TRANSONIC FLOW IN A
CONVERGING-DIVERGING NOZZLE
FINAL REPORT

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Lockheed
MISSILES & SPACE COMPANY
A GROUP DIVISION OF LOCKHEED AIRCRAFT CORPORATION
SUNNYVALE, CALIFORNIA
TRANSONIC FLOW IN A
CONVERGING-DIVERGING NOZZLE

FINAL REPORT

Contract NAS7-743

by

R. J. Prozan

APPROVED:

J. W. Benefield, Supervisor
Propulsion Section

G. D. Reny, Manager
Aeromechanics Dept.

J. S. Farrior
Resident Director
FOREWORD

This document presents the results of work performed by Lockheed Missiles & Space Company, Huntsville Research and Engineering Center, Huntsville, Alabama, under Contract NAS7-743, "Transonic Flow in a Converging-Diverging Nozzle."

The Contracting Agency is the National Aeronautics and Space Administration, Liquid Rocket Research and Technology Branch, Washington, D.C. The technical manager is Dr. R. F. Cuffel of the Jet Propulsion Laboratory, Pasadena, California.
SUMMARY

This document constitutes the final report of a study of steep angle nozzle inlets performed by Lockheed's Huntsville Research & Engineering Center under contract to the National Aeronautics and Space Administration. To be delivered with the final report is a computer program used to perform the calculations described herein.

Eight cases of interest to the National Aeronautics and Space Administration Jet Propulsion Laboratory, were analyzed and the results are presented in the main body of this report. These contours had steep inlets (75, 90 degrees) and very small nozzle wall radius of curvature to throat radius ratios (0.5, 0.25). The finite-difference techniques used in the analysis performed very well, particularly when the severity of the contours is considered.

Attached to the main body of this report are three appendixes. Appendix A discusses the theoretical approach to the finite difference solution reported herein. Appendix B is concerned with the computer program portion of the study. Appendix C contains the distribution list for this study.

The computer program described in Appendix B is considered to be extremely flexible, permitting several modes of operations for boundary conditions and grid structures. Many program control options are available for efficient utilization.
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**APPENDIXES:**

A: The Error Minimization Techniques as Applied to Transonic Flow in a Converging-Diverging Duct A-1

B: Converging-Diverging Nozzle Analysis Computer Program B-1

C: Distribution List for Contract NAS7-743 C-1
NOMENCLATURE

M  Mach number
M* Mach number on wall at minimum area
m  radial station identifier
n  axial station identifier
q  velocity
R  gas constant
T  temperature
x, r cylindrical coordinates
u, v axial and radial velocity components in cylindrical coordinates system

Greek

α  defined in text
γ  ratio of specific heats
ρ  density
ξ, ψ transformed coordinates

Subscripts and Superscripts

m radial index
m_T total number radial stations
n axial index
n_s minimum area axial station
n_T total number axial stations
' dimensional quantity
Section 1
INTRODUCTION

Transonic solutions for the converging-diverging region (throat) of rocket engines have occupied considerable attention in the scientific community. Contours in this region can greatly influence heat transfer and performance of the engines and, therefore, overall systems design. Analysis efforts in the past have centered around polynomial expansions of the potential function in the throat region. This approach, however, was limited until recently by throat radius of curvature to throat radius ratios that are generally exceeded in current designs. Moreover, these attempts did not consider the influence of inlet geometry for more than a short distance upstream of the physical throat.

Other attempts to describe these flow fields have utilized asymptotic time-dependent solutions to the finite difference equations of motion. Lockheed/Huntsville produced a program of this type which also considered non-isoenergetic flow. This program as well as programs produced by other organizations were plagued by stability problems inherent in the numerical analysis technique.

In an attempt to resolve the stability problem, Lockheed/Huntsville developed the Error Minimization technique. This numerical analysis approach has been successfully applied to transonic solutions with and without fuel striations. The unusual nozzle configurations of current interest to NASA/JPL, however, require a redevelopment of the coordinate system and the equations of motion in that system.

This study is divided into four tasks as follows:

1. Calculation of relatively steep inlet configurations which are within the capability of the existing program
2. Redevelopment of the equations of motion to consider inlet angles approaching 90 degrees
3. Execution of the computer program of Task 2, and
4. Documentation of the computer program and user's manual.

The first three items are discussed in the main body of this report and the last item is discussed in Appendix B.
Section 2
TECHNICAL DISCUSSION

The study effort identified in the Introduction of this report is divided, according to the Statement of Work, into three separate and distinct tasks (excluding documentation). In the interest of clarity, these tasks will be identified and discussed in chronological order.

TASK I - Statement

Utilizing the computer program described in Ref. 1, obtain the inviscid subsonic-transonic-supersonic flow field for four axisymmetric nozzle configurations. These are actually two nozzles each with different inlet configurations. The two nozzles have a steep convergent half-angle of approximately 75 deg. The divergent half angle is 15 deg and the throat radius is 0.8 in. One nozzle has a throat radius of curvature to throat radius ratio of 0.5 and the other 0.25. For each of these two nozzles the inlets will consist of:

- A conical inlet extending upstream to infinity.
- An axial flow inlet from a tube (contraction area ratio = 9.766) with transition to the convergent cone of the nozzle by means of a gradual turn which has a radius of curvature to throat radius ratio of 1.25.

To compute the flow field, an error-minimization technique is used to successively reduce the errors of the local finite-difference equations at the nodal points in a stretched Cartesian coordinate system.

TASK I - Discussion

The computer program described in Ref. 1 was not designed to handle the extremely steep inlet geometry which is the subject of this study. To
provide interim results, however, this task was defined with a configuration of interest which could possibly be treated with the existing analysis.

Figure 1 depicts the Task I nozzle configurations. An attempt was made to analyze these nozzles with the analysis method of Ref. 1 but the computer program was not capable of treating these severe cases. The inability of the existing analysis to treat the Task I configurations, however, was not a serious setback because these same configurations were to be analyzed using an improved solution developed under Task II.

**Task II — Statement**

Reformulate the inviscid flow problem discussed in Task I to adequately treat very steep inlet angles (up to and including 90 deg). This may be accomplished by a transformation applied to the conservation equations such that grid lines in the transformed plane map into circular arcs which are normal to the centerline and normal to conical walls of the nozzle. This program will be written in ASA standard FORTRAN V language and will be operational on a Univac computer.

**Task II — Discussion**

**Governing Equations:** To simplify the subsequent analysis it is assumed that the working gas in the nozzles to be analyzed is an ideal perfect gas and that the flow is isoenergetic, inviscid, adiabatic and isentropic. Under these assumptions the conservation equations in cylindrical coordinates reduce to:

\[
\frac{\partial (\rho' u')}{\partial x} + \frac{\partial (\rho' v')}{\partial r} + \frac{\rho' v'}{r} = 0 \quad \text{(conservation of mass)} \tag{1}
\]

\[
\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial r} = 0 \quad \text{(condition of irrotationality)} \tag{2}
\]
Fig. 1 - Task 1 Nozzle Geometries
From the above assumptions we may write

\[
\frac{\rho'}{\rho_c} = \left( \frac{T'}{T_c} \right)^{1/(y-1)}
\]  
(3)

where

\[
\frac{T'}{T_c} = 1 - \frac{y-1}{2yR} (u'^2 + v'^2)
\]  
(4)

For the sake of convenience a change of variables is made;

\[
\rho = \frac{\rho'}{\rho_c}; \quad T = \frac{T'}{T_c}; \quad u = \frac{u'}{\sqrt{yR T_c}}; \quad v = \frac{v'}{\sqrt{yR T_c}}
\]

The above equations become

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial r} + \frac{\partial v}{r} = 0 \quad (5a)
\]

\[
\frac{\partial v}{\partial x} - \frac{\partial u}{\partial r} = 0 \quad (5b)
\]

\[
\rho = T^{1/(y-1)} \quad (5c)
\]

\[
T = 1 - \frac{y-1}{2} (u^2 + v^2) \quad (5d)
\]

Consider the transformation to the coordinate system;

\[
x = x(\xi, \psi); \quad r = r(\xi, \psi)
\]
Under such a transformation Eqs. (5a) and (5b) become

\[
\frac{\partial r}{\partial \psi} \frac{\partial (\rho u)}{\partial \xi} - \frac{\partial x}{\partial \psi} \frac{\partial (\rho v)}{\partial \xi} + \frac{\partial x}{\partial \xi} \frac{\partial (\rho v)}{\partial \psi} - \frac{\partial r}{\partial \xi} \frac{\partial (\rho u)}{\partial \psi} + \frac{\partial \rho v}{r} = 0 \quad (6a)
\]

\[
\frac{\partial r}{\partial \psi} \frac{\partial v}{\partial \xi} - \frac{\partial r}{\partial \xi} \frac{\partial v}{\partial \psi} + \frac{\partial x}{\partial \xi} \frac{\partial u}{\partial \psi} - \frac{\partial x}{\partial \psi} \frac{\partial u}{\partial \xi} = 0 \quad (6b)
\]

where

\[
\alpha = \frac{\partial r}{\partial \psi} \frac{\partial x}{\partial \xi} - \frac{\partial r}{\partial \xi} \frac{\partial x}{\partial \psi}
\]

Figures 2 gives a schematic of the grid structure in the \( \xi, \psi \) plane (which is, of course, independent of the transformation). The grid is always centered and orthogonal even though the grid in the physical plane is not. The wall is always taken to correspond to \( \psi = 1 \) while the centerline is \( \psi = 0 \).

