQUATIONS

The basic equations used in the analysis will be derived for a rigid-body vehicle configuration. The pitch, yaw, and roll planes are assumed to be uncoupled; that is, disturbances in any of these planes will not be felt in the other two planes. This is a good approximation for symmetric or nearly symmetric vehicles. The autopilot effects (the effects of a real control system) are not included in the equations. Equations are derived separately for the pitch and yaw planes. However, the final results are very similar, and the same equations can be used with minor modifications to analyze both planes. The variables used are defined in appendix A.

Equations in Pitch Plane

Since the vehicle is roll-stabilized, the pitch plane is defined by the radius vector and the local horizontal in the launch azimuth direction. The basic vehicle configuration in the pitch plane is given in figure 23. The equations of motion are

$$\sum F_{\parallel V} = R_1 = m\dot{V} = T\cos(\theta - \gamma + \delta) - F_A\cos(\theta - \gamma) - N\sin(\theta - \gamma) - mg\sin\gamma \tag{B1a}$$

$$\sum F_{\perp V} = R_2 = mV\dot{\gamma} = T\sin(\theta - \gamma + \delta) - F_A\sin(\theta - \gamma) + N\cos(\theta - \gamma) - mg\cos\gamma \tag{B1b}$$

$$\sum M_{CG} = I\ddot{\theta} = Nl_a - Tl_c\sin\delta \tag{B1c}$$

$$\alpha = \theta - \gamma - \alpha_w \tag{B1d}$$

$$\sin\alpha_w = \frac{V_w}{V_r}\sin\gamma \tag{B1e}$$

where V_w is the wind velocity component in the pitch plane, and

$$N = N_1\alpha + N_2 \tag{B1f}$$

where the normal force per angle of attack $N_1 = QS_{ref}(dC_N/d\alpha)$, and the normal force due to the asymmetry of the vehicle $N_2 = QS_{ref}C_{N,0}$.

Linearizing equation (B1b) and evaluating the coefficients along the nominal trajectory (which is assumed to be a no-wind trajectory) gives

$$m\dot{\gamma}_n \, \Delta V + mV_n \, \Delta\dot{\gamma} = \Delta R_2 \tag{B2a}$$

where

$$\Delta R_2 = a_2 \, \Delta\theta - (R_1)_n \, \Delta\gamma + N_1 \cos \alpha_n \, \Delta\alpha + T \cos(\alpha + \delta)_n \, \Delta\delta \tag{B2b}$$

$$a_2 = T \cos(\alpha + \delta)_n - F_A \cos \alpha_n - N \sin \alpha_n \tag{B2c}$$

The nominal case is assumed to be a no-wind trajectory. Since the change in relative velocity from the nominal caused by the wind disturbance is small, the term $m\dot{\gamma}_n \, \Delta V$ in equation (B2a) can be neglected compared to $mV_n \, \Delta\dot{\gamma}$, and the equation reduces to

$$\Delta\dot{\gamma} = \frac{\Delta R_2}{mV_n} \tag{B3}$$

Linearizing equations (B1c) and (B1d) gives

$$\Delta\delta = \frac{1}{Tl_c \cos \delta_n} (N_1 l_a \, \Delta\alpha - I \, \Delta\ddot{\theta}) \tag{B4a}$$

$$\Delta\gamma = \Delta\theta - \Delta\alpha - \alpha_w \tag{B4b}$$

Eliminating $\Delta\delta$ and $\Delta\gamma$ in equations (B2b), (B3), (B4a), and (B4b) gives the change in attitude in terms of the change in angle of attack and wind-induced angle of attack; that is,

$$\frac{I \cos(\alpha + \delta)_n}{mV_n l_c \cos \delta_n} \Delta\ddot{\theta} + \Delta\dot{\theta} + a_1 \, \Delta\theta = \Delta\dot{\alpha} + c_1 \, \Delta\alpha + \dot{\alpha}_w + b_1 \alpha_w$$

This equation can be reduced further by assuming the inertial effects of the vehicle on the change in attitude to be small (i.e., I = 0). Then,

$$\Delta\dot{\theta} = a_1 \, \Delta\theta = \Delta\dot{\alpha} + c_1 \, \Delta\alpha + \dot{\alpha}_w + b_1 \alpha_w \tag{B5a}$$

where

$$a_1 = -\frac{W_e}{mV_n} \tag{B5b}$$

$$b_1 = \frac{(R_1)_n}{mV_n} = \frac{1}{mV_n}\left[T \cos(\alpha + \delta)_n - F_A \cos \alpha_n - N \sin \alpha_n - W_e\right] \tag{B5c}$$

$$c_1 = b_1 + \frac{N_1}{mV_n}\left[\cos \alpha_n + \frac{l_a \cos(\alpha + \delta)_n}{l_c \cos \delta_n}\right] \tag{B5d}$$

$$W_e = mg \sin \gamma_n \tag{B5e}$$

Equations (B5) are the basic equations used in the analysis.

Equations in Yaw Plane

The yaw plane is defined by the vector perpendicular to the launch azimuth plane at the vehicle's center of gravity and the projection of the longitudinal axis in the azimuth plane. The basic vehicle configuration in the yaw plane is given on figure 24. The equations of motion are

$$F_{\parallel V} = R_1 = m\dot{V} = T \cos(\theta - \gamma + \delta) - F_A \cos(\theta - \gamma) - N \sin(\theta - \gamma) - mg \sin \theta_p \cos \gamma \tag{B6a}$$

$$F_{\perp V} = R_2 = mV\dot{\gamma} = T \sin(\theta - \gamma + \delta) - F_A \sin(\theta - \gamma) + N \cos(\theta - \gamma) + mg \sin \theta_p \sin \gamma \tag{B6b}$$

$$M_{CG} = I\ddot{\theta} = Nl_a - Tl_c \sin \delta \tag{B6c}$$

$$\alpha = \theta - \gamma - \alpha_w \tag{B6d}$$

$$\sin \alpha_w = \frac{V_w}{V_r} \cos \gamma \tag{B6e}$$

$$N = N_1 \alpha + N_2 \tag{B6f}$$

Linearizing equation (B6b) and assuming the change in velocity caused by wind disturbance to be small gives

$$\Delta\dot{\gamma} = \frac{1}{mV_n} \Delta R_2 \tag{B7a}$$

where

$$\Delta R_2 = a_2 \Delta\theta - (R_1)_n \Delta\gamma + N_1 \cos \alpha_n \Delta\alpha + T \cos(\alpha + \delta)_n \Delta\delta \tag{B7b}$$

$$a_2 = T \cos(\alpha + \delta)_n - F_A \cos \alpha_n - N \sin \alpha_n \tag{B7c}$$

Linearizing equations (B6c) and (B6d) gives

$$\Delta\delta = \frac{1}{Tl_c \cos \delta_n} (N_1 l_a \Delta\alpha - I \Delta\ddot{\theta}) \tag{B8a}$$

$$\Delta\gamma = \Delta\theta - \Delta\alpha - \alpha_w \qquad \text{since } (\alpha_w)_n = 0 \tag{B8b}$$

Combining equations (B7) and (B8) and assuming the inertial effect to be negligible gives the attitude change in terms of the change in angle of attack and wind-induced angle of attack

$$\Delta\dot{\theta} + a_1 \Delta\theta = \Delta\dot{\alpha} + c_1 \Delta\alpha + \dot{\alpha}_w + b_1 \alpha_w \tag{B9a}$$

where

$$a_1 = -\frac{W_e}{mV_n} \tag{B9b}$$

$$b_1 = \frac{(R_1)_n}{mV_n} = \frac{1}{mV_n} \left[T \cos(\alpha + \delta)_n - F_A \cos \alpha_n - N \sin \alpha_n - W_e\right] \tag{B9c}$$

$$c_1 = b_1 + \frac{N_1}{mV_n}\left[\cos \alpha_n + \frac{l_a \cos(\alpha + \delta)_n}{l_c \cos \delta_n}\right] \tag{B9d}$$

$$W_e = mg \sin \theta_p \cos \gamma_n \tag{B9e}$$

Note that these equations are identical with equations (B5) derived in the pitch plane, with the exception of the equivalent weights given by equations (B5e) and (B9e). Therefore, the same equations may be used in the analysis in both pitch and yaw planes by using the appropriate equivalent weights given by equations (B5e) and (B9e), respectively.

