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STUDY OF ATMOSPHERIC EFFECTS ON OPTICAL COMMUNICATIONS AND OPTICAL SYSTEMS

by

William E. Webb, Project Director

Final Report on Contract NAS8-30507

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INTRODUCTION

The research which has been conducted during the past year under NASA Contract NAS8-30507 has consisted of two separate investigations. which, while related, are relatively independent of one another. The first of these consisted of a theoretical study of the tracking errors induced in an optical radar by atmospheric turbulence. Expressions for the magnitude of atmospherically induced angle of arrival fluctuations have been derived from Tatarski's theory of the propagation of light through randomly inhomogeneous media. These expressions have been compared with the available experimental data and found to agree within the experimental accuracy. We have also proposed a tracking system using multiple receivers which would reduce the apparent angle of arrival fluctuations without requiring large receiving apertures. The performance of such a receiver has been investigated theoretically and predictions of the RMS angular errors due to the earth's atmosphere have been made.

The second investigation consists of the reduction and analysis of certain government-supplied data. This data concerns the scintillation of a 10.6 micron laser beam and the signal-to-noise ratio in a frequency modulated CO_2 laser communications system. Techniques for reducing the subject data have been developed and software generated to implement the analysis on the IBM system 360-50 computer. The results of the analysis are not complete at this time, however. Preliminary results will be presented.

ATMOSPHERICALLY INDUCED ERRORS IN AN OPTICAL TRACKING SYSTEM

Introduction

We have investigated the errors introduced into a laser tracking system by atmospheric turbulence. The system being investigated is identical to those being used by Marshall Space Flight Center personnel to investigate atmospheric turbulence and is similar to systems which are proposed for early launch tracking of the Saturn V vehicle and for pointing deep-space optical communication systems. Theoretical expressions for the magnitude of the root mean square tracking errors (angle of arrival fluctuation) as a function of the aperture of the receiving optics have been derived. Two assumptions have been made in deriving these expressions, viz; (1) that the tracking system effectively measures the centroid of illuminance in the focal plane of a well-corrected lens, and (2) that the fluctuations in the amplitude of the incoming wave may be neglected compared to fluctuations in its phase.

An optical tracking system which was developed for Marshall Space Flight Center by Sylvania Electronics Corporation under Contract NAS8-20841 is currently being used by Marshall Space Flight Center personnel to investigate the effects of atmospheric turbulence. This system has been fully described in the literature [1]. Without becoming involved in the details of the operation of this tracker we may state that systems consist of receiving optics which focus an incoming laser beam onto the face of an image dissector tube. The image dissector measures the position of the focused spot as it moves across the face of the tube due to atmospheric turbulence. The output signal from the system is proportional to the x and y coordinates of this spot. This data is then analyzed to yield, among other things, the root mean square deviation of the apparent angle of arrival of the laser beam in terms of the azimuth and elevation angles. We shall derive a theoretical expression for the RMS deviation in the azimuthal angle. A similar analysis holds for the elevation.

The problem of defining the angle of arrival of distorted electromagnetic waves is discussed by Beckman [2]. Several authors have investigated the angle of arrival fluctuations of a laser beam propagated through the atmosphere using different definitions of the angle of arrival. Fried [3] has defined it as the angle of inclination of the plane which best fits the incoming wave front in a least mean square sense, while Heidbreder [4,5] has considered the problem in terms of the direction of maximum power radiation from a circular aperture antenna in a turbulent medium. Clearly for practical purposes one must take as the definition of the angle of arrival the quantity which is actually measured by the receiving system. In order to determine this quantity a careful analysis of the operation of the tracking system is required. If, for example, the system in question employs a quadrant detector with the output taken as the difference in signal from opposite quadrants then the apparent angle of arrival of the incoming beam as measured by the tracker will be proporational to the difference in illuminance on the two quadrants of the detector. For the case of a tracking system using an image dissector tube the situation is somewhat more complicated since the output will depend to a large extent on how the signal is processed. We will assume that the tracker measures the instantaneous position of the centroid of illuminance in the plane of the detector. While this assumption has not been fully verified by a detailed analysis of the particular tracking system in question it is felt that it should be sufficiently accurate for our purposes.

<u>Centroid of Illuminance</u>

Let x and y be Cartesian coordinates in the plane of the detector, x being taken in the horizontal direction and y vertically. With the assumptions discussed above the instantaneous azimuthal tracking error may clearly be expressed in terms of the moments of the intensity distribution on the face of the image dissector tube by $\theta_x(t) = \bar{x}/f$ where $\bar{x}$ is the x-coordinate of the centroid of the illuminance I(x,y) and f is the focal length of the system.

$$\theta_x(t) = \frac{1}{f}\frac{\iint xI(x,y)\,dydx}{\iint I(x,y)\;dydx} \tag{1}$$

We have assumed that in the absence of atmospheric disturbance the centroid of illumination is at the origin. The limits of integration will be assumed to be over the entire plane unless otherwise stated. I(x,y) may be expressed as the absolute square of the complex electric field quantity u(x,y).

$$I(x,y) = u(x,y)u^*(x,y) \tag{2}$$

For a well corrected optical system u(x,y) is given in terms of the electric field in the entrance pupil $U(\zeta,\eta)$ by the well known relation

$$u(x,y) = A\iint U(\zeta,\eta)\ \exp\ \{-\frac{2\pi i}{\lambda f}\ (x\zeta+y\eta)\}\ d\zeta d\eta \tag{3}$$

A quadratic phase factor has been suppressed in equation (3) as it will not effect the illuminance. The integration extends over the aperture of the system. We may define $U(\zeta,\eta)$ to be zero everywhere except in the aperture and extend the limits of integration over the entire ζ-η plane.

We now let $V = x/\lambda f$ and $W = y/\lambda f$ and combine equations (1) and (2).

$$\theta_x(t) = \lambda\ \frac{\iint v\,|u(v,w)|^2 dvdw}{\iint |u(v,w)|^2\ dvdw} \tag{4}$$

From equation (3) we have

$$\iint |u(v,w)|^2 dvdw = \iint [A\iint U(\zeta,\eta)e^{-2\pi i(v\zeta+w\eta)}d\zeta d\eta$$
$$\times\ A^*\iint U^*(\zeta',\eta')e^{+2\pi i(v\zeta'+w\eta')}d\zeta' d\eta']\ dvdw \tag{5}$$

which may be rewritten

$$\iint |u(v,w)|^2 dvdw = |A|^2\iiiint\!\!\iint U(\zeta,\eta)U^*(\zeta',\eta')$$
$$\times\ \exp\ \{-2\pi i[v(\zeta-\zeta') + w(\eta-\eta')]\}dvdwd\eta d\zeta d\eta' d\zeta' \tag{6}$$

Performing the integration on v and w we obtain

$$\iint |u(v,w)|^2 dvdw = |A|^2\iiiint U(\zeta,\eta)U^*(\zeta',\eta')$$
$$\times\ \delta(\zeta-\zeta')\delta(\eta-\eta')d\zeta d\eta d\zeta' d\eta' \tag{7}$$

so that the integration on ζ' and η' become trivial

$$\iint |u(v,w)|^2 dvdw = |A|^2 \iint U(\zeta,\eta)U^*(\zeta',\eta')d\zeta d\eta \tag{8}$$

Likewise the other integral in equation (3) is

$$\iint v|u(v,w)|^2 dvdw = |A|^2 \iiiiiint U(\zeta,\eta)U^*(\zeta',\eta') \\ \times v \exp\{-2\pi i[v(\zeta-\zeta') + w(\eta-\eta')]\ dvdwd\zeta d\eta d\zeta' d\eta' \tag{9}$$

On interchanging the order of integration this becomes

$$\iint v|u(v,w)|^2 dvdw = |A|^2 \iiiint U(\zeta,\eta)U^*(\zeta',\eta') \\ \times \left[\int \exp\{-2\pi iw(\eta-\eta')\}dw\right] \left[\int \exp\{-2\pi iv(\zeta-\zeta')\}vdv\right] d\zeta d\zeta' d\eta d\eta' \tag{10}$$

The integration on w leads to a delta function in the usual way. To perform the integration on v we note that

$$\delta(\zeta-\zeta') = \int e^{-2\pi i(\zeta-\zeta')v} dv \tag{11}$$

Differentiating with respect to ζ we have

$$\frac{\partial\delta(\zeta-\zeta')}{\partial\zeta} = -2\pi i \int e^{-2\pi iv(\zeta-\zeta')} vdv \tag{12}$$

or

$$\int e^{-2\pi iv(\zeta-\zeta')} vdv = \frac{i}{2\pi} \frac{\partial\delta(\zeta-\zeta')}{\partial\zeta} \tag{13}$$

Making use of this result, equation (10) becomes

$$\iint v|u(v,w)|^2 dvdw = \frac{i}{2\pi} |A|^2 \iiiint U(\zeta,\eta)U^*(\zeta',\eta') \times \delta(\eta-\eta')\frac{\partial\delta(\zeta-\zeta')}{\partial\zeta} d\eta d\eta' d\zeta d\zeta' \tag{14}$$

The integration of η is easily performed and the integration on ζ may be carried out by parts.

$$\iint v|u(v,w)|^2 dvdw = \frac{i}{2\pi} |A|^2 \iiint U(\zeta,\eta)U^*(\zeta',\eta') \frac{\partial\delta'(\zeta-\zeta')}{\partial\zeta} d\zeta d\zeta' d\eta \tag{15}$$

We take

$$u = U(\zeta,\eta') \qquad dv = \frac{\partial\delta(\zeta-\zeta')}{\partial\zeta} d\zeta$$

$$du = \frac{\partial U}{\partial\zeta} \qquad v = \delta(\zeta-\zeta') \tag{16}$$

then

$$\int U(\zeta,\eta') \frac{\partial\delta(\zeta-\zeta')}{\partial\zeta} d\zeta = U(\zeta,\eta')\Big|_{\zeta_1}^{\zeta_2} - \int \frac{\partial U(\zeta,\eta')}{\partial\zeta} \delta(\zeta-\zeta')d\zeta \tag{17}$$

$$\int U(\zeta,\eta') \frac{\partial\delta(\zeta-\zeta')}{\partial\zeta} d\zeta = U(\zeta,\eta')\Big|_{\zeta_1}^{\zeta_2} - \frac{\partial U(\zeta',\eta')}{\partial\zeta} \tag{18}$$

here ζ_2 and ζ_1 are the values of ζ at the edge of the aperture. In general for any aperture other than a rectangular one ζ_1 and ζ_2 will be functions of η. Substituting equation (18) into equation (15) we have

$$\iint v|u(v,w)|^2 dvdw = \frac{i|A|^2}{2\pi} \iint U^*(\zeta',\eta')$$

$$\times \quad U(\zeta_2,\eta')\delta(\zeta_2-\zeta') - U(\zeta_1,\eta')\delta(\zeta_1-\zeta') - \frac{\partial U(\zeta',\eta')}{\partial \zeta'} \quad d\zeta' d\eta' \qquad (19).$$

hence

$$\iint v|u(v,w)|^2 dvdw = \frac{i}{2\pi}|A|^2 \int \frac{1}{2}|U(\zeta_2,\eta)|^2 - \frac{1}{2}|U(\zeta_1,\eta)|^2$$
$$- \int U^*(\zeta,\eta)\, \frac{\partial U(\zeta,\eta)}{\partial \zeta} d\zeta \; d\eta \qquad (20)$$

The factor 1/2 in each of the first two terms arises from the fact that $U(\zeta,\eta)$ is discontinuous at ζ_1 and ζ_2 since it has been defined to be zero for all ζ and η outside the aperture. Since integrals of the form $\int f(x)\delta(x)dx$ are properly defined only if $f(x)$ is continuous at the point where the argument of the delta function vanishes we must exercise care in evaluating the integrals. First we perform a transformation of coordinates by letting $\zeta=\zeta-\zeta_1$ so that the argument of the delta function is zero at the origin. The limits of integration then extend from zero to infinity. We may now define $U(-\zeta) = U(\zeta)$ rather than zero. This does not effect the value of the integral since $U(-\zeta)$ is outside the range of integration. This integral may then be written as one-half the integral from minus infinity to plus infinity since the integrand is symmetrical. Hence the origin of the factor of one-half.

Now we write U as $|U|e^{i\phi}$ so that the last term in equation (20 becomes

$$\int |U| e^{-i\phi} \frac{\partial}{\partial \zeta} |U| e^{+i\phi} \, d\zeta \qquad (21)$$

which may be integrated by parts to yield

$$i\int |U(\zeta,\eta)|^2 \frac{\partial\phi}{\partial\zeta}\, d\zeta + \frac{1}{2}|U(\zeta_2,\eta)|^2 - \frac{1}{2}|U(\zeta_1,\eta)|^2 \tag{22}$$

Substituting into (20) we have

$$\iint v|u(v,w)|^2 dv dw = \frac{A^2}{2\pi} \iint |U(\zeta,\eta)|^2 \frac{\partial\phi}{\partial\zeta}\, d\zeta d\eta \tag{23}$$

so that equation (4) becomes

$$\theta_x(t) = \frac{\lambda}{2\pi} \frac{\iint_\Sigma |U(\zeta,\eta)|^2 \frac{\partial\phi}{\partial\zeta} d\zeta d\eta}{\iint_\Sigma |U(\zeta,\eta)|^2\, d\zeta d\eta} \tag{24}$$

Equation (24) is a quite general expression for the instantaneous tracking error. By squaring and taking an ensemble average over all physical realizations of the incoming wave one would obtain the RMS angular fluctuation of θ_x. Unfortunately, the resulting expressions cannot be readily evaluated in terms of the statistical functions which are normally used to describe the fluctuations of the atmosphere. In one important case, viz, if the amplitude fluctuation can be neglected in comparison to variations in the phase, then equation (24) may be evaluated. We shall first turn our attention to this case and later discuss methods of evaluating equation (24) when this approximation is not valid.

The Ray Optics Approximation

Let us assume that the random fluctuations in the amplitude of the field may be neglected in comparison to the fluctuation in the phase.

This is essentially the ray optics approximation. The validity of this approximation has been widely discussed in the literature [6-8]. With this approximation $|U(\zeta,\eta)|^2$ is a constant and may be taken outside of the integral sign in equation (24). We then have

$$\theta_x(t) = \frac{\lambda}{2\pi} \frac{\iint \frac{\partial\phi}{\partial\zeta}\, d\zeta d\eta}{\iint d\zeta d\eta} \tag{25}$$

where the integration extends over the aperture of the system. The integral in the denominator is just the area of the aperture, hence

$$\theta_x(t) = \frac{\lambda}{2\pi A} \int_{\eta_2}^{\eta_1} \int_{\zeta_2}^{\zeta_1} \frac{\partial\phi}{\partial\zeta}\, d\zeta d\eta \tag{26}$$

$$\theta_x(t) = \frac{\lambda}{2\pi A} \int_{\eta_2}^{\eta_1} [\phi(\zeta_1,\eta) - \phi(\zeta_2,\eta)]d\eta \tag{27}$$

where ζ_1, ζ_2, η_1 and η_2 are the appropriate limits of integration. We shall consider three cases: (a) a narrow slit of length a; (b) a rectangular aperture of length a and width b; and (c) a circular aperture of diameter D.

(a) Narrow slit aperture: For this case equation (27) reduces to

$$\theta_x(t) = \frac{\lambda}{2\pi a} [\phi(\tfrac{a}{2}) - \phi(-\tfrac{a}{2})] \tag{28}$$

Squaring θ_x and taking the ensemble average we obtain

$$\left\langle \theta_x^2 \right\rangle = \left(\frac{\lambda}{2\pi a}\right)^2 \quad [\phi(\frac{a}{2}) - \phi(-\frac{a}{2})]^2 \tag{29}$$

We recognize the quantity on the right as the phase structure function; hence the root mean square tracking error is

$$\theta_{RMS} = \frac{\lambda}{2\pi a} \sqrt{D_\phi(a)} \tag{30}$$

Following Tatarski [9] we take $D_\phi(a)$ for a plane wave as

$$D_\phi(r) = \sqrt{6.88} \left|\frac{r}{r_o}\right|^{5/3} \tag{31}$$

for values of r much larger than the inner scale of turbulence ℓ_o. Then

$$\theta_{RMS} = \frac{\lambda}{2\pi} \sqrt{6.88}\, r_o^{-5/6}\, a^{-1/6} \tag{32}$$

For $a \ll \ell_o$, D_ϕ is proportional to r^2 [10]. Clearly θ_{RMS} must be independent of the length for very short slits. From Tatarski's equation 7.101 we have after some manipulation

$$D_\phi = 4.21\, \ell_o^{-1/3}\, r_o^{-5/3}\, a^2 \tag{33}$$

Which yields a value

$$\theta_{RMS} = .326 \lambda r_o^{-5/6}\, \ell_o^{-1/6} \tag{34}$$

From equation (32) we find the value of (a) which will give this limit to be $5.2\ell_o$. Hence we may write (32) and (34) as

$$\theta_{rms} = .429\lambda r_o^{-5/6} a^{-1/6} \qquad a > 5\ell_o \tag{35a}$$

$$\theta_{rms} = .326\lambda r_o^{-5/6} \ell_o^{-1/6} \qquad a < 5\ell_o \tag{35b}$$

(b) A rectangular aperture: Now consider a rectangular aperture of length a and height b. If we take the origin of our coordinate system at the geometrical center of the aperture, then $\zeta_1 = \frac{1}{2}b$, $\zeta_2 = -\frac{1}{2}b$, $\eta_1 = \frac{1}{2}a$ and $\eta_2 = -\frac{1}{2}a$. Equation (27) then becomes

$$\theta_x(t) = \frac{\lambda}{2\pi ab} \int_{\eta_1}^{\eta_2} [\phi(\zeta_1,\eta) - \phi(\zeta_2,\eta)]d\eta \tag{36}$$

Squaring θ_x and writing the square of the integral as a double integral over η and η' we obtain

$$\theta_x^2 = \left(\frac{\lambda}{2\pi ab}\right)^2 \int_{\eta_1}^{\eta_2}\int_{\eta_1}^{\eta_2} [\phi(\zeta_1,\eta) - \phi(\zeta_2,\eta)][\phi(\zeta_1,\eta') - \phi(\zeta_2,\eta')]d\eta d\eta' \tag{37}$$

or

$$\theta_x^2 = \left(\frac{\lambda}{2\pi ab}\right)^2 \int_{\eta_1}^{\eta_2}\int_{\eta_1}^{\eta_2} [\phi(\zeta_1,\eta)\phi(\zeta_1,\eta') + \phi(\zeta_2,\eta)\phi(\zeta_2,\eta') - \phi(\zeta_1,\eta)\phi(\zeta_2,\eta\ ) - \phi(\zeta_2,\eta)\phi(\zeta_1,\eta')]d\eta d\eta' \tag{38}$$

We now take the ensemble average of θ_x^2 in equation (38). Interchanging the linear operations of averaging and integration we note that the resulting quantities in the right hand side of equation (38) are of the form as phase covariance functions.

$$C_\phi(\vec{r}_1 - \vec{r}_2) = \left\langle \phi(\vec{r}_1)\phi(\vec{r}_2) \right\rangle \tag{39}$$

