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INTERPLANETARY ENERGETIC PARTICLE DIFFUSION COEFFICIENT DETERMINATION USING RANDOM WALK FROM SHOCK WAVES

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ABSTRACT

A shock wave passing through the solar wind can apparently reflect energetic protons to build up an enhanced intensity in front of the shock wave. Values of the diffusion coefficient in interplanetary space and the reflection efficiency of the particles can respectively be calculated from the distance the enhanced region extends in front of the shock wave and the increase in intensity from the undisturbed intensity. Values of $\sim 10^{17}\ \frac{cm^2}{sec}$ and $\sim 90\%$ are found by fitting the data of Ogilvie and Arens (1970) and describe properties of 1 MeV protons with pitch angle $\approx 90^{\circ}$ scattering through $\approx 20^{\circ}$.

INTRODUCTION

Singer (1970), Ogilvie and Arens (1970), and Armstrong, Krimigis and Behannon (1970) have seen enhanced energetic proton intensities for 15-20 minutes before the passage of a shock wave passing through the solar wind. Axford and Reid (1963) proposed a Fermi acceleration mechanism using the interplanetary and earth's bow shock waves to produce such enhanced intensities.

Fermi acceleration between the interplanetary shock wave and scattering centers in the solar wind ahead of the shock wave (Van Allen and Ness, 1967; Fisk, 1970) could also produce such enhancements. Here we use random walk theory in the presence of the reflecting shock wave and scattering centers to calculate curves of enhanced intensity vs. distance in front of the shock wave. The scattering centers are magnetic field irregularities.

First we will ignore the directionality of the magnetic field and look at the enhanced region in the same way as one would an aerosol cloud in front of a moving screen which lets air molecules through but is impermeable to aerosol particles. We consider the case where the aerosol particles have speeds much larger than the screen speed. The particles bounce off the screen and random walk due to collisions with air molecules. Most particles will not move far from the screen because the net mean velocity (net distance traveled/time) of random walking (diffusing) particles decreases with time, so the screen will eventually catch up with each particle and reflect it again. So we will have a cloud in front of the screen which will not diminish with time.

Next, in this paper, we will consider reflection efficiencies of 1 MeV protons off shock waves and show how increased or decreased intensity regions can exist in front of shock waves.

Thirdly, we will talk briefly about the effects of the directionality of the magnetic field. Last we compare the calculations with experimental data and find a value for the diffusion coefficient of 1 MeV protons at 1 A.U.

CALCULATIONS

The enhanced intensity region in front of the shock wave should be time invariant since the 15-20 minute interval needed for the passage of the enhanced intensity region is much smaller than the time taken by the shock wave to travel from the vicinity of the sun to the satellite, near the earth. Buildup and loss rates are hypothesized to be equal because the peak intensity is only several or several tens of times higher than the background intensity that existed before the buildup. If the build-up rate were higher than the loss rate, we might expect a peak to back-ground ratio of at least several times

$$\frac{d_1}{d_2} = \frac{\frac{vL}{v+v_s}}{(v+v_s)\,t_L} \approx 50$$

since energization by elastic scattering off the moving shock wave would considerably enlarge this number. Here the approximation is made that the shock wave is planar, all particles are piled up, and

d_1 = distance shock wave has traveled through solar wind,

d_2 = distance enhanced region extends in front of shock wave,

v = shock wave speed through solar wind $\simeq 200\ \frac{km}{sec}$,

v_s = solar wind speed $\simeq 400\ \frac{km}{sec}$,

L = distance to vicinity of sun $\simeq 10^8$ km,

t_L = time taken for enhanced region to pass by satellite $\simeq$ 1000 sec.

The intensity in front of the shock wave, I_s should depend only on the distance from the shock wave, x, making $I_s(x)$ a function of one dimension.

