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EPHEMERIS OF A HIGHLY ECCENTRIC ORBIT: EXPLORER 28

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ABSTRACT

An ephemeris has been obtained for Explorer 28 (IMP 3) which agrees well with 2 years of radio observations and with SAO observations a year later. This ephemeris is generated over the 3 year lifetime by a numerical integration method utilizing a set of initial conditions at launch and without requiring further differential correlation. Because highly eccentric orbits are difficult to compute with acceptable accuracy and because a long continuous arc has been obtained which compares with actual data to a known precision, this ephemeris may be used as a standard for computing highly eccentric orbits in the Earth-Moon system.

Orbit improvement was used to obtain the initial conditions which generated the ephemeris. This improvement was based on correcting the energy by adjusting the semimajor axis to match computed times of perigee passage with the observed. This procedure may generate errors in semimajor axis to compensate for model errors in the energy; however this compensation error is also implicit in orbit determination itself.

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CONTENTS

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I. INTRODUCTION

Highly eccentric orbits offer unusual opportunities for fitting long arcs with high precision. One major advantage is that a high eccentricity orbit is seldom, or never, affected by the earth's atmosphere. Since the atmospheric drag force is poorly predicted, the uncertainties in close earth circular orbits have to be assumed to arise from this source, and indeed, close earth orbits are often used to derive the density of the atmosphere. In the case of high eccentricity orbits, however, precise determinations can be obtained and extrapolated forward with good accuracy.

A second major advantage is provided by the good resolution of the perigee position of the orbit. Because the raw data will yield a good definition of the position of perigee, the time of perigee passage becomes an important parameter in the orbit improvement. This parameter has the additional benefit of being readily derived from both the reduced observational data and the numerical computation, thus facilitating comparisons between the two. Further, the investigation of the long term behavior of this parameter yields some insights which may not be clearly visible in short arc fitting.

IMP 3 (Explorer 28) satellite is a good example of a high eccentricity orbit. Radio tracking has been obtained for over 120 orbits, that is, 2 years of the 3 year lifetime. This paper discusses a technique of improving the orbit by finding a long arc which is consistent with the available short arc data. An ephemeris has been obtained which gives a good fit to the available data (time of perigee passage). This ephemeris, when extrapolated ahead one year, proved capable of making predictions of accuracy such that SAO's Baker-Nunn network was able to acquire the satellite. The technique of obtaining a good fitting ephemeris is discussed in this paper.

Earth satellites in highly eccentric orbits ($e > .9$) are subject to large and erratic perturbations by the moon, as well as a strong solar perturbation. Also, the earth's oblateness term may cause a serious computational error, even though the secular effect of the oblateness is quite small. It can be difficult to distinguish between a legitimate perturbation and a numerical error introduced by improper selection of integration interval, or a programming error.

The IMP 3 (Explorer 28) orbit provides a good sample of the major perturbational effects in the earth-moon system. Since there is a total of 2 years data with which a numerical integration has been compared, it is suggested that the ephemeris of IMP 3 serve as a computational standard. For most applications the full accuracy of the ephemeris will not be required; however, a program comparison with the IMP 3 ephemeris will test the ability of the program under test to compute an orbit with high eccentricity and large semimajor axis in the Earth-Moon system within desired accuracy requirements.

The general history of the satellite is discussed in Section II. Section III gives a summary of the perturbations, the characteristic changes of specific orbital elements and the origins of the changes. Section IV describes the method of computation and discusses the short period effect of the earth's oblateness, which is the major source of computational errors and consumption of machine time. Section V explains the data which is employed in the comparison and indicates the technique involved in obtaining initial conditions which will give a satisfactory fit over the entire arc. Section VI gives the comparison between the data and the numerical integration. Section VII presents the ephemeris. The final section, Section VIII, presents the conclusions.

II. HISTORY OF SATELLITE

The satellite was launched on May 29, 1965. The initial orbital elements, as assumed in this report, are

$$a_0 = 21.726138 \text{ earth radii}$$

$$e_0 = 0.95251083$$

$$i_0 = 33^\circ\!.828446$$

$$w_0 = 135^\circ\!.72696$$

$$\Omega_0 = 138^\circ\!.45267$$

$$M_0 = 0^\circ\!.023663$$

or, in vector form,

$$x_0 = 6099.5844 \text{ km}$$

$$y_0 = 602.05128$$

$$z_0 = 2409.1608$$

$$\dot{x}_0 = 1.1047527 \text{ km/sec}$$

$$\dot{y}_0 = 9.8556127$$

$$\dot{z}_0 = -4.4520836$$

Epoch time is May 29, 1965 at 12:07 hours U.T. (a = semimajor axis, e = eccentricity, i = inclination, w = argument of perigee, Ω = right ascension of the ascending node, M = mean anomaly. The angular elements are taken with respect to the earth's equator.) The initial value of perigee is about 200 km above the earth's surface, and the apogee is about 240,000 km. The orbital period is about 140 hours, or 5.8 days.

Tracking data was acquired for the first two years of the lifetime. After the cession of radio transmission in March 1967, there were no observations of any kind until March 1968 when the Smithsonian Astrophysical Observatory was able to photograph the satellite with Baker-Nunn cameras (Figure 1). At this point, the perigee had been lowered enough to permit visual observations. These observations, which were taken during several perigee passages, on March 21, 27, April 8, May 7 and May 30, confirm the validity of the ephemeris during the unobserved period in the last year of the orbit.

III. PERTURBATIONS

The major perturbations on a highly eccentric satellite orbit are due to the gravitational fields of the sun and the moon. In addition the oblateness and higher order terms in the earth's gravitational field and the solar radiation pressure may produce a discernible effect on the ephemeris.

The height of perigee is the element which is critical to the success of the mission. The lunar-solar perturbations on perigee height are large and must be carefully considered when selecting the launch conditions to ensure an adequate lifetime for the mission (Reference 1-4). Both the gravitational fields of the sun and the moon cause the long term trend, often termed the free oscillation, in the eccentricity and inclination as a function of the argument of perigee (Reference 5-7). The effect of the long term trend on the perigee height of IMP 3 is shown in Figure 2. This long period trend rises for 1-1/2 years, reaching a maximum of 41,000 km from the center of the earth; then it reverses and declines until perigee goes below the ground after a total lifetime of a little over 3 years.

When a small portion of the lifetime is examined closely the changes in perigee height seem very irregular (Figure 3). During the lifetime, the difference between one given perigee height and the following one may be as little as 20. km, or as much as 2200. km. These seemingly random fluctuations around the long period trend are due solely to the moon's gravitational field. They occur because the semimajor axis of the satellite is not negligible in comparison with the moon's semimajor axis ($a/a' \simeq 1/3$). The satellite's apogee moves only slowly in inertial space during the lifetime of the satellite, while the moon is orbiting about the earth once every 28 days. Therefore when the satellite reaches apogee it may find the moon nearby; then on the next apogee pass 6 days later, the moon will have moved nearly 90° away. It is this variation in the apogee-lunar configuration which is responsible for the erratic quality of the changes of perigee height, and, also, of all of the orbital elements.

The variability of the lunar perturbation is of great consequence to the lifetime near the beginning or near the end of the lifetime. Between launch and the first return to perigee, the perigee of IMP 3 is raised from 200 km to 600 km, bringing the satellite above the regions where the earth's atmosphere will have any effect on the orbit.

Similarly, the erratic component in the lunar perturbation will determine on which orbit the satellite will be destroyed. At the 193rd perigee passage, the perigee height is 600 km. A very strong drop of 1100 km in perigee height occurs during the subsequent orbit, resulting in a predicted perigee height of 500 km below the earth's surface, or, actually in the impact of the satellite before perigee passage.

