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A SURVEY OF THE TRANSPORT PROPERTIES OF HELIUM AT HIGH MACH NUMBER WIND-TUNNEL CONDITIONS

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SUMMARY

A survey of the literature pertaining to the transport properties of He^4 has been made for high Mach number wind-tunnel conditions. Important experimental data are tabulated and correlation techniques examined. As noted by Monchick et al. (Physical Review, Aug. 1965), theoretical calculations of quantum-mechanical effects on viscosity and thermal conductivity (evaluated using the Lennard-Jones (12-6) potential) agree with the available low-temperature data ($<15^\circ$ K). It is pointed out that Reynolds number is often calculated by using Keesom's power-law relation ("Helium," 1942) for viscosity and that this procedure results in an error of about 54 percent at a typical helium-tunnel operating condition (Mach number of about 20 and static temperature of about 2.25° K). Equations for viscosity are presented, based on the quantum calculations, and this method eliminates such errors. Equations for thermal conductivity are also given. This paper is of particular use to those working with high Mach number helium wind tunnels.

INTRODUCTION

For more than a decade, fluid dynamic studies have been made in helium wind tunnels which reach free-stream Mach numbers of 20 and above. Basically, helium is used as a test medium because of the very low temperatures at which helium liquefies (see fig. 1 obtained from table 4.01 of ref. 1). This property greatly simplifies or eliminates both model and facility structural problems which would otherwise occur as a result of the high stagnation temperatures necessary with air as the test medium. As an important consequence of testing in helium, however, quantum effects become important (at free-stream static temperatures below about 8° K) and seriously affect the magnitude of the viscosity and thermal conductivity. Moreover, the empirical equations now commonly used to predict these transport properties are unreliable at such temperatures because of errors in the original experimental data. A survey of how these properties vary under helium-tunnel operating conditions has therefore been made. Other properties important in fluid dynamic studies – such as specific heat at constant pressure, specific-heat ratio, and Prandtl number – were also examined. The results are presented in the form of equations and tables for these variables along with tabulated experimental data. Earlier survey papers on the properties of helium are listed in references 2 to 4. The objective of this paper was to examine existing methods of calculating transport properties of He^4 at high Mach number wind-tunnel conditions and to recommend the most accurate methods. A secondary objective was to compile the latest and most pertinent data.

SYMBOLS

B,C	second and third virial coefficients, cm^3/mol and $\left(cm^3/mol\right)^2$, respectively
c_p	specific heat at constant pressure for helium, cal/gram-$^{\circ}$K; $c_{p,i} = \frac{5}{2} R$ (1 cal/gram-$^{\circ}$K = 4184 joules/kg-$^{\circ}$K = 1 Btu/lbm-$^{\circ}$F)
c_v	specific heat at constant volume for helium, cal/gram-$^{\circ}$K; $c_{v,i} = \frac{3}{2} R$
k	thermal conductivity, cal/cm-sec-$^{\circ}$K (1 cal/cm-sec-$^{\circ}$K = 418.4 joules/m-sec-$^{\circ}$K = 0.80636 Btu-in/ft^2-sec-$^{\circ}$F)
l	mean free path, cm
M	Mach number
p	static pressure, mm Hg
Pr	Prandtl number, $c_p \mu / k$
R	gas constant for helium, $R = 0.4965$ cal/gram-$^{\circ}$K
Re	Reynolds number
Re/x	unit Reynolds number
T	static temperature, $^{\circ}$K unless otherwise specified
T_t	total temperature, $^{\circ}$K
V	velocity
Z	compressibility factor
γ	ratio of specific heats for helium; $\gamma_i = \frac{5}{3}$

μ	viscosity, poise (1 poise = 0.1 N-sec/m^2 = 0.067197 lbm/ft-sec)
ρ	density

Subscripts:

i	ideal gas value
eq.7	calculated from equation (7)
quantum	calculated from quantum theory

RESULTS AND DISCUSSION

A survey has been made of some theoretical and empirical methods of calculating viscosity and thermal conductivity of gaseous He^4. This paper compares recent (ref. 5) low-temperature quantum-mechanical calculations with the available experimental data and recommends the temperature range where the quantum calculations should be used. Curve fits for viscosity and thermal conductivity were applied to the quantum-mechanical calculations, and the resulting equations are accurate within 1 percent. A general discussion of the results as applicable to high Mach number wind tunnels is also provided, particularly the now-common use (refs. 6 to 10) of a power-law relation to calculate the viscosity (and, therefore, Reynolds number) of helium.

The calculations of Monchick et al. (ref. 5) do not account for the pressure-dependent quantum symmetry effect which Hirschfelder et al. (ref. 11) mention as important at temperatures of 1° K and below. Quantum symmetry effects become important when the de Broglie wavelength of the molecules is of the order of magnitude of the mean free path. The de Broglie wavelength was therefore computed and compared with the mean free paths encountered in helium facilities, and it was concluded that the pressure-dependent quantum symmetry effect would not occur in high Mach number helium wind tunnels.

Viscosity

Although it has been recognized for many years that quantum effects are important in helium at low temperatures, accurate theoretical calculations have only recently become available (ref. 5). Keller (ref. 12) did make similar calculations over a more limited temperature range but had expressed doubts – since resolved – concerning the validity of the theory. Monchick's calculations agreed with those of Keller. Both methods were based on the Lennard-Jones (12-6) potential, and an extensive discussion of the theory involved can be found in references 5, 11, and 12.

