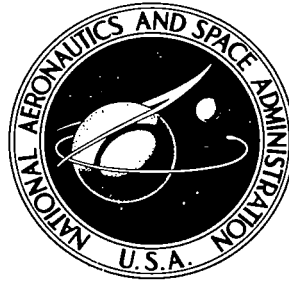


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# ERROR CONTROL USING A CONCATENATED CODE

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16. Abstract  <p>A coding scheme is described which achieves, in some cases, an improvement of two orders of magnitude in data quality for the infrared interferometer spectrometer experiment to be used on the Mariner Mars Probe 1971. This improvement concerns the existing biorthogonal channel. The coding scheme is a concatenation of a biorthogonal code with a generalized Hamming code.</p> <p>A detection philosophy is described which allows the experimenter to select a processing-decoding algorithm to maximize data quantity for a given probability of bit error threshold.</p>					
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# ERROR CONTROL USING A CONCATENATED CODE

by

Bernhard Dorsch\* and Warner H. Miller

*Goddard Space Flight Center*

## INTRODUCTION

A coding scheme is described which is a concatenation of a biorthogonal code with a generalized Hamming code. (Concatenation is the coding of a coded signal.) The generalized Hamming code is a systematic code which has a Hamming distance of three and is described by using elements of a Galois field  $GF(2^s)$ .

In this paper a calculation is made of the probability of bit error, assuming a particular detection philosophy. A comparison, with respect to probability of bit error, of this coding scheme with a binary symmetric uncoded channel is made. With the adopted detection philosophy, a trade-off between data confidence and data quantity can be controlled by the experimenter. The design of the encoder/decoder is also described.

This coding scheme originated from a coding effort as an aid to the infrared interferometer spectrometer experiment on Mariner Mars Probe 1971. This scheme gives the experimenter control of (1) probability of bit error and (2) data quantity. Also, this code yields a higher confidence level for data with respect to the existing Mariner telecommunication channel.

## CODING TECHNIQUE

This coding technique links two codes, an inner and an outer code. The outer coder uses a generalized Hamming code, while the inner coder uses a biorthogonal code. This linking technique achieves longer codes (blocks) for which the probability of error in some cases can be decreased by at least two orders of magnitude with respect to the inner or biorthogonal code.

In the spacecraft, the outer coder accepts serial source data symbols (bits) and reformats them into blocks of data symbols plus check symbols. Each block (Figure 1) consists of  $k$  information elements (words) and two check elements, each element of length  $s$ . The element length is chosen to match the order of the biorthogonal code.

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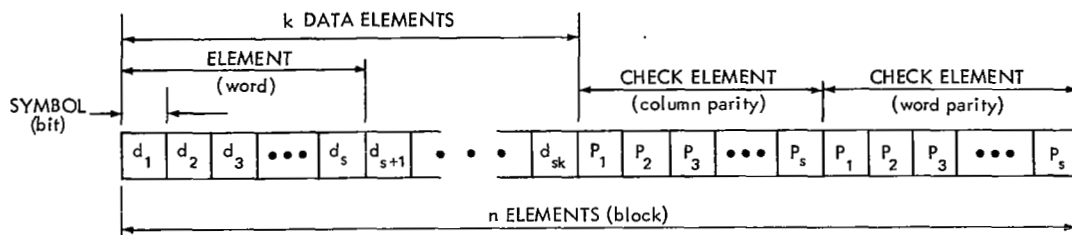


Figure 1—Block diagram for coding technique.

When decoding this concatenated code, the biorthogonal (inner) code is processed first by cross-correlation followed by a maximum-likelihood decision. When an error is made, it is on a word basis, resulting in bursts of bit errors. These burst lengths are related to the biorthogonal element length. For this reason, the outer code was designed as single-element-correcting.

A detailed description of the biorthogonal code will not be given because this is a relatively common code (see Reference 1). A description of the generalized Hamming code follows.

## A NEW GENERALIZED HAMMING CODE

The generalized Hamming code is described in terms of elements of a Galois field  $GF(2^s)$  of two check elements; one which determines the position of the erroneous element, and one which determines its error pattern.

### The Code

Each binary word  $b = (b_1, b_2, \dots, b_s)$  is regarded as one element  $b(z) = b_1 + b_2 z + \dots + b_s z^{(s-1)}$  of the Galois field  $GF(2^s)$ ;  $z$  is defined as a root of the primitive polynomial  $p(z) = 1 + b_1 z + b_2 z^2 + \dots + b_s z^s$  over  $GF(2)$ . The code is the null space of the matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & z & z^2 & \dots & z^{n-3} \end{bmatrix}.$$

The generalized Hamming distance of this code is three (each code word is separated by at least three *elements*) because of the linear independence of every two columns of  $H$ . (The elements  $1, z, z^2, \dots, z^{n-3}$  are distinct.) This requires  $(n-3) \leq (2^s - 2)$  or  $n \leq (2^s + 1)$ . Since this code has distance three, it is one-element-error correcting.

For example, if the element has a length of six symbols, the primitive polynomial is  $p(z) = 1 + z + z^6$  over  $GF(2)$  with the null space matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & z & z^2 & z^3 \end{bmatrix}.$$

## Encoding

Code words are the vectors  $a = (a_1, \dots, a_n)$ ,  $a_v$  of  $GF(2^s)$ , for which  $a \cdot H^T = 0$ ; i.e.,

$$\begin{bmatrix} a_1 + a_3 + a_4 + \dots + a_n = 0 \\ a_2 + a_3 + a_4 z + \dots + a_n z^{n-3} = 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} a_1 = a_3 + a_4 + \dots + a_n \\ a_2 = a_3 + a_4 z + \dots + a_n z^{n-3} \end{bmatrix}. \quad (1)$$

For example, let  $a_1$  and  $a_2$  be the check elements and  $a_3, a_4, a_5$ , and  $a_6$  the information elements. Define  $P_1$  through  $P_6$  and  $p_1$  through  $p_6$  as the check symbols and  $d_1$  through  $d_{24}$  as 24 information symbols.