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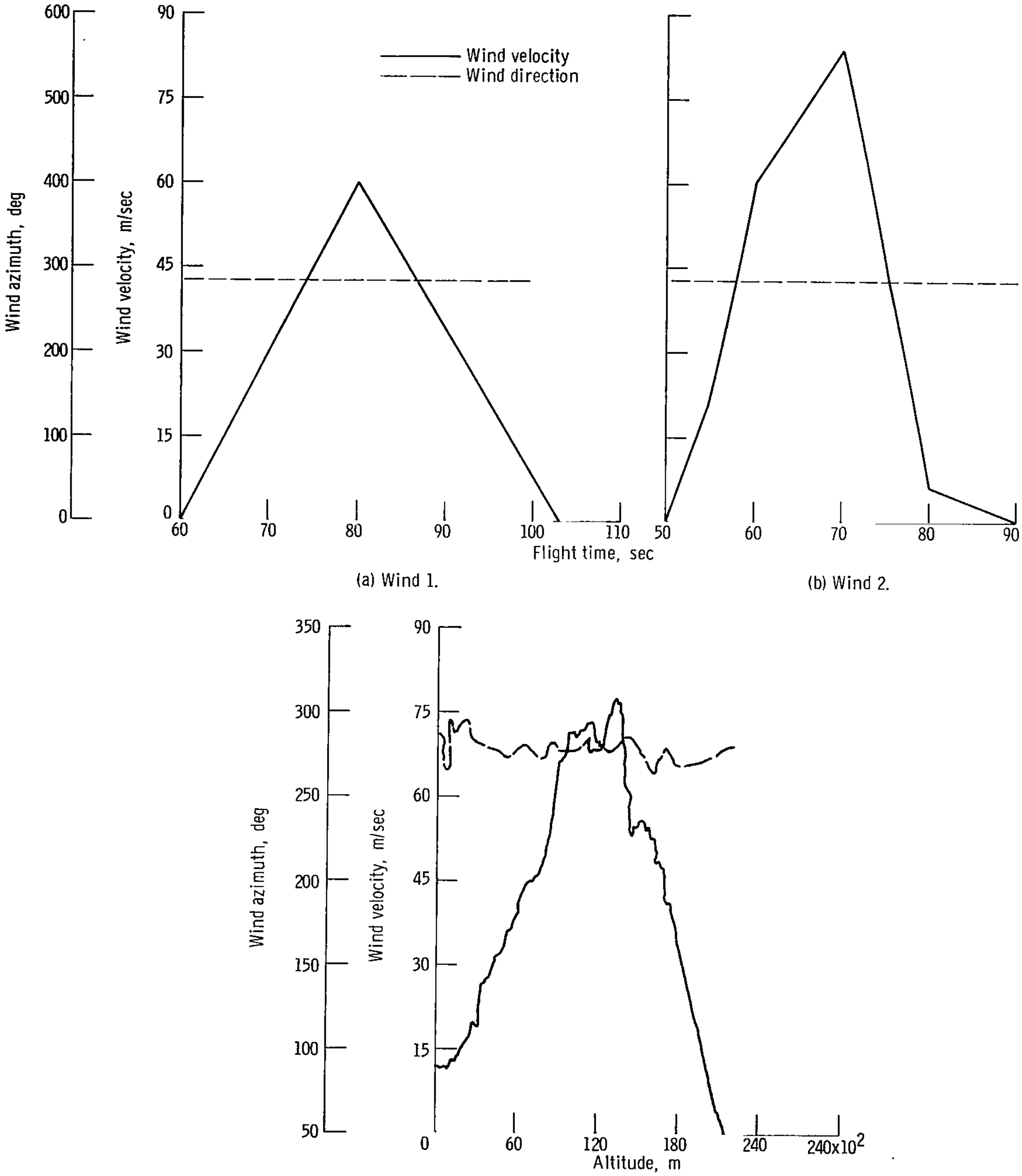
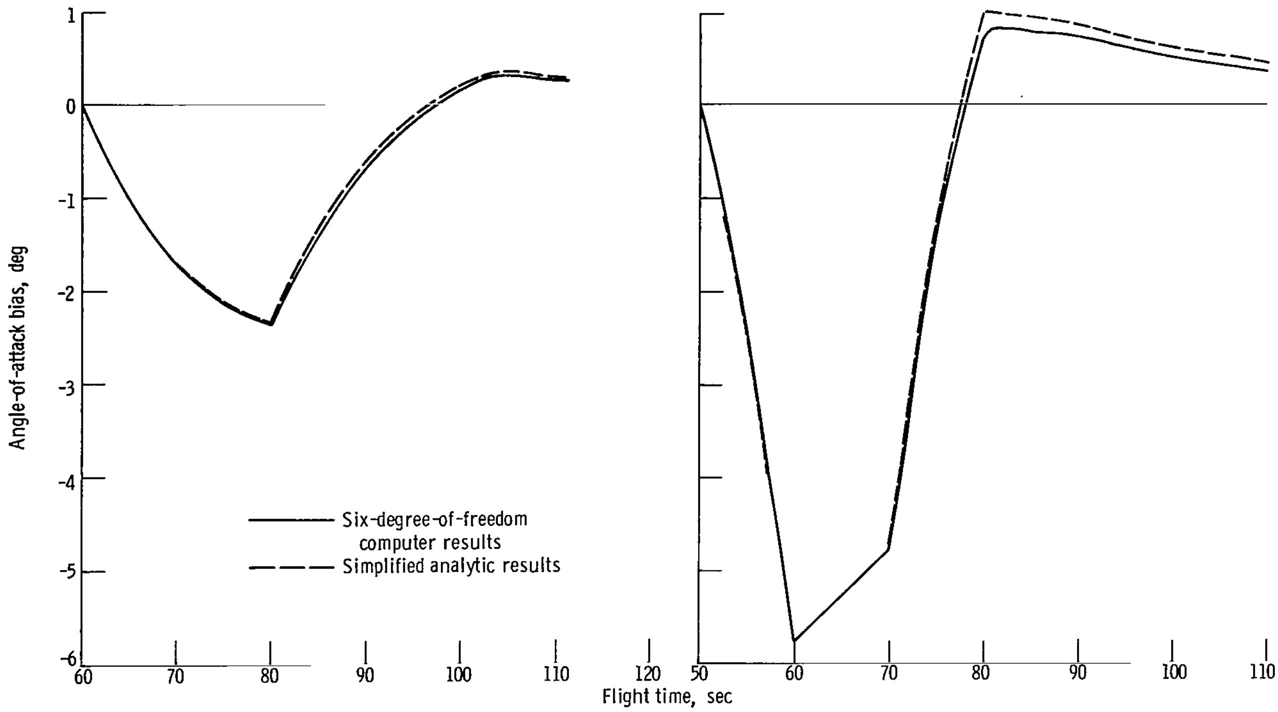


(a) Wind 1.

(b) Wind 2.

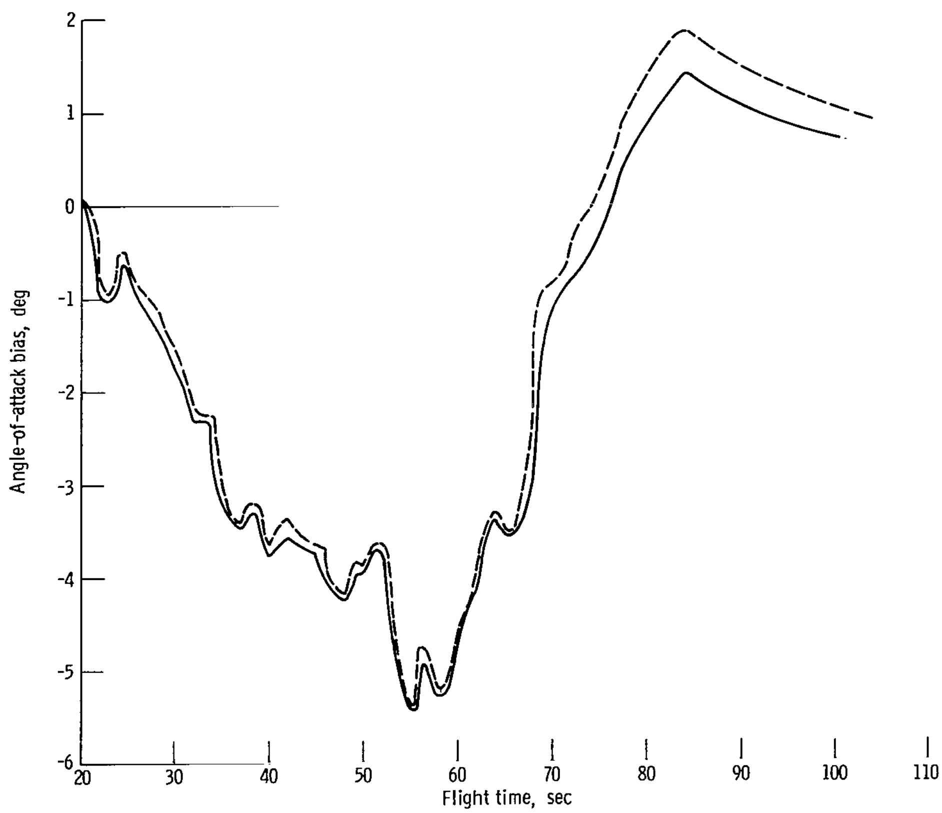
(c) Wind 3.

Figure 1. - Wind velocity and direction profiles.