Theoretical values of viscosity obtained from Monchick's calculations (second-order approximation) are presented in table 1 and plotted in figure 2 together with the available experimental data. The experimental results (presented in table 2) are those of Becker et al. (refs. 13 and 14), Coremans et al. (ref. 15), and Coremans' correction of Van Itterbeek's data (ref. 15). As can be seen in figure 2, the quantum theory offers an accurate prediction of the data at the lowest temperatures. At the higher temperatures ($T > 15^\circ$ K), a small divergence between theory and data occurs because the simple two-constant (12-6) potential used by Monchick does not accurately predict intermolecular forces over wide temperature ranges. At high temperatures, the repulsive forces begin to dominate molecular interactions and the potential becomes less accurate. A curve fit was made to the quantum calculated viscosity data of table 1 with the following results:

$0.2^\circ \text{ K} \leqq T \leqq 1.2^\circ \text{ K}$

$$\mu = \left(2.1630 - 26.665T + 120.54T^2 - 187.41T^3 + 126.82T^4 - 31.823T^5\right) \times 10^{-6} \qquad (1)$$

$1.2^\circ \text{ K} < T < 3.6^\circ \text{ K}$

$$\mu = \left(5.0200 - 3.2241T + 2.0308T^2 - 0.22351T^3\right) \times 10^{-6} \qquad (2)$$

$3.6 \text{ K} \leqq T \leqq 10^\circ \text{ K}$

$$\mu = \left(-1.5691 + 3.4167T - 0.10317T^2\right) \times 10^{-6} \qquad (3)$$

where viscosity is in poise and temperature is in °K.

Because of the lack of adequate theoretical calculations, a number of earlier researchers arrived at empirical equations for viscosity by using the then-available data. Keyes (ref. 16), for example, has provided three equations for the viscosity of helium:

1.64° K to 20° K

$$\mu = \frac{8.48\sqrt{T}}{1 + \frac{1.593}{T}} \times 10^{-6} \qquad (4)$$

20° K to 140° K

$$\mu = 5.272T^{0.6268} \times 10^{-6} \qquad (5)$$

140° K to 373° K

$$\mu = \frac{18.05\sqrt{T}}{1 + \frac{251/T}{10^{(50/T)}}} \times 10^{-6} \qquad (6)$$

In addition, Keesom (ref. 1, p. 107) gave a power-law relation for μ and stated that viscosity could be predicted within 1 percent by using the following equation:

4° K to 1100° K

$$\mu = 5.023T^{0.647} \times 10^{-6} \qquad (7)$$

This expression (sometimes referred to as Akin's equation for viscosity) is currently being used at temperatures below 4° K (refs. 6 to 10). It can be seen in figure 2 that use of equations (4) and (7) is not valid at temperatures below about 8° K since calculations made with these equations diverge from both the latest experimental data and also the quantum theory. The power-law relation, however, does agree with the quantum theory from about 8° K to about 15° K. At temperatures above 15° K, the power-law equation departs somewhat from the quantum theory but agrees with the experimental data. Thus, it is recommended that equation (7) be used above 8° K, whereas equations (1) to (3) should be used below 8° K. Since Mueller (refs. 6 and 7) used equation (7) to calculate Reynolds number at all temperatures, his charts of Reynolds number are in error at temperatures lower than 8° K; a correction technique which enables use of Mueller's charts is discussed in a subsequent section.

Thermal Conductivity

Quantum-mechanical calculations for thermal conductivity were made and are included in table 1. A curve fit applied to these data resulted in the following equations:

$$0.2^\circ\ \mathrm{K} \leqq T \leqq 1.2^\circ\ \mathrm{K} \qquad k = \left(-0.68450 - 0.54637T + 48.304T^2 - 63.865T^3 + 23.701T^4\right) \times 10^{-6} \tag{8}$$

$$1.2^\circ\ \mathrm{K} < T < 3.6^\circ\ \mathrm{K} \qquad k = \left(10.147 - 6.9399T + 4.1353T^2 - 0.45929T^3\right) \times 10^{-6} \tag{9}$$

$$3.6^\circ\ \mathrm{K} \leqq T \leqq 10^\circ\ \mathrm{K} \qquad k = \left(-2.9384 + 6.3590T - 0.19038T^2\right) \times 10^{-6} \tag{10}$$

where k is in cal/cm-sec-°K.

Ubbink and De Haas measured the thermal conductivity of He^4 from 1.62° K to 89.4° K in reference 17, and Ubbink made a later study of thermal conductivity in reference 18. Fokkens et al. (ref. 19) also measured thermal conductivity at temperatures less than 2° K. These measurements show good agreement with those of Ubbink and De Haas in figure 3 and these data are tabulated in table 3. Johnston and Grilly also measured the thermal conductivity of helium between 80° K and 380° K (ref. 20). Akin (ref. 21) has drawn a smoothed curve of thermal conductivity from 1.66° K to 873° K. This curve was based on the data of reference 17 from 1.66° K to 89.5° K and on Keesom's smoothed curve of thermal conductivity from 89.5° K to 373° K; from 373° K to 873° K, Akin calculated the thermal conductivity from viscosity data using kinetic theory and these data have been included in table 3. Keyes (ref. 16) has also correlated thermal conductivity data and has provided an equation which he states is valid within the following temperature limits:

15° K to 375° K
$$k = \frac{23.5\sqrt{T}}{1 + \frac{43.5/T}{10^{(10/T)}}} \times 10^{-6} \tag{11}$$

The different methods of calculating thermal conductivity are shown in figure 3. The quantum-mechanical method offers a good prediction of the data of Ubbink and De Haas (ref. 17) and Fokkens et al. (ref. 19) from 1° K to 4° K and also agrees with Akin's curve to 10° K. The good prediction of the viscosity data by the quantum theory from 1° K to 2° K also leads one to suspect that Fokkens' et al. thermal-conductivity data are somewhat high. The quantum calculation of thermal conductivity is therefore recommended for use below 10° K (eqs. (8) to (10)). Above this temperature, the quantum-mechanical calculations begin to deviate from the data correlation because the (12-6) model fails to closely approximate the intermolecular repulsion force. At temperatures of 1.66° K to 873° K, Akin's correlation may also be used to accurately determine thermal conductivity.

Effect of Pressure on Viscosity and Thermal Conductivity

The previous sections assumed that the mean free path of the gas molecules was small in comparison with the container size. In this region, called the Maxwellian or molar region, only mutual collisions of the molecules need be considered, not their interactions with the container walls (ref. 22). At low pressures, however, the fact that the mean free path can become significant in comparison with the container size would indicate that viscosity and thermal conductivity were dependent on pressure. As pressure approaches zero, viscosity and thermal conductivity approach zero and the rate at which this occurs depends on temperature. (See fig. 2.43 in ref. 1.) The equation used to calculate the mean free path of helium was obtained from the Chapman-Enskog relation (see eq. 2.85 in ref. 1) resulting in

$$l = \frac{4.29\mu\sqrt{T}}{p} \tag{12}$$

where l is in cm. Estimates of the mean free path were computed based on typical helium-tunnel flow conditions and compared with the dimensions of existing helium wind tunnels. From this comparison, it appears that tunnel free-stream properties are in the pressure-independent region.

