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PHASE I INTERIM REPORT

GENERALIZED PROPULSION SYSTEM MODEL

FOR

NASA MANNED SPACECRAFT CENTER

HOUSTON, TEXAS

Contract No. NAS9-10319

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17 April 1970

NASA CR 108418

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FOREWORD

This report is the first Interim Report for the Generalized Propulsion System Model Contract NAS9-10319. This report covers the work accomplished in Phase I during the period 12 January to 20 March 1970.

SUMMARY

The initial effort in this program was spent evaluating analytical techniques which may be used to characterize the behavior of pressure-fed rocket engine system. An existing Rocketdyne model, which is used to predict start and cutoff transients in pressure-fed rocket engines, is used as a starting point in this program. In order to supply the additional capability required by NASA, a study was made to determine the methods to be used to simulate two phase flow, a simple pressurization system, 600 Hertz frequency response, simulation of injector and thrust chamber at cutoff and components such as valves, cavitating venturies, orifices, etc.

After selection of the analytical techniques to be used an outline of the new program was made. This outline identifies the subroutine and function subprograms needed and allowed an estimate of program size to be made. In order to use the program on the NASA Computer, its size was reduced from 72,000 words to approximately 42,800.

INTRODUCTION

The purpose of the generalized Propulsion System Model program is to provide a general purpose, modular, digital computer program for simulating transient operation in pressure-fed rocket engine systems. In order to accomplish this, an existing Rocketdyne digital computer program will be modified so that it meets the requirements of the contract. The effort in this program has been divided into three phases. In the first phase an evaluation of the analytical techniques used to characterize the dynamic behavior of pressure-fed rocket propulsion systems was made. In Phase II the techniques selected in Phase I will be programmed and the program instruction documents written. In Phase III the use of the program will be demonstrated by modling the Apollo Ascent and Descent engines.

The digital model resulting from completion of this program will be flexible, and have options for calculating the performance of a number of different components. It describes the propellant line dynamics using the wave equations, and calculates the pressure and flow conditions when a portion of a line is being filled. The model allows a system to be broken down into a number of segments and, by proper combination, many different plumbing configurations may be simulated.

In between segments, components used in rocket engine feed systems can be installed by use of the input data and since the performance of these components is calculated entirely in subroutines, it is an easy task to add new components or change the existing ones.

The existing Rocketdyne pressure-fed rocket engine model is used to calculate the transient performance of various rocket engine systems. It does this by breaking the propellant feed system into a number of segments and calculating the flow and pressure in them. The segments can be joined by branch elements so that more than one flow path is available. Subroutines are used in the program to define the flows that exist at the end of a string of segments. As presently used the program is modified to represent a specific engine system, and thus it is not as flexible as desired.

In order to make this program meet the requirements of the contract, modifications will be made to it. The most commonly used features will be built into the program so that they may be called out by input data. In addition several new features will be added as follows:

1. A simplified pressurization system which includes gas flow, pressure, propellant-gas heat exchange, and tank inlet gas temperature.

2. Changes in feedline representation to include effects of two-phase flow.

3. Improvement of thrust chamber and injector response, including the addition of injection stream ballistic dynamics and injector orifice inertance.

4. Injector-thrust chamber blowdown after propellant valve closure.

5. Built-in components-valves, cavitating venturies, capped lines, accumulators, etc.

The use of subroutines to calculate the performance of different types of components will be continued, and improved by grouping of calculations, maintainance and modification of function will be simplified. The mathematical basis of the various program functions and a description of the digital computer model are given in the following sections. A list of symbols used is presented in Table I.

FEED SYSTEM DYNAMICS

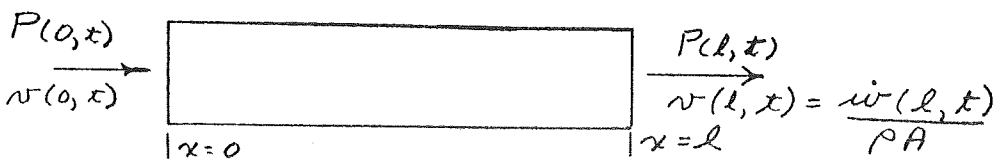
Wave Equation

In order to simulate the dynamics of a pressure fed rocket engine system, it is necessary to consider the dynamics of all of its components. The rocket engine feedline dynamics are involved in the start and cutoff transients and in feed system coupled instabilities, therefore, their correct representation in a dynamic model is both necessary and important. In general, two different methods of simulating feedlines are used, (1) lumped parameters, and (2) distributed parameters. In the first method, it is assumed that the liquid in a line moves as a solid lump; the fact that the fluid is compressible is handled by a storage term in the analytical expression. The distributed parameter representation assumes a continuous media and shows that disturbances are propagated by waves at the speed of sound in the fluid. Both methods can be used to give an adequate dynamic representation of the feedlines, however, it is felt that the wave equations (distributed parameter) are more practical for a general purpose model. The wave equations have theoretically unlimited frequency response, whereas the response of a lumped parameter system will depend on the number of elements into which it is broken. The wave equations are utilized in this model because they yield accurate frequency response independent of the line length assigned to the segment.

The equations used are the normal water hammer equations.

$$\frac{\partial P}{\partial x} = -K_1 \frac{\partial v}{\partial t} \quad \text{and} \quad \frac{\partial v}{\partial x} = -K_2 \frac{\partial P}{\partial t}$$

The solution of these equations involves terms which include past, present, and future values of velocity and pressure. By using Laplace transformation techniques and the equation of continuity, the above expressions may be solved for the pipe segment shown below



The resultant equations are:

$$\dot{w}(\ell, t) = \rho A v(t) = \dot{w}(t) \tag{1}$$

$$P(\ell, t) = -\frac{a}{Ag}\dot{w}(\ell, t) + 2P(0, t-\tau) + \frac{a}{Ag}\dot{w}(\ell, t-2\tau) - P(\ell, t-2\tau) \tag{2}$$

$$P(0, t) = P(t) \tag{3}$$

$$\dot{w}(0, t) = \frac{Ag}{a}P(0, t) + 2\dot{w}(\ell, t-\tau) - \frac{Ag}{a}P(0, t-2\tau) + \dot{w}(0, t-2\tau) \tag{4}$$

and since this solution involves only past history, we may solve for the flow and pressure at each end of a pipe segment directly.

To include the effect of fluid resistance, the segment pressure drop is lumped at the downstream end of the pipe. Thus:

$$P_R(\ell, t) = P(\ell, t) + R\,\dot{w}(\ell, t)\cdot|\dot{w}(\ell, t)| \tag{5}$$

Acoustic Velocity

The acoustic velocity of the fluid enters into the above calculation in two places; (1) directly in the constant relating flow to pressure, and (2) indirectly in the time delay value τ which equals $\frac{\ell}{a}$ seconds. The acoustic velocity of a fluid is a property of that fluid, however, its effective value is reduced by the elastic walls of a pipe or entrainment of gas and vapor in the liquid-two phase flow. Vapor in the liquid can come from two places. One is direct entrainment from mixing of gas and liquid in the propellant tank, and the second results from evolution of dissolved gas if the liquid is supersaturated.

Dissolved pressurant gas in liquids obeys Henry's law which says that the amount of dissolved gas at equilibrium is a linear function of pressure. Thus the equilibrium value of dissolved gas will decrease as pressure is reduced.