The orbital period of IMP 3 also shows an erratic variation between 139 hours and 143 hours (Table 1). This is an unusual occurrence in celestial mechanics; it happens because the satellite's trajectory extends out to 2/3 of the lunar distance. This phenomenon is predicted by numerical analysis, and is verified by the data. Because of the large and erratic variations, the term "orbital period" must be carefully defined; here, it is used to denote the time elapsed between one perigee passage and the next. "Perigee passage" is said to occur when the minimum value of the radius vector from the center of the earth is obtained.

There is also a short period perturbation by the earth's oblateness and higher harmonics. These perturbations are at a maximum at perigee and diminish as r^{-3}. Figure 4 shows a typical example of the short period oblateness perturbation on a prelaunch IMP orbit (Reference 2). The osculating elements at perigee may predict a markedly larger period than occurs according to the definition of the period above. All of the orbital elements will show very sharp changes during perigee passage, however, most elements do not show a secular change following perigee passage. As the perigee is raised away from the earth, this effect diminishes.

For most applications, therefore, it will be useful to find the minimum values of the radius vector. The difference between successive times of passage through the perigee should be used to compute the orbital period, rather than osculating elements, recognizing that this value will fluctuate 3 - 4 hours from one orbit to the next.

The inclination in a highly eccentric orbit can be perturbed through tens of degrees. The inclination of IMP 3 shows a long period trend, rising from 33° to over 50° (measured with respect to the plane of the earth's equator) and decreasing to less than 30° just before termination of the orbit (Figure 5). The tendency to rise rapidly initially and then to form a plateau is typical of highly eccentric orbits; the plateau may occur at very high inclinations.

The latitude of perigee moves from 22°N to -10°S during the satellite's lifetime (Figure 6). The longitude of the subsatellite point at perigee is random, because the satellite's orbital period is not commensurate with the rotation of the earth; the right ascension of the ascending node, which is fixed in inertial space, evolves gradually from +222° to +176° during the lifetime.

It should be commented that there is no evidence of a secular change in the average value of the semimajor axis. This remains an open question in celestial mechanics (Reference 8). Here the small apparent long term variation of the semimajor axis may be attributed to the earth's second harmonic coupled with the long-term evolution in perigee.

IV. COMPUTATION

A highly eccentric orbit provides a stringent test of a numerical integration program. Most major forces are required to compute the orbit accurately, along with careful computational techniques.

The major gravitational forces are the sun, the moon and the earth, including oblateness and higher order harmonics. The sun and the moon both perturb the perigee height strongly, and if either is inadvertently omitted, or incorrectly included, in a program, the perigee height will not be correctly computed.

The oblateness of the earth has a small effect on the long term evolution of the orbital parameters. However, there is a very sharp short period effect of the oblateness perturbation which occurs very near perigee. Extreme care must be taken to prevent the development of computational inaccuracies during this portion of the orbit. Because the effect is so localized, the computation is very sensitive to the size of the integration interval near perigee. It can happen that an interval which is satisfactory elsewhere in the orbit is far too large near perigee. In fact, most of the machine time necessary to compute an orbit is consumed during perigee passage. An improved method of calculating the oblateness perturbation at perigee would be very helpful in reducing machine time.

If the integration interval is not controlled carefully, the orbit will receive an apparent (and erroneous!) impulsive force which will alter the orbit. After a few such perigee passes, the numerical accuracy of the computation will have become irretrievably degraded. The osculating value of the semimajor axis is helpful in indicating the accuracy of the trajectory. In a correct computation, this value will rise sharply as perigee passage occurs, shortly after perigee, this quantity will drop back to the value shown just before perigee passage. This "spike" effect is to be expected, with the magnitude of the spike depending on the height and position of perigee.

If the integration interval is large compared with the spike, so that the spike is not computed correctly, the value of the semimajor axis after perigee passage will not return to the value before perigee. The effect is to add an incorrect impulsive change in the orbit.

Large changes in the semimajor axis between two perigee passes are not necessarily due to numerical error. As was explained in the previous section on perturbation, the moon may impose a substantial variation on the semimajor axis (that is, the orbital period). These variations should average out to near zero over a number of orbits. However, a drastic secular change in the semimajor axis should not occur if there is no substantial atmosphere drag and no passage around the moon occurs. For example, a decay of several earth radii in the semimajor axis would most likely indicate computational error.

The computation of the ephemeris has been performed with the ITEM (Interplanetary Trajectory Encke Method) program (Reference 10), using a Fortran IV - 360 version. This program is based on a modified Encke method, which numerically integrates the perturbations on a reference ellipse. The reference body is the earth, with permissible gravitational perturbations including the moon, sun, Jupiter, Venus, Mars, the earth's harmonics and the non-gravitational perturbation due to solar radiation pressure.

There is an option in the program to employ a regularized variable as an independent variable rather than the time. The regularized variable is an advantage in computing highly eccentric orbits, since the velocity of the satellite varies by a factor of 40 between apogee and perigee. Integration intervals of equal time would allow the satellite to go an excessive distance at perigee while barely changing at apogee. Therefore, a transformation to use β as the independent variable is made:

$$\frac{dt}{d\beta} = \frac{\rho}{\sqrt{\mu}}$$

where t = time in seconds, ρ the radius vector in km, and μ = 398603. km^3/sec^2 (gravitational constant for the earth). Then, at perigee, taking ρ = 7000 km, a value of $\Delta\beta$ = 1/16 will equal a Δt = .6 sec; while at apogee, ρ = 240,000 km, then for $\Delta\beta$ = 1/16, Δt = 25. sec. In other words, the use of $\Delta\beta$ will automatically contract the integration interval at perigee and expand it at apogee. The choice of $\Delta\beta$ = 1/16 yields the accuracy desired for this problem. This choice is indicated by the interval required to satisfactorily compute the earth's oblateness near perigee. A larger value of Δt could likely be used at apogee; but since most of the machine time is consumed performing the perigee computation the value of Δt at apogee is not a significant factor.

The value of $\Delta\beta$ = 1/16 was compared with results from an otherwise identical run utilizing $\Delta\beta$ = 1/32. The discrepancy in time of perigee passage between the two runs was an order of magnitude less than the differences between the computation and the data. Therefore, the value of $\Delta\beta$ = 1/16 was considered satisfactory.

V. DATA AND TECHNIQUE

Orbit improvement, rather than orbit determination, is used to obtain initial conditions which will produce an ephemeris valid over the entire two-year arc. A set of preliminary initial conditions are used to generate an ephemeris which is compared to the data and, from the differences between the data and the computation an improved set of intiial conditions is computed.

The "data" itself is not raw observations, but rather is extracted from computations which are the result of previous reductions. That is, a sequence of short arc orbit determinations was available and the objective was to try to obtain a good fit to all the short arcs simultaneously, rather than to attempt to match the original raw data as obtained from the range and range-rate tracking. The short arcs are in the form of World Maps (Reference 9), which express the longitude and latitude of the subsatellite point and the height above the earth's surface at intervals of one minute. These short arcs are determined over a period of about one month, that is, 5 or 6 satellite revolutions.

Two quantities were selected from the World Maps for the purpose of comparing the computation with the data: the time of perigee passage and the perigee height. These two parameters were of crucial significance in predicting the reentry of the orbit. Also, they provide a stringent test of the accuracy of the ephemeris.

The time of perigee passage yields a critical evaluation of the ability of the ephemeris to compute an instantaneous position in space. If there is a timing error in the ephemeris, for example originating from an incorrect estimation of the initial semimajor axis, the computed value of the position in orbit will increasingly deviate from the actual value, although all other orbital elements may be in good agreement with the "real" elements.