Van Itterbeek et al. (refs. 23 and 24) have measured viscosity under pressure-dependent conditions; these data (included as table 4) indicate a negligible effect of pressure on viscosity at typical helium-tunnel conditions. Ubbink and De Haas (ref. 17) varied pressure from 10 mm Hg to 760 mm Hg at temperatures from 14.7° K to 89.4° K and found no effect of pressure on thermal conductivity. (See table 3.) In a later paper, however, Ubbink found that thermal conductivity depended somewhat on pressure at 4.2° K and 20° K. (See ref. 18.) Ubbink's data obtained at 4.2° K are included as table 5.

Fokkens' et al. (ref. 19) measurements of thermal conductivity were specified as made at 0.15 mm Hg, but the authors also state they find no pressure dependence in this range. These conflicting experimental results regarding the existence of a pressure effect on viscosity and thermal conductivity at low pressures tend to indicate that the effect is small or nonexistent for helium-tunnel operating conditions.

Reynolds Number

The unit Reynolds number is defined as $\frac{Re}{x} = \frac{\rho V}{\mu}$. Mueller has provided charts of unit Reynolds number (refs. 6 and 7) for helium based on viscosity calculated from equation (7). These charts are in error for static temperatures below 8° K. A simple way to correct Mueller's charts for quantum effects is to use the correction factor given in figure 4. This correction factor was obtained by taking the ratio of viscosity predicted by equation (7) to the appropriate quantum equation (eq. (1), (2), or (3)):

$$\frac{\mu_{eq.7}}{\mu_{quantum}} = \frac{(Re)_{quantum}}{(Re)_{eq.7}} \tag{13}$$

Figure 4 illustrates the importance of including the quantum effects in Reynolds number calculations. At temperatures less than 8° K, significant errors are obtained by using the viscosity calculated from equation (7) to calculate Reynolds number. The maximum error of about 67 percent occurs at $T \approx 1.6^{\circ}$ K. At a typical helium-tunnel flow condition of $M = 20$ and $T_t \approx 300^{\circ}$ K ($T \approx 2.25^{\circ}$ K), use of equation (7) results in $(Re)_{quantum} = 1.54(Re)_{eq.7}$; thus, using Mueller's charts for this case would cause a 54-percent error. Equation (1), (2), or (3) is therefore recommended when calculating Reynolds numbers below 8° K. Above about 15° K, equation (7) should be used to calculate Reynolds number. (See discussion in section entitled "Viscosity.") Between 8° K and 15° K, both methods result in essentially the same answer, with equation (7) being the easiest to use as Mueller has presented Reynolds number based on this equation in graphical form.

Imperfect-Gas Effects

The virial form of the equation of state of a pure gas may be expressed by the relation

$$\frac{p}{\rho RT} = Z = 1 + B\rho + C\rho^2 + \ldots \tag{14}$$

where Z is the compressibility factor and B, C, etc. are functions only of temperature for a given gas and are called the second, third, etc. virial coefficients. If the imperfect-gas effects are small, then Z is close to unity. For low densities,

$$\frac{p}{\rho RT} = Z = 1 + B\rho \tag{15}$$

If B is known, equation (15) may be used to estimate the magnitude of the divergence from perfect-gas conditions. Kilpatrick et al. have calculated the second virial coefficients of helium including quantum effects with the Lennard-Jones (12-6) potential (ref. 25) and also with the exp-six potential (ref. 26). In addition, Keller (ref. 27) has made experimental measurements of the second virial coefficient and also critically examined previous measurements. In reference 26, it was shown that the exp-six potential gave the best agreement with the low-temperature experimental data (also see fig. 5 in ref. 27); therefore, the values of B given in reference 26 were used for the calculations made in the present paper. In order to obtain an expression for B, a curve fit was made to the calculations of reference 26 with the following result:

$$1^\circ\ \mathrm{K\ to}\ 7^\circ\ \mathrm{K} \qquad B = \frac{-330.74}{T^{1.5}} - \frac{119.17}{T^{0.5}} + 9.5945T^{0.5} \tag{16}$$

where B is in $\mathrm{cm^3/mol}$. (Actually, ref. 26 tabulates B for temperatures from 0.3° K to 60° K.)

In order to calculate the deviation from ideal-gas conditions at temperatures and pressures encountered in helium facilities, it is first necessary to obtain an expression for density. Solving the quadratic relation (eq. (15)) for ρ and substituting for R yields

$$\rho = \frac{-1 + \sqrt{1 + \frac{4Bp}{T}\left(1.6024 \times 10^{-5}\right)}}{2B} \tag{17}$$

where the constant 1.6024×10^{-5} gives ρ the units $\mathrm{mol/cm^3}$ for use with equation (15).

The deviation from the perfect-gas equation was then calculated by using equations (15) and (17) and the results are summarized in table 6. From this table, it is evident that the perfect-gas assumption is valid at conditions typical of high Mach number helium wind tunnels.

The departure of the heat capacities from the ideal-gas values was also checked. These quantities depend on the derivatives of the second virial coefficient. Equation (16) was used to obtain dB/dT and d^2B/dT^2 so that heat capacities could be computed at representative conditions. The equations used to calculate c_p and c_v were

$$\frac{c_p}{c_{p,i}} = 1 - \frac{2T}{5R}\frac{d^2B}{dT^2}\,p\left(7.951 \times 10^{-6}\right) \tag{18}$$

$$\frac{c_v}{c_{v,i}} = \frac{1}{c_{v,i}}\left\{c_p - R\left[1 + \frac{1}{R}\frac{dB}{dT}p\left(7.951 \times 10^{-6}\right)\right]\right\} \tag{19}$$

and are given, for example, in reference 1. (Also see ref. 9.) Results are included in table 6 where it is evident that the perfect-gas assumption is valid at helium-tunnel conditions.