The phase covariance C_ϕ does not properly exist for nonstationary systems. We will introduce it, however, for mathematical convenience in manipulation without regard to the convergence of the integrals which are implied. C_ϕ will later be replaced by the structure function D_ϕ which is properly defined. The use of the covariance notation lends mathematical simplicity and leads to results equivalent to those which can be derived by manipulating equation (38) into the form of structure functions. Thus our procedure is justified even though it may lack certain mathematical rigor. Making use of the fact that ϕ is a homogeneous and isotropic random variable, equation (38) can be written as

$$\left\langle \theta_x^2 \right\rangle = \left(\frac{\lambda}{2\pi ab}\right)^2 \int_{\eta_1}^{\eta_2} \int_{\eta_1}^{\eta_2} [2C_\phi(\eta-\eta') - 2C_\phi(\sqrt{(\zeta_1-\zeta_2)^2+(\eta-\eta')^2})]\,d\eta\,d\eta' \tag{40}$$

By the well known relation between the covariance and structure function, i.e.,

$$2C_\phi(r) = D_\phi(\infty) - D_\phi(r) \tag{41}$$

equation (40) may be put in the form

$$\left\langle\theta_x^2\right\rangle = \left(\frac{\lambda}{2\pi ab}\right)^2 \int_{\eta_1}^{\eta_2}\int_{\eta_1}^{\eta_2} \left\{D_\phi(\sqrt{(\zeta_1-\zeta_2)^2 + (\eta-\eta')^2} - D_\phi(\eta-\eta')\right\} d\eta d\eta' \tag{42}$$

$$\left\langle\theta_x^2\right\rangle = \left(\frac{\lambda}{2\pi ab}\right)^2 \quad 6.88r_o^{-5/3} \int_{\eta_1}^{\eta_2}\int_{\eta_1}^{\eta_2} [((\zeta_1-\zeta_2)^2 + (\eta-\eta')^2)^{5/6} - |\eta-\eta'|^{5/3}]d\eta d\eta' \tag{43}$$

We first remove the absolute value signs from equation (43). The integrand in the first term is everywhere positive so that it does not present a problem. In the second term we may make a change of variables by letting $x = \eta+a/2$ and $y = \eta'+a/2$. Noting that $\eta_1 = a/2$ and $\eta_2 = -a/2$, the second integral becomes

$$\int_o^a \int_o^a |x-y|^{5/3}\, dydx \tag{44}$$

Since the integrand is symmetrical about the line $x = y$ and is positive for all $x > y$, we may write it in the following form which may be directly integrated.

$$2 \int_o^a \int_o^x (x-y)^{5/3}\, dydx = \frac{9}{44}\, a^{\,11/3} \tag{45}$$

The first term in equation (43) may be evaluated by expanding the integrand in a Taylor's series and integrating term by term. Noting that $\zeta_2-\zeta_1 = b$, we have

$$\iint [(\zeta_1-\zeta_2)^2+(\eta-\eta')^2]^{5/6} d\eta d\eta' = b^{5/3} \int\int_{-a/2}^{a/2} (1+(\frac{\eta-\eta'}{b})^2)^{5/6} d\eta d\eta'$$

$$= b^{5/3} \int\int_{-a/2}^{+a/2} \left[1 + \frac{5}{6}(\frac{\eta-\eta'}{b})^2 - \frac{5}{6^2 \cdot 2!}(\frac{\eta-\eta'}{b})^4 + \frac{5\cdot 7}{6^3 \cdot 3!}(\frac{\eta-\eta'}{b})^6 - \frac{5\cdot 7\cdot 13}{6^4 \cdot 4!}(\frac{\eta-\eta'}{b})^8 + \ldots \right] d\eta d\eta' \quad (46)$$

From equations (44), (46) and (43), we obtain

$$\left\langle \theta_x^2 \right\rangle = \frac{\sqrt{6.88}}{r_o^{5/3}} (\frac{\lambda}{2\pi})^2 a^{-2} b^{-2} \left[-\frac{9}{44} a^{11/3} + a^2 b^{5/3} + 10 \sum_{n=1}^{\infty} (-1)^n \frac{C_n}{6^n n!(2n+1)(2n+2)} a^{2n+2} b^{5/3-2n} \right] \quad (47)$$

Where C_n = 1; 7; 7 x 13; 7 x 13 x 19; 7 x 13 x 19 x 25; etc. for n = 1, 2, 3, ... Equation (47) is valid for a less than b since for a greater than b the expansion in equation (46) does not converge. Now if we consider a rectangular aperture of length b in the direction along which the component of the tracking error is being measured and width a perpendicular to this direction, we define a shape parameter σ for the aperture as the ratio of a to b. Clearly σ must lie between zero and one for the series expansion to converge. Writing equation (47) in terms of σ and taking the square root we obtain for θ_{rms}

$$\theta_{rms} = \frac{\lambda}{2\pi}\sqrt{6.88}\; r_o^{-5/6} b^{-1/6}\left[1 - \frac{9}{44}\sigma^{5/3} + 10\sum_{1}^{\infty}(-1)^n \frac{C_n\;\sigma^{2n}}{6^n n!(2n+1)(2n+2)}\right]^{1/2} \quad (48)$$

We see that for a rectangular aperture the RMS tracking error varies with the b dimension (i.e., the direction along which the component of angle is measured) like the reciprocal one-sixth power. The dependence on the other dimension is very slight since the term in braces varies from unity for σ equal to zero (a narrow slit) to 0.96405 for σ equal to one (a square aperture). Clearly equation (48) agrees with our previous results for the case of a narrow slit.

We have evaluated the numerical coefficient in equation (48) for several intermediate values of the shape parameter. These results are given in Figure 1.

(c) Circular Aperture: For a circular aperture of radius R, equation (38) becomes

$$\theta_x^2 = \left(\frac{\lambda}{2\pi A}\right)^2 \int\int_{-R}^{R} [\phi(\zeta_1,\eta)\phi(\zeta_1',\eta') + \phi(\zeta_2,\eta)\phi(\zeta_2',\eta') - \phi(\zeta_1,\eta)\phi(\zeta_2',\eta') - \phi(\zeta_2,\eta)\phi(\zeta_1',\eta')]d\eta d\eta \quad (49)$$

Where $A = \pi R^2$ is the area of the aperture and

$$\zeta_1 = (R^2-\eta^2)^{1/2} \qquad \zeta_2 = -(R^2-\eta^2)^{1/2}$$

$$\zeta_1' = (R^2-\eta'^2)^{1/2} \qquad \zeta_2' = -(R^2-\eta'^2)^{1/2} \quad (50)$$

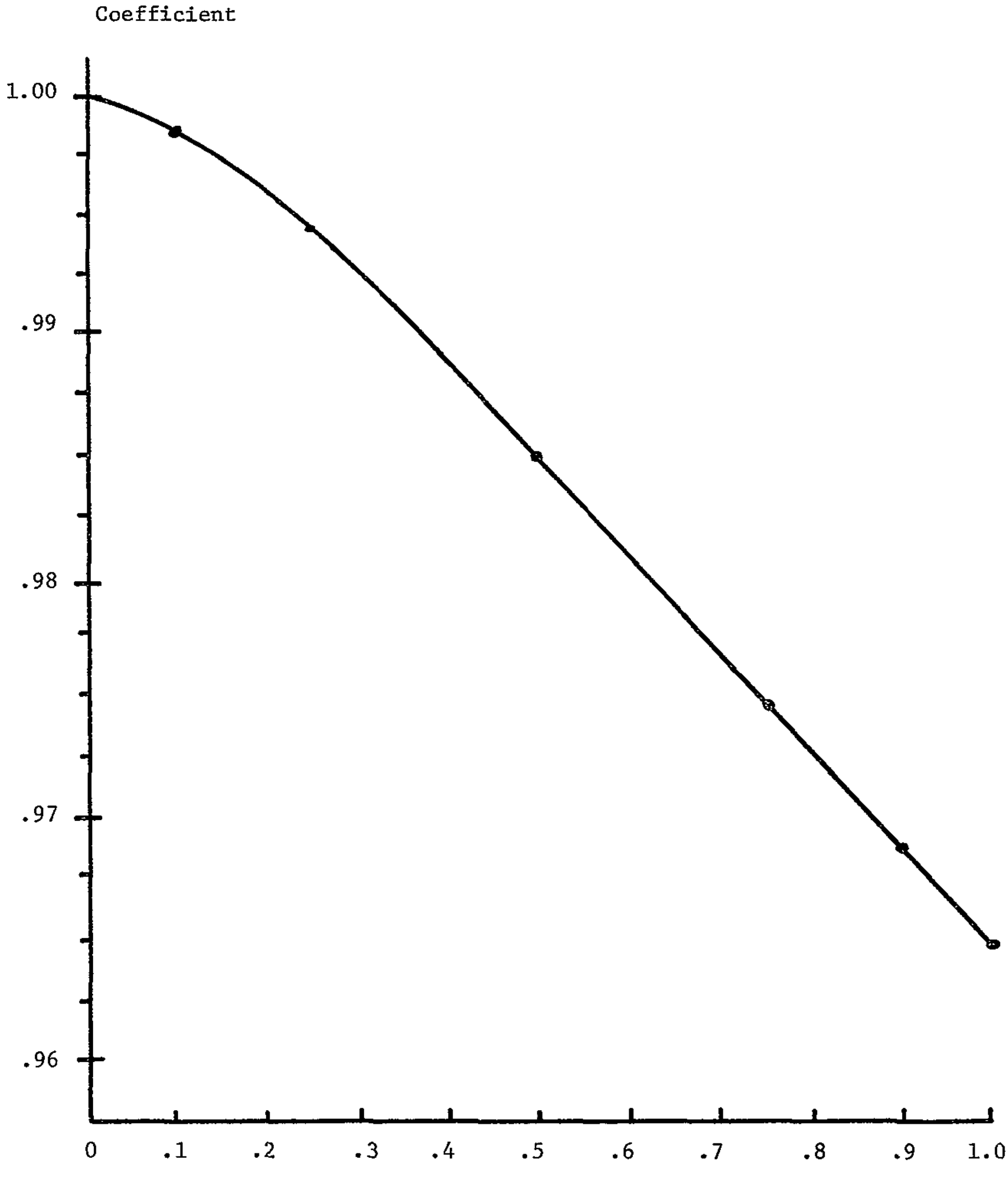


Figure 1. Numerical coefficient of equation (48) for a square aperture versus the shape parameter $\sigma = a/b$.

We now take the ensemble average of equation (49), making use of equations (31), (39), (41) and (50), and the homogeneity and isotropy of the structure function. We introduce new variables $x = R_\eta$ and $y = R_\eta'$ and obtain, after considerable manipulation

$$\left\langle \theta_x^2 \right\rangle = \left(\frac{\lambda}{2\pi R^2}\right)^2 \frac{6.88}{r_o^{5/3}} R^{11/3} \int_{-1}^{+1}\int_{-1}^{+1} \left\{ \left[\left(\sqrt{1-x^2} + \sqrt{1-y^2}\right)^2 + \left(x-y\right)^2\right]^{5/6} - \left[\left(\sqrt{1-x^2} - \sqrt{1-y^2}\right)^2 + \left(x-y\right)^2\right]^{5/6} \right\} dxdy \quad (51)$$

The root mean square tracking error is easily obtained by taking the square root of equation (51). Introducing D = 2R, the aperture diameter, and simplifying somewhat, we obtain:

$$\theta_{rms} = \frac{\sqrt{6.88}}{r_o^{5/6}} \frac{\lambda}{2\pi} \left[\frac{2^{7/12}}{\pi} (J^- - J^+)\right]^{1/2} D^{-1/6} \quad (52)$$

where

$$J^{\pm} = \int_{-1}^{+1}\int_{-1}^{+1} \left[1-xy \pm \sqrt{(1-x^2)(1-y^2)}\right]^{5/6} dydx \quad (53)$$

is a numerical coefficient which exactly corresponds to the shape coefficient which was introduced for the square aperture. The integrals of equation (53) were programmed for the IBM 360 computer. Using a 200 point Simpson's rule integration this yielded a value of .986 for the shape coefficient (the term in equation (52) contained in curly braces). Thus we see that the tracking error for a circular aperture corresponds very closely to that for a rectangular aperture with length to width ratio of 1/2.

Circular Aperture of Very Small Diameter

For apertures which are small compared to the inner scale of turbulence the structure function is given by equation (33) instead of equation (31). Thus for small apertures equation (51) becomes

$$\left\langle \theta_x^2 \right\rangle = \left(\frac{\lambda}{2\pi}\right)^2 \frac{4.21}{r_o^{5/3} \ell_o^{1/3}} R^4 \int_{-1}^{+1} \int_{-1}^{+1} \left\{ \left[\left(\sqrt{1-x^2} + \sqrt{1-y^2}\right)^2 + (x-y)^2 \right]^2 - \left[\left(\sqrt{1-x^2} - \sqrt{1-y^2}\right)^2 + (x-y)^2 \right]^2 \right\} dydx \qquad (54)$$

which reduces to

$$\left\langle \theta_x^2 \right\rangle = \left(\frac{\lambda}{2\pi}\right)^2 \frac{4.21}{r_o^{5/3} \ell_o^{1/3}} \cdot 16 \int_{-1}^{+1} \int_{-1}^{+1} (1-xy) \sqrt{(1-x^2)(1-y^2)}\, dydx \qquad (55)$$

Since the x and y integrations may be separated we write equation (55) as

$$\left\langle \theta_x^2 \right\rangle = \left(\frac{\lambda}{2\pi}\right)^2 \frac{67.36}{r_o^{5/3} \ell_o^{1/3}} \left\{ \left[\int_{-1}^{+1} \left(\sqrt{1-x^2}\right) dx \right]^2 - \left[\int_{-1}^{1} x \sqrt{1-x^2}\, dx \right]^2 \right\} \qquad (56)$$

The integrals in equation (56) may be evaluated directly yielding values of 2 and zero for the first and second terms respectively. Taking the square root to obtain the root mean square angular fluctuation we have

$$\theta_{rms} = \frac{\lambda}{\pi} \frac{\sqrt{67.36}}{r_o^{5/6} \ell_o^{1/6}} \tag{57}$$

Thus we see that for a circular aperture the angular fluctuation will be independent of aperture size for apertures which are small compared to the inner scale of turbulence.

Comparison With Calculations Based on Average Wave Front Tilt

Fried [3] had developed expressions for the average tilt of a wave front after passing through a turbulent atmosphere, by expanding the phase ϕ in a series of Modified Zernike Polynomials - i.e.,

$$\phi(x_1 y) = a_n F_n(x_1 y) \tag{58}$$

where

$$\begin{aligned} F_1 &= (\pi R^2)^{-1/2} & F_4 &= (\pi R^6/12)(x^2+y^2-R^2/2) \\ F_2 &= (\pi R^4/4)^{-1/2} x & F_5 &= (\pi R^6/6)(x^2-y^2) \\ F_3 &= (\pi R^4/4)^{-1/2} y & F_6 &= (\pi R^6/24) xy \end{aligned} \tag{59}$$

Here a_1 represents the average phase shift of the wave front, a_2 and a_3 its average tilt, a_4 the spherical deformation, a_5 and a_6 the astigmatic deformation, etc. The average tilt $\left\langle a_L^2 \right\rangle$ is given in terms of a_1 and a_2 by

$$\left\langle a_L^2 \right\rangle = \left\langle a_1^2 \right\rangle + \left\langle a_2^2 \right\rangle \tag{60}$$

Fried has derived an expression for $\left\langle a_L^2 \right\rangle$ by requiring that the polynomial expansion of ϕ (equation (58)) gives a best fit to the actual phase in a least mean square sense. We have found that Fried's work contains an error in that he has omitted a factor of $(\pi R^2)^{-1}$ in his equation 4.6a. Following through Fried's work we find that the correct expression for $\langle a_L^2 \rangle$ is

$$\left\langle a_L^2 \right\rangle = .883\pi R^2 \left(\frac{D}{r_o}\right)^{5/3} \tag{61}$$

The reader should compare this expression with equation 7.8a in Fried's paper.

With this correction in hand, we may proceed to calculate the apparent angle of arrival of the beam. Consider a plane wave traveling in approximately the z direction. Neglecting deformation of the wave front we may write its phase as

$$\phi = a_2F_2 + a_3F_3 \tag{62}$$

But the phase is also $(\frac{2\pi z}{\lambda})$ so that the equation of the isophase surface is

$$\frac{2\pi z}{\lambda} + a_2F_2(x) + a_3F_3(y) = \text{constant} \tag{63}$$

We let $\vec{K}$ be a vector normal to the isophase surface and $\vec{k}$ be the unit vector in the z direction. Substituting the values of F from equation (59) into equation (63) we may write $\vec{K}$ as

$$\vec{K} = \overrightarrow{\mathrm{GRAD}} \left[z + \frac{\lambda}{\pi^{3/2} R^2} \left(a_2 x + a_3 y \right) \right] \tag{64}$$

Since

$$|K| \cos\theta_T = \vec{K}_1 . \ \vec{K} \tag{65}$$

a straightforward calculation yields

$$\cos\theta_T = \left[\frac{\lambda^2}{\pi^3 R^4} \left(a_2{}^2 + a_3{}^2 \right) + 1 \right]^{-1/2} \tag{66}$$

Squaring both sides and noting that for small angles

$$\cos\theta_T = 1 - \frac{\theta_T^2}{2} + \ldots \tag{67}$$

and

$$\left[1 + \frac{\lambda^2}{\pi^3 R^4} \left(a_2{}^2 + a_3{}^2 \right) \right]^{-1/2} = 1 - \frac{\lambda^2}{2\pi^3 R^4} \left(a_2{}^2 + a_3{}^2 \right) + \ldots \tag{68}$$

We obtain

$$\theta_{RMS} = \sqrt{\left\langle \theta_T^2 \right\rangle} = \frac{\lambda}{\pi^{3/2} R^2} \sqrt{\left\langle a_L^2 \right\rangle} \tag{69}$$

Substituting from equation (61)

$$\theta_{RMS} = .414 \frac{\lambda}{r_o^{5/6}} D^{-1/6} \tag{70}$$

Chase [11] has noted that the constant (.883) in equation (61) should properly be .828 since there were numerical errors in Fried's claculations which were later corrected by Chase.

Heidbreder's calculations [4,5] based on the maximum of the radiation pattern of a circular aperture also led to a minus one-sixth power dependence on the aperture diameter but with a somewhat different proportionality constant. That the calculations of Fried, those of Heidbreder and ours all lead to the same functional dependence with only slightly different numerical constants is consistent with the conclusion (as Fried and others have pointed out) that wave front tilt is the predominant effect responsible for angle of arrival fluctuations.

Comparison With Experimental Results

Wyman [12] has reported experimental measurements of the RMS tracking errors as a function of receiver aperture over 3.2 and 6.4 Km paths. Figure 2 shows the root mean square angular fluctuations for both path lengths and several aperture sizes. Theoretical curves of angular fluctuations versus aperture size from equation (52) are also plotted in Figure 2. Here the correlation distance r_o has been chosen to produce the best fit with the experimental data.

The amount of data available is very minimal so that the theory cannot be said to be fully verified. Nevertheless the available data does fit a $D^{-1/6}$ power law to within the experimental error.

