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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1429

Operating Characteristics of Computer Programs for Nonlinear Transient Analysis

W. J. Schneider

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Preface

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Contents

I. Introduction . . . 1

II. Methods of Analysis . . . 1

A. The Nodal Method . . . 1

B. The State-Variable Method . . . 3

III. Integration Procedure . . . 3

IV. Component Models . . . 4

V. Transistor Models . . . 9

VI. General Discussion . . . 11

A. User-Oriented Input Language . . . 11

B. Modes of Analysis . . . 11

C. Circuit Size . . . 13

D. Analysis of the Comparison Circuit . . . 13

VII. Problem Solution . . . 13

A. ECAP . . . 13

1. Circuit preparation . . . 13

2. Solution times . . . 14

B. CIRCUS . . . 14

1. Circuit preparation . . . 14

2. Solution times . . . 19

C. MTRAC . . . 19

1. Circuit preparation . . . 19

2. Solution times . . . 19

D. SCEPTRE . . . 19

1. Circuit preparation . . . 19

2. Solution times . . . 40

VIII. Results . . . 40

A. Hand Calculations . . . 40

B. Interprogram Comparisons . . . 45

IX. Conclusions . . . 45

Nomenclature . . . 47

Bibliography . . . 48

Contents (contd)

Tables

1. SCEPTRE input listing 5
2. CIRCUS input listing 7
3. ECAP input listing 7
4. MTRAC input listing 9
5. Transistor model comparison 10
6. ECAP output 15
7. CIRCUS output 20
8. MTRAC output 36
9. SCEPTRE output 41
10. Output data comparison 46
11. Summary of test-circuit solution times 46

Figures

1. Circuit used for nodal-method example 2
2. Circuit used for state-variable method example . . . 3
3. Augmented Ebers–Moll model 9
4. Ebers–Moll model for ECAP 12
5. One-shot multivibrator 13
6. Multivibrator for ECAP 14
7. Multivibrator for CIRCUS 14
8. Multivibrator for MTRAC 19
9. Multivibrator for SCEPTRE 19

Abstract

Considerations relative to the selection and use of four computer programs for the simulation of electronic circuits (CIRCUS, ECAP, MTRAC, and SCEPTRE) are discussed. Various aspects of the analysis are considered, including versatility and simplicity of input, integration methods, and semiconductor modeling. A comparison of the programs and their pertinent operating characteristics is made in terms of the analysis of a single circuit by each of the programs under consideration. Specific results relative to computational speed are presented. Differences between the semiconductor models is presumed to be the dominant reason for differences in the numerical values obtained for the node voltages. The computer times required for this problem are summarized. For each program, the load, execution, and simulation times are listed from the actual computer run. On the basis of the ratio of simulation time to execution time, MTRAC is the most effective of the programs; CIRCUS is the least effective by a factor of nearly 300. The execution time is extrapolated to estimate the time required to complete 75×10^{-6} s of simulation time; on this basis, SCEPTRE would require 3 h, and CIRCUS would require almost three times that long.

Operating Characteristics of Computer Programs for Nonlinear Transient Analysis

I. Introduction

Computer analysis of electronic circuits is a tool that can be used to good advantage in design, reliability, and systems engineering. The Systems Division of JPL provides programming and computer services for this purpose to the Laboratory. To have not only the most versatile but the most efficient programs available, a survey and comparison of contemporary programs has been undertaken, the results of which are described herein.

The current survey was restricted to nonlinear transient analysis programs. After a preliminary screening, it was decided to restrict the survey to the ECAP, CIRCUS, SCEPTRE, and MTRAC[1] (or TRAC) programs. The exclusion of NET-1 and the GENERAL NETWORK ANALYSIS PROGRAM was necessary because of computer-system incompatibilities.

Surveys of computer programs for network analysis appear frequently in the literature. Attempting to provide complete coverage of the subject required many of the papers to be limited to a rather cursory discussion of the program details. The purpose of this survey is to provide in-depth information, but only on the most popular programs. The information presented herein is intended to aid the engineer in selecting a program for his particular application. The topics discussed are the mathematical methods of analysis, component models, semiconductor models, circuit data preparation, and results. The discussion is carried out in terms of the analysis of a test circuit by each of the programs under consideration.

II. Methods of Analysis

Two distinct methods of analysis are used by these programs. The MTRAC and ECAP programs use a nodal-solution matrix method, whereas CIRCUS and SCEPTRE use a state-variable method. For an illustration of these methods, the circuit shown in Fig. 1 will be referred to in the paragraphs that follow.

A. The Nodal Method

The nodal method requires that the current and voltage relationships for each element (see Fig. 1) be written in difference-equation form. The difference equations

[1]The TRAC program was originally the work of Autonetics, Inc. Stanford Research Laboratories developed the M version from the Autonetics original and renamed it MTRAC.

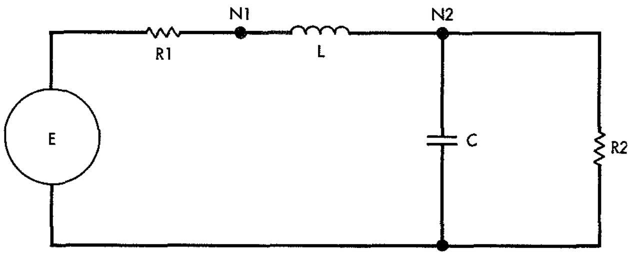


Fig. 1. Circuit used for nodal-method example

used in the International Business Machines (IBM) 7094 ECAP program[2] are as follows:

Capacitors:

$$I_{C_{tn}} = C\frac{dV_c}{d_t} = C\frac{V_{C_{tn}} - V_{C_{tn-1}}}{t_n - t_{n-1}} \tag{1a}$$

Inductors:

$$I_{L_{tn}} = \frac{1}{L}\int_{t_{n-1}}^{t_n} V_L dx = \frac{1}{L}\frac{V_{L_{tn-1}} + V_{L_{tn}}}{2}(t_n - t_{n-1}) + I_{L_{tn-1}} \tag{1b}$$

Resistors:

$$I_{R_{tn}} = \frac{1}{R}V_R = \frac{1}{R}V_{R_{tn}} \tag{1c}$$

The subscripts tn and $tn - 1$ are used to designate the values of the variables at the end of the nth and n − 1st step sizes, respectively.

Equations are then written for the current summation at each node. The difference equations are then substituted.

At node 1 (N1):

$$\begin{aligned}\sum_{N_1} I &= I_{R_2} - I_L + I_C = 0 \\ &= \frac{C}{\Delta t}(V_{2_{tn}} - V_{2_{tn-1}}) \\ &\quad - \frac{\Delta t}{2L}(V_{1_{tn}} - V_{2_{tn}} + V_{1_{tn-1}} - V_{2_{tn-1}}) \\ &\quad - I_{L_{tn-1}} + \frac{1}{R_2}V_{2_{tn}}\end{aligned}$$

[2]Hogsett, G. R., and Nieswanger, D. A., "7090/7094 ECAP" (interoffice memorandum), IBM–L.A. North/062, Los Angeles, Calif., June 1965. Also, see the Bibliography for documentation of computer programs.

At node 2 (N2):

$$\begin{aligned}\sum_{N_2} I &= I_{R_1} + I_2 = 0 \\ &= -\frac{1}{R_1}(E_{tn} - V_{1_{tn}}) \\ &\quad + \frac{\Delta t}{2L}(V_{1_{tn}} - V_{2_{tn}} + V_{1_{tn-1}} - V_{2_{tn-1}})\end{aligned}$$

The resulting equations are then arranged in matrix form:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \left(-\frac{\Delta t}{2L}\right) & \left(\frac{C}{\Delta t} + \frac{\Delta t}{2L} + \frac{1}{R_2}\right) \\ \left(\frac{1}{R_1} + \frac{\Delta t}{2L}\right) & \left(-\frac{\Delta t}{2L}\right) \end{bmatrix} \begin{bmatrix} V_{1_{tn}} \\ V_{2_{tn}} \end{bmatrix}$$

$$+ \begin{bmatrix} \left(-\frac{\Delta t}{2L}\right) & \left(-\frac{C}{\Delta t} + \frac{\Delta t}{2L}\right) \\ \left(\frac{\Delta t}{2L}\right) & \left(\frac{\Delta t}{2L}\right) \end{bmatrix} \begin{bmatrix} V_{1_{tn-1}} \\ V_{2_{tn-1}} \end{bmatrix}$$

$$+ \begin{bmatrix} -1 \\ +1 \end{bmatrix} I_{L_{n-1}} + \begin{bmatrix} -\frac{1}{R_1} \\ 0 \end{bmatrix} E_n$$

The TRAC and ECAP programs differ in that TRAC avoids the numerical differentiation used by ECAP to determine the capacitor current. In MTRAC, the basic differential equation

$$I_C = C\frac{dV_c}{dt}$$

is rearranged and solved by integration:

$$\int_{t_{n-1}}^{t_n} I_C(x)dx = C(V_{C_{tn}} - V_{C_{tn-1}})$$

$$\frac{I_{C_{tn}} + I_{C_{tn-1}}}{2}\Delta t = C(V_{C_{tn}} - V_{C_{tn-1}})$$

$$I_C = \frac{2C}{\Delta t}(V_{C_{tn}} - V_{C_{tn-1}}) - I_{C_{tn-1}}$$

(see Eq. 1).

All terms in TRAC are then consistent, giving exact results for constant dI/dt. The version of ECAP used by TRW Systems also incorporates the improved capacitor model.[3]

B. The State-Variable Method

The state-variable formulation results in the least number of differential equations for the representation of a specific circuit. The number of equations is equal to the order of complexity of the circuit. In a passive circuit, this complexity is equal to the number of reactive elements less the number of inductor cut-sets and capacitor loops.

A tree for the network must be formed that includes all of the nodes, but closes no loops. The double lines in Fig. 2 indicate the tree portion of the network.

The tree is drawn so as to include all capacitors, some of the resistors, but none of the inductors. (To achieve such a tree, SCEPTRE and CIRCUS do not allow circuits that contain inductor cut-sets or capacitor loops.) Equations are then written for the capacitor currents and inductor voltages. The objective of this method is to derive circuit equations in the form of a set of first-order differential equations, as shown below, where V_L is given by

$$-V_L = L\frac{dI_L}{dt} = V_C + V_R + E$$

$$\frac{dI_L}{dt} = \frac{R_2}{L}I_2 + \frac{V_C}{L} + \frac{1}{L}E$$

and V_C is given by

$$-I_C = -I_L + I_R = -C\frac{dV_C}{dt}$$

$$\frac{dV_C}{dt} = \frac{1}{C}I_L - \frac{1}{RC}V_C$$

using the I_L and V_C terms as variables (that is, the state variables). In this example, the general form

$$\begin{bmatrix} V_C \\ I_L \end{bmatrix} = [A]\begin{bmatrix} \dot{V}_C \\ \dot{I}_L \end{bmatrix} + [B]\,E$$

[3]Hass, B. A., Williams, G. J., and Jurkovich, F. F., "Summary of ECAP Revisions" and other interoffice correspondence, TRW Systems, Inc., Feb.–Oct. 1967.

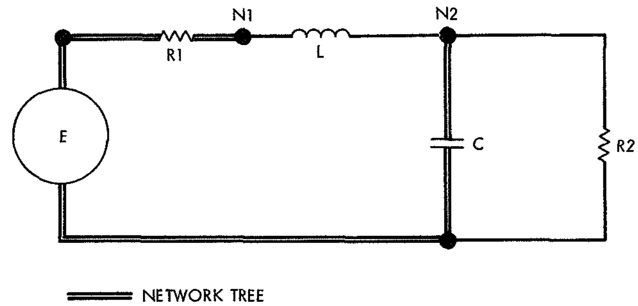


Fig. 2. Circuit used for state-variable method example

is obtained when the variables V_C and I_L are eliminated from the first and second equations, respectively. After rearranging, we have

$$\begin{bmatrix} V_C \\ I_L \end{bmatrix} = \begin{bmatrix} -\dfrac{R_1R_2C}{R_1+R_2} & -\dfrac{R_1L}{R_1+R_2} \\ \dfrac{R_LC}{R_1+R_2} & -\dfrac{L}{R_1+R_2} \end{bmatrix}\begin{bmatrix} \dot{V}_0 \\ \dot{I}_L \end{bmatrix} + \begin{bmatrix} \dfrac{R_1}{R_1+R_2} \\ \dfrac{1}{R_1+R_2} \end{bmatrix} E$$

III. Integration Procedure

In the MTRAC program, the transient solution is obtained by first filling the matrices with numerical values based on calculations using the current value of Δt (step size). Then a number of iterations are made to obtain a solution to the resulting set of nonlinear equations. If convergence cannot be achieved, the Δt is reduced and the process is restarted.

In the ECAP program, the process is similar to that for MTRAC except that continuous nonlinearities are not allowed as they are in MTRAC. Nonlinearities are incorporated into an ECAP problem by means of "switch" statements. Should a "switch" (a test of branch current direction) be activated, the time of switching is accurately determined, an appropriate change in an element value is made, and the solution is restarted. Step-size control is not automatic in ECAP.

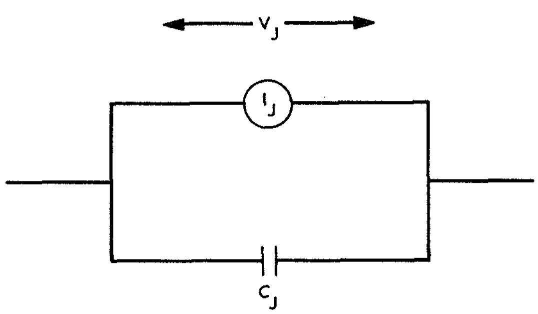
The SCEPTRE and CIRCUS programs both perform integration on the system of equations. A modification of the exponential method is used by CIRCUS; SCEPTRE permits the user to select either exponential, Runge–Kutta, or trapezoidal integrators as part of the problem input. The exponential method is recommended in the SCEPTRE user's manual for circuits with widely separated time constants. The step size taken by these integration methods varies as the smallest local time constant of the network. Nonlinear matrix **A** is updated at each step of the integration by a sequential calculation based on the state variables.

In the CIRCUS program, the only nonlinear elements allowed are diode junctions. More complex semiconductor models have been incorporated into the program by the use of diode junctions and dependent generators.

Diode junctions, as characterized by Ebers and Moll, are nonlinear in junction conductance and capacitance; thus,

$$J_D = I_S\left[\exp\left(\frac{q}{kTM}V_J\right) - 1\right]$$

$$C_J = \frac{C_D}{\left(1 - \frac{V_J}{V_0}\right)^n}$$



The independent variable for both equations (V_J) is a state variable. This fact allows the nonlinearities to be updated without a simultaneous solution of the circuit equations. As a consequence, diode junctions without the associated junction capacitance cannot be implemented by this procedure.

A much larger class of nonlinearities is allowed by SCEPTRE. However, because the nonlinear elements are updated by a sequential rather than a simultaneous equation-solution process, a number of subtle restrictions are placed upon the user. At each step size, the independent variable of the nonlinearity must be obtainable by sequential calculation from a state variable. Although there are exceptions, the user is cautioned by the SCEPTRE user's manual to use state variables to define nonlinearities. This condition is violated if the user specifies a diode nonlinearity without a diode-shunt capacitance. The SCEPTRE solution must use the diode voltage from the previous step size because there is no provision to solve for the diode voltage. The resulting error is dependent upon the steepness of the nonlinearity and is not easily assessed.

All junctions are thus required to have capacitances associated with them to avoid this problem (called a "computational delay"). Because forward-biased junctions typically have short time constants, there will always be a number of small local time constants. These small time constants will result in small integration step size and a relatively large number of calculations, particularly for simulation times much larger than the local time constants.

IV. Component Models

The SCEPTRE program accepts only elementary branches as component models. Every branch is user-specified and consists of an R, L, C, E, J, or M. A minor exception occurs in the M statement for a transformer because it defines a leakage inductance that is not explicit in the data. Each of these elements may be defined as a constant or as a function of other circuit variables. This feature is not available in any of the other programs.

The user may construct and store a model by the interconnection of one or more circuit elements. It may be called out as a whole whenever the user requires it. In Table 1, a model for transistor 2N2907[4] is written in the MODEL DESCRIPTION portion of the program. In this model, cards 4 through 8 are constant-value resistors and cards 9, 10, and 24 describe capacitor CE as follows:

$$\mathrm{CE} = \frac{6.4}{(0.8 + V_{CE})^{0.38}} + 4 \times 10^4 (J_E + 9.7 \times 10^{-6})$$

where

V_{CE} = voltage across capacitor CE

J_E = current of generator JE

[4]This is not a limiting complexity for SCEPTRE, but rather corresponds to the equations in the user's manual.

Table 1. SCEPTRE input listing

```
MODEL DESCRIPTION                                                       2N2907 1
 MODEL 2N2907 (TEMP) (E-B-C)                                            2N2907 2
  ELEMENTS                                                              2N2907 3
   RE,E -EI= 6.50407E-04                                                2N2907 4
   RI,EI-BI= 2.00000E 03                                                2N2907 5
   RN,CI-BI= 2.95578E 06                                                2N2907 6
   RC,C -CI= 1.00000E-03                                                2N2907 7
   RB,B -BI= 2.84041E-02                                                2N2907 8
   CE,BI-EI=EQUATION C ( 6.43091E 00, 8.00000E-01,VCE,                  2N2907 9
                         3.80000E-01, 4.03397E 01,JE, 9.73589E-06)      2N290710
   CC,BI-CI=EQUATION C ( 6.43091E 00, 8.00000E-01,VCC,                  2N290711
                         3.80000E-01, 3.45748E 02,JC, 1.94718E-05)      2N290712
   JE,BI-EI=DIODE EQUATION ( 9.73589E-06, 2.17130E 01)                  2N290713
   JC,BI-CI=DIODE EQUATION ( 1.94718E-05, 2.53333E 01)                  2N290714
   JN,CI-BI=EQUATION B (TABLE A ( JE),JE)                               2N290715
   JI,EI-BI= 4.93171E-01*JC                                             2N290716
  OUTPUTS                                                               2N290717
   PC,PB,IRB,IRC,PW,PLOT                                                2N290718
  DEFINED PARAMETERS                                                    2N290719
   PW=EQUATION P (VRC,VRN,VRI,VRE,VRB,IRC,IRB)                          2N290720
   PC=EQUATION S (VRC,VRN,VRI,VRE)                                      2N290721
   PB=EQUATION S (0.0,VRB,VRI,VRE)                                      2N290722
  FUNCTIONS                                                             2N290723
   EQUATION C (A,B,C,D,E,F,G)=(A/(B-C)**D+E*(F+G))                      2N290724
   EQUATION P (A,B,C,D,E,F,G)=((A+B-C-D)*F+(E-C-D)*G)                   2N290725
   EQUATION S (A,B,C,D)=(A+B-C-D)                                       2N290726
   EQUATION B (A,B)=(A*B)                                               2N290727
   TABLE    A                                                           2N290728
           .00000E-39, 9.86343E-01,                                     2N290729
          1.00000E-01, 9.86842E-01,                                     2N290730
          1.00000E 00, 9.90095E-01,                                     2N290731
          1.00000E 01, 9.90099E-01,                                     2N290732
          1.50000E 02, 9.90099E-01,                                     2N290733
          5.00000E 02, 9.80392E-01,                                     2N290734
          1.00000E 03, 9.80392E-01,                                     2N290735
          2.00000E 03, 9.80392E-01,                                     2N290736
          4.00000E 03, 9.80392E-01,                                     2N290737
          8.00000E 03, 9.80392E-01                                      2N290738
 MODEL 1N914  (TEMP ) (A-C  )
    ELEMENTS
     RC   ,1    -C     =   .002
     CA   ,A    -1     =EQUATION C (24.1,0.9,VCA,0.5,18.1,JA,2.9E-6)
     JA   ,A    -1     =DIODE EQUATION (2.9E-6,24.1)
     RA   ,A    -1     =  1.1  E+3
    OUTPUTS
     IRC,PV,PLOT
    DEFINED PARAMETERS
     PV   =EQUATION S (VRA,VRC)
    FUNCTIONS
     EQUATION C (A,B,C,D,E,F,G    )=(A/(B-C)**D+E*(F+G))
     EQUATION S (A,B) =(A+B)
CIRCUIT DESCRIPTION
        MULTIVIBRATOR TEST CIRCUIT
  ELEMENTS
     R1   ,N6   -N2    = 10.
     R2   ,N6   -N3    =120.
     R3   ,N4   -N1    = 10.
     R4   ,N6   -N4    = 10.
     R5   ,N5   -N1    = 22.
     C1   ,N3   -N2    =500.
     C2   ,N4   -N1    =100.
     D1   ,N2   -N7           =MODEL 1N914   (TEMP)
     T1   ,N0   -N1    -N2   =MODEL 2N2907   (TEMP)
     T2   ,N0   -N3    -N4   =MODEL 2N2907   (TEMP)
     E1   ,N0   -N6    = 15.
     E2   ,N0   -N5    =- 3.
     E3   ,N0   -N7    =TABLE E3
  OUTPUTS
          PVN2,PVN3,PVN4,PVN5,XSTPSZ,XSTPNO,PLOT
     E3,PLOT
  FUNCTIONS
     TABLE E3
        .    , 15.,
       10.   ,  0.,
      110.   ,  0.,
     1110.   , 15.,
     2000.   , 15.
     EQUATION S (A,B,C,D)=(A+B-C-D)
     DEFINED PARAMETERS
     PVN2 =EQUATION S (E1,0.0,VR1,0.0)
     PVN3 =EQUATION S (E1,0.0,VR2,0.0)
     PVN4 =EQUATION S (E1,0.0,VR4,0.0)
     PVN5 =EQUATION S (E1,0.0,VR4,VR3)
  RUN CONTROLS
     STOP TIME              =75.E+3
     MINIMUM STEP SIZE  = 1.E-30
     RUN INITIAL CONDITIONS
     STARTING STEP SIZE = 1.E-15
     COMPUTER TIME LIMIT =500.
     MAXIMUM PRINT POINTS=175
 END
```

The equation above includes both transition and diffusion capacitances. In turn, generator JE is described by

$$J_E = 9.7 \times 10^{-6} \exp(0.21 \times V_{JE})$$

where V_{JE} is the voltage across generator JE.