Because of the pressure drop in the feedlines, the pressure will decrease as the propellant flows toward the engine, and there is a tendency for gas to be evolved. The rate at which the gas is evolved appears to be at a rate proportional to its excess. Thus

$$\frac{d}{dt} G(x) = -\gamma \left(G(x) - G_s \right) \tag{6}$$

Knowing the amount of gas in the liquid, the effective acoustic velocity of the mixture may be calculated. A number of different approaches exist for this calculation, as presented in ref. (a). The one used in this model assumes isentropic compression of the gas. The change in volume of the liquid can be written:

$$d V_l = -\frac{V_l}{\beta} dP \tag{7}$$

and for the gas

$$d V_g = -\frac{V_g}{k P} dP \tag{8}$$

Defining a constant α as

$$\alpha = \frac{V_g}{V_l} \tag{9}$$

the following relation is obtained

$$\frac{dV_t}{V_t} = \frac{-dP}{\left[\frac{1+\alpha}{\beta + kP}\right]} \tag{10}$$

The bracketed term is the compressibility of the mixture. The density of the mixture can be shown to be:

$$\rho_m = \frac{\alpha \rho_g + \rho_l}{(1+\alpha)} \tag{11}$$

The acoustic velocity of a liquid in an elastic pipe is

$$a = \sqrt{\frac{1}{\frac{\rho}{g}\left(\frac{1}{\beta} + \frac{Dc}{eE}\right)}} \tag{12}$$

Using the above expressions for density, compressibility, and acoustic velocity, we obtain:

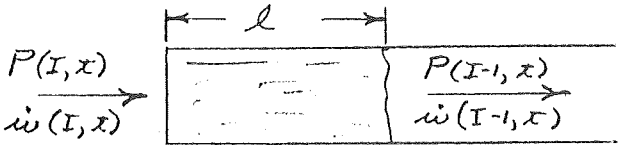
$$a = \left[\frac{1}{\frac{\rho_m}{1+\alpha}\left(\frac{1}{\rho_\ell a_\ell^2} + \frac{\alpha}{\rho_g a_g^2} + \frac{1+\alpha}{g} \cdot \frac{Dc}{Ee} \right)} \right]^{\frac{1}{2}} \qquad (13)$$

This expression will be used along with the wave equations to define the characteristics of a feed system with two phase flow. For an all liquid system $\alpha = 0$ and the same equation can be used.

Priming

The priming transient is that one during which the empty feed system lines are filled with propellant. When a propellant valve is opened or a burst diaphragm ruptures, the liquid is forced into the empty line segments. The model assumes that no flow leaves a segment until it is filled. This is not completely true in reality, but the assumption greatly simplifies the modeling and correlates well with test data. In a vacuum start, the propellant will vaporize in the unfilled portions of the manifold, and a certain amount of this vapor will flow out of the engine. Again this effect is neglected in the present model as the gas outflow is only a small percentage of the liquid inflow.

The priming equations consider the forces acting on a compressible "slug" of liquid. The slug has the same cross sectional area as the pipe segment being filled and the length of the slug is determined from the amount of fluid in the partially filled element.



Writing a force balance on the fluid slug as,

$$P(I-1,x) - P = \frac{\ell}{A_g} \frac{d}{dt} \dot{w} \qquad (14)$$

where P is a dummy pressure which represents a net value acting on the slug. Storage of fluid in the slug due to compressibility is then represented as

$$\dot{w}(I,t) - \dot{w}(I-1,t) = \frac{\rho A \ell}{\beta} \frac{d}{dt} P \tag{15}$$

The normal lumped resistance will be distributed so that

$$P(I-1,t) = P - R\left[\dot{w}(I-1,t)\right]^2 \tag{16}$$

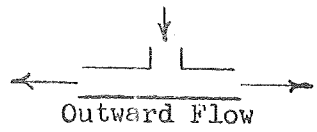
and using the water hammer equation at the junction of the two segments gives;

$$P(I,t) = \frac{a}{Ag}\left[\dot{w}(I,t-2\tau) - \dot{w}(I,t)\right] + 2P(I+1,t-\tau) - R\left[\dot{w}(I,t-2\tau)^2 + \dot{w}(I,t)^2\right] \tag{17}$$

The downstream pressure is assumed to be given and thus the above equations can be solved for the flows and pressures in the segment being filled. The flow into the segment is integrated to determine if the segment is full, and flags the start of flow into the next downstream segment when filling has been completed.

Branch Lines

The equations derived thus far consider only the flow from one pipe into another and has not covered the problem of branched lines. Branched lines can be classified in three catagories, (1) outward flow, (2) inward flow, and (3) a combination of both. To handle branched lines it is assumed that the branch has zero internal volume and that its flow is incompressible. Thus the pressures of all segments which meet at a branch are set equal, and the continuity of flow is used to provide the necessary equations to allow the calculations of flow and pressure in each branch. For the outward flow branch, priming of the system will work as the single feed line will fill normally and when it is full, the downstream branches can be filled as long as some initial split of the flow is provided. The inward flow branch will have a problem in priming if one of the feedlines primes before the other. The model is set up to provide a normal direction


Outward Flow


Inward Flow

of flow in a segment and this information is used in the priming of lines. To handle the inward flow branch, the program will prevent flow out of the branch until enough propellant has been accumulated to fill all segments which normally feed the branch. Thus all segments upstream of the branch will be primed before fluid flows out of the branch.

PRESSURIZATION SYSTEM

In a pressure fed rocket system the propellant tank pressurization system has a direct effect on the performance of the system. For this reason, two different pressurization schemes are provided with this model. In one, the tank pressure may be programmed as a function of time; in the second a simplified pressurization system is simulated. The first system needs no further explanation. In the second, the basic elements of the system consist of the storage tank, pressure regulator, heat exchangers, propellant tank ullage volume, and general items such as valves, fittings, and a filter. Equations used for the various components are derived below:

Tank Equations

Figure 1 shows a typical tank with gas flow in and out, liquid outflow, and heat addition. The scheme of subscripts shown there is generally applicable to any series of tanks or other significant volumes in which compressibility is important, although the final model will have only one propellant tank. Application of mass conservation, energy conservation, and the equation of state relationships will yield the desired thermodynamic conditions within such a tank at any instant.

By conservation of mass for the gas system,

$$\frac{d}{dt} W_i = \dot{w}_i - \dot{w}_{i+1} \tag{18}$$

where $\dot{W}_i$ is the stored weight, and $\dot{w}_i$ and $\dot{w}_{i+1}$ are gas inflow and outflow respectively. Integration of this equation yields the instantaneous stored weight of gas in the i^{th} tank or compressible volume.