The time of perigee passage provides a particularly useful parameter for evaluating the timing of the ephemeris because it is very well-defined in a highly eccentric ellipse and occurs when the best geometrical definition is obtained because the satellite is closest to the earth. In fact, it may be that the original orbit determination process will force the solution to be correct at this position by making compensatory errors elsewhere. Certainly, the ephemeris in this paper was obtained by a deliberate forcing of the solution to be accurate at the time of perigee passage.

Since the World Map quantities are listed at intervals of one minute, an interpolation was found desirable to provide more accurate values of the perigee time and position. This allows a better and more convenient comparison with the numerical integration, which computes values at the instant of perigee passage.

The height of perigee is also a well-defined parameter. It provides a measure of the eccentricity and the variations of the eccentricity. It is critical to the lifetime of the satellite. Also, a good agreement in the height of perigee guarantees that the perturbing forces of the sun and the moon are correctly computed. The values of perigee were also obtained by interpolation from the World Maps.

In addition, the latitude and longitude of the perigee position are compared. The latitude indicates the accuracy of the angular orbital elements. The longitude is a redundant measure of the timing error and the angular elements. The major usefulness of the longitude for a comparison program is to ensure that the rotation of the earth is included correctly. The initial orbital elements, obtained from the injection conditions as determined from the early orbits following launch, were used to initiate a numerical integration of the orbits. The values of the time of perigee passage for every orbit were obtained from the computation and subtracted from the "observed" values. When these differences, or residuals, of the first 5 or 10 orbits were examined, a linear growth in the time of perigee passage would appear due to the accumulation of error when the orbital period assumed in the computation differs slightly from the actual period. The residuals between the observed and computed time of perigee passage on the ith orbit after launch

$$\Delta t_{p_i} = t_{p_{i\,obs}} - t_{p_{i\,comp}}$$

were summed by least squares to yield a change Δp in the orbital period P_0 initially assumed

$$\Delta p = \left[\frac{\Sigma t_{p_{i\,comp}} \Delta t_{p_i}}{\Sigma t^2_{p_{i\,comp}}} \right] P_0$$

which was converted into an improved value of the semimajor axis, a_0', by use of Kepler's law

$$a_0' = a_0 + \Delta a = a_0 + 2/3\,\Delta p \,\frac{a_0}{P_0}$$

(time is measured from launch). The numerical computation was performed again, utilizing the new initial value of the semimajor axis, a_0'. This process is repeated until the residuals no longer exhibit a linear trend; then the computation may be extended over more orbital passages, and, if necessary, the linear trend may again be removed. Only a few iterations are necessary. It was found that by using 25 orbits, all linear accumulations were removed and that iterations through additional data would yield no further improvement.

The values of the perigee height residuals were used to adjust the initial eccentricity. The major effect of changing the initial eccentricity is to move the base line of the residuals in perigee height up or down.

It was found that a change in the initial value of the eccentricity, or of one of the angular elements (inclination, argument of perigee, or the right ascension of the ascending node) necessitated a change in the initial value of the semimajor axis although the value of the semimajor axis prior to the change in another orbital element had yielded satisfactory residuals in the time of perigee passage. This coupling of the semimajor axis with the other orbital elements is apparently due to the oblateness of the earth which strongly influences the energy of the orbit. (The oblateness causes a difference between the anomalistic and the osculating value of the period at perigee of about 4 hours on the IMP 3 orbital period of 140 hours.) However, satisfactory values of the semimajor axis can be obtained equally well after small changes in the orbital elements, providing that first the remaining elements are modified, and then the iterations are performed with only an input change to the semimajor axis until the procedure converges.

Due to this coupling of the corrections to the semimajor axis with corrections to other orbital elements, the initial conditions which provide satisfactory residuals in the time of perigee passage are not unique. For example, little change in the residuals is produced, if either set of orbital elements

	Set 1	Set 2
a_0	21.726218 earth radii	21.726138 earth radii
ω_0	135°.73696	135°.72696
e_0	.95251083	Same
i_0	33.828445	Same
Ω_0	-138.45267	Same

is used to initiate the numerical computation. It might be argued that this occurs because the time of perigee passage is only one observational parameter for testing the orbit and that consideration of two or more variables might provide a better fit of the orbit; however, this "observed" quantity is particularly sensitive and will in fact partially determine the choice of other quantities once a semimajor axis has been chosen. Further, not only corrections to the orbital elements but also model errors such as the value of the earth's harmonics, may be eliminated by an appropriate choice of the semimajor axis. It should be stressed that the semimajor axis in Set 2 could not be substituted into Set 1 and produce satisfactory residuals. A change of .5 km in a Set 1 would produce a substantial linear term in the residuals. The correlation between the semimajor axis and numerous other quantities constitutes a major difficulty in long-arc orbit analysis.

VI. COMPARISON BETWEEN DATA AND COMPUTATION

The residuals in the time of perigee passage provide the most critical evaluation of the accuracy of the ephemeris. The data from the first 25 orbits were used to adjust the initial value of the semimajor axis; then the numerical integration was extended without further correction through for the 120 orbits during which there was radio coverage. The difference between the observed time of perigee passage and the computed time for each orbit is shown in Figure 7. These residuals show no linear trend during the 120 orbits, indicating that 25 orbits are sufficient to define the semimajor axis (or, more rigorously expressed, to define the total energy).

The residuals in the second portion of the 120 orbits exhibit a systematic deviation which resembles a low frequency sinusoidal. This deviation is currently unexplained but may arise from one of the following sources:

1. a small error in one of the initial conditions
2. an error or omission in the model in the numerical integration, for example, an omission of a higher order harmonic in the earth's gravitational field, or a poor choice of the value used in computing the solar radiation pressure force
3. an error in the numerical integration.

Of these choices, 2, is most likely. It can be difficult to locate the model error when the model is as complex as is required for this ephemeris.

The largest residuals are no greater than 1 minute in time of perigee passage, compared with an orbital period of 140 hours. This is an indication of the accuracy of the ephemeris. The systematic trend in the residuals shows the data itself is highly accurate. Since there is no discernible scatter in the data, the random error in the data cannot be evaluated, but it is certainly much less than one minute of time. It seems probable that this type of data is independent of systematic errors and that the systematic trend exhibited in the residuals is due to computation error.

The residuals in the height of perigee are shown in Figure 8. Some estimate of how much of this deviation is due to the data can be made by examining overlapping data arcs of 4 or 5 orbits; the residuals often change sharply when the arc is changed. Occasionally, there is a perigee passage which is computed during the overlapping position of two adjacent arcs. There are differences between the overlapping arcs in the perigee height amounting to 6 km early in the lifetime and as large as 20 km toward the end of the data (when the perigee is near maximum distances from the earth); therefore, the data is at least that inaccurate. There appears to be a systematic trend as well; this is most likely due to a model error in the computation. The latitude of perigee residuals are less than .07°.

Although the telemetry ceased operating after two years, there are visual observations confirming the latter part of the ephemeris. As the perigee height decreased, it became possible to acquire the satellite with Baker-Nunn cameras.

After almost one year without any type of observations, this ephemeris was used to predict the location of the satellite during perigee passage. One several occasions, the SAO tracking network acquired a satellite in the predicted location with the expected visual magnitude and with a spin rate compatible with the satellite's characteristics (Figure 1). Although it is difficult to compare the observations precisely with the ephemeris, it is felt that the acquisition of the satellite implies that the time of perigee passage is correct to within ± 3 minutes.

The precision of the ephemeris and the fact that the last perigee before reentry was 600 km above ground while the reentry "perigee" was 500 km below ground led to the hope that the reentry phase might be observed. A group on the island of Gan in the Indian Ocean attempted to observe the event (Reference 11); however, there was a heavy cloud cover over the area that night and the reentry was not observed.

