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ANALYSIS OF A HYBRID PHASE-LOCK LOOP

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16. Abstract

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SUMMARY

A model for a hybrid phase-lock loop has been derived by using Fokker-Planck techniques. The hybrid loop operates on both the data and carrier components of the received signal to provide the phase estimate required for coherent detection. The loop is optimized with respect to the system design parameters, and the performance of a bipolar phase-shift-keyed communication system is evaluated. The results are compared with systems which employ Costas-type loops or conventional phase-lock loops. The hybrid loop gives superior performance for all signal-to-noise ratios, with a maximum improvement of 1.5 dB for low data-rate systems, and, additionally, is satisfactory over a wider range of signal-to-noise ratios than either the Costas or conventional phase-lock loops.

INTRODUCTION

Binary phase-shift-keyed (PSK) communication systems transmit information in the form $s(t) = \sqrt{2}\, A \cos\,(\omega t + a(t)\theta_o + \theta)$, $a(t) = \pm 1$. To perform coherent detection, it is required that the receiver determine the carrier phase θ prior to detection. If the signal $s(t)$ contains a residual component at the carrier frequency, a phase-lock loop can be used to track the carrier phase. On the other hand, the modulated portion of the signal also contains information about the carrier phase; systems for generating a phase reference directly from the modulation component have been described by Costas (ref. 1).

This report is concerned with a phase-estimation scheme that combines the phase estimate from the carrier component with the phase estimate from the modulation component. It is mechanized by employing a combination of the conventional phase-lock loop and the Costas-type phase-lock loop. This combination is called a hybrid phase-lock loop (hereinafter referred to as hybrid loop). The remaining sections of this report are devoted to a description and theoretical analysis of the hybrid loop. Fokker-Planck techniques are applied to measure its performance and the results are used to evaluate the error probability of a binary, bipolar communication system. Detection probabilities for the hybrid loop are compared with those for equivalent systems which use a phase-lock loop or a Costas-type loop.

THE HYBRID-LOOP

The received signal is assumed to be of the form

$$y(t) = s(t) + n(t) \tag{1}$$

with

$$S(t) = \sqrt{2}A \sin (\omega t + x(t) \cos^{-1} m + \theta) \tag{2}$$

and

$$n(t) = n_c(t) \cos \omega t + n_s(t) \sin \omega t \tag{3}$$

where A^2 is the received signal power; $x(t)$, a bipolar modulated square-wave subcarrier of unit magnitude; m, the modulation index which adjusts the power allocation between the modulated and carrier portions of the total signal; θ, the phase shift introduced during transmission and assumed unknown at the receiver; and $n(t)$, the additive noise component.

Expanding s(t) gives

$$s(t) = \sqrt{2}Am \sin (\omega t+\theta) + \sqrt{2}A(1-m^2)^{1/2} x(t) \cos (\omega t+\theta). \tag{4}$$

From Eq. (4) we note that the data component with power $A^2(1-m^2)$ is

$$s_d(t) = \sqrt{2}A(1-m^2)^{1/2} x(t) \cos (\omega t+\theta) \tag{5}$$

and the residual carrier component, which will be termed the reference signal, is

$$s_r(t) = \sqrt{2}Am \sin (\omega t+\theta). \tag{6}$$

The physical significance of the parameter m may be seen from Eq. (6) where m may be defined as the square root of the ratio of the power in the reference signal to the total transmitted power, i.e., $m = (P_r/P)^{1/2}$. It is worth a digression at this point to briefly review several schemes for generating a phase reference at the receiver that will lead us to the technique under consideration here.

For the case in which m > 0, Viterbi (ref. 2) has performed an analysis by Fokker-Planck techniques of a scheme whereby the carrier or reference component can be tracked by a phase-lock loop. An experimental evaluation of this method has been made by Lindsey and Charles (ref. 3). Stiffler (ref. 4) and Lindsey (ref. 5), moreover, have considered the question of optimizing the modulation index m to minimize total system-error probability.

For m = 0, in the absence of a reference component, Costas and squaring-loop mechanizations (refs. 1 and 6, respectively) have been proposed. Didday and Lindsey (ref. 6) have demonstrated the mathematical equivalence of these two systems and have applied Fokker-Planck methods to analyze their behavior. Bussgang and Leiter (ref. 7) have derived the performance of a communication system, which uses a reference signal and an optimum detection scheme. This is equivalent to employing a maximum likelihood estimate of the phase in a correlator-type detector. Finally, Van Trees (ref. 8) has considered a system, based on linear tracking theory, that is similar to the one proposed here but is restricted to high signal-to-noise- ratios to permit linearization of the loop equations.