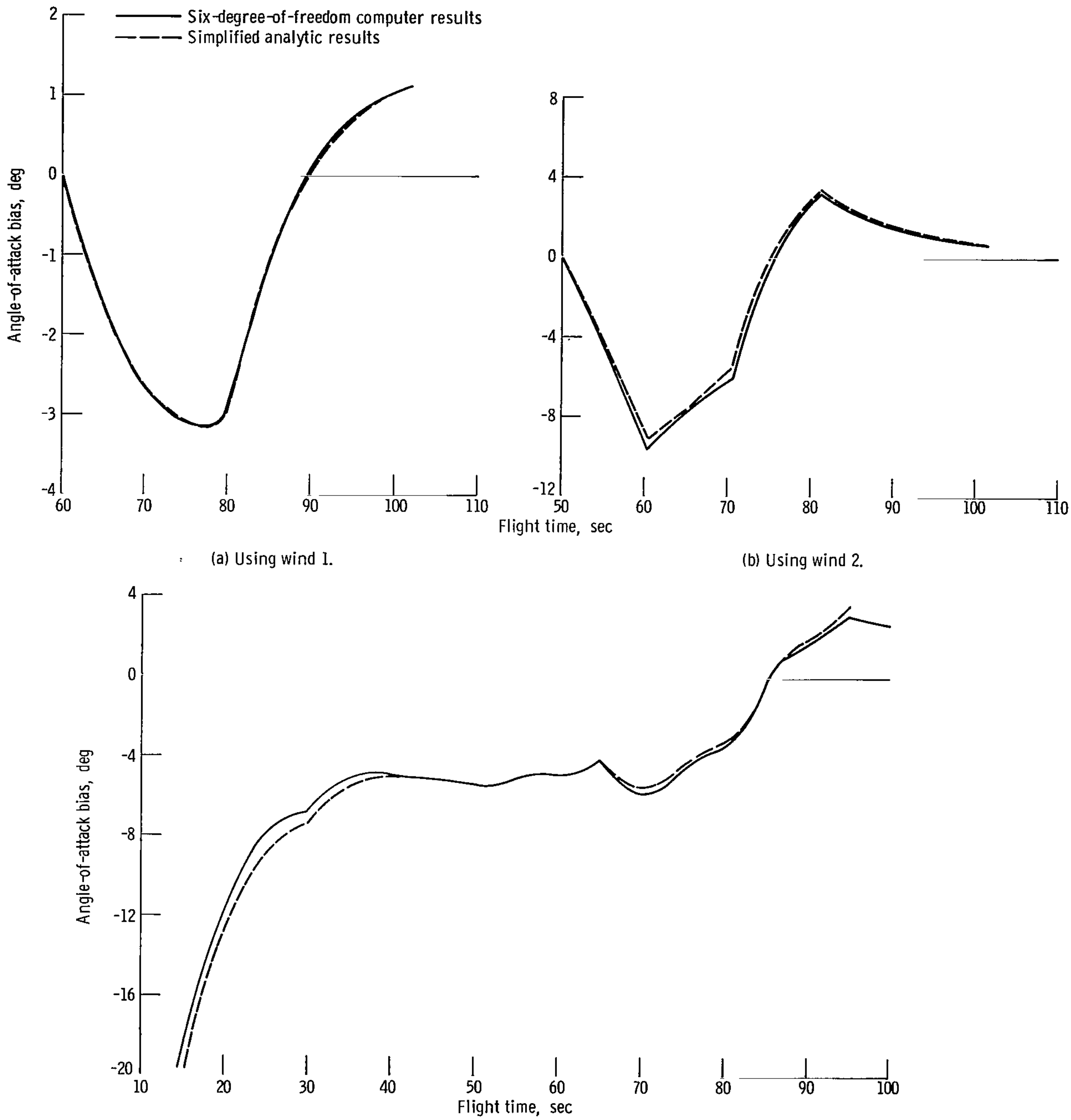
(a) Using wind 1.

(b) Using wind 2.



(c) Using wind 3.

Figure 2. - Change in angle of attack from nominal, for configuration I.

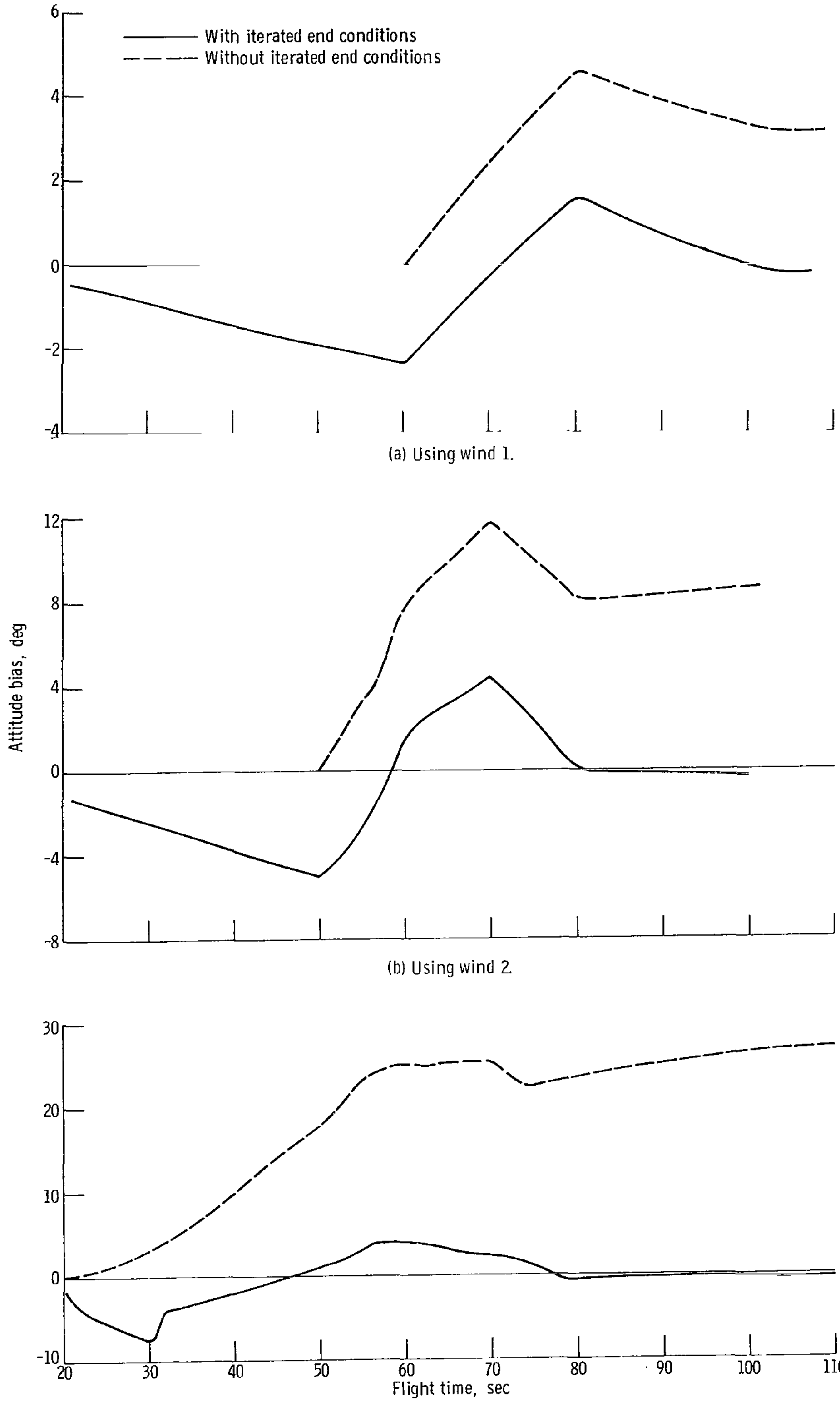


(a) Using wind 1.

(b) Using wind 2.

(c) Using wind 3.

Figure 3. - Change in angle of attack from nominal, for configuration II.



(a) Using wind 1.

(b) Using wind 2.

(c) Using wind 3.

Figure 4. - Simplified analytic attitude bias from nominal, for configuration I.