VII. EPHEMERIS

The computed ephemeris is presented here for use as a standard in checking other programs. The ephemeris (Table II) is presented in two forms: 1) the osculating orbital elements are listed at perigee passage (to within $\underline{10^{-4}}$ hours) for every 10th perigee passage during the satellite's lifetime: 2) the osculating orbital elements are listed every 30 days during the first year. (The tabulation is actually within a fraction of an hour of 30 days. Usually the osculating elements change little during this time span.) The orbit number is defined by taking the "0th" orbit to be at launch.

The geodetic longitude and latitude of the subsatellite point at perigee and the perigee distance in km from the center for the earth for the first 25 orbits are given in Table III.

An ephemeris listing osculating elements at every perigee passage may be obtained from the author.

A researcher interested in comparing his program with this ephemeris would most likely have one of two objectives:

1. The ability of this program to compute the evolution of perigee height. If this is the researcher's objective, precision computation of the time of perigee passage is not necessary. The evolution of perigee can be computed by one of three methods:

   a. a special perturbations method such as employed here, which relies completely on numerical integration;

b. a "mixed" method which utilizes both analytic expressions and numerical integration. An example of this is the Halphen method as employed by Musen (Reference 12), and applied by Smith (Reference 13). (This method computes only the long term evolution. Terms depending on the moon's longitude are omitted.)

c. a strictly analytic method, such as developed by Lidov (Reference 6) as employed by Renard (References 1, 3). It should be pointed out that the IMP 3 orbit is a more stringent test than necessary for high eccentricity orbits with smaller semimajor axes.

2. The ability of this program to track the satellite around its orbit. The accurate computation of time of perigee passage is a necessary, although not sufficient, condition to yield the correct position of the satellite. If there are model errors which are compensated for by an incorrect value of the semimajor axis, there will be systematic errors in the residuals near, but not at, the perigee passage which are not necessarily evident in the computation of t_p.

As has been discussed above, the data indicates that there probably are model errors in the generation of this ephemeris. The researcher may compensate for these and for model differences between his program and ITEM by adjusting the initial value of the semimajor axis according to the procedure described above.

VIII. CONCLUSIONS

An ephemeris has been obtained which represents the "long" (monthly) arcs of a high eccentricity satellite with good accuracy. Probably with further study, and with the improved data now available on current satellites, even better precision can be obtained.

The ephemeris can be used as a standard to evaluate a program's ability to compute either the long-term evolution of a high eccentricity orbit in the Earth-Moon system, or to track a satellite along its orbit.

It may be possible to predict the longitude, latitude and time of reentry of future high eccentricity orbits far in advance of the event. The ability to predict will depend on the values of the final perigee heights. The IMP 3 was particularly favorable for prediction because of the large decrease in the perigee height during the final orbit.

Long arc determinations may be useful in evaluating the methods and the results of short arc determinations. In particular, it may be that ambiguities in the energy determination similarly affect short arc determinations so that equally good fitting initial conditions could be obtained with low errors ascribed to each but with larger differences between the two sets than the errors. Also, model errors may be compensated for by changes in the initial conditions producing an illusion of a good fit. It may be worthwhile to determine model errors by identifying observable quantities which are particularly sensitive to a model quantity—such as the application of resonant orbits to the earth's harmonics.

The time of perigee passage is an exceptionally sensitive parameter when it is available for large number of orbits and is therefore to be recommended for utilization in long arc studies. It insures that the instantaneous position of the satellite corresponds closely to the predicted position. Further, it appears that this quantity is determined very accurately when raw data has been reduced to produce a short arc of 5 or 6 orbits. The accuracy of the data is inferred because there is no strong discontinuity in the residuals from short arc between one short arc and the next.

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Table 1

Variation of Time Between Perigee Passages
Due to Random Lunar Perturbation

Perigee Passage	Time in Hours Since Last Passage
1	139.866
2	140.590
3	138.860
4	139.385
5	139.604
6	138.773
7	140.609
8	139.770
9	140.350
10	140.309

Table II

Elements at Every 10th Perigee

Orbit #	t (hrs)	a	e	i	ω	Ω
0	0.0	21.72613525	0.95251077	33.82844543	135.72695923	-138.45266724
~~10~~	~~1398.11596680~~	~~21.59710693~~	~~0.92810524~~	~~37.18685913~~	~~141.50469971~~	~~-145.33018494~~
20	2796.23583984	21.42175293	0.91221130	46.37657166	149.66542053	-154.98464966
~~30~~	~~4200.94921875~~	~~21.38430786~~	~~0.86355197~~	~~47.76776123~~	~~153.06774902~~	~~-156.87995911~~
40	5597.82812500	21.55686951	0.82488447	45.92031860	153.33386230	-156.60366821
~~50~~	~~6997.53125000~~	~~21.37214661~~	~~0.79930210~~	~~51.88568115~~	~~157.95069885~~	~~-160.68077087~~
60	8400.21875000	21.45301819	0.76386106	51.02879333	161.24218750	-160.61872864
~~70~~	~~9800.16015625~~	~~21.49591064~~	~~0.73197979~~	~~49.62825012~~	~~163.62460327~~	~~-160.99873352~~
80	11198.65234375	21.33445740	0.72345817	53.15490723	167.15707397	-162.61418152
~~90~~	~~12605.19921875~~	~~21.56539917~~	~~0.70308542~~	~~51.74876404~~	~~172.07267080~~	~~-162.71607971~~
100	14004.22656250	21.39897156	0.69893336	50.94969177	175.89070129	-163.33935547
~~110~~	~~15405.55859375~~	~~21.41853333~~	~~0.71008831~~	~~52.52732849~~	~~-179.71098328~~	~~-164.39640808~~
120	16810.25781250	21.47793579	0.72564995	52.17111206	-175.49523926	-164.40739441
~~130~~	~~18210.67187500~~	~~21.51316833~~	~~0.74451661~~	~~48.99206543~~	~~-171.54043579~~	~~-165.59844971~~
140	19611.01562500	21.34976196	0.77660877	51.18333435	-169.77943420	-166.01838684
~~150~~	~~21016.96093750~~	~~21.50984192~~	~~0.81164545~~	~~50.30612183~~	~~-166.38722229~~	~~-166.35319519~~
160	22416.37890625	21.50439453	0.84867328	44.95545959	-162.40332031	-169.23683167
~~170~~	~~23814.10156250~~	~~21.42352295~~	~~0.87818670~~	~~45.91520691~~	~~-162.10961914~~	~~-169.41687012~~
180	25220.67187500	21.48927307	0.92911297	44.93495178	-159.29423523	-170.77110291
~~190~~	~~26617.92187500~~	~~21.86282349~~	~~0.94948208~~	~~31.09628296~~	~~-149.09939575~~	~~177.58930969~~

Elements Every 30 Days During First Year

Time (hrs)	a	e	i	ω	Ω
0.0	21.72613525	0.95251077	33.82844543	135.72695923	-138.45266724
720.11962891	21.38981628	0.93976802	35.53506470	139.79541724	-142.00456238
1440.03735352	21.46665955	0.92782646	37.28845215	141.66471863	-145.57542419
~~2160.13720703~~	~~21.49863516~~	~~0.92217571~~	~~41.73361206~~	~~146.31529236~~	~~-151.14788819~~
2880.47485352	21.53030396	0.90132010	47.79040527	151.06108093	-156.43797302
3600.46435547	21.53904724	0.87746084	47.08518982	151.91322327	-156.17785645
4320.14843750	21.41691589	0.85928148	46.86953735	152.63725281	-156.22853088
5040.00000000	21.46520996	0.83706790	46.15867615	153.09681702	-156.43241882
5760.08203125	21.51409912	0.82594895	46.55622864	153.57153320	-157.08666992
~~6480.32031250~~	~~21.39477539~~	~~0.81902146~~	~~50.13539124~~	~~155.80899048~~	~~-159.36437988~~
7200.33203125	21.40110779	0.79618001	52.42835999	158.73838806	-161.00299072
7920.13671875	21.54339600	0.77252102	51.13845825	160.49586487	-160.67384338
8640.20312500	21.47477722	0.75874907	50.13275146	161.36207581	-160.26983643
9360.13281250	21.37338257	0.74508071	50.32829285	162.69363403	-160.76251221