The model is used twice in the CIRCUIT DESCRIPTION portion of the input—for T1 and T2. It is necessary to list only the nodes and model designation (as has been done on card 2) to use the model.

The SCEPTRE program calculates only element voltages and currents; therefore, the user must write expressions for all auxiliary variables. FORTRAN coding rules for these expressions must be followed because they are incorporated verbatim into the simulation code. The expressions for node voltages, power (PW), collector-emitter voltage (PC), and base-emitter voltage (PB) are included in this model.

The CIRCUS program is designed to accept only "ordinarily available" circuit components R, L, C, and K[5] for transformers, diodes, and a number of discrete semiconductor-device models. This represents a considerable simplification over SCEPTRE, both in program structure and in ease to the user. The CIRCUS input data (Table 2) demonstrate this simplification (only one-half as much code is necessary to write in the same problem). DEVICE PARAMETERS corresponds to the SCEPTRE MODEL DESCRIPTION (see Table 1) in that it contains only the model designation and a parameter list, but no topology is required. Node voltages are calculated by CIRCUS and are available for output if requested.

This simplification is accompanied by a decreased capability. For example, it is not possible for CIRCUS to represent a nonlinear resistor, a saturable inductor, or an operational amplifier.

The ECAP program allows the elementry branches R, L, and C, and transformer M. In addition, each branch may include a current generator or voltage source. The current generators are allowed to be linearly dependent on a branch voltage or current—g_m (transconductance) or β (current gain). Also, each branch may have two element values. The value used is chosen on the basis of the current direction in a designated "switch" branch. Arranging a number of "switched" branches allows nonlinear elements to be realized by step approximations.

The procedure for implementing step approximations is illustrated in Table 3. The first 46 statements have been used to construct a transistor model similar to those already illustrated for the SCEPTRE and CIRCUS programs. Two junction conductances (B12, B16, B20, and B24 together with B31, B3, B5, and B7 form one) and two diffusion capacitances are represented by four breakpoint approximations. Because ECAP does not have a stored model capability, the two transistors (92 statements) are written as part of the circuit description.

The MTRAC program generally resembles CIRCUS in that the model topology is stored internally; in some cases, however, the MTRAC models are considerably more complex. In MTRAC, for example, R may be time-dependent, L and C may have series and shunt resistors, and L may also be current-dependent. Transistor and diode models are similar to those of CIRCUS. No dependent sources or operational amplifiers are available in MTRAC or CIRCUS. A core model is included in MTRAC that provides a hyperbolic hysteresis loop and a nonlinear, magnetomotive-force-dependent, time derivative of flux.

One of the differences between MTRAC and the other programs is its use of a fixed-input format as opposed to the free-field input of CIRCUS and SCEPTRE and the quasi-free-field input of ECAP. This is apparent in Table 4, in which it should be noted that card number 59010 designates the transistor type (= 0 for *n–p–n*), followed by the base, collector, and emitter node designations (1, 2, and G...0). The three cards that follow give model parameters. Card number 59050 describes the second transistor as *n–p–n* connected to nodes 3, 4, and G...0. The number one in the fifth field indicates that the second transistor has the same model parameters as those of the first transistor.

The MTRAC program has the interesting feature that component models remain intact during the simulation; that is, the nodal-solution matrix sees only the model nodes and not the model elements. This is not true of any of the other programs under consideration.

[5]Coupling coefficient $K = (M^2/L_1L_2)^{1/2}$.

Table 2. CIRCUS input listing

```
     -MULTIVIBRATOR TEST PROBLEM-
  DEVICE PARAMETERS
   TRANSISTOR,  2N2907,  NPN    ,
          RB, 2.84041E 01,    RC, 1.00000E 00,    RE, 6.50407E-01,
        PHI1, 8.00000E-01,    N1, 3.80000E-01,    A1, 6.43091E-12,
        PHI2, 8.00000E-01,    N2, 3.80000E-01,    A2, 6.43091E-12,
      THETAN, 2.17130E 01,   IES, 9.73589E-09,
      THETAI, 2.53333E 01,   ICS, 1.94718E-08
         TCN, 1.00000E-04,        1.59236E-09
         TCI, 1.00000E-04,        1.59236E-08
          BI, 1.00000E-04,        9.73054E-01
          BN, 0.00000E-39,        1.00000E-04,
              1.00000E-03,        1.00000E-02,
              1.50000E-01,        5.00000E-01,
              1.00000E 00,        2.00000E 00,
              4.00000E 00,        8.00000E 00,
              7.22222E 01,        7.50000E 01,
              1.00000E 02,        1.00000E 02,
              1.00000E 02,        5.00000E 01,
              5.00000E 01,        5.00000E 01,
              5.00000E 01,        5.00000E 01
DIODE,   1N914,
RB,     2.,        RS,        1.1E6,  A,     2.4E-11,
PHI,    .9,        N,         .5,     IS,    2.9E-9,
THETA,   21.5,      KD,        18.1E-6
    END
      PV1   ,7     ,0     ,15., 0.,0.,10.E-9,100.E-9,1000.E-9,1.E-3
      V2    ,5     ,0     ,-3.0
      V1    ,6     ,0     , 22.5
      D1    ,7     ,2     , 1N914
      T2    ,3     ,4     ,0      , 2N2907
      C2    ,4     ,1     ,100.  E-12
      C1    ,3     ,2     ,500.  E-12
      R5    ,5     ,1     , 22.  E+3
      R4    ,6     ,4     , 10.  E+3
      R3    ,4     ,1     , 10.  E+3
      R2    ,6     ,3     ,120.  E+3
      T1    ,1     ,2     ,0      , 2N2907
      R1    ,6     ,2     , 10.  E+3
      INTERVALS,75.E-10, 3.E-06
     PRINT,PV1,VN2, VN3, VN4, VN1
      DIAGNOSTICS
      PLOT,PV1,VN2, VN3, VN4, VN1
      EXECUTE
      END
END OF JOB
```

Table 3. ECAP input listing

```
C      MULTIVIBRATOR TEST CIRCUIT
       TRANSIENT ANALYSIS
B 1    N( 3, 0),R= 6.50407E-01                                  2N2907 0
B 2    N( 4, 3),R= 2.00000E 06                                  2N2907 2
B 3    N( 4, 5),R= 2.95578E 09                                  2N2907 3
B 4    N( 1, 4),R= 2.84041E 01                                  2N2907 4
B 5    N( 2, 5),R= 1.00000E 00                                  2N2907 5
B 6    N( 6, 0),R= 1.00000E 07                                  2N2907 6
B 7    N( 7, 0),R= 1.00000E 07                                  2N2907 7
B 8    N( 6, 0),R= 1.00000E 03                                  2N2907 8
B 9    N( 7, 0),R= 1.00000E 03                                  2N2907 9
B10    N( 6, 0),C= 1.59236E-12                                  2N290710
B11    N( 7, 0),C= 1.59236E-11                                  2N290711
B12    N( 4, 3),R=( 1.00000E 07, 3.93642E 05),E=-0.00000E-39    2N290712
B13    N( 4, 5),R=( 1.00000E 07, 9.87000E 04),E=-0.00000E-39    2N290713
B14    N( 4, 3),C=( 1.75000E-12, 3.24771E-13)                   2N290714
B15    N( 4, 5),C=( 1.75000E-12, 3.24771E-13)                   2N290715
T 1    B(12, 6),BETA=-1.00000E 00                               2N290716
T 2    B(13, 7),BETA=-1.00000E 00                               2N290717
S 1    B=12,(12,14),OFF                                         2N290718
S 2    B=13,(13,15),OFF                                         2N290719
B16    N( 4, 3),R=( 1.00000E 07, 7.75838E 03),E=-1.80000E-01    2N290720
B17    N( 4, 5),R=( 1.00000E 07, 1.03332E 03),E=-1.80000E-01    2N290721
B18    N( 4, 3),C=( 1.75000E-12, 8.60770E-13)                   2N290722
B19    N( 4, 5),C=( 1.75000E-12, 8.60770E-13)                   2N290723
T 3    B(16, 6),BETA=-1.00000E 00                               2N290724
T 4    B(17, 7),BETA=-1.00000E 00                               2N290725
S 3    B=16,(16,18),OFF                                         2N290726
S 4    B=17,(17,19),OFF                                         2N290727
B20    N( 4, 3),R=( 1.00000E 07, 1.55741E 02),E=-3.60000E-01    2N290728
B21    N( 4, 5),R=( 1.00000E 07, 1.08107E 01),E=-3.60000E-01    2N290729
B22    N( 4, 3),C=( 1.75000E-12, 1.72278E-12)                   2N290730
B23    N( 4, 5),C=( 1.75000E-12, 1.72278E-12)                   2N290731
T 5    B(20, 6),BETA=-1.00000E 00                               2N290732
T 6    B(21, 7),BETA=-1.00000E 00                               2N290733
S 5    B=20,(20,22),OFF                                         2N290734
S 6    B=21,(21,23),OFF                                         2N290735
B24    N( 4, 3),R=( 1.00000E 07, 3.12631E 00),E=-5.40000E-01    2N290736
B25    N( 4, 5),R=( 1.00000E 07, 1.13102E-01),E=-5.40000E-01    2N290737
B26    N( 4, 3),C=( 1.75000E-12, 3.88683E-12)                   2N290738
```

Table 3 (contd)

```
B27   N( 4, 5),C=( 1.75000E-12, 3.88683E-12)                          2N290739
T 7   B(24, 6),BETA=-1.00000E 00                                       2N290740
T 8   B(25, 7),BETA=-1.00000E 00                                       2N290741
S 7   B=24,(24,26),OFF                                                 2N290742
S 8   B=25,(25,27),OFF                                                 2N290743
T 9   B( 8, 3),BETA=-9.86343E-01                                       2N290744
T10   B( 9, 2),BETA=-4.93171E-01                                       2N290745
B28   N(10, 0),R= 6.50407E-01                                          2N290745
B29   N(11,10),R= 2.00000E 06                                          2N290747
B30   N(11,12),R= 2.95578E 09                                          2N290748
B31   N( 8,11),R= 2.84041E 01                                          2N290749
B32   N( 9,12),R= 1.00000E 00                                          2N290750
B33   N(13, 0),R= 1.00000E 07                                          2N290751
B34   N(14, 0),R= 1.00000E 07                                          2N290752
B35   N(13, 0),R= 1.00000E 03                                          2N290753
B36   N(14, 0),R= 1.00000E 03                                          2N290754
B37   N(13, 0),C= 1.59236E-12                                          2N290755
B38   N(14, 0),C= 1.59236E-11                                          2N290756
B39   N(11,10),R=( 1.00000E 07, 3.93642E 05),E=-0.00000E-39            2N290757
B40   N(11,12),R=( 1.00000E 07, 9.87000E 04),E=-0.00000E-39            2N290758
B41   N(11,10),C=( 1.75000E-12, 3.24771E-13)                           2N290759
B42   N(11,12),C=( 1.75000E-12, 3.24771E-13)                           2N290760
T11   B(39,33),BETA=-1.00000E 00                                       2N290761
T12   B(40,34),BETA=-1.00000E 00                                       2N290762
S 9   B=39,(39,41),OFF                                                 2N290763
S10   B=40,(40,42),OFF                                                 2N290764
B43   N(11,10),R=( 1.00000E 07, 7.75838E 03),E=-1.80000E-01            2N290765
B44   N(11,12),R=( 1.00000E 07, 1.03332E 03),E=-1.80000E-01            2N290766
B45   N(11,10),C=( 1.75000E-12, 8.60770E-13)                           2N290767
B46   N(11,12),C=( 1.75000E-12, 8.60770E-13)                           2N290768
T13   B(43,33),BETA=-1.00000E 00                                       2N290769
T14   B(44,34),BETA=-1.00000E 00                                       2N290770
S11   B=43,(43,45),OFF                                                 2N290771
S12   B=44,(44,46),OFF                                                 2N290772
B47   N(11,10),R=( 1.00000E 07, 1.55741E 02),E=-3.60000E-01            2N290773
B48   N(11,12),R=( 1.00000E 07, 1.08107E 01),E=-3.60000E-01            2N290774
B49   N(11,10),C=( 1.75000E-12, 1.72278E-12)                           2N290775
B50   N(11,12),C=( 1.75000E-12, 1.72278E-12)                           2N290776
T15   B(47,33),BETA=-1.00000E 00                                       2N290777
T16   B(48,34),BETA=-1.00000E 00                                       2N290778
S13   B=47,(47,49),OFF                                                 2N290779
S14   B=48,(48,50),OFF                                                 2N290780
B51   N(11,10),R=( 1.00000E 07, 3.12631E 00),E=-5.40000E-01            2N290781
B52   N(11,12),R=( 1.00000E 07, 1.13102E-01),E=-5.40000E-01            2N290782
B53   N(11,10),C=( 1.75000E-12, 3.88683E-12)                           2N290783
B54   N(11,12),C=( 1.75000E-12, 3.88683E-12)                           2N290784
T17   B(51,33),BETA=-1.00000E 00                                       2N290785
T18   B(52,34),BETA=-1.00000E 00                                       2N290786
S15   B=51,(51,53),OFF                                                 2N290787
S16   B=52,(52,54),OFF                                                 2N290788
T19   B(35,30),BETA=-9.86343E-01                                       2N290789
T20   B(36,29),BETA=-4.93171E-01                                       2N290790
B55   N(16,2),R=10000
B56   N(16,8),R=120000
B57   N(9,1),R=10000
B58   N(16,9),R=10000
B59   N(17,1),R=22000
B60   N(8,2),C=500E-12
B61   N(9,1),C=100E-12
B62   N(0,15),R=0.01
B63   N( 2,15),R=( 1.00000E 07, 3.93642E 05),E=-0.00000E-39
B64   N( 2,15),R=( 1.00000E 07, 7.75838E 03),E=-1.80000E-01
B65   N( 2,15),R=( 1.00000E 07, 1.55741E 02),E=-3.60000E-01
B66   N( 2,15),R=( 1.00000E 07, 3.12631E 00),E=-5.40000E-01
B67   N( 2,15),C=( 1.75000E-12, 3.24771E-13)
B68   N( 2,15),C=( 1.75000E-12, 8.60770E-13)
B69   N( 2,15),C=( 1.75000E-12, 1.72278E-12)
B70   N( 2,15),C=( 1.75000E-12, 3.88683E-12)
S17   B=63,(63,67),OFF
S18   B=64,(64,68),OFF
S19   B=65,(65,69),OFF
S20   B=66,(66,70),OFF
B71   N(0,16),R=0.01,E=15.
B72   N(0,17),R=0.01,E=-3.
E62   (1),15.00,00.0,00.0,00.3,00.6,00.9,01.2,01.5,01.8,02.1,02.4,02.7,
     *03.0,03.3,03.6,03.9,04.2,04.5,04.8,05.1,05.4,05.7,06.0,06.3,06.6,
     *06.9,07.2,07.5,07.8,08.1,08.4,08.7,09.0,09.3,09.6,09.9,10.2,10.5,
     *10.8,11.1,11.4,11.7,12.0,12.3,12.6,12.9,13.2,13.5,13.8,14.1,14.4,
     *14.7,15.0,15.0,15.0
      TIME STEP=5E-8
      OUTPUT INTERVAL =10
      INITIAL TIME=0.
      FINAL TIME =75.E-06
      PRINT,NV
*PLOT NV(8),LINEAR,TIME,LINEAR
*PLOT NV(9),LINEAR,TIME,LINEAR
*PLOT NV(15),LINEAR,TIME,LINEAR
*PLOT NV(1),LINEAR,TIME,LINEAR
*PLOT NV(2),LINEAR,TIME,LINEAR
      EQUILIBRIUM
      EXECUTE
```

Table 4. MTRAC input listing

```
***   ONE-SHOT MULTIVIBRATOR         COMPARISON CIRCUIT     ***
        1200                                                               49980
         350           *                                                   49981
MVTEST                                                     1           1   49990
 ,0,0,1,0,0,0,0,0,0,0,0            0          10                           49991
           6          10           3                                       49992
  .0000001  0.0000001                                                      49993
           4           0          -1           3           1           1   50000
                       1                                                   50010
VN1        1VN2        2VN3        3VN4        4                           50020
                                                                           50021
                                                                           50030
                                                                           50031
                                                                           50032
                                                                           50033
TIME        MICROSECONDS 1.0     -06 0.0     -06 75.     -06PLOT NO.1      50040
V IN        VOLTS        1.0         0.0         20.0       PLOT NO. 1     50041
TIME        MICROSECONDS 1.0     -06 0.0     -06 75.     -06PLOT NO. 2     50042
VN2         VOLTS        1.0         0.0         20.0       PLOT NO. 2     50043
TIME        MICROSECONDS 1.0     -06 0.0     -06 75.     -06PLOT NO. 3     50044
VN3         VOLTS        1.0         0.0         20.0       PLOT NO. 3     50045
TIME        MICROSECONDS 1.0     -06 0.0     -06 75.     -06PLOT NO. 4     50046
VN4         VOLTS        1.0         0.0         20.0       PLOT NO. 4     50047
  .075   -05 0.075   -03                                                   50100
                                                                           50110
                                                                           50120
                                                                           50130
 300.0                                                                     50200
                                                                           53000
           4           0                                                   53010
 15.         0.0     -06 0.0         10.     -09 0.0         110.    -09   53011
 15.         1110.   -09                                                   53012
           1-          1                                                   53013
 15.                                                                       53014
           1-          1                                                   53015
 3.                                                                        53016
                                                                           53500
                                                                           53600
                                                                        I  54000
           5                                                            NR 55000
S          2           2 10000.                                         R1 55010
S          2           3 120000.                                        R2 55020
           4           1 10000.                                         R3 55030
S          2           4 10000.                                         R4 55040
S          3           1 22000.                                         R5 55050
           2                                                            NC 56000
           3           2 500.    -12 .01                                C1 56010
           4           1 100.    -12 .0002                              C2 56020
                                                                        NL 57000
                                                                        NZ 70000
           1                                                            ND 58000
           2S          1                                                D1L58010
 2.9     -09 1.78       1.1      +06 26.7    -12 0.9         .842    -06D1 58010
           2                                                            NQ 59000
                       1           2G          0                        Q1 59010
 7.22222E 01 9.73054E-01 1.59236E-09 1.59236E-08 1.94718E-08 1.51816E 002N2907 1
 7.18997E-12 8.00000E-01 2.95578E 09 9.73589E-09 1.77129E 00 7.18997E-122N2907 2
 8.00000E-01 2.00000E 06 5.00000E-03 5.00000E-03                        2N2907 3
                       3           4G          0           1            T2 59050
```

V. Transistor Models

The MTRAC and CIRCUS programs allow only the Ebers–Moll transistor model. For the purpose of comparing the operation of all four programs, the Ebers–Moll model has also been used for ECAP and SCEPTRE. This model, depicted in Fig. 3, is augmented by bulk resistances in the emitter, base, and collector leads. In the programs being compared, the models incorporated are subsets of this configuration. The differences between the models are indicated in Table 5.