TRACKING SYSTEM USING MULTIPLE APERTURES

From our previous results and from the available experimental data

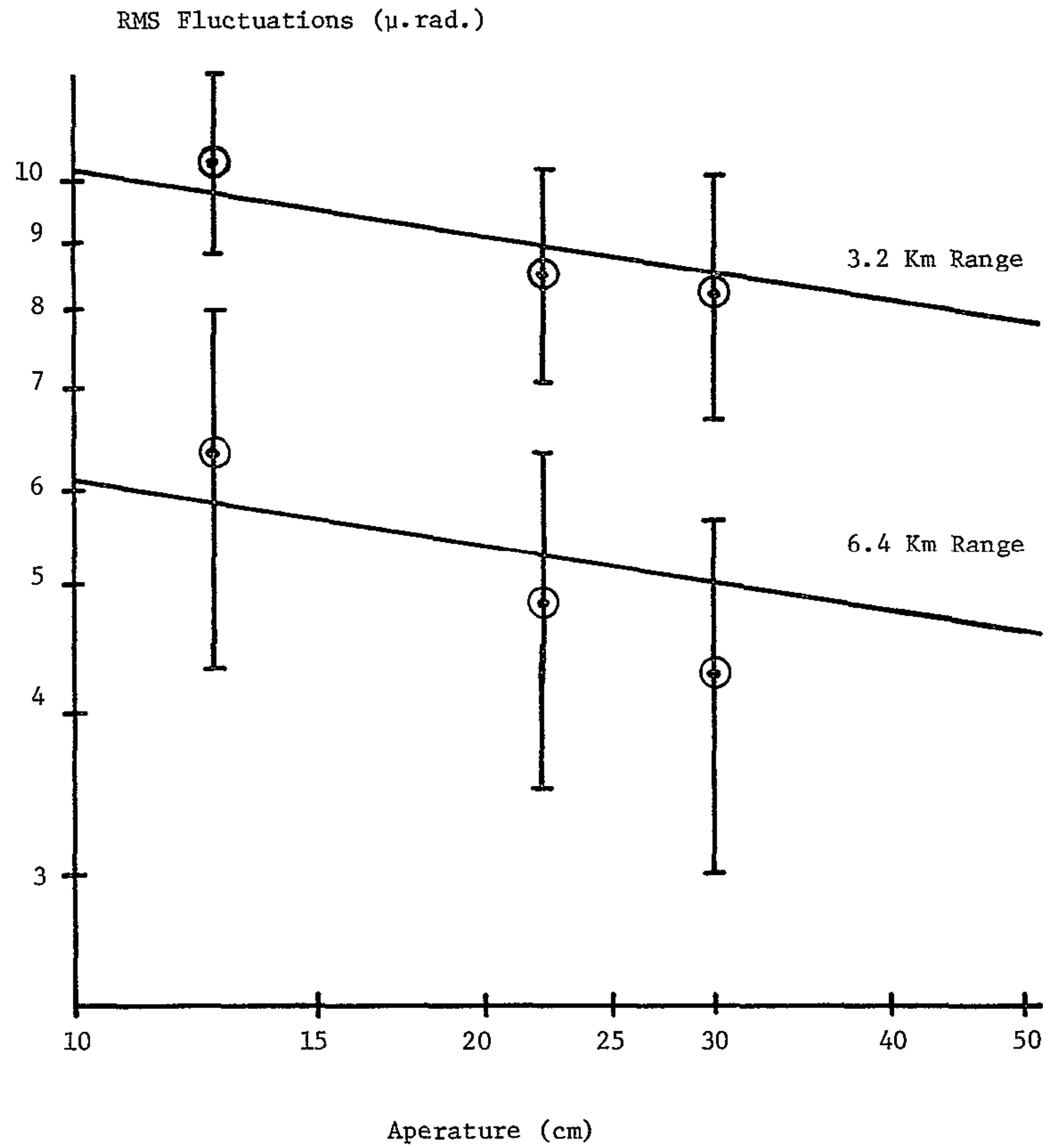


Figure 2. RMS angular fluctuations in microradians as a function of receiving aperture diameter for 3.2 and 6.4 kilometer paths. Experimental data after Wyman, Reference 12.

it is clear that when one increases the receiving aperture of an optical tracking system the atmospherically induced errors are decreased due to averaging over a larger part of the incoming wave front. Since the critical factor in reducing the atmospheric errors is the linear dimensions involved, it seems likely that an effective increase in accuracy could be obtained by using two tracking systems with small receiving apertures located some distance apart instead of a larger single aperture system. Since large diameter optics are extremely expensive the two aperture systems might well be more economical than a larger single system.

For this reason we have studied the error in a double receiver tracking system. In the process of our investigation we have devised an experiment to test the validity of our theoretical calculations which should overcome some of the difficulties previously encountered in measurements of atmospheric effects. This experiment will be discussed later.

Theoretical

Let us consider two identical optical tracking systems, such as have been previously described, located in close proximity to each other and observing a common target. Each tracker will see an apparent angular motion of the target due to atmospheric fluctuations which we will denote by $\theta_1(t)$ and $\theta_2(t)$ respectively. Let us suppose that we may observe either the "relative tracking error", $\theta_r(t)$, which we shall define as the instantaneous difference in $\theta_1(t)$ and $\theta_2(t)$, or the "average tracking error", $\theta_a(t)$, which we shall define as the average of $\theta_1(t)$ and $\theta_2(t)$. As shown in Figure 3 we take ξ and η to be cartesian coordinates

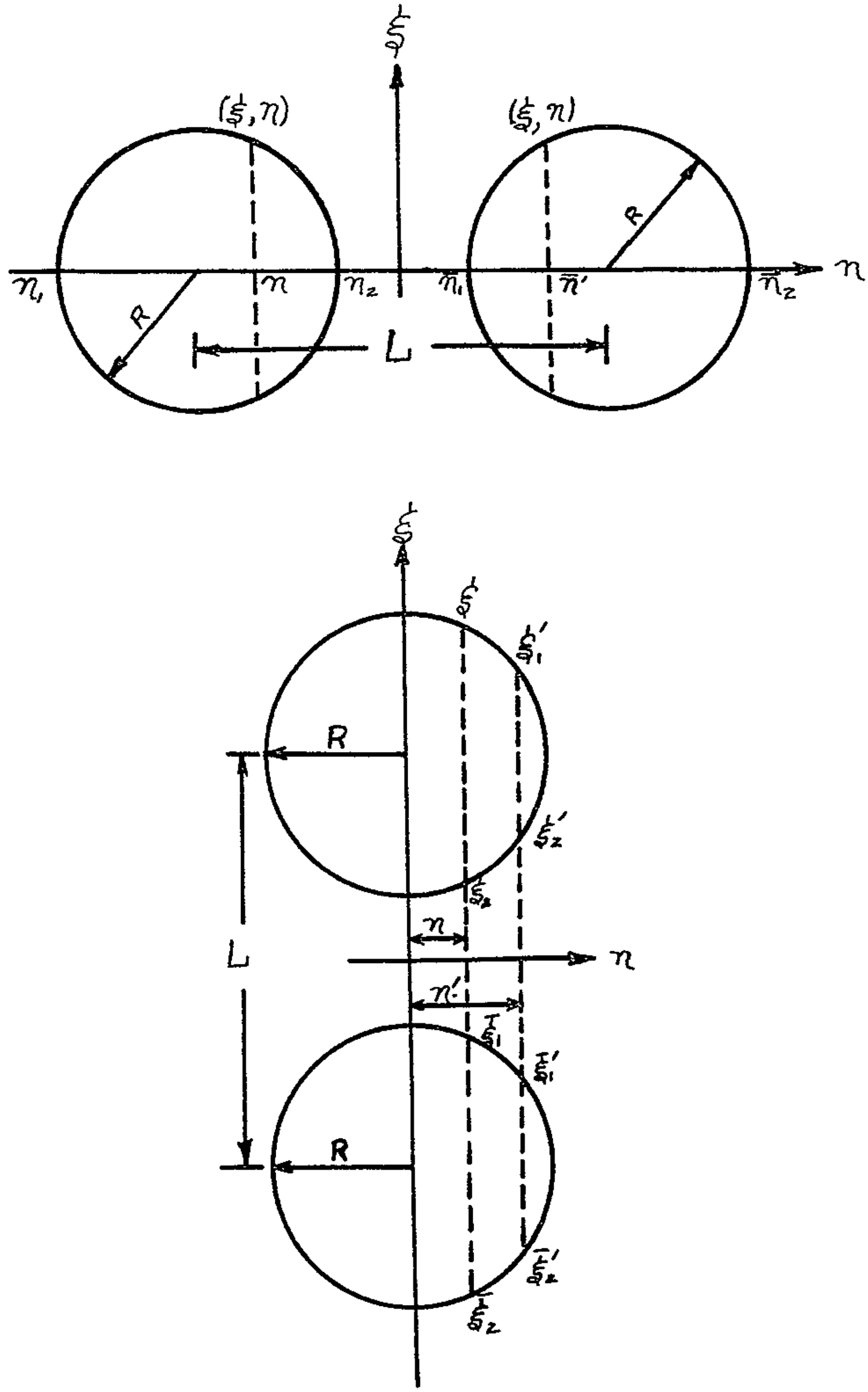


Figure 3. Geometry of the two aperature receiver

in the plane of the receivers' apertures and let R be the radius of each aperture and L the distance between their centers. If θ_1 and θ_2 are the components of the instantaneous angular tracking errors of each system along the η axis then, as has been shown,

$$\theta_1(t) = \frac{\lambda}{2\pi A}\int_{\eta_1}^{\eta_2}[\Phi(\xi_1,\eta,t) - \Phi(\xi_2,\eta,t)]\, d\eta \tag{71}$$

$$\theta_2(t) = \frac{\lambda}{2\pi A}\int_{\overline{\eta}_1}^{\overline{\eta}_2}[\Phi(\overline{\xi}_1,\overline{\eta},t) - \Phi(\overline{\xi}_2,\overline{\eta},t)]\, d\eta \tag{72}$$

where η_1, η_2, $\overline{\eta}_1$, and $\overline{\eta}_2$ are the appropriate limits of integration for a circular aperture, ξ_1 and ξ_2 are functions of η, and A is the area of a single aperture.

We have defined the relative tracking error as

$$\theta_r = \theta_1 - \theta_2 \tag{73}$$

so that

$$\theta_r = \frac{\lambda}{2\pi A}\left\{\int_{\eta_1}^{\eta_2}[\Phi(\xi_1,\eta,t) - \Phi(\xi_2,\eta,t)]\, d\eta - \int_{\overline{\eta}_1}^{\overline{\eta}_2}[\ (\overline{\xi}_1,\eta,t) - \Phi(\overline{\xi}_2,\eta,t)]\, d\eta\right\} \tag{74}$$

Making use of the fact that the centers of the apertures are located at $(0,\pm\frac{L}{2})$ respectively, we obtain the relations

$$\xi_1 = \sqrt{R^2-(\eta - \frac{L}{2})^2} \qquad \xi_2 = - \sqrt{R^2-(\eta - \frac{L}{2})^2}$$

$$\overline{\xi}_1 = \sqrt{R^2-(\eta + \frac{L}{2})^2} \qquad \overline{\xi}_2 = - \sqrt{R^2-(\eta + \frac{L}{2})^2} \tag{75}$$

and

$$\eta_1 = \frac{L}{2} - R \qquad \eta_2 = \frac{L}{2} + R$$

$$\overline{\eta}_1 = - \frac{L}{2} - R \qquad \overline{\eta}_2 = - \frac{L}{2} + R \tag{76}$$

Squaring equation (74) one obtains

$$\begin{aligned}
\theta_r^{\,2}(t) = [\frac{\lambda}{2\pi A}]^2 \quad & \int_{\eta_1}^{\eta_2}\int_{\eta_1}^{\eta_2} \Big(\Phi(\xi_1,\eta,t) - \Phi(\xi_2,\eta,t)\Big) \\
& \qquad \times \Big(\Phi(\xi_1',\eta',t) - \Phi(\xi_2',\eta',t)\Big)\, d\eta d\eta \\
& + \int_{\overline{\eta}_1}^{\overline{\eta}_2}\int_{\overline{\eta}_1}^{\overline{\eta}_2} \Big(\Phi(\overline{\xi}_1,\eta,t) - \Phi(\overline{\xi}_2,\eta,t)\Big) \\
& \qquad \times \Big(\Phi(\overline{\xi}_1',\eta',t) - \Phi(\overline{\xi}_2',\eta',t)\Big)\, d\eta d\eta' \\
& - \int_{\eta_1}^{\eta_2}\int_{\overline{\eta}_1}^{\overline{\eta}_2} \Big(\Phi(\xi_1,\eta,t) - \Phi(\xi_2,\eta,t)\Big) \\
& \qquad \times \Big(\Phi(\overline{\xi}_1',\eta',t) - \Phi(\overline{\xi}_2',\eta',t)\Big)\, d\eta d\eta'
\end{aligned}$$

$$- \int_{\bar{\eta}_1}^{\bar{\eta}_2} \int_{\eta_1}^{\eta_2} \left\{\Phi(\bar{\xi}_1,\eta,t) - \Phi(\bar{\xi}_2,\eta,t)\right\}$$

$$\times \left\{\Phi(\xi_1',\eta',t) - \Phi(\xi_2',\eta',t)\right\} d\eta d\eta' \qquad (77)$$

We expand the products under the integral sign in equation (77) and take the ensemble average of both sides. The 16 terms resulting on the right hand side are recognized as phase covariance functions [13]. Using the procedure we have derived in the first section we replace these by appropriate phase structure functions using the relation

$$2C_\Phi = D_\Phi(\infty) - D_\Phi(r) \qquad (78)$$

or

$$D_\Phi(r) = 2C_\Phi(0) - 2C_\Phi(r) \qquad (79)$$

Performing these operations and substituting from equation (75) we obtain for the mean square of the relative tracking error

$$\langle\theta_r{}^2\rangle = \left(\frac{\lambda}{2\pi A}\right)^2 \int_{\eta_1}^{\eta_2} \int_{\eta_1}^{\eta_2} D_\Phi\left\{\sqrt{(\xi_1 - \xi_2')^2 + (\eta-\eta')^2}\right\}$$

$$-D_\Phi\left\{\sqrt{(\xi_1 - \xi_1')^2 + (\eta-\eta')^2}\right\} d\eta d\eta'$$

$$+ \int_{\bar{\eta}_1}^{\bar{\eta}_2} \int_{\bar{\eta}_1}^{\bar{\eta}_2} D_\Phi\left[\sqrt{(\bar{\xi}_1 - \bar{\xi}_2')^2 + (\eta-\eta')^2}\right]$$

$$+D_\Phi\left[\sqrt{(\bar{\xi}_1 - \bar{\xi}_1')^2 + (\eta-\eta')^2}\right] \quad d\eta d\eta'$$

$$- \int_{\eta_1}^{\eta_2} \int_{\bar{\eta}_1}^{\bar{\eta}_2} D_\Phi\left[\sqrt{(\xi_1 - \bar{\xi}_2')^2 + (\eta-\eta')^2}\right]$$

$$+D_\Phi\left[\sqrt{(\xi_1 - \bar{\xi}_1')^2 + (\eta-\eta')^2}\right] \quad d\eta d\eta'$$

$$- \int_{\bar{\eta}}^{\bar{\eta}_2} \int_{\eta_1}^{\eta} D_\Phi\left[\sqrt{(\bar{\xi}_1 - \xi_1')^2 + (\eta-\eta')^2}\right] \tag{80}$$

$$+D_\Phi\left[\sqrt{(\bar{\xi}_1 - \xi_1')^2 + (\eta-\eta')^2}\right] \quad d\eta d\eta'$$

As before we take the structure function as

$$D_\phi(r) = 6.88 \left|\frac{r}{r_o}\right|^{\frac{5}{3}} \tag{81}$$

where r_o is the wave correlation distance. Combining equations 75, 80 and 81 and making a change of variables in each integral of the form

$$x = \frac{1}{R}\ (\eta \pm \frac{L}{2}) \tag{82a}$$

$$y = \frac{1}{R}\ (\eta \pm \frac{L}{2}) \tag{82b}$$

we have

$$\langle\theta_r^2\rangle = \left(\frac{\lambda}{2\pi A}\right)^2 \frac{6.88\ R^{11/3}}{r_o^{5/3}}$$

$$\times\ 2 \int_{-1}^{1} \int_{+1}^{1} \left\{ \left[\left(\sqrt{1-x^2} + \sqrt{1-y^2} \right)^2 + (x-y)^2 \right]^{5/6} \right.$$

$$- \left[\left(\sqrt{1-x^2} - \sqrt{1-y^2} \right)^2 + (x-y)^2 \right]^{5/6}$$

$$- \left[\left(\sqrt{1-x^2} + \sqrt{1-y^2} \right)^2 + (x-y+\tfrac{L}{R})^2 \right]^{5/6}$$

$$\left. + \left[\left(\sqrt{1-x^2} - \sqrt{1-y^2} \right)^2 + (x-y+\tfrac{L}{R})^2 \right]^{5/6} \right\} dydx \qquad (83)$$

Here we have taken the signs in equations (82a) and (82b) as (+,+) respectively in the 1st integral, (-,-) in the second, (-,+) in the third and (+,-) in the last.

Equation (83) may be put into the form

$$\langle\theta_r^2\rangle = 2\left[\frac{\lambda}{2\pi A}\right]^2 \frac{6.88\ R^{11/3}}{r_o^{5/3}} \left[I_p^+(0) - I_p^-(0) - I_p^+(L/R) + I_p^-(L/R) \right] \qquad (84)$$

where

$$I_p^{\pm}(L/R) = \int_{-1}^{+1} \int_{-1}^{+1} \left[\left(\sqrt{1-x^2} \pm \sqrt{1-y^2} \right)^2 + (x-y+L/R)^2 \right]^{5/6} dxdy \qquad (85)$$

Comparing this to our previous expression for the tracking error due to a single circular aperture, $\langle\theta_x^2\rangle$, we see that the first two terms in equation (84) are just $2\langle\theta_x^2\rangle$, while the last two terms are the reduction in error obtained by using a two aperture system, i.e.,

$$<\theta_r^2> = 2<\theta_x^2> - 2\left[\frac{\lambda}{2\pi A}\right]^2 \frac{6.88\ R^{11/3}}{r_o^{5/3}} \left[I_p^+(L/R) - I_p^-(L/R)\right] \tag{86}$$

Examination of equation (84) shows that for L equal to zero (the two systems sharing a single aperture), $<\theta_r^2>$ is zero. This is clearly necessary since for this case (if it were physically realizable) $\theta_1(t)$ is identical to $\theta_2(t)$ for all t. For very large L, $I_p^+(L/R) = I_p^-(L/R)$, and equation (86) reduces to

$$<\theta_r^2> = 2<\theta_x^2> \tag{87}$$

or

$$\theta_{rms}\ \text{(Double Aperture)} = \sqrt{2}\ \theta_{rms}\ \text{(Single Aperture)} \tag{88}$$

We next turn our attention to the component of angular tracking error in the ξ direction (i.e., at right angles to the line of centers of the two apertures). An analysis similar to that performed above leads to an equivalent expression for this component, i.e.,

$$<\theta_r^2> = 2\left[\frac{\lambda}{2\pi A}\right]^2 \frac{6.88\ R^{11/3}}{r_o^{5/3}} \left[I_s^+(0) - I_s^-(0) - I_s^+(L/R) + I_s^-(L/R)\right] \tag{89}$$

where $I^\pm(L/R)$ is now given by

$$I_s^\pm(L/R) = \int_{-1}^{+1}\int_{-1}^{+1} \left[\left(\sqrt{1-x^2} \pm \sqrt{1-y^2} + \frac{L}{R}\right)^2 + (x-y)^2\right]^{5/6} dydx \tag{90}$$

As before this reduces to zero for L equal to zero and approaches the square root of twice the RMS error of a single aperture for large L.