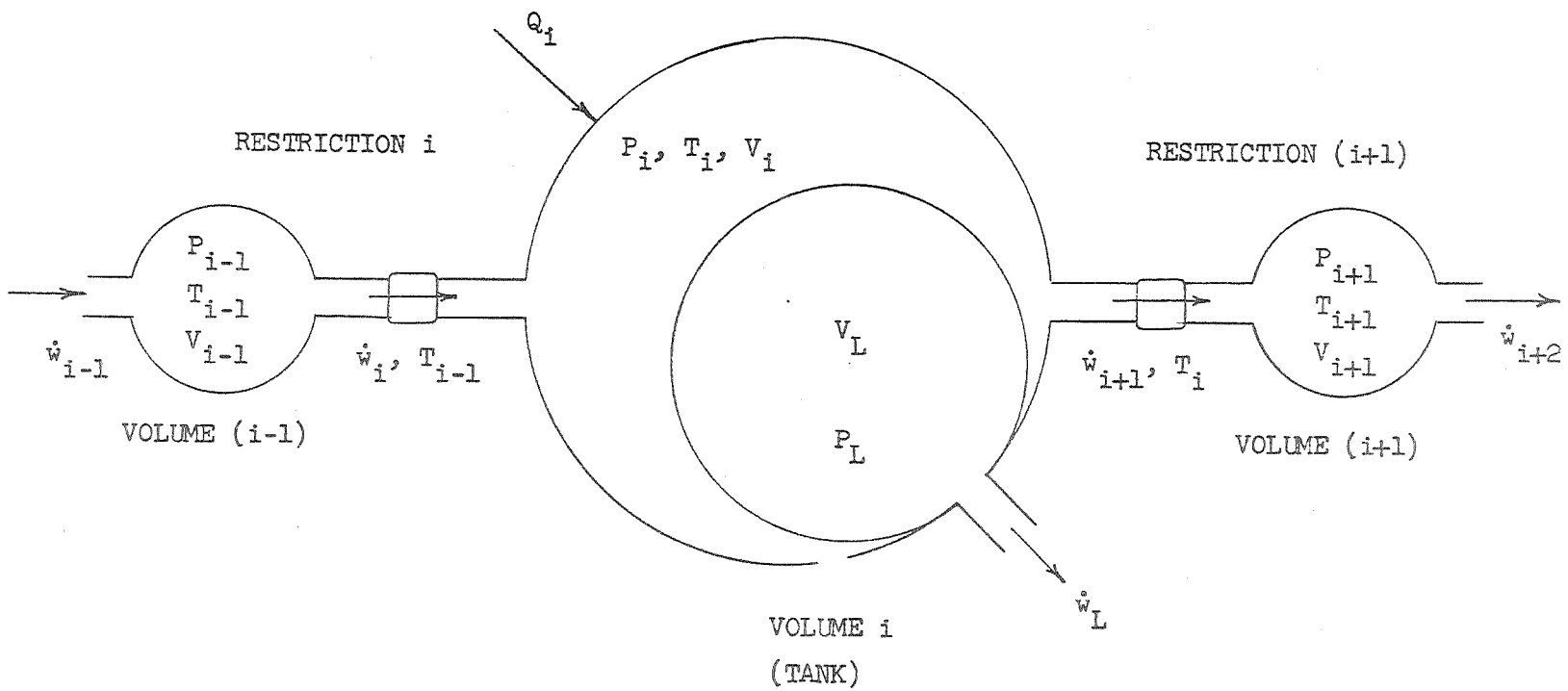


FIGURE 1 - SCHEMATIC SHOWING GENERALIZED FEEDSYSTEM TANK (COMPRESSIBLE VOLUME) AND IDENTIFICATION

By conservation of energy with respect to the gas system in this tank, (ref. b), neglecting potential and kinetic energy,

$$\frac{d}{dt} U_{TOT} = -\Delta H\dot{w} + Q_i - W_i \tag{19}$$

where U_{TOT} is the total internal energy of the gas mass. H represents the enthalpy per unit weight, and the Δ represents the change from inlet to outlet. Q_i is heat added, including convection from the walls or liquid interface, mass transfer from the liquid interface, or from an immersed heat exchanger. W_i is the external work, i.e. the work done by the gas on the liquid surface.

The external work is

$$W_i = P_i \frac{d}{dt} V_i = P_i \frac{\dot{w}_{\ell i}}{\rho_\ell} \tag{20}$$

where it is noted that the rate of change of gas volume, V_i, is simply the negative of the rate of change of liquid volume.

Substituting equation (20) into (19) and applying the relationships

$$U_{TOT} = C_V W T$$
$$H = C_P T$$
$$k = C_P / C_V$$

it is found that,

$$\frac{d}{dt} T_i = \frac{1}{W_i}\left[\dot{w}_i (kT_{i-1} - T_i) - (k-1)\dot{w}_{i+1} T_i - \frac{P_i \dot{w}_\ell}{\rho_\ell C_V} + \frac{Q_i}{C_V}\right] \tag{21}$$

This equation can be integrated to yield the instantaneous temperature of the gas within the i^{th} tank. Note that the absence of liquid outflow or inflow, or heat addition can be represented simply by striking out the associated term in equation (21).

The instantaneous volume occupied by the gas is found from

$$\frac{d}{dt} V_i = \frac{\dot{w}_\ell}{\rho_\ell} \tag{22}$$

Equations (18) and (22) are sufficient to determine the density of the gas in the tank, which is a thermodynamic property. A second thermodynamic

property, temperature, is obtained from equation (21). Assuming a single phase, these two properties are sufficient to determine all other properties, specifically pressure. That is

$$P_i = P_i\left(\frac{W_i}{V_i}, T_i\right) \tag{23}$$

which is an equation of state.

If it is assumed at this point that the gas is at a temperature sufficiently above its critical temperature, the equation of state can be approximated by an ideal gas form. That is

$$P_i = Z \frac{W_i}{V_i} \frac{R_o}{M} T_i \tag{24}$$

where Z is a compressibility factor. The compressibility factor can be taken as constant for a sufficiently small change in temperature and pressure, such as would be expected during an engine start period. The validity of this approximation must be examined in each case, making reference to reduced properties charts, or other representations of actual gas behavior. The alternative would be to include an equation of state in tabular form. The remainder of this development will employ equation (24).

Flow Device Equations

Many flow devices can be represented by the compressible flow orifice equation. A convenient form of this equation is, for the flowrate through the i^{th} device

$$w_i = \frac{C_i P_{i-1} A_i}{\sqrt{T_{i-1}}} \Phi\left(\frac{P_i}{P_{i-1}}\right) \tag{25}$$

where Φ is the compressible flow function. Letting the pressure ratio be α, Φ can be expressed as,

$$\Phi = \left[\frac{2kg}{R(k-1)}\left(\alpha^{\frac{2}{k}} - \alpha^{\frac{k+1}{k}}\right)\right]^{\frac{1}{2}}, \quad \alpha \geq \alpha_c \tag{26}$$

$$\Phi = \Phi_M = \Phi(\alpha_c), \quad \alpha < \alpha_c$$

$$\alpha_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \tag{27}$$

In this equation k and R are the specific heat ratio and gas constant for the gas being used in this system.

Filters in the gas flow system are generally laminar flow devices, and therefore obey a linear flow-pressure loss relationship. This can be expressed as

$$\dot{w}_i = K \rho \Delta P_i$$

Assuming an ideal gas density

$$\rho = \frac{P}{RT}$$

results in

$$\dot{w}_i = \frac{K_i P_i}{R T_{i-1}} (P_{i-1} - P_i) \tag{28}$$

The proportionality constant K_i can be evaluated if the flow temperature and pressure conditions are known at any nominal point, i.e.

$$K_i = \left(\frac{\dot{w} R T}{P \Delta P} \right)_m \tag{29}$$

Pressure loss through a heat exchanger core is generally expressed in a square-law relationship (Ref. 2), i.e.

$$\dot{w}_i = \left(\dot{w} \sqrt{\frac{T}{P \Delta P}} \right)_m \left(\frac{P_{i-1} (P_{i-1} - P_i)}{T_{i-1}} \right)^{\frac{1}{2}} \tag{30}$$

Again, the proportionality constant has been evaluated by reference to some nominal point at which the pressure, temperature, and flowrate are all known.

A pressure regulator in the system can be simulated with equation (25), if it is noted that the flow area is a variable dependent on diaphragm or reference pressure. This relationship can be developed by equating diaphragm force, F_d, to the force exerted by the reference spring, F_r. If it is assumed that the spring has a force deflection curve which is linear over the range of poppet travel, the reference spring force is

$$F_R = F_{RO} + K_1 x \quad , \quad 0 \leq x \leq x_{MAX} \tag{31}$$

where F_{ro} is the spring force when the regulator is closed, and X is the poppet position, measured from the closed position. The local spring slope is K_1, and diaphragm force is represented as;

$$F_d = (P_R - P_a) A_e \tag{32}$$

where P_r and P_a are the reference and ambient pressures respectively, and A_e is the effective diaphragm area. For small poppet travel the seat area is a linear function of X, i.e.

$$A_s = K_2 x \quad , \qquad 0 \leq x \leq x_{MAX} \tag{33}$$

By equating diaphragm and spring forces, and substituting equation (33), there results,

$$A_s = \left(P_R - P_a - \frac{F_{Ro}}{A_e} \right) \frac{K_2 A_e}{K_1} \quad , \quad 0 \leq A_s \leq A_{MAX} \tag{34}$$

In a simple system, the reference pressure is the pressure immediately downstream of the seat, but it could as well be supplied from some other downstream point by means of a sensing line.