Recalling that \( \rho = \rho(u,v) \) we may rewrite Eq. (6a)

\[
\begin{align*}
\rho u_\xi - x_\psi v_\xi + x_\xi v_\psi - r_\xi u_\psi + \frac{\alpha v}{r} \\
- \frac{1}{T} \left[ (\rho u - x_\psi v) [uu_\xi + vv_\xi] + [x_\xi v - r_\xi u] [uu_\psi + vv_\psi] \right] = 0 \quad (7)
\end{align*}
\]

Using the nomenclature of Fig. 3 and using central differences the finite-difference analogs to Eqs. (7) and (6b) are written:
Fig. 2a - Grid Structure in m, x Plane

Fig. 2b - Grid Structure in $\xi$, $\psi$ Plane

Fig. 3 - Finite Difference Grid
Equations (8a) and (8b) must be satisfied for every grid point, (at least every interior grid point) in the field.

To complete the solution, equations governing the behavior of the wall, centerline, inlet and outlet points are needed. Unfortunately these equations are not currently embodied in the steady state theory, and approximate relations must be used.

\[
\bar{r}_\psi (u_2 - u_1) - \bar{x}_\psi (v_2 - v_1) + \bar{x}_\xi (v_4 - v_3) - \bar{r}_\xi (u_4 - u_3) + \frac{\bar{\alpha} v^o}{r^o} \\
- \frac{1}{T^o} \left\{ \left[ \bar{r}_\psi u^o - \bar{x}_\psi v^o \right] \left[ u^o (u_2 - u_1) + v^o (v_2 - v_1) \right] \right. \\
+ \left[ \bar{x}_\xi v^o - \bar{x}_\xi u^o \right] \left[ u^o (u_4 - u_3) + v^o (v_4 - v_3) \right] \right\} = 0 \quad (8a)
\]

\[
\bar{r}_\psi (v_2 - v_1) - \bar{x}_\psi (u_2 - u_1) - \bar{x}_\xi (u_4 - u_3) = 0 \quad (8b)
\]

where

\[
\bar{r}_\psi = r_4 - r_3; \quad \bar{x}_\psi = x_4 - x_3
\]

\[
\bar{r}_\xi = r_2 - r_1; \quad \bar{x}_\xi = x_2 - x_1
\]

and where

\[
\bar{\alpha} = \bar{r}_\psi \bar{x}_\xi - \bar{r}_\xi \bar{x}_\psi
\]
The upper wall condition is approximated by a linear extrapolation of the square of speed from adjacent points;

\[ q^2|_{\psi=1} = 2q^2|_{1-\Delta\psi} - q^2|_{1-2\Delta\psi} \]  \hspace{1cm} (9a)

and is used in conjunction with the tangency conditions,

\[ v|_{\psi=1} = \tan\theta_w u|_{\psi=1} \]  \hspace{1cm} (9b)

For the centerline, use is made of symmetry to yield (second order extrapolation);

\[ u|_{\psi=0} = \frac{1}{3} \left( 4u|_{\Delta\psi} - u|_{2\Delta\psi} \right) \]  \hspace{1cm} (9c)

and the tangency condition is given by

\[ v|_{\psi=0} = 0 \]  \hspace{1cm} (9d)

The inlet and outlet grid station will be discussed later.

**Numerical Solution of Governing Equations:** The previously derived relations must now be satisfied over the entire flow field. It should be noted that the governing equations are now nonlinear algebraic equations where \( u, v \) are independent variables rather than the original partial differential equations in which the spatial coordinates were independent.

The Error Minimization technique, first reported in Ref. 1 is used to effect the numerical solution. A discussion of this technique with application to this problem may be found in Appendix A, while a discussion of the computer program generated in this study may be found in Appendix B.
Task III – Statement

Utilizing the computer program developed under Task II, obtain the subsonic-transonic-supersonic solutions for eight nozzle configurations. Actually there are four nozzles, each with two different inlets. Two of the nozzles are the same as described in Task I. The other two differ only by the convergent half angle which is 90 deg instead of 75 deg. With each of these four nozzles the inlets described in Task I or their equivalents will be used. Figure 4 illustrates the additional nozzle configurations to be analyzed.

In general, as expected, the dominant parameter in these configurations is the radius ratio (radius of curvature of the throat/radius of the throat). Accordingly the configurations with identical radius ratios are presented together. Figure 5 gives the computer program results for the 0.5 radius ratio nozzles while Fig. 6 gives the results for the 0.25 radius ratio nozzles. Plotted in these figures are the wall and centerline Mach number distributions.
Fig. 4 - Task 2 Additional Nozzle Geometries
Mach Number vs.
Axial Location 0.5
Radius Ratio Nozzles
($\gamma = 1.4$)
Flow Coef. = 0.98

Fig. 5 - 0.5 Radius Ratio Nozzle Solutions
Mach Number vs Axial Location
0.25 Radius Ratio Nozzles
($\gamma = 1.4$)
Flow Coef. = .965

Fig. 6 - 0.25 Radius Ratio Nozzle Solutions
Section 3
RESULTS

The most striking result is, of course, that the Mach number distributions are virtually identical on the wall for those regions in which the contour is common to all cases on both the 0.5-radius-ratio and 0.25-radius-ratio nozzles. Although the scaling or weighting techniques described in Appendix A would tend to produce such a result this effect cannot be rationalized solely on that basis.

It can be concluded that the throat is indeed the boundary condition and that the inlet and outlet do indeed respond to the throat influence but only within certain bounds, i.e., a throat distribution which requires an impossible inlet distribution is prevented. As such the throat is not an absolute boundary condition. This conclusion is supported by the fact that in an elliptic region there is a mutual interplay or exchange of information.

Figure 7 illustrates the effect of radius ratio and inlet angle on the throat flow angle distribution for the reservoir inflow cases. The corresponding axial inflow results are again virtually identical. As can be seen, a noticeable alteration of the flow angle distribution occurs due to an inlet angle change from 75 deg to 90 deg for the 0.25-radius-ratio case. This effect is barely perceptible on the 0.5-radius-ratio case.

It also should be noted (Fig.6) that there exists a tendency to produce waves at the juncture of the throat circle and the exit cone. Whether this is caused by the large velocity gradients in this region or by the nozzle envelope shock wave has not yet been determined. Attempts to introduce smoothing utilizing the second derivatives of the velocity components (similar to pseudo-viscous terms in unsteady asymptotic analyses) have been only moderately successful.
Fig. 7 - Throat Flow Angle vs Radial Position for Reservoir Inflow Cases
Conclusions

The accuracy of the techniques used in this study have been verified for moderate inlet angles and throat radius ratios by Ref. 1. It is reasonable to expect that the generalization of the grid distribution system will, with judicious use enhance rather than detract from the accuracy. It is further to be expected that the accuracy of the results will improve with a finer grid system than used in the example calculations. The predictions presented in this report are consistent and reasonable. The precision of the results must be determined by comparison with experimental data.

Future Work

It is felt that this study has been a demonstration of the power of finite-difference techniques. Unfortunately, the science of numerical methods is imperfect. The conservation equations adequately describe the interior of the flow but boundary conditions are ill defined. Studies currently in progress at Lockheed/Huntsville inevitably lead to the conclusion that the major stumbling block in finite-difference analysis is the adequate knowledge and treatment of boundary conditions.

For numerical techniques to truly come into their own it is imperative that the proper boundary condition relationships be found. It is suggested that studies investigating these boundary conditions are a necessary next step in the pursuit of numerical solutions to complex flow fields.
REFERENCE

Appendix A
THE ERROR MINIMIZATION TECHNIQUES
AS APPLIED TO TRANSONIC FLOW IN A
CONVERGING-DIVERGING DUCT
NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, B, C, D</td>
<td>defined in text</td>
</tr>
<tr>
<td>G</td>
<td>solution merit function</td>
</tr>
<tr>
<td>g</td>
<td>local contribution to total error</td>
</tr>
<tr>
<td>\overline{T}</td>
<td>unit vector</td>
</tr>
<tr>
<td>n_s</td>
<td>minimum area axial station</td>
</tr>
<tr>
<td>q</td>
<td>velocity</td>
</tr>
<tr>
<td>\overline{s}</td>
<td>solution vector</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>u, v</td>
<td>velocity components in cylindrical systems</td>
</tr>
<tr>
<td>x, r</td>
<td>cylindrical coordinates</td>
</tr>
<tr>
<td>\rightarrow</td>
<td>replace operation</td>
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**Greek**

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</thead>
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<tr>
<td>\gamma</td>
<td>ratio of specific heats</td>
</tr>
<tr>
<td>\delta</td>
<td>step modifier</td>
</tr>
<tr>
<td>\epsilon</td>
<td>residual or error</td>
</tr>
<tr>
<td>\nu</td>
<td>any independent variable</td>
</tr>
<tr>
<td>\xi, \psi</td>
<td>transformed system coordinates</td>
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<tr>
<td>\Gamma</td>
<td>scale function</td>
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**Subscripts and Superscripts**

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<tr>
<td>m</td>
<td>radial station index</td>
</tr>
<tr>
<td>m_T</td>
<td>maximum radial station index</td>
</tr>
<tr>
<td>n</td>
<td>axial station index</td>
</tr>
<tr>
<td>n_T</td>
<td>maximum axial station</td>
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DISCUSSION