The hybrid phase-lock loop is shown in Figure 1. As stated in the Introduction, it is a combination of the Costas and conventional phase-lock loops. By setting the gain a_2 to zero, the hybrid loop reduces to a conventional phase-lock loop which derives the phase estimate from the carrier component of y(t). Setting a_1 to zero gives a Costas-loop mechanization which extracts the phase from the data component of y(t). In essence,

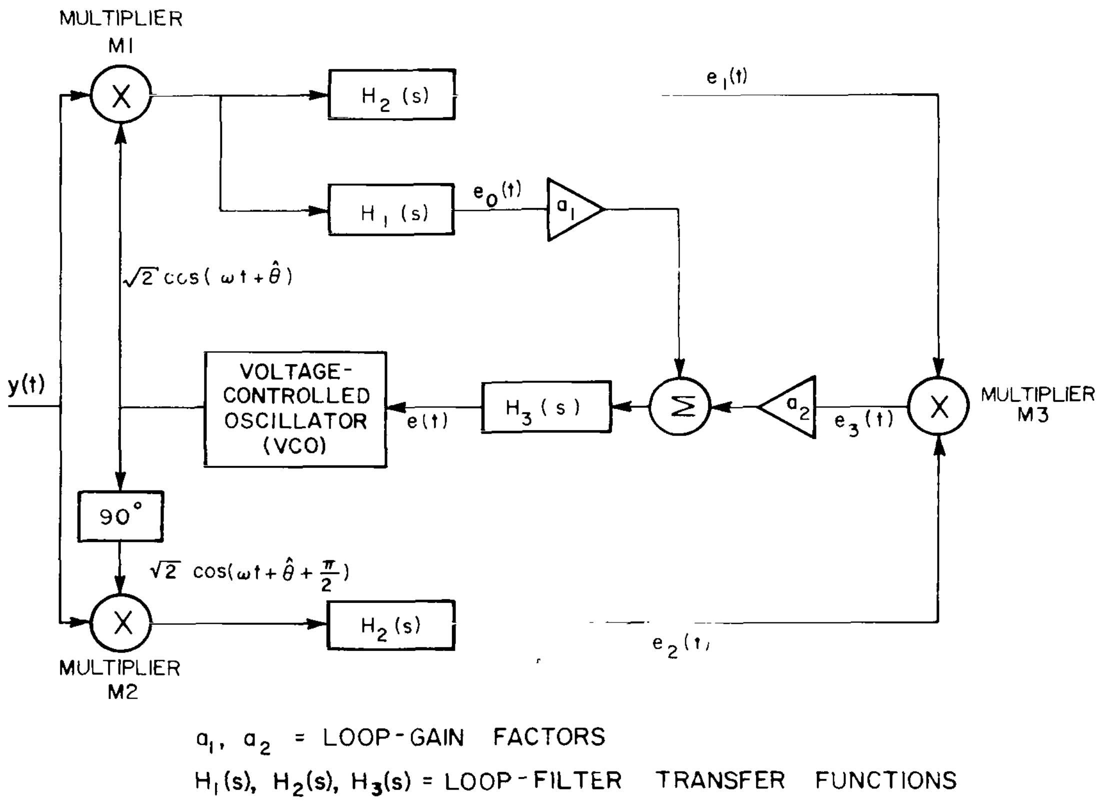

Figure 1.- Block diagram of hybrid loop

the hybrid loop combines the error signals from both loop branches to drive the voltage-controlled oscillator (VCO). The loop filters with transfer functions $H_1(s)$, $H_2(s)$, and $H_3(s)$ remove certain double frequency and modulation components resulting from the multiplication operations, and are otherwise assumed to be flat over the frequency range of the loop.

In addition to the usual system design parameters, the hybrid loop must be optimized with respect to a_1 and a_2, the two gain factors which provide the relative weighting of the hybrid-loop error signals. Also, the modulation index m must be determined to permit proper allocation of the transmitter power between the data and carrier components.

ANALYSIS OF THE HYBRID LOOP

In the configuration depicted in Figure 1, the outputs of multipliers M1 and M2 after filtering by $H_1(s)$ and $H_2(s)$ are given by

$$e_o(t) = Am \sin (\theta-\hat{\theta}) + \frac{n_c(t)}{\sqrt{2}} \cos \hat{\theta} - \frac{n_s(t)}{\sqrt{2}} \sin \hat{\theta},$$

$$e_1(t) = A(1-m^2)^{1/2} x(t) \cos (\theta-\hat{\theta}) + \frac{n_c(t)}{\sqrt{2}} \cos \hat{\theta} - \frac{n_s(t)}{\sqrt{2}} \sin \hat{\theta},$$

and

$$e_2(t) = A(1-m^2)^{1/2} x(t) \sin (\theta-\hat{\theta}) - \frac{n_c(t)}{\sqrt{2}} \sin \hat{\theta} - \frac{n_s(t)}{\sqrt{2}} \cos \hat{\theta}, \qquad (7)$$

where it is assumed that $H_1(s)$ and $H_2(s)$ remove all double-frequency components due to the multiplication. It is further assumed that the bandwidth of $H_1(s)$ is sufficiently narrow to remove the modulated subcarrier $x(t)$ and that $H_2(s)$ removes the low-frequency term from the carrier component. According to these assumptions it would be required that the data subcarrier $x(t)$ have insignificant spectral components near the origin.

The output of multiplier M3 is given by

$$e_3(t) = \frac{A^2(1-m^2)^{1/2}}{2} \sin 2\phi + \frac{n_c(t)}{\sqrt{2}} A(1-m^2)^{1/2} x(t) \sin(\phi-\hat{\theta})$$

$$- \frac{n_s(t)}{\sqrt{2}} A(1-m^2)^{1/2} x(t) \cos(\phi-\hat{\theta}) - \frac{n_s(t) n_c(t)}{2} \cos 2\hat{\theta}$$

$$+ \left[\frac{n_s^2(t)}{4} - \frac{n_c^2(t)}{4}\right] \sin 2\hat{\theta} \qquad (8)$$

where $\phi = \theta - \hat{\theta}$. The VCO error voltage is

$$e(t) = a_1 e_o(t) + a_2 e_3(t) \qquad (9)$$

where the factors a_1 and a_2 combine the estimates from the two sections of the loop. Let us denote the VCO constant by K, which also includes any additional loop gains; then the loop differential equation may be written

$$\frac{d\phi}{dt} = -K\, e(t) \qquad (10)$$

in which the detuning of the VCO from the carrier frequency ω is assumed negligible.