Heat Exchangers

Heat exchangers in the pressurization system can be simulated most easily by the effectiveness vs net-transfer-unit method described in Ref. (c). Figure (2) shows a typical heat exchanger. In a pressurization system, the gas is the colder fluid and is being heated by a secondary flow. Therefore the gas inlet and outlet temperatures are referred to as $T_{c\ in}$ and $T_{c\ out}$, and the cooling fluid inlet and outlet temperatures are $T_{h\ in}$ and $T_{h\ out}$, following the terminology of Ref. (2). The secondary or heating fluid flowrate is designated $\dot{w}_h$, whereas the gas flowrate is designated $\dot{w}_g$. The method is independent of the type of the two flows. That is, the heating flow can be either liquid or gaseous.

Capacity rates are defined for both flows as

$$C_h = \dot{w}_h C_{Ph} \qquad \text{(hot fluid)} \tag{35}$$

$$C_c = \dot{w}_G C_{PG} \qquad \text{(cold fluid)}$$

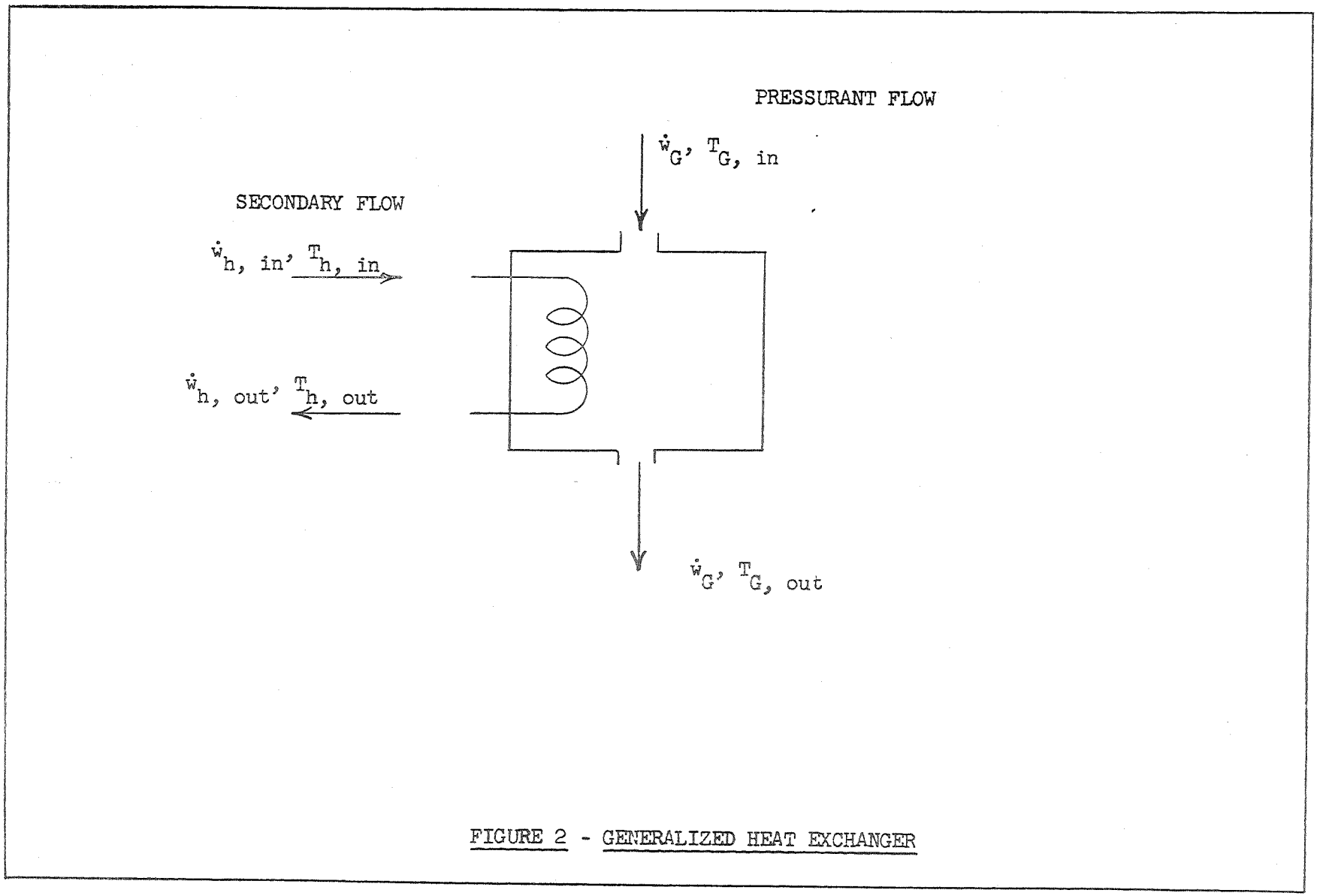


FIGURE 2 - GENERALIZED HEAT EXCHANGER

It is necessary to distinguish between the C which is maximum and that which is minimum in assessing the heat exchanger performance. Therefore, let,

$$C_m = MIN\,(C_h, C_c) \qquad C_M = MAX\,(C_h, C_c) \tag{36}$$

where the functions MIN(a,b) and MAX(a,b) are defined as the minimum and maximum of the arguments (a,b) respectively. These functions are standard library functions for most digital programming languages.

The number of transfer units, or NTU, is defined as

$$NTU = \frac{A_h U_h}{C_m} = \frac{A_c U_c}{C_m} \tag{37}$$

where AU is the transfer area times the average over-all conductance between the two flows. It can be evaluated either for the hot or the cold fluid side, since conservation of energy requires them to be equal.

Finally, the heat exchanger effectiveness, ε, can be determined from charts (Ref. c), or can be approximated by closed-form expressions. In general,

$$\varepsilon = \varepsilon(\text{NTU},\ C_m/C_M,\ \text{Flow arrangement}) \tag{38}$$

For many systems, the hot fluid will be liquid and the cold gas, in which case, due to the low mass flowrate and C_p of the gas relative to the liquid,

$$\frac{C_m}{C_M} << 1$$

and ε becomes

$$\varepsilon = 1 - e^{-NTU} \tag{39}$$

for all flow arrangements. This is also the case for a heat exchanger submerged in a constant temperature fluid, since C_M is then essentially infinite.

Once the effectiveness of the heat exchanger is known, the heat transferred is found from

$$Q = C_m (T_{h\,IN} - T_{c\,IN})\,\varepsilon \tag{40}$$

The outlet temperature of the two flows is then

$$T_{c\,out} = T_{c\,in} + \frac{Q}{C_c} \tag{41}$$

$$T_{h\,out} = T_{h\,in} + \frac{Q}{C_h} \tag{42}$$

If the heat exchanger is submerged in a propellant or pressurant tank, the Q determined from equation (40) is employed in the tank temperature equation, rather than in equation (42).

From the above it is apparent that the heat supplied from a particular heat exchanger to the pressurant flow is easily determined from upstream temperatures, fluid flowrates, and fluid properties. That is, functionally,

$$Q = f(\dot{w}_h, C_{Ph}, \dot{w}_c, C_{Pc}, AU, T_{h\,in}, T_{c\,in}) \tag{43}$$

which can most efficiently be developed as a digital subroutine program.

Solution for Particular System

The equations developed above can be applied to any combination of elements, thereby describing a large number of different pressurization systems. A typical system is shown in Fig. (3). This system has three heat exchangers, the second of which is submerged in the storage tank. The propellant tank has a liquid outflow, $\dot{w}_L$, which is assumed to be given. Other known data include the initial conditions in the two tanks, the flowrate for heat exchangers, system geometry and pressurant properties.

Since gas storage volumes in the flow system are small relative to the two tanks, storage will be neglected in the flow system. This implies that all flow devices have the same instantaneous flowrate, $\dot{w}_G$. Thus equation (25) is applied at the start valve, regulator, and fittings, equation (28) is applied at the filter, and equation (21) at the heat exchangers, all with the same flowrate. The occurrence of this single variable in all of these equations makes it necessary to solve the equations iteratively.