Prandtl Number

The Prandtl number is expressed by the relation $Pr = \frac{c_p \mu}{k}$. Prandtl number was obtained from the quantum-mechanical calculations and the results are shown in figure 5. The theoretical Prandtl number determined in this manner is constant at $Pr = 2/3$ until a temperature of about 2° K is reached; below this temperature, Prandtl number drops slightly and approaches a value of 0.638 at 0.2° K.

Cook (ref. 28) smoothed both Becker's et al. (refs. 13 and 14) viscosity data and Ubbink's and De Haas' (ref. 17) thermal-conductivity data for temperatures of 1.5° K to 4° K (in 1/2° increments) and from 15° K to 20° K (in 1° increments); these smoothed experimental points are included in figure 5 and table 7. The experimental data at $T = 1.5^{\circ}$ K to 4° K result in $Pr \approx 0.64$ (though there is considerable scatter in this range), whereas the data at $T = 15^{\circ}$ K to 20° K result in $Pr \approx 0.68$. Prandtl number was also calculated from Keesom's power-law equation for viscosity and Akin's correlation of thermal conductivity data (table 3) from 22.55° K to 366.5° K, and the results shown in figure 5 give $Pr \approx 0.688$. (Also see table 7.)

CONCLUSIONS

A review has been made of the transport properties of helium for application to high Mach number helium wind-tunnel conditions. Important experimental data are summarized and tabulated along with empirical techniques. The conclusions obtained were as follows:

1. For typical facility operating conditions (static temperatures of 2.25° K and static pressures of 0.2 mm Hg to 0.5 mm Hg), it was found that theoretical calculations of viscosity and thermal conductivity made by using the quantum-mechanical calculations offer good predictions of these variables. Reynolds numbers evaluated by using the quantum viscosity are therefore accurate, whereas Reynolds numbers evaluated by Keesom's power-law equation ("Helium," 1942) for viscosity are significantly in error below about 8° K. Above about 15° K, the quantum results become inaccurate and Keesom's power-law equation is more reliable.

2. Viscosity and thermal conductivity are probably independent of pressure at typical helium-tunnel conditions.

3. Ideal gas values for heat capacity may be used at typical helium-tunnel conditions.

4. Prandtl number is constant at about 0.688 above temperatures of about 15° K. From 15° K to about 1° K, Prandtl number may drop to perhaps 0.64 to 0.62.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., March 24, 1970.

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TABLE 1.- VISCOSITY AND THERMAL CONDUCTIVITY CORRECTED FOR QUANTUM EFFECTS

T, °K	μ, poise	k, cal/cm-sec-°K	T, °R
0.2	0.3450×10^{-6}	0.6702×10^{-6}	0.36
.4	1.710	3.321	.72
.6	3.040	5.702	1.08
.8	3.543	6.754	1.44
1.0	3.631	6.933	1.80
1.2	3.698	7.000	2.16
1.4	3.865	7.260	2.52
1.6	4.138	7.730	2.88
1.8	4.490	8.370	3.24
2.0	4.911	9.143	3.60
2.4	5.901	10.99	4.32
2.8	7.008	13.05	5.04
3.2	8.162	15.21	5.76
3.6	9.310	17.34	6.48
4.0	10.43	19.43	7.20
4.5	11.76	21.89	8.10
5.0	13.00	24.21	9.00
5.5	14.17	26.40	9.90
6.0	15.25	28.41	10.8
7.0	17.25	32.18	12.6
8.0	19.07	35.59	14.4
9.0	20.77	38.78	16.2
10.0	22.36	41.75	18.0
12.0	25.33	47.33	21.6
14.0	28.13	52.56	25.2
16.0	30.71	57.41	28.8
20.0	35.59	66.52	36.0
25.0	41.15	76.92	45.0
30.0	46.31	86.57	54.0
35.0	51.14	95.60	63.0
40.0	55.72	103.7	72.0
50.0	64.26	120.2	90.0
60.0	72.21	135.1	108.0
80.0	86.79	162.3	144.0
100.0	100.1	187.3	180.0

TABLE 2.- SUMMARY OF VISCOSITY DATA

Source	T, °K	μ, poise	p, mm Hg
Becker et al. (ref. 13)	1.29	3.40×10^{-6}	0.5
	1.68	4.18	.5
	1.99	4.83	.5
	2.13	5.32	.5
	2.22	5.38	.5
	2.68	6.60	.5
	2.92	7.32	.5
	3.17	7.87	.5
	3.65	9.46	.5
	4.00	10.20	.5
	4.20	10.78	.5
Becker and Misenta (ref. 14)	14.12	28.0×10^{-6}	2.5
	15.45	29.4	2.5
	17.04	31.0	2.5
	18.70	33.1	2.5
	20.33	35.2	2.5
Coremans et al. (ref. 15)	1.44	4.25×10^{-6}	
	1.88	5.26	
	2.79	7.67	
	3.00	7.91	
	3.22	8.44	
	3.48	9.50	
	4.19	11.29	
	4.19	11.34	
	4.21	11.34	
Coremans' smoothed values (ref. 15)	20	34.9×10^{-6}	
	30	45.4	
	40	54.7	
	50	63.1	
	60	71.0	
	70	78.5	
	80	85.6	

TABLE 3.- SUMMARY OF THERMAL-CONDUCTIVITY DATA

Source	T, °K	k, cal/cm-sec-°K	p, mm Hg
Ubbink and De Haas (ref. 17)	1.62	7.9 $\times 10^{-6}$	4
	2.00	9.2	4
	2.78	13.3	4
	3.25	15.8	4
	3.94	19.5	4
	14.7	52.6	10 to 760
	16.8	56.1	10 to 760
	18.0	58.5	10 to 760
	20.1	62.4	10 to 760
	20.9	63.5	10 to 760
	76.3	146	10 to 760
	89.4	165	10 to 760
Fokkens et al. (ref. 19)	1.100	7.98 $\times 10^{-6}$	0.15
	1.304	8.08	.15
	1.547	8.29	.15
Akin (ref. 21)	2.05	9.095 $\times 10^{-6}$	
	5.38	26.04	
	10.94	43.82	
	16.49	54.98	
	22.05	66.14	
	27.61	76.89	
	33.16	86.81	
	88.72	165.4	
	144.3	227.4	
	199.8	278.2	
	255.4	327.4	
	366.5	413.4	
	477.6	492.4	
	588.7	564.3	
	699.8	630.0	
	810.9	692.4	
	866.5	723.5	