The CIRCUS and MTRAC programs do not use series-resistance element R_E, R_B, or R_C. This omission is important primarily in the saturation region. Because the forward-biased emitter and collector junction voltages in the Ebers–Moll model differ by only a few millivolts, the dominant characteristic is that of the series resistance.

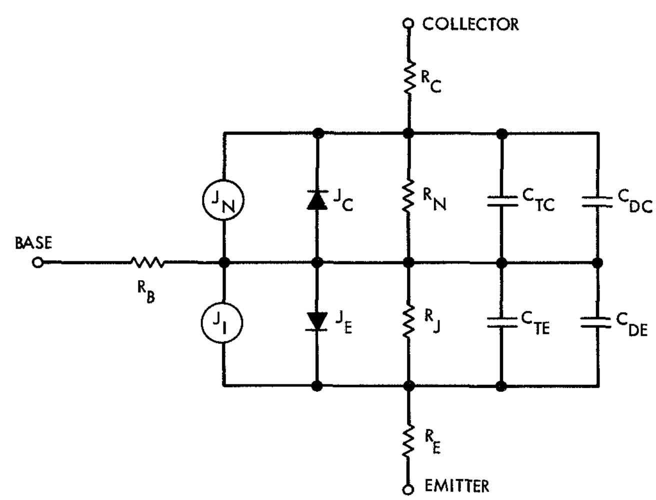


Fig. 3. Augmented Ebers–Moll model

Table 5. Transistor model comparison

Element	Program			
	CIRCUS	ECAP	MTRAC	SCEPTRE
R_B	R_B	R_b	NA[a]	R_B
R_E	R_E	R_e	NA	R_E
R_C	R_C	R_c	NA	R_C
R_I	NA[a]	R_I	R_{EL}	R_I
R_N	NA	R_N	R_{CL}	R_N
C_{TC}	$\frac{a_1}{(\phi_1 - V_{BC})^{n_1}}$	$\frac{C_{oc}}{(V_{OC} - V_{BC})^{nc}}$	$\frac{C_{co}}{(1 - V_{BC/V_{DC}})^{0.5}}$	$\frac{C_{oc}}{(V_{OC} - V_{BC})^{n_c}}$
C_{TE}	$\frac{a_2}{(\phi_2 - V_{BE})^{n_2}}$	$\frac{C_{OE}}{(V_{OE} - V_{BE})^{ne}}$	$\frac{C_{ED}}{(1 - V_{BE/V_{DE}})^{0.5}}$	$\frac{C_{OE}}{(V_{OE} - V_{BE})^{n_e}}$
C_{DC}	$\theta_I T_{CI}(I_{DC} + I_{CS})$	NA	$I_C = T_I \frac{dI_{DC}}{dt}$	$\theta_C T_C(I_{DC} + I_{CS})$
C_{DE}	$\theta_N T_{CN}(I_{DE} + I_{ES})$	NA	$I_C = T_N \frac{dI_{DE}}{dt}$	$\theta_e T_E(I_{DE} + I_{ES})$
I_{DC}	$I_{CS}(\exp \theta_N V_{bc} - 1)$	$I_{CS}(\exp \theta_N V_{bc} - 1)$	$I_{CS}(\exp V_{bc}/M_e\theta - 1)$	$I_{CS}(\exp \theta_c V_{bc} - 1)$
I_{DE}	$I_{ES}(\exp \theta_s V_{be} - 1)$	$I_{ES}(\exp \theta_I V_{be} - 1)$	$I_{ES}(\exp V_{be}/M_c\theta - 1)$	$I_{ES}(\exp \theta_e V_{be} - 1)$
I_{IC}	$\frac{I_{DE}}{(1/\beta N^{-1})}$	$I_{DE}\,\alpha_n$	$\frac{I_{DE}}{(1/h_{FEN} - 1)}$	$I_{DE}\,\alpha_N$
I_{IE}	$\frac{I_{DC}}{(1/\beta I^{-1})}$	$I_{DC}\,\alpha_I$	$\frac{I_{DC}}{(1/h_{FEI} - 1)}$	$I_{DC}\,\alpha_I$
Auxiliary relationships	$T_{CI} = f(I_{DC})$ $T_{EI} = f(I_{DE})$ $\beta_I = f(I_{DC})$ $\beta_N = f(I_{DE})$	$\alpha_n = \frac{\alpha_{NL}}{(1 + ST_N)}$ $\alpha_I = \frac{\alpha_{IL}}{(1 + ST_I)}$	$\theta = \frac{kT}{q} = 0.026$ V	$\alpha_N = f(I_{DE})$

[a]NA = not applicable.

Should the resistors be considered necessary by a user of CIRCUS or MTRAC, they may be added externally to the model.

Implementation of junction time constants will now be considered. The transistor capacitances C_{TE} and C_{TC} are equivalent in each of the programs, as are the expressions for the diode currents J_C and J_E. The expression for transition capacitance allows a singularity at $V_{BC} = V$. Because B is generally larger than 0.75 V, corresponding to a diode current of 10^{13} I_{CS}, it is well beyond the expected operating range of transistor currents.

The SCEPTRE and CIRCUS programs use equivalent form for the diffusion capacitance; that is,

$$C_{DC} = \frac{q}{kTM_C} T_C(J_C + J_N)$$

for the collector junction and

$$C_{DE} = \frac{q}{kTM_E} T_E(J_E + J_I)$$

for the emitter junction. This capacitance is defined in terms of the increment of charge resulting from an increment in junction voltage:

$$C_{DC} = \frac{\Delta Q}{\Delta V_C}$$

The charge increment can also be described in terms of the lifetime τ:

$$\Delta Q = \tau \Delta J_C \qquad (2)$$

Hence,

$$C_{DC} = \tau \frac{dJ_C}{dV_C} \tag{3}$$

The dI_{DC}/dV_C term can be evaluated from the diode-current equation:

$$J_C = I_{CS}(\exp q/kTM_C - 1)$$

$$\frac{dJ_C}{dV_C} = \frac{q}{kTM} I_s \exp\left(\frac{q}{kTM}\right)$$

Finally,

$$C_{DC} = \tau \frac{q}{kTM}(J_D + I_s)$$

The current in the diffusion capacitance is given by

$$I_{DC} = C_{DC}\frac{dV_C}{dt}$$

If Eq. (3) is substituted,

$$I_{DC} = \tau \frac{dJ_C}{dV_C}\frac{dV_C}{dt} = \tau \frac{dJ_C}{dt} \tag{4}$$

The MTRAC program uses this expression to determine the magnitude of a current generator that replaces the diffusion capacitance.

The frequency-domain representation of the diffusion capacitance will now be considered. For clarity, the expression for emitter current is simplified to

$$I_E = J_E + I_{DC}$$

since I_{RN} and J_N are small under normal bias conditions. By substitution of Eq. (4), this becomes

$$I_E = J_E + \tau \frac{dJ_E}{dt}$$

using operational notation

$$I_E = J_E(1 + s\tau)$$

where s = complex operator $\sigma + j\omega$.

The normal current gain of the simplified transistor at low frequencies is given by

$$\alpha_{NL} = \frac{I_C}{J_E}$$

At higher frequencies, the gain is

$$\alpha_{NH} = \frac{I_C}{J_E(1 + s\tau)} = \alpha_{NL}\frac{1}{1 + s\tau}$$

The effect of the diffusion capacitor is to shunt a portion of the total emitter current away from the active (or gain-producing) elements of the model. This is equivalent to the single-pole representation of the frequency dependence of current gain. This means has been used in the ECAP transistor model (Fig. 4) to deal with the diffusion capacitance.

VI. General Discussion

There is considerable room for personal preference and difference of opinion regarding the features that should be included in circuit-analysis programs and their relative value. Some of the features often mentioned, as they relate to the programs under consideration, are considered in the paragraphs that follow.

A. User-Oriented Input Language

The engineers who use the ECAP, CIRCUS, SCEPTRE, and MTRAC programs are seldom interested in computer programming, nor do they have time to learn it. The input language is the interface between the engineer and the machine, and it must be understandable *in engineering terms*. Input languages vary from the most flexible (SCEPTRE) to the least flexible (MTRAC).

Each input language, however flexible, requires absolute conformity to its own set of rules. For example: in SCEPTRE, some expressions are lifted directly from the user input and placed in the SIMUL8 program code; hence, all applicable FORTRAN rules are enforced and FORTRAN errors not caught by SCEPTRE are picked up by the FORTRAN compiler.

The fixed format of MTRAC has a disadvantage in that one cannot troubleshoot the input deck by inspection. Each card must be individually checked for format and sequence. If there is an error, however, the description of the fault is quite explicit. More elaborate programs often produce seemingly irrelevant error messages. (Differences in input languages will become more apparent in later illustrations.)

B. Modes of Analysis

The modes of analysis required reflect the users' needs; hence, they vary widely. Although SCEPTRE, CIRCUS,

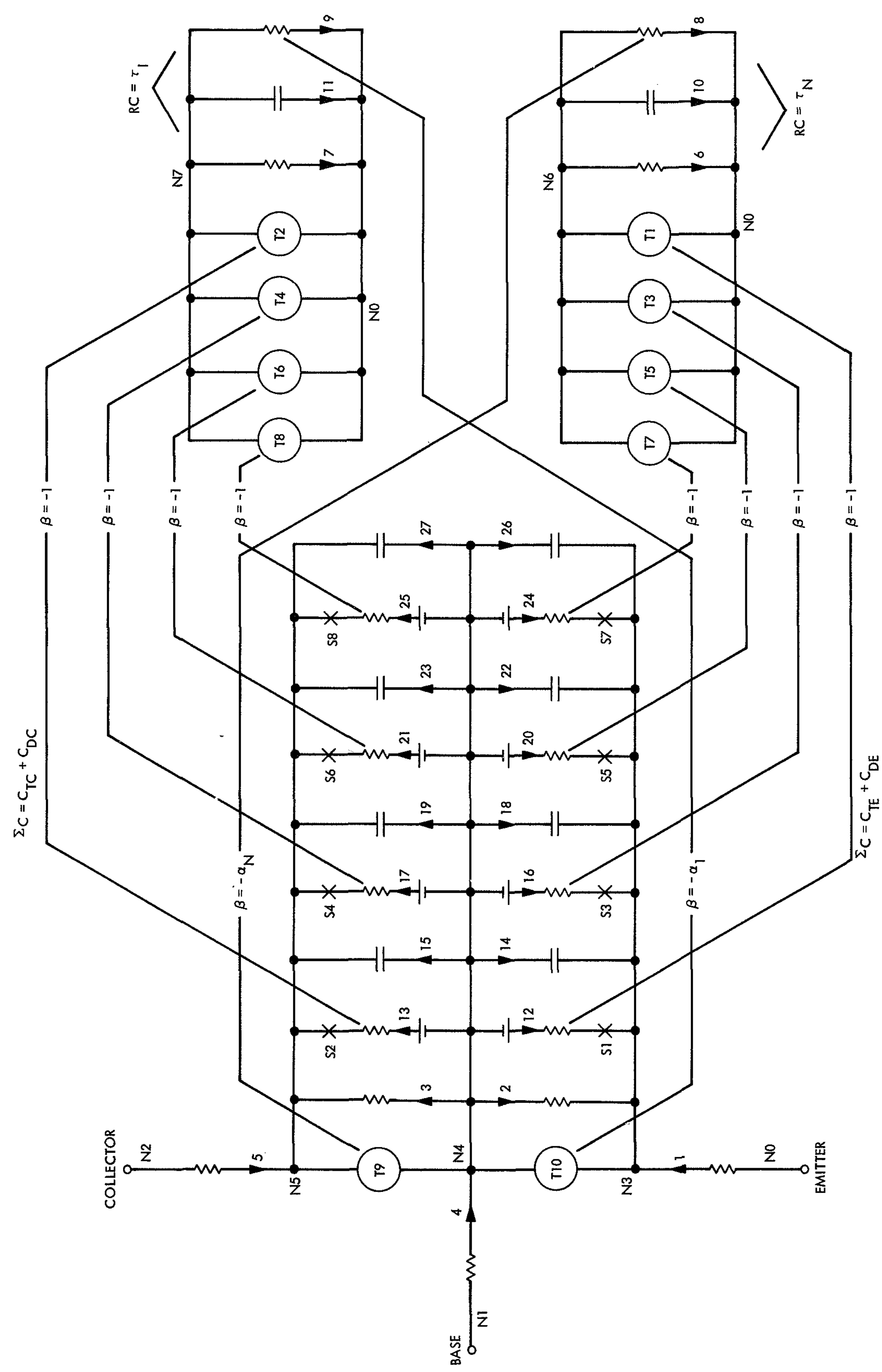


Fig. 4. Ebers–Moll model for ECAP

and MTRAC are basically transient-analysis programs, each includes an initial-condition solution capability. The ECAP program, despite its awkward mechanization of nonlinearities, has the broadest capability. This is because of its sensitivity and inclusion of linear ac analysis and "worst-case" analysis. Other programs must be used, however, to determine operational impedances and transfer functions.

C. Circuit Size

Of the group of programs being considered, CIRCUS and SCEPTRE are potentially capable of accepting the largest circuits. This is because the state-variable formulation leads to the minimum number of variables by which the circuit may be represented. Since CIRCUS incorporates variable dimensioning, it is probably the larger of the two. The SCEPTRE program has the restriction of requiring a vector $\mathbf{X}(\mathbf{m})$ notation for all elements if more than 70 elements are used. The dimensions in ECAP and MTRAC are considerably smaller. Their solution matrices are 50×50 and 60×60, respectively. The dimension of the nodal matrix corresponds to the number of unknown nodes.

D. Analysis of the Comparison Circuit

The comparison of the operation of CIRCUS, ECAP, MTRAC, and SCEPTRE described herein was made by having each of the four programs analyze the same circuit. The circuit chosen was the one-shot multivibrator shown in Fig. 5.

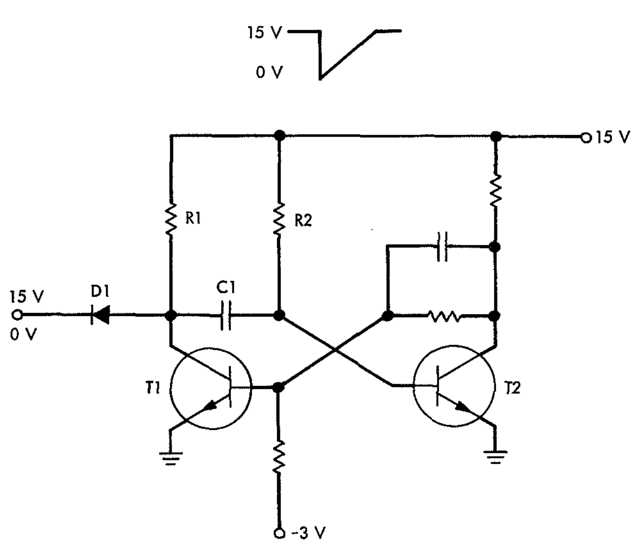


Fig. 5. One-shot multivibrator

This multivibrator circuit has one stable state that occurs because T2 is normally *on* because of the current in resistor R2. Since T1 is directly coupled from T2, T2 is expected to have the opposite state. The input transient waveform is coupled through D1 and C1 to T2, turning it off. As T2 turns off, T1 is turned on. This unstable state is maintained until the charge on capacitor C1 is reversed. The regenerative action then causes a return to the stable state. The length of the cycle is approximated by $T = 0.5\,R_2C_1$. A cycle length of approximately 30 μs is expected in this example.

This circuit includes two difficult problems that are often encountered in computer-circuit analysis. The first is the requirement for transistor and diode models that simulate these devices in three regions of operation: (1) saturation (both junctions forward-biased), (2) cut off (both junctions reverse-biased), and (3) active (base–emitter junction forward-biased and base–collector junction reverse-biased). The second difficulty is that of widely separate characteristic roots. The current gain of the transistor has its dominant root at approximately $2 \times 10^{0.9}$ s. This is a factor of 1.5×10^4 from the time period of the time-varying source used to trigger the multivibrator.

VII. Problem Solution

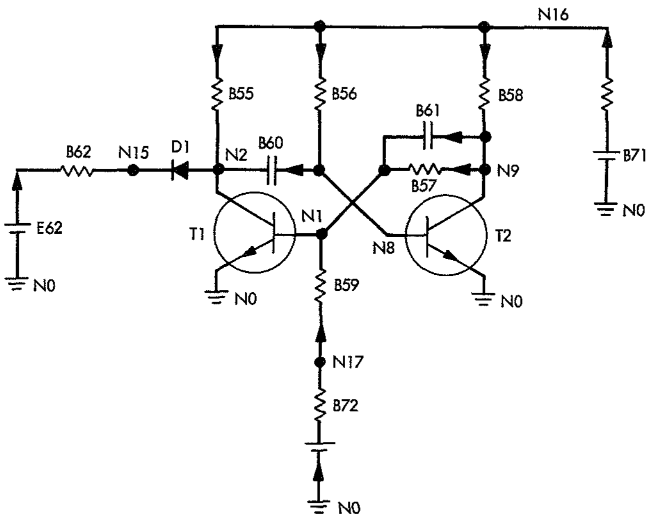
The input circuit and input data requirements for each of the programs are described in this section. Individual program restrictions and capabilities are not explored herein, however; such detail can be found only in the respective users' manuals. The discussion in the paragraphs that follow applies to the problem at hand, but nevertheless it is illustrative of the overall capability in each case.

A. ECAP

The circuit as prepared for ECAP is shown in Fig. 6. See Table 3 for the ECAP input listing.

1. Circuit preparation. The ECAP program requires sequential branch and node numbers. The 0 node is always ground. In this example, N3–N7 and N10–N14 are internal to the transistor models.

The descriptions of the voltage sources, as well as the pulse input to the circuit, are made in branch statements. Branches 71, 72, and 62 have been used for this purpose. It should be noted that a small resistance must be included as the branch element for each source. Branch 62



ELEMENTS INTERNAL TO THE MODELS

ELEMENT TYPE	T1	T2	D1
BRANCHES	1-27	28-54	63-70
NODES	0, 1-7	0, 8-14	2, 15
SWITCHES	1-8	9-16	17-19
TRANSISTORS	1-10	11-20	—

Fig. 6. Multivibrator for ECAP

includes the time-varying input that is specified by the E62 statement. The voltage values occur 10 step sizes apart, as specified by the (10),. field on the E62 card.

The ECAP program requires that the user choose the step size. In this case, the choice was 50×10^{-9} s, and no effort was made to pick an optimum step size. This particular version of ECAP allows selective printing and plotting, as indicated. The plot-interval statement will cause outputs to occur at $10 \times 50 \times 10^{-9}$ s $= 5 \times 10^{-6}$ for a total of 150 points. This was necessary because the plotting routine is restricted to 500 points per plot.

2. Solution times. The solution times for ECAP are as follows:

(1) Load time (includes compiling one subroutine): 183 s.

(2) Execution time (computer running time): 374 s.

(3) Simulation time (circuit operation time): 75.05×10^{-6} s.

The plotted outputs are shown in Table 6.

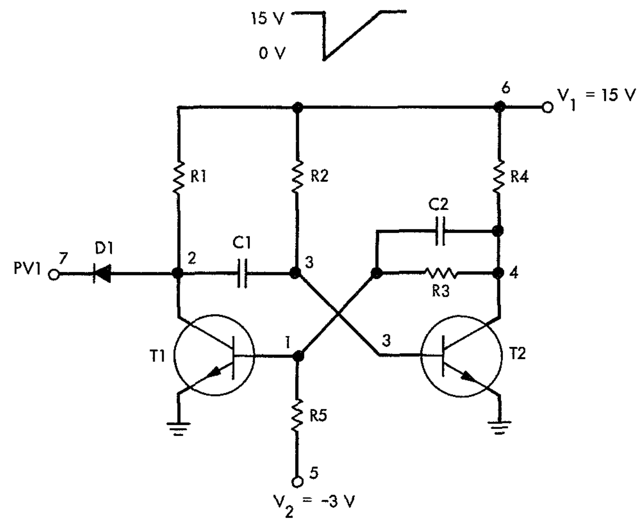


Fig. 7. Multivibrator for CIRCUS

B. CIRCUS

The circuit as prepared for CIRCUS is shown in Fig. 7. See Table 2 for the CIRCUS input listing.