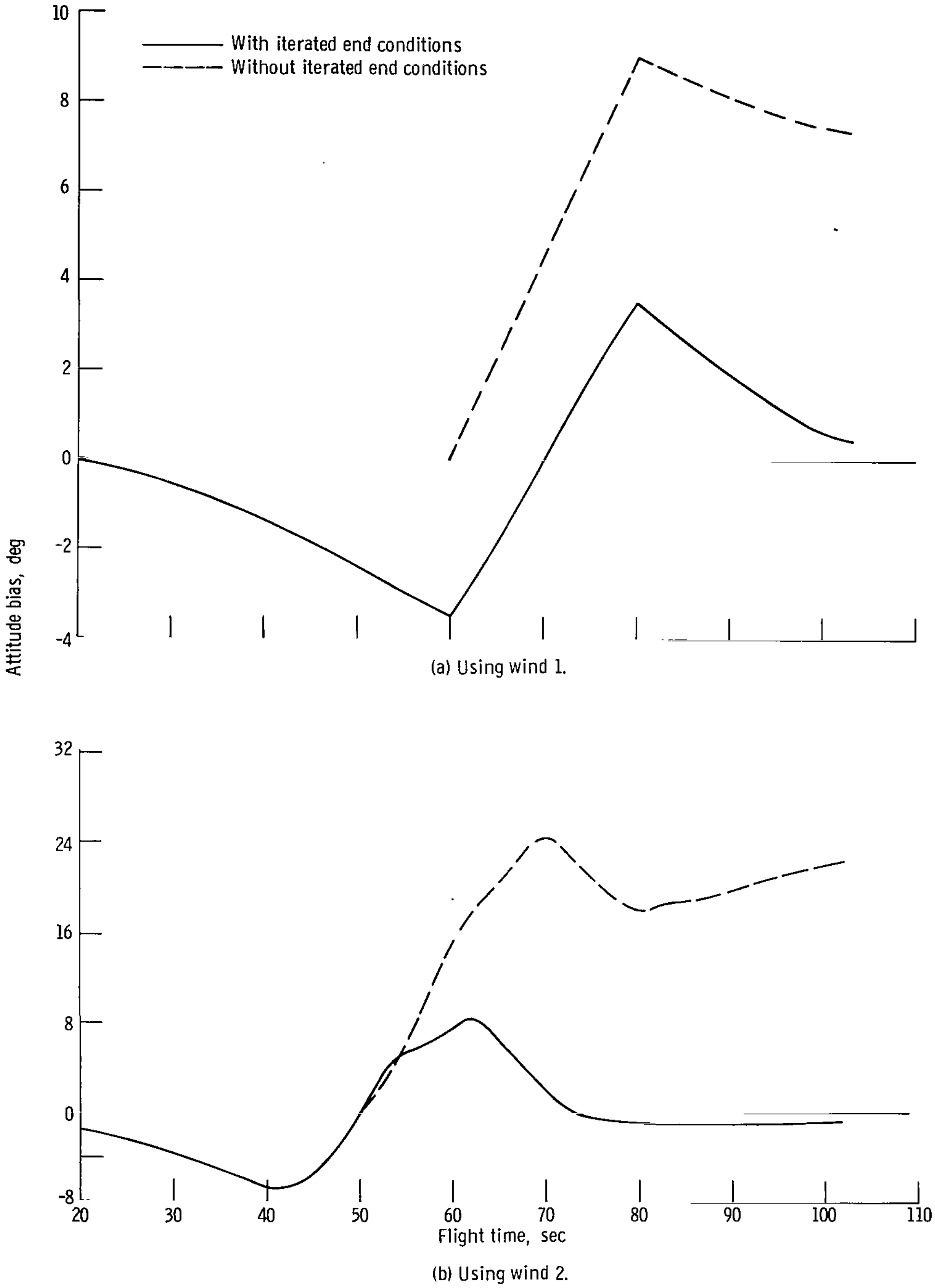
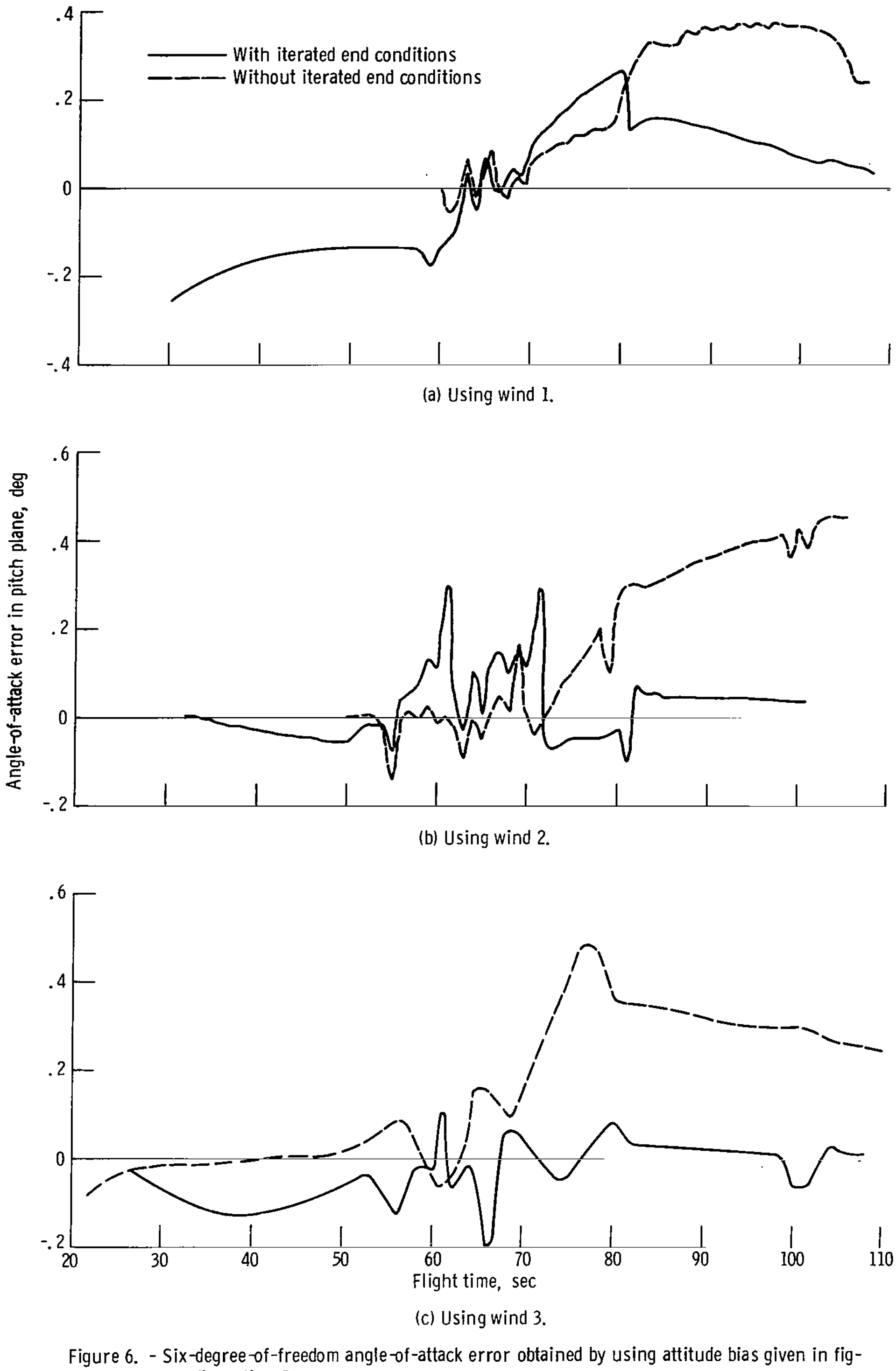


Figure 5. - Simplified analytic attitude bias from nominal, for configuration II.



(c) Using wind 3.

Figure 6. - Six-degree-of-freedom angle-of-attack error obtained by using attitude bias given in figure 4, for configuration I.