From equation (84) (or (89)) we may find the ratio of the relative tracking error between two apertures to the RMS error in a single aperture system of the same diameter, i.e.,

$$\frac{\sqrt{<\theta_r^2>}}{\sqrt{<\theta_x^2>}} = 2\left[\frac{I^+(0) - I^-(0) - I^+(L/R) + I^-(L/R)}{I^+(0) - I^-(0)}\right]^{1/2} \tag{91}$$

where $I^{\pm}(L/R)$ denotes either $I_p(L/R)$ or $I_s(L/R)$ (given by equation (85) or equation (90)) depending on whether the angle being measured is along the line of center or at right angles to it.

The quantities $I^{\pm}(L/R)$ have been evaluated on the IBM 360 computer using a 201 point Simpson's Rule integration for values of L/R from 2 to 500. The results are given in Table I and Figure 4.

We have defined the average tracking error $\theta_a(t)$ as

$$\theta_a(t) = \frac{\theta_1(t) + \theta_2(t)}{2} \tag{92}$$

The RMS value of this quantity is easily found by a straightforward modification of the previous calculation.

$$\frac{\sqrt{<\theta_A^2>}}{\sqrt{<\theta_x^2>}} = \frac{1}{2}\left[\frac{I^+(0) - I^-(0) + I^+(L/R) - I^-(L/R)}{I^+(0) - I^-(0)}\right]^{1/2} \tag{93}$$

This quantity has also been evaluated as a function of (L/R), the results being given in Table I and plotted in Figure 5.

Discussion of Results

A double tracking system such as has been described might operate by taking the average of the output of the two trackers as the actual position of the target. From Table I it can be seen that for a separation of 10 radii between the centers of the two apertures, the RMS tracking error $<\theta_A^2>^{1/2}$ is approximately 86% of the error for a single aperture and for a separation of 100 radii the error is 78%. Since the RMS tracking error for a single aperture varies as the reciprocal one-sixth power of the diameter we see that for 10 radii separation the error will be the same as that in a single system whose aperture is 2.5 times larger. That is a double system with two 20 cm. diameter receivers located 1 meter apart will have no more error than a single receiver with a 50 cm. diameter objective. Likewise if the receivers were 10 meters apart the system should be subject to tracking errors approximately the same as an 88 cm. diameter single receiver. Whether or not the gain in accuracy in a double receiver system would justify the additional complexity will probably depend on the requirements of the particular system in question.

Experimental Verification of the Theory

One of the major problems encountered in the experimental investigations of atmospheric effects is that the statistics of the atmosphere are non-stationary. Thus we are always attempting to gather statistical data on a system whose statistical properties are constantly changing. For example, if one attempts to measure the angular fluctuations of a beam propagating through the atmosphere as a function of receiver aperture size he finds it difficult to collect enough data to be

TABLE I

L/R	AVERAGE ANGULAR FLUCTUATION		RELATIVE ANGULAR FLUCTUATION	
	PARALLEL	PERPENDICULAR	PARALLEL	PERPENDICULAR
2	0.9553	0.8980	0.5912	0.8799
3	0.9316	0.8701	0.7268	0.9858
4	0.9147	0.8546	0.8084	1.0385
5	0.9019	0.8441	0.8637	1.0723
6	0.8919	0.8362	0.9045	1.0967
7	0.8839	0.8300	0.9352	1.1155
8	0.8769	0.8249	0.9613	1.1305
9	0.8710	0.8206	0.9824	1.1430
10	0.8659	0.8169	1.0003	1.1536
50	0.8039		1.1894	
100	0.7849		1.2392	
500	0.7534	0.7384	1.3151	1.3487
999	0.7441		1.3361	

statistically significant in a short enough time to insure that the atmospheric donditions have not changed during the course of the experiment. The alternate approach of collecting data over a long period of time, say many months, and then assuming that the average results represent some sort of hypothetical "average atmosphere" is not only time consuming and expensive but also is unsatisfying since the variation in atmospheric

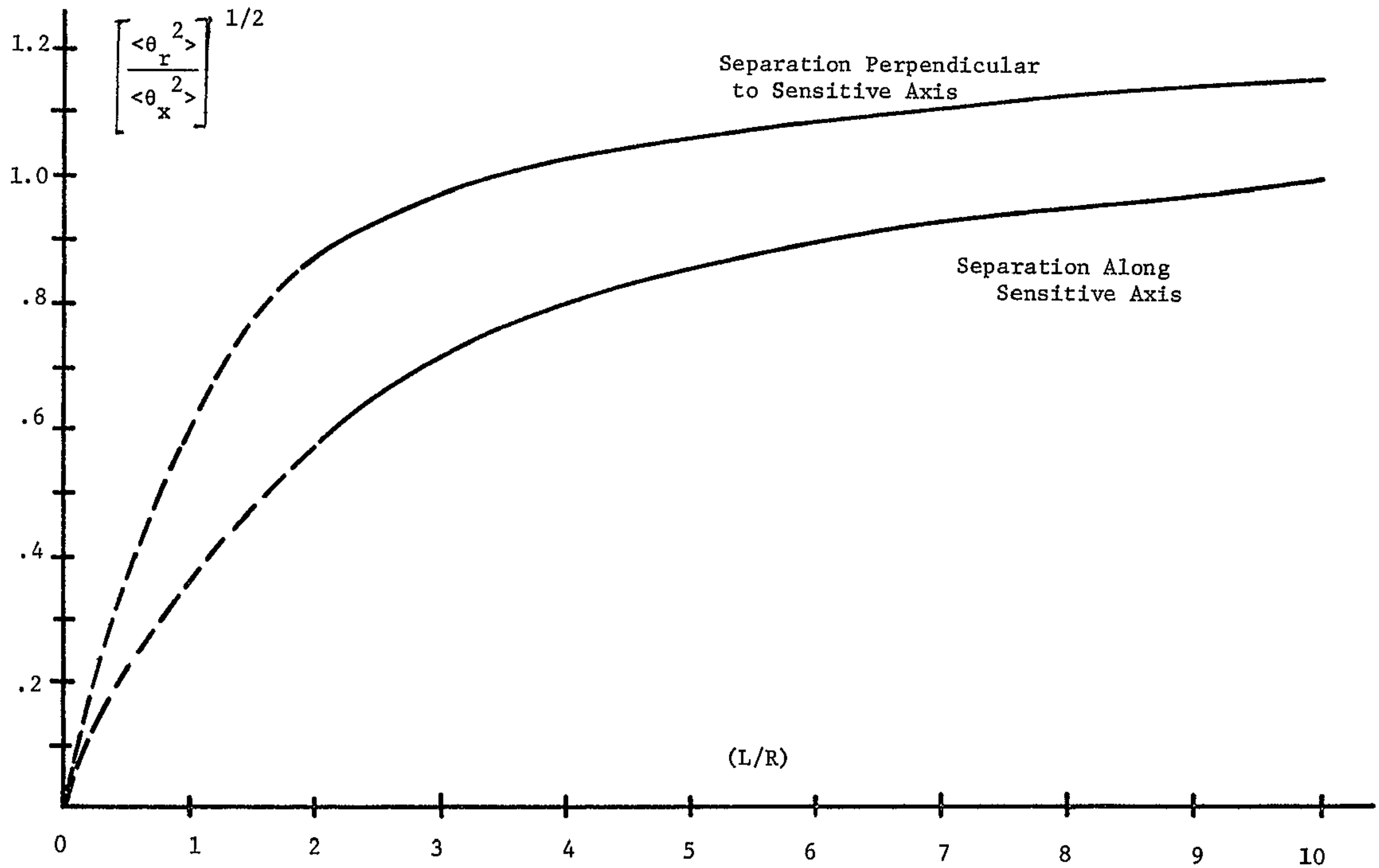


Figure 4. Ratio of Relative Angular Fluctuations to Fluctuations in Single Aperture

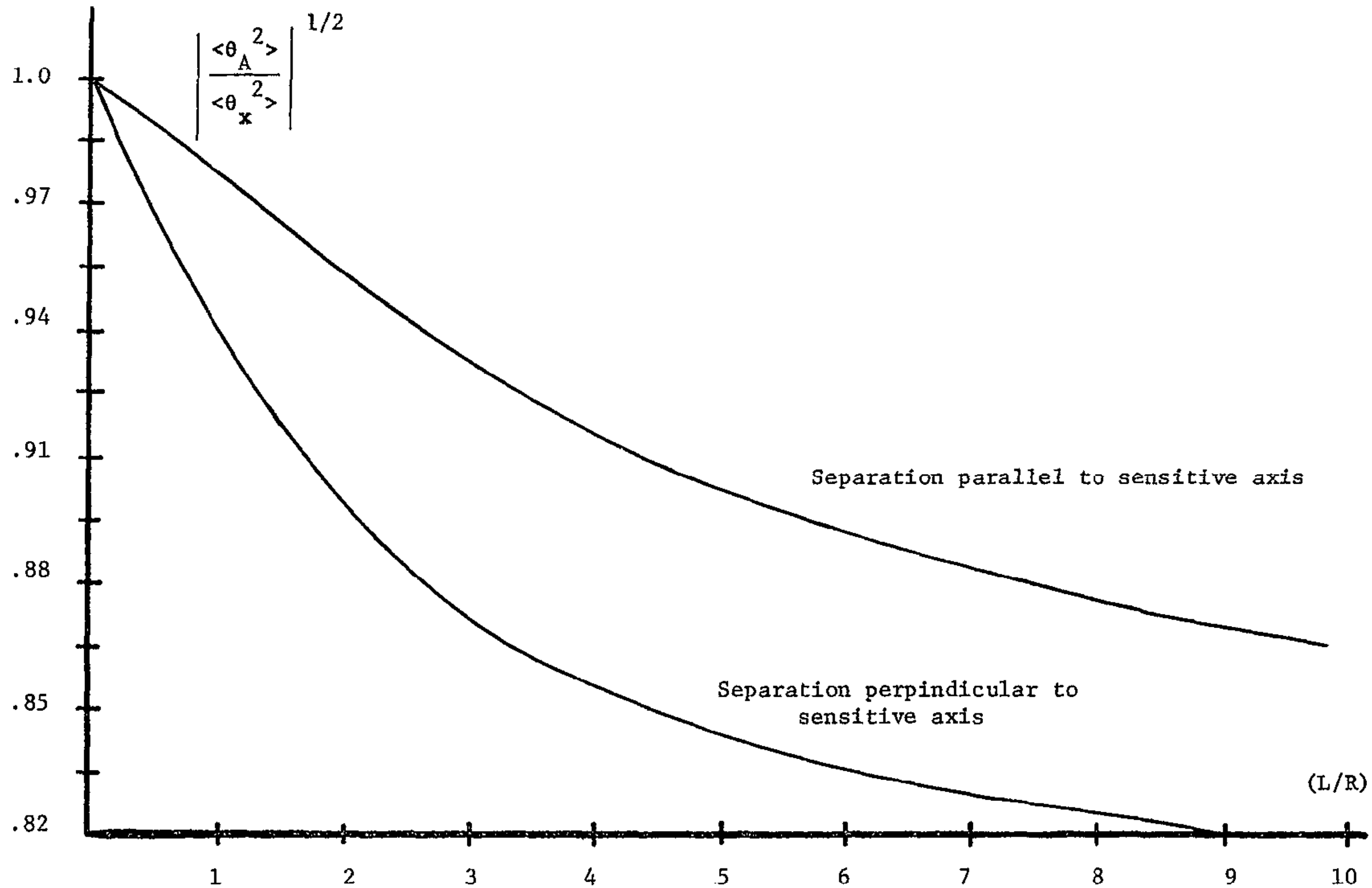


Figure 5. Ratio of Average Angular Fluctuations to Fluctuations in A Single Aperture

turbulence from day to day may be as large as or larger than the effect which one wishes to observe. Furthermore one is also tempted to discard data which does not agree with the theory on the grounds that "the atmosphere must have changed during the run." The validity of data subject to such subjective interpretation must always be suspected. One would like an experiment which would confirm the essential validity of the theoretical model of the atmosphere yet not be dependent upon the strength of the turbulence at a particular time.

We believe that a two aperture tracking system would provide such an experimental opportunity. The scale of turbulence and the turbulence strength enter into our calculations only through the single parameter r_o, the correlation distance. This parameter has been eliminated from equations (91) and (93). Thus an experiment in which the RMS relative and/or average angular fluctuations were measured simultaneously with the individual angular fluctuations in each tracker should provide a sensitive test of the theory independent of the value of r_o. Any inaccuracy in the theoretical model of the atmosphere, such as a departure of the atmospheric fluctuations from a log-normal distribution, should be easily demonstrated.

GAUSSIAN DISTRIBUTION OF BEAM INTENSITY

We have also considered the effects of a Gaussian distribution of intensity of the laser beam on the apparent angular fluctuation. A distribution of intensity across the beam of the form

$$I = I_o e^{-\sigma r^2} \tag{94}$$

was assumed. From equation (94) we have that

$$\theta_x(t) = \frac{\lambda}{2\pi} \frac{\iint_\Sigma |u(\xi.\eta)|^2 \frac{\partial\Phi}{\partial\xi}\, d\xi\, d\eta}{\iint_\Sigma |u(\xi.\eta)|^2\, d\xi\, d\eta} \tag{95}$$

which yields

$$\theta_x(t) = \frac{\lambda}{2\pi} \frac{\iint_\Sigma e^{-\sigma(\xi^2+\eta^2)} \frac{\partial\Phi}{\partial\xi}\, d\xi\, d\eta}{\iint_\Sigma e^{-\sigma(\xi^2+\eta^2)}\, d\xi\, d\eta} \tag{96}$$

where σ is a parameter which describes the beam width and the integration extends over the receiving aperture. The integral in the numerator of equation (96) may be integrated by parts on ξ to eliminate the derivative $\partial\Phi/\partial\xi$. The resulting expression is then squared and the ensemble average taken in the usual way. After more manipulation the resulting expression can be recognized as phase structure functions. Unfortunately this expression contains 16 terms each of which is a multiple integral of the product of D_ϕ and an exponential function of the coordinates squared. Sufficient computer time has not been available to evaluate these quite complex expressions.

EFFECTS OF ATMOSPHERIC TURBULENCE ON A CO_2 LASER COMMUNICATIONS SYSTEM

Introduction

The second phase of the research conducted during the course of this project involves the reduction and analysis of certain government supplied data concerning the effects of atmospheric turbulence on a CO_2 heterodyne communications system. This data was collected during the

Summer of 1969 at Marshall Space Flight Center and was transmitted to the University of Alabama for analysis in September, 1969. We have generated the necessary computer software to reduce the data and have begun to analyze it. Due to the large amount of data and the limited time available this analysis is not yet complete. We expect, however, to be able to complete the analysis in the near future. In this report the problems encountered in analyzing the data are discussed; the computer programs which have been written are described; and preliminary results are given.

Experimental

The atmospheric experiments were performed by the project director in association with MSFC personnel at MSFC and are therefore not a part of this project. We will therefore not attempt a complete discussion of the experimental procedures in this report but will give only a brief description necessary to describe the data which was analyzed. A complete discussion of the experiment is being prepared by the project director and MSFC personnel and will be published in the near future as a NASA Technical Report.

The subject data consist of measurements of the scintillation and heterodyne signal of a CO_2 laser beam propagated over a 3.2 Km path between Madkin Mountain and the Astrionics Laboratory building, both located on Redstone Arsenal, Alabama. The laser transmitter was a 2 watt, stabalized CO_2 laser with a 10 cm., off-axis Cassegrainian collimator. The laser could be frequency modulated by driving one of the cavity mirrors mounted on a piezoelectric pusher. The receiver consisted of a 10 cm. aperture off-axis Cassegrainian telescope, a local oscillator laser, combining optics, and a Hg-CdTe detector. The receiver was fitted with removable aperture stops so that its aperture could be varied from 2 cm. to 10 cm. The transmitter and receiver were constructed for MSFC by the Minneapolis-Honeywell Corporation and have been described in the literature [14].

Signal-to-noise measurements were made by extracting the 10 MHz beat note between the received signal and local oscillator after it had passed through one stage of IF amplifications. The signal was then detected with a simple diode circuit and the resulting voltage recorded on magnetic tape.

Scintillation measurements were made by turning off the local oscillator laser and chopping the transmitted beam at 90 Hz by means of a mechanical chopper located in the transmitter. The output of the detector was amplified and recorded directly on a 14 channel magnetic tape. Thus the recorded signal consisted of an amplitude-modulated square wave whose amplitude was proportional to the instantaneous power being received. In this way fluctuations in background illumination were eliminated.

Twelve channels of each analog magnetic tape were used for data, the other two channels being reserved for identification and timing purposes. The tapes were digitized at a sampling rate of 1 KHz in order to reduce the time required for A/D conversion. The resulting digital tapes were recorded in a multiplex format with five channels of the analog tape on each digital tape.

Between June 15 and August 31, 1969, approximately 750 observations of scintillation were made at all hours of the day and under as wide a variety of weather conditions as possible. Each observation consisted of about 90 seconds of recorded data from which a 60 second segment near the middle of each run was selected for digitization. The time of day, temperature, humidity, wind speed and general weather conditions were recorded for each observation.

Data Reduction

In order to reduce the data a program has been written for the IBM 360 Model 50 computer. The principal problems which were encountered in writing this program concerned formating the data for the computer and extracting the amplitude of the square wave. The latter proved to be somewhat difficult since the sampling rate during digitization could not be accurately synchronized with the period of the square wave. The sampling rate of 1 KHz and the chopping rate of 90 Hz should yield approximately 10 samples per cycle of the square wave. In actuality it was found that the number of samples per cycle varied between 10 and 12 due to the sampling rate not being an integral multiple of the square wave frequency. It was therefore necessary to design a program which would determine whether a particular data point was a base point (i.e., from the part of the square wave when the laser beam was blocked by the chopper) or a signal point (when power was being received from the laser beam). The problem was further compounded by the fact that the rise and fall times of the square wave were non-negligible so that about one percent of the data points were sampled during the switching transient and should therefore be discarded. Furthermore it was found that some of the data contained an occasional noise spike which should be eliminated. It was decided that the elimination of these spikes would not adversely effect the validity of the analysis so provisions for eliminating them were also included in the program.