TABLE 4.- SUMMARY OF PRESSURE-DEPENDENT VISCOSITY DATA

Source	T, °K	p, mm Hg	μ, poise
Van Itterbeek and Van Paemel (ref. 23)	290.1	760	195.5 × 10^{-6}
		.01031	5.22
		.00593	2.90
		.00333	1.67
	90.27	760	91.0 × 10^{-6}
		15.4	90.7
		1.876	86.18
		.885	81.62
		.415	72.96
		.226	62.09
		.1056	45.94
		.0456	27.59
		.0209	15.01
		.00826	6.69
		.00418	3.667
	20.41	760	35.06 × 10^{-6}
		3.91	34.64
		.897	34.15
		.206	31.72
		.0882	28.54
		.0400	23.23
		.01716	15.54
		.00643	7.73
		.00250	3.463
	14.62	760	29.37 × 10^{-6}
		3.32	29.37
		.471	28.54
		.1227	26.04
		.0489	22.10
		.01526	14.29
		.00452	6.443
Van Itterbeek et al. (ref. 24)	4.19	712.5	12.41 × 10^{-6}
		315.8	11.81
		36.3	11.61
		11.3	11.58
	4.21	602.2	12.47 × 10^{-6}
		298.6	11.99
		153.4	11.68
		43.7	11.63
		17.6	11.60
	3.00	181.8	8.41 × 10^{-6}
		94.9	8.35
		47.0	8.33
		12.7	8.31
		5.0	8.28

TABLE 5.- SUMMARY OF PRESSURE-DEPENDENT THERMAL-CONDUCTIVITY DATA AT 4.2° K

[Source: Ubbink (ref. 18)]

p, mm Hg	k, cal/cm-sec-°K
8.10	20.36×10^{-6} 20.36 20.39
17.4	20.59×10^{-6} 20.59
46.2	20.75×10^{-6} 20.80 20.75 20.75 20.77
101.6	21.04×10^{-6} 21.00 21.00
206	21.14×10^{-6} 21.17 21.14
344	21.29×10^{-6} 21.29 21.35
499	21.49×10^{-6} 21.48 21.48
620	21.77×10^{-6} 21.77 21.79
474	21.46×10^{-6} 21.44 21.43 21.48 21.48 21.46
264.4	21.20×10^{-6} 21.24 21.25
93.7	21.00×10^{-6} 21.00 21.00
43.4	20.92×10^{-6} 20.92 20.92
6.3	20.36×10^{-6} 20.36
32.6	20.90×10^{-6} 20.92 20.90
120.5	21.15×10^{-6} 21.15 21.11
124.5	20.97×10^{-6} 20.98 21.04 21.03
181	21.15×10^{-6} 21.12

TABLE 6.- EFFECT OF TEMPERATURE AND PRESSURE ON IDEAL-GAS BEHAVIOR

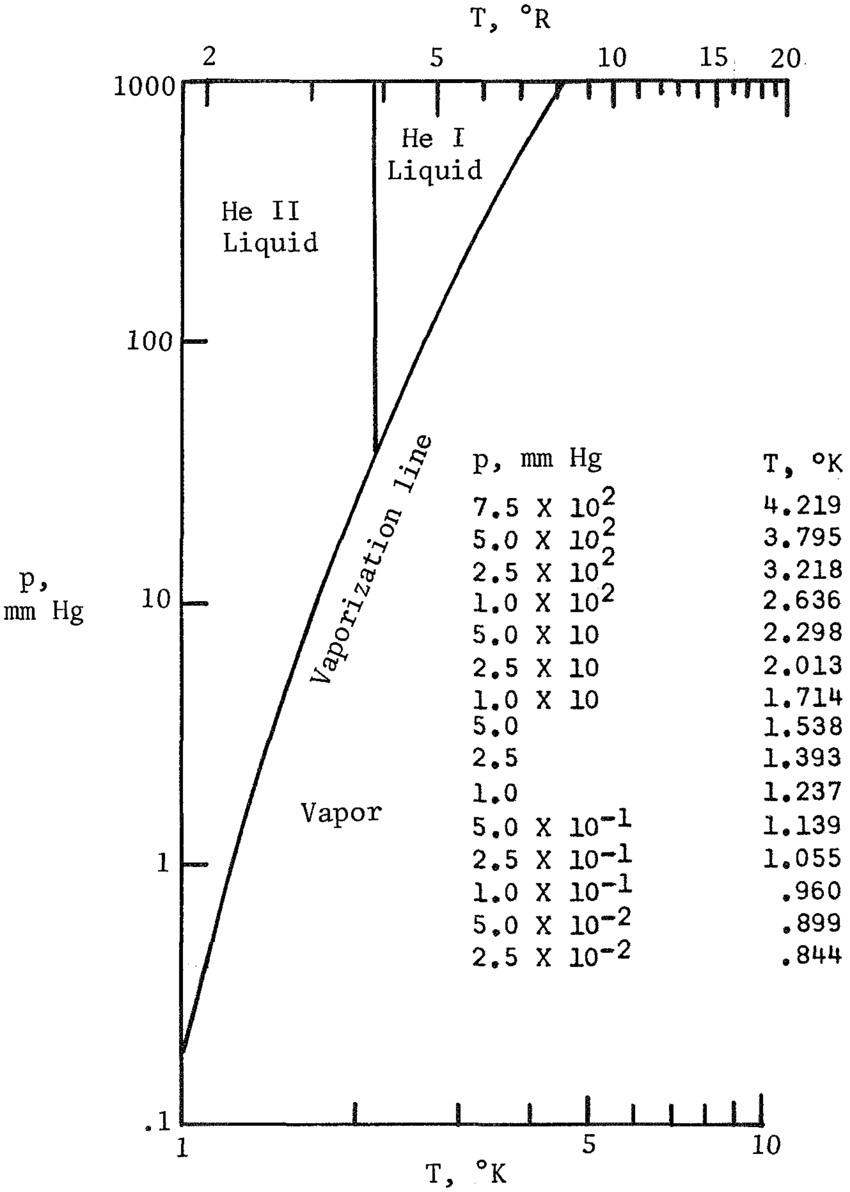
T, ^{o}K	B, cm^3/mol	p, mm Hg	$\frac{p}{\rho RT}$	$\frac{c_p}{c_{p,i}}$	$\frac{\gamma}{\gamma_i}$
1.0	-442.4	0.1	0.999	1.001	1.000
		1.0	.993	1.002	1.001
3.0	-116.24	0.1	1.000	1.000	1.000
		1.0	.999	1.001	1.000
5.0	-61.19	0.1	1.000	1.000	0.999
		1.0	1.000	1.000	1.000