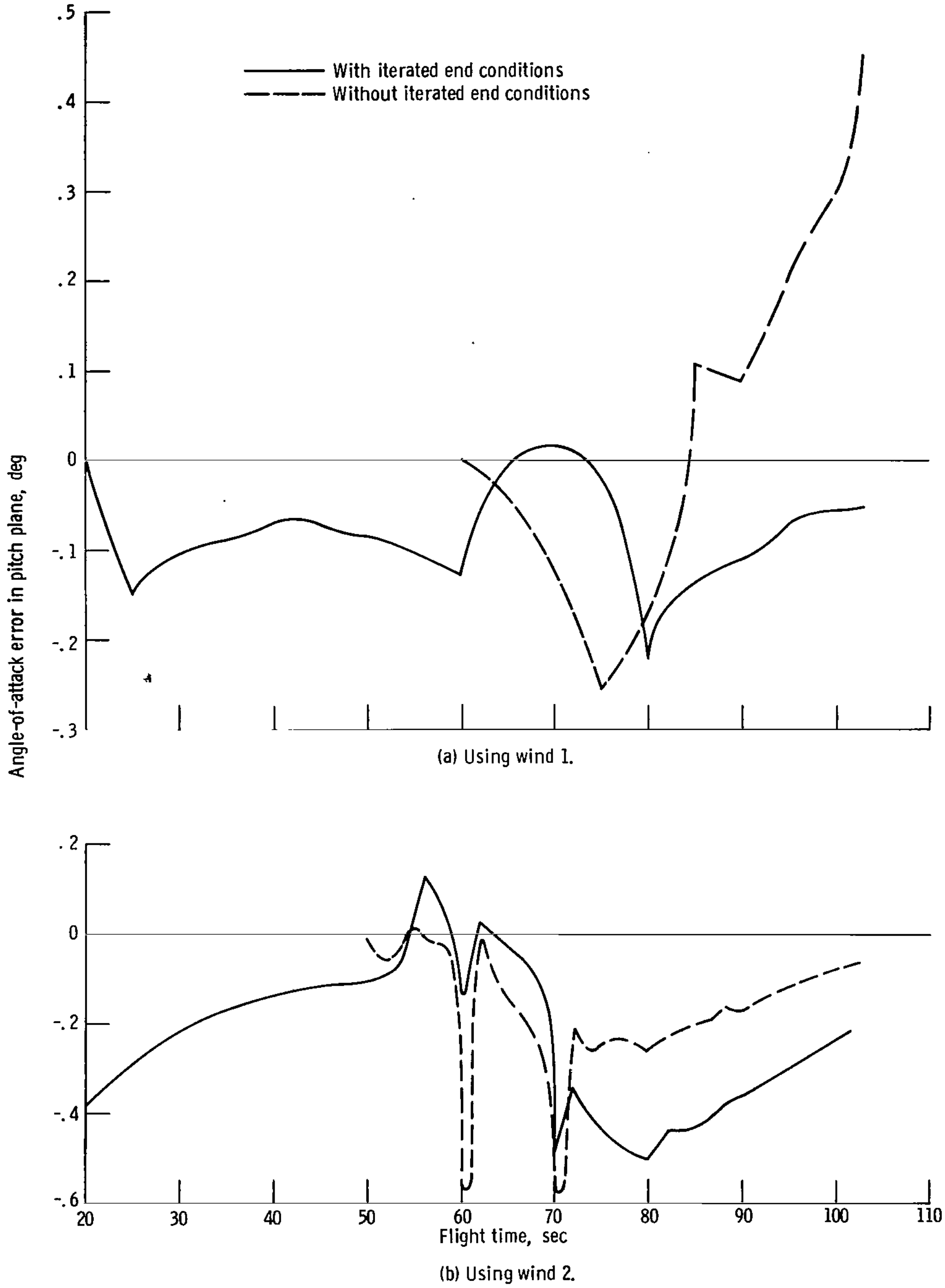
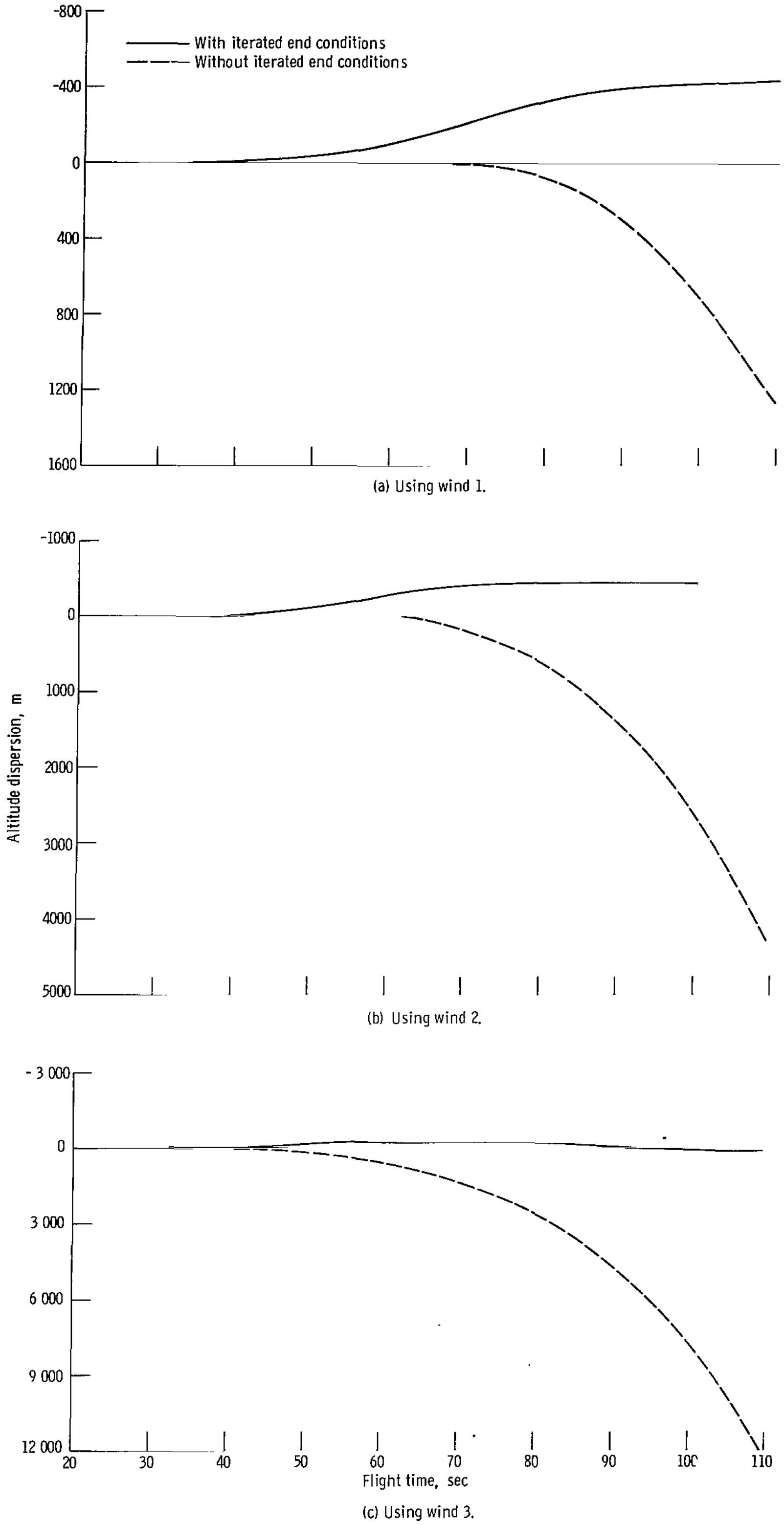


Figure 7. - Six-degree-of-freedom angle-of-attack error obtained by using attitude bias given in figure 5, for configuration II.

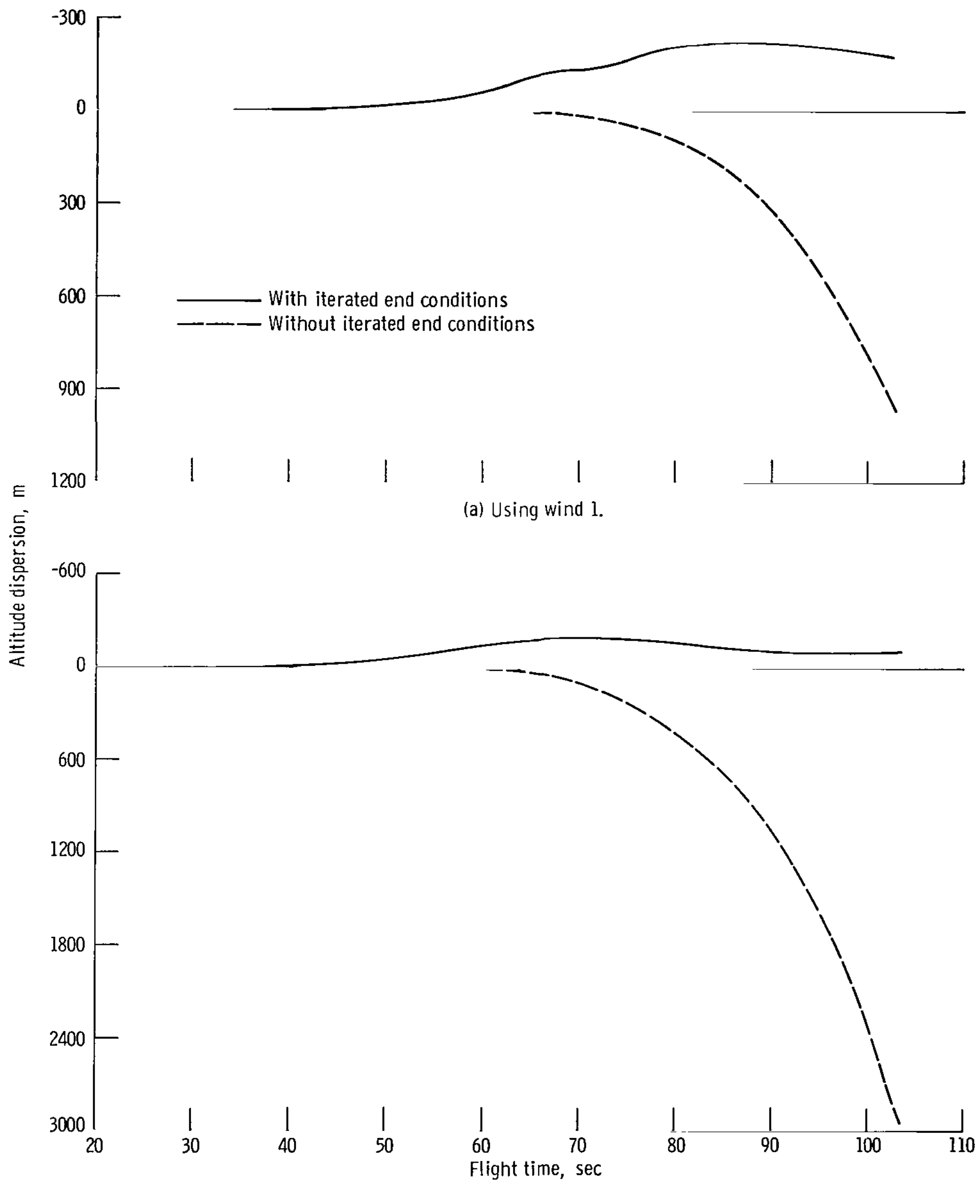


(a) Using wind 1.

