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THE GENERATION OF X RAYS IN PLANETARY NEBULAE

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The evolution of planetary nebulae and the production of X rays will be summarized from the literature. Planetary nebulae emit continuous radiation which originates by free-free transitions involving kinetic energy losses of free electrons in the electrostatic fields of ions. These free-free emissions are responsible for the radiation observed in the radio frequency spectra. They also contribute in the visible and infrared regions. Therefore, if X rays are generated by planetary nebulae, it can be assumed that they will also be produced by free-free transitions. However, free-free radiation will be measured along with free-bound radiation. A formula is given for the intensity of the combined continuum for an ionized gas with a Maxwellian distribution of the electron velocities.

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LIST OF SYMBOLS

Symbol	Definition
χ_0	ionization energy of hydrogen
c	velocity of light
E	initial energy of the free electron
E'	final energy of the free electron
e	charge of the electron
ϵ_0	permittivity of free space
$f(T, \nu)$	functional containing $g_n(\nu)$
$g(E, \nu)$	Gaunt factor
$g(T, \nu)$	weighted average of g for a Maxwellian distribution
$g_n(\nu)$	Gaunt factor for the transition from the free level E to the bound level E'
h	Planck's constant
$\hbar$	atomic unit of angular momentum
k	Boltzmann's constant
m	mass of the electron
$m(\nu)$	principal quantum number of the lowest bound level to which emission at the frequency ν can occur
N_i	ion density
N_e	electron density

LIST OF SYMBOLS (Concluded)

Symbol	Definition
ν	frequency of emitted radiation
ν_0	frequency of the hydrogen ionization limit
$P(T, \nu)$	power density emitted in a combined continuum per sec^{-1} bandwidth
$P_{ff}(T, \nu)$	power density emitted in free-free encounters per sec^{-1} bandwidth
T	temperature of gas
$\bar{v}$	mean velocity of electron
Z	atomic number

TECHNICAL MEMORANDUM X-53996

THE GENERATION OF X RAYS IN PLANETARY NEBULAE

SUMMARY

The evolution of planetary nebulae and the production of X rays will be summarized from the literature. Planetary nebulae emit continuous radiation which originates by free-free transitions involving kinetic energy losses of free electrons in the electrostatic fields of ions. These free-free emissions are responsible for the radiation observed in the radio frequency spectra. They also contribute in the visible and infrared regions. Therefore, if X rays are generated by planetary nebulae, it can be assumed that they will also be produced by free-free transitions. However, free-free radiation will be measured along with free-bound radiation. A formula is given for the intensity of the combined continuum for an ionized gas with a Maxwellian distribution of the electron velocities.

INTRODUCTION

In a statistical study which was undertaken to determine the degree of correlation existing between the positions of X-ray sources and classical cepheids, galactic novae, planetary nebulae, and Wolf-Rayet stars, the correlation coefficients obtained for the cepheids and Wolf-Rayet stars were small and those obtained for the galactic novae and planetary nebulae were high. It can be concluded that X-ray sources are more strongly associated with disk Population II objects than with extreme Population I objects. X rays are produced in supernovae as synchrotron radiation and in novae as deceleration (free-free) radiation. Planetary nebulae emit continuous radiation which originates by free-free transitions involving kinetic energy losses of free electrons in the electrostatic fields of ions. These free-free emissions are responsible for the radiation observed in the radio frequency spectra. They also contribute in the visible and infrared regions. Therefore, if X rays are generated by planetary nebulae, it can be assumed that they will be produced by free-free transitions also. Distant encounters between protons and electrons, involving small losses of energy, are more numerous than close approaches which produce large energy changes. Therefore, the free-free emission is more important at lower frequencies or longer wavelengths.

THE EVOLUTION OF PLANETARY NEBULAE

To maintain the energy balance in a planetary nebula, the star has to possess a degenerate core with a contracting outer region. The most highly evolved stars approach the white dwarf state. Since the mass lost during the formation of the planetary nebula occurs at an early stage, the surrounding shell is too faint to be detected in later stages. A significant number of the white dwarfs has evolved through the stage of planetary nebulae.

To estimate chemical composition of a planetary nebula is:

Element	H	He	C	N	O	F	Ne	Na	S	Cl	Ar	K	Ca
log N	12.00	11.25	8.7	8.5	9.0	5.2	8.2	6.0	8.0	6.5	6.9	5.8	6.2

The planetary nebulae have an original mass of about 1.2 $M_\odot$ with a lifetime, after nuclear reactions begin, of 7×10^9 years. The star-nebula systems observed as planetary nebulae represent the gravitational contraction phase following nuclear fuel burning in stars of $M = 1.2\ M_\odot$. Less massive stars do not have to lose mass in this phase and will not be identified as planetary nebulae. The more massive stars may evolve in such a manner that the mass loss occurs slowly, failing to produce the bright shells.