1. Circuit preparation. The CIRCUS program requires sequential node numbers beginning with 0 for ground or reference. The usual *from–to* node sequence for elements is not used by CIRCUS; instead, current flow is considered positive when flowing from the higher node number to the lower.

Constant sources are written in with a V_{name} statement. Time-varying sources are restricted to specific types, including the pulse source (PV1) used herein. Descriptors following the node numbers in the PV1 statement are: initial voltage, pulse voltage, delay time, rise time, duration, fall time, and period. The CIRCUS program provides a wide range of circuit and model functions that are available for output. Only the node voltages are requested in this example.

The INTERVALS statement selects both the time between printed outputs and the circuit-simulation stop time. Moreover, this statement also sets the starting step size for the integration process. Because internal restrictions prevent indefinite reduction of step size, a reasonably good estimate of the starting step size is necessary. In the initial run of this multivibrator circuit (see Fig. 7), a print interval of 75×10^{-9} s was chosen. The CIRCUS program was unable to converge the first integration step in 840 s of execution.

Table 6. ECAP output

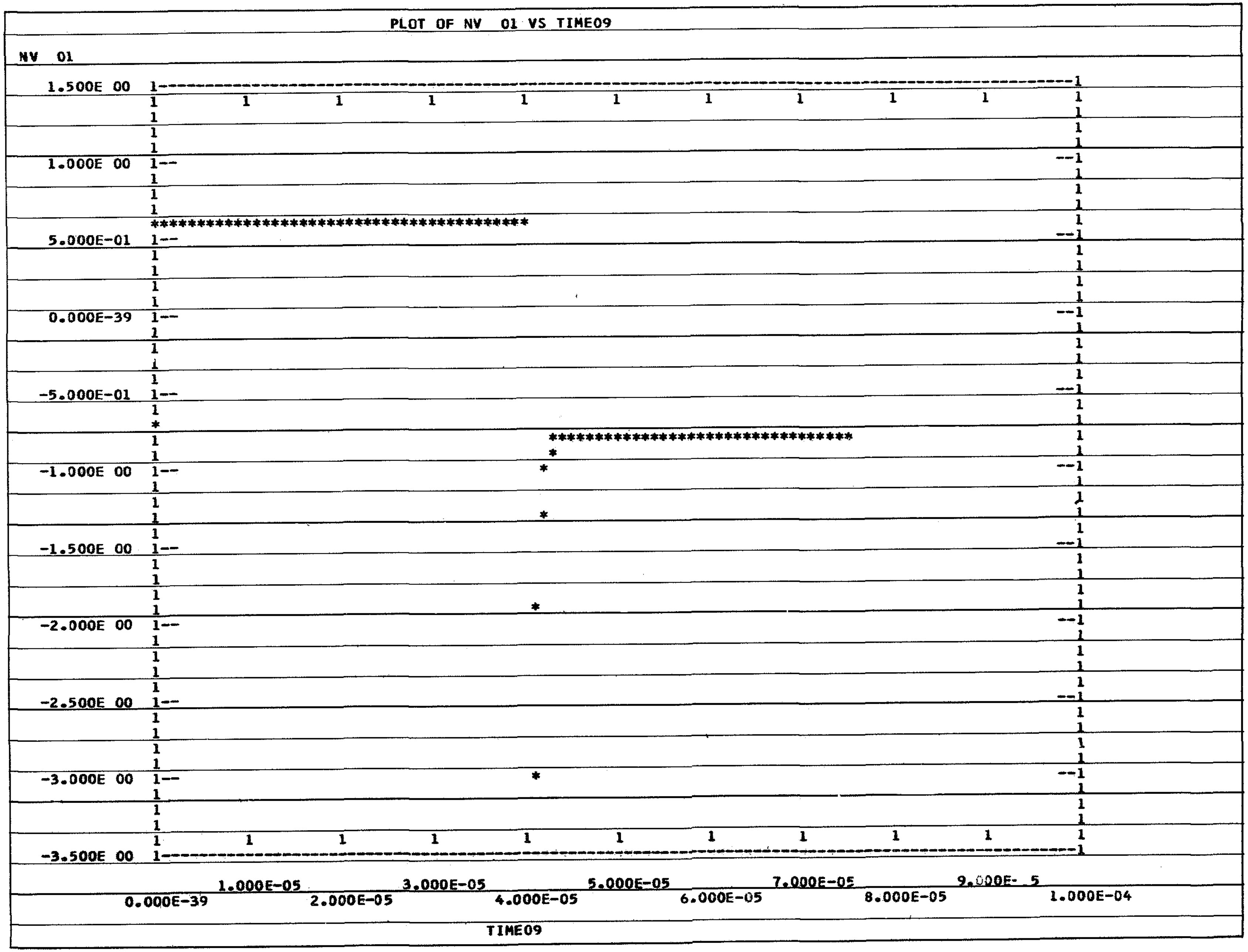


Table 6 (contd)

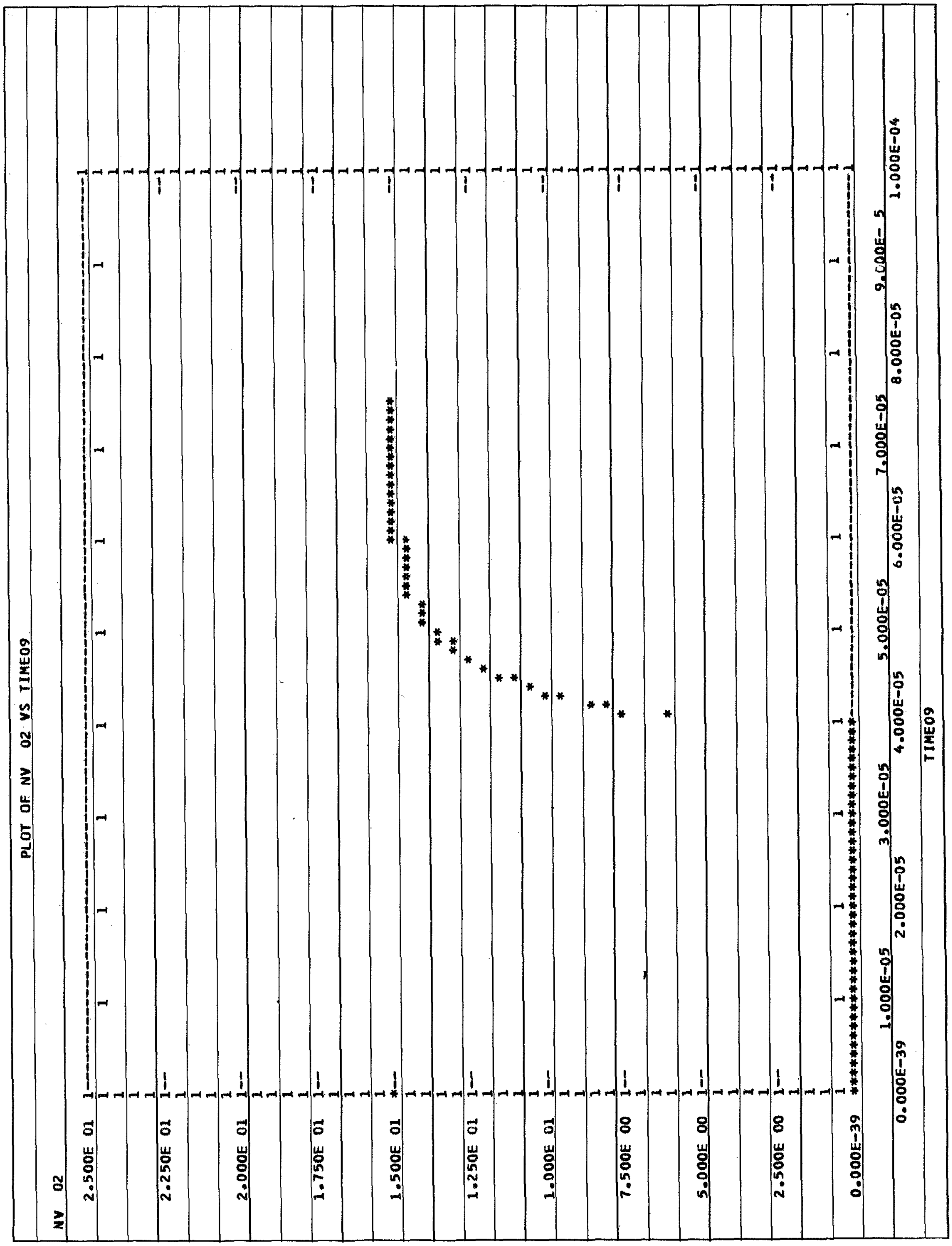


Table 6 (contd)

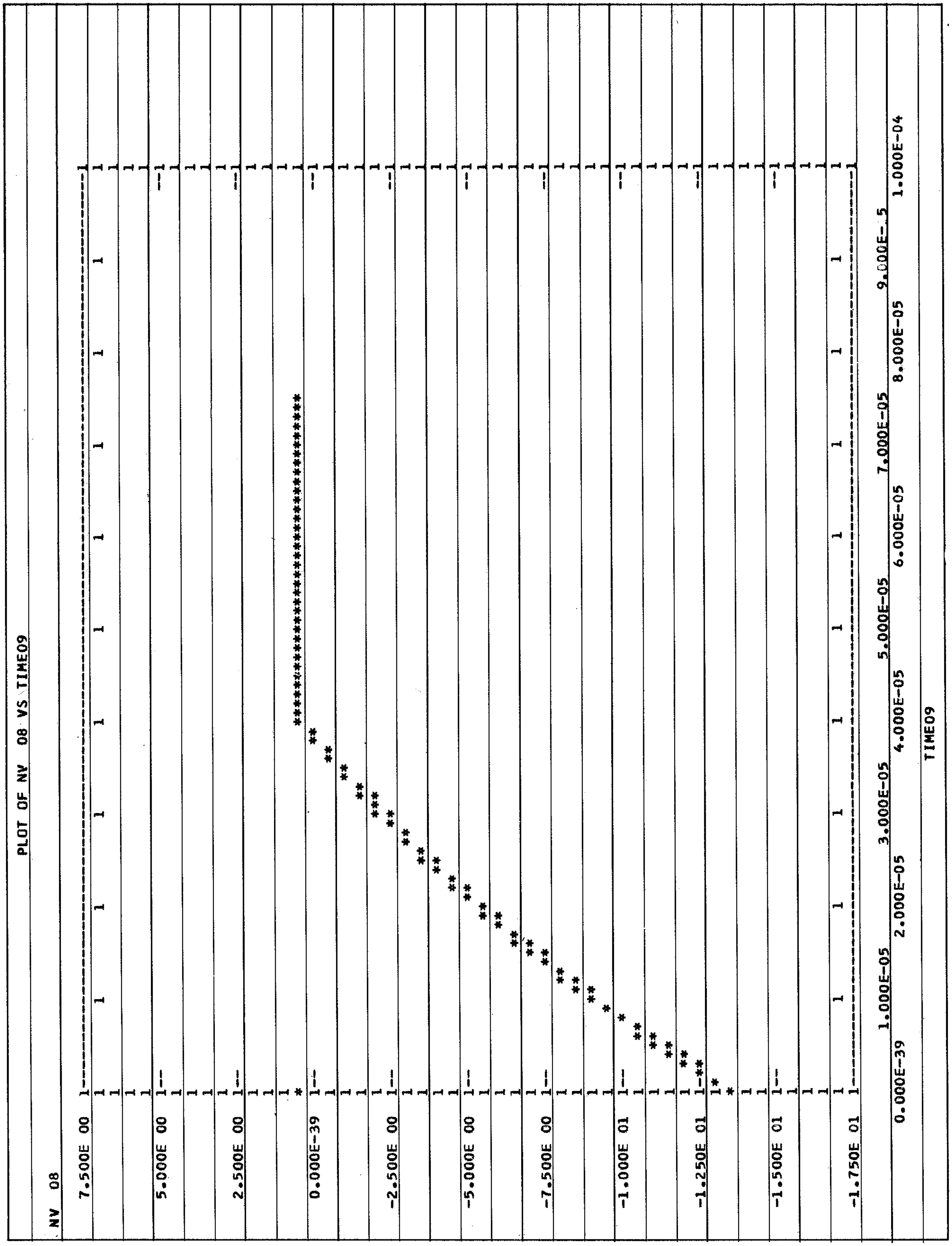
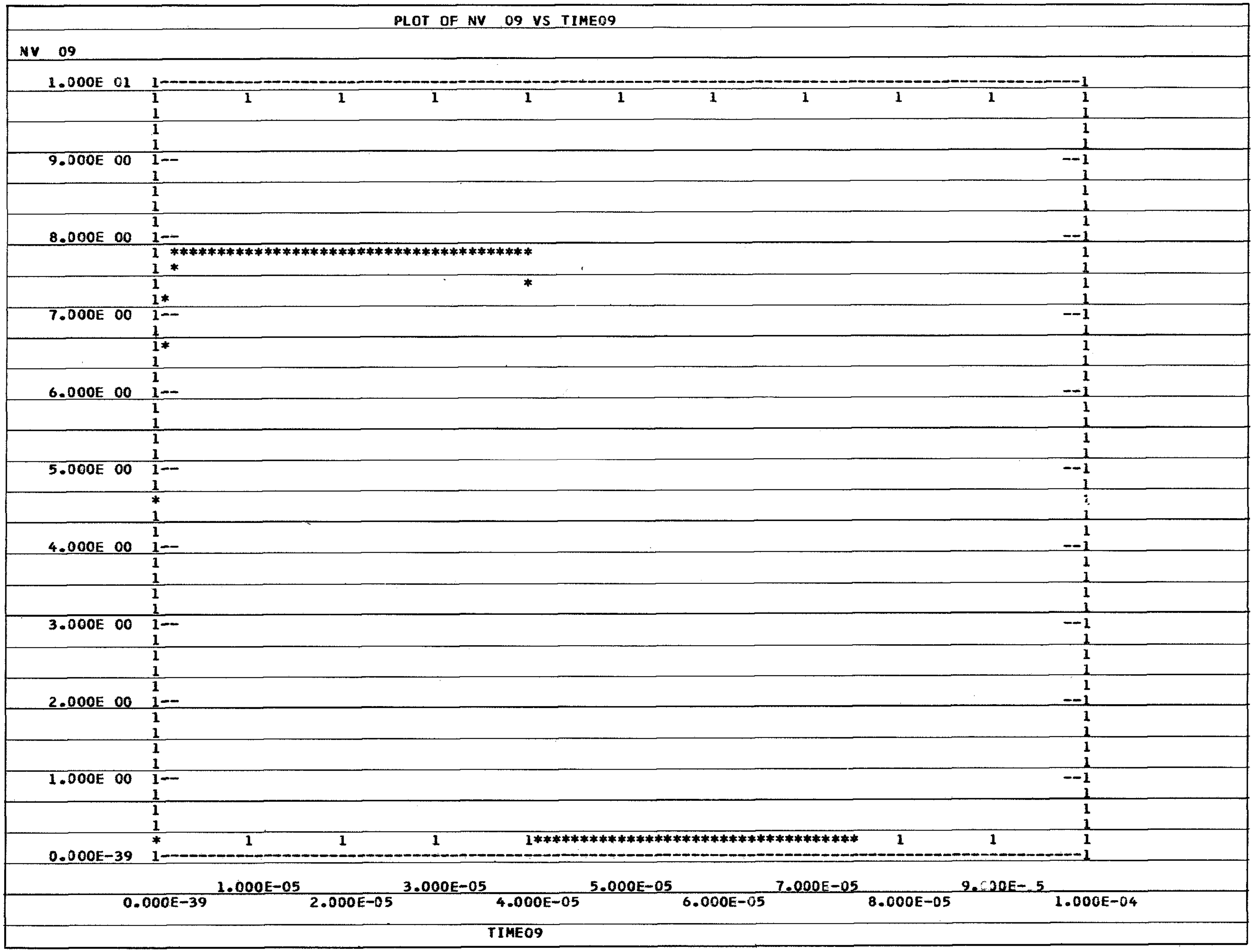


Table 6 (contd)



The second run was successfully made with a print interval of 7.5×10^{-9} s. For this run, the simulation time was set to 3×10^{-6} s; as a result, 400 points were printed and plotted. Because this plot routine plots 120 abscissa points on each plot, four plots were required. The output plots are shown in Table 7.

2. Solution times. The solution times for CIRCUS are as follows:

(1) Load time: 52 s.

(2) Execution time: 1250 s.

(3) Simulation time 3×10^{-6} s.

(4) Execution time (extrapolated to 75×10^{-6} s simulation): 31,250 s.

C. MTRAC

The circuit as prepared for MTRAC is shown in Fig. 8. See Table 4 for the MTRAC input listing.

1. Circuit preparation. The MTRAC program requires sequential node numbers starting at 0 for ground. Element designations are implied by the relative order of the input cards. Card 55000 (rightmost column) gives the number of resistors in the circuit. Card 55030 is for the third resistor and describes R3 connected from node 4 to node 1. The card that follows describes R4 connected from source 2 to node 4. The card numbers are for the users' benefit; they are not read by the computer.

Sources are also numbered and their numbers are also implied from the relative order of input. Card 50000 has already indicated that there are three sources. Card 53010 indicates that the first source is time-varying and that four sets of voltage–time coordinates will be given. That source 2 is dc is indicated by the -1 on card 53013. The value (15 V) is indicated by the card that follows.

Output requests, for nodes 1 through 4, are made on card 50020. As with CIRCUS, a great number of internal parameters are available for output, but the user must write these requests into subprogram TREQ, using FORTRAN.

2. Solution times. The solution times for MTRAC are as follows:

(1) Load time: 83 s.

(2) Execution time: 102 s.

(3) Simulation time: 75×10^{-6} s.

The MTRAC output plots are shown in Table 8.

D. SCEPTRE

The circuit as prepared for SCEPTRE is shown in Fig. 9. See Table 1 for the SCEPTRE input listing.