$$\begin{aligned} a_1 &= P_1 + P_2 z + P_3 z^2 + P_4 z^3 + P_5 z^4 + P_6 z^5 \\ a_2 &= p_1 + p_2 z + p_3 z^2 + p_4 z^3 + p_5 z^4 + p_6 z^5 \\ a_3 &= d_1 + d_2 z + d_3 z^2 + d_4 z^3 + d_5 z^4 + d_6 z^5 \\ a_4 &= d_7 + d_8 z + d_9 z^2 + d_{10} z^3 + d_{11} z^4 + d_{12} z^5 \\ a_5 &= d_{13} + d_{14} z + d_{15} z^2 + d_{16} z^3 + d_{17} z^4 + d_{18} z^5 \\ a_6 &= d_{19} + d_{20} z + d_{21} z^2 + d_{22} z^3 + d_{23} z^4 + d_{24} z^5. \end{aligned}$$

Calculation of  $P$  and  $p$  using Equations 1 and  $p(z) = 1 + z + z^6 = 0$  yields the parity equations:

$$\begin{aligned} P_1 &= d_1 + d_7 + d_{13} + d_{19} \\ P_2 &= d_2 + d_8 + d_{14} + d_{20} \\ P_3 &= d_3 + d_9 + d_{15} + d_{21} \\ P_4 &= d_4 + d_{10} + d_{16} + d_{22} \\ P_5 &= d_5 + d_{11} + d_{17} + d_{23} \\ P_6 &= d_6 + d_{12} + d_{18} + d_{24} \end{aligned} \quad (2)$$

and

$$\begin{aligned}
 p_1 &= d_1 + d_{12} + d_{17} + d_{22} \\
 p_2 &= d_7 + d_2 + d_{18} + d_{23} + d_{12} + d_{17} + d_{22} \\
 p_3 &= d_{13} + d_8 + d_3 + d_{24} + d_{18} + d_{23} \\
 p_4 &= d_{19} + d_{14} + d_9 + d_4 + d_{24} \\
 p_5 &= d_{20} + d_{15} + d_{10} + d_5 \\
 p_6 &= d_6 + d_{11} + d_{16} + d_{21}.
 \end{aligned} \tag{3}$$

### Decoding

**Syndrome:** Let  $a' = a + f = (a_1 + f_1, \dots, a_n + f_n)$  be the erroneous block, where  $f$  represents the error pattern. The Syndrome  $q = a' \cdot H^T$  of a linear code is related to the error pattern  $f$  by the equation

$$q = (a + f) \cdot H^T = f \cdot H^T,$$

because  $a \cdot H^T = 0$ . For the above described generalized Hamming code,

$$a' \cdot H^T = q_1, q_2,$$

$$q_1 = f_1 + f_3 + f_4 + \dots + f_n,$$

and

$$q_2 = f_2 + f_3 + f_4 z + \dots + f_n z^{n-3}.$$

If  $q_1 \neq 0$ ,  $q_2 \neq 0$ , an error is detected. For  $q_1 = q_2 = 0$ , two situations are possible:

1. No error occurred.
2. The error is not detectable.

**Correction:** For the Hamming distance of three, correction can be performed if a single element is disturbed.

If an information element is disturbed ( $v \geq 3$ ),

$$q_1 = f_v \neq 0,$$

$$q_2 = f_v z^{v-3} \neq 0.$$



$q_1$  indicates the error pattern of the disturbed element. The minimum number  $u$  for which  $q_2 z^{-u} = q_1$  is  $u = v - 3$ . Thus  $u + 3 = v$  is the position of the erroneous element.

If a check element is disturbed ( $v = 1$  or  $v = 2$ ), either  $q_1 = f_v \neq 0$ ,  $q_2 = 0$ , or  $q_1 = 0$ ,  $q_2 = f_v \neq 0$ .

If no element is disturbed,  $q_1 = q_2 = 0$ .

## Hardware

### Encoding

Source symbols are serially shifted into two shift registers, ( $Y_1, Y_2$ ), where the parity elements  $a_1$  and  $a_2$  are generated; see Figure 2. The "flip-flops" ( $Y_{k,v}$ ) of the feedback shift register  $Y_k$ , store the vectors  $y_k = (y_{k,0} \cdots y_{k,s-1})$ , or in polynomial representation,  $y_k(z) = y_{k,0} + y_{k,1}z + \cdots + y_{k,s-1}z^{s-1}$ .

Starting with the contents  $y_1 = y_2 = 0$ , the following operations are performed:

- $s$  shifts of the input and the registers  $Y_1$  and  $Y_2$  to the right with  $S_1 = 0$  add the elements  $a_v$  to  $y_1$  and  $y_2$ . Simultaneously  $a_v$  comes to the output.
- One shift of  $Y_2$  to the right with  $S_1 = 1$  multiplies  $y_2$  by  $z \bmod p(z)$ .

Repeating the operation  $n-2$  times yields  $y_1 = a_1$  and  $y_2 = a_1$  by Equation 1. After switching  $S_2$ ,  $2s$  shifts of both  $Y_1$  and  $Y_2$  ( $S_1 = 0$ ) to the right causes  $a_2$  and  $a_1$  to come to the output.

### Decoding

See Figure 2. Decoding circuitry consists of hardware similar to the encoding hardware with the addition of a word detector ( $y_1 = y_2$ ). Also,  $Y_2$  must be capable of being shifted to the left.