The program which we have written to divide the data into base and signal points operates basically as follows. One record, containing 2000 characters, is read from magnetic tape. These 2000 characters represent 200 sample values from each of 5 data channels. Each sample value is a 10 bit binary number plus sign bit occupying two tape characters. The 400 characters corresponding to the channel being analyzed are converted to internal floating point format and are stored in an array. A second record is read from tape, converted, and stored in a second array. To begin the analysis twenty data points from the first of the array are selected and the maximum and minimum are found. Two limits, L_1 and L_2, are then set by the relations

$$L_1 = A_{max} - P_1(A_{max} - A_{min}) \tag{97}$$

$$L_2 = A_{min} + P_2(A_{max} - A_{min}) \tag{98}$$

where A_{max} and A_{min} are the maximum and minimum of the first twenty points and P_1 and P_2 are constants between zero and one-half. Since the signal was inverted when it was recorded on analog tape, the base line is greater than the signal, hence a particular point greater than L_1 is considered a base point, if it is less than L_2 it is considered a signal point. Points lying between L_1 and L_2 are assumed to be from the transient portion of the wave form and is neglected.

The computer is programmed to take each point successively and determine if it is a base point, a signal point or neither. As a preliminary to processing the first twenty points are scanned and the beginning of a base line segment of the wave form is found. Then new limits are set on the next 15 points and they are scanned and grouped into 3 arrays, a base line segment, a signal segment and a second base line segment. Each array may contain up to 7 points. At this time the amplitude of the square wave is computed for the group of signal points (as will be described later) and stored. The second group of base points is transferred into the first array, new limits are set using the next 10 data points, a new group of signal points and base points are found to fill the second and third arrays, and finally their amplitudes are computed. This process is continued until the 200 points from the first record have been used. At this time the 200 points from the second tape record are transferred into the array formally occupied by the first record and a third record is read from tape. Processing then continues until all points in the data set have been processed.

Data Processing Irregularities

As previously mentioned, several irregularities in the data are possible and a number of checks have been built into the program to provide for them. These checks are as follows:

1) During the search for either base points or signal points more than 10 consective points are found.

2) After completing a search less than 3 base or signal points have been found.

3) More than 3 consective points satisfying neither the base or signal point criteria are found.

Any of these three conditions indicate that the waveform is departing drastically from a modulated square wave and appropriate action should be taken. For conditions one and three the program skips 10 data points, or approximately one cycle of the square wave, and begins processing again. For condition two the computation of amplitudes are surpressed and processing continues. As a further check, if the total number of errors in any record exceeds five the entire record is omitted.

During processing a record is kept of each time an irregularity was encountered and this information is printed in tabular form at the completion of processing. Clearly if an excessive number of irregularities occurs in a given run the results of that run must be suspect.

Computation of Amplitudes

After each cycle of the waveform has been processed to divide the data points into base points and signal points the amplitude is computed. Three methods for computing the amplitude have been tried. The first method took the base line for a group of signal points as the average of all the points in the group of base points preceding it and the one following it. That is, the background during the half-cycle in which the laser beam intensity was recorded is taken as the average background recorded during the half-cycle immediately preceding the signal and the half cycle immediately following it. This average base line was subtracted from each signal point and the difference taken as the amplitude of the laser beam at that instance.

The second type of amplitude calculation considered was to reconstruct the base line during the period when the signal was recorded by fitting a least-mean-square curve to the base line points on either side. This method was found to give very erratic results and was abandoned.

The third method consisted of taking the difference in the first signal point in a group and the last base point preceding it as an amplitude. The difference in the last signal point in the group and the first base point following it gives a second amplitude. This method gives only two amplitudes per cycle but has the advantage that they are evenly spaced.

The final computer program contained both the first and third type amplitude calculation, the one to be used being selected by a parameter read during execution. An option is also provided which will either store all amplitudes along with the time which the amplitude occurred or will, instead of storing the amplitude, count the number of times an amplitude lying in a given range occurs. The former yields received beam intensity as a function of time while the latter gives the probability distribution function for the intensity fluctuations. Either is saved for whatever analysis one wishes to perform on the data.

Program Checkout and Adjustment of Parameters

In order to test the program a segment of data was printed from the magnetic tape and was inspected. Each data point was classified as either a signal point, a base point, or a bad point from the transient portion of the waveform. This classification was purely subjective, yet in inspecting the data there was usually no questions as to how a particular point should be categorized. The same data was then fed into the computer. Print statements were added to the program to list each point and indicate how the computer classified it. The program was run several times varying the parameters P_1 and P_2 (equations (97) and (98)) between runs and the results compared with the subjective analysis. Data having "bad places" in it was also processed and the results compared with our judgement as to whether or not a segment should be omitted. On the basis of these comparisons the parameters P_1 and P_2 were set at 0.05 and 0.10, respectively. That is a point within 5% of the maximum base point or 10% of the minimum signal point would be retained while points between these limits were discarded. These limits seemed to allow the computer to make very nearly the same decisions as we would have made had we analyzed the data by hand.

It is felt that due to the statistical nature of the analysis whether or not a few points are discarded as bad when they should have been retained will not appreciable effect the results. Preliminary analysis of the data seems to confirm that the results are not too sensitive to small changes in the values of the limits.

Calculations of the Scintillation Statistics

The final segment of the data analysis program accetps the probability density function for the intensity fluctuations which has been generated and computes the scintillation statistics. The program computes and lists the class mark for the intensity and the corresponding value of the log-amplitude defined by

$$\ell_i = \frac{1}{2}(\ln)\ I_i/\overline{I} \qquad (99)$$

where ℓ_i and I_i are the log-amplitude and the intensity for the ith class interval, and $\overline{I}$ is the mean intensity. The frequency for each class and the cumulative probability are also listed.

It has been customary in the literature to test the hypothesis of log-normality of scintillation data by plotting the cumulative probability function of the log-amplitudes against a "probability scale" such that if the data is log-normal the resulting curve will be a straight line. Not only is this procedure not a very sensitive test of a statistical distribution but it is also very time consuming when a large quantity of data is to be processed. We have therefore included in the program a chi square test on both a normal and a log-normal distribution function. These tests provide a quick and sensitive means of testing the hypothesis of log-normal scintillation.

The statistical analysis routine also computes the mean, standard deviation, skewness, and kurtosis for both the intensity distribution and the log-amplitude distribution. From the standard deviation of the log amplitudes the atmospheric structure constant will be computed. The skewness and kurtosis are computed to give an additional check on log-normality.

Description of Program

A listing of the computer programs described above along with a sample output are included in Appendix A, and a sample output is shown in Appendix B. For reference, Appendix C lists a program for generating log-normal random numbers which was used in checking the chi square test subroutines.

Atmospheric Structure Constant

The log amplitude variance $C_\ell(0)$ for a plane wave is given in terms of the atmospheric structure constant C_n by [15].

$$C_\ell(0) = 0.309\ k^{7/6}\ Z^{11/6}\ C_n^{\ 2} \tag{100}$$

where k is the wave number and Z the length of the path through the atmosphere. For a spherical wave the corresponding expression is [16]

$$C_\ell(0) = 0.124\ k^{7/6}\ Z^{11/6}\ C_n^{\ 2} \tag{101}$$

In our preliminary analysis we have neglected the finite aperture of the receiver and treated it as a point source. Equations (100) or (101) can then be used directly to obtain $C_n^{\ 2}$ by noting that the standard deviation of the log-amplitude distribution which we have computed is the square root of $C_\ell(0)$. It is well known however that a finite receiving aperture has the effect of averaging the intensity fluctuation from various parts of the wave front thereby reducing the variance of the scintillation. This effect has been investigated by Fried [17]. From Figure 2 in Fried's paper we note that for large scintillation conditions and for the path lengths and apertures used in the experiment, this effect will be significant.

In order to allow for aperture averaging we may use the expressions given by Fried [14,16], viz.

$$\sigma_s^{\ 2} = \left[\frac{\pi}{4}\, D^2\right]^2 \Theta \quad C_I(0) \tag{102}$$

where $\sigma_s^{\ 2}$ is the signal variance which corresponds to the square of the standard deviation of the intensity fluctuation, D is the diameter of the receiving aperture, and Θ is an aperture averaging factor given by

$$\Theta = \frac{16}{\pi D^2} \int_0^D \rho\, d\rho \, \frac{\exp\,[4C_\ell(\rho)]-1}{\exp\,[4C_\ell(0)]-1} \cdot H(\rho/D) \tag{103}$$

$H(\rho/D)$ is the optical transfer function of a circular aperture.

$$H(\rho/D) = \cos^{-1}(\rho/D) - (\rho/D)\,[1 - (\rho/D)^2]^{1/2} \tag{104}$$

and $C_\ell(\rho)$ is the log-amplitude co-variance given by

$$C_\ell(\rho) = C_\ell(0) \sum_{n=0}^{\infty} \left[a_n + b_n \left(\frac{k\rho^2}{4z}\right)\right]$$

$$\times \left[\left(\frac{k\rho^2}{4z}\right) /(2n)!\right] - 7.53034\left(\frac{k\rho^2}{4z}\right)^{5/6} \tag{105}$$

In the last expression a_n and b_n are the expansion coefficients for the modified confluent hypergeometric function ${}_1F_1(-\frac{11}{6};\ \underline{1};\ ix)$ and are given by

$$a_o = 1 \qquad\qquad b_o = 6.84209 \tag{106}$$

and the recursion relations

$$a_n = -a_{n-1}\left[(2n - 23/6)(2n - 17/6)/(2n-1)(2n)\right] \tag{107a}$$

$$b_n = -b_{n-1}\left[(2n - 17/6)(2n - 11/6)(2n-1)/(2n)(2n+1)^2\right] \tag{107b}$$

With the additional relation that the intensity variance $C_I(0)$ in equation (102) is related to the log variance by

$$C_I(0) = I_o^2 \left[\exp[4C_\ell(0)] - 1\right] \tag{108}$$

equations (102 - 108) specify $C_\ell(0)$ in terms of σ_s^2 and I_o^2. Since I_o and σ_s are just the mean and standard deviation of the received intensity we may compute $C_\ell(0)$ and then using equation (101) find the atmospheric structure constant.

Combining the above equations we have

$$\left(\frac{\sigma_s}{I_o}\right)^2 = \pi D^2 \left[\exp\left[4C_\ell(0)\right] - 1\right]$$

$$\times \int_o^D \rho\, d\rho\, \frac{\exp[4f\,\frac{k\rho}{4z}\cdot C_\ell(0)] - 1}{\exp[4\,C_\ell(0)] - 1}\, \acute{H}(\rho D) \tag{109}$$

where $f(k\rho/4z)$ is the summation given in equation (105).

Since σ_s/I_o is an experimentally determined constant, equation (109) is an integral equation for $C_\ell(0)$. We have programmed the IBM 360 computer to solve equation (109). The technique used is to evaluate the integral in equation (109) for a number of trial values of $C_\ell(0)$ using a fourth order Runga-Kutta integration. This gives a table of σ_s/I_o as a function of $C_\ell(0)$. From this table the value of $C_\ell(0)$ corresponding to the measured value of σ_s/I_o is determined using Lagrange-Hermite interpolation formula.

Preliminary Results

At the present time approximately 60 of the 750 scintillation measurements have been processed. This includes computation of the cumulative probability curve, the moments of the intensity distribution, the moments of the log-amplitude distribution, and a chi square test for normal and log-normal distributions. Aperture averaging effects have not yet been considered nor has the power spectral density.

From the data thus far analyzed several definite trends are apparent. It must be emphasized, however, that these results are based on a cursory inspection of a small part of the data so that any conclusions drawn at this time must be considered as only tentative. We shall, however, discuss some of these results briefly.

1) Aperture Averaging. The scintillation data consist of groups of runs in which the receiver aperture was varied from 2 cm. to 10 cm. The runs within a particular group were made in as rapid succession as possible in order to minimize the probability that the atmospheric statistics would change between runs. It was hoped that in this way the effects of aperture averaging could be separated from the effects of nonstationary atmospheric statistics.

Preliminary analysis of the data has indicated definite aperture averaging effects. Thus far, however, insufficient data has been analyzed to fully verify the accuracy of the theoretical prediction of the aperture averaging effect found in the literature.

We have written computer programs with which to calculate the aperture averaging and to compute the log-amplitude variance allowing for these effects. These calculations will be completed in the near future.

2) Probability Distribution. Both Fried [18] and Patton [19] have reported departures from the log-normal distribution at 10.6 microns. Fried has attributed these results to instrumental errors in the measuring equipment. It has been, therefore, one of our primary objectives to either verify the log-normal distribution for the scintillation of a CO_2 laser beam or to offer definite evidence that the fluctuations are not log normal.

Our preliminary results seem to favor a log-normal distribution but unfortunately they are not conclusive. Out of the first 31 runs analyzed, 22 fit a log-normal distribution, 5 a normal distribution, and 4 fit neither. Much of the data analyzed has chi square values for a log-normal distribution that are well within the 95% confidence limits customarily used as a criterion for a fit. That is, the probability that random data would have fit a log-normal distribution as well is less than 5%. The cumulative probability distribution for two such runs is shown in Figures 6 and 7. Figure 6 clearly shows that the data fits a log-normal distribution better than a normal distribution. In Figure 7 the distinction is not as clear. In fact from the curves one would have difficulty in deciding between a normal and a log-normal model. The value of chi square for the log-normal model was 65 as compared to a chi square of 256 for the normal model, clearly indicating a better fit to the log-normal curve. In Figure 6, where the distinction is much greater, the chi square values were 90 and 1027 for the log-normal and normal models respectively.

Some of the data, such as shown in Figure 8, shows a much better fit to a normal distribution than to a log-normal one. Although firm conclusions cannot be drawn from the limited amount of data so far investigated, it seems that this occurs when a larger aperture is used. A possible explanation for a normal distribution could be that if the aperture is larger than the correlation distance then the intensity incident upon the detector sum of uncorrelated intensity fluctuations across the aperture. Involving the central limit theorem we may argue that if the aperture larger than the correlation distance the distribution function should be normal while for apertures small compared to the correlation distance it will be log-normal. For intermediate size aperture one would expect a distribution somewhere between normal and log-normal. It is hoped that the complete analysis of all the available data will clarify this point.

It has been found that an occasional run will have a distribution which departs radically from both the normal and the log-normal model. Often these runs are characterized by a large negative skewness. No explanation can be offered at this time.

3) Structure Constant. The atmospheric structure constant has been computed for a number of the runs, neglecting the finite aperture size. The values obtained are within the range of values expected from theory. These values will not be reported at this time since it is felt that they will be effected significantly by the aperture averaging effects.

Continuation of the Project

As has been emphasized above, the results which we are reporting at this time are based upon analysis of only a small part of the available data. A proposal has been submitted for extension of this project. It is expected that when the analysis is completed the available data will yield (1) verification of the distribution function, (2) accurate values for the atmospheric structure constant, (3) power spectral densities, (4) confirmation of the theoretical predictions of aperture averaging and (5) information concerning the variation of the structure constant and spectral density with time of day and with weather conditions.

The preliminary analysis has indicated that there is every reason to expect that much useful information may be extracted from the available data.

FIGURES 6 - 9

The cumulative probability distribution for the illuminance and the log amplitudes showing (Fig. 6) a typical small aperture run in which the scintillation is log-normal, (Fig. 7) a run in which the scintillation is intermediate between normal and log-normal, and (Fig. 8) a run in which it tends toward a normal distribution. Figure 9 shows an example of an occasional run in which the data departs radically from both the normal and log-normal distributions.

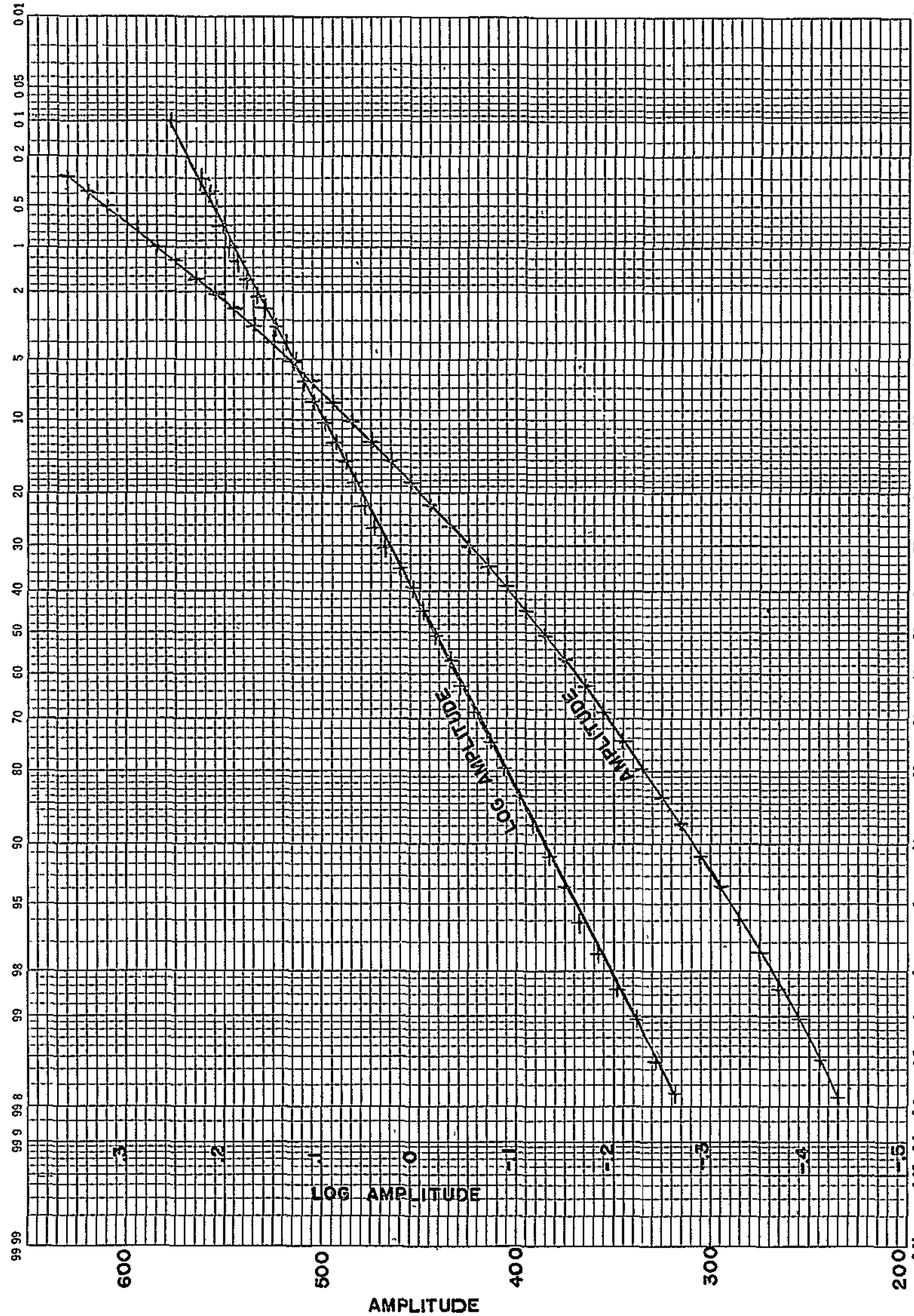
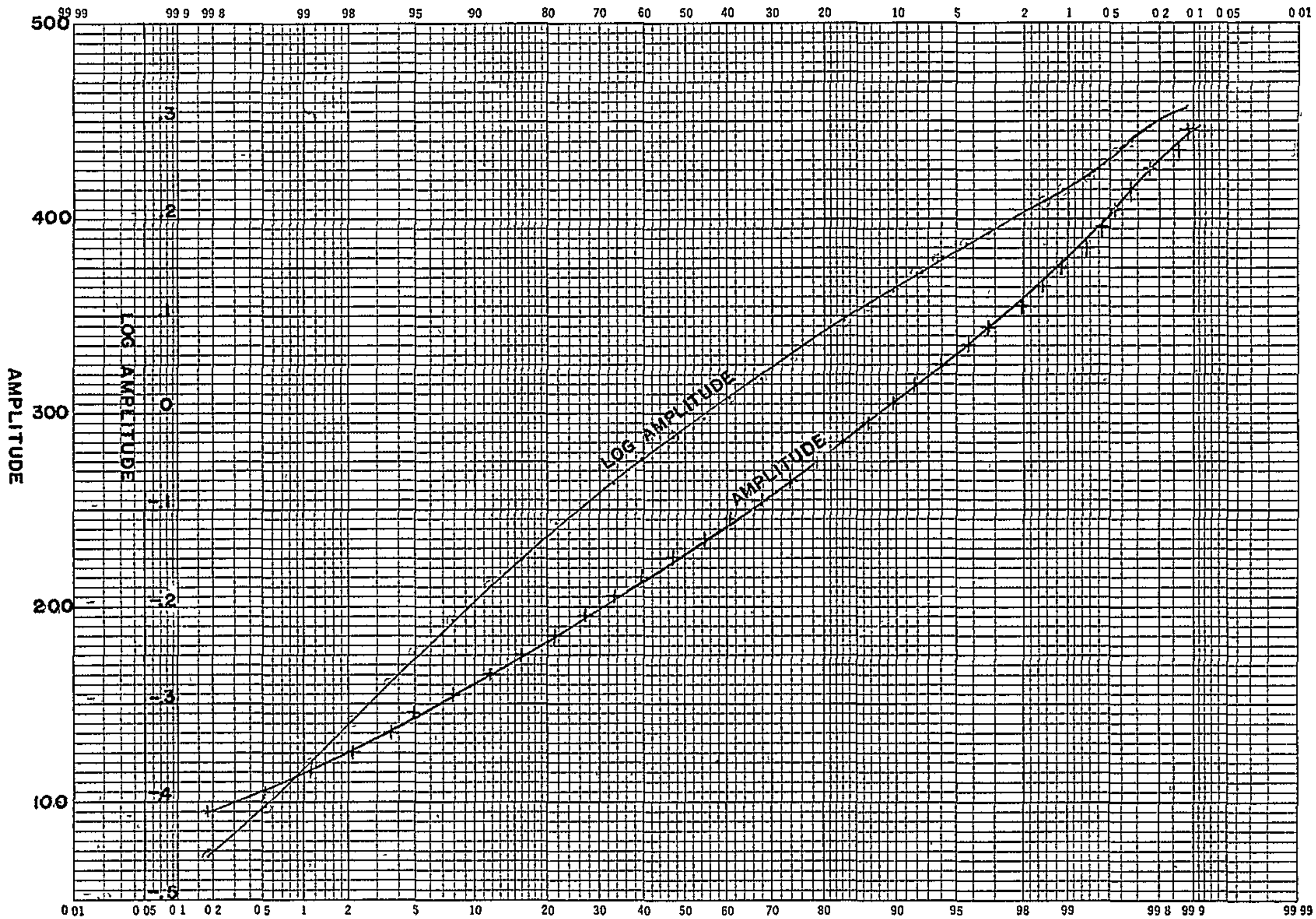