Equation (10) has the form $\dot{\phi} = F(\phi,t)$, and the corresponding Fokker-Planck equation is, in the stationary case,

$$\frac{1}{2} \frac{\partial^2}{\partial\phi^2} K_2(\phi)\, p(\phi) - \frac{\partial}{\partial\phi} K_1(\phi)\, p(\phi) = 0. \qquad (11)$$

Here

$$K_1(\phi) = E\, F(\phi,t), \qquad (12)$$

$$K_2(\phi) = \int_{-\infty}^{\infty} E\; F(\phi,t)\; F(\phi,t+\tau) - K_1^2 \; d\tau . \tag{13}$$

E denotes ensemble averaging, and the assumption is made that the correlation time τ_c of the noise process is much smaller than the loop time constants.* Physically, this is equivalent to the requirement that the loop bandwidth is much smaller than the bandwidth of the noise in the loop. Details of this concept are given in reference 9.

If we now assume that $\phi(t)$ and $x(t)$ are slowly varying so that $x(t) = x(t+\tau)$, then

$$K_1(\phi) = E\; F(\phi,t) = -E\; Ke(t)$$

$$= -K\left[a_1 Am \sin\phi + a_2 \frac{A^2(1-m^2)}{2} \sin 2\phi\right] \tag{14}$$

and

$$K_2 = \int_{-\infty}^{\infty} \frac{K^2}{2} \left\{a_1^2 R_{nn}(\tau) + \frac{a_2^2}{2}\left[R_{nn}^2(\tau) + 2A^2(1-m^2)R_{nn}(\tau)\right]\right\} d\tau . \tag{15}$$

Substituting Eqs. (14) and (15) into Eq. (11) and solving for $p(\phi)$, we have

$$p(\phi) = C \exp \frac{2K}{K_2}\left[a_1 Am \cos\phi + a_2 \frac{A^2(1-m^2)}{4} \cos 2\phi\right] \tag{16}$$

where C is determined by the condition

---

*The correlation time of the random process $n(t)$ is defined as

$$\tau_c = \int_0^{\infty} \frac{|R_{nn}(\tau)|}{R_{nn}(0)} \, d\tau$$

where $R_{nn}(\tau)$ is the correlation function of $n(t)$.

$$\int_{-\pi}^{\pi} p(\phi)\, d\phi = 1 \tag{17}$$

Next, let the envelope of the input noise process have a flat power-spectrum density with magnitude N_o W/Hz and cutoff frequency ω_i Hz. The noise correlation function is

$$R_{n_c n_c}(\tau) = R_{n_s n_s}(\tau) = 2N_o\omega_i \frac{\sin 2\pi\omega_i\tau}{2\pi\omega_i\tau}, \tag{18}$$

and substitution of Eq. (18) into Eq. (15) gives

$$K_2 = \frac{K^2 N_o}{2}\left[a_1^2 + a_2^2 N_o \omega_i + a_2^2 A^2 (1 - m^2)\right]. \tag{19}$$

From the linear phase-lock loop theory, the equivalent noise bandwidth ω_L of the loop is defined as

$$2\omega_L = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} |H(s)|^2\, ds. \tag{20}$$

The expression for H(s) is found by linearizing Eq. (10) giving

$$H(s) = -\frac{KA\left[a_1 m + a_2 A(1-m^2)\right]}{s + KA\left[a_1 m + a_2 A(1-m^2)\right]}, \tag{21}$$

which yields

$$\omega_L = \frac{KA}{4}\left[a_1 m + a_2 A(1-m^2)\right] \text{ Hz}. \tag{22}$$

With the substitution from Eqs. (20) and (22) into Eq. (16), we have

$$p(\phi) = C \exp \frac{D}{E}\left[\cos\phi + \frac{\rho\gamma}{4}\cos 2\phi\right] \tag{23}$$

where

$$D = \alpha\delta[1+\rho\gamma]m^2 \tag{24}$$

and

$$E = 1 + \rho^2\left[1 + \frac{1}{\alpha\delta\beta(1-m^2)}\right][1-m^2], \tag{25}$$

in which expressions

$$\alpha = \frac{A^2}{N_o R} \qquad \delta = \frac{R}{\omega_L}$$

$$\rho = \frac{a_2 A}{a_1} \qquad \gamma = \frac{1-m^2}{m}$$

$$\omega_L = \frac{Ka_1 Am}{4}[1+\rho\gamma] \qquad \beta = \frac{\omega_L}{\omega_i}. \tag{26}$$

With a_2 set equal to zero, or $\rho = 0$, Eq. (23) reduces to

$$p(\phi) = C \exp \alpha\delta m^2 \cos \phi, \tag{27}$$

which is the phase-error probability density function for a conventional phase-lock loop as described in references 2 and 3. If a_1 is set equal to zero, or $\rho = \infty$, then Eq. (23) reduces to

$$p(\phi) = C \exp D \cos 2\phi, \tag{28}$$

with

$$D = \frac{\alpha\delta}{4\left[1 + \frac{1}{\alpha\delta\beta}\right]}, \tag{29}$$

which corresponds to the probability density function for the Costas loop (ref. 6).