1. Circuit preparation. Node designations are arbitrary in SCEPTRE; any six-character alphameric is acceptable.

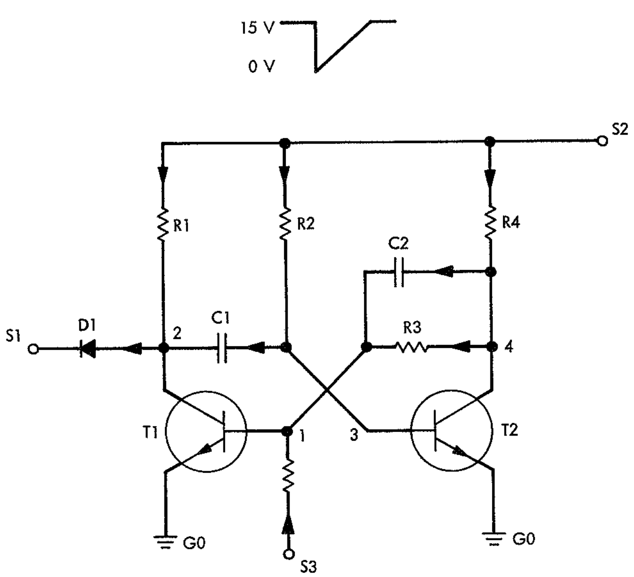


Fig. 8. Multivibrator for MTRAC

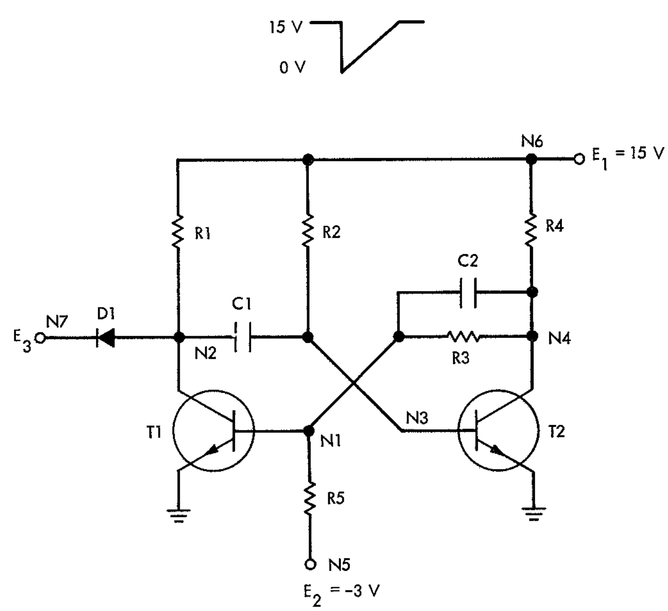


Fig. 9. Multivibrator for SCEPTRE

Table 7. CIRCUS output

```
                      PLOT OF VN1     VS. TIME                    'MULTIVIBRATOR TEST PROBLEM'
 9.40E-01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 7.60E-01 I       ********    I         I         I         I         I         I         I         I         I         I         I
 5.80E-01 I      *  I    **********************************************************************************************************
 4.00E-01 I     *                                                                                                                 I
 2.20E-01 I                                                                                                                       I
 4.00E-02 +---                                                                                                                 ---+
-1.40E-01 I                                                                                                                       I
-3.20E-01 I                                                                                                                       I
-5.00E-01 I                                                                                                                       I
-6.80E-01 I    *                                                                                                                  I
-8.60E-01 *---                                                                                                                 ---+
-1.04E 00 I                                                                                                                       I
-1.22E 00 I                                                                                                                       I
-1.40E 00 I                                                                                                                       I
-1.58E 00 I                                                                                                                       I
-1.76E 00 +---                                                                                                                 ---+
-1.94E 00 I                                                                                                                       I
-2.12E 00 I   *                                                                                                                   I
-2.30E 00 I                                                                                                                       I
-2.48E 00 I                                                                                                                       I
-2.66E 00 +---                                                                                                                 ---+
-2.84E 00 I                                                                                                                       I
-3.02E 00 I                                                                                                                       I
-3.20E 00 I                                                                                                                       I
-3.38E 00 I                                                                                                                       I
-3.56E 00 +---                                                                                                                 ---+
-3.74E 00 I                                                                                                                       I
-3.92E 00 I  *                                                                                                                    I
-4.10E 00 I                                                                                                                       I
-4.28E 00 I                                                                                                                       I
-4.46E 00 +---                                                                                                                 ---+
-4.64E 00 I                                                                                                                       I
-4.82E 00 I                                                                                                                       I
-5.00E 00 I                                                                                                                       I
-5.18E 00 I                                                                                                                       I
-5.36E 00 +---                                                                                                                 ---+
-5.54E 00 I                                                                                                                       I
-5.72E 00 I                                                                                                                       I
-5.90E 00 I                                                                                                                       I
-6.08E 00 I                                                                                                                       I
-6.26E 00 +**-                                                                                                                 ---+
-6.44E 00 I                                                                                                                       I
-6.62E 00 I                                                                                                                       I
-6.80E 00 I                                                                                                                       I
-6.98E 00 I                                                                                                                       I
-7.16E 00 +---                                                                                                                 ---+
-7.34E 00 I                                                                                                                       I
-7.52E 00 I                                                                                                                       I
-7.70E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-7.88E 00 I*        I         I         I         I         I         I         I         I         I         I         I         I
-8.06E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
      0.000E-39           1.500E-07           3.000E-07           4.500E-07           6.000E-07           7.500E-07           9.000E-07
                7.500E-08           2.250E-07           3.750E-07           5.250E-07           6.750E-07           8.250E-07
```

Table 7 (contd)

```
                     PLOT OF VN1     VS. TIME                 'MULTIVIBRATOR TEST PROBLEM'

  9.40E-01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
  7.60E-01 I         I         I         I         I         I         I         I         I         I         I         I         I
  5.80E-01 *************************************************************************************************************************
  4.00E-01 I                                                                                                                       I
  2.20E-01 I                                                                                                                       I
  4.00E-02 +---                                                                                                                 ---+
 -1.40E-01 I                                                                                                                       I
 -3.20E-01 I                                                                                                                       I
 -5.00E-01 I                                                                                                                       I
 -6.80E-01 I                                                                                                                       I
 -8.60E-01 +---                                                                                                                 ---+
 -1.04E 00 I                                                                                                                       I
 -1.22E 00 I                                                                                                                       I
 -1.40E 00 I                                                                                                                       I
 -1.58E 00 I                                                                                                                       I
 -1.76E 00 +---                                                                                                                 ---+
 -1.94E 00 I                                                                                                                       I
 -2.12E 00 I                                                                                                                       I
 -2.30E 00 I                                                                                                                       I
 -2.48E 00 I                                                                                                                       I
 -2.66E 00 +---                                                                                                                 ---+
 -2.84E 00 I                                                                                                                       I
 -3.02E 00 I                                                                                                                       I
 -3.20E 00 I                                                                                                                       I
 -3.38E 00 I                                                                                                                       I
 -3.56E 00 +---                                                                                                                 ---+
 -3.74E 00 I                                                                                                                       I
 -3.92E 00 I                                                                                                                       I
 -4.10E 00 I                                                                                                                       I
 -4.28E 00 I                                                                                                                       I
 -4.46E 00 +---                                                                                                                 ---+
 -4.64E 00 I                                                                                                                       I
 -4.82E 00 I                                                                                                                       I
 -5.00E 00 I                                                                                                                       I
 -5.18E 00 I                                                                                                                       I
 -5.36E 00 +---                                                                                                                 ---+
 -5.54E 00 I                                                                                                                       I
 -5.72E 00 I                                                                                                                       I
 -5.90E 00 I                                                                                                                       I
 -6.08E 00 I                                                                                                                       I
 -6.26E 00 +---                                                                                                                 ---+
 -6.44E 00 I                                                                                                                       I
 -6.62E 00 I                                                                                                                       I
 -6.80E 00 I                                                                                                                       I
 -6.98E 00 I                                                                                                                       I
 -7.16E 00 +---                                                                                                                 ---+
 -7.34E 00 I                                                                                                                       I
 -7.52E 00 I                                                                                                                       I
 -7.70E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 -7.88E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 -8.06E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

       9.000E-07           1.050E-06           1.200E-06           1.350E-06           1.500E-06           1.650E-06           1.800E-06
                 9.750E-07           1.125E-06           1.275E-06           1.425E-06           1.575E-06           1.725E-06
```

Table 7 (contd)

```
                        PLOT OF VN1    VS. TIME                 'MULTIVIBRATOR TEST PROBLEM'

 9.40E-01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 7.60E-01 I         I         I         I         I         I         I         I         I         I         I         I         I
 5.80E-01 *************************************************************************************************************************
 4.00E-01 I                                                                                                                       I
 2.20E-01 I                                                                                                                       I
 4.00E-02 +---                                                                                                                 ---+
-1.40E-01 I                                                                                                                       I
-3.20E-01 I                                                                                                                       I
-5.00E-01 I                                                                                                                       I
-6.80E-01 I                                                                                                                       I
-8.60E-01 +---                                                                                                                 ---+
-1.04E 00 I                                                                                                                       I
-1.22E 00 I                                                                                                                       I
-1.40E 00 I                                                                                                                       I
-1.58E 00 I                                                                                                                       I
-1.76E 00 +---                                                                                                                 ---+
-1.94E 00 I                                                                                                                       I
-2.12E 00 I                                                                                                                       I
-2.30E 00 I                                                                                                                       I
-2.48E 00 I                                                                                                                       I
-2.66E 00 +---                                                                                                                 ---+
-2.84E 00 I                                                                                                                       I
-3.02E 00 I                                                                                                                       I
-3.20E 00 I                                                                                                                       I
-3.38E 00 I                                                                                                                       I
-3.56E 00 +---                                                                                                                 ---+
-3.74E 00 I                                                                                                                       I
-3.92E 00 I                                                                                                                       I
-4.10E 00 I                                                                                                                       I
-4.28E 00 I                                                                                                                       I
-4.46E 00 +---                                                                                                                 ---+
-4.64E 00 I                                                                                                                       I
-4.82E 00 I                                                                                                                       I
-5.00E 00 I                                                                                                                       I
-5.18E 00 I                                                                                                                       I
-5.36E 00 +---                                                                                                                 ---+
-5.54E 00 I                                                                                                                       I
-5.72E 00 I                                                                                                                       I
-5.90E 00 I                                                                                                                       I
-6.08E 00 I                                                                                                                       I
-6.26E 00 +---                                                                                                                 ---+
-6.44E 00 I                                                                                                                       I
-6.62E 00 I                                                                                                                       I
-6.80E 00 I                                                                                                                       I
-6.98E 00 I                                                                                                                       I
-7.16E 00 +---                                                                                                                 ---+
-7.34E 00 I                                                                                                                       I
-7.52E 00 I                                                                                                                       I
-7.70E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-7.88E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-8.06E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

      1.800E-06           1.950E-06           2.100E-06           2.250E-06           2.400E-06           2.550E-06           2.700E-06
                1.875E-06           2.025E-06           2.175E-06           2.325E-06           2.475E-06           2.625E-06
```

Table 7 (contd)

```
                            PLOT OF VN1    VS. TIME                  'MULTIVIBRATOR TEST PROBLEM'

  9.40E-01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
  7.60E-01 I         I         I         I         I         I         I         I         I         I         I         I         I
  5.80E-01 *****************************************         I         I         I         I         I         I         I         I
  4.00E-01 I                                                                                                                       I
  2.20E-01 I                                                                                                                       I
  4.00E-02 +---                                                                                                                 ---+
 -1.40E-01 I                                                                                                                       I
 -3.20E-01 I                                                                                                                       I
 -5.00E-01 I                                                                                                                       I
 -6.80E-01 I                                                                                                                       I
 -8.60E-01 +---                                                                                                                 ---+
 -1.04E 00 I                                                                                                                       I
 -1.22E 00 I                                                                                                                       I
 -1.40E 00 I                                                                                                                       I
 -1.58E 00 I                                                                                                                       I
 -1.76E 00 +---                                                                                                                 ---+
 -1.94E 00 I                                                                                                                       I
 -2.12E 00 I                                                                                                                       I
 -2.30E 00 I                                                                                                                       I
 -2.48E 00 I                                                                                                                       I
 -2.66E 00 +---                                                                                                                 ---+
 -2.84E 00 I                                                                                                                       I
 -3.02E 00 I                                                                                                                       I
 -3.20E 00 I                                                                                                                       I
 -3.38E 00 I                                                                                                                       I
 -3.56E 00 +---                                                                                                                 ---+
 -3.74E 00 I                                                                                                                       I
 -3.92E 00 I                                                                                                                       I
 -4.10E 00 I                                                                                                                       I
 -4.28E 00 I                                                                                                                       I
 -4.46E 00 +---                                                                                                                 ---+
 -4.64E 00 I                                                                                                                       I
 -4.82E 00 I                                                                                                                       I
 -5.00E 00 I                                                                                                                       I
 -5.18E 00 I                                                                                                                       I
 -5.36E 00 +---                                                                                                                 ---+
 -5.54E 00 I                                                                                                                       I
 -5.72E 00 I                                                                                                                       I
 -5.90E 00 I                                                                                                                       I
 -6.08E 00 I                                                                                                                       I
 -6.26E 00 +---                                                                                                                 ---+
 -6.44E 00 I                                                                                                                       I
 -6.62E 00 I                                                                                                                       I
 -6.80E 00 I                                                                                                                       I
 -6.98E 00 I                                                                                                                       I
 -7.16E 00 +---                                                                                                                 ---+
 -7.34E 00 I                                                                                                                       I
 -7.52E 00 I                                                                                                                       I
 -7.70E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 -7.88E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 -8.06E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

      2.700E-06           2.850E-06           3.000E-06           3.150E-06           3.300E-06           3.450E-06           3.600E-06
                2.775E-06           2.925E-06           3.075E-06           3.225E-06           3.375E-06           3.525E-06
R-PLOT
END L6
```

Table 7 (contd)

```
                    PLOT OF VN2     VS. TIME                 'MULTIVIBRATOR TEST PROBLEM'

 1.78E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 1.74E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.70E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.66E 01 I                                                                                                                       I
 1.62E 01 I                                                                                                                       I
 1.58E 01 +---                                                                                                                 ---+
 1.54E 01 *                                                                                                                       I
 1.50E 01 I                                                                                                                       I
 1.46E 01 I                                                                                                                       I
 1.42E 01 I                                                                                                                       I
 1.38E 01 +---                                                                                                                 ---+
 1.34E 01 I                                                                                                                       I
 1.30E 01 I                                                                                                                       I
 1.26E 01 I                                                                                                                       I
 1.22E 01 I                                                                                                                       I
 1.18E 01 +---                                                                                                                 ---+
 1.14E 01 I                                                                                                                       I
 1.10E 01 I                                                                                                                       I
 1.06E 01 I                                                                                                                       I
 1.02E 01 I                                                                                                                       I
 9.80E 00 +---                                                                                                                 ---+
 9.40E 00 I                                                                                                                       I
 9.00E 00 I                                                                                                                       I
 8.60E 00 I                                                                                                                       I
 8.20E 00 I                                                                                                                       I
 7.80E 00 +---                                                                                                                 ---+
 7.40E 00 I                                                                                                                       I
 7.00E 00 I                                                                                                                       I
 6.60E 00 I                                                                                                                       I
 6.20E 00 I                                                                                                                       I
 5.80E 00 +---                                                                                                                 ---+
 5.40E 00 I                                                                                                                       I
 5.00E 00 I                                                                                                                       I
 4.60E 00 I                                                                                                                       I
 4.20E 00 I*                                                                                                                      I
 3.80E 00 +---                                                                                                                 ---+
 3.40E 00 I                                                                                                                       I
 3.00E 00 I                                                                                                                       I
 2.60E 00 I                                                                                                                       I
 2.20E 00 I                                                                                                                       I
 1.80E 00 +---                                                                                                                 ---+
 1.40E 00 I                                                                                                                       I
 1.00E 00 I                                                                                                                       I
 6.00E-01 I************                                                                                                           I
 2.00E-01 I            ************************************************************************************************************
-2.00E-01 +---                                                                                                                 ---+
-6.00E-01 I                                                                                                                       I
-1.00E 00 I                                                                                                                       I
-1.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.80E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-2.20E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

      0.000E-39           1.500E-07           3.000E-07           4.500E-07           6.000E-07           7.500E-07           9.000E-07
                7.500E-08           2.250E-07           3.750E-07           5.250E-07           6.750E-07           8.250E-07
```

Table 7 (contd)

```
                    PLOT OF VN2    VS. TIME                    'MULTIVIBRATOR TEST PROBLEM'

 1.78E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 1.74E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.70E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.66E 01 I                                                                                                                       I
 1.62E 01 I                                                                                                                       I
 1.58E 01 +---                                                                                                                 ---+
 1.54E 01 I                                                                                                                       I
 1.50E 01 I                                                                                                                       I
 1.46E 01 I                                                                                                                       I
 1.42E 01 I                                                                                                                       I
 1.38E 01 +---                                                                                                                 ---+
 1.34E 01 I                                                                                                                       I
 1.30E 01 I                                                                                                                       I
 1.26E 01 I                                                                                                                       I
 1.22E 01 I                                                                                                                       I
 1.18E 01 +---                                                                                                                 ---+
 1.14E 01 I                                                                                                                       I
 1.10E 01 I                                                                                                                       I
 1.06E 01 I                                                                                                                       I
 1.02E 01 I                                                                                                                       I
 9.80E 00 +---                                                                                                                 ---+
 9.40E 00 I                                                                                                                       I
 9.00E 00 I                                                                                                                       I
 8.60E 00 I                                                                                                                       I
 8.20E 00 I                                                                                                                       I
 7.80E 00 +---                                                                                                                 ---+
 7.40E 00 I                                                                                                                       I
 7.00E 00 I                                                                                                                       I
 6.60E 00 I                                                                                                                       I
 6.20E 00 I                                                                                                                       I
 5.80E 00 +---                                                                                                                 ---+
 5.40E 00 I                                                                                                                       I
 5.00E 00 I                                                                                                                       I
 4.60E 00 I                                                                                                                       I
 4.20E 00 I                                                                                                                       I
 3.80E 00 +---                                                                                                                 ---+
 3.40E 00 I                                                                                                                       I
 3.00E 00 I                                                                                                                       I
 2.60E 00 I                                                                                                                       I
 2.20E 00 I                                                                                                                       I
 1.80E 00 +---                                                                                                                 ---+
 1.40E 00 I                                                                                                                       I
 1.00E 00 I                                                                                                                       I
 6.00E-01 I                                                                                                                       I
 2.00E-01 *************************************************************************************************************************
-2.00E-01 +---                                                                                                                 ---+
-6.00E-01 I                                                                                                                       I
-1.00E 00 I                                                                                                                       I
-1.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.80E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-2.20E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

    9.000E-07           1.050E-06           1.200E-06           1.350E-06           1.500E-06           1.650E-06           1.800E-06
              9.750E-07           1.125E-06           1.275E-06           1.425E-06           1.575E-06           1.725E-06
```

Table 7 (contd)

```
                PLOT OF VN2    VS. TIME                  'MULTIVIBRATOR TEST PROBLEM'

 1.78E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 1.74E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.70E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.66E 01 I                                                                                                                       I
 1.62E 01 I                                                                                                                       I
 1.58E 01 +---                                                                                                                 ---+
 1.54E 01 I                                                                                                                       I
 1.50E 01 I                                                                                                                       I
 1.46E 01 I                                                                                                                       I
 1.42E 01 I                                                                                                                       I
 1.38E 01 +---                                                                                                                 ---+
 1.34E 01 I                                                                                                                       I
 1.30E 01 I                                                                                                                       I
 1.26E 01 I                                                                                                                       I
 1.22E 01 I                                                                                                                       I
 1.18E 01 +---                                                                                                                 ---+
 1.14E 01 I                                                                                                                       I
 1.10E 01 I                                                                                                                       I
 1.06E 01 I                                                                                                                       I
 1.02E 01 I                                                                                                                       I
 9.80E 00 +---                                                                                                                 ---+
 9.40E 00 I                                                                                                                       I
 9.00E 00 I                                                                                                                       I
 8.60E 00 I                                                                                                                       I
 8.20E 00 I                                                                                                                       I
 7.80E 00 +---                                                                                                                 ---+
 7.40E 00 I                                                                                                                       I
 7.00E 00 I                                                                                                                       I
 6.60E 00 I                                                                                                                       I
 6.20E 00 I                                                                                                                       I
 5.80E 00 +---                                                                                                                 ---+
 5.40E 00 I                                                                                                                       I
 5.00E 00 I                                                                                                                       I
 4.60E 00 I                                                                                                                       I
 4.20E 00 I                                                                                                                       I
 3.80E 00 +---                                                                                                                 ---+
 3.40E 00 I                                                                                                                       I
 3.00E 00 I                                                                                                                       I
 2.60E 00 I                                                                                                                       I
 2.20E 00 I                                                                                                                       I
 1.80E 00 +---                                                                                                                 ---+
 1.40E 00 I                                                                                                                       I
 1.00E 00 I                                                                                                                       I
 6.00E-01 I                                                                                                                       I
 2.00E-01 *************************************************************************************************************************
-2.00E-01 +---                                                                                                                 ---+
-6.00E-01 I                                                                                                                       I
-1.00E 00 I                                                                                                                       I
-1.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.80E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-2.20E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

  1.800E-06           1.950E-06           2.100E-06           2.250E-06           2.400E-06           2.550E-06           2.700E-06
            1.875E-06           2.025E-06           2.175E-06           2.325E-06           2.475E-06           2.625E-06
```