An evolutionary track of log $\frac{L}{L_\odot}$ against log T on the Hertzsprung-Russell diagram is shown in Figure 1. The positions of radius $R_0 = 0.062$ and $R = 0.1,\ 0.2,$ and 0.4 parsec are indicated. Since the nebulae are expanding, evolution takes place in the direction shown by the arrows.

The electron density of a nebula, and the luminosity and temperature of its central star vary with the time t. Initially N_e is large, the luminosity L equals 60 $L_\odot$, the temperature T equals 32 000° K, and the nebula is optically thick. The star evolves to $L = 25\ 000\ L_\odot$, $T = 60\ 000°$ K, the nebula expands, and the optical thickness decreases. At a time t_0, radius R_0, the nebula becomes optically thin. For $t_0 < t$, N_e continues to decrease because of expansion. Then the temperature rises to $10^{5°}$ K at approximately constant L, and the optical thickness continues to decrease during this phase. Thereafter L decreases rapidly; the temperature remains at $10^{5°}$ K until the luminosity declines to 100 $L_\odot$. Finally, the decrease in luminosity resulting from the onset of degeneracy becomes more important than the expansion, and the optical depth increases. At a radius R_i, the nebula again becomes optically thick. $R_0 = 0.06$ pc and $R_1 = 0.6$ parsec.

The results of Figure 1, the main sequence, the horizontal branch, and the white dwarfs are drawn in the H-R diagram of Figure 2. Using an expansion velocity of the nebula of 20 km sec^{-1}, a time of 1.7×10^4 years is obtained for the evoluation from $R_0 = 0.06$ pc to $R = 0.4$ parsec. The entire track shown in Figure 2 is described in approximately 5×10^4 years. Tracks for two models, calculated by Hayashi and collaborators for Russell mixture stars evolving without nuclear processes, are also included. An initial contraction at constant luminosity is arrested by the effects of degeneracy and followed by cooling at constant radius. These models show some measure of agreement with the results for the central stars of planetaries, because the luminosity drop in the later stages of their evolution is a consequence of degeneracy, and because the final stage of their evolution points to the white dwarfs. The average mass of the central stars of planetaries, as well as of white dwarfs, is assumed to be $0.6\ M_{\odot}$. The average nebular mass is $0.6\ M_{\odot}$. These stars belong to the disk population (Tables 1 and 2).

There exists no satisfactory explanation of the processes leading to mass ejection and to the increase of the radii of the central stars during the initial stage of their evolution.

THE PRODUCTION OF X RAYS

The radiation emitted during encounters of electrons with ions in a fully ionized gas of low density is called "deceleration radiation." The term "free-free transitions" is customary for the same process in the astrophysical literature. The formulas for the deceleration radiation by a Maxwellian distribution of electrons will be given.

The initial and final energies of a free electron may be represented by the expressions:

$$E = \tfrac{1}{2} m v^2 \qquad \text{and} \qquad E' = E - h\nu \quad .$$

The free-free emitted power per unit volume of a gas is obtained by integrating a quantity proportional to g/v over a distribution function proportional to the expression:

$$e^{-\frac{E}{kT}} v^2 \, dv \quad .$$

It is then found that the weighted average of g for a Maxwellian distribution may be expressed in two ways:

$$\bar{g}(T, \nu) = e^{-\frac{h\nu}{kT}} \int_{h\nu}^{\infty} g(E, \nu) \, e^{-\frac{E}{kT}} \, d\frac{E}{kT}$$

and

$$g(T, \nu) = g(E' + h\nu, \nu) \, e^{-\frac{E'}{kT}} \, d\frac{E'}{kt} \quad .$$

The equation for the power density per sec^{-1} bandwidth per steradian emitted in free-free encounters is:

$$\frac{P_{ff}(T, \nu)}{4T_1} = \frac{8Z^2 e^6}{3(4\pi\epsilon_0)^3 m^2 c^3} \left(\frac{2\pi m}{3kT}\right)^{\frac{1}{2}} N_i N_e \bar{g}(T, \nu) \, e^{-\frac{h\nu}{kT}} \quad .$$

This formula yields the free-free emission. It contains the exponential factor, because free-free emission of frequency ν can occur only if the initial electron energy exceeds $h\nu$. However, deceleration radiation can be emitted by electrons of any initial velocity. In free-bound radiation, the electron is captured by the ion. The two emissions will be measured together.