PERFORMANCE OF THE HYBRID LOOP

In order to gain some insight into the degree of improvement offered by the hybrid loop, we will first compare the variance of the hybrid-loop phase error with the variance of the phase error for the Costas and conventional phase-lock loops. This will be done under the assumption that the probability density function of the phase error can be approximated by the Gaussian density function. Then, we will obtain exact results for the performance of correlation detectors.

Approximating cos ϕ by $(1 - \phi^2)/2$ in Eqs. (23), (27), and (28), we find the variance of the three loops to be as follows:

1. Hybrid Loop

$$\sigma_H^2 = \frac{1 + \rho^2(1-m^2)\left[1 + \frac{1}{\alpha\delta\beta(1-m^2)}\right]}{\alpha\delta[m+\rho(1-m^2)]^2} , \tag{30}$$

2. Conventional Phase-lock Loop

$$\sigma_P^2 = \frac{1}{\alpha\delta m^2} , \tag{31}$$

3. Costas Loop

$$\sigma_C^2 = \frac{\left[1 + \frac{1}{\alpha\delta\beta(1-m^2)}\right]}{\alpha\delta(1-m^2)} . \tag{32}$$

The optimum value of ρ for combining the loop outputs can be found from Eq. (30) to be:

$$\rho_{opt} = \frac{\alpha\delta\beta(1-m^2)}{m\left[1+\alpha\delta\beta(1-m^2)\right]} = \frac{\sigma_P^2}{\sigma_C^2} \cdot \frac{1-m^2}{m} . \tag{33}$$

Thus, for fixed m, the different loop outputs are combined according to the ratio of the phase-error variances from each section of the loop. If Eq. (33) is substituted for ρ in Eq. (30), we have for the hybrid loop

$$\sigma_H^2 = \frac{1}{\alpha\delta} \frac{[1+\alpha\delta\beta(1-m^2)]}{\left[\alpha\delta\beta(1-m^2)^2 + m^2[1+\alpha\delta\beta(1-m^2)]\right]} = \frac{\sigma_P^2 \cdot \sigma_C^2}{\sigma_P^2 + \sigma_C^2} . \quad (34)$$

From Eq. (34), we see that the variance of the hybrid loop is always less than that of the Costas or phase-lock loop. Also, the maximum improvement is a factor of 2, which occurs when $\sigma_P^2 = \sigma_C^2$.

The variance of the phase error is plotted in Figures 2, 3, and 4 for various combinations of the loop parameters. Examination of the curves indicates that the performance of the phase-lock loop is superior at low signal-to-noise ratios, while that

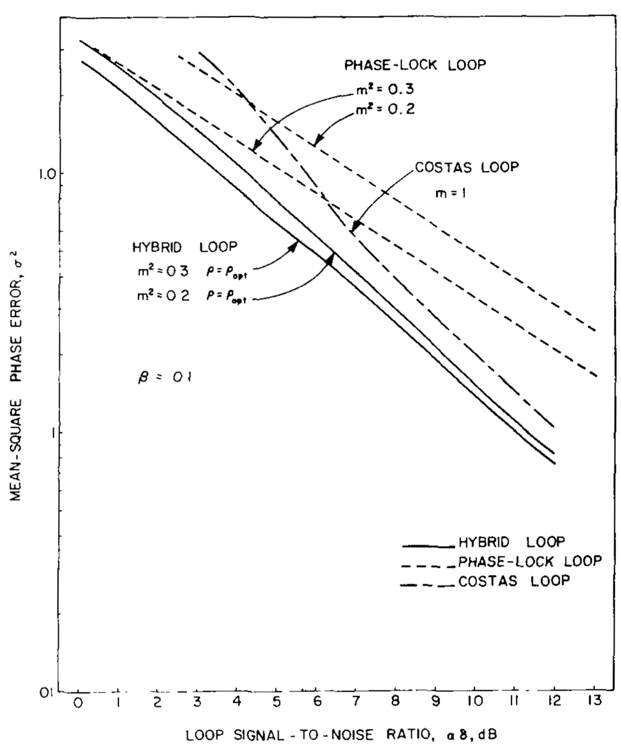

Figure 2.- Mean-square phase error σ^2 versus loop signal-to-noise ratio $\alpha\delta = A^2/N_o\omega_L$ for hybrid, Costas, and conventional phase-lock loops with $\beta = \omega_L/\omega_i = 0.1$

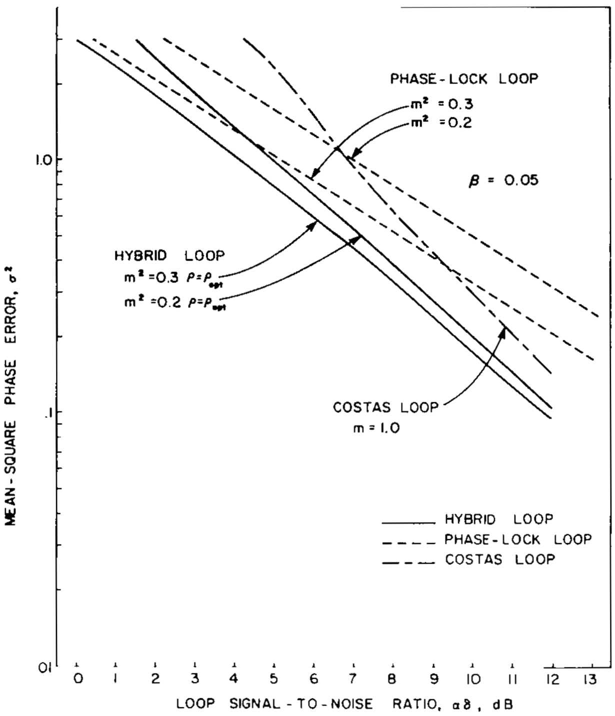

Figure 3.- Mean-square phase error σ^2 versus loop signal-to-noise ratio $\alpha\delta = A^2/N_o\omega_L$ for hybrid, Costas, and conventional phase-lock loops with $\beta = \omega_L|\omega_i = 0.05$