Diffusion Coefficient

The steady state solution in front of the shock wave should roughly approximate the distribution obtained by a random walk process from a stationary point after a time T such that the mean random walk distance equals the distance traveled by the shock wave. The random walk distribution and mean distance are

$$P(x,T) = \frac{1}{\sqrt{\pi D T}}\, e^{-\frac{x^2}{4DT}}$$

and

$$\bar{x} = \int_{x=0}^{\infty} \frac{1}{\sqrt{\pi D T}}\, e^{-\frac{x^2}{4DT}}\, x\, dx$$

$$= 2\sqrt{\frac{DT}{\pi}}$$

where

$$D = \tfrac{1}{2} V \ell$$

is the diffusion coefficient. ℓ is the mean free path and V is the speed of a particle. We set

$$v T = \bar{x}$$

so

$$T = \frac{4D}{\pi v^2}$$

and

$$P(x,T) = \frac{v}{2D} e^{-\frac{\pi v^2 x^2}{16 D^2}} .$$

Figure 1 shows how good an approximation this curve is compared to the more exact distribution, $\psi(x,0)$, calculated below:

Since the intensity in front of the shock wave, $I_s(x)$, is unchanging with time, we want a solution such that

$$\psi(x+\delta, \tau)_{x>\delta} = \psi(x,0)_{x>0}$$

where the shock wave moves a distance δ in time τ as shown in Figure 2. We let all particles random walk with perfect reflection off the shock wave to conserve the total number of particles since we first assume the pre-shock undisturbed interplanetary intensity, I_b, is zero. Later, when we calculate the reflection efficiency, we will make the loss by absorption into the shock wave equal to the gain by picking up new particles from the undisturbed intensity and by energization from reflection off the moving shock wave in order to conserve the total number of particles. We now let the shock wave jump a small distance δ at time t=0 and sweep up all the particles in the space traversed. These particles then random walk from x=δ during the time from t=0 to t=τ. Meanwhile particles at x > δ random walk in the presence of a reflecting barrier at x=δ. Using random walk theory,

$$\psi(x,\tau)_{x>\delta} = f\left(\psi(\ ,0)_{x>0}\right)$$

$$= \int_{x'=\delta}^{\infty} c\left[e^{-\frac{(x-x')^2}{B}} + e^{-\frac{(2x_1-(x'-x))^2}{B}}\right]\psi(x',0)\,dx'$$

$$+ 2e^{-\frac{(x-\delta)^2}{B}}\int_{x'=0}^{\delta} c\,\psi(x',0)\,dx'$$

where $$c = \frac{1}{2\sqrt{\pi D\tau}},$$

$$B = 4D\tau,$$

$$\delta = v\tau$$

and $$x_1 = x' - \delta.$$

The bracketed factor in the first integral gives the distribution for random walk with a reflecting barrier (Chandrasekhar, 1943). The second integral is the source of particles that random walk from $x=\delta$.

The approximation of the shock wave jumping and putting the source at $x=\delta$ should give the same result as a smoothly progressing shock wave because the random walk motion of particles is much faster than the shock wave speed at first; i.e.,

$$\frac{\partial \bar{x}}{\partial t} = \frac{\partial}{\partial t} 2 \int_{y=0}^{\infty} y \frac{1}{2\sqrt{\pi D t}} e^{-\frac{y^2}{4Dt}} dy \gg v$$

for $\qquad t \leq \tau \ll \tau_{coll}$

where $\qquad y = x - \delta$

and τ_{coll} is the time in which the average number of collisions each particle makes with the shock wave is 1 as shown in Figure 3. If the solution for $\psi(x,0)$ satisfies

$$\left.\frac{\partial \psi(x,0)}{\partial t}\right|_{x=0} \quad \text{is finite}$$

this approximation will be valid.