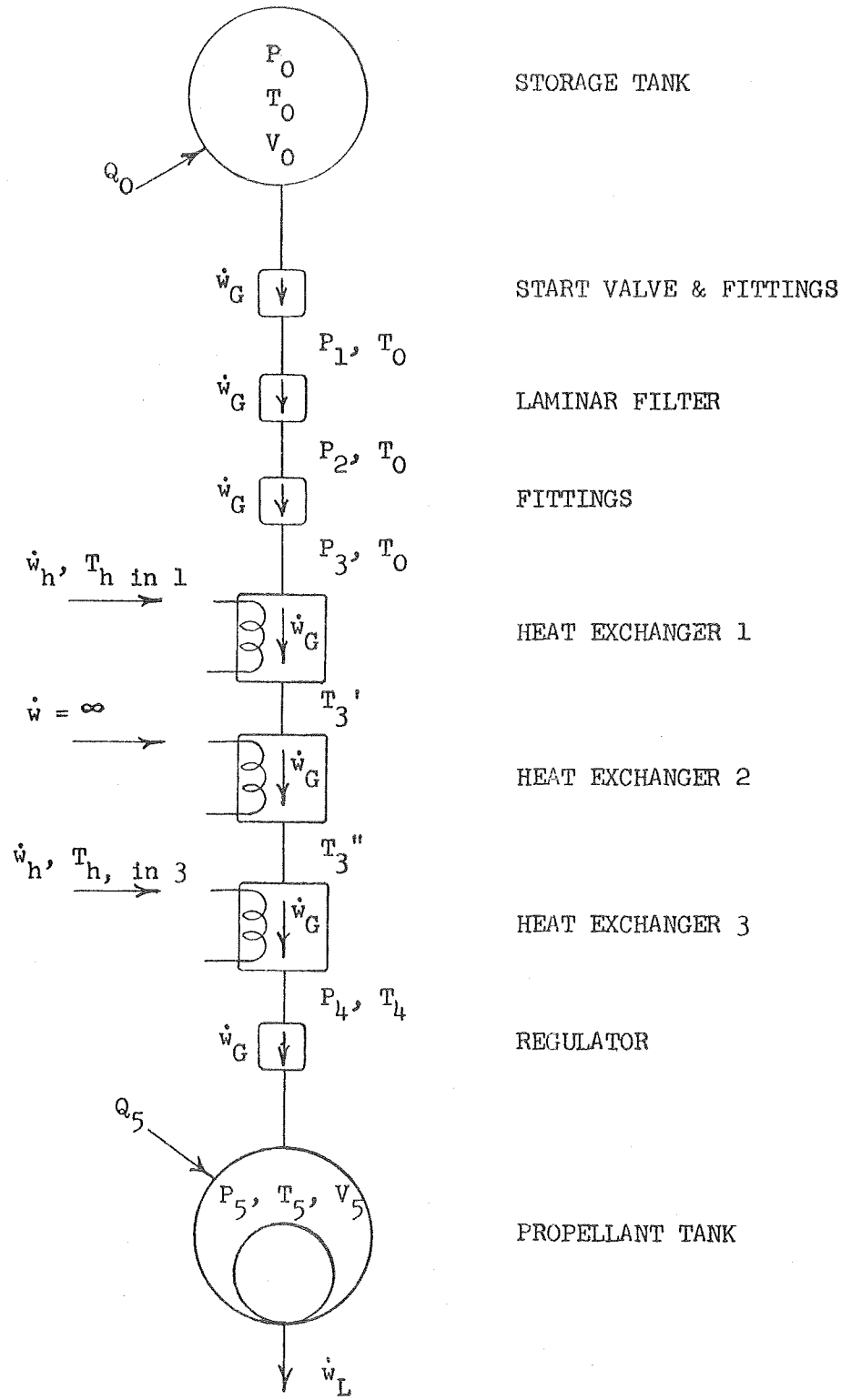


FIGURE 3 - TYPICAL PRESSURIZATION SYSTEM

COMBUSTION CHAMBER

The combustion chamber model describes the reaction of the propellant, their storage in the combustion chamber, and the continuity between flow in and out. This can be expressed mathematically as

$$M = \int (\dot{w}_{IN} - \dot{w}_{OUT})\, dt \tag{44}$$

The mass M, of reacted propellant is related to the chamber pressure by the perfect gas law

$$P_c V_c = M R_c T_c \tag{45}$$

The flowrate out of the chamber is also related to the chamber pressure and can be written

$$\dot{w}_{OUT} = \frac{P_c A_t}{\eta_{C^*} \frac{C^*}{g}} \tag{46}$$

If equations (44) and (45) are differentiated they can be combined with equation (46) to give

$$\frac{d}{dt} P_c = \frac{R_c T_c}{V_c} \dot{w}_{IN} - \frac{R_c T_c}{V_c} \cdot \frac{A_t}{\eta_{C^*} \frac{C^*}{g}} \tag{47}$$

which is the desired form.

The constants in equation (47) vary with mixture ratio and chamber pressure. This variation is supplied from a table which contains RcTc, C^*/g, and η_{C^*} as functions of oxidizer fraction $MR/(1+MR)$ and a correction factor $C_f = (P_c / P_{BASE})^{\epsilon}$. These constants are determined from thermochemical calculations for the propellant combination being used. The value of $\dot{w}_{in}$ is the sum of the fuel and oxidizer flowrate which is presently being converted into combustion products, and the quotient of these values determines the mixture ratio used to find chamber properties.

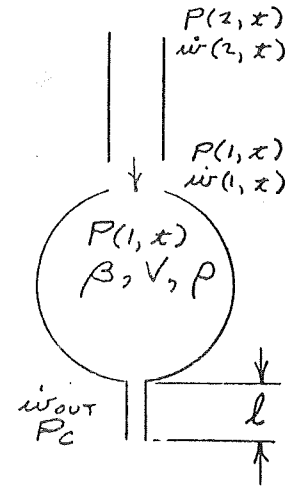
The propellant presently burning is not the propellant leaving the injection orifices. Instead, it is propellant that has finally reached the combustion zone after traveling some finite distance from the injector. The time required for the propellant to travel to the combustion is known as flight time or transport time, and is often assumed to be a constant. Reference (d) discusses

the transport process and shows that the flight time should be a variable dependent on the injection velocity. If it is assumed that the combustion zone is stationary the time delay can be computed from the velocity which is a function of the injector differential pressure. Thus

$$\tau_D = \frac{\ell}{v} = \frac{\ell}{\sqrt{2g\,\Delta P/\rho}} \tag{48}$$

INJECTOR

The injector dynamics are included by treating the injector as a lumped compressible volume as shown in the figure at the right. The flow and pressure at the bottom of the pipe segment are related by the equation:



$$P(1,t) = \frac{a}{Ag}\left(\dot{w}(1,t-2\tau) - \dot{w}(1,t)\right) + 2P(2,t-\tau) - P(1,t-2\tau) - R\left(\dot{w}(1,t)^2 + \dot{w}(1,t-2\tau)^2\right) \tag{49}$$

The flow out is controlled by the differential pressure across the injector and the resistance and inertia of the injector orifices; thus

$$P(1,t) - P_C = R\,\dot{w}_{OUT}^2 + \frac{\ell}{Ag}\frac{d}{dt}P(1,t) \tag{50}$$

Storage of propellant in the injector manifold can be represented by the following equation.

$$\dot{w}_{OUT} = \dot{w}_{IN} - \frac{V\rho}{\beta}\frac{d}{dt}P(1,t) \tag{51}$$

Since Pc is determined by flow into the combustion chamber some time prior to the present (transport time delay), it can be calculated directly. Then the above equations, which involve only 3 unknowns, may be calculated directly.