**Syndrome:** By repetition of the encoding procedure the following are obtained:

$$y_1 = a_3' + a_4' + \cdots + a_n'$$

$$y_2 = a_3' + a_4' + \cdots + a_n' z^{n-3}.$$

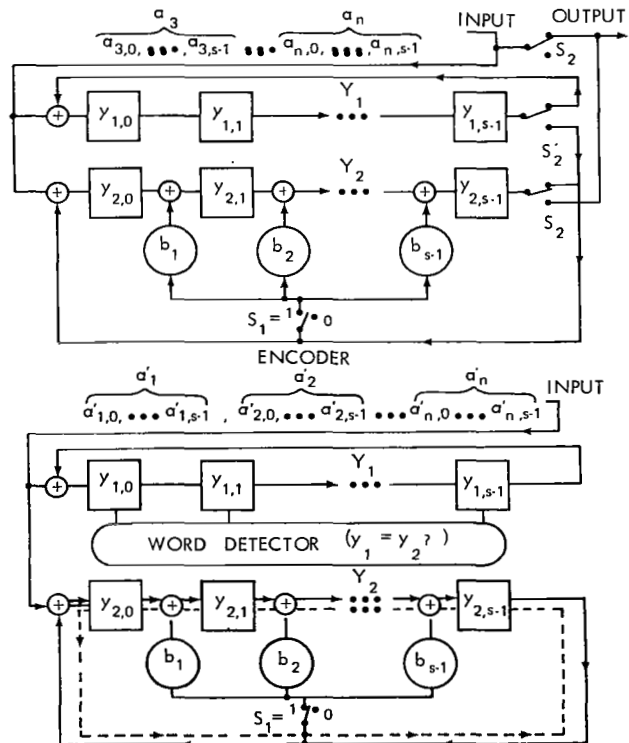


Figure 2—Hamming code encoder and decoder.

The first  $s$  parity bits ( $a_2'$ ) are shifted into  $Y_2 (S_1 = 0)$ ; the next  $s$  parity bits ( $a_1'$ ) are shifted into  $Y_1$ ; this forms the total syndrome.

$$y_1 = a_1' + a_3' + a_4' + \dots + a_n' = q_1,$$

$$y_2 = a_2' + a_3' + a_4' z + \dots + a_n' z^{n-3} = q_2.$$

The information elements ( $a_3', \dots, a_n'$ ) are stored temporarily.

Correction Technique: An error in an information element results in

$$y_1 = f_v \neq 0,$$

$$y_2 = f_v z^{v-3} \neq 0.$$

Multiplying  $y_2$  by  $z^{-u}$  (performed by  $u$  shift of  $Y_2$  to the left with  $S_1 = 1$ ) gives  $y_2 z^{-u} = y_1$  at the position  $v = u + 3$  with  $a_v'$  shown to be the element in error.

If a check element is disturbed, the condition  $y_2 z^{-u} = y_1$  will never be achieved, and no information element is assumed to be in error.

If no error occurs, the condition  $y_2 z^{-u} = y_1$  will be achieved by  $u = 0$ , and  $y_1 = y_2 = 0$  will be added to  $a_3'$ .

If more elements are wrong, no error correction is attempted and this data is erased.

## QUANTITATIVE RESULTS

At the receiver, the noisy data is processed first by an inner decoder and then by an outer decoder. The inner decoder uses correlation and a maximum-likelihood detector to detect the binary elements of length " $s$ ". The probability of element error  $p(s)$  at the output of the inner decoder can be calculated from the expression

$$p(s) = 1 - \int_{-(2s\text{SNR})^{1/2}}^{\infty} \frac{e^{-v^2/2}}{\sqrt{2\pi}} dv \left[ \int_{-\{v+(2s\text{SNR})^{1/2}\}}^{\{v+(2s\text{SNR})^{1/2}\}} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz \right]^{2^{(s-1)}-1},$$

where  $s$  is the number of symbols per element, and SNR is the signal-to-noise ratio defined as signal energy per symbol per single-sided noise power density (see Reference 1). From the inner decoder the detected binary signal with errors is passed to the outer decoder for parity checks on each block of data.

Parity checks are performed by a detector and then by the corrector. The following algorithm is used in the processing of each block of data:

1. The detector detects blocks in which there are known errors, and passes these blocks to the corrector. The remaining blocks, considered to possess the highest confidence, are passed to the user.
2. The corrector corrects blocks which have  $t$  or fewer element errors, where  $t$  is the maximum number of possible correctable error elements. It then discards those blocks which it determines to have more than  $t$  element errors.

If the user requires his data to have the highest possible confidence, only the data from the detector is used. However, if the user is willing to sacrifice confidence for data quantity, the corrector data may be included with the detector data. Of course, the corrector data can always carry a flag with it, indicating its lower confidence level.

## Synchronization

In this paper, element and block synchronization are assumed to be perfect, and it is assumed that the biorthogonal code vectors are synchronized with the block. Even though synchronization is assumed, no loss in generality occurs in the following derivation because element synchronization, which is needed to perform the correlation and maximum-likelihood decision, can easily be derived from biorthogonal code vectors. This element synchronization property of the biorthogonal code set can be enhanced by using a co-set code to maximize the comma freedom property (see Reference 2). Block synchronization can be acquired by minimizing the discard rate. Synchronization of the biorthogonal code vectors with the block is by design.

## Basic Equations

In Figure 3, probability of bit error at the detector output is defined as  $P_A$ , the probability of bit error at the corrector output is  $P_B$ , and the combined probability of bit error is  $P_C$ .  $A_1$  through  $A_5$  are defined to be the data quantities (normalized to unity) at the indicated places in the figure.  $P_A$ ,  $P_B$ ,  $P_C$ , and  $A_1$  through  $A_5$  are calculated below. First, however, some expressions from which the error detection/correcting capability can be determined must be developed to relate the generalized Hamming distance to the block length.

The generalized Hamming code vector has a length of  $n$  elements,  $k$  information elements

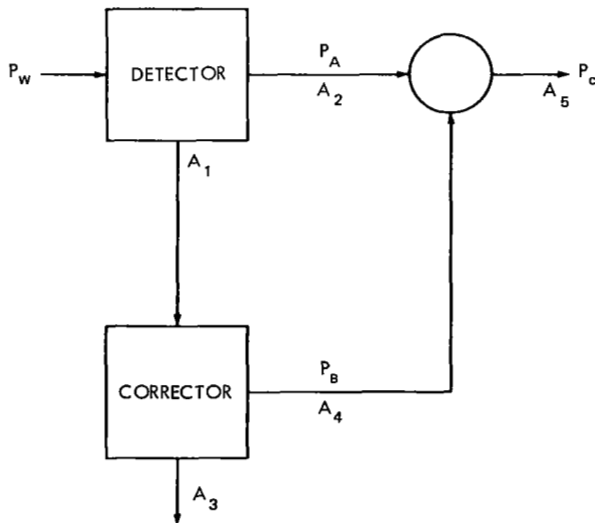


Figure 3—Detector—corrector processing.

and  $m$  check elements. Each element length consists of  $s$  symbols. The described code is a specific case of a Reed-Solomon code, in which the minimum distance,  $d$ , between elements is related to the number of check elements,  $m$ , by  $d = m + 1$ . The maximum number of correctable elements per block,  $t$ , is related to the check element by  $m = 2t$ .