A.1 INTERIOR POINTS

In the main body of this report it was shown that the finite-difference analogs to the conservation of mass and the irrotationality condition were:

\[ \bar{\nabla} \left( u_2 - u_1 \right) - \bar{\nabla} \left( \gamma_1 - \gamma_2 \right) + \bar{\nabla} \left( \gamma_4 - \gamma_3 \right) - \bar{\nabla} \left( u_4 - u_3 \right) + \frac{\partial}{\partial x} \left( \frac{\gamma_2}{\lambda} \right) \]

\[ - \frac{1}{T_0} \left\{ \[ \bar{\nabla} \left( u_0 - \bar{\nabla} \psi \right) \left[ u_0 \left( u_2 - u_1 \right) + \gamma_2 \left( \gamma_2 - \gamma_1 \right) \right] \right\} \]

\[ + \left[ \bar{\nabla} \psi \left( \gamma_0 - \bar{\nabla} \psi \right) \right] \left[ u_0 \left( u_4 - u_3 \right) + \gamma_0 \left( \gamma_4 - \gamma_3 \right) \right] \right\} = 0 \quad (A.1) \]

\[ \bar{\nabla} \left( \gamma_2 - \gamma_1 \right) - \bar{\nabla} \left( \gamma_4 - \gamma_3 \right) + \bar{\nabla} \left( u_4 - u_3 \right) - \bar{\nabla} \left( u_4 - u_3 \right) = 0 \quad (A.2) \]

where

\[ \bar{\nabla} \psi = \psi_4 - \psi_3 \quad \bar{\nabla} \psi = \psi_4 - \psi_3 \]

\[ \bar{\nabla} \psi = \lambda \psi_3 \quad \bar{\nabla} \psi = \psi_3 - \psi_1 \]

\[ \bar{\nabla} \psi \psi_3 = \lambda \psi_3 \psi_3 - \bar{\nabla} \psi \psi_4 \]

For an arbitrary initial estimate of the flow conditions equations (1) and (2) will be in error at each interior grid point in the field. If an interior point is described by subscripts m,n then the errors in Eqs. (A.1) and (A.2) are \( \xi_{1,m,n} \) and \( \xi_{2,m,n} \), respectively.

A-2
The problem reduces to one of driving the errors at every point in the flow field to zero (or at least reducing them substantially).

Now let

\[ q_{m,n} = \xi_1 \gamma_{m,n} + \xi_2 \gamma_{m,n} \]  

(A.3)

Obviously \( q_{m,n} \) is positive definite and is zero only when the errors are zero. Further let

\[ G = \sum_{n=1}^{m-1} \sum_{m=2}^{n-1} q_{m,n} \]  

(A.4)

Again if \( q_{m,n} \) is zero for every \( m,n \) then \( G \) is a positive definite function which has an absolute minimum at the desired solution point.

From calculus

\[ dG = \nabla G \cdot d\xi = \nabla G \cdot \xi d\xi \]

where \( d\xi \) is an infinitesimal displacement along the unit vector \( \xi \). For any given displacement \( d\xi \), \( dG \) is a maximum when

\[ \frac{\nabla G \cdot \xi}{|\nabla G|} = 1 \]  

then

\[ dG = \nabla G \cdot \xi = |\nabla G| d\xi \]

but the largest possible change in \( G \) is

\[ dG \geq G^{(t)} - G^{(d)} = -G^{(d)} \]

A-3
if the minimum was found on the next step then

\[ dS = \frac{-\mathcal{G}}{|\nabla \mathcal{G}|} \]

while

\[ d\mathcal{S} = \mathcal{G}_d dS = \frac{-\mathcal{G} \cdot \nabla \mathcal{G}}{|\nabla \mathcal{G}|^2} \]

of course, \( d\mathcal{S} \) is the change in the independent vector, so that

\[ S^{(l+1)} = S^{(l)} \left( 1 - \left( \frac{\mathcal{G} \cdot \nabla \mathcal{G}}{|\nabla \mathcal{G}|^2} \right)^{(l)} \right) \]

The above expression is a first-order multivariable counterpart of the familiar Newton-Raphson recursion formula. It is quite possible that the value of \( \mathcal{S}^{(l+1)} \) may be greater than \( \mathcal{S}^{(l)} \). It may be recalled that the step length was calculated under the optimistic assumption that the next value of \( \mathcal{G} \) would be zero. It is in fact immaterial to the technique how long a step is taken as long as \( \mathcal{S}^{(l+1)} \) is less than \( \mathcal{S}^{(l)} \). A step modifier is therefore added to the above recursion formula to achieve the final form used in the error minimization technique.

\[ S^{(l+1)} = S^{(l)} \left( 1 - \left( \frac{\mathcal{G} \cdot \nabla \mathcal{G}}{|\nabla \mathcal{G}|^2} \right)^{(l)} \right) \]

In the computer program discussed in Appendix B the initial value of \( \mathcal{S} \) was chosen as unity and each time that a new value of \( \mathcal{G} \) was calculated that was greater than the previous value, \( \mathcal{S} \) was set to 75% of its previous value. No large amount of scientific thought went into this choice, and experimentation and investigation of the behavior of \( \mathcal{S} \) might well significantly reduce the computation time.

A-4
Repetitive use of Eqs. (A.1) and (A.2) over the entire flow field allow the evaluation of the merit function $G$ for the current relaxation step. In order to apply Eq. (A.5), however, to alter the independent vector, the gradient of $G$ must be calculated. Now

$$\frac{\partial G}{\partial u_{ij}} = \sum_{m=2}^{r-1} \sum_{n=2}^{r-1} \frac{\partial^2 G}{\partial u_{ij} \partial u_{mn}}\quad (i \neq 1, n \neq 1, m \neq 1)$$

Because of the five point influence this reduces to

$$\frac{\partial G}{\partial u_{mn}} = \frac{\partial^2 G}{\partial u_{mn} \partial u_{m+1,n}} + \frac{\partial^2 G}{\partial u_{mn} \partial u_{m-1,n}} + \frac{\partial^2 G}{\partial u_{mn} \partial u_{m,n+1}} + \frac{\partial^2 G}{\partial u_{mn} \partial u_{m,n-1}} \quad (A.6)$$

In the actual calculation, it becomes very inefficient to calculate the above expression directly. It is however very easy to calculate the following terms while at point $(m,n)$

$$\frac{\partial G}{\partial u_{m-1,n}}, \frac{\partial G}{\partial u_{m+1,n}}, \frac{\partial G}{\partial u_{m,n+1}}, \frac{\partial G}{\partial u_{m,n-1}}, \frac{\partial G}{\partial u_{m+1,n+1}}, \frac{\partial G}{\partial u_{m-1,n-1}}$$

where $(m,n)$, $(m+1,n)$, $(m,n+1)$, $(m,n-1)$, $(m+1,n+1)$ are identified as points 3, 4, 0, 2, 1 in Eqs. (A.1) and (A.2). It also can be seen that

$$\frac{\partial G}{\partial u} = \frac{\partial G}{\partial u_{1,1}} + \frac{\partial G}{\partial u_{2,1}} + \frac{\partial G}{\partial u_{1,2}}$$

where $u$ is any variable in the system.

Equations (A.1) and (A.2) may then be differentiated with respect to the velocity components. For simplicity Eq. (A.1) is rewritten

A-5
\begin{align*}
& - \frac{1}{T_0} \left\{ AB + CD \right\} \\
& \text{where}
\begin{align*}
& A = \bar{R}_u u_0 - \bar{K}_u v_0 ; \quad B = u_0 (u_1 - u_1) + v_0 (r_3 - r_1) \\
& C = \bar{K}_u v_0 - \bar{R}_u u_0 ; \quad D = u_0 (u_4 - u_3) + v_0 (v_2 - v_3)
\end{align*}

\text{Then:}
\begin{align*}
& \frac{\partial \xi_{1(0)}}{\partial u_0} = \frac{\partial \xi_{1(0)}}{\partial v_0} = 0 \\
& \frac{\partial \xi_1}{\partial u_1} = - \bar{K}_u + \bar{h}_u A = - \frac{\partial \xi_1}{\partial u_0} \\
& \frac{\partial \xi_1}{\partial r_1} = \bar{K}_u + v_0 A = - \frac{\partial \xi_1}{\partial r_0} \\
& \frac{\partial \xi_3}{\partial u_1} = - \bar{K}_u = - \frac{\partial \xi_3}{\partial u_0} \\
& \frac{\partial \xi_3}{\partial r_1} = - \bar{K}_u = - \frac{\partial \xi_3}{\partial r_0} \\
& \frac{\partial \xi_4}{\partial u_1} = \bar{K}_u + \bar{h}_u C = - \frac{\partial \xi_4}{\partial u_0} \\
& \frac{\partial \xi_4}{\partial u_3} = \bar{K}_u + \bar{h}_u C = - \frac{\partial \xi_4}{\partial u_0}
\end{align*}

(A.7a) \\
(A.7b) \\
(A.7c) \\
(A.7d) \\
(A.7e) \\
(A.7f) \\
(A.7g) \\
(A.7h)
As a sweep is made through all interior points the gradient is constructed by summing the contributions in the appropriate fashion, i.e.,

\[
\frac{\partial \mathbf{E}(\xi)}{\partial \eta} = \frac{\partial \mathbf{E}(\xi)}{\partial \eta_n} = -\frac{\partial \mathbf{E}(\xi)}{\partial \eta_4}
\]

(A.7i)

After the sweep of the entire flow field has been completed, the gradient vector will have been properly formed. Any confusion concerning this process will be clarified by examining subroutine EMT in Appendix B.