(b) Using wind 2.

(c) Using wind 3.

Figure 8. - Six-degree-of-freedom altitude deviation from nominal attitude bias given in figure 4, for configuration I.



(b) Using wind 2.

Figure 9. - Six-degree-of-freedom altitude deviation from nominal, using attitude bias given in figure 5. for configuration II.

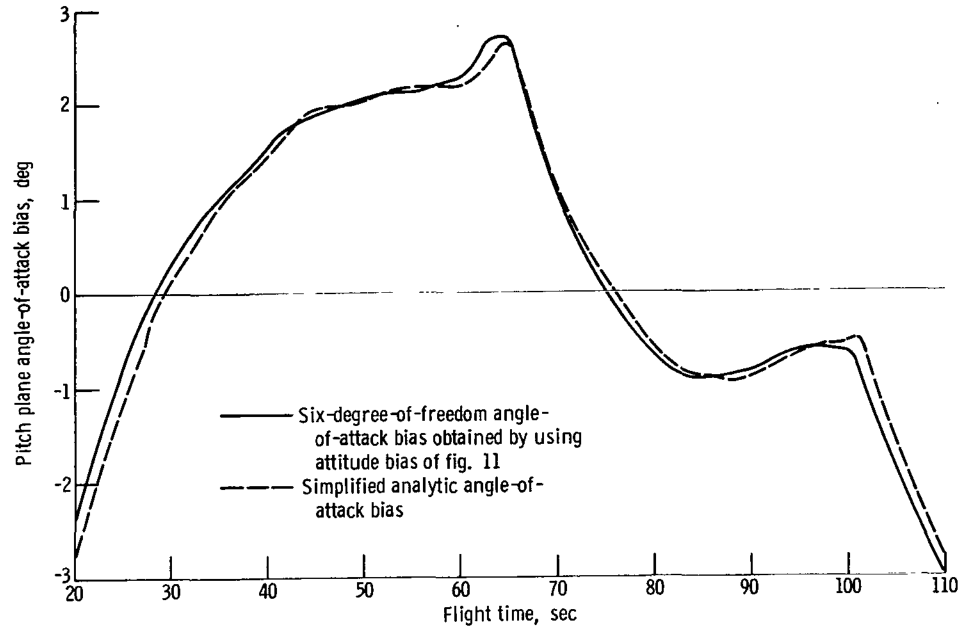


Figure 10. - Pitch plane angle-of-attack bias obtained by using the simplified analytic equations and the sample of 100 March winds, for configuration I.

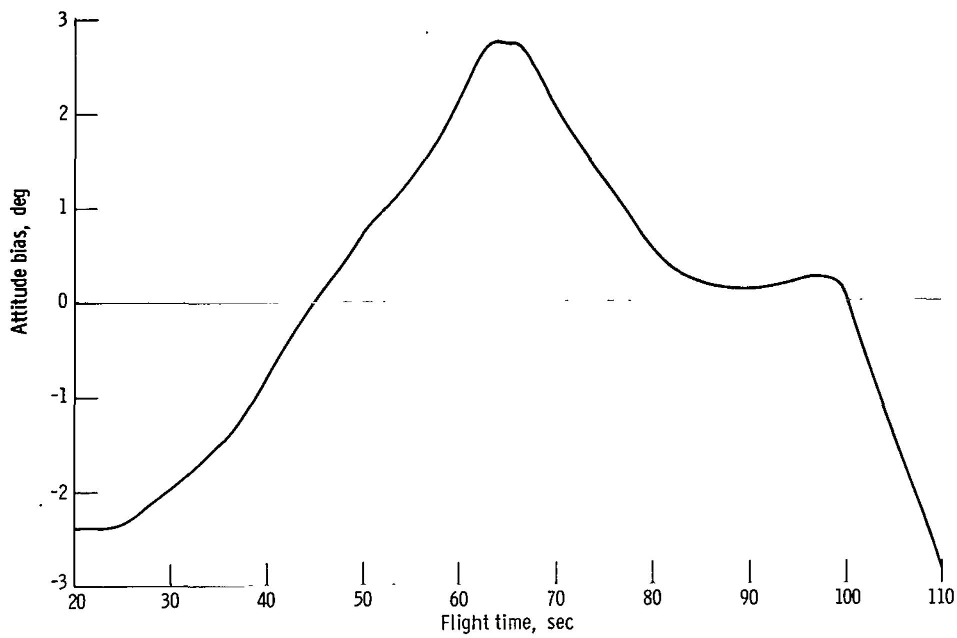


Figure 11. - Simplified analytic attitude bias using angle-of-attack bias of figure 10, for configuration I.

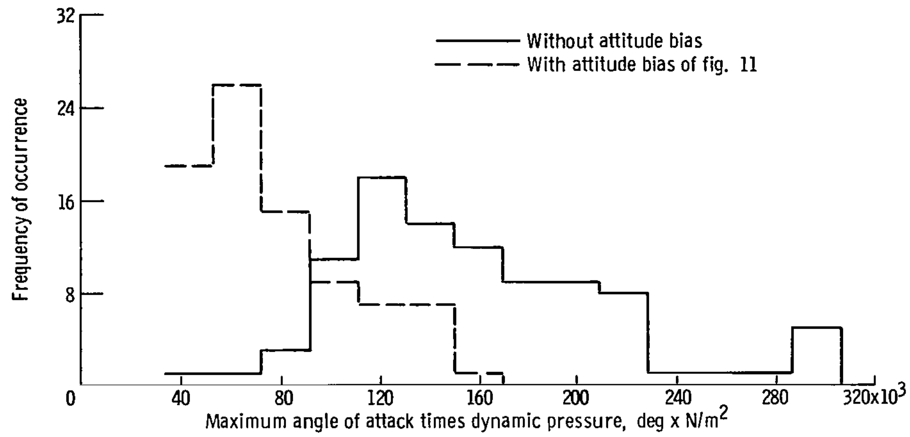


Figure 12. - Histogram for distribution of 100 March winds obtained by using simplified analytic equations, for configuration I.

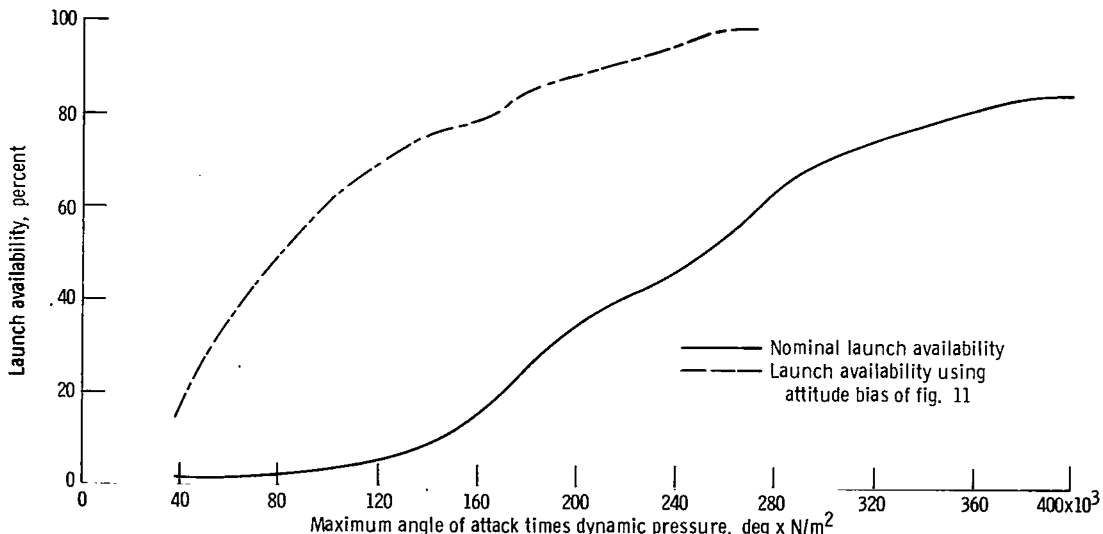


Figure 13. - Launch availability derived by using simplified analytic equations with and without attitude bias of figure 11, for configuration I.