We approximate

$$\begin{aligned} \psi(x',0) &= \psi(x'+\delta, \tau) \\ &= \psi(x',\tau) + \frac{\partial \psi(x',\tau)}{\partial x'} \delta \end{aligned}$$

and integrate by parts:

$$\int_{x'=\delta}^{\infty} c\left[e^{-\frac{(x-x')^2}{B}}+e^{-\frac{(x+x'-2\delta)^2}{B}}\right]\frac{\partial\psi(x',\tau)}{\partial x'}\,\delta\,dx'$$

$$=\delta c\int_{x'=\delta}^{\infty}\left[e^{-\frac{(x-x')^2}{B}}+e^{-\frac{(x+x'-2\delta)^2}{B}}\right]d\psi(x',\tau)$$

$$=\delta c\left\{\left[e^{-\frac{(x-x')^2}{B}}+e^{-\frac{(x+x'-2\delta)^2}{B}}\right]\psi(x',\tau)\Big|_{x'=\delta}^{\infty}\right.$$

$$\left.-\int_{x'=\delta}^{\infty}\psi(x',\tau)\left[e^{-\frac{(x-x')^2}{B}}\frac{2(x-x')}{B}+e^{-\frac{(x+x'-2\delta)^2}{B}}\left(-\frac{2(x+x'-2\delta)}{B}\right)\right]dx'\right\}$$

$$=\delta c\left\{-2e^{-\frac{(x-\delta)^2}{B}}\psi(\delta,\tau)-\int_{x'=\delta}^{\infty}\psi(x',\tau)\left[e^{-\frac{(x-x')^2}{B}}\frac{2(x-x')}{B}+e^{-\frac{(x+x'-2\delta)^2}{B}}\left(-\frac{2(x+x'-2\delta)}{B}\right)\right]dx'\right\}$$

$$\psi(x,\tau)=2ce^{-\frac{(x-\delta)^2}{B}}\left\{\int_{x'=0}^{\delta}\psi(x',\tau)dx'-\delta\psi(\delta,\tau)\right\}+c\int_{x'=0}^{\infty}\psi(x',\tau)\left[\left(1-\frac{2\delta(x-x')}{B}\right)e^{-\frac{(x-x')^2}{B}}+\left(1+\frac{2\delta(x+x'-2\delta)}{B}\right)e^{-\frac{(x+x'-2\delta)^2}{B}}\right]dx'$$

$$=2ce^{-\frac{(x-\delta)^2}{B}}\left\{\delta\psi\left(\frac{\delta}{2},0\right)-\delta\psi(\delta,\tau)\right\}+c\int_{x'=0}^{\infty}\psi(x',\tau)\left[\left(1-\frac{2\delta(x-x')}{B}\right)e^{-\frac{(x-x')^2}{B}}+\left(1+\frac{2\delta(x+x'-2\delta)}{B}\right)e^{-\frac{(x+x'-2\delta)^2}{B}}\right]dx'$$

Using $\psi(x,\tau)=\psi(x-\delta,0)=\psi(y,0)$,

$$\psi(y,0)=2ce^{-\frac{y^2}{B}}\delta\left\{\psi(x=0,0)+\frac{\partial\psi(x,0)}{\partial x}\Big|_{x=0}\frac{\delta}{2}-\psi(x=0,0)\right\}+c\int_{y'=0}^{\infty}\psi(y',0)\left[\left(1-\frac{2\delta(y-y')}{B}\right)e^{-\frac{(y-y')^2}{B}}+\left(1+\frac{2\delta(y+y')}{B}\right)e^{-\frac{(y+y')^2}{B}}\right]dy'$$

$$=\delta^2ce^{-\frac{y^2}{B}}\frac{\partial\psi(x,0)}{\partial x}\Big|_{x=0}+c\int_{y'=0}^{\infty}\psi(y',0)\left[\left(1-\frac{2\delta(y-y')}{B}\right)e^{-\frac{(y-y')^2}{B}}+\left(1+\frac{2\delta(y+y')}{B}\right)e^{-\frac{(y+y')^2}{B}}\right]dy'$$

$$=c\int_{y'=0}^{\infty}\psi(y',0)\left[\left(1-\frac{2\delta(y-y')}{B}\right)e^{-\frac{(y-y')^2}{B}}+\left(1+\frac{2\delta(y+y')}{B}\right)e^{-\frac{(y+y')^2}{B}}\right]dy'$$

if we neglect terms of order δ^2.