Table 7 (contd)

```
                       PLOT OF VN2    VS. TIME                    'MULTIVIBRATOR TEST PROBLEM'
 1.78E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 1.74E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.70E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.66E 01 I                                                                                                                       I
 1.62E 01 I                                                                                                                       I
 1.58E 01 +---                                                                                                                 ---+
 1.54E 01 I                                                                                                                       I
 1.50E 01 I                                                                                                                       I
 1.46E 01 I                                                                                                                       I
 1.42E 01 I                                                                                                                       I
 1.38E 01 +---                                                                                                                 ---+
 1.34E 01 I                                                                                                                       I
 1.30E 01 I                                                                                                                       I
 1.26E 01 I                                                                                                                       I
 1.22E 01 I                                                                                                                       I
 1.18E 01 +---                                                                                                                 ---+
 1.14E 01 I                                                                                                                       I
 1.10E 01 I                                                                                                                       I
 1.06E 01 I                                                                                                                       I
 1.02E 01 I                                                                                                                       I
 9.80E 00 +---                                                                                                                 ---+
 9.40E 00 I                                                                                                                       I
 9.00E 00 I                                                                                                                       I
 8.60E 00 I                                                                                                                       I
 8.20E 00 I                                                                                                                       I
 7.80E 00 +---                                                                                                                 ---+
 7.40E 00 I                                                                                                                       I
 7.00E 00 I                                                                                                                       I
 6.60E 00 I                                                                                                                       I
 6.20E 00 I                                                                                                                       I
 5.80E 00 +---                                                                                                                 ---+
 5.40E 00 I                                                                                                                       I
 5.00E 00 I                                                                                                                       I
 4.60E 00 I                                                                                                                       I
 4.20E 00 I                                                                                                                       I
 3.80E 00 +---                                                                                                                 ---+
 3.40E 00 I                                                                                                                       I
 3.00E 00 I                                                                                                                       I
 2.60E 00 I                                                                                                                       I
 2.20E 00 I                                                                                                                       I
 1.80E 00 +---                                                                                                                 ---+
 1.40E 00 I                                                                                                                       I
 1.00E 00 I                                                                                                                       I
 6.00E-01 I                                                                                                                       I
 2.00E-01 *****************************************                                                                               I
-2.00E-01 +---                                                                                                                 ---+
-6.00E-01 I                                                                                                                       I
-1.00E 00 I                                                                                                                       I
-1.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.80E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-2.20E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
      2.700E-06           2.850E-06           3.000E-06           3.150E-06           3.300E-06           3.450E-06           3.600E-06
                2.775E-06           2.925E-06           3.075E-06           3.225E-06           3.375E-06           3.525E-06
```

Table 7 (contd)

```
                   PLOT OF VN3    VS. TIME               'MULTIVIBRATOR TEST PROBLEM'

 3.20E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 2.80E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 2.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 2.00E 00 I                                                                                                                       I
 1.60E 00 I                                                                                                                       I
 1.20E 00 +---                                                                                                                 ---+
 8.00E-01 I                                                                                                                       I
 4.00E-01 *                                                                                                                       I
-2.68E-07 I                                                                                                                       I
-4.00E-01 I                                                                                                                       I
-8.00E-01 +---                                                                                                                 ---+
-1.20E 00 I                                                                                                                       I
-1.60E 00 I                                                                                                                       I
-2.00E 00 I                                                                                                                       I
-2.40E 00 I                                                                                                                       I
-2.80E 00 +---                                                                                                                 ---+
-3.20E 00 I                                                                                                                       I
-3.60E 00 I                                                                                                                       I
-4.00E 00 I                                                                                                                       I
-4.40E 00 I                                                                                                                       I
-4.80E 00 +---                                                                                                                 ---+
-5.20E 00 I                                                                                                                       I
-5.60E 00 I                                                                                                                       I
-6.00E 00 I                                                                                                                       I
-6.40E 00 I                                                                                                                       I
-6.80E 00 +---                                                                                                                 ---+
-7.20E 00 I                                                                                                                       I
-7.60E 00 I                                                                                                                       I
-8.00E 00 I                                                                                                                       I
-8.40E 00 I                                                                                                                       I
-8.80E 00 +---                                                                                                                 ---+
-9.20E 00 I                                                                                                                       I
-9.60E 00 I                                                                                                                       I
-1.00E 01 I                                                                                                                       I
-1.04E 01 I*                                                                                                                      I
-1.08E 01 +---                                                                                                                 ---+
-1.12E 01 I                                                                                                                       I
-1.16E 01 I                                                                                                                       I
-1.20E 01 I                                                                                                                       I
-1.24E 01 I                                                                                                                       I
-1.28E 01 +---                                                                                                                 ---+
-1.32E 01 I                                                                                                                       I
-1.36E 01 I                                                                                                                       I
-1.40E 01 I  *******                                                                       ****************************************
-1.44E 01 I**       ***********************************************************************                                       I
-1.48E 01 +---                                                                                                                 ---+
-1.52E 01 I                                                                                                                       I
-1.56E 01 I                                                                                                                       I
-1.60E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.64E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.68E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

   0.000E-39           1.500E-07           3.000E-07           4.500E-07           6.000E-07           7.500E-07           9.000E-07
             7.500E-08           2.250E-07           3.750E-07           5.250E-07           6.750E-07           8.250E-07
```

Table 7 (contd)

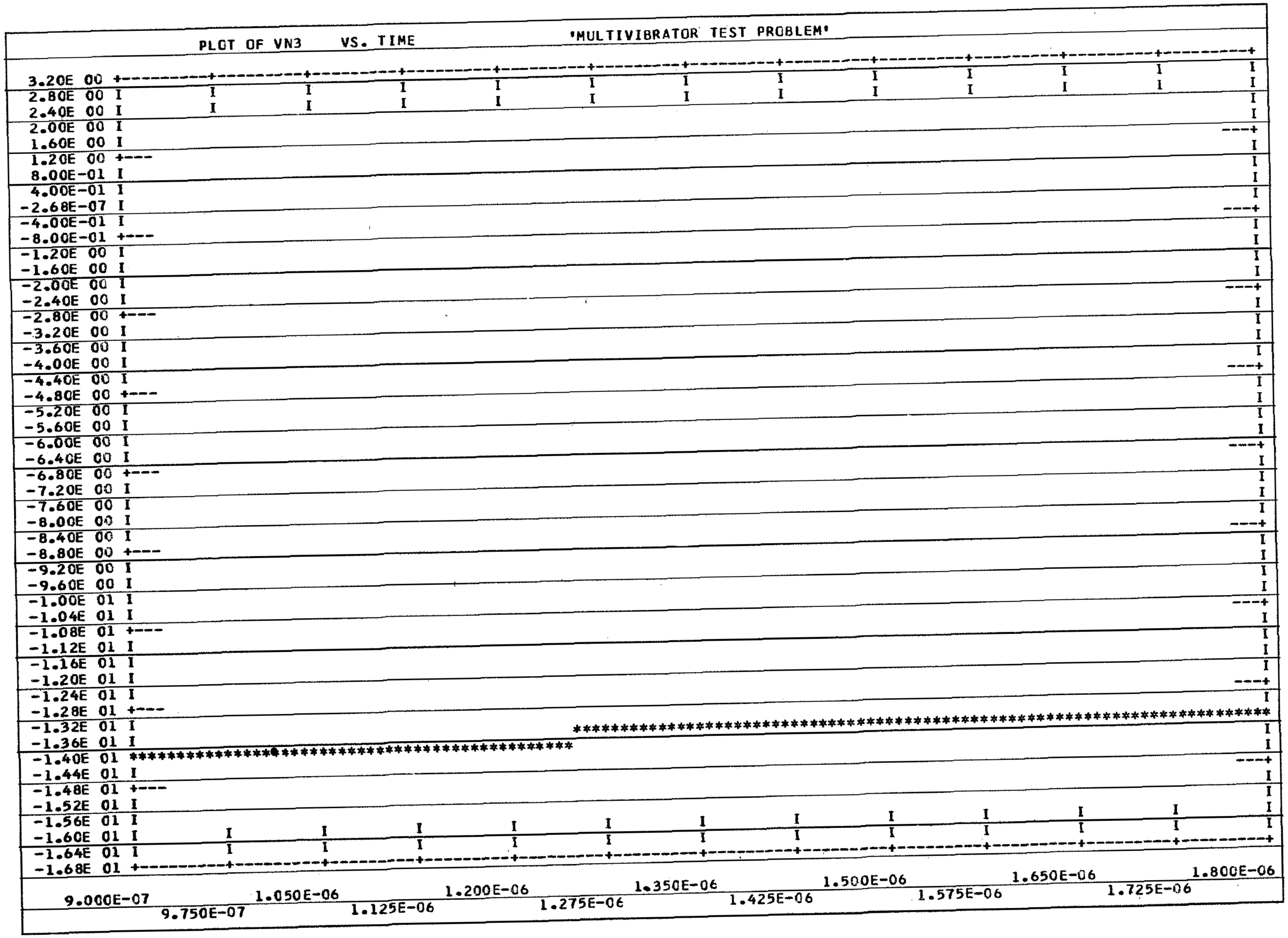


Table 7 (contd)

```
                   PLOT OF VN3    VS. TIME             'MULTIVIBRATOR TEST PROBLEM'

 3.20E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 2.80E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 2.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 2.00E 00 I                                                                                                                       I
 1.60E 00 I                                                                                                                       I
 1.20E 00 +---                                                                                                                 ---+
 8.00E-01 I                                                                                                                       I
 4.00E-01 I                                                                                                                       I
-2.68E-07 I                                                                                                                       I
-4.00E-01 I                                                                                                                       I
-8.00E-01 +---                                                                                                                 ---+
-1.20E 00 I                                                                                                                       I
-1.60E 00 I                                                                                                                       I
-2.00E 00 I                                                                                                                       I
-2.40E 00 I                                                                                                                       I
-2.80E 00 +---                                                                                                                 ---+
-3.20E 00 I                                                                                                                       I
-3.60E 00 I                                                                                                                       I
-4.00E 00 I                                                                                                                       I
-4.40E 00 I                                                                                                                       I
-4.80E 00 +---                                                                                                                 ---+
-5.20E 00 I                                                                                                                       I
-5.60E 00 I                                                                                                                       I
-6.00E 00 I                                                                                                                       I
-6.40E 00 I                                                                                                                       I
-6.80E 00 +---                                                                                                                 ---+
-7.20E 00 I                                                                                                                       I
-7.60E 00 I                                                                                                                       I
-8.00E 00 I                                                                                                                       I
-8.40E 00 I                                                                                                                       I
-8.80E 00 +---                                                                                                                 ---+
-9.20E 00 I                                                                                                                       I
-9.60E 00 I                                                                                                                       I
-1.00E 01 I                                                                                                                       I
-1.04E 01 I                                                                                                                       I
-1.08E 01 +---                                                                                                                 ---+
-1.12E 01 I                                                                                                                       I
-1.16E 01 I                                                                                                                       I
-1.20E 01 I                                                                                                                       I
-1.24E 01 I                                                                                                                       I
-1.28E 01 +---                                                                                                      ***************
-1.32E 01 I               ******************************************************************************************              I
-1.36E 01 ****************                                                                                                        I
-1.40E 01 I                                                                                                                       I
-1.44E 01 I                                                                                                                       I
-1.48E 01 +---                                                                                                                 ---+
-1.52E 01 I                                                                                                                       I
-1.56E 01 I                                                                                                                       I
-1.60E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.64E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.68E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

   1.800E-06           1.950E-06           2.100E-06           2.250E-06           2.400E-06           2.550E-06           2.700E-06
             1.875E-06           2.025E-06           2.175E-06           2.325E-06           2.475E-06           2.625E-06
```

Table 7 (contd)

```
           PLOT OF VN3     VS. TIME                  'MULTIVIBRATOR TEST PROBLEM'
 3.20E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 2.80E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 2.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
 2.00E 00 I                                                                                                                       I
 1.60E 00 I                                                                                                                       I
 1.20E 00 +---                                                                                                                 ---+
 8.00E-01 I                                                                                                                       I
 4.00E-01 I                                                                                                                       I
-2.68E-07 I                                                                                                                       I
-4.00E-01 I                                                                                                                       I
-8.00E-01 +---                                                                                                                 ---+
-1.20E 00 I                                                                                                                       I
-1.60E 00 I                                                                                                                       I
-2.00E 00 I                                                                                                                       I
-2.40E 00 I                                                                                                                       I
-2.80E 00 +---                                                                                                                 ---+
-3.20E 00 I                                                                                                                       I
-3.60E 00 I                                                                                                                       I
-4.00E 00 I                                                                                                                       I
-4.40E 00 I                                                                                                                       I
-4.80E 00 +---                                                                                                                 ---+
-5.20E 00 I                                                                                                                       I
-5.60E 00 I                                                                                                                       I
-6.00E 00 I                                                                                                                       I
-6.40E 00 I                                                                                                                       I
-6.80E 00 +---                                                                                                                 ---+
-7.20E 00 I                                                                                                                       I
-7.60E 00 I                                                                                                                       I
-8.00E 00 I                                                                                                                       I
-8.40E 00 I                                                                                                                       I
-8.80E 00 +---                                                                                                                 ---+
-9.20E 00 I                                                                                                                       I
-9.60E 00 I                                                                                                                       I
-1.00E 01 I                                                                                                                       I
-1.04E 01 I                                                                                                                       I
-1.08E 01 +---                                                                                                                 ---+
-1.12E 01 I                                                                                                                       I
-1.16E 01 I                                                                                                                       I
-1.20E 01 I                                                                                                                       I
-1.24E 01 I                                                                                                                       I
-1.28E 01 *****************************************                                                                            ---+
-1.32E 01 I                                                                                                                       I
-1.36E 01 I                                                                                                                       I
-1.40E 01 I                                                                                                                       I
-1.44E 01 I                                                                                                                       I
-1.48E 01 +---                                                                                                                 ---+
-1.52E 01 I                                                                                                                       I
-1.56E 01 I                                                                                                                       I
-1.60E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.64E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
-1.68E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

      2.700E-06           2.850E-06           3.000E-06           3.150E-06           3.300E-06           3.450E-06           3.600E-06
                2.775E-06           2.925E-06           3.075E-06           3.225E-06           3.375E-06           3.525E-06
```

Table 7 (contd)

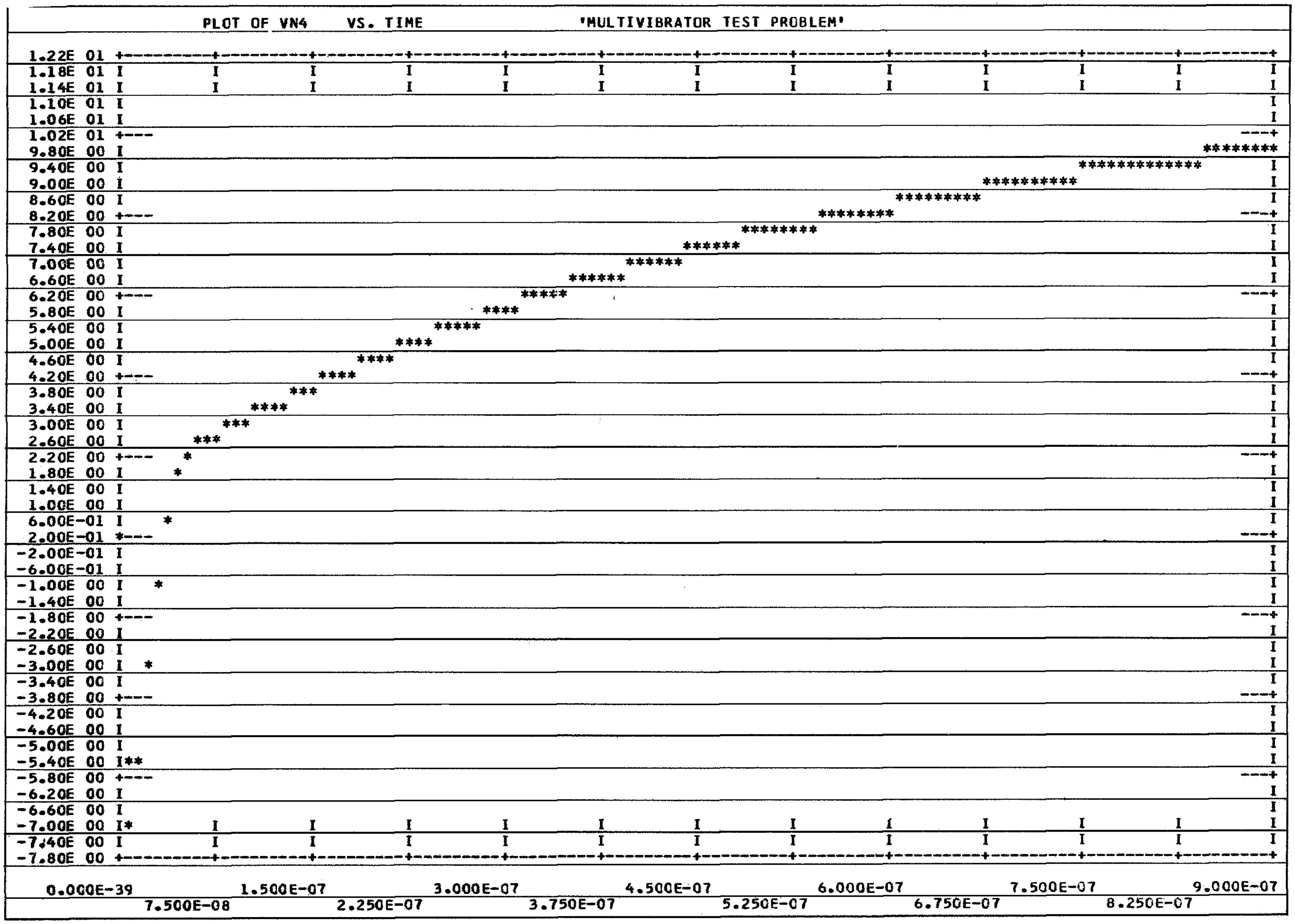


Table 7 (contd)

```
                     PLOT OF VN4    VS. TIME              'MULTIVIBRATOR TEST PROBLEM'
 1.22E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 1.18E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.14E 01 I         I         I         I         I         I         I         I         I         I         I      **************
 1.10E 01 I                                                        **************************************************             I
 1.06E 01 I                            ****************************                                                               I
 1.02E 01 +---     ********************                                                                                        ---+
 9.80E 00 *********                                                                                                               I
 9.40E 00 I                                                                                                                       I
 9.00E 00 I                                                                                                                       I
 8.60E 00 I                                                                                                                       I
 8.20E 00 +---                                                                                                                 ---+
 7.80E 00 I                                                                                                                       I
 7.40E 00 I                                                                                                                       I
 7.00E 00 I                                                                                                                       I
 6.60E 00 I                                                                                                                       I
 6.20E 00 +---                                                                                                                 ---+
 5.80E 00 I                                                                                                                       I
 5.40E 00 I                                                                                                                       I
 5.00E 00 I                                                                                                                       I
 4.60E 00 I                                                                                                                       I
 4.20E 00 +---                                                                                                                 ---+
 3.80E 00 I                                                                                                                       I
 3.40E 00 I                                                                                                                       I
 3.00E 00 I                                                                                                                       I
 2.60E 00 I                                                                                                                       I
 2.20E 00 +---                                                                                                                 ---+
 1.80E 00 I                                                                                                                       I
 1.40E 00 I                                                                                                                       I
 1.00E 00 I                                                                                                                       I
 6.00E-01 I                                                                                                                       I
 2.00E-01 +---                                                                                                                 ---+
-2.00E-01 I                                                                                                                       I
-6.00E-01 I                                                                                                                       I
-1.00E 00 I                                                                                                                       I
-1.40E 00 I                                                                                                                       I
-1.80E 00 +---                                                                                                                 ---+
-2.20E 00 I                                                                                                                       I
-2.60E 00 I                                                                                                                       I
-3.00E 00 I                                                                                                                       I
-3.40E 00 I                                                                                                                       I
-3.80E 00 +---                                                                                                                 ---+
-4.20E 00 I                                                                                                                       I
-4.60E 00 I                                                                                                                       I
-5.00E 00 I                                                                                                                       I
-5.40E 00 I                                                                                                                       I
-5.80E 00 +---                                                                                                                 ---+
-6.20E 00 I                                                                                                                       I
-6.60E 00 I                                                                                                                       I
-7.00E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-7.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-7.80E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
      9.000E-07           1.050E-06           1.200E-06           1.350E-06           1.500E-06           1.650E-06           1.800E-06
                9.750E-07           1.125E-06           1.275E-06           1.425E-06           1.575E-06           1.725E-06
```