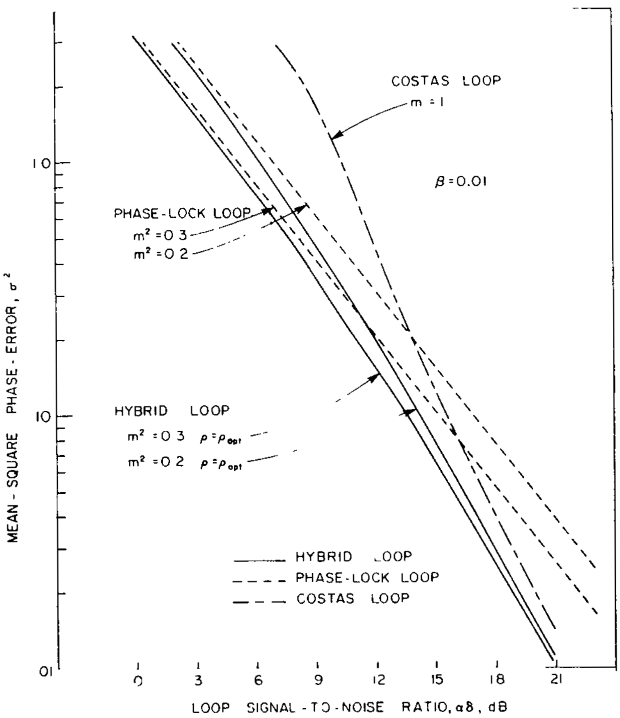

Figure 4.- Mean-square phase error σ^2 versus loop signal-to-noise ratio $\alpha\delta = A^2/N_o\omega_L$ for hybrid, Costas, and conventional phase-lock loops with $\beta = \omega_L|\omega_i = 0.01$

of the Costas loop is better at high signal-to-noise ratios. The performance of the hybrid loop is found to be superior to either the Costas or conventional phase-lock loop, with the maximum improvement at the point where the plots of σ_p^2 and σ_C^2 intersect. Of practical importance is the fact that the performance of the hybrid loop is better over the entire range of signal-to-noise ratios in contrast to the Costas loop, which is poor at low signal-to-noise ratios, and the phase-lock loop, which is poor at high signal-to-noise ratios. Consequently, the hybrid loop will be less sensitive to variations in the signal-to-noise ratio over its operating range.

We now want to compute the performance of a correlation detector which uses a hybrid loop for the phase estimate. The correlation detector computes

$$\lambda = \int_0^T y(t)\ x(t)\ \cos\ (\omega t+\hat{\theta})dt. \tag{35}$$

If $\lambda > 0$, the decision is made that the binary data is +1, and -1 if $\lambda < 0$. It is easy to show that the detector-error probability $P(\varepsilon)$, conditioned on ϕ, is

$$P(\varepsilon|\phi) = \int_{\sqrt{\frac{2A^2(1-m^2)}{N_oR}}\cos\phi}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx\ . \tag{36}$$

The average error probability is found by averaging $P(\varepsilon|\phi)$ over $p(\phi)$ given by Eq. (23). That is,

$$P(\varepsilon) = \int_{-\pi}^{\pi} P(\varepsilon|\phi)\ p(\phi)d\phi. \tag{37}$$

The system is optimized with respect to m and ρ, where ρ_{opt} and m_{opt} are those values of m and ρ which minimize $P(\varepsilon)$. A computer program was written to determine ρ_{opt} and m_{opt}. The results are presented in Figures 5, 6, and 7. Figures 8, 9, and 10 show the corresponding error probabilities.

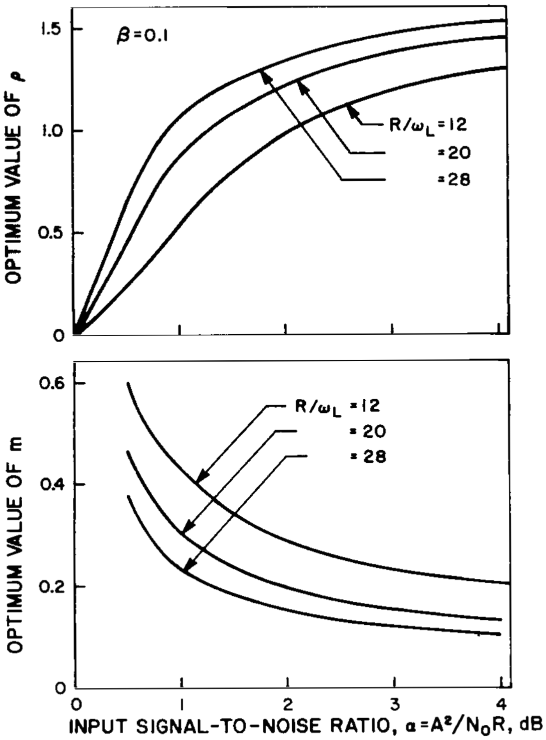

Figure 5.- Optimum parameter ρ and optimum modulation index m versus input signal-to-noise ratio $\alpha = A^2/N_oR$ for hybrid loop with $\beta = \omega_L | \omega_i = 0.1$