TABLE 7.- EFFECT OF TEMPERATURE ON PRANDTL NUMBER*

$[c_p = 1.241 \text{ cal/gram-}^{o}\text{K}]$

T, ^{o}K	μ, poise	k, cal/cm-sec-^{o}K	Pr
1.5	3.82×10^{-6}	7.5×10^{-6}	0.632
2.0	4.88	9.3	.651
2.5	6.10	11.7	.647
3.0	7.41	14.4	.639
3.5	8.78	17.3	.630
4.0	10.20	20.2	.627
15	28.90	53.2	.674
16	29.90	55.0	.675
17	31.05	56.8	.678
18	32.25	58.5	.684
19	33.50	60.3	.689
20	34.80	62.1	.695
22.05	37.17	66.14	.697
27.61	42.99	76.89	.694
33.16	48.40	86.81	.692
88.72	91.48	165.4	.686
144.3	125.32	227.4	.684
199.8	154.69	278.2	.690
255.4	181.32	327.4	.687
366.5	229.04	413.4	.688

*Data from 1.5^{o} K to 20^{o} K from Cook's smoothed values of Becker's viscosity data and Ubbink's and De Haas' thermal conductivity data; data from 22.05^{o} K to 366.5^{o} K from Keesom's power-law equation for viscosity and Akin's correlation of thermal-conductivity data.



p, mm Hg	T, °K
7.5×10^{2}	4.219
5.0×10^{2}	3.795
2.5×10^{2}	3.218
1.0×10^{2}	2.636
5.0×10	2.298
2.5×10	2.013
1.0×10	1.714
5.0	1.538
2.5	1.393
1.0	1.237
5.0×10^{-1}	1.139
2.5×10^{-1}	1.055
1.0×10^{-1}	.960
5.0×10^{-2}	.899
2.5×10^{-2}	.844

Figure 1.- Diagram of state for He^4 (from ref. 1).

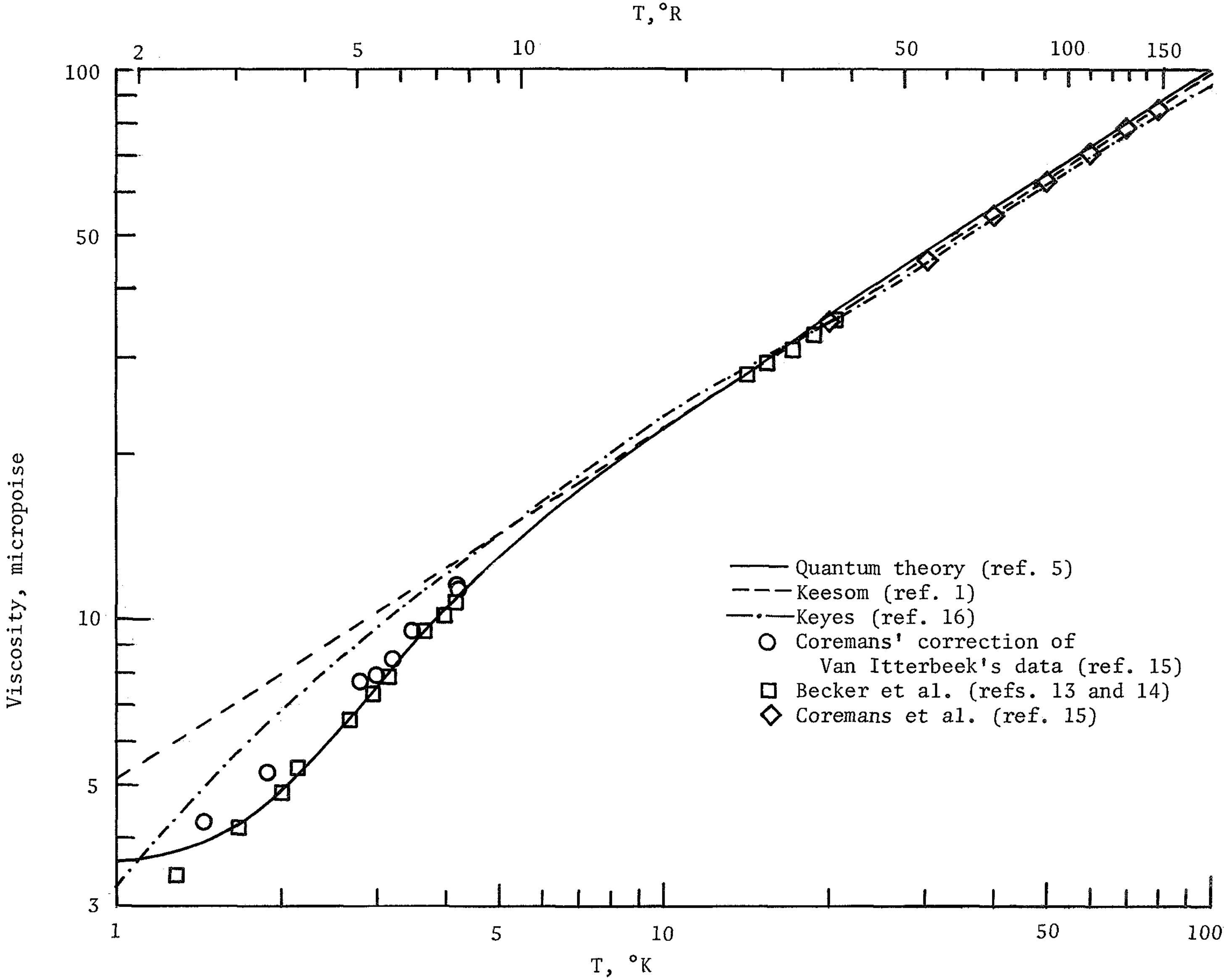


Figure 2.- Effect of temperature on viscosity.