CUTOFF

Rocket engine cutoff is initiated by closing one or both of the main propellant valves. As the valves start to close the resistance of the valve segment will increase and flow will start to decrease. When the valve effective area reaches zero, the resistance will be set to a value of 10^{31} to act as a flag in the program to signify valve closure. While the valve or valves are closing, the model will operate normally. When the injector manifold is finally separated from the propellant tank, the state of the propellants will be determined by the energy in the system downstream of the closed propellant valves. At valve closure, the quantities of propellant liquid, vapor, and pressurant gas may be determined from known properties of the propellant.

Flow out of the injector is represented by equation (50), and the equations of state for the gas and liquid phase can be represented as follows:

$$M_\ell = \frac{V_\ell \rho_\ell}{\beta} P_M \tag{52}$$

and

$$M_v = M_g + M_{\ell v} = \frac{P_g V_g}{R_o T} + \frac{P_{\ell v} V_g}{R_o T} \tag{53}$$

The liquid vapor pressure is a function of temperature and can be approximated by

$$\mathcal{L}og_{10} P_{\ell v} = K_1 T + K_2 \tag{54}$$

The total pressure in the injector is the sum of the partial gas pressures, thus:

$$P_M = P_g + P_{\ell v} = P_g + \mathcal{L}og_{10}^{-1}(K_1 T + K_2) \tag{55}$$

After engine cutoff, heat soakback will raise the temperature of the injector and there will be heat transfer to the propellant. However, as the pressure in the injector falls more of the liquid will vaporize which will tend to

lower the propellant temperature in the manifold. The propellant temperature is therefore found from the following heat balance:

$$T(x)\left(C_p M_l + C_{Pg} M_g + C_{Pev} M_{ev}\right)_{x=x} = T(x-1)\left(C_p M_l + C_{Pg} M_g + C_{Pev} M_{ev}\right)_{x=x-1} + Q\left(T_{INJ} - T_{(x-1)}\right)\Delta x - \left(M_{ev(x)} - M_{ev(x-1)}\right) H_v \tag{56}$$

The concentration of dissolved pressurant gas in the propellant is determined using the same equation as used for the derivation of effective acoustic velocity

$$\frac{d}{dt} G = -\gamma \left(G - G_s\right) \tag{57}$$

To solve these equations it is assumed that the propellant mixture is homogeneous and therefore if $\dot{w}$ is known, the mass of propellant left in the injector can be calculated. Thus, the iteration technique available in the existing program can be used which is of the predictor-corrector variety. First Pm is estimated, and if Pc is known, $\dot{w}$ can be calculated followed by the state of the propellants in the injector. Using these data, the pressure estimate can be checked using equation (55).

At the start of the cutoff transient, the combustion chamber pressure is calculated in a normal fashion as previously described. As the propellant valves close combustion will continue down to some limiting value, which will have to be established. After combustion ceases, the thrust chamber will blow down to a pressure level which will be maintained by the vaporization rate of the propellants. Flow out of the chamber during this period will be assumed to be propellant vapor plus any pressurization gas which is present. In terms of mass storage,

$$\dot{w}_{IN} - \dot{w}_{OUT} = \frac{V_c}{R_c T_c} \frac{d}{dt} P_c \tag{58}$$

The flow in is that calculated in equation (50) above, and flow out will be determined by the compressible flow equation. This will allow calculation of the cutoff transient when unchoking of the exit nozzle occurs due to

existance of an ambient pressure. The values of Rc and k needed in this calculation will be only approximate values; Tc will be the same as the injector temperature. The pressure calculated during this phase of the cutoff is generally so low that the above approximations will not cause much difference in the calculation of thrust or impulse.

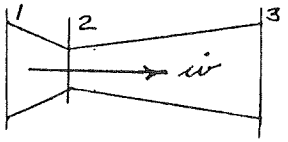
MISCELLANEOUS COMPONENTS

Cavitating Venturi

These components are used in rocket engine systems to control flowrate and their operation is based on Bernoulli's Equation given below

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2g} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2g} \tag{59}$$

Using the equation of continuity equation (59) becomes,



$$\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2} = \frac{\dot{w}_2^2}{2\rho_2^2 A_2^2 g} - \frac{\dot{w}_1^2}{2\rho_1^2 A_1^2 g} \tag{60}$$

For an incompressible liquid, the above equation can be simplified to

$$P_1 - P_2 = \frac{\dot{w}^2}{2g\rho}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) \tag{61}$$

From this equation it may be seen that for constant geometry and inlet pressure P_1, as the flowrate increases the pressure P_2 decreases until it equals the vapor pressure of the fluid. At this point, cavitation occurs in the throat of the Venturi and further increases in flow are not possible without an increase in inlet pressure P_1. The pressure and flow on either side of the Venturi may be determined by the equations (2) and (21) developed previously. Thus:

$$P_1 = f(\dot{w}_1) \tag{62}$$

The pressure on the downstream side of the Venturi for no cavitation will be

$$P_3 = f(\dot{w}_1) \tag{63}$$

This latter function may be either the priming equation or the wave equation depending on whether the segment is filled. P_1 and P_3 are related to the pressure recovery of the Venturi

$$P_3 = K P_1 \tag{64}$$

Using the above equations, the flow through the Venturi and the inlet and outlet pressures may be calculated assuming no cavitation. By solving equation (61) for P_2 it can be determined if cavitation is occurring. If the calculated value of P_2 is less than the vapor pressure, cavitation is occurring and the flow and pressure must be calculated using equations (61) and (62). Using this flow, P_3 can be calculated from equation (63).

If there is gas entrained with the liquid, the gas will expand and the density can no longer be assumed to be constant. Thus, equation (60) must be used and the values of P_1 and P_2 determined as functions of pressure. If a perfect gas is assumed,

$$\rho_g = \frac{P}{RT} = \frac{W_g}{V_g} \tag{65}$$

and since the ratio of the masses of gas and liquid are known,

$$B = \frac{W_g}{W_\ell} \quad \text{and} \quad \rho_\ell = \frac{W_\ell}{V_\ell} \tag{66}$$

The equivalent density of the mixture will be

$$\rho_m = \frac{W_\ell + W_g}{V_\ell + V_g} = \frac{(1 + B)}{\frac{1}{\rho_\ell} + \frac{BRT}{P}} \tag{67}$$

It is assumed that two phase flow in the pipe segments is isothermal but sudden transients are adiabatic. Thus,

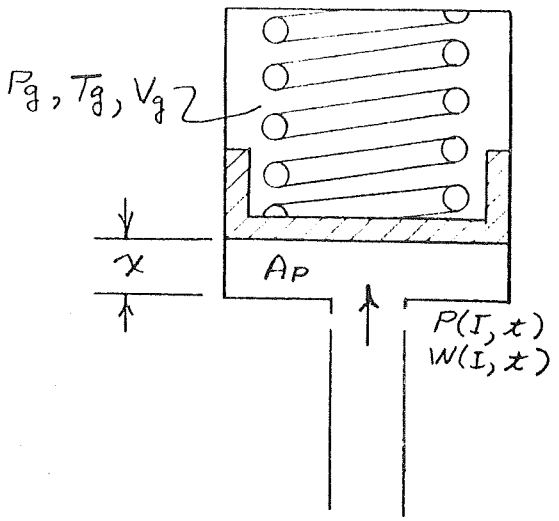
$$\rho_1 = \frac{1 + B}{\frac{1}{\rho_\ell} + \frac{BRT_1}{P_1}} \quad \text{and} \quad \rho_2 = \frac{1 + B}{\frac{1}{\rho_\ell} + \frac{BRT_1}{P_1}\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}} \tag{68}$$

Using these values of density, the performance of the cavitating Venturi can be calculated. The technique is similar to that used with no gas. First the pressures and flows assuming no cavitation are calculated; second, using the calculated value of P_1 and the vapor pressure for P_2, the maximum

allowable flow is determined. If this flow is less than that calculated in the first step, cavitation occurs and equations (60) and (62) must be solved for the value of P_1 and $\dot{w}$. After these values are obtained, P_3 may be calculated.