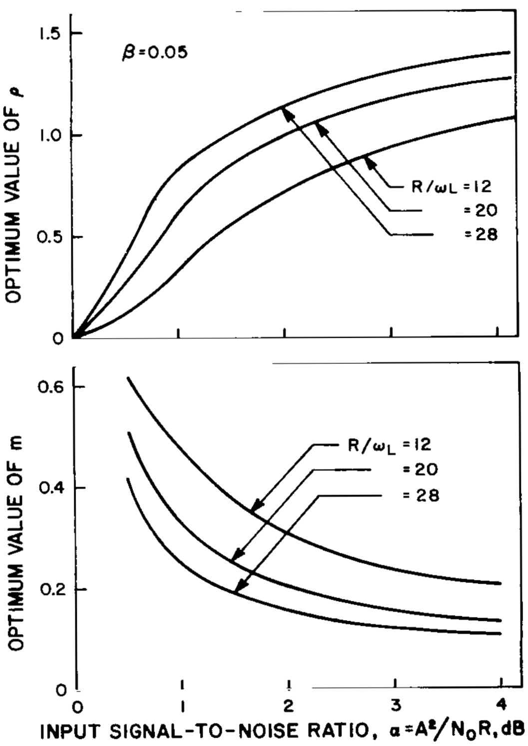

Figure 6.- Optimum parameter ρ and optimum modulation index m versus input signal-to-noise ratio $\alpha = A^2/N_oR$ for hybrid loop with $\beta = \omega_L | \omega_i = 0.05$

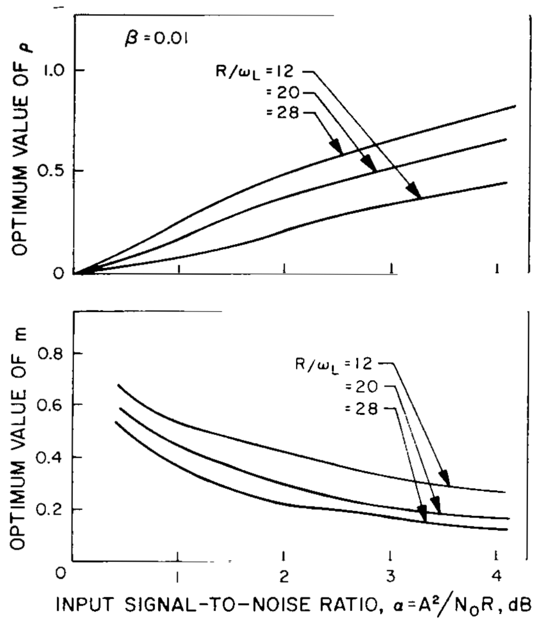

Figure 7.- Optimum parameter ρ and optimum modulation index m versus input signal-to-noise ratio $\alpha = A^2/N_0R$ for hybrid loop with $\beta = \omega_L|\omega_i = 0.01$

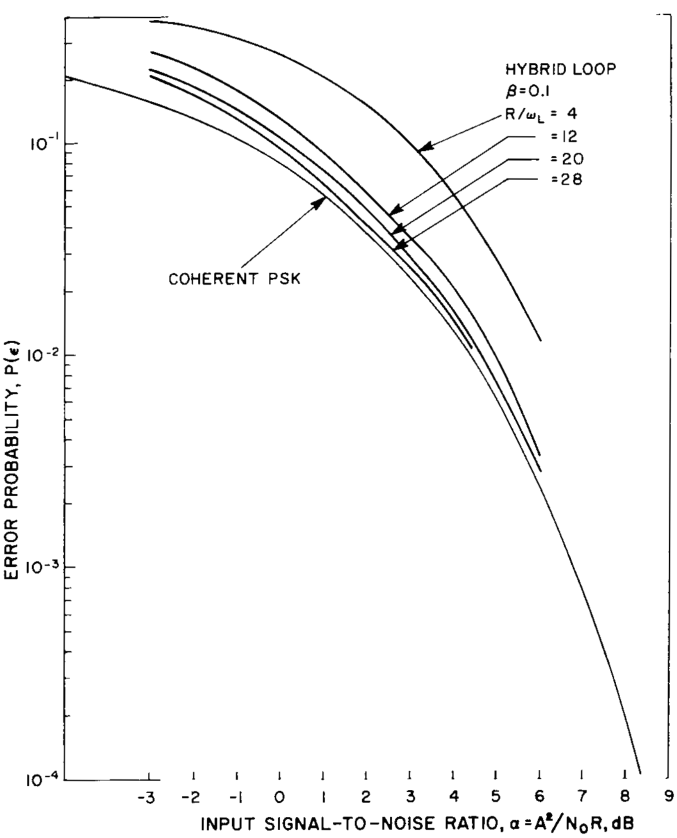

Figure 8.- Receiver error probability $P(\varepsilon)$ versus input signal-to-noise ratio $\alpha = A^2/N_0R$ for hybrid loop with $\beta = \omega_L|\omega_i = 0.1$