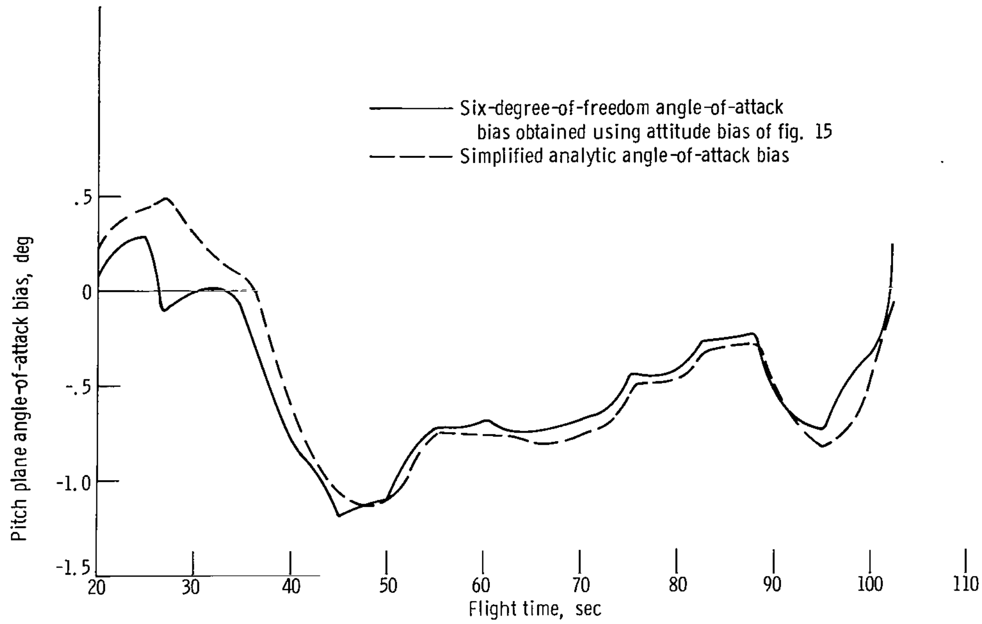


Figure 14. - Pitch-plane angle-of-attack bias obtained by using the simplified analytic bias and the sample of 100 March winds, for configuration II.

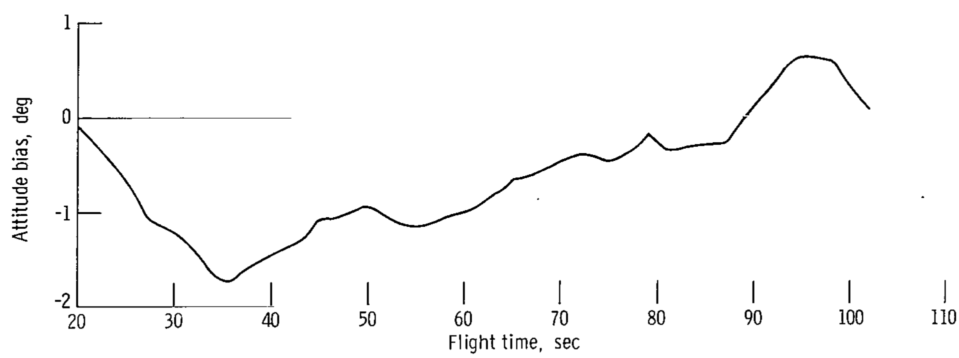


Figure 15. - Simplified analytic attitude bias using angle-of-attack bias of figure 14, for configuration I.

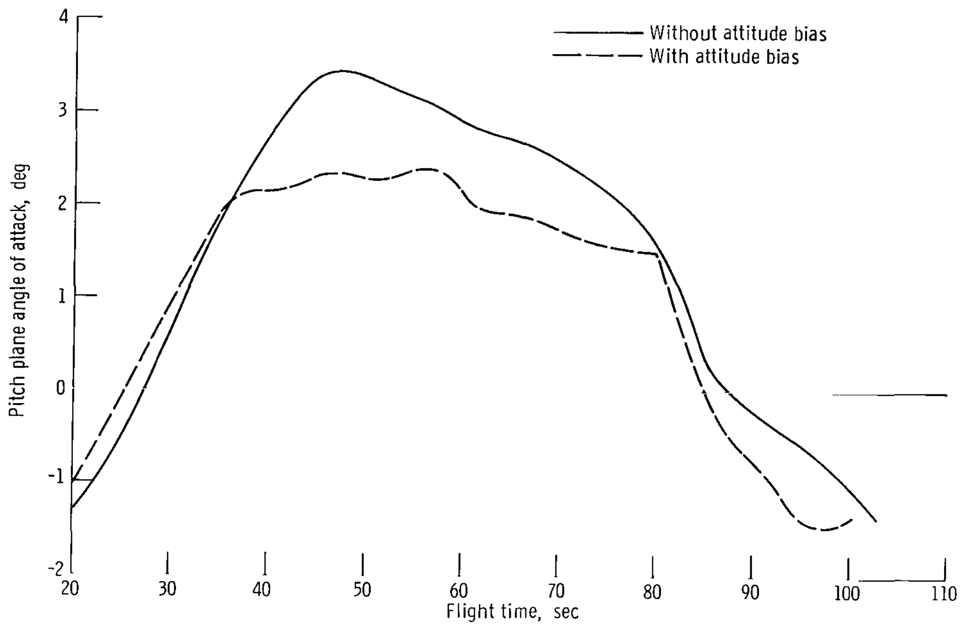


Figure 16. - Six-degree-of-freedom no-wind total angle of attack with and without attitutde bias of figure 15, for configuration II.

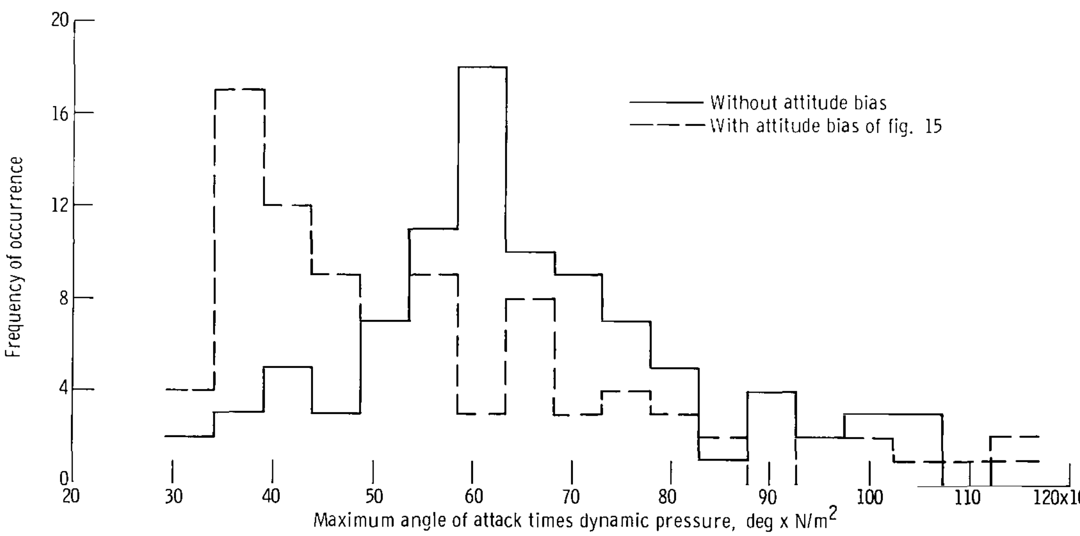


Figure 17. - Histogram for distribution of 100 March winds obtained by using the simplified analytic equations, for configuration II.

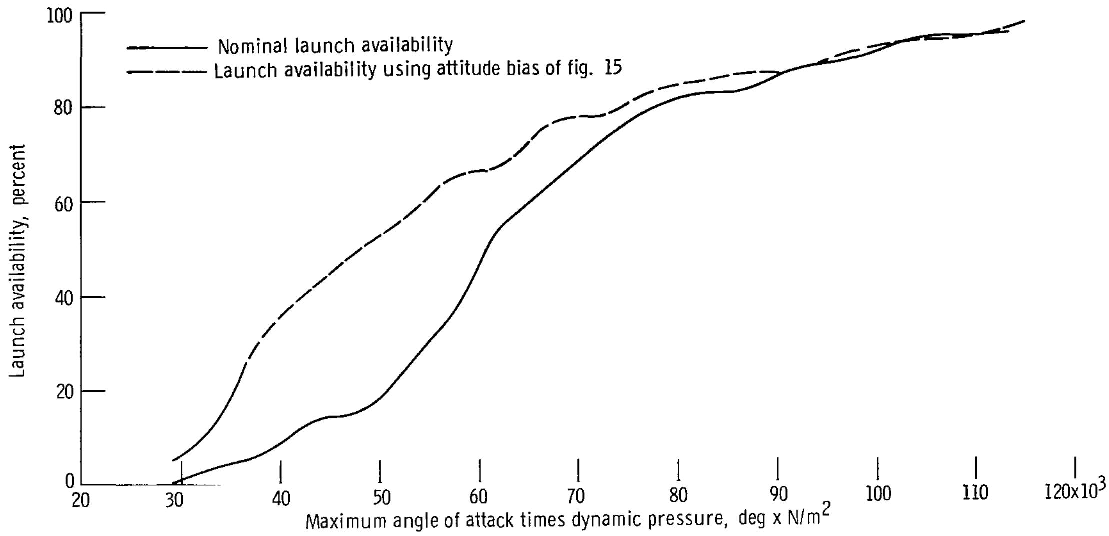


Figure 18. - Launch availability derived by using the simplified analytic equations with and without attitude bias of figure 15, for configuration II.