The free-bound radiation differs from the free-free emission by the quantization of the final energy. The inclusion of the free-bound emission will cancel the exponential factor in the power density formula, approximately. The successive entry of more free-bound continua yields a saw-tooth function which remains constant on the average, so that the exponential drop of the free-free emission is nearly cancelled by the free-bound contributions. At low frequencies or at high temperatures the free-bound contribution becomes negligible. Applying the relevant Gaunt factor, the emission spectrum from a gas for a combined continuum is given by the expression:

$$\frac{P(T,\nu)}{4\pi} = N_i N_e C \bar{v} e^{-\frac{h\nu}{kT}} \left[\bar{g}(T,\nu) + f(T,\nu)\right] \frac{h}{kT}$$

$$C = \frac{4\pi Z^2 e^6}{3^{\frac{3}{2}} (4\pi \epsilon_0)^3 mhc^3}$$

$$v = \left(\frac{8kT}{\pi m}\right)^{\frac{1}{2}}$$

$$f(T,\nu) = 2\theta \sum_{\lambda = m(\nu)}^{\infty} n^{-3} e^{\frac{\theta}{n^2}} g_n(\nu)$$

$$\theta = \frac{h\nu_0 Z^2}{kT}$$

$$\nu_0 = \frac{\chi_0}{h}$$

$$\chi_0 = \frac{m e^4}{2(4\pi\epsilon_0)^2 \hbar^2}$$

This formula gives the intensity of the combined continuum for an ionized gas with a Maxwellian distribution of the electron velocities. The values of $(\bar{g} + f)$ are given in Table 3 for $\theta = 1$, which is equivalent to $T = 1.58 \times 10^{5}$ °K for hydrogen. The free-bound continuum becomes increasingly important from $\lambda = 1\mu$ down. $\bar{g}(T, \nu)$, alone, is to be computed only if:

$$h\nu << kT \quad .$$

$\bar{g}(T, \nu) + f(T, \nu)$ is the quantity which has to be compared with experimental emission data.

In the absence of computed values of $f(T, \nu)$, $\bar{g}(T, \nu)$ can be used if the exponential factor is dropped.

Evaluating the total power for 100 Å, which is the long-wave limit for X rays, the following values are substituted into the equation:

C	$\bar{v}$	T	N	$\bar{g} + f$
joule m^2	m sec^{-1}	°K	m^{-3}	
1.82948×10^{-45}	2.46936×10^{6}	1.58×10^{5}	4.169×10^{8}	6.58

The result obtained is:

$$\frac{P(T, \nu)}{4\pi} = 1.73 \times 10^{-40} \text{ watt } m^{-3} \text{ sterad}^{-1} (\text{sec}^{-1} \text{ bandwidth})^{-1} \quad .$$

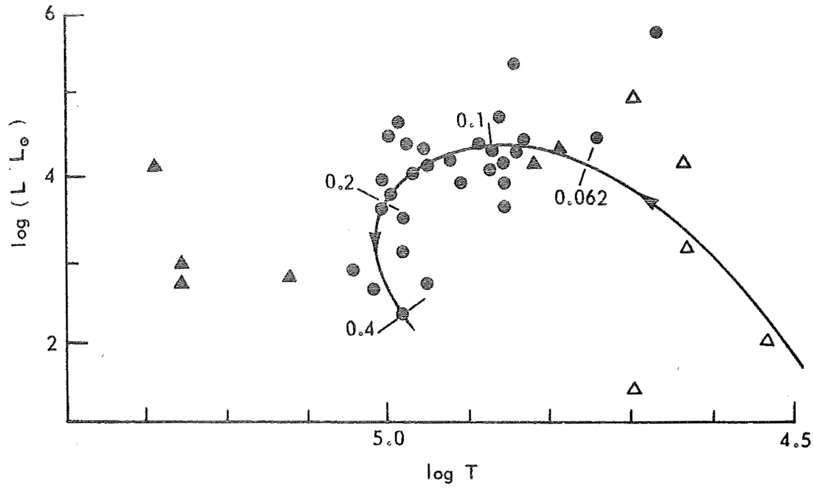

Figure 1. The Hertzsprung-Russell diagram for the central stars of planetaries, (values of nebular radii, in parsecs, are indicated) [1].