1. The following is a calculation to determine the normalized data quantities  $A_1$  and  $A_2$ , and the probability,  $P_A$ , of bit error at the detector output.

The probability,  $P_0$ , that a block of  $m$  elements has no errors is:

$$P_0 = (1 - p)^m$$

where  $p$  is the probability of element error. One property of the code is that all errors of the minimum distance  $d$  can be detected. Therefore, the probability,  $P_3$ , that  $d$  or more element errors exist in the block is:

$$P_3 = \sum_{j=2t+1}^n \binom{n}{j} p^j (1 - p)^{n-j}$$

Then the probability,  $P_4$ , of undetectable errors is that fraction of  $P_3$  which is:

$$P_4 = 2^{-ms} P_3.$$

The quantity of data,  $A_2$ , at the detector output can now be calculated from the above expression:

$$A_2 = P_0 + 2^{-ms} P_3.$$

The probability,  $P_A$ , of bit error at the detector output is:

$$P_A = \frac{2^{-ms} P_3}{P_0 + 2^{-ms} P_3} \left( \frac{2t + 1}{n} \right) P_{WB},$$

where the probability of block error is the first term of  $P_A$ , given immediately below.

$$\frac{2^{-ms} P_3}{P_0 + 2^{-ms} P_3}$$

The approximate conditional probability of word error, given that a block is in error, the second term of  $P_A$ , is

$$\frac{2t + 1}{n},$$

and  $P_{WB}$  represents the conditional probability of bit error, given that a word is in error. For a biorthogonal inner coder with  $s$  greater than 5,  $P_{WB}$  is approximately equal to  $1/2$ .

Blocks of data which are known to contain errors are passed to the corrector to be reprocessed. The quantity of data  $A_1$  is equal to  $1 - A_2$ .

2. The following is a calculation of the quantities of corrected data,  $A_4$ , and discarded data,  $A_3$ , and the probability,  $P_B$ , of a bit error in the corrected data.

Assume that all blocks of data with  $t$  or fewer elements in error are correctable. Then the probability,  $P_1$ , that there are only correctable errors in a block is determined from the binomial expression

$$P_1 = \sum_{j=1}^t \binom{n}{j} p^j (1-p)^{n-j}.$$

The probability,  $P_2$ , of uncorrectable error in a block (the probability that more than  $t$  elements in a block are in error) can be determined from

$$P_2 = \sum_{j=t+1}^n \binom{n}{j} p^j (1-p)^{n-j}.$$

The fraction,  $H$ , of uncorrectable data that is passed by the corrector is

$$H = \frac{\binom{n}{1}(2^s - 1) + \binom{n}{2}(2^s - 1)^2 + \dots + \binom{n}{t}(2^s - 1)^t}{(2^{ms} - 1)}$$

The fraction of all incorrect data which is passed by the corrector is  $P_2 \cdot H$ . The data quantity,  $A_4$ , at the corrector output can be predicted from the expression

$$A_4 = A_1 \left( \frac{P_1 + HP_2}{P_1 + P_2} \right).$$

The data quantity,  $A_3$ , which is discarded as uncorrectable is given by

$$A_3 = A_1 \left( 1 - \frac{P_1 + HP_2}{P_1 + P_2} \right).$$

The probability of bit error at the corrector output is given by

$$P_B = \frac{HP_2}{P_1 + HP_2} \left( \frac{2t + 1}{n} \right) P_{WB}.$$

where the probability that the block is in error, the first term of  $P_B$ , is

$$\left[ \frac{HP_2}{P_1 + HP_2} \right]$$

and an approximation of the conditional probability of word error, given that the block is in error, the second term of  $P_B$  is

$$\left( \frac{2t+1}{n} \right).$$

$P_{WB}$  is the conditional probability of bit error given a word is in error.

3. The quantity of data,  $A_5$ , and the probability,  $P_C$ , of bit error of the combined data from the detector and the corrector are

$$A_5 = A_2 + A_4$$

and

$$P_C = \frac{P_A A_2 + P_B A_4}{A_2 + A_4}.$$

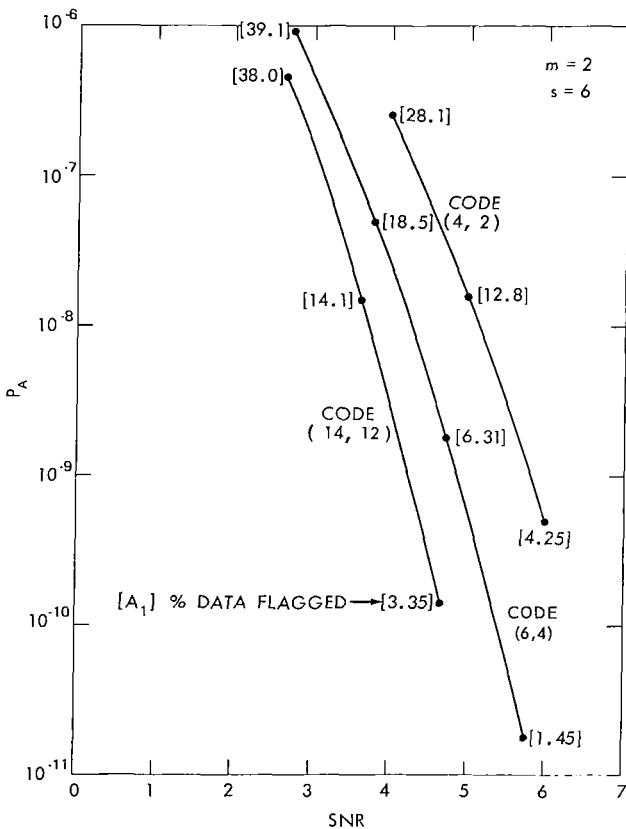


Figure 4—Probability of bit error at detector output for various SNR with  $m = 2, s = 6$ .