A.2 BOUNDARY POINTS

As discussed in the main body of the report, an approximation to the wall boundary condition is

\[
q^{+}_{\eta,\xi} = q^{+}_{\eta,\xi} - q^{+}_{\eta,\xi}
\]

(A.8)

where points 1, 2, 3 correspond to \(\psi = 1\), \(1 - \Delta \psi\), \(1 - 2\Delta \psi\) respectively. Now

\[
q^{\eta} = u^{\eta} + v^{\eta}
\]
so that the differential of Eq. (A.8) is

\[ \partial u_{1,n} \, du_{1,n} + 2 \partial r_{1,n} \, dr_{1,n} = 4 \left( u_{2,n} \, du_{2,n} + r_{3,n} \, dr_{3,n} \right) \]

\[ - 2 \left( u_{3,n} \, du_{3,n} + r_{3,n} \, dr_{3,n} \right) \]

but

\[ r_{1,n} = \tan \theta_w \, u_{1,n} \]

and

\[ \partial r_{1,n} = \tan \theta_w \, du_{1,n} \]

so that

\[ u_{1,n} \, du_{1,n} \left( 1 + \tan^2 \theta_w \right) = 2 \left( u_{2,n} \, du_{2,n} + r_{3,n} \, dr_{3,n} \right) \]

\[ - \left( u_{3,n} \, du_{3,n} + r_{3,n} \, dr_{3,n} \right) \]

It can be seen from Eq. (A.5) that the change in each independent variable can be found from the recursion formula. Therefore

\[ \frac{\partial \xi}{\partial u_{1,n}} = \left\{ 2 \left( u_{1,n} \frac{\partial \xi}{\partial u_{1,n}} + r_{3,n} \frac{\partial \xi}{\partial r_{3,n}} \right) - \left( u_{3,n} \frac{\partial \xi}{\partial u_{3,n}} + r_{3,n} \frac{\partial \xi}{\partial r_{3,n}} \right) \right\} \left/ \left( 1 + \tan^2 \theta_w \right) u_{1,n} \right. \] (A.9)

In order to retain the proper tangency relationship

\[ \frac{\partial \xi}{\partial r_{1,n}} = \tan \theta_w \frac{\partial \xi}{\partial u_{1,n}} \] (A.10)

Similarly, the centerline conditions yield

\[ \frac{\partial \xi}{\partial u_{1,n}} = \frac{1}{8} \left( 4 \frac{\partial \xi}{\partial u_{1,n}} - \frac{\partial \xi}{\partial u_{3,n}} \right) \] (A.11)
\[ \frac{\partial \psi}{\partial n} = 0 \quad \text{(A.12)} \]

where points 1, 2, 3 correspond to $\psi_0$, $\Delta \psi$, $2\Delta \psi$ respectively.

It may be noticed that components of the gradient surface are calculated at all interior points plus all surrounding or non-interior points. Equations (A.9) through (A.12) ignored or destroyed this information at the wall and centerline. At the inlet and outlet however use is made of this information to alter the variables at these stations such that the function $G$ is minimized. This is consistent with the assumption that these surfaces are not boundary conditions.

Equations (A.1) and (A.2) are not altered if they are multiplied by some scale factor. In this problem it is advantageous to multiply the equations by a scale factor which emphasizes the throat. The scale used was

\[ I_n^* = \frac{1}{\sqrt{1 - n - n_s}} \]

where $n_s$ is the throat station. The local error terms are redefined

\[ \xi_{e_{1,m,n}} = \int_n \xi_{i,m,n} \quad ; \quad \xi_{e_{2,m,n}} = \int_n \xi_{i,m,n} \]

and all pertinent equations are altered appropriately. It is felt that the proper formulations currently under investigation should automatically provide this type of scaling or emphasis of the throat. For the time being the above approach is justifiable from the numerical viewpoint. The scaling tends to sacrifice the end regions in order to improve the throat region. It is this flow of information away from the throat which implicitly causes the throat to act as the boundary condition.
Using the governing equations and process described above the nozzle solution described in Appendix B yields the results shown in Fig. (A.1) for the 90 deg inlet, 0.25 radius ratio, reservoir inflow case. It can be seen that the results are wavy in the supersonic region. The reason for this effect is not known and is certainly not a physical phenomenon. The problem may be due to the rapid turning of the flow through the low transonic being suddenly restricted to flowing parallel to the conical exit. This recompression effect due to the discontinuity in second derivatives is well known and gives rise to a nozzle shock wave.

In any event a method has been incorporated into the solution to smooth the results. This is a user-selected option and is generally not employed until the unsmoothed relaxation equations have approached a final solution. In this way it is possible to calculate the unsmoothed solution as well as the smoothed. The smoothing is accomplished by introducing error terms based on the second derivative;

\[ \xi_3 \eta, \eta = \eta_1 + \eta_1 - 2 \eta_0 \]
\[ \xi_4 \eta, \eta = \nu_1 + \nu_1 - 2 \nu_0 \]
\[ \xi_5 \eta, \eta = \xi_1 + \xi_1 - 2 \xi_0 \]
\[ \xi_6 \eta, \eta = \nu_1 + \nu_1 - 2 \nu_0 \]  \( \text{(A.13)} \)

Driving, or attempting to drive, the second derivative to zero implies that the functions must be linear, certainly an unsatisfactory situation. The magnitude of these terms is kept small, however, so that the smoothing does not introduce errors in the differential equation solution. Scaling Eqs. (A.13) by 0.03 yields the second set of results on Figs. A.1 and A.2.

A-10
Effect of Smoothing on 0.5 Radius Ratio, 90 degree Reservoir Inflow Case

Fig. A-1 – Effects of Smoothing on .5 Radius Ratio Solution
Effect of Smoothing on .25 Radius Ratio, 90° Reservoir Inflow Case

- △ Unsmoothed Wall Data
- ◊ Unsmoothed Centerline Data
- ▼ Smoothed Wall Data
- ◊ Smoothed Centerline Data

Fig. A-2 – Effect of Smoothing on .25 Radius Ratio Solutions
A.3 CONCLUSION

The Error Minimization analysis is straightforward, simple and rapid. It is certainly an attractive technique for numerical analysis of the partial differential equations of motion. The problem areas of boundary conditions, need for smoothing (pseudo-viscosity?) lie with the theory and not with the solution technique. Future improvements would seem to depend largely upon developments in the theory since the numerical solution seems adequate.
Appendix B
CONVERGING-DIVERGING NOZZLE
ANALYSIS COMPUTER PROGRAM
DISCUSSION

A computer program has been written to carry out the calculations discussed in the preceding sections of this report. This Appendix is written to explain the inner working of this program as well as its input-output.

GENERAL

The program is written in FORTRAN V language and with suitable alteration of dimension statements, can be utilized on machines with 16K (or greater) core locations. Input and output during execution is confined to the standard input and "listable" tapes. Conversion from machine to machine should therefore be readily accomplished. The program is an assembly of special or restricted purpose subroutines. These subroutines may be conveniently grouped as follows:

<table>
<thead>
<tr>
<th>Subroutine Name</th>
<th>Subroutine Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAIN</td>
<td>Driver</td>
</tr>
<tr>
<td>INPUT</td>
<td>Input</td>
</tr>
<tr>
<td>SETUP1</td>
<td>Grid preparation</td>
</tr>
<tr>
<td>SETUP2</td>
<td></td>
</tr>
<tr>
<td>SETUP3</td>
<td></td>
</tr>
<tr>
<td>SETUP4</td>
<td></td>
</tr>
<tr>
<td>TRANSX</td>
<td>Flowfield initialization</td>
</tr>
<tr>
<td>TRANSR</td>
<td>Error minimization control</td>
</tr>
<tr>
<td>TRANSO</td>
<td></td>
</tr>
<tr>
<td>WALL</td>
<td></td>
</tr>
<tr>
<td>START</td>
<td></td>
</tr>
<tr>
<td>LOGIC</td>
<td></td>
</tr>
</tbody>
</table>

B-2
Before proceeding with an explanation of each subroutine it may be helpful to describe basically the organization of the computer program. The code was, as much as possible, written such that variable names were descriptive. This fact should help in the understanding of the program.

A main routine is written which acts as a driver for the entire subsequent calculation. Multiple or stacked cases are possible since the main program recycles to initiate the input routine. If all cases have been processed this attempt to read will terminate the job.

The sole function of the input routine is to get the necessary information into core and does essentially no processing. Assisting the input routine are four setup subroutines which are selected based on the input instructions and data required. Each of these routines upon completion of their function, returns to the input routine which in turn returns to the main program.

At that point all input requirements are satisfied and the physical grid and transformation derivatives have been determined. The next step is to make an initial flowfield estimate. For this estimate, subroutine START is called.

Upon completion of the initialization, the relaxation process is begun by entering subroutine LOGIC. This routine repetitively employs subroutines EMT and BOUND until a terminal condition is reached. Periodically the output subroutine is called. Upon termination of the calculation, control is returned to the main program which reinitializes the input.
SUBROUTINE DESCRIPTIONS

Subroutine: MAIN

This routine is an unchanging main program that seldom if ever needs recompiling.