Table 7 (contd)

```
                     PLOT OF VN4    VS. TIME                  'MULTIVIBRATOR TEST PROBLEM'

 1.22E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 1.18E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.14E 01 *************************************************************************************************************************
 1.10E 01 I                                                                                                                       I
 1.06E 01 I                                                                                                                       I
 1.02E 01 +---                                                                                                                 ---+
 9.80E 00 I                                                                                                                       I
 9.40E 00 I                                                                                                                       I
 9.00E 00 I                                                                                                                       I
 8.60E 00 I                                                                                                                       I
 8.20E 00 +---                                                                                                                 ---+
 7.80E 00 I                                                                                                                       I
 7.40E 00 I                                                                                                                       I
 7.00E 00 I                                                                                                                       I
 6.60E 00 I                                                                                                                       I
 6.20E 00 +---                                                                                                                 ---+
 5.80E 00 I                                                                                                                       I
 5.40E 00 I                                                                                                                       I
 5.00E 00 I                                                                                                                       I
 4.60E 00 I                                                                                                                       I
 4.20E 00 +---                                                                                                                 ---+
 3.80E 00 I                                                                                                                       I
 3.40E 00 I                                                                                                                       I
 3.00E 00 I                                                                                                                       I
 2.60E 00 I                                                                                                                       I
 2.20E 00 +---                                                                                                                 ---+
 1.80E 00 I                                                                                                                       I
 1.40E 00 I                                                                                                                       I
 1.00E 00 I                                                                                                                       I
 6.00E-01 I                                                                                                                       I
 2.00E-01 +---                                                                                                                 ---+
-2.00E-01 I                                                                                                                       I
-6.00E-01 I                                                                                                                       I
-1.00E 00 I                                                                                                                       I
-1.40E 00 I                                                                                                                       I
-1.80E 00 +---                                                                                                                 ---+
-2.20E 00 I                                                                                                                       I
-2.60E 00 I                                                                                                                       I
-3.00E 00 I                                                                                                                       I
-3.40E 00 I                                                                                                                       I
-3.80E 00 +---                                                                                                                 ---+
-4.20E 00 I                                                                                                                       I
-4.60E 00 I                                                                                                                       I
-5.00E 00 I                                                                                                                       I
-5.40E 00 I                                                                                                                       I
-5.80E 00 +---                                                                                                                 ---+
-6.20E 00 I                                                                                                                       I
-6.60E 00 I                                                                                                                       I
-7.00E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-7.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-7.80E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

      1.800E-06           1.950E-06           2.100E-06           2.250E-06           2.400E-06           2.550E-06           2.700E-06
                1.875E-06           2.025E-06           2.175E-06           2.325E-06           2.475E-06           2.625E-06
```

Table 7 (contd)

```
                PLOT OF VN4    VS. TIME                    'MULTIVIBRATOR TEST PROBLEM'

 1.22E 01 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+
 1.18E 01 I         I         I         I         I         I         I         I         I         I         I         I         I
 1.14E 01 *****************************************         I         I         I         I         I         I         I         I
 1.10E 01 I                                                                                                                       I
 1.06E 01 I                                                                                                                       I
 1.02E 01 +---                                                                                                                 ---+
 9.80E 00 I                                                                                                                       I
 9.40E 00 I                                                                                                                       I
 9.00E 00 I                                                                                                                       I
 8.60E 00 I                                                                                                                       I
 8.20E 00 +---                                                                                                                 ---+
 7.80E 00 I                                                                                                                       I
 7.40E 00 I                                                                                                                       I
 7.00E 00 I                                                                                                                       I
 6.60E 00 I                                                                                                                       I
 6.20E 00 +---                                                                                                                 ---+
 5.80E 00 I                                                                                                                       I
 5.40E 00 I                                                                                                                       I
 5.00E 00 I                                                                                                                       I
 4.60E 00 I                                                                                                                       I
 4.20E 00 +---                                                                                                                 ---+
 3.80E 00 I                                                                                                                       I
 3.40E 00 I                                                                                                                       I
 3.00E 00 I                                                                                                                       I
 2.60E 00 I                                                                                                                       I
 2.20E 00 +---                                                                                                                 ---+
 1.80E 00 I                                                                                                                       I
 1.40E 00 I                                                                                                                       I
 1.00E 00 I                                                                                                                       I
 6.00E-01 I                                                                                                                       I
 2.00E-01 +---                                                                                                                 ---+
-2.00E-01 I                                                                                                                       I
-6.00E-01 I                                                                                                                       I
-1.00E 00 I                                                                                                                       I
-1.40E 00 I                                                                                                                       I
-1.80E 00 +---                                                                                                                 ---+
-2.20E 00 I                                                                                                                       I
-2.60E 00 I                                                                                                                       I
-3.00E 00 I                                                                                                                       I
-3.40E 00 I                                                                                                                       I
-3.80E 00 +---                                                                                                                 ---+
-4.20E 00 I                                                                                                                       I
-4.60E 00 I                                                                                                                       I
-5.00E 00 I                                                                                                                       I
-5.40E 00 I                                                                                                                       I
-5.80E 00 +---                                                                                                                 ---+
-6.20E 00 I                                                                                                                       I
-6.60E 00 I                                                                                                                       I
-7.00E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-7.40E 00 I         I         I         I         I         I         I         I         I         I         I         I         I
-7.80E 00 +---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+---------+

   2.700E-06           2.850E-06           3.000E-06           3.150E-06           3.300E-06           3.450E-06           3.600E-06
             2.775E-06           2.925E-06           3.075E-06           3.225E-06           3.375E-06           3.525E-06
```

Table 8. MTRAC output

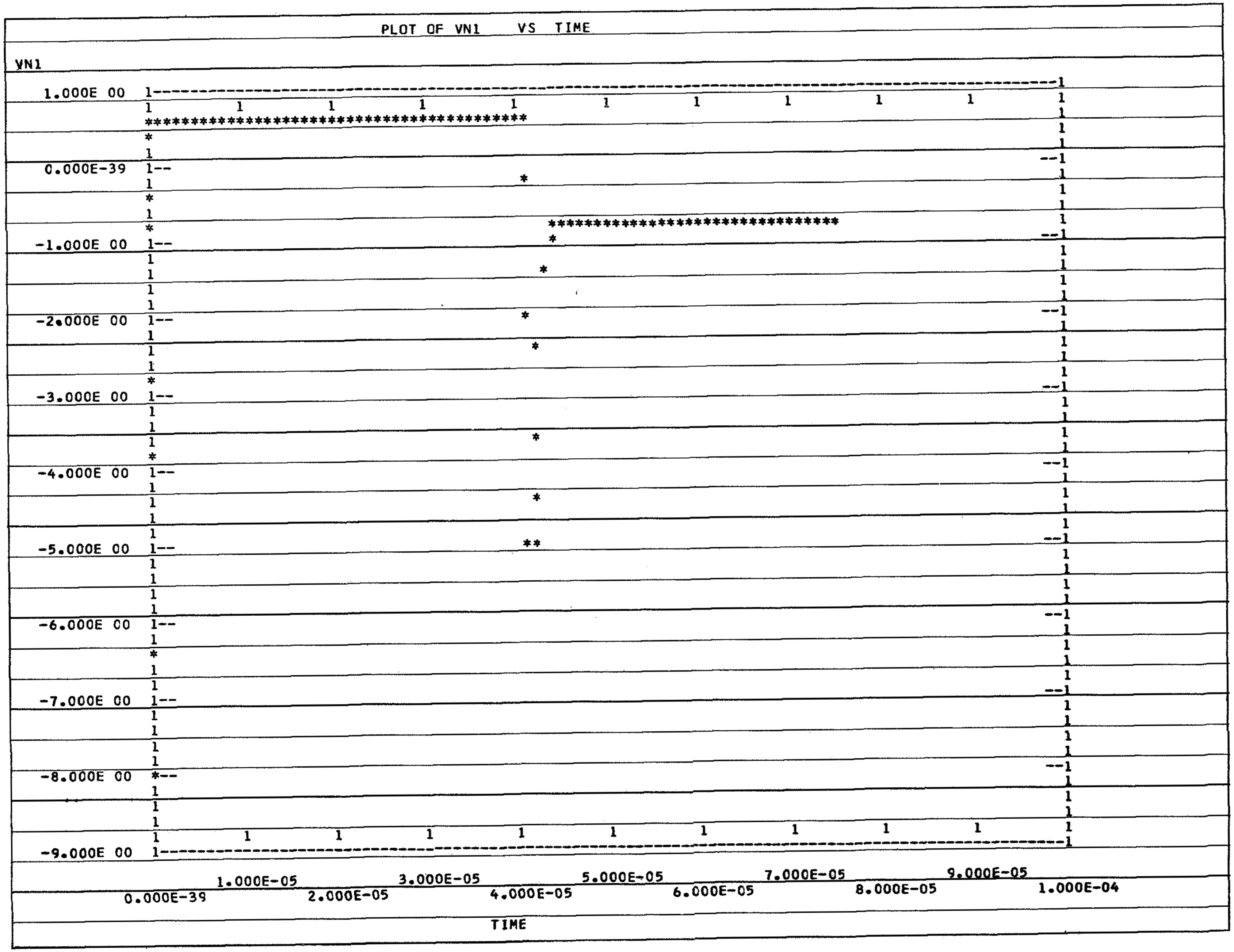


Table 8 (contd)

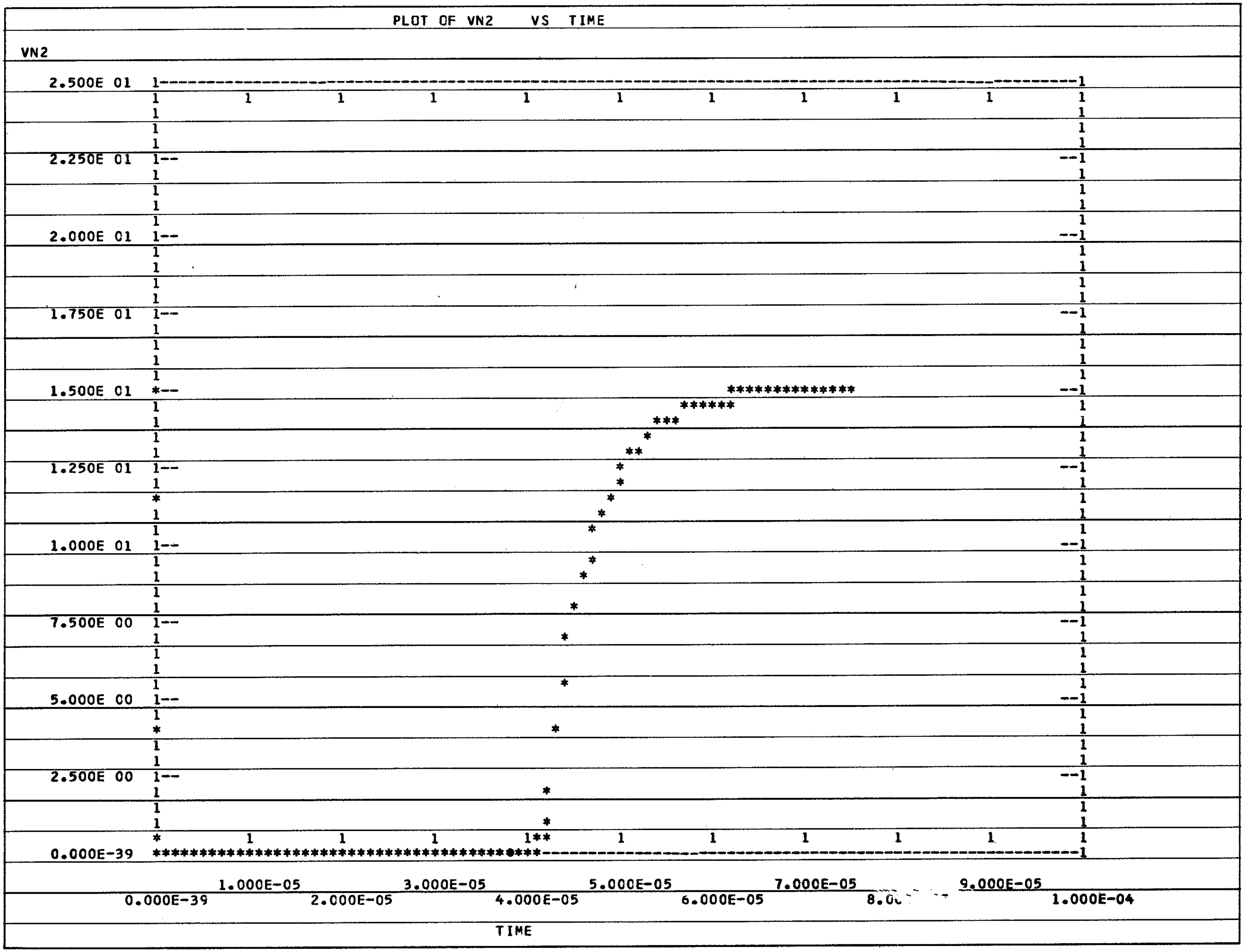


Table 8 (contd)

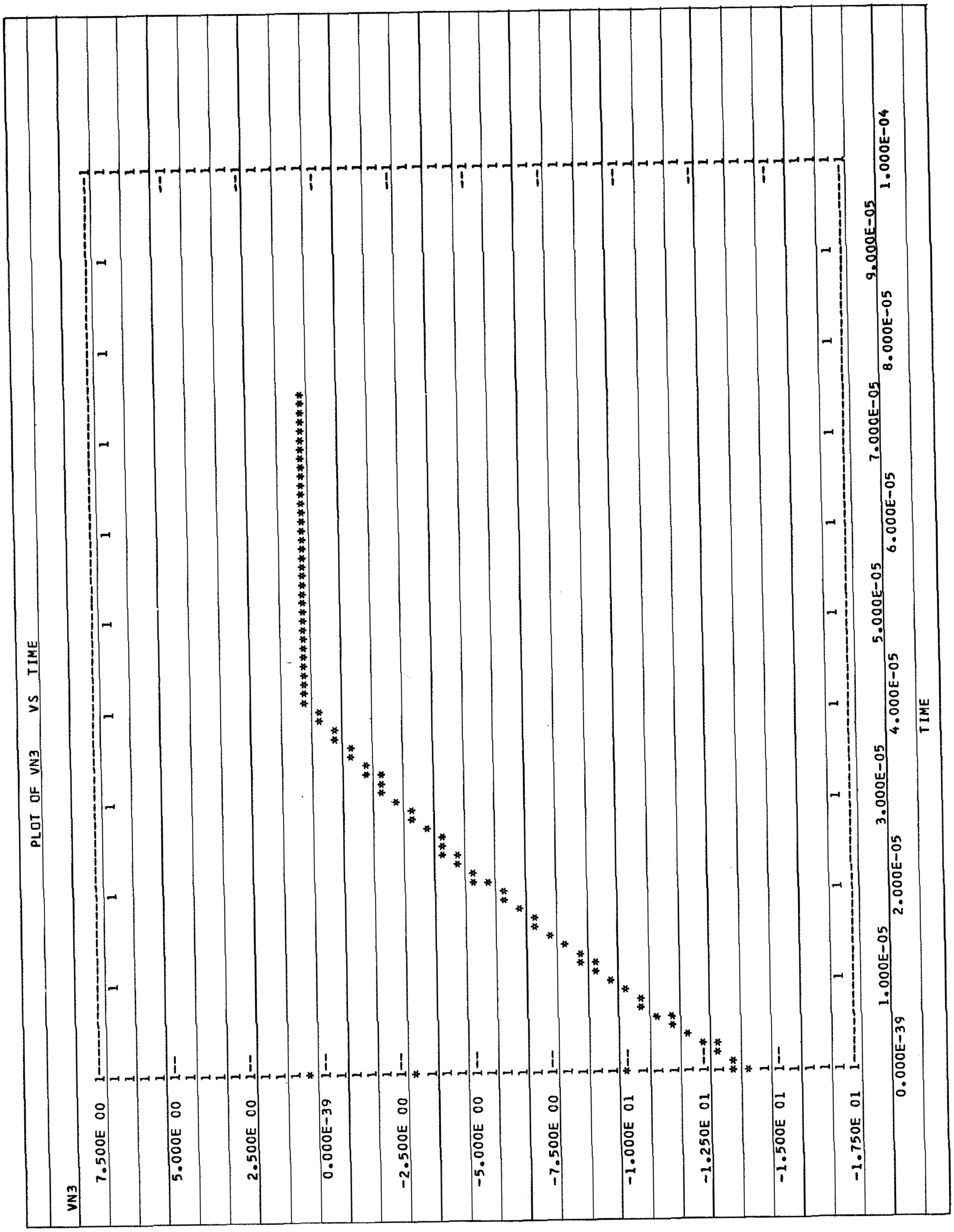


Table 8 (contd)

```
                                        PLOT OF VN4     VS  TIME
VN4
 1.250E 01  1---------------------------------------------------------------------------------------------------1
            1         1         1         1         1         1         1         1         1         1         1
            1                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
 1.000E 01  1--                                                                                               --1
            1                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
            1  **************************************                                                           1
 7.500E 00  1-*                                      *                                                        --1
            1*                                       *                                                          1
            1*                                                                                                  1
            1                                        *                                                          1
            1                                                                                                   1
 5.000E 00  *--                                                                                               --1
            1                                                                                                   1
            *                                        *                                                          1
            1                                                                                                   1
            *                                                                                                   1
 2.500E 00  *--                                                                                               --1
            *                                                                                                   1
            *                                                                                                   1
            *                                                                                                   1
            1                                        *                                                          1
 0.000E-39  *--                                       **********************************                      --1
            1                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
            *                                                                                                   1
-2.500E 00  1--                                                                                               --1
            *                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
-5.000E 00  1--                                                                                               --1
            *                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
            *                                                                                                   1
-7.500E 00  1--                                                                                               --1
            1                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
-1.000E 01  1--                                                                                               --1
            1                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
            1         1         1         1         1         1         1         1         1         1         1
-1.250E 01  1---------------------------------------------------------------------------------------------------1
                   1.000E-05           3.000E-05           5.000E-05           7.000E-05           9.000E-05
         0.000E-39           2.000E-05           4.000E-05           6.000E-05           8.000E-05           1.000E-04
                                                 TIME
```

As in the other programs, the element currents are considered positive in the *from–to* node sequence. The voltage drop across a source is in the same direction as the internal current, whereas, for a passive element, the voltage is opposite to the internal current flow. In the description of E_1, node N6 (the *to* node) is the positive terminal; in the description of R3, node N4 (the *from* node) is presumed to be positive. In this regard, SCEPTRE differs from the other programs under consideration.

The time-variable source (here E_3) is written in as a set of time–voltage coordinates (see Table 3). The relationship could as easily have been described by a user-written function subprogram. Any algebraic function available in FORTRAN could have been used; e.g., log (t), sin (V_{C_3}), etc.

Outputs other than element voltages or currents must be calculated from user-supplied equations. In the present case, the voltage at node N2 had to be calculated by using voltage E_1 and the voltage drop calculated for resistor R1.

One might expect that

$$E_1 + VR_1 + VN_2 = 0$$

yet this is not so. For SCEPTRE,

$$E_1 = VR_1 + VN_2$$

Hence,

$$VN_2 = E_1 - VR_1$$

This situation often leads to errors in defining output terms and dependencies.

The number of output points can be defined independently of the step size or simulation time. In this example, the MAXIMUM PRINT POINTS has been set to 175. This particular run terminated after less than 3×10^{-6} s simulation time because an internal limit of 20,000 integration passes had been exceeded. The pass limit is easily changed, but it is convenient to this analysis as it is.

2. ***Solution times.*** The solution times for SCEPTRE are as follows:

(1) Load time (first pass): 102 s.

(2) Execution time (first pass): 37 s.

(3) Load time (second pass): 51 s.

(4) Execution time (second pass): 416 s.

(5) Step size (maximum mode): 1.125×10^{-9} s.

(6) Simulation time (20,000 passes): 2.728×10^{-6} s.

(7) Simulation time (extrapolated to 75×10^{-6} s): 11,473 s.

The SCEPTRE output plots are shown in Table 9.

VIII. Results

No closed-form solution is available for the problem above. The correctness of a solution can be estimated by comparing the results with hand-calculated estimates and by comparison with the results obtained by the other programs.

A. Hand Calculations

At time zero:

(1) Transistor T1 is turned off and draws neither base nor collector current.