Table III

Perigee Conditions of IMP 3

Orbit #	Longitude	Latitude	Perigee Distance
0	-62.923	21.584	6580.71
1	-9.685	22.923	6958.01
2	35.600	22.868	7639.24
3	107.008	22.826	7934.76
4	170.485	22.772	7916.74
5	-129.317	22.632	8207.35
6	-56.606	22.702	8268.73
7	-11.625	22.342	9820.28
8	46.015	22.377	9717.20
9	94.992	22.296	9661.32
10	144.594	22.184	9903.49
11	-154.982	22.354	9704.73
12	-98.018	22.165	10545.22
13	-45.654	22.159	10333.99
14	-0.865	22.002	10406.69
15	49.720	21.928	10603.66
16	92.709	21.899	11274.44
17	164.642	21.858	11518.69
18	-132.246	21.752	11510.26
19	-72.908	21.522	11821.87
20	-3.905	21.511	11994.70
21	34.274	20.957	14236.42
22	91.518	20.925	14374.23
23	140.187	20.707	14616.92
24	-170.119	20.477	15117.77
25	-110.129	20.531	15266.92
26	-62.626	20.146	16903.51
27	17.246	20.093	17047.93
28	21.661	19.850	17467.76
29	67.623	19.760	17981.45
30	108.915	19.636	18610.51
31	178.324	19.623	19224.81
32	-120.011	19.574	19340.30
33	-61.749	19.394	19774.83
34	7.705	19.505	20005.67
35	44.442	18.951	22246.86
36	108.657	19.085	22306.37
37	164.422	19.018	22345.00
38	-140.016	18.918	22686.98
39	-72.790	19.120	22505.68
40	-21.777	18.839	24077.20
41	28.835	18.919	23867.32
42	72.005	18.764	23926.82
43	116.979	18.705	24167.27
44	164.859	18.590	24248.45
45	123.378	18.574	24670.92
46	-60.096	18.480	24575.04
47	-1.378	18.206	24821.60
48	65.037	18.165	25001.31
49	97.373	17.231	27235.73
50	166.366	17.204	27358.16

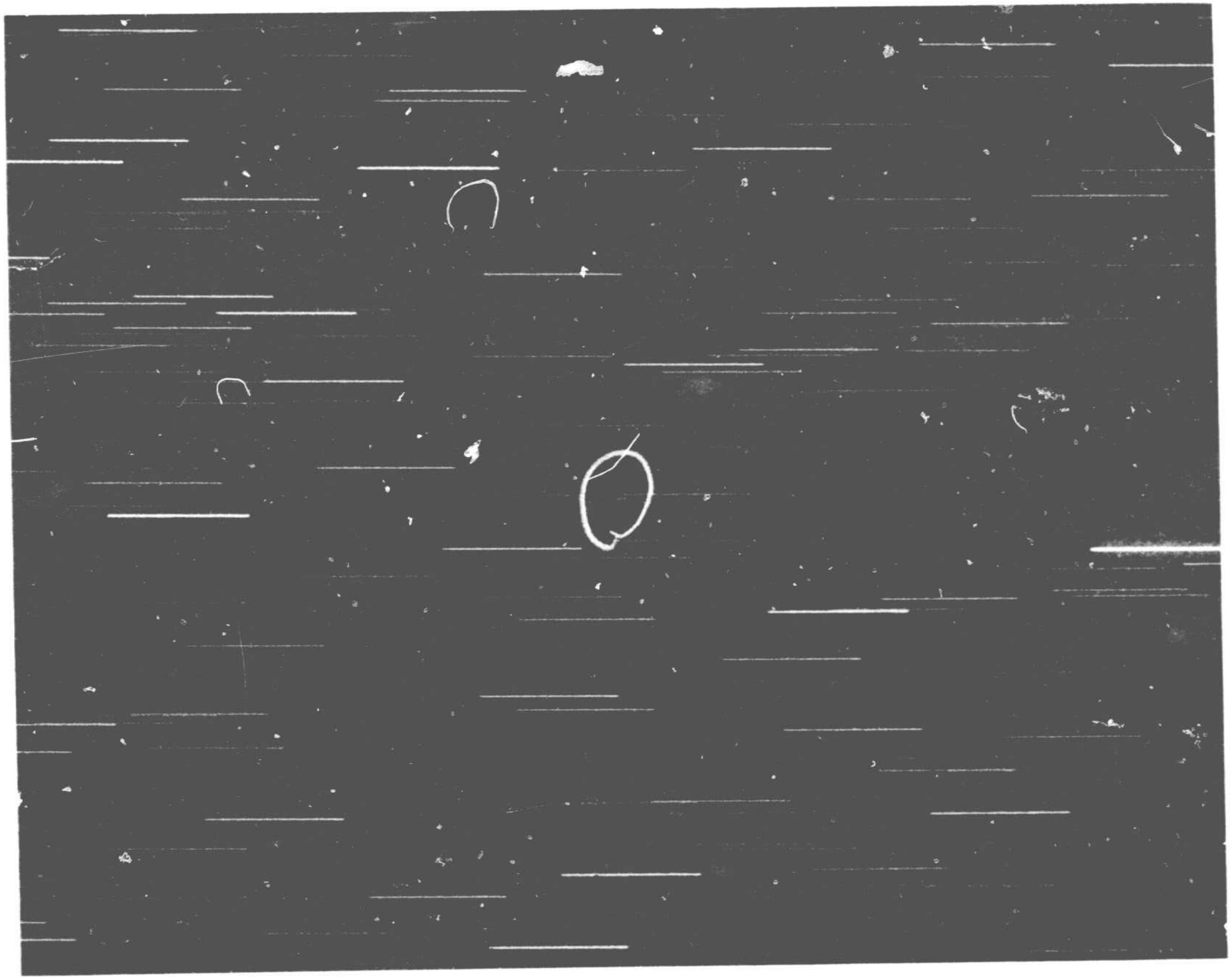

Figure 1. Photograph of IMP-III taken by Smithsonian Astrophysical Observing Station at Olifantsfontein, South Africa, on March 27, 1968 at 18:24:26.5 UT