Subroutine: INPUT

This subroutine is the basic input routine. The necessary input sequence is as follows:

Card 1, Format (20A4), header information, appears at the top of each page
Card 2, Format (16I5), options card, controls execution of program

Field 1, ΔΔΔΔX, X = number of boundary equations
Field 2, ΔΔXYZ, X = 1, automated setup (see SETUP1)
X = 2, input and distribution (see SETUP2)
X = 3, input all points (see SETUP3)
X = 4, input circular arcs (see SETUP4)
Y = 0, tube radial transformation (see TRANSR)
Y = 1, sine radial transformation (see TRANSR)
Y 1 (see TRANSR)
Z = 0 linear axial transformation (see TRANSX)
Z = 1 tangent axial transformation (see TRANSX)
Z 1 (see TRANSX)

Field 3, ΔΔΔXX, XX = number ψ cuts
Field 4, ΔΔΔXX, XX = number of ξ cuts
Field 5, ΔΔXX, XX = number of ξ cuts downstream of throat
Field 6, XXXXX, XXXXX = number of calculation steps/print step
Field 7, XXXXX, XXXXX = number of calculation steps to termination
Field 8, ΔΔΔΔX, X = number of ξ cuts/printed page
Field 9, ΔΔΔXY, X = 0, alter upstream boundary
X = 1, do not alter upstream boundary
Y = 0, alter downstream boundary
Y = 1, do not alter downstream boundary

Field 10, XXXXX, number of calculation steps before smoothing employed.
Card 3, Format (5E10.6), gas reservoir conditions

Field 1-4, chamber pressure, temperature, gas constant, gamma

Card(s)*4, Format (II, 9X, 6E10.6), wall equations data

Field 1, X, X=1 conic eq. \( r = a \left\{ \sqrt{b + cx + dx^2} + e \right\} \)

\( X = 2 \) poly \( r = ax^4 + bx^3 + cx^2 + dx + e \)

\( X = 3 \) expon. \( r = a \left( 1 + e^{-bx} \right) + c \)

Field 2-7 a, b, c, d, e, axial limit to which equation applies.

Card 5, Format (5E10.6), linear axial stretching factor, damping factor

Card(s)6, see subroutines SETUP1, SETUP2, SETUP3, SETUP4

Subroutine: SETUP1

This subroutine calculates the type of grid distribution shown in the sketch below. The actual distributions \( r(\xi, \psi) \), \( X(\xi, \psi) \) are determined by TRANSR, and TRANSX, respectively.

*Input as many cards as boundary equations beginning with equation farthest upstream
This type of grid distribution involves the least effort on the part of the user and should be satisfactory for many current rocket engines. Extremely high entrance angles should not be attempted with this distribution for obvious reasons.

An additional input parameter must be supplied to be read by this subroutine. This parameter is $\Delta \xi$ and for the tangent stretching transformation $\Delta \xi$ must be chosen such that the maximum value of $|\xi|$ is less than $\pi/2$.

Card 6, Format (E10.6), $\Delta \xi$

Subroutine: SETUP2

This subroutine gives much the same distribution as that above except that the user cannot find a convenient transformation $X = \text{TRANSX}(\xi, \psi)$ and therefore wishes to input this transformation in a tabular form. Starting with the most upstream station input, list of axial locations for $\xi$ data surfaces.

Cards 6, Format (E10.6), (X(N), N = 1, NTOT)

Subroutine: SETUP3

This subroutine reads the grid setup in its entirety. The wall equations need not be input with this option.

Cards 6, Format (8E10.6), (X(M, N), R(M, N), M = 1, MTOT)

Subroutine: SETUP4

Referring to the sketch on the following page it can be seen that the grid distribution is determined along circular arcs. Generally these arcs are normal to the wall and center line. Occasionally this is not possible nor practical, particularly in the case of the axial inflow. It is for this reason that the wall angle is also input along with the basic information needed to determine the grid.
All \( \xi \) stations are input in order starting from the left. For all stations but the throat station a card containing the following information is supplied:

Cards 6, Format (8E10.6), \( r_s \), \( x_s \), \( \theta_s \), \( \theta_w \)

where \( x_s \) is the center of the circular arc, \( r_s \) is the radius of the arc, \( \theta_s \) is the included angle (degrees) of the arc and \( \theta_w \) is the wall angle (degrees) at arc wall intersection. The throat is a special case and requires a card with the throat radius in field one with the remaining fields blank.

It is not necessary to input the wall boundary conditions with this setup option.

Function TRANSX

This function provides one of two selectable transformations on call. They are

\[
x = C \xi \\
x = C \tan\left(\frac{\pi}{2} \xi \right)
\]

where \( C \) (axial stretching factor) was input (see Subroutines INPUT)

B-7
The function is designed such that other transformations may be easily added.

Function TRANSR

This function provides one of two selectable transformations on call. They are

\[ r = R_w(x) \psi \]

where \( R_w(x) \) is the wall radius determined from the boundary equations

\[ r = R_w(x) \sin \left( \frac{\pi}{2} \psi \right) \]

This function is designed such that other transformations may easily be added.

Subroutine TRANSD

The purpose of this calculation is to provide the derivatives of the transformations that have been supplied. These derivatives are taken numerically.

Using centered differences

\[ r_\psi = \frac{r_4 - r_3}{2\Delta \psi} \quad ; \quad x_\psi = \frac{x_4 - x_3}{2\Delta \psi} \]

B-8
\[ r_\xi = \frac{r_2 - r_1}{2\Delta \xi}; \quad x_\xi = \frac{x_2 - x_1}{2\Delta \xi} \]

The information which is actually retained for subsequent use is

\[ 2\Delta \psi r_\psi, 2\Delta \psi x_\psi, 2\Delta \xi x_\xi, 2\Delta \xi x_\xi \]

As it stands, this subroutine could be removed with an attendant savings in core since block common GRIDD contains information which can easily be recomputed as necessary.

**Subroutine WALL**

This subroutine calculates \( R_w(x) \) using one of the three input equations described in Subroutine INPUT. Other functions could be included easily if desired.

**Subroutine START**

It is necessary to initialize the flow components to some reasonable values before beginning the relaxation process. The better the initial guess the faster one can expect an acceptable final solution. It is necessary to use some variation of one dimensional flow theory in order to make this initial approximation to the solution. With the grid distributions described in SETUP1, SETUP2, SETUP3 the constant \( \xi \) surface is initialized to have a constant Mach number, which if the flow were axial, would conserve the one-dimensional value of the choking mass flow.

With SETUP4, however, it is possible to make a better initial guess since the circular arcs (for \( \theta_s = \theta_w \)) correspond to constant Mach number surfaces for source or sink flow.
Subroutine LOGIC

The descent process described in Appendix A is implemented by this subroutine. This subroutine also prints a message each time the descent process is unsuccessful.

Subroutine EMT

This subroutine utilizes the equations derived in the main body of the report and the derivative equations found in Appendix A to calculate the error function and its derivatives for the entirety of the interior flow field points.

Subroutine BOUND

This subroutine utilizes the equations derived in the main body of the report and the derivatives equations found in Appendix A to calculate the derivative of the error function for the boundary points. If desired this subroutine can prevent either the inlet or the outlet or both surfaces from changing during the relaxation process. This is done by zeroing out the error function derivatives corresponding to these surfaces (see Field 9 of Card 2).

Subroutine OUTPUT

Periodically (determined by input) the OUTPUT subroutine is called. Its function is to output current values of flow variables over the entire flow field. The magnitude of the error function ($\theta$) as well as the descent step length ($\Delta$) are printed for this output relaxation step. The velocity components are given in ft/sec while the flow angle is given in degrees. The axial and radial position are output in the same units (any consistent units) which were input. The Mach number, and pressure and temperature ratios are, of course, dimensionless.

The mass flux through each surface and the wall thrust integral (initialized to zero at the leftmost station) are also calculated. The mass flow is divided by the one-dimensional value and is output as the mass flow coefficient. The thrust increment is divided by the chamber pressure and the throat area and is therefore also output as a dimensional quantity.
It is possible (see Subroutine INPUT) to control the number of surfaces output per page so that an unnecessary amount of paper is not generated.

See Subroutine LOGIC for an additional print message.

Subroutine ITSUB

This general purpose routine is used to perform the iterative solution to a function of one variable, i.e., determine $x$ such that $f(x) = 0$.

Function RHO

This function calculates the density as a function of the velocity components.

Sample Input

The infinite reservoir, 0.25 radius of curvature, 90-deg inlet case discussed in the main body of the report is given below as a sample of the input required.

<table>
<thead>
<tr>
<th>JPL 90 DEG INLET</th>
<th>0.25 RC</th>
<th>RESERVOIR INFLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 400 11 19 8</td>
<td>400 2000 2 200</td>
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</tr>
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<td>+04+1</td>
<td>+04+1768</td>
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<td>+01+3</td>
<td>-01</td>
</tr>
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</table>

MASS FLOW COEFFICIENT = 0.9683E00 THRLST INCREMENT = -0.7859E01
C MAIN

100 CALL INPUT
CALL START
CALL LOGIC
GO TO 100
END

SUBROUTINE SETUP1
COMMON/OPTION/KOPT(16)
COMMON/GRID/X(11*21)*R(11*21)
10 FORMAT(E10.6)
MTOT=KOPT(3)
NTOT=KOPT(4)
ND=KOPT(5)
READ(5,10)DXI
XI=-FLOAT(NTOT-ND)*DXI
DPSI=1.0/FLOAT(MTOT-1)
DO 100 N=1,NTOT
XI=XI+DXI
PSI=1.0+DPSI
DO 100 M=1,MTOT
PSI=PSI-DPSI
X(M*N)=TRANSX(XI*PSI)
R(M*N)=TRANSR(XI*PSI)
100 CONTINUE
RETURN
END
SUBROUTINE SETUP2
COMMON/OPTION,KOPT(16)
COMMON/GRID/X(11*21),R(11*21)
DIMENSION XIN(21)
10 FORMAT(8E10.5)
MTOT=KOPT(3)
NTOT=KOPT(4)
READ(5,10)(XIN(N)*N=1,NTOT)
DPS=1./FLOAT(MTOT-1)
DO 100 N=1,NTOT
XI=XIN(N)
PSI=1.+DPSI
DO 100 M=1,MTOT
PSI=PSI-DPSI
X(M*N)=TRANSX(XI*PSI)
R(M*N)=TRANSR(XI*PSI)
100 CONTINUE
RETURN
END