Iterative solutions to $\psi(y,0)$ might be expected to converge if an initial distribution that goes to zero for large x slowly evolves to one final distribution as the shock wave progresses. This fact is borne out by solutions obtained by iteration on a computer and shown in Figure 1. $\frac{\partial\psi(x,0)}{\partial x}\Big|_{x=0}$ is also seen to be finite, justifying the approximations made near the origin.

Reflection Efficiency

Since we hypothesize a steady state solution, we require that the total number of particles in the enhanced region be a constant. If the intensity in the enhanced region is

$$I_s(E) = \mu E^{-\gamma}$$

where E is energy and μ and γ are constants and each particle has its energy multiplied k times upon reflecting off the shock wave, the intensity would be increased by $k^{\gamma-1}$ after all particles have been reflected one time. If ϵ is the efficiency for reflection, we need

$$k^{\gamma-1}\,\epsilon = 1$$

for no net increase in intensity since we want a steady state solution. In general ϵ will not satisfy this equation; define ϵ_o such that

$$k^{\gamma-1}\,\epsilon_o = 1.$$

We will first work with ϵ_o in order to gain understanding of the problem and then find a solution for ϵ.

The equation for ε_0 is not entirely correct if there is a background, $I_b(E)$, before the enhanced region arrives. The shock wave must let as many protons pass through it as it both sweeps up and creates by energization if the enhanced region is to remain unchanging. If $I_{s-}(E)$ is the intensity per unit energy interval that is moving toward the shock wave just in front of it as seen in the solar wind rest frame and $\tau \ll \tau_{coll}$, $I_{s-}(E)(\bar{V}_\perp + v)\tau$ is the number per unit energy interval that hit the shock wave in time τ. $\bar{V}_\perp$ and $\bar{V}_\parallel$ are the average velocities of the particles comprising I_{s-} perpendicular to and parallel to the shock wave front.

Approximation, $I_{s-}(E)(\bar{V}_\perp + v)\tau\varepsilon_0 k^{\gamma-1}$ is the intensity per unit energy interval receding from the shock wave after striking it in time τ. If the number of particles is to remain constant,

$$I_{s-}(E)(\bar{V}_\perp + v)\tau - I_{s-}(E)(\bar{V}_\perp + v)\tau\varepsilon_0 k^{\gamma-1} = I_b(E)v\tau.$$

If $$\bar{V}_\perp \gg v$$

and ε_0 is large, then

$$I_{s-}(E) \doteq \frac{1}{2} I_s(E)$$

and $$\varepsilon_0 k^{\gamma-1} \doteq \frac{I_s(E)\bar{V}_\perp - 2I_b(E)v}{I_s(E)\bar{V}_\perp}$$

Hudson (1965) and Sonnerup (1969) have devised models that describe particles reflecting off shock waves. Here we will use a simple approach and find k as follows: For elastic reflection and isotropic fluxes

$$\bar{V}_{\perp}^{p} = \bar{V}_{\perp} + 2v \quad , \qquad \bar{V}_{\parallel}^{p} = \bar{V}_{\parallel}$$

$$E^{p} = \frac{1}{2} m V^{p^2}$$

$$= \frac{1}{2} m \left(\bar{V}_{\parallel}^{p^2} + \bar{V}_{\perp}^{p^2} \right)$$

$$\simeq E + \frac{2}{\sqrt{2}} m v V$$

$$k = \frac{E^{p}}{E} \simeq 1 + \frac{2\sqrt{2}\, v}{V}$$

if $\quad v \ll V \quad \text{and} \quad \bar{V}_{\parallel} \simeq \bar{V}_{\perp} \simeq \frac{V}{\sqrt{2}}$

and where superscript p denotes post collision.