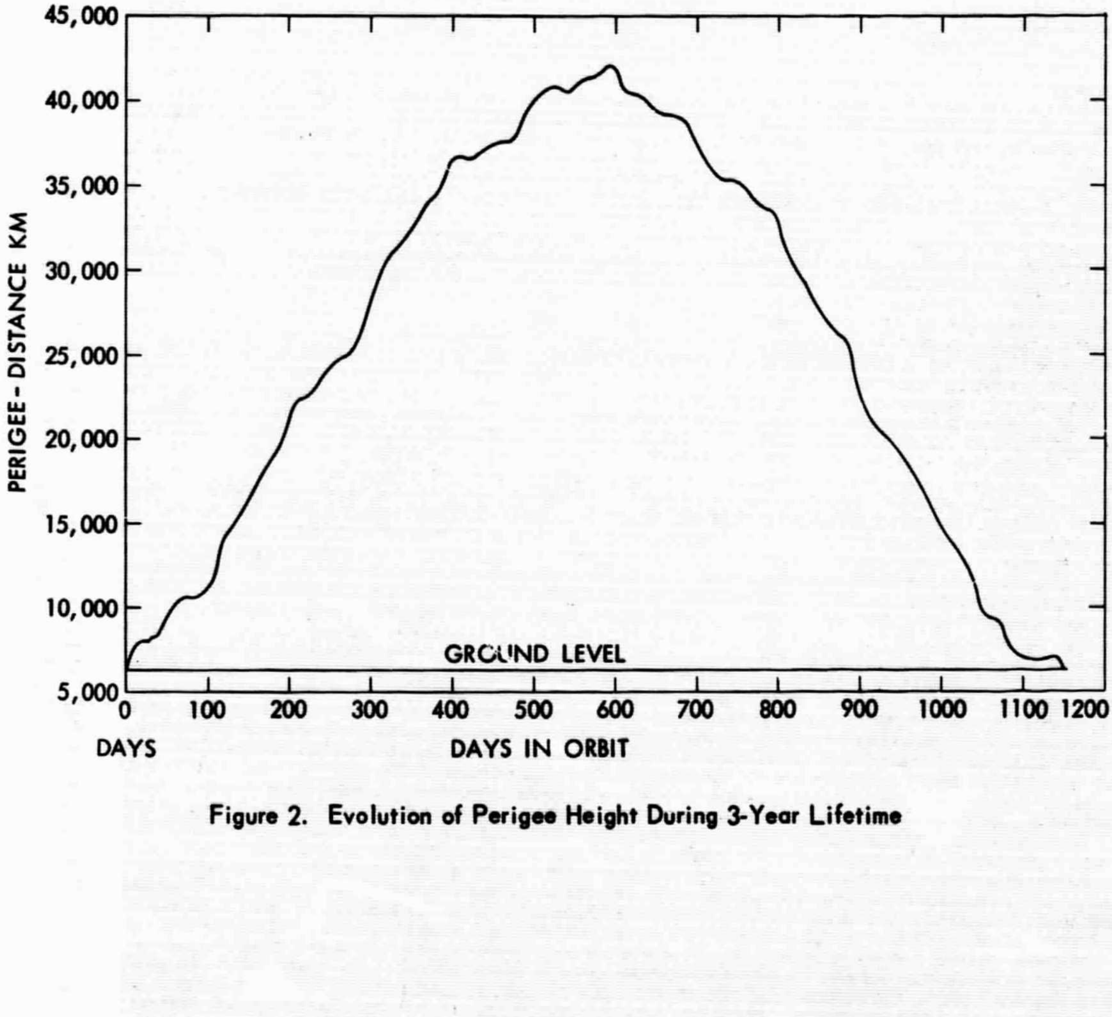

Figure 2. Evolution of Perigee Height During 3-Year Lifetime

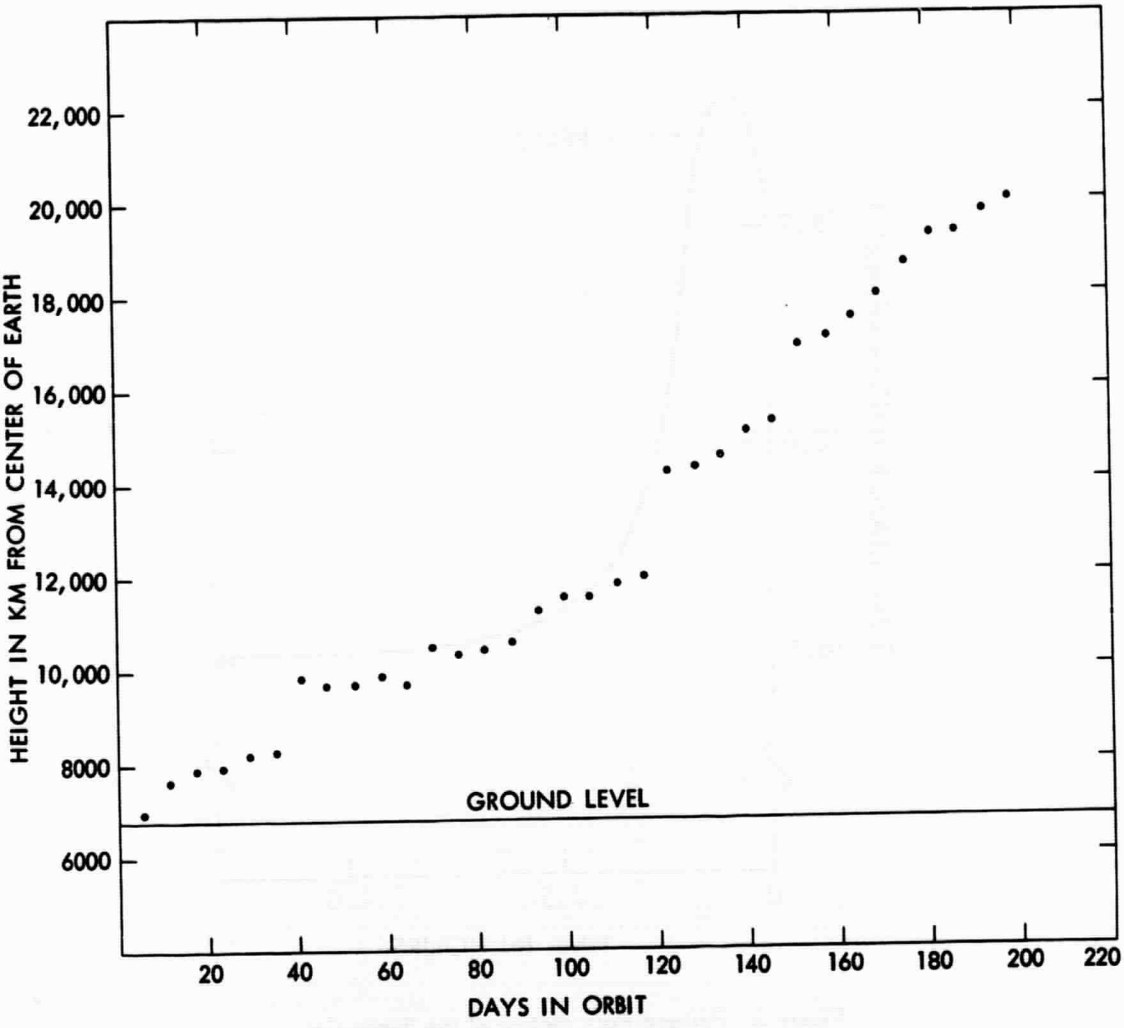

Figure 3. Random Effect of Short-Period Perturbations on Perigee Height

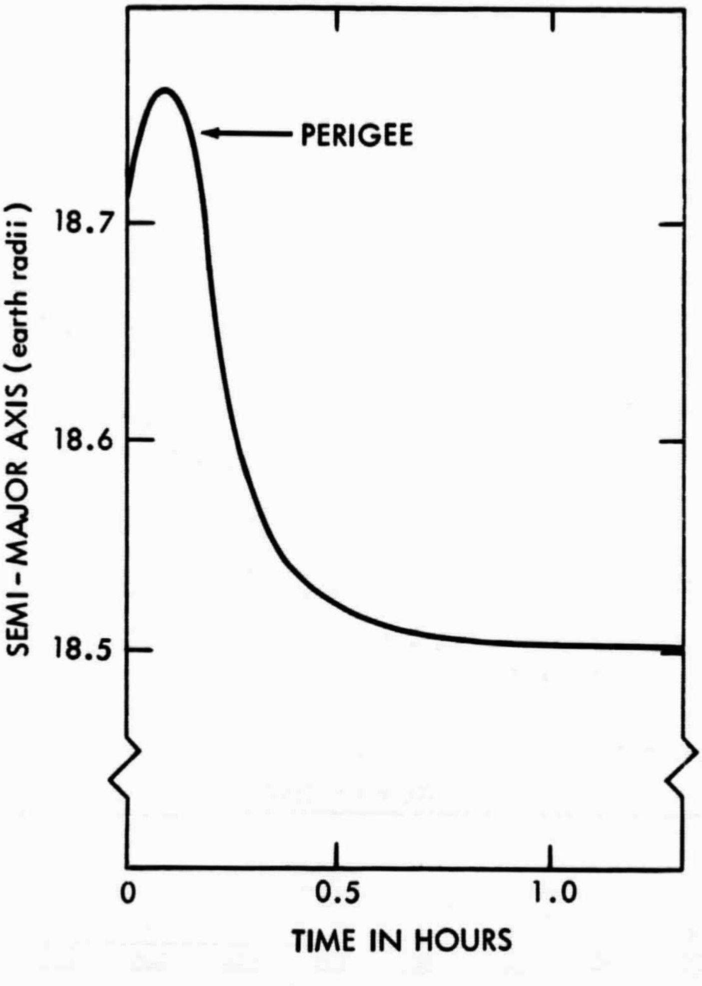