If the channel information rate is to remain constant after coding, an increase in channel bandwidth is required to send the check elements. This increase in bandwidth will result in a lower effective SNR. The change in SNR from the situation when check elements are sent, to that when no check elements are sent, can be computed from the equation

$$\Delta \text{SNR} = 10 \log \frac{m}{m+k},$$

where  $(m+k)/m$  is the increased bandwidth factor required by the Hamming code vectors.

The probability,  $P_A$ , of bit error at the detector output varies as a function of SNR as shown in Figure 4. For these curves, at a constant two check elements per block, the block length  $k$  was varied from two to twelve elements.

This generated the codes (4,2), (6,4), (10,8), and (14,12). The first number in the parentheses represents the total number of elements per block ( $m+k$ ), and the second number represents the number of information elements per block ( $k$ ). It was also assumed that the inner coder used a bi-orthogonal code of length six. The numbers shown in brackets in Figure 4 denote the percentage of data having detected errors. Figure 5 shows a family of iso-probabilistic curves in which the SNR can be determined from a given block length and a given bit error rate. The curves of Figure 5 are a replot of the data used in Figure 4.

The next set of data was obtained after the operations of detection and correction had been performed. Figure 6 shows a family of curves in which the probability,  $P_c$ , of bit error for the combined outputs of the detector and corrector varies as a function of SNR. Figure 7 is a family of iso-probabilistic curves in which the SNR can be determined from a given block length and a given bit error rate.

Codes were selected to maintain a constant information-to-transmitted symbol rate of  $2/3$ . This was achieved by varying both the number of check elements and the number of information elements per block. Codes considered were (6,4), (12,8), and (18,12). Table 1 shows the probability,  $P_A$ , of bit error at the detector output and the percentage of detected errors for various SNR and codes.

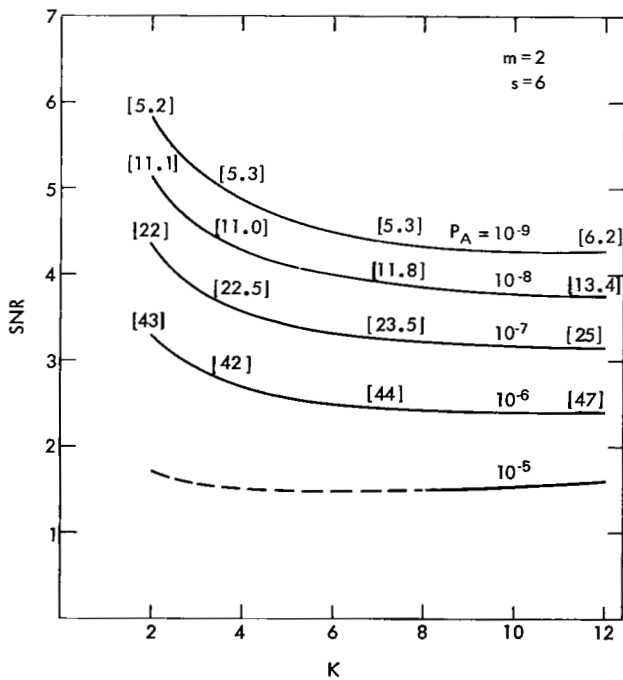


Figure 5—Iso-probabilistic curves as a function of  $k$  with  $m = 2$ ,  $s = 6$ ; for error detection only.

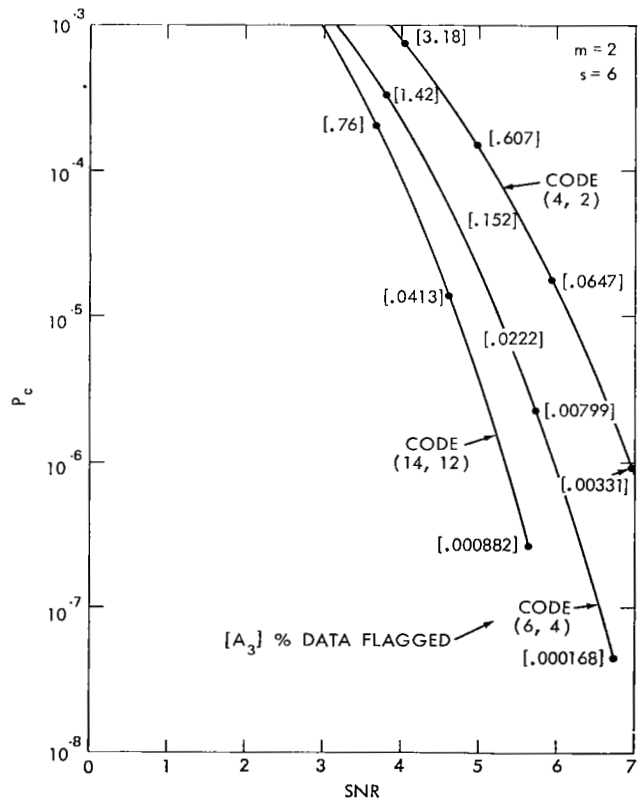


Figure 6—Probability of bit error after error correction for various SNR with  $m = 2$ ,  $s = 6$ .