As an example, the velocities of a 1 MeV proton and a typical shock wave are

$$V = 1.38 \times 10^{4} \ \frac{km}{sec} \quad , \qquad v = 200 \ \frac{km}{sec} \ ,$$

yielding $\quad k = 1.061 .$

If $\quad \gamma = 3 \quad \text{and} \quad \frac{I_s(E)}{I_b(E)} = 10 \ ,$

$$\epsilon_0 = 0.88 .$$

Such a large value for ϵ_0 can seem plausible for both thick and thin shock waves. For a thick wave draw an analogy with grazing incidence energetic electrons scattering off a metal surface, where reflection efficiencies ~ 99% can often occur. An electron will scatter through a small angle. If it scatters out of the block it is free; if it scatters in it still has a large probability of subsequently scattering out before losing all its energy. If the magnetic field inside the shock wave in our problem is much more turbulent than in the solar wind ahead of it, the protons will strongly scatter inside the wave until they become "free" by exiting from the wave. These protons, of course, do not gain or lose much energy and hence there is practically no limit on the time they can stay in the wave. If the thickness of the shock wave is larger than the mean free path, l, of a proton, the protons will predominately exit on the side they entered; i.e., the front of the shock wave.

If the shock wave is much thinner than the gyroradius of a particle, the particles will hardly notice the shock wave and have a very low reflection coefficient. However, almost as many particles will overtake the shock wave and pass through the wave from the back to the front side, making the effective reflection coefficient $\lesssim 1$ if the intensity on the back side is equal to the undisturbed intensity on the front side. But now we cannot calculate an effective energization factor. More precise calculations considering magnetic fields will be considered later. We will proceed using the thick shock wave model.

Returning to the actual reflection coefficient, ϵ must fall within certain limits in order to give an unchanging region in front of the shock wave. If

$$\epsilon = \epsilon_0 < \left(1 - \frac{2\sqrt{2}\,v}{V}\right) k^{\gamma-1}$$

there will be a lower intensity in front of the shock wave than in the background; i.e., the shock wave acts as a sink for particles. The decrease seen by Van Allen and Ness (1967) may have been due to an absorbing shock wave. If

$$\epsilon > k^{1-\gamma}$$

a steady state solution cannot exist. Particles are created by energization faster than they are lost into the shock wave. The enhanced region will constantly increase as the shock wave progresses. So ϵ must satisfy

$$\left(1 - \frac{2\sqrt{2}\,U}{V}\right) k^{1-\gamma} < \epsilon < k^{1-\gamma}$$

for a steady state solution with

$$I_s(E) > I_b(E).$$

ϵ most certainly almost never falls within these close limits and hence steady state enhanced regions in front of shock waves as here described should seldom exist. Several possibilities might explain the data:

1) The enhanced intensity is not unvarying but is increasing and the reason it is not extremely large is that ϵ or D changes with distance from the sun so that the enhanced intensity has been increasing for only a short while.
2) Particles leak around the edge of the shock wave.
3) The shock wave surface is irregular with regions that act as sinks (low ϵ) to moderate the growth of enhanced intensities in other regions (high ϵ).

4) The Axford-Reid Fermi acceleration mechanism is the only mechanism that causes enhanced intensities in front of shock waves.

5) There is a finite source of low energy particles caused by decreasing ϵ with decreasing energy.

Let us consider the fifth possibility. We suppose that below some energy, E_o, the shock wave does not reflect and energize particles. If interplanetary space were devoid of particles with energy $> E_o$ except for one narrow band of energies of width

$$\Delta E' = (l_2(E') - 1)\,E' \quad \text{at} \quad E = E'$$

where the intensity (per unit energy per unit distance) is $I_b(E')$, the shock wave would see a steady source of particles as it swept through space. These particles would be energized by the shock wave to produce a distribution in front of the wave. $\Delta E'$ was chosen so that after the first reflection the new intensity would fall adjacent to the original intensity as shown in Figure 4.