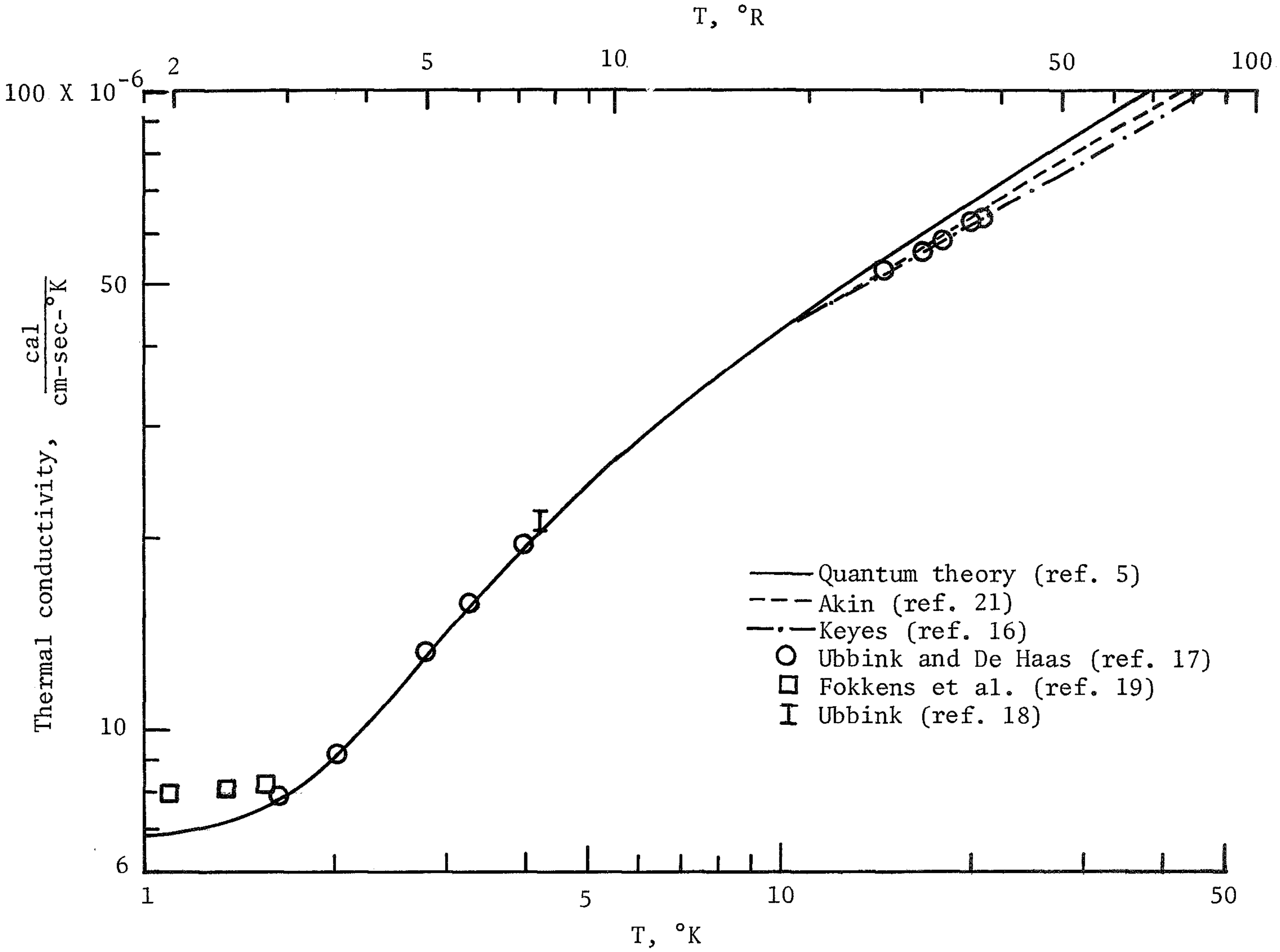


Figure 3.- Effect of temperature on thermal conductivity.