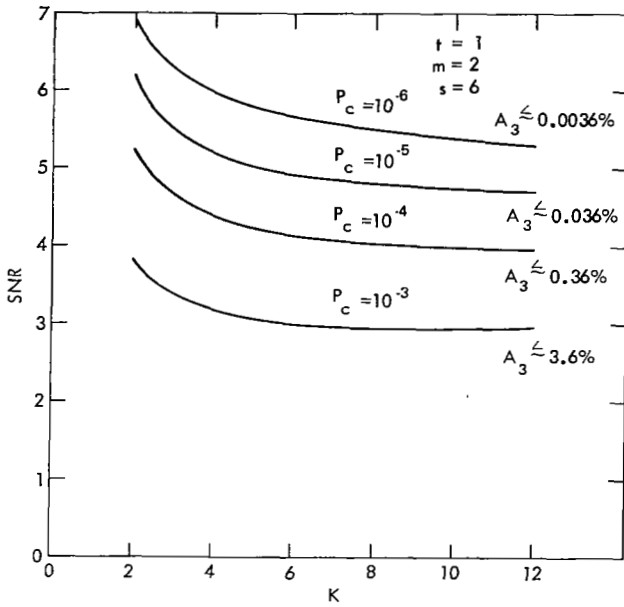


Figure 7—Iso-probabilistic curves as a function of k with  $m = 2, s = 6$ ; with error corrections.

Probability of bit error for the combined outputs of the detector and corrector is shown in Figure 8 for various SNR and rate-2/3 codes. The numbers in brackets indicate the percentage of data that is detected in error.

### APPLICATION OF CODE (6,4) TO THE MARINER MARS SPACECRAFT

Mariner Mars Probe 1971 is an orbital mission on which an infrared interferometer spectrometer (IRIS) experiment (Reference 3) and a television experiment are to provide information about the Martian atmosphere and surface. The spacecraft telecommunication channel was designed primarily for the television experiment, which does not require a data confidence as great as that of the IRIS experiment. Under worst-case conditions, the confidence required by the IRIS experiment is almost two orders of magnitude greater than that which the telecommunication channel

Table 1  
Probability,  $P_A$ , of Undetected Bit Error and the Percentage of Detected Errors for Various Rate-2/3 Codes as a Function of SNR.

Code	SNR	$P_A$	$A_1$ in %
(6,4)	2.76 db	$8.35 \times 10^{-7}$	39.1
	3.76	$5.26 \times 10^{-8}$	18.5
	4.76	$1.60 \times 10^{-9}$	6.3
	5.76	$1.76 \times 10^{-11}$	1.4
	6.76	$5.33 \times 10^{-14}$	0.21
(12,8)	2.76 db	$5.20 \times 10^{-11}$	62.9
	3.76	$5.20 \times 10^{-13}$	33.6
	4.76	$1.54 \times 10^{-15}$	12.2
	5.76	$8.46 \times 10^{-19}$	2.8
	6.76	$5.19 \times 10^{-23}$	0.42
(18,12)	2.76 db	$3.61 \times 10^{-15}$	77.4
	3.76	$5.80 \times 10^{-18}$	45.9
	4.76	$1.69 \times 10^{-21}$	17.7
	5.76	$4.54 \times 10^{-26}$	4.28

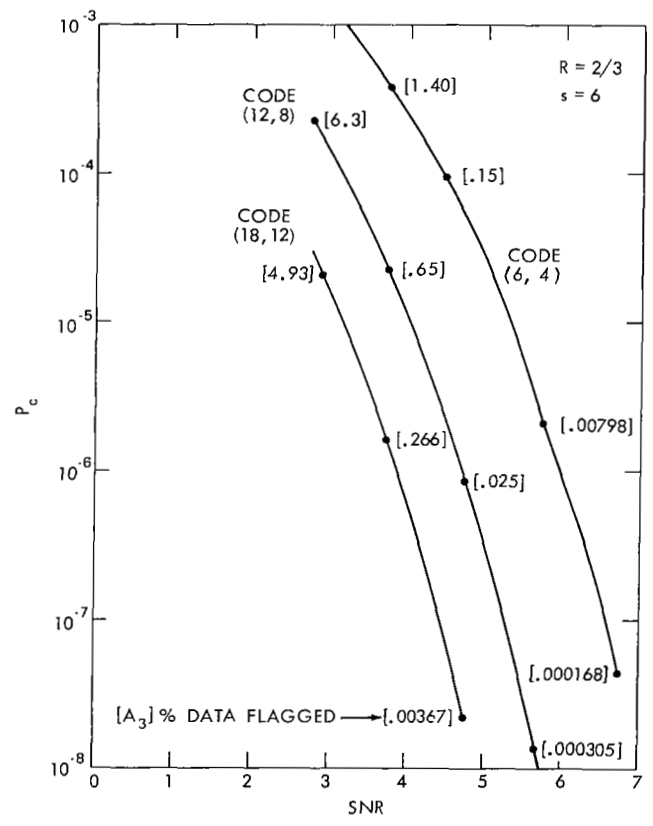


Figure 8—Probability of bit error after error correction for rate 2/3 codes,  $s = 6$ , for various SNR.



provides. This higher confidence requirement of the IRIS experiment was met by concatenating the IRIS source data with a (6,4) Hamming code before using the existing Mariner sixth order bi-orthogonal code.

Performing a Fourier transformation on the IRIS data should give a radiation spectrum from the interferogram. A typical infrared interferometer interferogram of an atmospheric emission for a warm blackbody calibration is shown in Figure 9. The spectrum which was derived from that interferogram is shown in Figure 10. Also shown in Figure 10 is the effect of an error on the spectrum. Note that information about the atmospheric content is masked by an oscillatory wave.