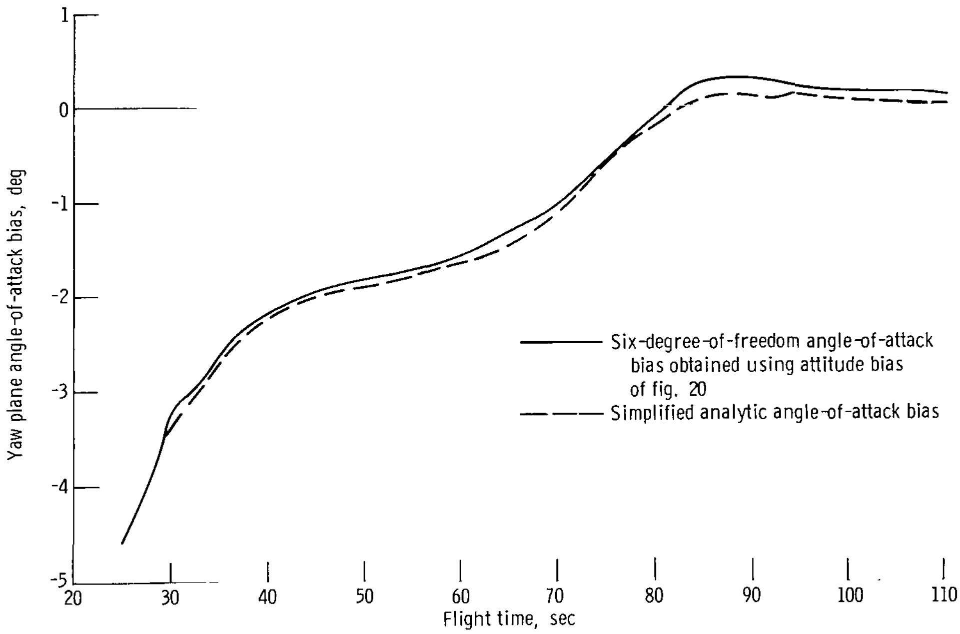


Figure 19. - Yaw plane angle-of-attack bias obtained by using the simplified analytic equations and the sample of 100 March winds, for configuration I.

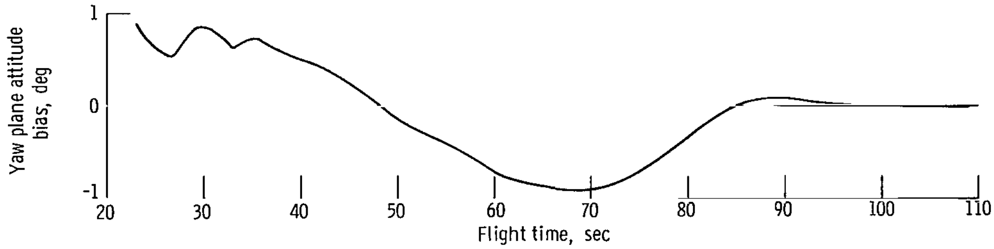


Figure 20. - Simplified analytic attitude bias in yaw plane using angle-of-attack bias of figure 19, for configuration I.

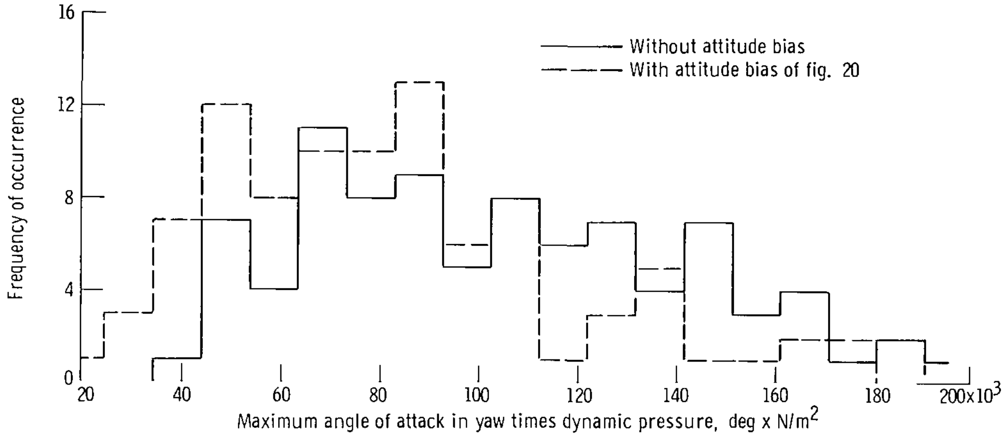


Figure 21. - Histogram for distribution of 100 March winds obtained by using the simplified analytic equations, for configuration I.

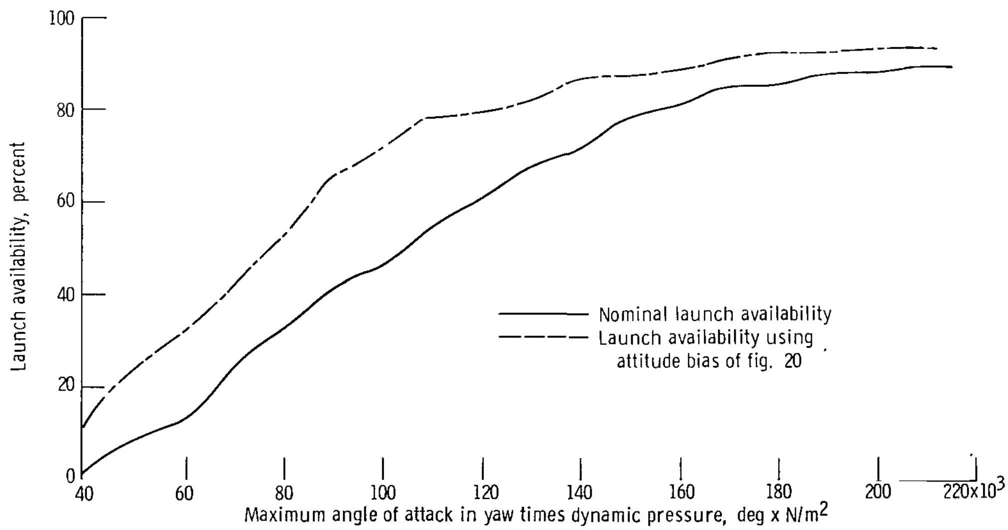


Figure 22. - Launch availability derived by using simplified analytic equations with and without attitude bias of figure 20, for configuration I.

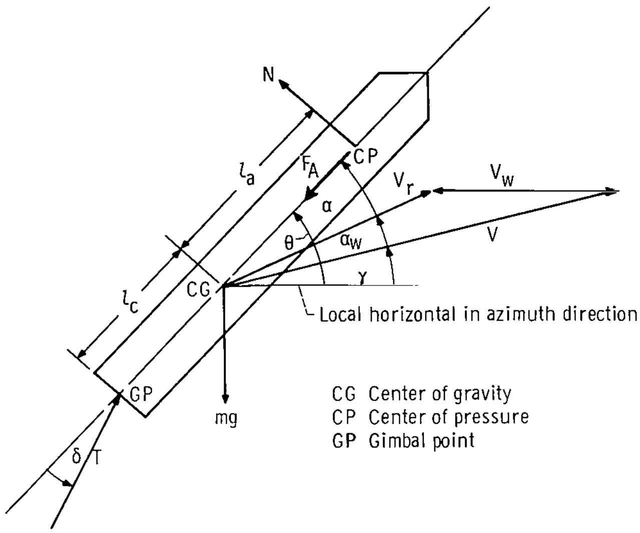


Figure 23. - Basic vehicle configuration in pitch plane.

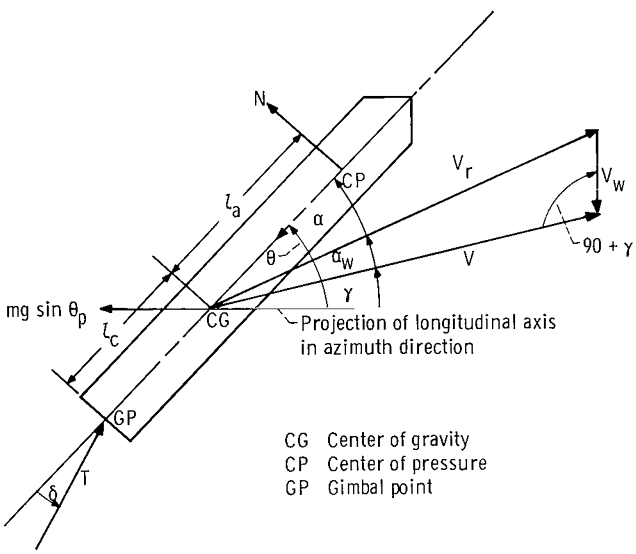


Figure 24. - Basic vehicle configuration in yaw plane.

NASA-Langley, 1970 —— 30 E-5590