SUBROUTINE SETUP3
COMMON/OPTION,KOPT(16)
COMMON/GRID/X(11*21),R(11*21)
10 FORMAT(8E10.6)
MTOT=KOPT(3)
NTOT=KOPT(4)
DO 100 N=1,NTOT
READ(5,10)(X(M+N)*R(M+N)*M=1,MTOT)
100 CONTINUE
RETURN
END
SUBROUTINE SETUP
COMMON/OPTICN/KOPT(16)
COMMON/GRID/X(11*21),R(11*21)
COMMON/SOURCE/RS(21),XS(21),THETAS(21),THETWA(21)
DATAPI02/1.*5707963/
10 FORMAT(8E10.6)
MTOT=KOPT(3)
NTOT=KOPT(4)
ND=KOPT(5)
NS=NTOT-ND
DPSI=1./FLOAT(MTOT-1)
DO100=1,MTOT
READ(N*10)RS(N),XS(N),THETAS(N),THETWA(N)
THETAS(N)=THETAS(N)/57.3
THETWA(N)=THETWA(N)/57.3
PSI=1.+DPSI
DO100=M+MTOT
PSI=PSI-DPSI
R(M+N)=RS(N)*SIN(ABS(THETAS(N)))*SIN(P102*PSI)
IF(N.EQ.NS)R(M+N)=RS(N)*SIN(P102*PSI)
F=1.
IF(THETAS(N).LT.0.)F=-1.
X(M+N)=XS(N)+F*SQRT(RS(N)**2-R(M+N)**2)
IF(N.EQ.NS)X(M+N)=0.
100 CONTINUE
RETURN
END

FUNCTION TRANSX(XI,PSI)
COMMON/OPTICN/KOPT(16)
COMMON/MISG/XSCALE,DAMP
DATAPI02/1.*5707963/
I=KOPT(2)-10.*(KOPT(2)/10)
IF(I.NE.0)GO TO 100
TRANSX=XSCALE*X1
GOTO 1000
100 IF(I.NE.1)GO TO 200
TRANSX=XSCALE*SIN(P102*X1)/COS(P102*X1)
GOTO 1000
200 CONTINUE
1000 RETURN
END
FUNCTION TRANSR(XI, PSI)
COMMON/OPTION/KOPT(16)
DATAP102/1.5707963/
I=KOPT(2)/10-10*(KOPT(2)/100)
IF(I.NE.0) GO TO 100
X=TRANSX(XI,* PSI)
CALL WALL (RSCALE*X)
TRANSR=RScale*PSI
GO TO 1000
100 IF(I.NE.1) GO TO 200
X=TRANSX(XI, PSI)
CALL WALL (RScale*X)
TRANSR=RScale*SIN(P102*PSI)
GO TO 1000
200 CONTINUE
1000 RETURN
END

SUBROUTINE TRANSO
COMMON/GRID/DXDXI(11.21)*DXDPSI(11.21)*DRDXI(11.21)*DRDPSI(11.21)
COMMON/GRID/X(11.21)*R(11.21)
COMMON/OPTION/KOPT(16)
MTOT=KOPT(3)
NTOT=KOPT(4)
DO 1000 N=1,NTOT
NM1=N-1
NP1=N+1
IF(N.EQ.1) NM1=N
IF(N.EQ.NTOT) NP1=N
DO 1000 M=1,MTOT
MM1=M-1
MP1=M+1
IF(M.EQ.1) MM1=M
IF(M.EQ.MTOT) MP1=M
DXDXI(M+N)=Y(M+NP1)-X(M+NM1)
DXDPSI(M+N)=X(MM1+N)-X(MP1+N)
DRDXI(M+N)=R(M+NP1)-R(M+NM1)
DRDPSI(M+N)=R(MM1+N)-R(MP1+N)
1000 CONTINUE
RETURN
END

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SUBROUTINE WALL(R*X)
COMMON/WALLFO/IWALL(10),WALLCO(10*6)
COMMON/OPTION/KOPT(16)
DIMENSION A(5)
NBOUND=KOPT(1)
DO 100 N=1,NBOUND
IF(WALLCO(N*6)*GT*X)GO TO 200
100 CONTINUE
N=NBOUND
200 DO 300 J=1,5
300 A(J)=WALLCO(N+J)
IW=IWALL(N)
GO TO (400,500,600)IW
400 R=A(1)*(SQRT(A(2)+A(3)*X+A(4)*X*X)+A(5))
GO TO 700
500 R=((A(1)*X+A(2))*X+A(3))*X+A(4))*X+A(5)
GO TO 700
600 E=EXP(-A(2)*X*X)
R=A(1)*(1-E)+A(3)
GO TO 700
700 RETURN
END

FUNCTION RHO(U*V)
COMMON/REST/PC*TC+RC*GAMMAC+RHOC+AC
GM1=GAMMAC-1.
RHO=(1.*-5.*GM1*(U*U+V*V))**(1./GM1)
RETURN
END
SUBROUTINE ITSUB (FOFY, Y, SAVE, CONV, NTIMES)
SUBROUTINE FOR REGULI FALSI ITERATION CONTROL

DIMENSION SAVE(8)
N1 = SAVE(3) ++ 1
FOFXCK = SAVE(8)
FOFX = FOFY
X = Y
IF (ABS (FOFX) - CONV <= 0) GOTO 110
ITIME = SAVE(1) + ++ 1
GOTO (10, 30, 50, 70) * ITIME
10 N1 = 1
   ITIME = 2
   FOFXCK = FOFX
   SAVE(8) = FOFXCK
   IF (FOFX <= 0) GOTO 50
30 IF (FOFX <= 0) GOTO 70
   IF (FOFXCK * GE * FOFX) GOTO 35
   SAVE(2) = -1 * SAVE(2)
   X = X - 2 * SAVE(2)
   GOTO 90
35 SAVE(4) = X
   SAVE(5) = FOFX
   X = X - SAVE(2)
   GOTO 90
50 ITIME = 3
   IF (FOFX GT 0) GOTO 70
   IF (FOFXCK LE FOFX) GOTO 55
   SAVE(2) = -1 * SAVE(2)
   X = X + 2 * SAVE(2)
   GOTO 90
55 SAVE(6) = X
   SAVE(7) = FOFX
   X = X + SAVE(2)
   GOTO 90
70 ITIME = 4
   N1 = SAVE(3)
IF(FOFX.LT.0.)GOTO75
SAVE(4)=X
SAVE(5)=FOFX
GOTO80
75 SAVE(6)=X
SAVE(7)=FOFX
80 X=SAVE(4)-SAVE(5)*((SAVE(6)-SAVE(4))/(SAVE(7)-SAVE(5)))
90 IF(N1.GE.INTIM)GOTO100
   N1=N1+1
   SAVE(3)=N1
GOTO120
100 INTIME=6
GOTO120
110 INTIME=5
   SAVE(4)=X
   SAVE(5)=FOFX
   SAVE(6)=X
   SAVE(7)=FOFX
120 SAVE(1)=FLOAT(INTIME)+1
   Y=X
RETURN
END
SUBROUTINE OUTPUT(L)
COMMON/OPTION /KOPT(16)
COMMON/GRID/X(11*21)*R(11*21)
COMMON/FLOW/U(11*21)*V(11*21)
COMMON/REST/PC*TC*RC*GAMMAC*RHOC*AC
COMMON/HEADER/HEAD(20)
COMMON/MISCD/G*GM*DEL
COMMON/GRIDD/DXDXI(11*21)*DXPSI(11*21)*DRDXI(11*21)*DRPSI(11*21)
DATAP/3e14159/
10 FORMAT(1H1*20A4)
20 FORMAT(17HORELAXATION STEP *15*3H G=*E12*4*7H DELTA=*E12*4)
30 FORMAT(1H0*11X*1HR*11X*1HX*11X*1HU*11X*1HV*11X*1HM*7X*5HTHETA
1*8X*4HP/PC*8X*4HT/TC)
40 FORMAT(1H0*8E12*4)
50 FORMAT(1H0*22HMASS FLOW COEFFICIENT=*E12*4*18H THRUST INCREMENT=*E12*4)
MTOT=KOPT(3)
NTOT=KOPT(4)
NPAGE=KOPT(8)
NS=NTOT-KOPT(5)
AS=PI*R(1*NS)*R(1*NS)
US=SQRT(2e/(GAMMAC+1.))
RHOS=RHO(USIO-)
WONED=RHOS*US*AS
1GGM1=GAMMAC/(GAMMAC-1.)
P1=0.
00 DO 2000 N=1*NTOT
IF(N-NPAGE*(N/NPAGE)*EQ.1)WRITE(6*10)HEAD
WRITE(6*20)L*G*DEL
WRITE(6*30)
W=0.
IF(N*EQ.1)GOTO500
P2=(1.-((GAMMAC-1.)*S*(U(1*N)**2+V(1*N)**2))**GGM1
P1=(1.-((GAMMAC-1.)*S*(U(1*N-1)**2+V(1*N-1)**2))**GGM1
PINTEG=PINTEG+P2*P1*5*PI*(R(1*N)**2-R(1*N-1)**2)/AS
500 DO 1000 M=1*MTOT
UP=U(M*N)*AC
VP=V(M*N)*AC