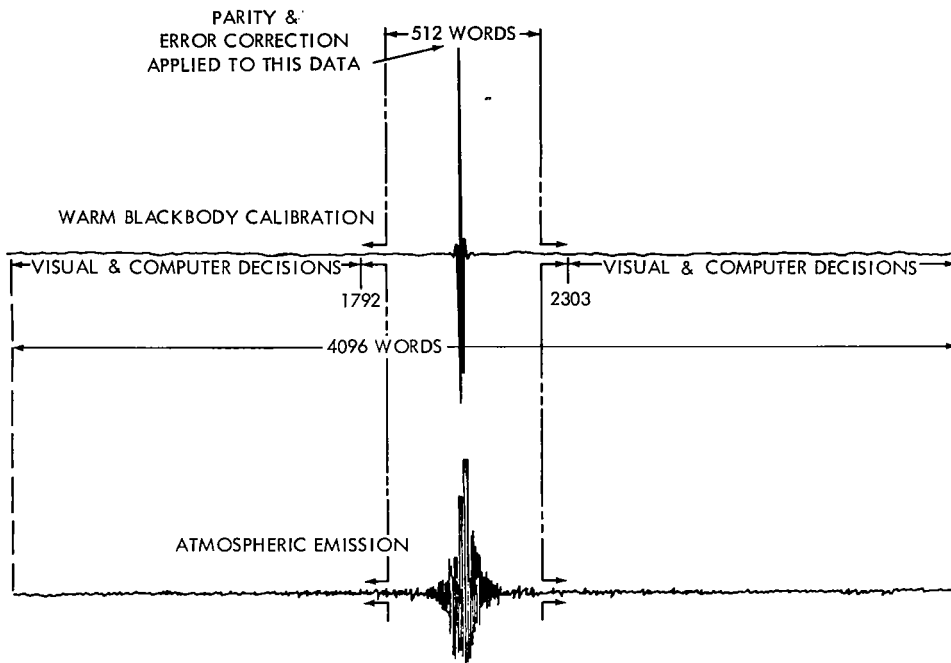


Figure 9—Infrared interferometer interferogram.

High data confidence is only required for the center portion of the interferogram, for this data has little redundancy. Therefore, it is this center portion of the data which is concatenated-coded. One error here could cause the frame to be discarded or misinterpreted. Coding is not required for the two side portions of the interferogram because this data is highly redundant so that errors are easily detected at the ground processor before the Fourier transformation is performed.

The IRIS experiment data is formatted to start each frame with 64 words of housekeeping

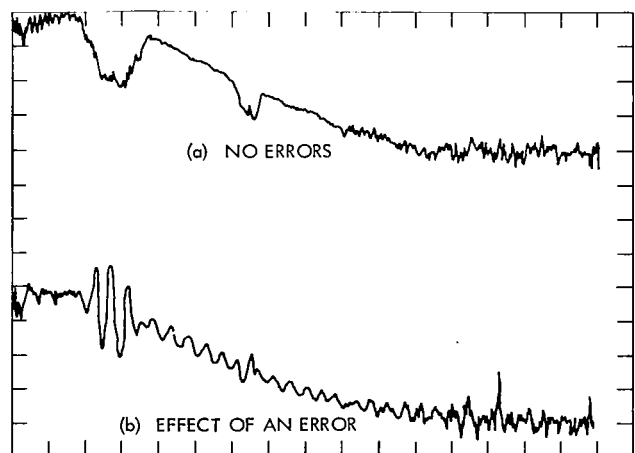


Figure 10—Spectrum of interferogram.

data. This is followed by 4096 words of the interferometer data and 64 more words of housekeeping data. The remaining unused 501 words are set to zeros. These 501 words make the IRIS frame compatible with the TV frame. Each word of the frame consists of 12 bits.

Coding requirements specified by the experimenter were:

1. The probability of bit error was not to exceed  $1 \times 10^{-5}$  even under worst-case conditions.
2. The IRIS data format was not to be disturbed.
3. An encoder failure was not to affect the experiment data.

Requirement 1 was satisfied by choosing the Hamming code (6,4). For schemes using just the biorthogonal code and the concatenation of the (6,4) code with the biorthogonal code, a plot of probability of bit error versus the time that the spacecraft is in Martian orbit is shown in Figure 11. These curves are shown both for the nominal telecommunication channel and for the worst-case conditions. Since the check elements utilize part of the unused 501 words at the end of the frame, no additional bits or bandwidth penalties occur. These curves were drawn under the assumption that the IRIS data words are not synchronized to the biorthogonal code vectors. If the system is synchronized, the bit error rate will decrease approximately by a factor of 10 (see Figure 12).

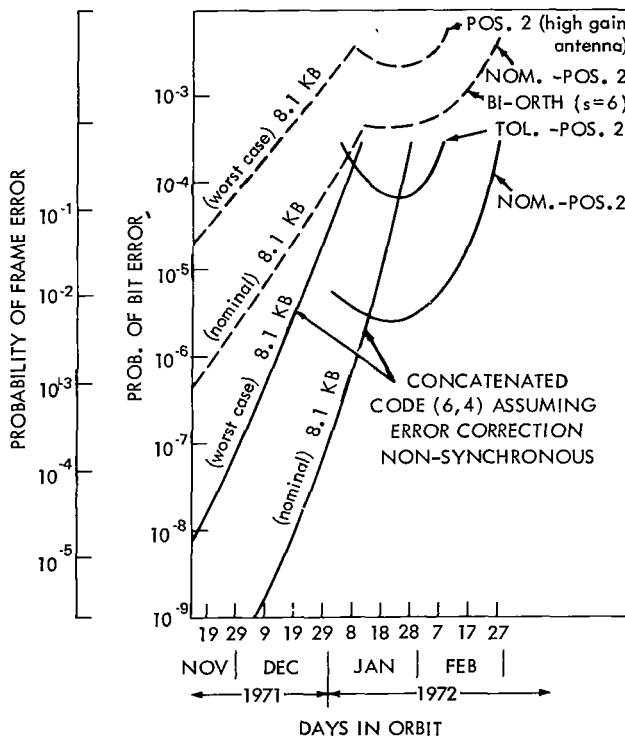


Figure 11—Comparison of coding techniques for Martian orbit.