Figure 4. Perigee Perturbation of the Semimajor Axis of an IMP-Type Satellite

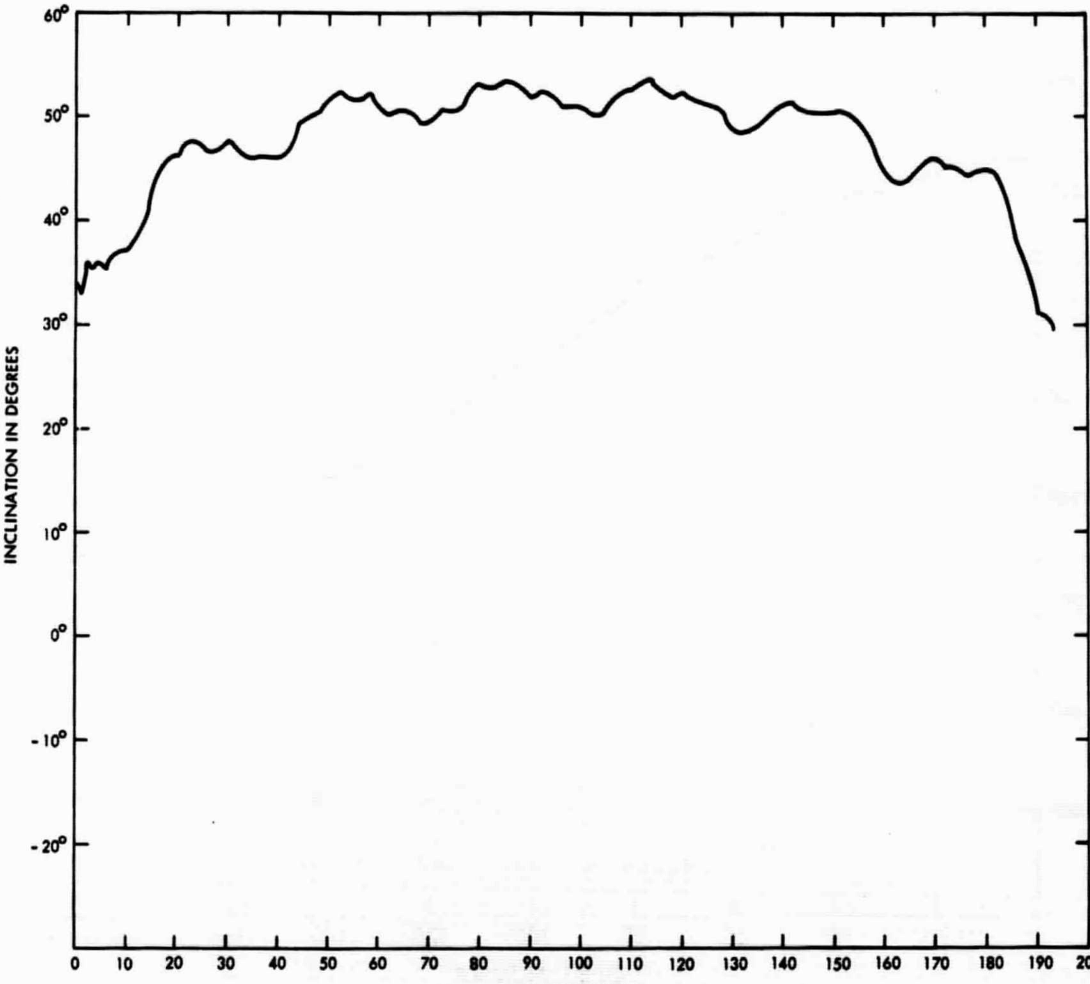

Figure 5. Evolution of Inclination of IMP III

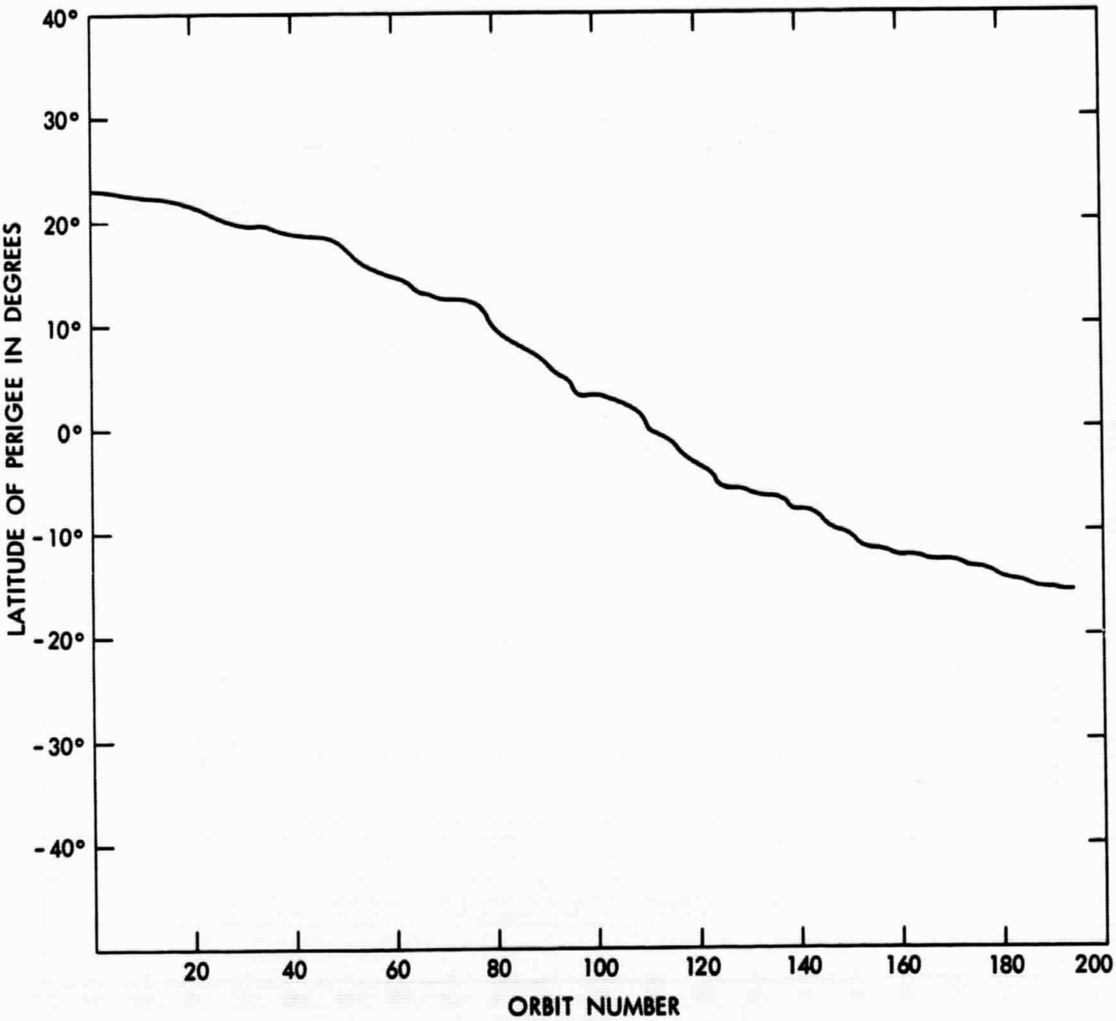

Figure 6. Latitude of Perigee During Lifetime of IMP 3.