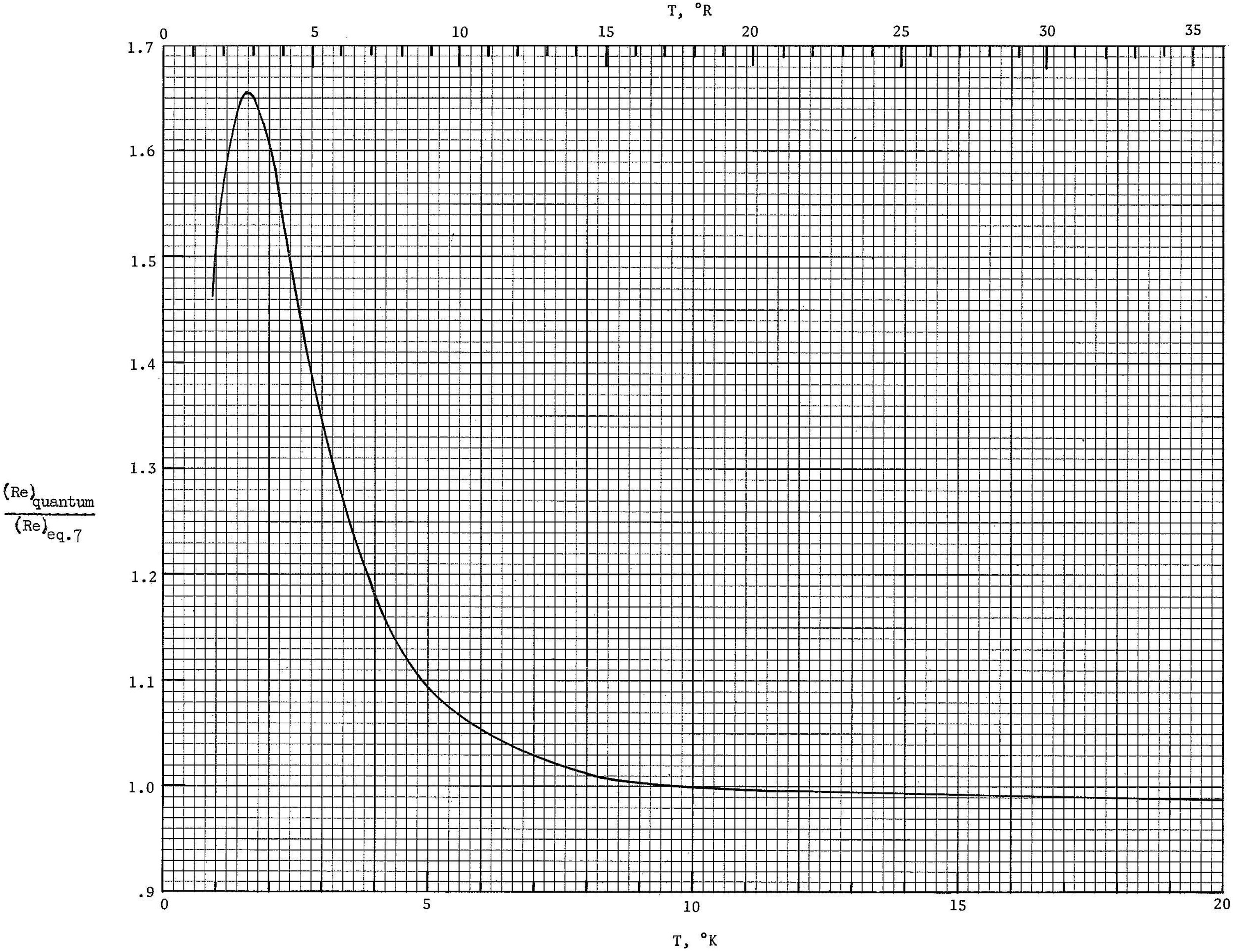


Figure 4.- Ratio of Reynolds number based on quantum viscosity to Reynolds number based on power-law viscosity as a function of temperature.

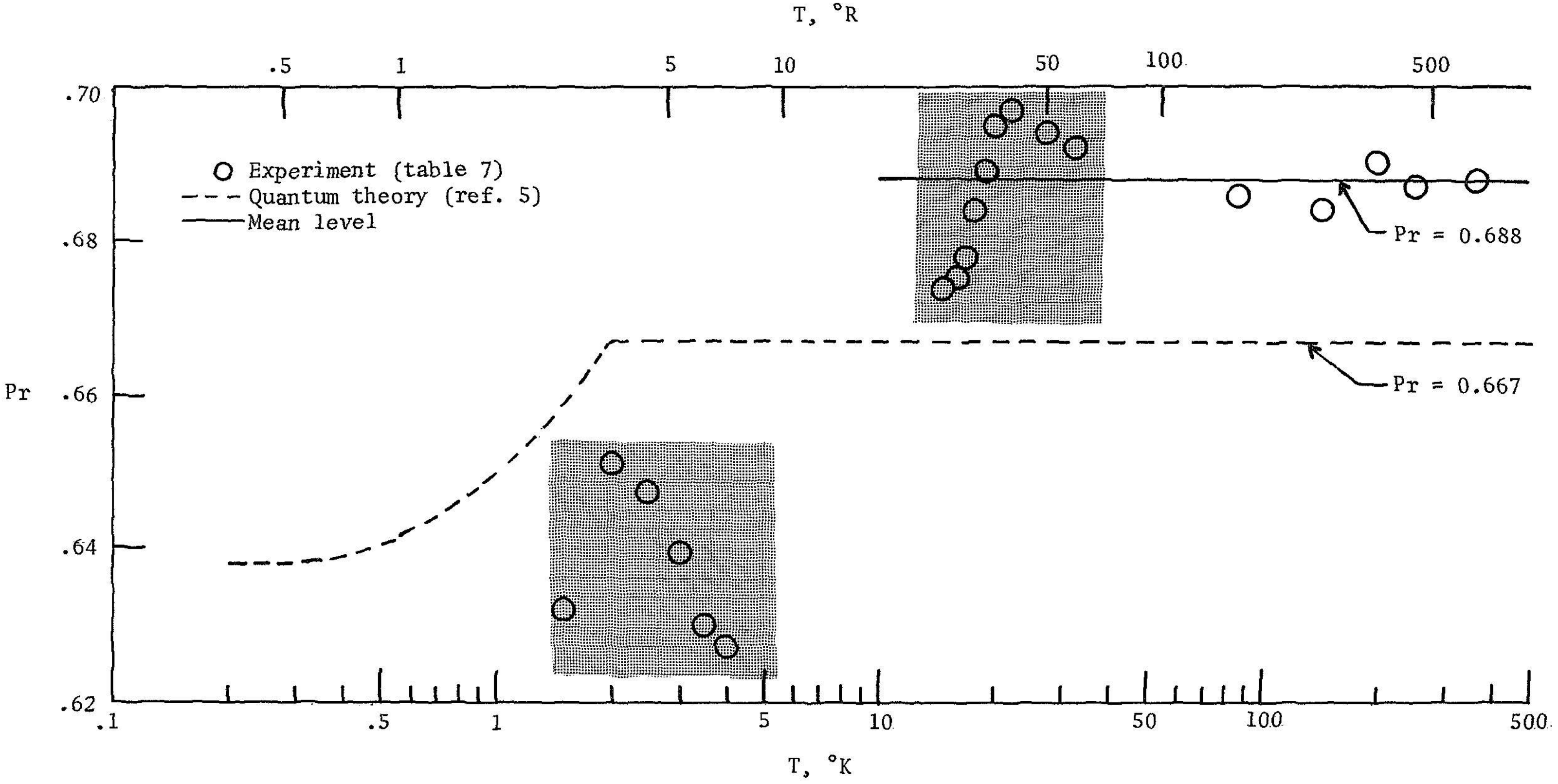


Figure 5.- Effect of temperature on Prandtl number.

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