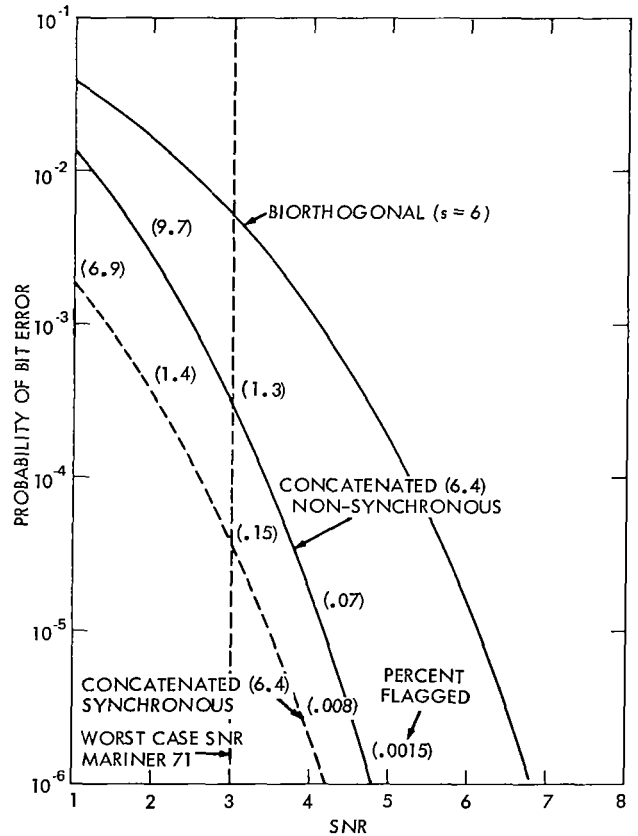


Figure 12—Probability of bit error after correction vs SNR for concatenated (6,4) synchronous and non-synchronous systems as referenced to a biorthogonal ( $s = 6$ ) code.

Requirement 2 was satisfied by not changing the real-time sending of biorthogonal-encoded interferometer data. The 501 unused words in the original format were partially filled with the check words. The real-time data were simultaneously biorthogonally coded and sent to the Hamming encoder. The resulting Hamming check elements were stored until the 501-word block time occurred, whereupon they were biorthogonally encoded and transmitted.

Requirement 3 was satisfied by designing the encoder so that if failure occurred, the check elements would be in error and the data confidence level would revert to that which the biorthogonal code vectors would normally provide.

## Encoding

The (6,4) Hamming encoder receives the IRIS source symbols and reformats them so that each code vector (block) consists of two 12-symbol data elements and one 12-symbol check element. This encoding can be accomplished with two 6-symbol shift registers and feedback networks as shown in Figure 13. One of the registers generates a 6-symbol column check element; the other register generates a 6-symbol word check element.

The encoding procedure is accomplished in four steps:

1. Set the registers to zero state.
2. Shift 24 information symbols simultaneously into the two shift registers and to the biorthogonal encoder. The binary status of the shift registers after the 24th information symbol has been shifted, constituting the complement of the check elements transmitted.
3. Disable the feedback circuitry.
4. Shift the contents of these shift registers into storage. This storage will be read into the biorthogonal encoder during the previously unused 501 word times.

## Decoding

To decode the (6,4) Hamming code vectors, a circuit similar to the encoder can be used with the exception that the circuit should be able to shift to the left as well as to the right. The decoding procedure is accomplished in four steps:

1. Set the registers to zero state.
2. Shift 24 information symbols into the two shift registers.
3. Compare the states of the shift registers to the transmitted check symbols.

(a) If the compared elements agree or if only one of the elements disagrees, the 24 information symbols are assumed correct.

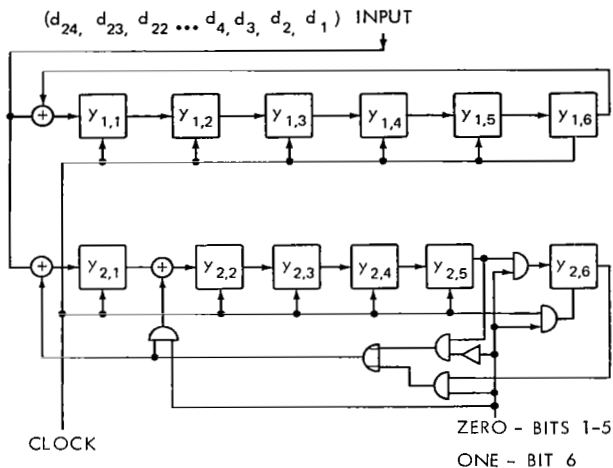


Figure 13—Hamming (6,4) encoder.

- (b) If both check elements disagree, an error is assumed to have occurred in the 24 information symbols.
- 4. If 3(a) occurs, the registers are reset to zero and the procedure is repeated on the next 24 information symbols.
- 5. If 3(b) occurs, the binary states of the registers are compared to the transmitted check symbols.
  - (a) If the patterns agree, the errors are assumed to be in the last information element of the block, and the symbol positions which indicated an error are corrected. The 24 information symbols are assumed error-free, and step 1 is repeated.
  - (b) If the patterns disagree, the contents in the word check element shift register are shifted one position to the left. Comparisons are again made. If the error patterns correlate, the error is assumed to occur in the previous information element of the block. The registers which indicated an error are those positions which are corrected. After correction, the block is assumed error-free, and step 1 is repeated. Corrected blocks may be flagged as having higher probability of bit error than the uncorrected data.
  - (c) If the patterns disagree, the contents of the word-check element shift register are shifted one position to the left, and comparisons are again made. If the error patterns agree, the previous information element of the block is assumed in error. If the error patterns disagree, one more shift is attempted. If the patterns do not correlate after the third shift, the error is assumed to have occurred in more than one word, and the block is flagged as having an uncorrectable error.

## CONCLUSION

One advantage of concatenated coding is that each experimenter can be provided with his own reliability (confidence level). The telecommunications channel may be designed for the lowest confidence level not loading the channel with the experiments requiring higher confidence. The experimenter requiring higher confidence would be given extra bits for coding.

With a concatenated code as used on Mariner 71, which consists of a biorthogonal ( $s = 6$ ) and a generalized Hamming code (6,4), the following characteristics can be noted:

- (1) The probability of bit error at nominal signal level as received from Mars is improved by two orders of magnitude if the generalized Hamming code elements (words) are *not* synchronized to the biorthogonal code elements and by three orders if the two codes are synchronized.
- (2) The experimenter's data format is not disturbed since the parity check elements are inserted into the 501 unused words at the end of the experimenter's frame.
- (3) The experiment's data is unaffected if the algebraic coder fails.

- (4) The decoding of the algebraic code can be achieved, without a computer, with simple hardware similar to that used in the spacecraft plus a six symbol comparator.
- (5) The decoding procedure allows the experimenter to select a processing scheme which will maximize data quantity for a given probability of bit error threshold.

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