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QSQ=UP*UP+VP*VP
T=TC-(GAMMAC-1*e)*5*QSQ/(GAMMAC*RC)
EMP=SQRT(QSQ/(GAMMAC*RC*T))
THETA=57.3*ATAN(VP/UP)
TOTC=T/TC
POPC=(TOTC)**(GAMMAC/(GAMMAC-1*e))
WRITE(6,40)R(M,N)*X(M,N)*UP*VP*EMP*THETA*POPC*TOTC
RHOP=POPC/TOTC
MM1=M-1
MP1=M+1
IF(M*EQ.1)MM1=M
IF(M*EQ.MTOT)MP1=MTOT
DA=PI*(R(MM1,N)+R(MP1,N))*5
W=W+DA*RHOP*(U(M,N)*DRDPSI(M,N)-V(M,N)*DXDPSI(M,N))
1000 CONTINUE
WRAT=W/WONED
WRITE(6,50)WRAT*PINTEG
2000 CONTINUE
RETURN
END
SUBROUTINE INPUT
COMMON/HEADER/HEAD(20)
COMMON/OPTION/KOPT(16)
COMMON/REST/PC*TC*RC*GAMMAC*RHOC*AC
COMMON/WALLFO/IWALL(10)*WALLCO(10*6)
COMMON/MISCG/XSCALE*DAMP
10 FORMAT(20A4)
20 FORMAT(16A5)
30 FORMAT(5E10.6)
40 FORMAT(11*9X*6E10.6)
50 FORMAT(1H1+20A4)
60 FORMAT(5H OPT(*I2*2H)t*
70 FORMAT(4H PC=*E~~.~*~~HTCE*E~~~~*~HRC=*E~~~~*~HGAMMAC=*E~~~~)
80 FORMAT(6HOIWALL+11X+1HA+11X+1HB+11X+1HC+11X+1HD+11X+1HE+9X+3HMAX)
90 FORMAT(1H +4X*I1+6E12+4)
   READ(5+10)HEAD
   WRITE(6*50)HEAD
   READ(5+20)KOPT
   DO 100 I=1,10
   WRITE(6*60)I*KOPT(I)
100 CONTINUE
   READ(5+30)PC*TC*RC*GAMMAC
   WRITE(6*70)PC*TC*RC*GAMMAC
   RHOC=PC/(RC*TC)
   AC=SQRT(GAMMAC*RC*TC)
   IB=KOPT(I)
   IF(IB.EQ.0)GOTO200
   READ(5+40)(IWALL(I)*~WALLCO~I~J~J~~~~~*I~~~I~~
   WRITE(6*80)
   WRITE(6*90)(IWALL(I)*~WALLCO(I*J)*J~1*6)*I=1*IB)
200 READ(5+30)XSCALE*DAMP
   I=KOPT(2)/100
   IF(I.EQ.1)CALL SETUP1
   IF(I.EQ.2)CALL SETUP2
   IF(I.EQ.3)CALL SETUP3
   IF(I.EQ.4)CALL SETUP4
   CALL TRANSD
   RETURN

END
SUBROUTINE START
COMMON/GRID/X(11,21)*R(11,21)
COMMON/FLOW/U(11,21)*V(11,21)
COMMON/REST/PC*TC*RC*GAMMAC*RHOC*AC
COMMON/OPTION/KOPT(16)
COMMON/GRIDD/DXDXI(11,21)*DXDPSI(11,21)*DRDXI(11,21)*DRDPSI(11,21)
COMMON/ENDC/ULEND(11)*VLEND(11)*UREND(11)*VREND(11)
COMMON/SOURCE/RSS(21)*XSS(21)*THETAS(21)*THETWA(21)
DIMENSION SAVE(8)
DATAPI/3.14159/
MTOT=KOPT(3)
NTOT=KOPT(4)
ND=KOPT(5)
NS=NTOT-ND
US=SQRT(2.0/(GAMMAC+1.0))
RHOS=RHO(US)*0.0
WONED=PI*RHOS*US*R(1*NS)*R(1*NS)
U(1*NS)=US
NSM1=NS-1
IF(KOPT(2)/100.0*EQ.4)GOTO6000
IF(NSM1*EQ.0)GOTO1050
UW=US*0.9
DO 1000 I=1,NSM1
SAVE(1)=I
SAVE(2)=0.049*UW
N=NS-I
RW=R(1*N)
AP1=PI*RW*RW
100 CALL ITSUB(RHO(UW)*UW*A-WONED+UW*SAVE*0.001*WONED*99)
IF(UW*GT*US)UW=US
IBR=SAVE(I)
GOTO (100,100,100,100,200,200)*IBR
200 U(I*N)=UW
1000 CONTINUE
1050 UW=US*1.1
DO 2000 I=1,ND
SAVE(1)=I
SAVE(2)=0.049*US

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N=NS+1
RW=R(1+N)
A=*PI*RW
1100 CALL ITSUB(RHO(UW*O*)*UW*A-WONED*UW*SAVE*001*WONED*99)
   IF(UW*LT*US)UW=US
   IBR=SAVE(1)
   GO TO (1100,1100,1100,1100,1200,1200,1200)
1200 U(1*N)=UW
2000 CONTINUE
   DO 3000 N=1,NTOT
   Q=U(1*N)
   DO 3000 M=1,MTOT
   DS=SQRT(DXDXI(M*N)**2+DRDXI(M*N)**2)
   U(M*N)=Q*DXDXI(M*N)/DS
   V(M*N)=Q*DRDXI(M*N)/DS
3000 CONTINUE
   N=NTOT
   TANTW=DRDXI(1*N)/DXDXI(1*N)
   THETAW=ATAN(TANTW)
   IF(THETAW*LT*0.05)GOTO4000
   RW=R(1*N)
   XS=-RW/TANTW
   DO3500 M=1,MTOT
   RP=R(M*N)
   RS=SQRT(XS*X+RP*RP)
   THETA=ATAN(RP*TANTW/RW)
   A=*2.*PI*RS*RS*(1-COS(THETAW))/COS(THETA)
   TANTP=SIN(THETA)/COS(THETA)
   SAVE(1)=1.
   SAVE(2)=0.05*UW
3100 VW=UW*TANTP
   CALL ITSUB(RHO(UW*VW)*UW*A-WONED*UW*SAVE*001*WONED*99)
   IBR=SAVE(1)
   GOTO(3100,3100,3100,3100,3200,3200,3200)
3200 U(M*N)=UW
   V(M*N)=VW
3500 CONTINUE
4000 CONTINUE
DO5000 M=1*MTOT
ULEND(M)=U(M+1)
VLEND(M)=V(M+1)
UREND(M)=U(M+NTOT)
VREND(M)=V(M+NTOT)

5000 CONTINUE
GOTO 7000

6000 DO6100 M=1*MTOT
U(M*NS)=US

6100 V(M*NS)=0
Q=9*US
D06200 I=1*NS1
IF(Q*GT*9*US)Q=9*US
N=NS-I
A=2*PI*RSS(N)*RSS(N)*(1-COS(THETAS(N)))
SAVE(1)=1
SAVE(2)=0.05*US

6050 CALL ITSUB(RHO(Q*O*Q)*Q*A-WONED*Q*SAVE*0.001*WONED*99)
IF(Q*GT*US)Q=US
IBR=SAVE(1)
GOTO(6050*6050*6050*6050*6150*6150*IBR)

6150 D06200 M=1*MTOT
TANT=R(M*N)/(X(M*N)-XSS(N))
THET=ATAN(TANT)*THETWA(N)/THETAS(N)
U(M*N)=Q*COS(THET)
V(M*N)=Q*SIN(THET)

6200 CONTINUE
Q=1.1*US
D06400 I=1*ND
IF(Q*LT*1.1*US)Q=1.1*US
N=NS+I
A=2*PI*RSS(N)*RSS(N)*(1-COS(THETAS(N)))
SAVE(1)=1
SAVE(2)=0.05*US

6250 CALL ITSUB(RHO(Q*O*Q)*Q*A-WONED*Q*SAVE*0.001*WONED*99)
IF(Q*LT*US)Q=US
IBR=SAVE(1)
GOTO(6250*6250*6250*6250*6300*6300*IBR)

6300 DO6400 M=1*MTOT
TANT=R(M*N)/(X(M*N)-XSS(N))
THET=ATAN(TANT)*THETWA(N)/THETAS(N)
U(M*N)=Q*COS(THET)
V(M*N)=Q*SIN(THET)

6400 CONTINUE
7000 CONTINUE
RETURN
END
SUBROUTINE BOUND

COMMON/FLOW/U(11*21)*V(11*21)
COMMON/DECENT/DGDU(11*21)+DGDV(11*21)
COMMON/GRID/DX/DXPSI(11*21)+DRDXI(11*21)+DRDPSI(11*21)
COMMON/OPTION/KOPT(16)
COMMON/MISC/G*GM1*DEL
COMMON/ENDC/UEND(11)*VEND(11)+UREND(11)*VREND(11)
COMMON/GRID/X(11*21)*R(11*21)
MTOT=KOPT(3)
NTOT=KOPT(4)
NTOTM1=NTOT-1
DO300 N=2*NTOTM1
DX=DXDXI(1*N)
DR=DRDXI(1*N)
RHS=2*(U(2*N)+DGDV(2*N)+DGDV(2*N)-U(3*N)*DGDU(3*N)+V(3*N)
1)*DGDV(3*N))
IF(ABS(DX)*CT*ABS(DR))GOTO100
DGDV(1*N)=RHS/(V(1*N)*(1+DX*DX/(DR*DR)))
DGDU(1*N)=DGDV(1*N)*DX/DR
GOTO200