ACCUMULATOR

Since accumulators are often used to lower surge pressures or smooth out pressure fluctuations, a model of a simple accumulator will be provided in the program. This element will be mounted on the end of a pipe segment as shown in the sketch. The liquid pressure in the accumulator will be the same as that at the end of the pipe. Thus, the wave equation (2) can be used to obtain



$$P(I,t) = f(W(I,t)) \tag{69}$$

The accumulator functions when liquid flows into it and moves the piston so as to compress the gas behind the piston and the spring; stops are provided to limit its motion. Depending on the initial pressure behind the piston and preload on the spring, some initial pressure level must generally be reached before the piston will move; also in some cases, the piston or the spring may be missing. The equations are derived for the most general case.

The gas pressure behind the accumulator will depend on the initial value to which it is charged and its instantaneous volume assuming isentropic compression.

$$P_g(t) = P_o \left(\frac{V_o}{V_g(t)} \right)^k \tag{70}$$

A force balance on the piston yields

$$W_P \ddot{x} = (P(I,t) - P_g(t))A_P - K_s(x_o + x) \qquad (71)$$

The gas pressure and liquid flow are related to piston position so that:

$$P_g(t) = P_o\left(\frac{V_o}{V_o - xA_P}\right)^k \qquad (72)$$

$$\dot{x} = \frac{w(I,t)}{A_P}$$

These equations can be combined to produce a non-linear second order differential equation in of a form

$$M_P \ddot{x} = K_1 - K_2\dot{x} - K_3\dot{x}|\dot{x}| - K_4\left(\frac{1}{1-K_5x}\right)^k \qquad (73)$$

where the capital K's represent constants. There are several techniques available for solving the above equation, however, an iterative technique will be used so that no special cases develop if one or more of the constants are zero. Secondly, the results will be checked to ensure that x does not exceed its limits, and if this occurs, the flow will be set to zero.

VALVES, FITTINGS, ORIFICES, ETC.

These components are all characterized by the fluid resistance they add to the system. This resistance is handled in two different manners in the model which depends on whether an element is primed or not. For liquid filled lines, the resistance of the pipe and any other component is lumped at the downstream end of the element. Valve resistance can be varied as a function of time for ten different values in either the fuel or oxidizer feed system. When lines are being filled, the resistance of a segment is distributed along the length of an element so that the friction loss will affect the priming. If the resistance is concentrated at an orifice, the priming rate of a line will be reduced too much. This will be prevented during priming by removing the orifice resistance from that line segment until after it is primed.

GENERAL STRUCTURE

The equations previously described will be programmed into a digital computer program. The general outline of this program is shown in Fig. 4. The program contains a main program which reads all data, zeros arrays, and controls the flow of calculations through the required subroutines. To use the program, the system to be modeled is broken up into segments, and the required valves, cavitating Venturies, orifices, etc. are noted. The model will have the capability to handle 60 segments, 10 valves, 2 cavitating Venturies, 2 accumulators, and 2 dead end terminations in both the fuel and oxidizer systems. Provisions will also be available to handle 4 individual combustion chambers. Additional components may be added by use of subroutines.

The segments and components in the system to be modeled will be sequenced so that their location in the system will be known. Data on the length, area, pressure drop, and wall stiffness must be supplied by the user to describe the line segments. Data to describe the physical characteristics of the miscellaneous components, thrust chambers, and pressurization system will also have to be supplied. After the data to describe the physical system to be modeled is supplied, information to describe the function to be simulated must be furnished. This will generally be a description of a valve or valves opening or closing, change in area of a cavitating Venturi, or change in tank pressure. These functions will allow simulation of engine start, shutdown, or throttling or a combination of all three.

After the data has been supplied to the program, the program will calculate the initial conditions-pressure, flow, etc., in the system. Then the calculation will start. Time will be incremented, valve areas set to their new values, tank pressure calculated, acoustic velocity and density calculated so as to include gas dilution effects, and then pressures and flows throughout the system will be calculated at each segment. Time will be incremented again and the process repeated until the transient has been run its allotted time. At intermittent intervals, data on flow, pressure, etc., will be outputed so that the performance of the engine system can be obtained.

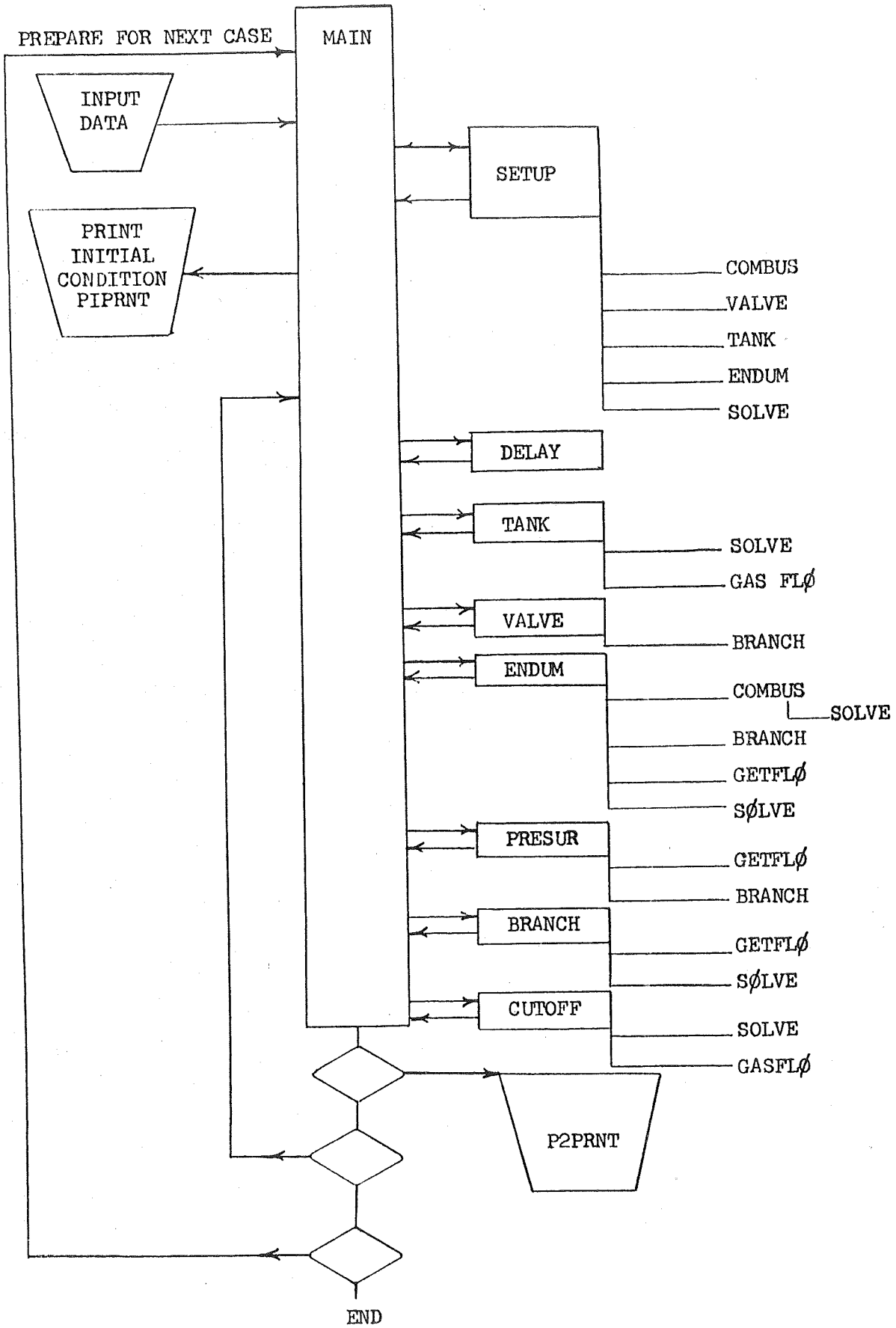


FIGURE 4 - OUTLINE OF GENERALIZED PROPULSION SYSTEM MODEL COMPUTER PROGRAM

SUBROUTINES

Figure 1 shows that the program is made up of a number of subroutines. During the Phase I work, the general nature of each was outlined so that its role in the overall program is identified. Of course as the programming progresses in Phase II, further changes may be recognized and the list may be modified. Input data will be supplied in three different ways. The majority of the data to describe an engine system will be read in in a formated data set. These data-lengths-areas-tabular data describing components- will be read only once. Subsequent data will be read using the Namelist Routine which lends itself to handling stacked cases. Comment cards will be read in with an "A" Format so that the output may be identified.