Figure 6



AMPLITUDE
LOG AMPLITUDE
500
400
300
200
10.0
.3
.2
.1
0
-.1
-.2
-.3
-.4
-.5
99 99
99 9
99 8
99
98
95
90
80
70
60
50
40
30
20
10
5
2
1
0 5
0 2
0 1
0 05
0 01
0 05
0 1
0 2
0 5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99 8
99 9
99 99
LOG AMPLITUDE
AMPLITUDE

Figure 7

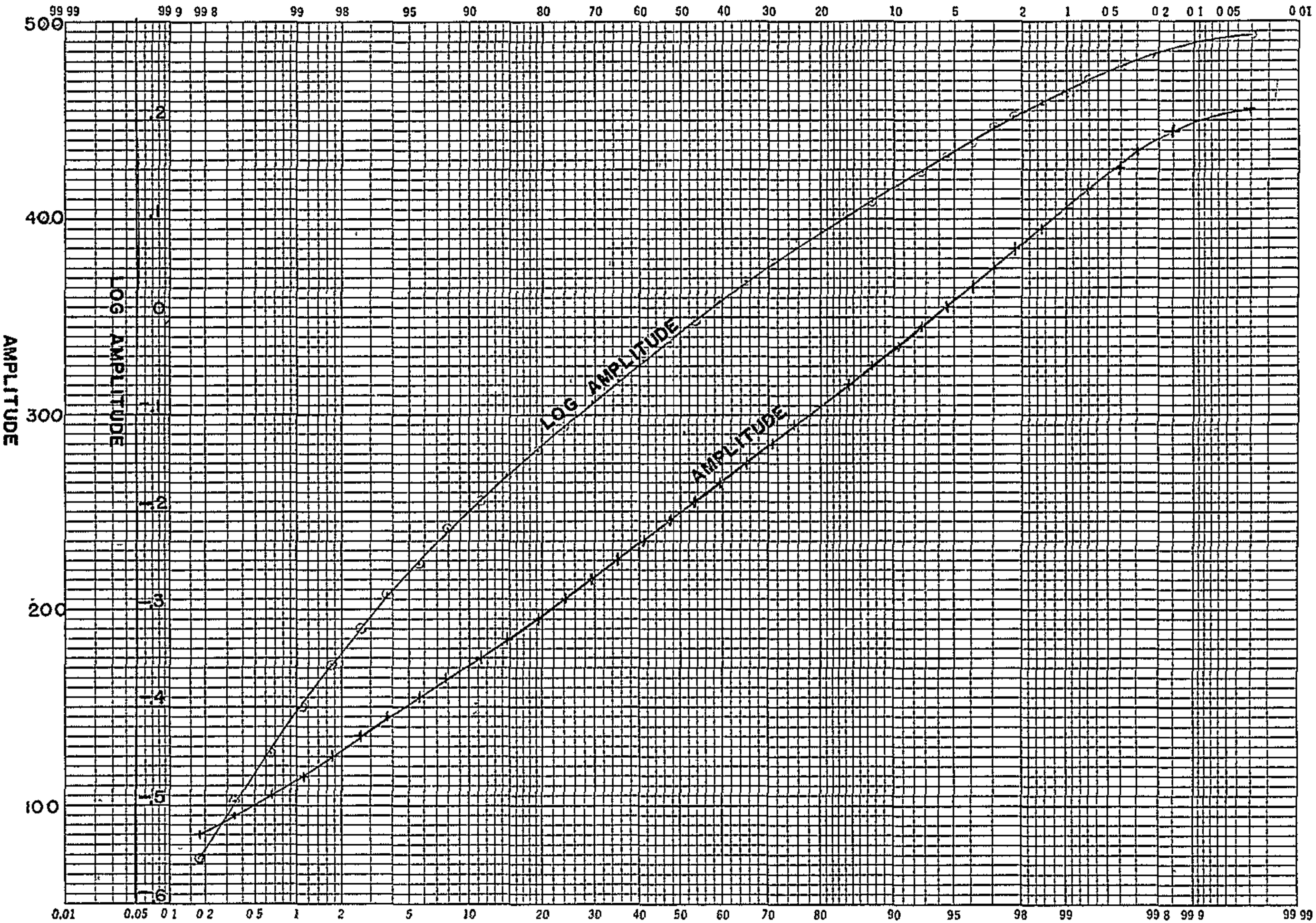

AMPLITUDE
LOG AMPLITUDE
500
400
300
200
100
.2
.1
0
-.1
-.2
-.3
-.4
-.5
-.6
LOG AMPLITUDE
AMPLITUDE
0.01
0.05
0 1
0 2
0 5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99 8
99 9
99 99

Figure 8

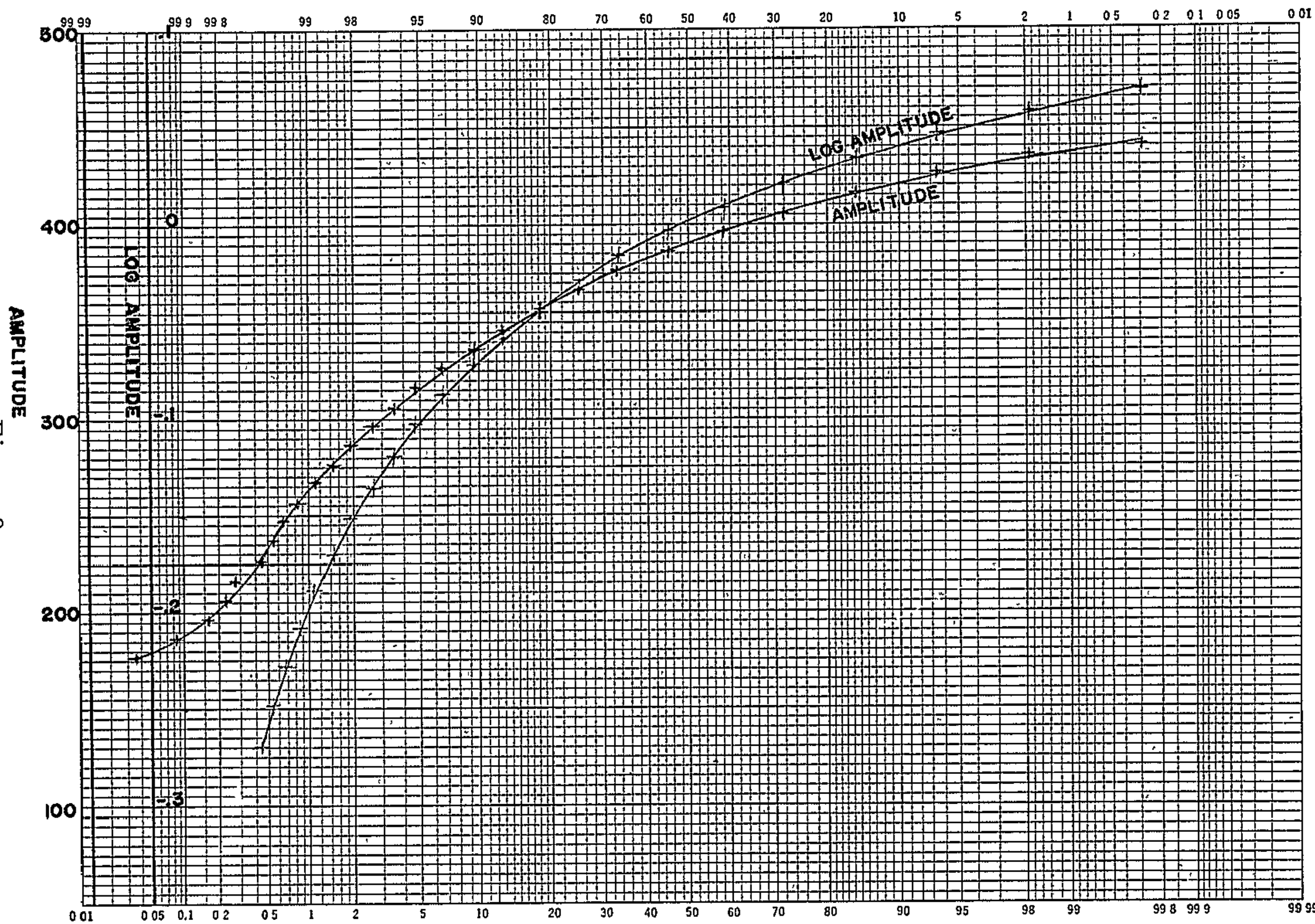

AMPLITUDE
LOG AMPLITUDE
LOG AMPLITUDE
AMPLITUDE
500
400
300
200
100
0
-.1
-.2
-.3
99 99 | 99 9 | 99 8 | 99 | 98 | 95 | 90 | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 10 | 5 | 2 | 1 | 0 5 | 0 2 | 0 1 | 0 05 | 0 01
0 01 | 0 05 | 0.1 | 0 2 | 0 5 | 1 | 2 | 5 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 95 | 98 | 99 | 99 8 | 99 9 | 99 99

Figure 9

(3) NCI - Specifies the number of class intervals to be used in the statistics routine.

(4) NOSCAN- Number of multiplexed variables contained in the record that belong to a given data set. (For example, 5 multiplexed variables, 200 points per data set).

(5) NOCHAN- Number of variables multiplexed

(6) TOL1 - Sets tolerance on base limits for base point selection.

(7) TOL2 - Sets tolerance on signal limits for signal point selection.

(8) TOL3 - Number of signal base, and amplitude points required for a cycle to be completed.

(9) L1 - Number of sequential searches for points allowed before cycle is aborted.

(10) L2 - Number of base points required for projected signal point search.

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APPENDIX A

This appendix further describes the computer program employed to reduce the atmospheric data. Included is a list of the FORTRAN statements of the program.

The program reads data from digital magnetic tapes with the use of three special FORTRAN subroutines, NTRAN, MOVE, TRNSL. These subroutines are used together to convert the data format on the tape to a workable format for FORTRAN processing.

The data is written on the tape in blocks called files. Each file contains 300 records of data plus an identification record at the first of each file. This record consists of 12 tape characters in a binary called decimal format. The 12 characters contain the following information; tape number in characters 1-4; scale factor in 5; type file in 6(0 = data file, 1 = calibration file); number of variables multiplexed in 7-8; number of scans in 9-12. The first 4 characters in each data record gives the time. The remaining 2000 binary characters in the record yield 1000 numbers in a multiplexed format, i.e., every fifth number belongs to the same data group. The program unpacks and stores the desired group in an array to be used in processing. The first file of each tape is a calibration file.

The program requires 10 control parameters to be read in as data at the first of each tape run. These are:

(1) IBTYP - Determines method to be used to calculate the signal points. IBTYP = 0 will have the program calculate amplitude points by taking the difference between the average of the base points of the cycle and the signal points. IBTYP = 2 will have the program to calculate 2 amplitude points by subtracting the first signal point from the base point preceding it and the last signal point from the base point following it. IBTYP = 3 will cause only 1 data point to be calculated by subtracting the first signal point from the base point preceding it.