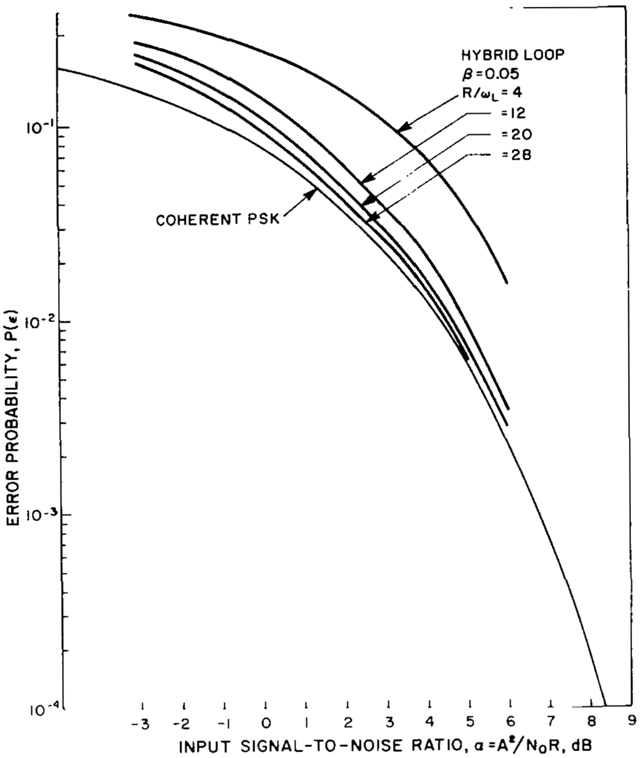

Figure 9.- Receiver error probability $P(\varepsilon)$ versus input signal-to-noise ratio $\alpha = A^2/N_oR$ for hybrid loop with $\beta = \omega_L|\omega_i = 0.05$

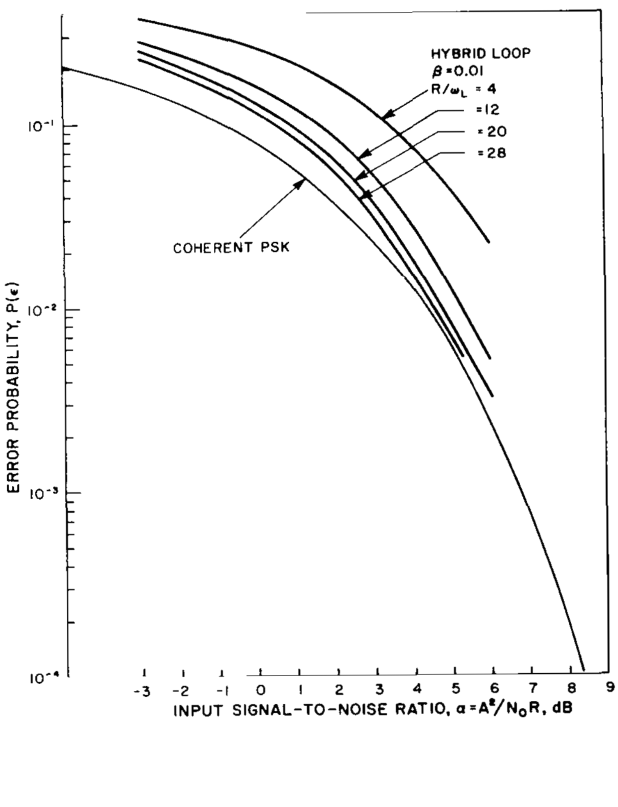

Figure 10.- Receiver error probability $P(\varepsilon)$ versus input signal-to-noise ratio $\alpha = A^2/N_oR$ for hybrid loop with $\beta = \omega_L|\omega_i = 0.01$

In Figures 11, 12, and 13, the hybrid loop is compared with equivalent systems which use a Costas or conventional phase-lock loop. The error probability for the phase-lock loop system is given by Eq. (37), with $p(\phi)$ given by Eq. (27). The phase-lock loop system is optimized with respect to m. For the Costas loop, the phase estimate exhibits a phase ambiguity of π radians. Consequently, a realistic comparison of this system must include the additional degradation due to this difficulty. One scheme for resolving the problem employs differential encoding of the transmitted data (ref. 2). The equation giving the error probability for this system is

$$P(\varepsilon) = \int_{-\pi/2}^{\pi/2} 2\ P(\varepsilon|\phi)[1-P(\varepsilon|\phi)]\ p(\phi)d\phi \tag{38}$$

with $p(\phi)$ as defined by Eq. (28), and $P(\varepsilon|\phi)$ by Eq. (36), with $m = 0$.