After reading the input data, subroutine SETUP will be entered and the initial conditions that exist in the system being modeled will be calculated. This routine is used in the existing program, but does not have the required flexibility since it is used for a unique system. It will call other routines throughout the program to solve for the conditions which exist in a particular component.

Upon return to the main program subroutine PlPRNT, a new program, will be called, and system data and initial conditions will be printed. The main program will then begin the sequential calculation of the transient to be simulated.

Subroutine DELAY is the first routine entered at each time interval. It is an existing routine which will be extensively modified to handle two phase flow. In this routine, the gas concentration, the value of effective acoustic velocity, and fluid density in each segment will be calculated. The time delay flow and pressure terms will then be obtained from stored data so that the wave equations can be solved for pressure in each segment.

Subroutine TANK calculates the pressures and flows in the pressurization system. This will be done using an iterative technique. Also included in this new subprogram will be provisions to program tank pressure as a function of time.

Subrouting VALVE will compute the propellant valve opening and closing characteristics. These data will be supplied in tables of valve effective area versus time. The subroutine will also compute the flow and pressure through the valves, and areas in variable area cavitating Venturies. To accomplish these functions, the existing subroutine will be rewritten.

Subroutine ENDUM is a new routine that will calculate flow and pressure at the end of line segments. To do this it will have to calculate performance of components like accumulators, cavitating Venturies, and injector manifolds. The performance of these components will be solved in specific sections of this routine so that individual portions could be removed and used as separate subroutines, or additional ones may be added. It will also call subroutine CØMBUS in which the combustion dynamics are described. This routine is one which is presently used, although it will be modified to allow use of a variable time delay.

The main calculation of flow and pressure in the feed system segments will be handled in subroutines PRESUR and BRANCH. Subroutine PRESUR will calculate flow and pressure between single segments which are completely filled with propellant, and call subroutine BRANCH to fill partially full segments. Filling of segments will be done in subroutine BRANCH which will also calculate flow and pressure in branch elements. These subroutines exist in the present program and will be modified for use in the new program.

Subroutine CUTØFF will calculate the flow and pressure downstream of the main propellant valves after their closure. It will also control the calculation of combustion chamber pressure during this period. This program exists as a separate program, and will have to be modified to use it as a subroutine in this program.

Subroutine P2PRNT will control the printing of calculated data during execution of the program. This will be a new subroutine which will print selected data, and write other data on tape for retrieval at a later time.

Subroutine SØLVE is a special purpose routine designed to predict new values to be used in the iterative solution of equations. It accepts data on the estimated and resultant calculated value of a variable and predicts a new value of the estimated variable. It is called by all subroutines which use the iterative technique for solution of their equations.

GASFLØ and GETFLØ are function subprograms. GASFLØ calculates the flowrate of a gas through an orifice for known pressures and temperatures. It will be used to calculate the pressure in the combustion chamber after combustion ceases during engine cutoff. GETFLØ is a routine for finding the root of a second order equation of the form

$$A x|x| + B x + C = 0$$

This equation is used in several places in the program and hence this function subprogram was written.

INPUT - OUTPUT

Because of the varied nature of data required by the program, data input will be done in three different ways. The original program used the IBM Namelist routine which has proved to be useful and flexible. However, with the large amount of tabular data required in the program, it seems that use of a Formated input would be simpler. Thus the Namelist routine will be used to read in control information and data for stacked cases. To load information on the system being modeled, the option of a Formated input will be available. Briefly, this data consists of, (1) information on each segment in the model defining its length, area, wall compliance, and pressure drop, (2) the table used to determine combustion chamber pressure which contains C*/g, the product of the gas constant and temperature, and combustion efficiency as a function of mixture ratio, (3) the time effective area profiles of the ten fuel and oxidizer valves and four cavitating Venturies, and (4) data to define the physical properties of all the other system components. This tabular data which describes the system will be the initial data read into the programs. Then, the Namelist Routine will be called and miscellaneous data can be inputed along with minor configuration changes to the

tabular data. Finally "comment" cards will be read so that data output may be identified.

Data output will be controlled in two subroutines and thus may be easily modified to suit the needs of individual users. The first of these subroutines will output the input data and the initial conditions set up in the program. This will provide the user with information useful in finding errors in data input, and can be used to identify differences between individual cases. The second subroutine will provide output of data at specified time intervals. This output will print 8 parameters chosen by the user, in a formated listing. Thirty additional parameters will be written on tape and may be used at a later time to provide additional output. This output could be in the form of additional listings or pictorial display.

COMPUTER CORE REQUIREMENTS

The present program is used on an IBM 360 Model 65 and requires 72,000 words of core. Since the NASA computer has space for only 53,000 words, it is necessary to reduce the size of the program. The present program stores data to be plotted and printed at the end of a run, in an array. This array contains 300 points for each of 38 parameters plus a corresponding value of time for each of the 300 points. Since data will be written on tape as the program runs, this array is no longer needed. The subroutine and functions to plot the data are not being supplied with the program, and therefore can be deleted from the present core requirements. These two items will reduce the program size by approximately 19,000 words. To bring it down to a size which allows for some margin, another 10,000 words will be eliminated by changing the segmentation in the model. The present model contains 72 fuel and oxidizer segments and keeps 100 past values of flow and pressure to provide data for the waterhammer time delay terms. This means that the longest segment can be no more than 50 times as long as the shortest. Practical experience has shown that this limit is never reached and a more reasonable value would be 25. Thus the NASA program will

store only 50 past values of flow and pressure. In addition, one other parameter must be stored as a function of time to allow the inclusion of a variable time delay due to gas dilution. This will be the value of acoustic velocity. To provide some additional space, the number of segments used will be reduced from 72 to 60. This change will result in the saving of approximately another 11,000 words, and should allow adequate room for program expansion through minor modifications as desired.

TABLE I

DEFINITION OF SYMBOLS

A	area	square inches
C*	characteristic velocity	in/sec
CP, C_v	heat capacity	BTU/pound
D	diameter	inches
E	Young's Modulus	psi
G	ratio of disolved gas weight to fluid weight	---
G_s	equilibrium value of G	---
H	heat of vaporization	BTU/lb
K	constant	---
M	molecular weight	---
P	pressure	psia
R	resistance	$psi/(pps)^2$
R_o	universal gas constant	psia in^3
T	temperature	degrees R
U	internal energy	BTU/pound
V	volume	cubic inches
W	total mass	pounds
Z	compressibility factor	---
c	pipe end restraint factor	---
e	pipe wall thickness	inches
e	base of natural logarithms	---
g	gravitational constant	386 in/sec/sec
k	ratio of specific heats	---
l	length	inches
t	time	seconds
v	velocity	inches/second
x	distance	inches
$\dot{w}$	flowrate	pounds/second
β	fluid compressibility	psi

Table 1 (cont'd)

α	ratio of volume of gas to volume of liquid	---
γ	time constant	seconds^{-1}
ε	heat exchanger effectiveness	---
ρ	density	pounds/cubic inch
τ	time delay constant	seconds
η_c*	combustion efficiency	

SUBSCRIPTS

c	combustion chamber
g	gas
i	sequence number
l	liquid
lv	liquid vapor
m	mixture
k	piston
S	spring
t	throat
v	vapor

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