(2) IFLAG - The value of this parameter determines the type chi square test to be used on the data. If IFLAG = 1, no test will be performed. If IFLAG = 2, the data will be compared to a normal distribution. IFLAG = 3 will call for a log normal test, and IFLAG = 4 will call for both log normal and normal tests to be performed.

```
0001            INTEGER FILES,FILCK
0002            COMMON P(1000),AUX(1000),INDO(10),NCNT(300),JOVF,JUNF,
               *BASE(2,10),SIG(10),IBLOC(2,10),KK,MM,XL90,XL10
0003            COMMON/AAA/IPRINT
0004            COMMON/RLC/IRTN
0005            COMMON/RAT/IRCEND
0006            COMMON/PR/IE(7,300)
0007            COMMON/UUU/NPNCH
0008            DIMENSION IBUF(1000)
0009            DIMENSION IA(3),IB(5)
0010            CALL RID(I1,I2,IB)
0011            IF(I1)155,150,155
0012        155 PRINT 157,FILCK
0013        157 FORMAT(' ***  END OF FILE  ***',I8)
0014        150 IF(I2)158,161,158
0015        158 PRINT 159
0016        159 FORMAT('  ***  READ ERROR  ***')
0017        161 CONTINUE
0018            NTAPE=IB(1)
0019            PRINT 160,IB
0020        160 FORMAT('  TAPE NUMBER=',IA4,
               1'  IDCAL=',IA4,
               1'  BASE=',IA4,
               1'  NUMBER OF CHANNELS=',IA4,
               1'  NUMBER OF SCANS=',IA4)
0021            FILCK=1
0022            READ(5,3350)IPRINT,NPNCH
0023       3350 FORMAT(2I10)
          C     READ INSTRUCTIONS AND TOLERANCES
0024            READ(5,360) IBTYP,IFLAG,NCI,NOSCAN,NOCHAN,L1,L2
0025        360 FORMAT(7I10)
0026            READ(5,1991) TOL1,TOL2,TOL3
0027       1991 FORMAT(3E10.0)
0028            PRINT 361,IBTYP,IFLAG,NCI,NOSCAN,NOCHAN,TOL1,TOL2,TOL3,L1,L2
0029        361 FORMAT('  TYPE BASE CALCULATION  ********************************
               ******  *****  IBTYP      *****      'I10/,
               *'  TYPE STATISTICS CALLED FOR  ********************************
               ****** IFLAG        *****      ',I10/,
               *'  NUMBER OF CLASS INTERVALS  *********************************
               ****** NCI          *****      ',I10/,
               *'  NUMBER OF SCANS PER RECORD  ********************************
               ****** NOSCAN       *****      ',I10/,
               *'  NUMBER OF CHANNELS ON TAPE  ********************************
               ****** NOCHAN       *****      ',I10/,
               *'  TOLERANCE ON BASE LIMITS  **********************************
               ****** TOL1         *****      ',F10.3/,
               *'  TOLERANCE ON SIGNAL LIMITS  ********************************
               ****** TOL2         *****      ',F10.3/,
               *'  NUMBER OF SIGNAL AND BASE POINTS REQUIRED PER CYCLE  ********
               ****** TOL3         *****      ',F10.3/,
               *'  NUMBER OF PASSES REQUIRED TO ABORT CYCLE  ******************
               ****** L1           *****      ',I10/,
               *'  NUMBER OF BASE POINTS REQUIRED FOR SIGNAL POINT SEARCH  *****
               ****** L2           *****      ',I10)
          C     READ DATA FOR TAPE PROCESSING
0030        511 READ(5,882)FILES,ISTART,ICHNO,IRCEND
```

```
0031            DO 8892 IJ=1,7
0032            DO 8892 JI=1,300
0033       8892 IE(IJ,JI)=0
0034            IRECNT=ISTART-1
0035            IEXIT=0
0036        987 IREC=0
0037        882 FORMAT(4I3)
0038            PRINT 901,FILES,ICHNO,ISTART,IRCEND
0039        901 FORMAT(1H1,'     PROCESSING FILE ***********',I5/,
               *'      CHANNEL *******************************',I5/,
               *'      BEGIN AT RECORD ***********************',I5/,
               *'      STOP PROCESSING AT RECORD ************',I5)
0040             IRIN=1
0041            IF(FILES)991,10,10
0042         10 IF(FILES.GT.FILCK) GO TO 9
0043            CALL NTRAN(1,10)
0044            GO TO 9
0045        991 PRINT 992
0046        992 FORMAT(10X,'******  PROGRAM STOP *****')
0047            STOP
0048          9 CALL REDREC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
          C     MOVE TAPE TO DESIRED FILE
0049            IF(FILCK-FILES)9,887,887
          C     READ RECORD, STORE RECORD IN ARRAY IBUF
0050        887 CALL REDREC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
          C     MOVE TAPE TO DESIRED RECORD
0051            IF(IREC-ISTART)887,889,889
          C     STORE CONTENTS OF IBUF INTO MAIN ARRAY P
0052        889 DO 888 L=1,N
0053        888 P(L)=IBUF(L)
          C     READ NEXT RECORD INTO IBUF
0054            CALL RED REC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
          C     STORE CONTENTS OF IBUF INTO AJXILIARY ARRAY AUX
0055            DO 500 L=1,N
0056        500 AUX(L)=IBUF(L)
0057            GO TO 501
0058        502 DO 503 L=1,N
0059        503 P(L)=AUX(L)
0060            CALL RED REC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
          C     STORE IBUF INTO AUX
0061            DO 504 L=1,N
0062        504 AUX(L)=IBUF(L)
0063        501 CALL AVG(N,L1,L2,IRECNT,TOL1,TOL2,TOL3,NCI,IBTYP)
          C     STOP PROCESSING ON DESIRED RECORD
0064            IF(IEXIT) 661,661,507
0065        661 IF(IREC-IRCEND) 502,221,221
0066        221 DO 2293 I=1,N
0067       2293 P(I)=AUX(I)
0068            IEXIT=1
0069            GO TO 501
          C     PRINT ERROR TABLE
0070        507 CALL PRINT(1)
0071            CALL STAT(NTAPE,ICHNO,FILES,NCI,IFLAG)
0072            GO TO 511
0073            END
```

```
0001             SUBROUTINE AVG(NK,L1,L2,IREC,TOL1,TOL2,TOL3,NCI,IBTYP)
0002             COMMON P(1000),AUX(1000),INDO(10),NCNT(300),JOVF,JUNF,
                *BASE(2,10),SIG(10),IBLOC(2,10),L,M,XL90,XL10
0003             COMMON/AAA/IPRINT
0004             COMMON/RLC/IRTN
0005             COMMON/PR/IE(7,300)
0006             COMMON/CBA/IBCNT(2),ISCT
0007             IREC=IREC+1
0008             IST=IST + 1
0009             INP=0
0010             INDX=1
0011             GO TO (150,11,12,150,776),IRTN
0012        150  IRUN=1
0013             IRP=0
0014             IST=0
0015             L=1
0016             M=20
0017             IF(IRTN.EQ.4)  GO TO 209
0018             DO 1050 I=1,300
0019       1050  NCNT(I)=0
0020             JOVF=0
0021             JUNF=0
0022             NDATA=0
0023        209  CALL LIMIT(TOL1,TOL2,NK)
           C     SEARCH FOR FIRST BASE POINT- HOLD VALUE OF INDX
0024        776  DO 1 I=L,M
0025             IF(I.GT.NK) GO TO 2000
0026             IF(P(I)-XL90)1,3,3
0027          3  INDX=I
0028             GO TO 4
0029          1  CONTINUE
0030             IE(1,IREC)=IE(1,IREC)+1
0031             IRTN=4
           C     RETURN TO DRIVER FOR NEXT RECORD
0032             RETURN
0033       2000  L=1
0034             M=M-NK
0035             IRTN=5
0036             RETURN
0037          4  M=INDX+15
0038             L=INDX
0039             CALL LIMIT(TOL1,TOL2,NK)
0040             IBC=1
0041             IBCNT(1)=0
0042             IBCNT(2)=0
0043             ISCT=0
0044         10  LC=0
0045             NP=0
           C     BASE POINT SEARCH
0046         11  IF(P(INDX)-XL90) 12,13,13
0047         13  IBCNT(IBC)=IBCNT(IBC) +1
0048             I=IBCNT(IBC)
           C     STORE BASE POINT
0049             BASE(IBC,I)=P(INDX)
0050             IBLOC(IBC,I)=INDX
0051             IF(IBCNT(IBC)-10) 14,401,401
```

```
0052        401  IRTN=4
0053             IE(2,IREC)=IE(2,IREC)+1
0054             RETURN
0055         14  INDX=INDX + 1
0056             IF(INDX-NK) 11,11,158
0057        158  IRTN=2
           C     NORMAL RETURN TO DRIVER FOR NEXT RECORD
0058             RETURN
           C     SIGNAL POINT SEARCH
0059         12  IF(P(INDX)-XL10) 20,20,15
0060         15  IF(LC)16,16,21
0061         16  NP=NP+1
           C     CHECK NUMBER OF UNSUCESSFUL PASSES
0062             IF(NP-L1) 1944, 1944, 208
0063       1944  IF(ISCT.EQ.0) GO TO 14
0064             GO TO 70
0065         21  IF(P(INDX)-XL90) 22,30,30
0066          30 IF(IRUN)10,10,31
           C     SET BASE COUNT INDX CONDITION
0067         31  IF(IBC-1)34,33,34
0068         33  IBC=2
0069             GO TO 10
0070         34  IBC=1
0071         20  IF(IRUN) 23,23,24
           C     CHECK LOOP CONDITION
0072         23  IF(LC) 40,40,29
0073         24  IF(IBC-1) 25,25,40
0074         25  LC=1
0075             NP=0
0076         29  ISCT=ISCT + 1
           C     STORE SIGNAL POINT
0077             SIG(ISCT)=P(INDX)
0078             INDO(ISCT)=INDX
0079             IF(ISCT-10) 26,501,501
0080        501  IRTN=4
0081             IE(3,IREC)=IE(3,IREC)+1
0082             RETURN
0083         26  INDX=INDX+1
0084             IF(INDX-NK) 12,12,162
0085        162  IRTN=3
           C     NORMAL RETURN TO DRIVER FOR NEXT RECORD
0086             RETURN
0087         22  NP=NP+1
0088             IF(NP-L1) 26,26,208
           C     CHECK NUMBER OF BASE AND SIGNAL POINTS
0089         40  IF(IBCNT(1).LT.TOL3) GO TO 222
0090             IF(IBCNT(2).LT.TOL3) GO TO 222
0091             IF(ISCT.LT.TOL3) GO TO 224
0092             GO TO 100
0093         222 IE(4,IREC)=IE(4,IREC)+1
0094             GO TO 555
0095         224 IE(5,IREC)=IE(5,IREC)+1
0096             GO TO 555
           C     ENTER ERROR RECYCLE
0097         208 IE(6,IREC)=IE(6,IREC)+1
0098         555 L=INDX
```

```
FORTRAN IV G LEVEL  1, MOD 4            AVG            DATE = 69290          13/17/51            PAGE 0003
  0099                      M=INDX+20
  0100                      IRUN=1
  0101                      IF(IPRINT.EQ.0) GO TO 742
  0102                      PRINT 666,IREC,INDX
  0103               666    FORMAT(1X,'IREC=',I10,10X,'INDX=',I10)
  0104                      IF(IREC.EQ.IRP) GO TO 742
  0105                      PRINT 333,(P(I),I=1,NK)
  0106               333    FORMAT(1X,/,(1X,10F10.3))
  0107                      IRP=IREC
  0108               742    CONTINUE
  0109                      INP=INP+1
  0110                      IF(INP-5)209,921,921
  0111               921    IE(7,IREC)=IE(7,IREC)+1
  0112                      IRTN  = 4
  0113                      RETURN
                    C       POINT SEARCH CYCLE COMPLETE
  0114               100    SUM1=0.0
  0115                      SUM2=0.0
  0116                      I=IBCNT(1)
                    C       PREPROSSING FOR HISTOGRAM FOLLOWS
  0117                      DO 101 J=1,I
  0118               101    SUM1=SUM1+ BASE(1,J)
  0119                      I=IBCNT(2)
  0120                      DO 102 J=1,I
  0121               102    SUM2=SUM2 +BASE(2,J)
  0122                      BA=(SUM1+SUM2)/(IBCNT(1) +IBCNT(2))
  0123                      CALL HIST(BA,NK,NCI,IBTYP,IRUN)
                    C       SET BASE COUNT INDX CONDITION
  0124                43    IF(IBC-1) 44,45,44
  0125                45    IBC=2
  0126                      GO TO 46
  0127                44    IBC=1
  0128                46    ISCT=0
  0129                      IBCNT(IBC)=0
  0130                      LC=1
  0131                      NP=0
  0132                      IRUN=0
  0133                      INDX2=INDX+9
  0134                      L=INDX
  0135                      M=INDX2
  0136                      CALL LIMIT(TOL1,TOL2,NK)
  0137                      GO TO 12
                    C       LOOK INTO NEXT RECORD FOR SIGNAL POINT
  0138                70    IF(IBCNT(IBC)-L2) 14,72,72
  0139                72    L=INDX
  0140                      M=INDX + 9
  0141                      CALL LIMIT(TOL1,TOL2,NK)
  0142                      IF(P(INDX)-XL10) 20,20,14
  0143                      END
```

```
C       HISTROGRAM
              SUBROUTINE HIST(BA,IDUM,NCI,IBTYP,IRUN)
              COMMON P(1000),AUX(1000),INDO( 10),NCNT(300),JOVF,JUNF,
             *BASE(2,10),SIG(10),IBLOC(2,10),KK,MM,XL90,XL10
              DIMENSION Y(40), X(40), LOC(40), A(15), V(15), B(200)
              COMMON/CBA/IBCNT(2),N
C             COMBINE BASE 1 AND BASE 2
              IF(IBTYP)100,100,200
          200 IF(IRUN.EQ.1) MMM=1
              KKK=1
              LLL=1
              IF(MMM.EQ.1) LLL=2
              IF(MMM.EQ.0) KKK=2
          100 IJ=N
              DO 1 I=1,N
              IF(IBTYP.EQ.0) GO TO 202
          400 IF(I-1) 402,401,402
          401 NTEMP=IBCNT(KKK)
              BA=BASE(KKK,NTEMP)
              GO TO 202
          402 IF(I-IJ) 1,403,1
          403 BA=BASE(LLL,1)
          202 J=BA-SIG(I)
              J=(NCI*J)/1000     +1
              IF(J-1)2,3,3
        2     JUNF=JUNF+1
              GO TO 1
        3     IF(J-300)5,5,4
        4     JOVF=JOVF+1
              GO TO 1
        5     NDATA=NDATA+1
              NCNT(J)=NCNT(J)+1
              IF(IBTYP.EQ.3) GO TO 50
            1 CONTINUE
              IF(MMM.EQ.0) MMM=1
              IF(MMM.EQ.1) MMM=0
           50 RETURN
              END
```

```
0001              SUBROUTINE LIMIT(TOL1,TOL2,NK)
0002              COMMON P(1000),AUX(1000),INDO(10),NCNT(300),JOVF,JUNF,
                 *BASE(2,10),SIG(10),IBLOC(2,10),KK,MM,XL90,XL10
0003              IK=KK
0004              IM=MM
0005              ID=0
0006              IF(IK-NK) 502,501,666
0007        666   PRINT 3000
0008       3000   FORMAT('  IK GREATER THAN NK   ')
0009        501   AMAX=P(IK)
0010              AMIN=AMAX
0011              ID=IM-NK
0012              GO TO 381
0013        502   IF(IM-NK) 360,360,361
0014        361   ID=IM-NK
0015              IM=NK
0016        360   AMAX=P(IK)
0017              AMIN=AMAX
0018              DO 350 J=IK,IM
0019              IF(AMAX-P(J)) 301,302,302
0020        301   AMAX=P(J)
0021        302   IF(AMIN-P(J)) 350,303,303
0022        303   AMIN=P(J)
0023        350   CONTINUE
0024              IF(ID) 380,380,381
0025        381   AMAXP=AUX(1)
0026              AMINP=AMAXP
0027              DO 650 J=1,ID
0028              IF(AMAXP-AUX(J)) 601,602,602
0029        601   AMAXP=AUX(J)
0030        602   IF(AMINP-AUX(J)) 650,603,603
0031        603   AMINP=AUX(J)
0032        650   CONTINUE
0033              IF(AMAX-AMAXP)700,701,701
0034        700   AMAX=AMAXP
0035        701   IF(AMIN-AMINP)380,380,703
0036        703   AMIN=AMINP
0037        380   A=AMAX-AMIN
0038              XL90=AMAX-TOL1*A
0039              XL10=AMIN+ TOL2*A
0040              RETURN
0041              END
```

```
                 C      STATISTICS
0001                    SUBROUTINE STAT(NTAPE,NCH,NFILE,NCI,IFLAG)
                 C       IFLAG = 1   NO CHI SQUARE TEST
                 C              2   CHI SQUARE TEST ON NORMAL DISTRIBUTION
                 C              3  CHI SQUARE TEST ON LOG NORMLL DISTRIBUTION
                 C              4  CHI SQUARE TEST ON BOTH
0002                    COMMON P(1000),AUX(1000),INDD(10),NCNT1(300),JOF,JUF,BASE(2,10),
                       *PIG(10),IBLOC(2,10),KK,MM,XL90,XL10
0003                    COMMON/UUU/NPNCH
0004                    DIMENSION Y(300),XLN(300),Q(4),R(4)
0005                    DIMENSION NCNT(300)
0006                    DO 60 I=1,300
0007                 60 NCNT(I)=NCNT1(I)
0008                    C=1000/NCI
                 C        FIND THE HIGHEST  AND LOWEST CLASS INTERVAL
0009                    DO 1 I=1,NCI
0010                    IF(NCNT(I)) 1,1,2
0011                  2 ILO=I
0012                    GO TO 3
0013                  1 CONTINUE
0014                  3 DO 4 I= ILO,NCI
0015                    IF(NCNT(I))4,4,5
0016                  5 IHI=I
0017                  4 CONTINUE
                 C        FIND NUMBER OF POINTS AND FLOAT NCNT
0018                    N=0
0019                    DO 6 I=ILO,IHI
0020                    Y(I)=NCNT(I)
0021                  6 N=N+NCNT(I)
                 C       PRINT HEADINGS
0022                    WRITE(6,101) NTAPE,NCH,NFILE
0023                    WRITE(6,102)
                 C       DEFALT DUE TO TOO FEW CLASS INTERVALS
0024                    NN= IHI-ILO
0025                    IF(NN-10) 50,50,51
0026                 50 WRITE(6,110)
0027                    RETURN
                 C       FIND AVERAGE AMPLITUDE
0028                 51 XN=N
0029                    AVE=0.00
0030                    DO 8 I=ILO,IHI
0031                    XI=I
0032                  8 AVE=AVE+Y(I)*(XI-0.5)*C
0033                    AVE=AVE/XN
                 C       COMPUTE CUMULATIVE PROBABILITIEC AND LOG AMPLITUDES
0034                    SUM=0.00
0035                    DO 10 I=ILO,IHI
0036                    XI=I
0037                    XI=(XI-0.5)*C
0038                    XLN(I)=0.5*ALOG(XI/AVE)
0039                    SUM=SUM+Y(I)
0040                    CP=SUM/XN
0041                 10 WRITE(6,103)  XI,XLN(I),Y(I),CP
0042                    WRITE(6,111) JOF, JUF
                 C       COMPUTE MOMENTS ABOUT THE MEAN
0043                    XLA=0.00
```

```
0044            DO 20 I=ILO,IHI
0045         20 XLA=XLA+Y(I)*XLN(I)
0046            XLA=XLA/XN
0047            DO 21 I=2,4
0048            Q(I)=0.00
0049         21 R(I)=0.00
0050            DO 22 I=ILO,IHI
0051            XI=I
0052            DO 22 J=2,4
0053            Q(J)=Q(J)+Y(I)*((XI-0.5)*C-AVE)**J
0054         22 R(J)=R(J)+Y(I)*(XLN(I)-XLA)**J
0055            DO 23 J=2,4
0056            Q(J)=Q(J)/XN
0057         23 R(J)=R(J)/XN
0058            NT=IHI-ILO+1
0059            WRITE(6,104) NT
0060            SIG=SQRT(Q(2))
0061            SIGL=SQRT(R(2))
0062            SKEW=0.5*Q(3)/(SIG**3)
0063            SKL = 0.5*R(3)/(SIGL**3)
0064            XKUR=((Q(4)/(Q(2)**2))-3.0)/2.0
0065            XKURL = ((R(4)/(R(2)**2))-3.0)/2.0
          C      PRINT MOMENTS
0066            WRITE(6,105) AVE,SIG,SKEW,XKUR,XLA,SIGL,SKL,XKURL,N
0067            IF(NPNCH .EQ. 0) GO TO 810
0068            PUNCH 800,NTAPE,NCH,NFILE,AVE,SIG,SIGL,XLA
0069        800 FORMAT(A4,I2,I4,4E14.4)
0070        810 CONTINUE
0071            GO TO (31,32,42,32) ,IFLAG
0072         31 RETURN
          C      NO CHI SQUARE TEST REQUESTED
          C
          C      CHI SQUARE TEST
0073         32 CALL CHI(CSQ,Y,ILO,IHI,C,NUSE,AVE,XLA,SIG ,XN,0,SIG)
0074            WRITE(6,106) CSQ,NUSE
          C      PRINT CHI SQUARE     NORMAL
0075            GO TO (31,31,42,42)     ,IFLAG
0076         42 CALL CHI(CSQ,Y,ILO,IHI,C,NUSE,AVE,XLA,SIG ,XN,1,SIGL)
0077            WRITE(6,106) CSQ,NUSE
          C      PRINT CHI SQUARE     LOG NORMAL
0078            RETURN
0079        101 FORMAT('1',5X,'TAPE NUMBER  ',A4,5X,'TRACK',I3,5X,'FILE',I3)
0080        102 FORMAT('0',16X,'AMPLITUDE',10X,'LOG AMPLITUDE',12X,'COUNT',
               1  8X,'CUMULATIVE PROBABILITY'/)
0081        103 FORMAT(7X,4E21.6)
0082        104 FORMAT('0',10X,'NUMBER OF CLASS INTERVALS =',I6)
0083        105 FORMAT('0',17X,'AVERAGE',9X,'STANDARD DEVIATION',8X,'SKEWNESS',
               *13X,'KURTOSIS'/7X,4E21.6/7X,4E21.6/'0',10X,'NUMBER OF DATA POINTS'
               *,I10)
0084        106 FORMAT('0',10X,'CHI SQUARE=',E14.6/11X'NUMBER OF CLASS INTERVALS
               1USED =',I5)
0085        110 FORMAT('0',5X,'TOO FEW CLASS INTERVALS  '/
               1 '0',5X,'  EXECUTION OF STATISTICS CALCULATION SUSPENDED')
0086        111 FORMAT('0',5X,'NUMBER OF OVERFLOWS',I6/
               *6X,'NUMBER OF UNDERFLOWS',I5)
0087            END
```

```
                C       CHI SQUARE TEST
0001                   SUBROUTINE CHI(CSQ,Y1,ILO,IHI,C,NUSE,AVE,XLA,SD,XN,NTYP,SX)
0002                   DIMENSION Y(300,3) , Y1(300)
0003                   DO 1 I=1,300
0004                 1 Y(I,1)=0.00
0005                   NUSE=0
0006                   KM=(ILO+IHI)/2
0007                   J=1
0008                   Y(J,2)=AVE-10.0*SD
                C        GROUP CLASS INTERVALS ON LOW END
0009                   DO 2 I= ILO,KM
0010                   Y(J,1)=Y1(I) + Y(J,1)
0011                   IF(Y(J,1)-5.0) 2,2,3
0012                 3 Y(J,3)=C*I
0013                   NUSE = NUSE + 1
0014                   J = J + 1
0015                   Y(J,2) = C*I
0016                 2 CONTINUE
                C       GROUP CLASS INTERVALS ON HIGH SIDE
0017                   I = IHI
0018                   Y(J,3) = AVE + 10.0*SD
0019                 6 IF(I-KM) 10,10,4
0020                 4 Y(J,1) = Y(J,1) + Y1(I)
0021                   IF(Y(J,1)-5.0) 11,11,5
0022                 5 Y(J,2) = C*(I-1)
0023                   NUSE = NUSE + 1
0024                   J = J + 1
0025                   Y(J,3) = C*(I-1)
0026                11 I = I-1
0027                   GO TO 6
                C       COMPUTE THEORITICAL PROBABILITY
0028                10 CSQ = 0.00
0029                   DO 30 I=1,NUSE
0030                   XLL = Y(I,2)
0031                   XUL=Y(I,3)
0032                24 CALL SIMP( FTH,XLL,XUL,21,NTYP,AVE,SX,XLA)
                C       COMPUTE CHI SQUARE
0033                   IF(FTH) 31,31,30
0034                31 WRITE(6,100) I
0035               100 FORMAT('0     ZERO VALUE OF THEORITICAL PROBABILITY IN',I5,' TH
                      1 INTERVAL'/ 6X,'EXECUTION OF CHI SQUARE TEST DISCONTINUED')
0036                   RETURN
0037                30 CSQ=CSQ+((Y(I,1)-XN*FTH)**2)/(XN*FTH)
0038                   RETURN
0039                   END
```

```
                C       SIMPSONS RULE
                C      SUBPROGRAM FOR SIMPSONS RULE INTEGRATION
                C
                C
0001                   SUBROUTINE SIMP(SUM,FLL,FUL,N,NTYP,A,B,C)
                C       INTEGRAND FUNCTION   REMOVE WHENCHANGING FUNCTION
0002                   PBF(X,A,S) = (1.0/(S*SQRT(6.28318)))*EXP(-0.5*(((X-A)/S)**2))
0003                   FNP=N-1
0004                   DELX=(FUL-FLL)/FNP
0005                   SUM=0.0
0006                   SUM1=0.0
0007                   SUM2=0.0
0008                   DO 1 I=1,N
0009                   FK=I-1
0010                   X=FK*DELX+FLL
                C      CALL FOR INTERGRAND SUBROUTINE HERE
0011                   IF(NTYP) 101,101,110
0012               101 VAL=PBF(X,A,B)
0013                   GO TO 102
0014               110 IF(X) 20,20,100
0015                20 VAL=0.00
0016                   GO TO 102
0017               100 XX=0.5*ALOG(X/A)
0018                   VAL=PBF(XX,C,B)
0019                   VAL=0.5*VAL/X
0020               102 CONTINUE
                C
0021                   IF(I.EQ.1.OR.I.EQ.N) GO TO 2
0022                   J=MOD(I,2)
0023                   IF(J)3,4,3
0024                 3 SUM1=SUM1+VAL
0025                   GO TO 1
0026                 4 SUM2=SUM2+VAL
0027                   GO TO 1
0028                 2 SUM=SUM+VAL
0029                 1 CONTINUE
0030                   SUM=SUM+2.0*SUM1 + 4.0*SUM2
0031                   SUM=SUM*DELX/3.0
0032                   RETURN
0033                   END
```

```
      SUBROUTINE RID(I3,I4,IC)
      INTEGER IB(5),IA(3), BUF(501),FLCNT,TTB(3),BLK,FILES,FILCK,
     *IBUF(1),IC(1)
      M=1
      I1=0
      I2=0
    5 CALL NTRAN(1,2,3,IA,K,2,-2004,BUF,L,22)
    2 IF(K+1)1,2,3
    3 CALL MOVE(IB,1,IA,1,4)
      CALL TRNSL(IB,4,TTB)
      DATA TTB/'0123456789'/
      DATA BLK /' '/
      IB(2)=BLK
      CALL MOVE(IB(2),1,IA,5,1)
      CALL TRNSL(IB(2),1,TTB)
      IB(3)=BLK
      CALL MOVE(IB(3),1,IA,6,1)
      CALL TRNSL(IB(3),1,TTB)
      IB(4)=BLK
      CALL MOVE(IB(4),1,IA,7,2)
      CALL TRNSL(IB(4),2,TTB)
      CALL MOVE(IB(5),1,IA,9,4)
      CALL TRNSL(IB(5),4,TTB)
      GO TO (6,7), M
    6 DO 14 I=1,5
   14 IC(I)=IB(I)
      I3=I1
      I4=I2
      RETURN
    1 IF(K.EQ.-2) GO TO 4
      I2=1
      CALL NTRAN(1,22)
      GO TO (6,7),M
    4 I1=1
      CALL NTRAN(1,22)
      GO TO (6,7),M
      ENTRY REDREC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
      M=2
   13 IREC=IREC+1
      CALL NTRAN(1,22)
    9 IF(L+1)8,9,10
   10 TIME=BUF(1)
      N=(L-4)/(2*NOCHAN)
      K=3+2*ICHNO
      DO 11 I=1,N
      IBUF(I)=0
      J=0
      CALL MOVE(J,4,BUF(1),K,1)
      CALL MOVE(IBUF(I),4,BUF(1),K+1,1)
      IBUF(I)=IBUF(I) + 64*J
      IF(IBUF(I).GE.1024) IBUF(I)=1024-IBUF(I)
   11 K=K+2*NOCHAN
      CALL NTRAN(1,2,-2004,BUF,L)
      RETURN
    8 IF(L.EQ.-2) GO TO 12
      PRINT 100
```

```
  100 FORMAT('0******  READ ERROR  ******')
      CALL NTRAN(1,22)
      CALL NTRAN(1,2,-2004,BUF,L)
      GO TO 13
   12 CONTINUE
      FILCK=FILCK+1
      CALL NTRAN(1,22)
      GO TO 5
    7 CONTINUE
      IREC=0
      GO TO 13
      END
```

```
      SUBROUTINE PRINT(NNN)
      COMMON/PR/IE(7,300)
      COMMON/RAT/IRCEND
      DIMENSION IESUM(7)
      PRINT 3
    3 FORMAT(1H1,50X,'DATA PROCESSING IRREGULARTIES',/,10X,'ERROR CODES
     *FOLLOW',////,
     *10X,'NO BASE POINTS FOUND IN BASE SEARCH **********  1'/,
     *10X,'NUMBER OF BASE POINTS EXCEEDS 10  ************  2'/,
     *10X,'NUMBER OF SIGNAL POINTS EXCEEDS 10 ***********  3'/,
     *10X,'NUMBER OF BASE POINTS INSUFFICIENT ***********  4'/,
     *10X,'NUMBER OF SIGNAL POINTS INSUFFICIENT *********  5'/,
     *10X,'NUMBER OF PASSES EXCEEDS LIMIT L1 ************  6'/,
     *10X,'NUMBER OF ERRORS IN RECORD EXCEEDS 5 *********  7')
      DO 555 I=1,7
  555 IESUM(I)=0
      PRINT 1
    1 FORMAT(10X,'ERROR',11X,'1',13X,'2',13X,'3',13X,'4',13X,'5',13X,
     *'6',13X,'7')
      PRINT 100
  100 FORMAT(10X,'RECORD',///)
      DO 55 I=1,IRCEND
      DO 55 K=1,7
   55 IESUM(K)=IESUM(K)+IE(K,I)
      DO 2 I=1,IRCEND
      DO 12 K=1,7
      IF(IE(K,I)) 15,12,15
   12 CONTINUE
      GO TO 2
   15 WRITE(6,5) I,(IE(K,I),K=1,7)
    5 FORMAT(10X,I3,10X,I4,10X,I4,10X,I4,10X,I4,10X,I4,10X,I4,10X,I4)
    2 CONTINUE
      PRINT 20
   20 FORMAT(10X,//,10X,'ERROR CODE',10X,'NUMBER OF ERRORS')
      DO 50 K=1,7
   50 PRINT21,K,IESUM(K)
   21 FORMAT(10X,I6,14X,I9)
      RETURN
      END
```