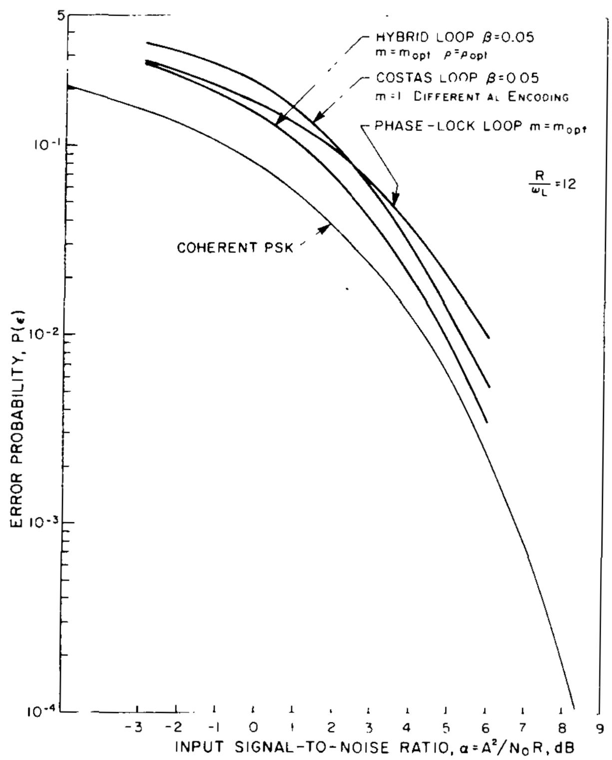

Figure 11.- Comparison of error probabilities versus signal-to-noise ratio for hybrid, Costas, and conventional phase-lock loops with $\beta = \omega_L|\omega_i = 0.05$ and $\delta = R/\omega_L = 12$

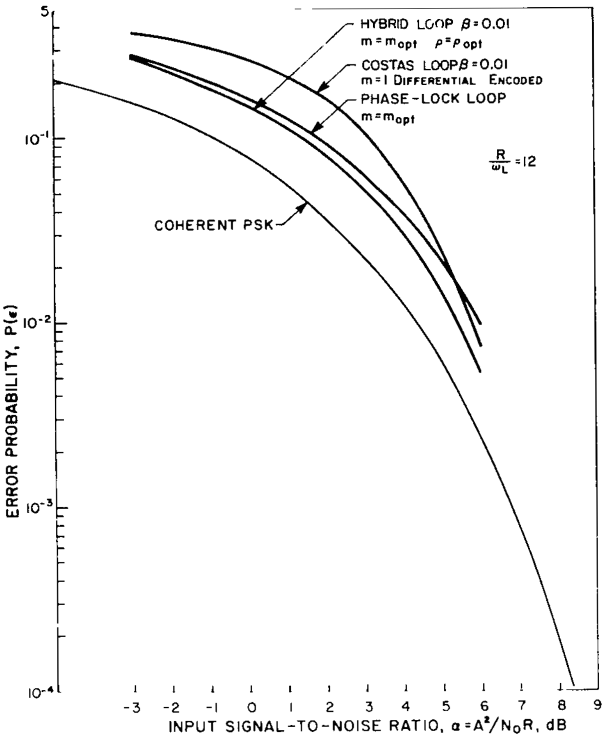

Figure 12.- Comparison of error probabilities versus signal-to-noise ratio for hybrid, Costas, and conventional phase-lock loops with $\beta = \omega_L | \omega_i = 0.01$ and $\delta = R/\omega_L = 12$

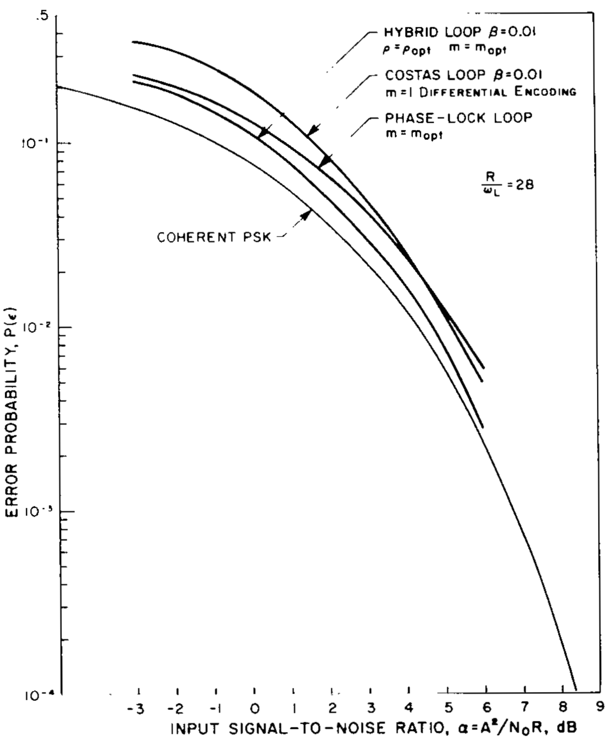

Figure 13.- Comparison of error probabilities versus signal-to-noise ratio for hybrid, Costas, and conventional phase-lock loops with $\beta = \omega_L | \omega_i = 0.01$ and $\delta = R/\omega_L = 28$

CONCLUSION

The hybrid loop offers significant improvements when compared with equivalent systems utilizing a phase-lock loop or Costas loop. The first improvement is that for low data-rate systems, $R/\omega_L < 20$, the hybrid-loop performance is less sensitive to variations in the signal-to-noise ratio. The same is not true for the Costas loop, which degrades rapidly for signal-to-noise ratios less than 3 dB, or the conventional phase-lock loop, which is inferior for signal-to-noise ratios greater than 3 dB. The hybrid loop is, in fact, uniformly better than either of these methods and would give additional protection to systems which operate under wide variations of the signal-to-noise ratio.

In addition, the hybrid loop uses the total received signal power to provide a phase estimate. Depending on the operating signal-to-noise ratio, the hybrid loop can offer a gain improvement in performance of about 1.5 dB when compared with systems which employ a phase-lock loop or Costas loop.

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