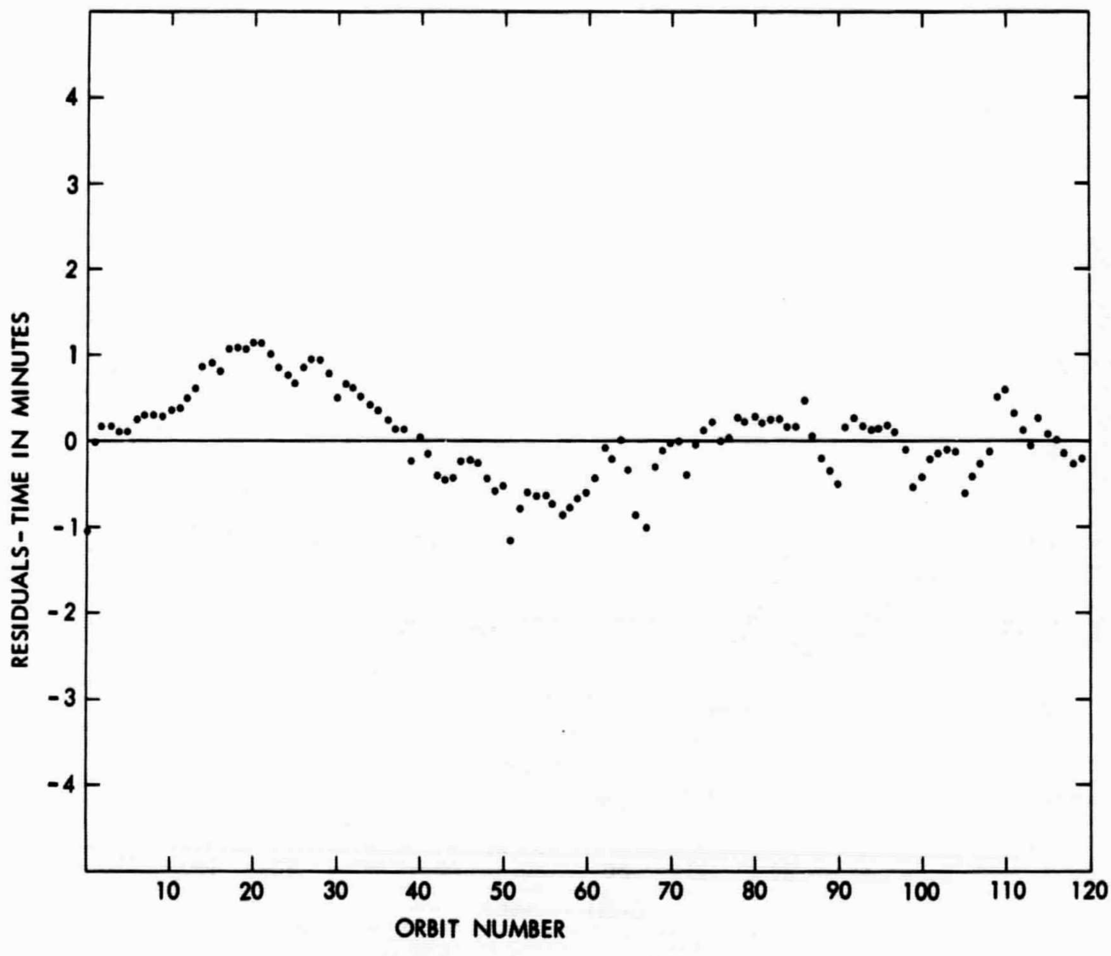

Figure 7. Residuals in Time of Perigee Passage

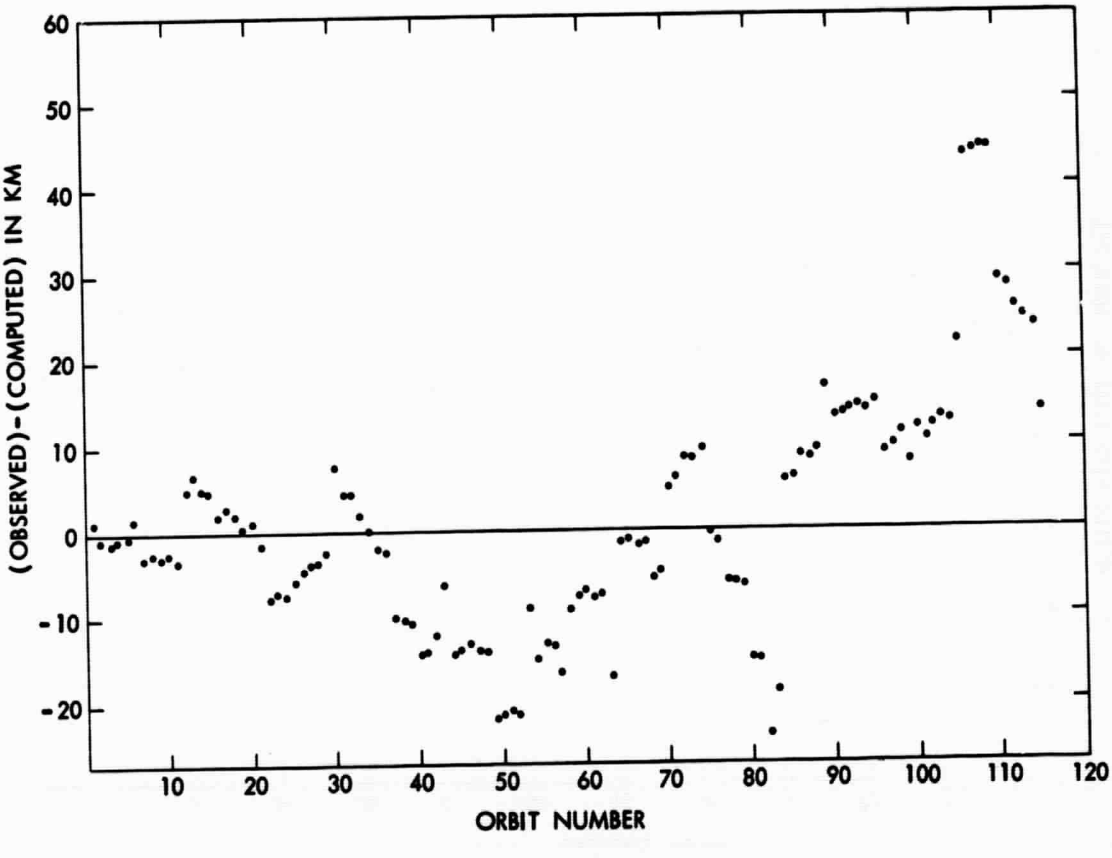

Figure 8. Residuals in Perigee Height