APPENDIX B

This program generates a set of N random numbers having a log-normal distribution and a pre-selected mean and standard deviation. The program is in the form of a FORTRAN IV subroutine.

Theory: By definition a log-normal random deviate is one whose logarithms are normal random deviates. Thus if (X_i) is a set of log-normal random numbers then their must exist a set of normal random numbers (y_i) related to the X_i by

$$y_i = \ln X_i \qquad \text{B1}$$

Equation B1 may be generalized by the addition of appropriate scaling factors. i.e. we may let

$$y_i = a \ln X_i + b \qquad \text{B2}$$

Now by choosing the mean and variance of the (y_i) and the values of the scale factors a and b it is possible to generate a set of (X_i) having any desired mean and variance from a set of normal deviates (y_i). Solving B2 for X_i we have

$$X_i = \exp\left(\frac{y_i - b}{a}\right) \qquad \text{B3}$$

Since we wish to specify only two parameters, viz., the mean and standard deviation of the (X_i) it seems reasonable to assume that we will need only two parameters in equation B3. We therefore let a = 1 and take the mean of the (y_i) to be zero. B3 then becomes

$$X_i = \exp(-b) \exp(y_i) \qquad \text{B4}$$

taking the average of both sides of equation B4 we have

$$\bar{X} = \exp(-b)\ \overline{\exp(y_i)} \qquad \text{B5}$$

and also taking the second moment of (X_i) about zero

$$\overline{X^2} = \exp\ (-b)\ \overline{\exp\ (2y_i)} \qquad \text{B6}$$

the averages of the exponential functions in equation B5 and B6 can be evaluated easily

$$\overline{\exp(ny_i)} = (2\pi t^2)^{-1/2} \int_{-x}^{x} \exp(ny) \cdot \exp(y^2/2\sigma)\,dy \qquad \text{B7}$$

Combining equation B5,B6 and B7 we obtain expressions which may be solved for the scale factor b and the required standard deviation of the (y_i)

$$\sigma^2 = \ln\ (\mu/\bar{X}^2) \qquad \text{B8}$$

and

$$\exp(-b) = \bar{X}\exp(-\sigma^2/2) \qquad \text{B9}$$

where μ is the second moment of the (X_i) about zero.

Program: The log-normal generator makes use of the normal random number generator included in the IBM Scientific Subroutine Package for the 360 computer. This routine (GAUSS) generates normal random numbers with any required mean and standard deviation. Coding for the program is shown in the accompanying listing. The argument list is as follows:

AVE The required mean.

VAR The required standard deviation

Y A vector of log-normal random numbers returned by the subroutine. Y is dimensioned by the calling program

N The number of random numbers to be generated

IX A "seed" required by GAUSS. IX must be a 5 digit odd integer.

Statements 003 to 006 compute the required standard deviation for the Gaussian-random numbers and the proper scaling factor. Statements 007 to 009 call GAUSS compute a log-normal random number from equation B5.

The log-normal random number generator has been used to test the statistics subroutines used in our data analysis program. A Calling Program which will provide the subroutine STAT and CHI with either normal or log-normal data is given.

```
FORTRAN IV G LEVEL 1, MOD 4          LOGN          DATE = 69290          14/18/08          PAGE 0001

      SUBROUTINE LOGN( AVE,VAR,Y,N,IX)
      DIMENSION Y(1)
      VAR = VAR**2 + AVE**2
      SIG = ALOG(VAR/AVE**2)
      ZBAR=EXP(SIG/2.0)
      SIG = SQRT(SIG)
      DO 1 I=1,N
      CALL GAUSS( IX,SIG,0.0,X)
    1 Y(I)=(AVE/ZBAR)*EXP(X)
      RETURN
      END
```

```
FORTRAN IV G LEVEL 1, MOD 4            MAIN              DATE = 69290         14/18/08              PAGE 0001

                COMMON NCNT(300)
               DIMENSION Y(5000)
            20 READ(5,100) AVE,VAR, N, IX   , IT
               IF(IT) 10,11,12
            10 STOP
            11 WRITE(6,103)
           103 FORMAT('1 LOG NORMAL DATA')
               CALL LOGN(AVE,VAR,Y,N,IX)
               GO TO 13
            12 WRITE(6,104)
          104  FORMAT('1 NORMAL DATA')
               DO 14 I=1,N
               CALL GAUSS(IX,VAR,AVE,Z)
            14 Y(I)=Z
            13 DO 2 I=1,100
             2 NCNT(I) = 0
               L=0
               M=0
               DO 1 I=1,N
               J=Y(I)/10.00+1
               IF(J.LT.1) L=L+1
               IF(J.GT.100) M=M+1
               IF(J.LT.1.OR.J.GT.100)  GO TO 1
               NCNT(J)=NCNT(J)+1
             1 CONTINUE
               CALL STAT(0,0,0,100,4,L,M)
               GO TO 20
           100 FORMAT(2F10.5, 3I10)
               END
```

APPENDIX C

The following is a typical computer print out for a file of data. The control parameters are listed at the first of each new tape run. After a file has been processed, the irregularities found in each record are listed in tabular form. The statistical calculations made on the data is then printed in a tabular form and identified as to it's tape, track, and file number for later reference.

```
TAPE NUMBER=3397  IOCAL=3      BASE=1      NUMBER OF CHANNELS=05     NUMBER OF SCANS=0200
TYPE BASE CALCULATION  *************************************  ***** IBTYP      *****          2
TYPE STATISTICS CALLED FOR  ********************************  ***** IFLAG      *****          4
NUMBER OF CLASS INTERVALS  *********************************  ***** NCI        *****        100
NUMBER OF SCANS PER RECORD  ********************************  ***** NOSCAN     *****        200
NUMBER OF CHANNELS ON TAPE  ********************************  ***** NOCHAN     *****          5
TOLERANCE ON BASE LIMITS  **********************************  ***** TOL1       *****      0.050
TOLERANCE ON SIGNAL LIMITS  ********************************  ***** TOL2       *****      0.100
NUMBER OF SIGNAL AND BASE POINTS REQUIRED PER CYCLE  ********  ***** TOL3       *****      3.000
NUMBER OF PASSES REQUIRED TO ABORT CYCLE  ******************  ***** L1         *****      0.000
NUMBER OF BASE POINTS REQUIRED FOR SIGNAL POINT SEARCH  *****  ***** L2         *****          5
```

DATA PROCESSING IRREGULARTIES

ERROR CODES FOLLOW

NO BASE POINTS FOUND IN BASE SEARCH ********** 1
NUMBER OF BASE POINTS EXCEEDS 10 ************ 2
NUMBER OF SIGNAL POINTS EXCEEDS 10 *********** 3
NUMBER OF BASE POINTS INSUFFICIENT *********** 4
NUMBER OF SIGNAL POINTS INSUFFICIENT ********* 5
NUMBER OF PASSES EXCEEDS LIMIT L1 ************ 6
NUMBER OF ERRORS IN RECORD EXCEEDS 5 ********* 7

ERROR / RECORD	1	2	3	4	5	6	7
3	0	0	0	0	1	0	0
6	0	0	0	1	0	1	0
13	0	0	0	1	0	0	0
18	0	0	0	0	1	0	0
28	0	0	0	0	1	0	0
35	0	0	0	0	1	0	0
44	0	0	0	0	1	0	0
45	0	0	0	0	1	0	0
51	0	0	0	3	0	1	0
54	0	0	0	0	1	0	0
68	0	0	0	0	1	0	0
72	0	0	0	0	2	0	0
75	0	0	0	0	1	0	0
79	0	0	0	0	2	0	0
85	0	0	0	0	1	0	0
87	0	0	0	0	1	0	0
101	0	0	0	0	1	0	0
128	0	0	0	0	1	0	0
153	0	0	0	0	0	1	0
154	0	0	1	0	0	0	0
190	0	0	1	1	0	0	0
191	0	0	0	1	0	0	0
209	0	0	0	0	1	0	0
229	0	0	0	0	1	0	0
247	0	0	0	0	1	0	0
251	0	0	0	0	1	0	0
252	0	0	0	0	1	0	0
256	0	0	0	0	1	0	0
277	0	0	1	1	0	0	0

ERROR CODE	NUMBER OF ERRORS
1	0
2	0
3	3
4	6
5	23
6	1
7	0

TAPE NUMBER 3397 TRACK 4 FILE 5

4

AMPLITUDE	LOG AMPLITUDE	COUNT	CUMULATIVE PROBABILITY
0.115000E 03	-0.387997E 00	0.200000E 01	0.191058E-03
0.125000E 03	-0.346306E 00	0.300000E 01	0.477646E-03
0.135000E 03	-0.307826E 00	0.120000E 02	0.162400E-02
0.145000E 03	-0.272096E 00	0.520000E 02	0.659151E-02
0.155000E 03	-0.238750E 00	0.880000E 02	0.149981E-01
0.165000E 03	-0.207490E 00	0.166000E 03	0.308559E-01
0.175000E 03	-0.178070E 00	0.267000E 03	0.563622E-01
0.185000E 03	-0.150285E 00	0.396000E 03	0.941918E-01
0.195000E 03	-0.123963E 00	0.506000E 03	0.142530E 00
0.205000E 03	-0.989579E-01	0.674000E 03	0.206916E 00
0.215000E 03	-0.751439E-01	0.734000E 03	0.277035E 00
0.225000E 03	-0.524127E-01	0.856000E 03	0.358808E 00
0.235000E 03	-0.306702E-01	0.849000E 03	0.439912E 00
0.245000E 03	-0.983380E-02	0.881000E 03	0.524073E 00
0.255000E 03	0.101689E-01	0.867000E 03	0.606897E 00
0.265000E 03	0.294018E-01	0.806000E 03	0.683894E 00
0.275000E 03	0.479222E-01	0.699000E 03	0.750669E 00
0.285000E 03	0.657815E-01	0.581000E 03	0.806171E 00
0.295000E 03	0.830243E-01	0.503000E 03	0.854222E 00
0.305000E 03	0.996928E-01	0.411000E 03	0.893485E 00
0.315000E 03	0.115823E 00	0.294000E 03	0.921570E 00
0.325000E 03	0.131449E 00	0.249000E 03	0.945357E 00
0.335000E 03	0.146602E 00	0.183000E 03	0.962839E 00
0.345000E 03	0.161309E 00	0.123000E 03	0.974589E 00
0.355000E 03	0.175596E 00	0.970000E 02	0.983856E 00
0.365000E 03	0.189486E 00	0.580000E 02	0.989396E 00
0.375000E 03	0.203000E 00	0.460000E 02	0.993791E 00
0.385000E 03	0.216159E 00	0.270000E 02	0.996370E 00
0.395000E 03	0.228980E 00	0.170000E 02	0.997994E 00
0.405000E 03	0.241483E 00	0.100000E 02	0.998949E 00
0.415000E 03	0.253676E 00	0.300000E 01	0.999236E 00
0.425000E 03	0.265581E 00	0.300000E 01	0.999522E 00
0.435000E 03	0.277210E 00	0.500000E 01	0.100000E 01

NUMBER OF OVERFLOWS 0
NUMBER OF UNDERFLOWS 0

NUMBER OF CLASS INTERVALS = 33

AVERAGE	STANDARD DEVIATION	SKEWNESS	KURTOSIS
0.243956E 03	0.470818E 02	0.179144E 00	0.190105E-01
-0.392637E-02	0.950000E-01	-0.837189E-01	-0.471106E-01

NUMBER OF DATA POINTS 10468

CHI SQUARE= 0.250713E 03
NUMBER OF CLASS INTERVALS USED = 29

CHI SQUARE= 0.652032E 02
NUMBER OF CLASS INTERVALS USED = 29

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APPENDIX D

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