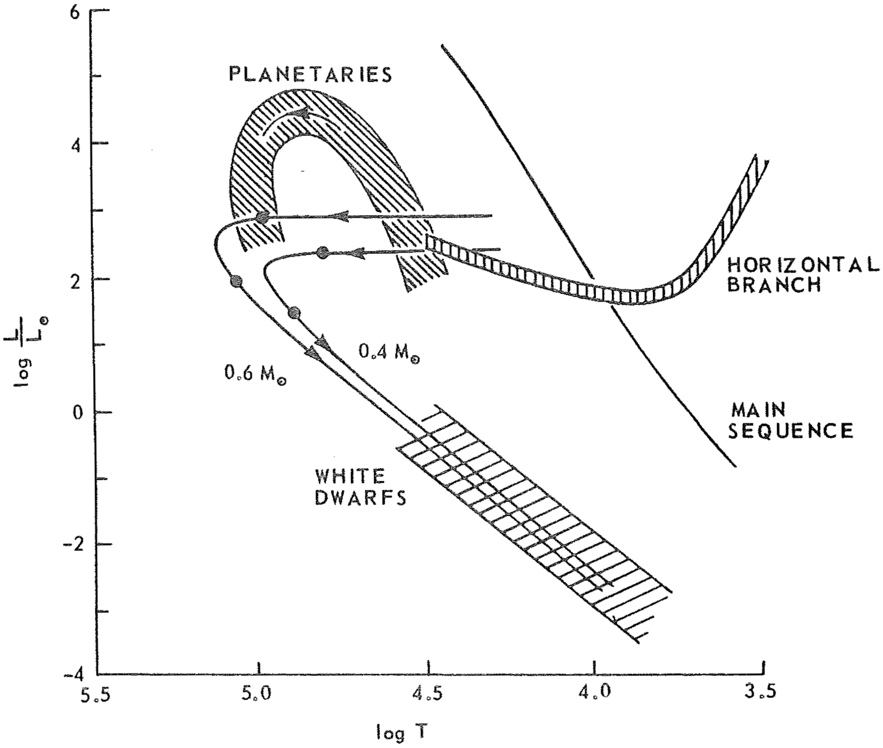

Figure 2. The Hertzsprung-Russell diagram showing main sequence, horizontal branch, planetary nebulae, and white dwarfs [1].

TABLE 1. OORT'S CLASSIFICATION OF STELLAR POPULATIONS [2].

Population	Halo Population II	Intermediate Population II	Disk Population		Intermediate Population I	Extreme Population I
Some Typical Members	Subdwarfs Glob. Clusters RR Lyr Var, With Per. $> 0.^{d}3$	High-Vel. F-M Long-Per. Var.	Planet. Neb Bright Red Giants Novae	Weak-Line Stars	Strong-Line Stars A Stars Me Dwarfs	Gas Supergiants T Tauri Stars
$\overline{\lvert z \rvert}$ (parsecs)	2000	700	450	300	160	120
$\overline{\lvert Z \rvert}$ (km/sec)	75	25	18	15	10	8
Axial Ratio	2	5	$\simeq 25$			100
Concentration Toward Center	Strong	Strong	Strong		Little	Little
Distribution	Smooth	Smooth	Smooth		Patchy, Spiral Arms	Extremely Patchy, Spiral Arms
$Z_{h.e.}$ (Schwarzschild)	0.003	0.01		0.02	0.03	0.04
Age (10^9 years)	6	6.0 to 5.0	5	1.5 to 5	0.1 to 1.5	< 0.1
Total Mass (10^9 ⊙)	16	47	47	47	5	2

TABLE 2. CLASSIFICATION OF STELLAR POPULATIONS [2].

Halo Population II	Intermediate Population II	Disk Population	Older Population I	Extreme Population I
Subdwarfs	High-velocity stars with z-velocities >30 km/sec	Stars of galactic nucleus	A-type stars	Gas, Young stars associated with the present spiral structure
Globular clusters of high z-motion RR Lyrae stars with periods longer than 0.4 days	Long-period variables with periods <250 days and spectral types earlier than M5e	Planetary nebulae Novae RR Lyrae stars with periods <0.4 days Weak-line stars	Strong-line stars	Supergiants Cepheids T Tauir stars Galactic clusters of Trumpler's class I

TABLE 3. AVERAGE GAUNT FACTORS FOR $\theta = 1$ [3].

$\log (\nu/\nu_0 Z^2)$	$\bar{g} + f$	$\log (\nu/\nu_0 Z^2)$	$\bar{g} + f$	$\log (\nu/\nu_0 Z^2)$	$\bar{g} + f$
-9	11.47	-1.500	2.21	+0	6.15
-8.5	10.83	-1.398	2.12	+0.05	6.30
-8	10.20	-1.398	2.13	+0.10	6.43
-7.5	9.56	-1.204	1.99	+0.15	6.55
-7	8.93	-1.204	2.02	+0.20	6.65
-6.5	8.29	-1.097	1.94	+0.25	6.75
-6	7.66	-0.954	1.83	+0.30	6.85
-5.5	7.02	-0.954	1.91	+0.45	7.02
-5	6.39	-0.900	1.88	+0.60	7.04
-4.5	5.75	-0.750	1.80	+0.81	6.87
-4	5.12	-0.602	1.72	+1.10	6.31
-3.5	4.48	-0.602	2.00	+1.50	5.14
-3	3.85	-0.500	1.97	+1.84	4.05
-2.5	3.23	-0.330	1.91		
-2	2.66	-0	1.81		

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APPROVAL

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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