(2) Transistor T2 is turned on; base voltage is estimated at 0.6 V; collector voltage is estimated at 0.2 V.

$$\text{Voltage node 1} = \frac{\dfrac{0.2}{104} - \dfrac{3}{2.2 \times 10^4}}{\dfrac{1}{104} + \dfrac{1}{2.2 \times 10^4}}$$

$$= \frac{0.2 - 1.36}{1.45} = -0.8\,\text{V}$$

$$\text{Node 2} = 15.0$$

$$\text{Node 3} = 0.6$$

$$\text{Node 4} = 0.2$$

After triggering:

(1) Transistor T1 is turned on; base voltage is estimated at 0.6 V; collector voltage is estimated at 0.2 V.

(2) Transistor T2 is turned off and draws neither base nor collector current.

$$\text{Voltage node 1} = +0.6$$

$$\text{Node 2} = +0.2$$

$$\text{Node 3} = 15 - (30 - 0.6 - 0.2)\, e - \frac{t}{60 \times 10^{-6}}$$

Table 9. SCEPTRE output

```
                                 PLOT OF PVN2   VS TIME

PVN2

  2.500E 01  1---------------------------------------------------------------------------------------------------1
             1         1         1         1         1         1         1         1         1         1         1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
  2.250E 01  1--                                                                                               --1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
  2.000E 01  1--                                                                                               --1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
  1.750E 01  1--                                                                                               --1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
  1.500E 01  *--                                                                                               --1
             *                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
             *                                                                                                   1
  1.250E 01  1--                                                                                               --1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
  1.000E 01  1--                                                                                               --1
             *                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
  7.500E 00  1--                                                                                               --1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
  5.000E 00  1--                                                                                               --1
             *                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
             1                                                                                                   1
  2.500E 00  1--                                                                                               --1
             1                                                                                                   1
             *                                                                                                   1
             1                                                                                                   1
             ***       1         1         1         1         1         1         1         1         1         1
  0.000E-39  1-******************************************************--------------------------------------------1

                    5.000E 02           1.500E 03           2.500E 03           3.500E 03           4.500E 03
          0.000E-39           1.000E 03           2.000E 03           3.000E 03           4.000E 03           5.000E 03

                                                 TIME
```

Table 9 (contd)

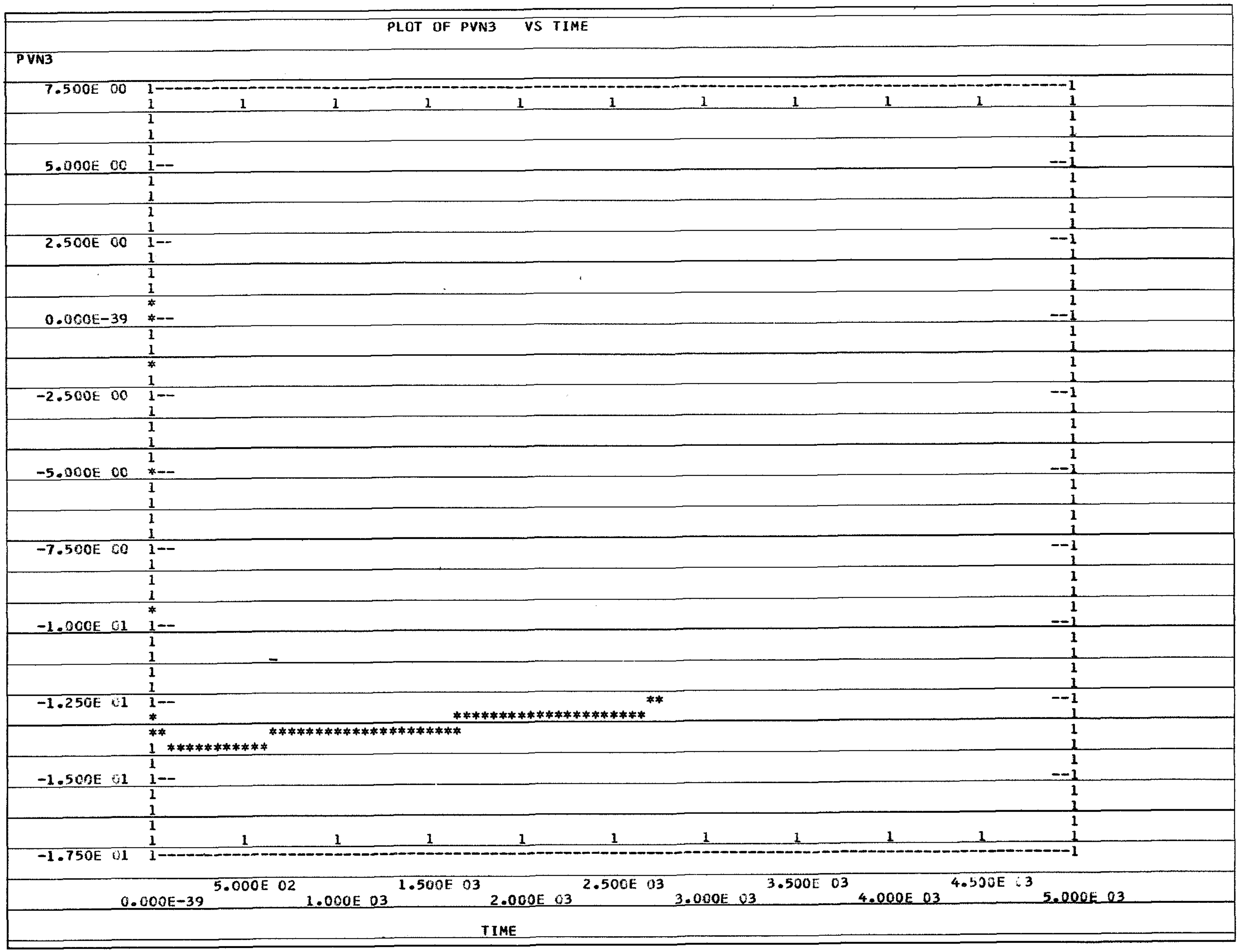


Table 9 (contd)

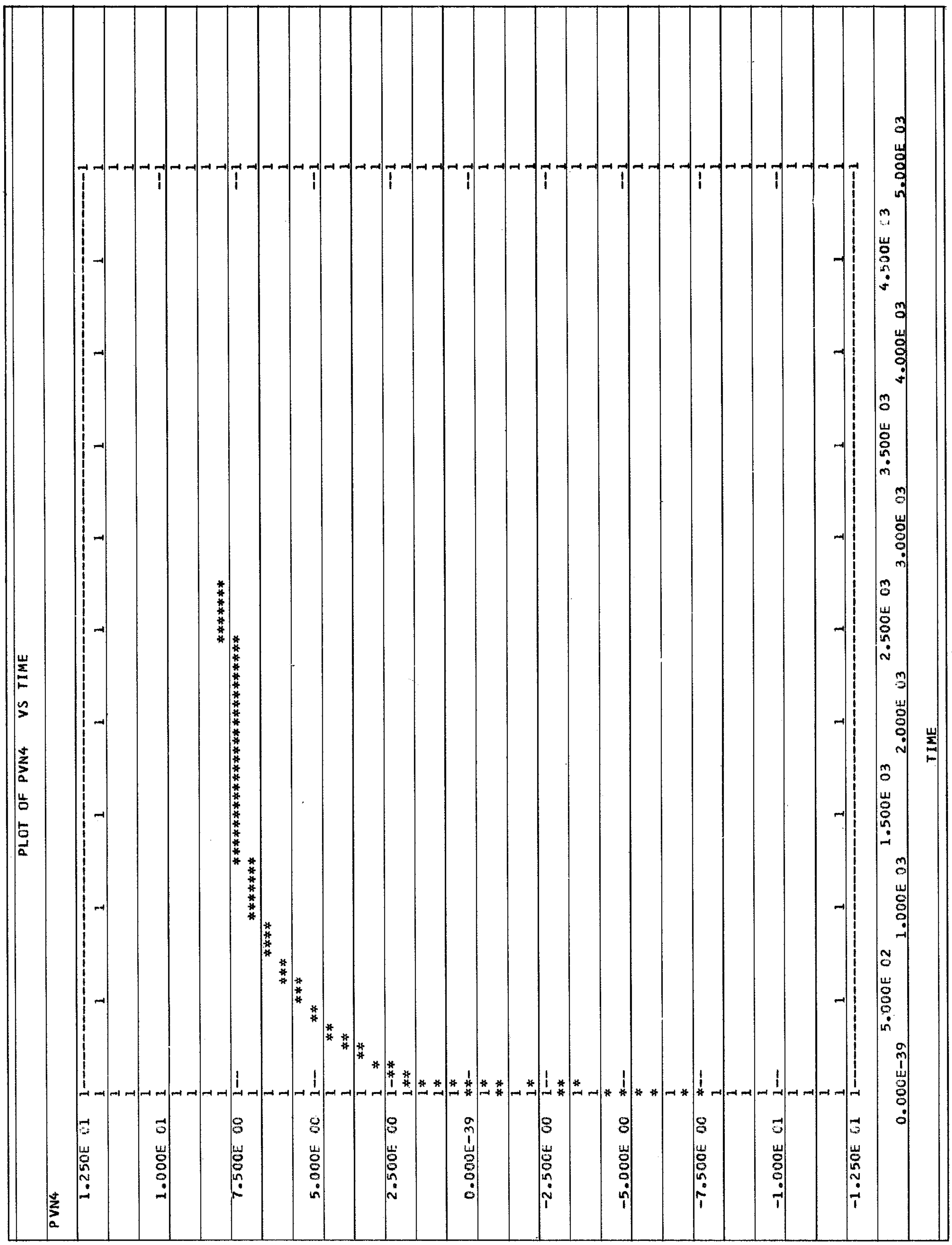


Table 9 (contd)

```
                        PLOT OF PVN5    VS TIME
PVN5
  1.000E 00 1---------------------------------------------------------------------------------------------------1
            1         1         1         1         1         1         1         1         1         1         1
            1*******************************************************                                            1
            1                                                                                                   1
            1*                                                                                                  1
  0.000E-39 1--                                                                                               --1
            1                                                                                                   1
            1*                                                                                                  1
            1                                                                                                   1
            **                                                                                                  1
 -1.000E 00 *--                                                                                               --1
            1*                                                                                                  1
            1                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
 -2.000E 00 1*-                                                                                               --1
            *                                                                                                   1
            1*                                                                                                  1
            1                                                                                                   1
            1                                                                                                   1
 -3.000E 00 1--                                                                                               --1
            1*                                                                                                  1
            1                                                                                                   1
            1                                                                                                   1
            1*                                                                                                  1
 -4.000E 00 *--                                                                                               --1
            1                                                                                                   1
            1                                                                                                   1
            1*                                                                                                  1
            1                                                                                                   1
 -5.000E 00 1--                                                                                               --1
            1                                                                                                   1
            *                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
 -6.000E 00 *--                                                                                               --1
            1                                                                                                   1
            *                                                                                                   1
            1                                                                                                   1
            *                                                                                                   1
 -7.000E 00 1--                                                                                               --1
            1                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
            *                                                                                                   1
 -8.000E 00 *--                                                                                               --1
            *                                                                                                   1
            1                                                                                                   1
            1                                                                                                   1
            1         1         1         1         1         1         1         1         1         1         1
 -9.000E 00 1---------------------------------------------------------------------------------------------------1
                  5.000E 02           1.500E 03           2.500E 03           3.500E 03           4.500E 03
        0.000E-39           1.000E 03           2.000E 03           3.000E 03           4.000E 03           5.000E 03

                                                  TIME
```

where

$$t = 0.5 \times 10^{-6}, \qquad VN_3 = 13.96$$

$$= 1.0 \times 10^{-6}, \qquad = 13.72$$

$$= 2.0 \times 10^{-6}, \qquad = 13.24$$

$$= 2.7 \times 10^{-6}, \qquad = 12.92$$

$$\text{Node } 4 = \frac{\dfrac{0.6}{10^4} + \dfrac{15}{10^4}}{\dfrac{1}{10^4} + \dfrac{1}{10^4}} = 7.8\text{ V}$$

B. Interprogram Comparisons

Comparisons of data points between the programs must be done with reservation because the transistor models are not identical in the four cases. It would have been possible to make the semiconductor models more nearly identical; however, the purpose here is a comparative survey of the programs and, to that end, different models are illustrative of the different capabilities.

The results may be compared by the plots of the node voltages, although this approach is hampered by different scaling of the axes. (The SCEPTRE line-printer plot routine has been used for MTRAC and ECAP; CIRCUS uses its own routine.) Only the MTRAC and ECAP programs completed the simulation of the multivibrator period in the time available. The CIRCUS and SCEPTRE plots are incomplete in this regard. The mechanism of the plot routine has expanded the time scale to fill the available space.

A further comparison can be made using Table 10. Here the node voltages have been excerpted from the listings at five times that were available from each of the programs. The SCEPTRE and CIRCUS programs have a great many points in this area, whereas ECAP and MTRAC used relatively large step sizes. The values used in Table 10 for ECAP and MTRAC were obtained by interpolation where necessary (Table 11).

IX. Conclusions

The effective use of a digital computer for circuit analysis requires of the engineer a great deal more than the data ordinarily contained in a schematic diagram. He must conform to the format requirements of the program he selects; that is, he must learn the input notation—the functions that control the analysis and the output. He must be prepared to supply model data far more detailed than those data explicitly required for synthesis. More importantly, he must know the mechanizations of the program so that he may choose the one best suited to his particular circuit.

This report attempts to provide insight into the problems that must be faced in the computer simulation of circuit analysis. The input format and topological constraints of the programs, the mathematics of the simulation, and the derivation of the models have been discussed in the perspective of a single problem as an aid in understanding the differences between these programs. The actual programs thus set up have been analyzed, and the results compared. The result is four programs for the prospective circuit.

To summarize, the numerical values obtained for the node voltages are quite consistent between the programs, differing by only a few percent in most cases. The primary program-dependent differences are the simplicity and versatility of the input and the cost of the analysis. Most engineers would probably consider the ECAP input most difficult because of the necessity of stepwise linear approximations of nonlinear elements. Also, CIRCUS is the least flexible, whereas SCEPTRE, in allowing nearly any conceivable dependency or nonlinearity, is the most flexible.

In regard to the cost of analysis, MTRAC provided the least expensive analysis of the example circuit. However, caution must be used because this relative effectiveness will surely be circuit-dependent. An additional consideration may be important where other computers are used. This arises because some computer facilities make additional charges for the use of peripheral tape units. (The SCEPTRE program normally requires three tape units; CIRCUS requires two; ECAP and MTRAC require only one apiece.)

Table 10. Output data comparison[a]

Time, μs	Program	Output at indicated node, V				Voltage at signal input, V
		B1	C1	B2	C2	
0.0	CIRCUS	−0.785	14.9	0.555	0.188	15.0
	ECAP	−0.741	14.86	0.5361	0.1820	
	MTRAC	−0.8048	15.00	0.5506	0.1888	
	SCEPTRE	−0.8042	14.99	0.5576	0.1896	
	Manual	−0.8	15.0	0.6	0.2	
0.5	CIRCUS	6.58	0.176	−13.8	5.37	5.85
	ECAP	6.5726	0.1749	−13.48	4.596	
	MTRAC	6.5868	0.1436	−13.82	5.425	
	SCEPTRE	6.6128	0.1472	−13.81	5.363	
	Manual	6.6	0.2	−13.96	7.8	
1.0	CIRCUS	0.593	0.179	−13.5	6.86	13.8
	ECAP	0.5725	0.1750	−13.48	4.597	
	MTRAC	0.5815	0.1471	−13.56	6.846	
	SCEPTRE	0.6085	0.1511	−13.56	6.888	
	Manual	0.6	0.2	−13.72	7.8	
2.0	CIRCUS	0.596	0.181	−13.0	7.67	15.0
	ECAP	0.5629	0.1784	−12.73	7.590	
	MTRAC	0.5768	0.1485	−13.08	7.695	
	SCEPTRE	0.5972	0.1520	−13.08	7.674	
	Manual	0.6	0.2	−13.26	7.8	
2.7	CIRCUS	0.597	0.181	−12.7	7.79	15.0
	ECAP	0.5624	0.1787	−12.39	7.724	
	MTRAC	0.5763	0.1489	−12.74	7.774	
	SCEPTRE	0.5967	0.1523	−12.74	7.771	
	Manual	0.6	0.2	−12.92	7.8	
—	—	End of cycle				
40.5	ECAP	From change in VC2				
41.43	MTRAC	From change in VC2				

[a]Node voltages at 0.5-μs intervals were obtained by interpolation.

Table 11. Summary of test-circuit solution times

Program	Actual times, s				Estimated execution for 75-μs simulation, s
	Load time	Execution time (T_E)	Simulation time (T_S)	Ratio (T_S/T_E)	
CIRCUS	52	1250	3×10^{-6}	2,400	31,250
ECAP	183[a]	374	75.05×10^{-6}	201,900	374
MTRAC	83	102	75×10^{-6}	735,300	102
SCEPTRE	190[b]	416	2.72×10^{-6}	6,500	11,473

[a]Includes incidental compilation of one subroutine.

[b]Includes generation of subroutine SIMUL8 used for the solution of the test circuit.

Nomenclature

B_i	ith branch in an ECAP problem
C	capacitance
C_D	diode diffusion capacitance
C_{DC}	collector–base diode diffusion capacitance
C_{DE}	emitter–base diode diffusion capacitance
C_E	equivalent emitter capacitance = $C_{DE} + C_{TE}$
C_T	diode transition capacitance
C_{TC}	collector–base diode transition capacitance
C_{TE}	emitter–base diode transition capacitance
C_0	junction diffusion capacitance at zero junction voltage (V_J)
E	voltage developed by source E
e	base on natural log
GM	magnitude of a voltage-controlled current source in ECAP program
g_m	transconductance
h_{FEI}	grounded emitter current gain, inverse mode
h_{FEN}	grounded emitter current gain, normal mode
I	current in a circuit element
I_B	base terminal current
I_C	collector terminal current
I_{CS}	collector–base diode saturation current
$I_{C_{tn}}$	current in capacitor C at end of nth interval
I_{DC}	current in collector–base diode diffusion capacitance
I_E	emitter terminal current
I_{ES}	emitter–base diode saturation current
I_J	diode junction current
I_S	diode junction saturation current
J_C	collector–base diode current generator
J_D	diode current generator
J_E	emitter–base diode current generator
$J_I = \alpha_I \times J_C$	inverse-mode transistor active current generator
$J_N = \alpha_I \times J_E$	normal-mode transistor active current generator
k	Boltzmann constant = 1.38047×10^{-23} W-s/°K
K_{ij}	coupling coefficient between windings i and j of a transformer
L	inductance of inductor L
L_i	inductance of ith winding of a transformer
M	diode junction constant
M_C	collector junction constant
M_E	emitter junction constant
M_{ij}	mutual inductance between windings i and j of a transformer
MMF	magnetomotive force
n	diode junction capacitance
n_C	collector junction capacitance
n_E	emitter junction capacitance
n–p–n	negative carrier junction transistor
p–n–p	positive carrier junction transistor
Q	charge on a capacitor
q	charge on an electron = 1.6×10^{-19} C
R_B	equivalent base terminal series resistance of a transistor
R_C	equivalent collector terminal series resistance of a transistor
R_E	equivalent emitter terminal series resistance of a transistor
R_i	resistance of resistor Ri
s	complex frequency in Laplace transform
T	storage time constant
T	temperature of junction in °K (appears only in q/kT)
t	time
T_i	designation of ith transistor

Nomenclature (contd)

t_i time at end of ith interval

V_C collector–base diode junction voltage, positive for a forward-biased n–p–n transistor

V_{CE} voltage across total emitter capacitance CE; appears only in

$$CE = \frac{6.4}{(0.8 + V_{CE})^{0.38}} + 4 \times 10^4 (JE + 9.7 \times 10^{-6})$$

V_D diode diffusion capacitance offset voltage

V_{DC} collector diffusion capacitance offset voltage

V_{DE} emitter diffusion capacitance offset voltage

V_E emitter–base diode junction voltage, positive for a forward-biased p–n–p transistor

V_i voltage at ith node of a circuit

V_{ij} voltage at node i relative to voltage at node j

V_J voltage drop across a junction

V_j diode junction voltage, positive for a forward-biased junction

V_{JE} voltage across emitter junction current generator

x integration dummy variable

α_I value of current-dependent current generator of transistor, inverse mode

α_N value of current-dependent current generator of transistor, normal mode

α_{NH} high-frequency value of α_N

α_{NL} low-frequency value of α_N

Δ used to indicate an incremental change of variable that it supersedes

τ majority carrier lifetime

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