100 DGDU(1*N)=RHS/(U(1*N)*(1+DR*DR/(DX*DX)))
DGDV(1*N)=DGDU(1*N)*DR/DX

200 DGDU(MTOT+N)=(4*DGDU(MTOT-1*N)-DGDU(MTOT-2*N))/3
DGDV(MTOT+N)=0

300 CONTINUE
IF(KOPT(9)/10*EQ.0)GOTO500
DO400 M=1*MTOT
DGDU(M*1)=0

400 DGDU(M*1)=0

500 IF(KOPT(9)-10*(KOPT(9)/10)*EQ.0)GOTO700
DO600 M=1*MTOT
DGDU(M*MTOT)=0

600 DGDU(M*MTOT)=0

700 CONTINUE
RETURN
END
SUBROUTINE LOGIC

COMMON/DECENT/DGDU(11*21)*DGDV(11*21)
COMMON/LASTEP/DGDU(11*21)*DGDV(11*21)
COMMON/LOW/U(11*21),V(11*21)
COMMON/OPTION/KOPT(16)
COMMON/MISCD/G•GM1•DEL
COMMON/MISG/XSCALE•DAMP
COMMON/SMOOTH/SMULT

10 FORMAT(1H•3E12•4)

L=-1
MTOT=KOPT(3)
NTOT=KOPT(4)
NTOTM1=NTOT-1
MTOTM1=MTOT-1
SMULT=0•
ISM=KOPT(10)
GM1=1•E+16
DEL=1•

100 GRAD=0•
G=0•
DO 200 N=1•NTOT
DO 200 M=1•MTOT
DGDU(M•N)=0•

200 DGDV(M•N)=0•
CALL EMT
CALL BOUND
IF(G+GE+GM1)GO TO 1000
GM1=G
L=L+1
DO 300 N=1•NTOT
DO 300 M=1•MTOT

300 GRAD=GRAD+DGDU(M•N)*DGDU(M•N)+DGDV(M•N)*DGDV(M•N)
IF(L+GE+KOPT(6)*(L/KOPT(6)))CALL OUTPUT(L)
IF(L+GE+ISM)GOTO600
IF(L+GE+KOPT(7))GOTO2000

400 SCALE=G/GRAD
450 CONTINUE
DO 500 N=1•NTOT
DO 500 M=1*MTOT
DGDU1(M*N)=DGDU(M*N)
DGDV1(M*N)=DGDV(M*N)
U(M*N)=U(M*N)-SCALE*DEL*DGDU(M*N)
V(M*N)=V(M*N)-SCALE*DEL*DGDV(M*N)
500 CONTINUE
GO TO 100

600 SMULT=DAMP
GM1=1.E+16
GO TO 100

1000 IF(DEL.LT.1.E-8)GOTO1500
WRITE(6,10)G,GM1,DEL
DO 1100 N=1*NTOT
DO 1100 M=1*MTOT
DGDU(M*N)=DGDU1(M*N)
DGDV(M*N)=DGDV1(M*N)
U(M*N)=U(M*N)+SCALE*DEL*DGDU(M*N)
V(M*N)=V(M*N)+SCALE*DEL*DGDV(M*N)
1100 DEL=.75*DEL
G=GM1
GOTO450

1500 CALL OUTPUT(L)
2000 RETURN
END
SUBROUTINE EMT
COMMON/GRID/IX(11,21)*IXDPSI(11,21)*IXDPSI(11,21)
COMMON/DECENT/DGDU(11,21)*DGDV(11,21)
COMMON/FLOW/U(11,21)*V(11,21)
COMMON/MISC/G+GM1*DEL
COMMON/OPTION/KOPT(16)
COMMON/REST/PC+TC+RC+GAMMAC+RHOC+AC
COMMON/GRID/X(11,21)*R(11,21)
COMMON/SOOTH/SMULT
MTOT=KOPT(3)
NTOT=KOPT(4)
MTOT=MTOT-1
NTOT=NTOT-1
NS=NTOT-KOPT(5)
GAMM1=GAMMAC-1.
DO 1000 N=2+NTOTM1
FNEXP=ABS(FLOAT(N-NS))
FNEXP=SQRT(FNEXP)
S=1./EXP(FNEXP)
DO 1000 M=2+MTOTM1
NM1=N-1
NP1=N+1
MM1=M-1
MP1=M+1
U0=U(M*N)
V0=V(M*N)
RHOO=RHO(U0*V0)
U1=U(M*NM1)
V1=V(M*NM1)
U2=U(M*NP1)
V2=V(M*NP1)
U4=U(MM1*N)
V4=V(MM1*N)
U3=U(MP1*N)
V3=V(MP1*N)
RPS1=R(MM1*N)-R(MP1*N)
RXI=R(M*NP1)-R(M*NM1)
XPS1=X(MM1*N)-X(MP1*N)

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\[ X_{XI} = X(M+NP1) - X(M+NM1) \]
\[ \text{ALPHA} = RPSI \times X_{XI} - RXI \times XPSI \]
\[ R = R(M+N) \]
\[ SA = 6.2832 \times R0 \times \text{SQRT}(RPSI \times RPSI + XPSI \times XPSI) \]
\[ SA = SA / (R(1+N) \times R(1+N)) \]
\[ \text{TRHO} = 1 + \text{RHOO} \times \text{GAMMA} \]
\[ \text{ORHOSQ} = 1 / (\text{TRHO} \times \text{TRHO}) \]
\[ T1 = RPSI \times (U2 - U1) - XPSI \times (V2 - V1) + X_{XI} \times (V4 - V3) - RXI \times (U4 - U3) + \text{ALPHA} \times V0 / R0 \]
\[ T2 = RPSI \times U0 - XPSI \times V0 \]
\[ T3 = U0 \times (U2 - U1) + V0 \times (V2 - V1) \]
\[ T4 = X_{XI} \times V0 - RY1 \times U0 \]
\[ T5 = U0 \times (U4 - U3) + V0 \times (V4 - V3) \]
\[ T6 = T2 \times T3 + T4 \times T5 \]
\[ E1MN = T1 - \text{TRHO} \times T6 \]
\[ E2MN = RPSI \times (V2 - V1) - RXI \times (V4 - V3) - X_{XI} \times (U4 - U3) \]
\[ E1MN = E1MN + S \]
\[ E2MN = E2MN + S \]
\[ E1MN = E1MN \times SA \]
\[ G = G + 5 \times (E1MN \times E1MN + E2MN \times E2MN) \]
\[ E1MN = E1MN + S \]
\[ E1MN = E1MN \times S \]
\[ E2MN = E2MN \times S \]
\[ SM = S \times \text{SMULT} \]
\[ E3MN = U2 + U1 - 2 \times U0 \]
\[ E4MN = V2 + V1 - 2 \times V0 \]
\[ E5MN = U4 + U3 - 2 \times U0 \]
\[ E6MN = V4 + V3 - 2 \times V0 \]
\[ E3MN = E3MN \times SM \]
\[ E4MN = E4MN \times SM \]
\[ E5MN = E5MN \times SM \]
\[ E6MN = E6MN \times SM \]
\[ G = G + 5 \times (E3MN \times E4MN + E3MN \times E5MN + E5MN \times E6MN + E6MN \times E6MN) \]
\[ E3MN = E3MN \times SM \]
\[ E4MN = E4MN \times SM \]
\[ E5MN = E5MN \times SM \]
\[ E6MN = E6MN \times SM \]
\[ DGDU(M+N) = DGDU(M+N) - E1MN \times (GAMMA \times U0 \times T6 \times ORHOSQ \times 1 + \text{TRHO} \times (RPSI \times T3 + T2 \times (U2 - U1) - RXI \times T5 + T4 \times (U4 - U3))) \]
\[
2 - E^3_{MN} * 2 - E^5_{MN} * 2 \\
DGDV(M_N) = DGDV(M_N) - E^1_{MN}((-\alpha / \rho_0 + \gamma_1^M V_0 T_6 \Omega_{HOSQ} \\
1 + \rho_0 (-x_{PSI} T_3 + 2 (V_2 - V_1) + x_{XI} T_5 + T_4 (V_4 - V_3) )) \\
2 - E^4_{MN} * 2 - E^6_{MN} * 2 \\
DU_2 = E^1_{MN} (x_{PSI} T_2 \rho_0 + E^2_{MN} x_{PSI} \\
DV_2 = E^1_{MN} (-x_{PSI} T_2 \rho_0 + E^2_{MN} x_{PSI} \\
DU_4 = E^1_{MN} (-x_{XI} T_4 \rho_0 + E^2_{MN} x_{XI} \rho_0 \\
DV_4 = E^1_{MN} (x_{XI} T_4 \rho_0 + E^2_{MN} x_{XI} \rho_0 \\
DGDU(M_{NP1}) = DGDU(M_{NP1}) + DU_2 \\
1 + E^3_{MN} \\
DGDV(M_{NP1}) = DGDV(M_{NP1}) + DV_2 \\
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DGDU(M_{NM1}) = DGDU(M_{NM1}) - DU_2 \\
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DGDV(M_{NM1}) = DGDV(M_{NM1}) - DV_2 \\
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DGDU(M_{MM1N}) = DGDU(M_{MM1N}) + DU_4  \\
1 + E^5_{MN} \\
DGDV(M_{MM1N}) = DGDV(M_{MM1N}) + DV_4 \\
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DGDU(M_{MP1N}) = DGDU(M_{MP1N}) - DU_4 \\
1 + E^5_{MN} \\
DGDV(M_{MP1N}) = DGDV(M_{MP1N}) - DV_4 \\
1 + E^6_{MN} \\
1000 \text{ CONTINUE} \\
\text{RETURN} \\
\text{END}
\]
Appendix C

DISTRIBUTION LIST FOR CONTRACT NAS7-743
### DISTRIBUTION LIST FOR FINAL REPORT

**CONTRACT NAS7-743**

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