The fraction of particles in the enhanced region at any energy striking the shock wave in unit time is

$$\frac{1}{\tau_{coll}(E)} = \frac{1}{2} I_s(E,0)\,V \Big/ \int_0^{\infty} I_s(E,x)\,dx \qquad (1)$$

where we also consider the shape of the enhanced region. For an unchanging enhanced region in front of the shock wave, each of the blocks in Figure 4 must receive particles from the next lower block at the same rate as it loses to the next higher block:

$$\tau_{coll}(E)\, I_s(E,0)\,\Delta E = \text{constant}.$$

Using the analytic approximate expression P(x,1) for the enhanced intensity distribution $I_s(E,x)$ we get from (1)

$$\frac{1}{2} P(0,T) V = \frac{v V}{4D} = \frac{v}{2\ell} = \frac{1}{\tau_{coll}(E)} \quad .$$

In the time $\tau_{coll}(E)$ the average number of collisions each particle makes with the shock wave is 1. In this time we energize all particles with energy E in the enhanced region once on the average. In the time $\tau_{coll}(E')$ we admit $I_b(E')\, v\, \tau_{coll}(E')$ particles from the undisturbed region into the enhanced region. So all particles move to the right one block in Figure 4 and we obtain an approximate intensity at the shock wave by using P(x,T)

$$\int_0^\infty I_s(E,x)\,dx = \frac{\tau_{coll}(E)}{\tau_{coll}(E')} I_b(E')\, v\, \tau_{coll}(E') \frac{E'}{E} \frac{k(E')-1}{k(E)-1} R(E,E'), \; E > E'$$

$$I_s(E,0) \simeq 2 I_b(E') \frac{v}{V} \frac{E'}{E} \frac{k(E')-1}{k(E)-1} R(E,E'), \quad E > E'.$$

The factor containing k(E) accounts for overlapping blocks after the first reflection in Figure 4.

$$R(E,E') = \epsilon^N \leq 1$$

is the efficiency for particles starting at energy E' to be energized to E without being lost by absorption into the shock wave, where N is the number of reflections and ϵ is assumed to be independent of energy. We can approximate R(E,E') for $\epsilon \approx 1$ and $k \approx 1$:

$$k^N E' = E$$

$$N \ln k \simeq N(k-1) \simeq \ln \frac{E}{E'}$$

$$R(E,E') = \epsilon^{N}$$

$$= \epsilon^{\frac{\ln \frac{E}{E'}}{k-1}}$$

$$= \left(\epsilon^{\left(\frac{1}{k-1}\right)}\right)^{\ln \frac{E}{E'}}$$

$$= \left(\frac{E}{E'}\right)^{\ln \epsilon^{\frac{1}{k-1}}} = \left(\frac{E}{E'}\right)^{\frac{\ln \epsilon}{k-1}}$$

$$= \left(\frac{E}{E'}\right)^{\frac{\epsilon-1}{k-1}} = \left(\frac{E'}{E}\right)^{\frac{1-\epsilon}{k-1}} .$$

Now, actually the source of particles is not at one narrow band of energies but is the undisturbed interplanetary intensity,

$$I_b(E') = I_b(E)\left(\frac{E'}{E}\right)^{-\alpha}$$

assuming a power law spectrum. We obtain

$$I_s(E,0) = \int_{E'=E_0}^{E} 2 I_b(E') \frac{v}{V} \frac{E'}{E} \frac{k(E')-1}{k(E)-1} R(E,E') \frac{dE'}{\Delta E'}$$

$$= 2 I_b(E)\left(\frac{1}{E}\right)^{-\alpha+1} \frac{1}{k(E)-1} E^{-\frac{1-\epsilon}{k-1}} \frac{v}{V} \int_{E_0}^{E} E'^{-\alpha} E'^{\frac{1-\epsilon}{k-1}} dE'$$

$$= 2 I_b(E) E^{\alpha-1} \frac{V}{2\sqrt{2}\,v} E^{-\frac{1-\epsilon}{k-1}} \frac{v}{V} \int_{E_0}^{E} E'^{\frac{1-\epsilon}{k-1}-\alpha} dE'$$

$$= \frac{1}{2\sqrt{2}} I_b(E) E^{-\frac{1-\epsilon}{k-1}+\alpha-1} \int_{E_0}^{E} E'^{\frac{1-\epsilon}{k-1}-\alpha} dE'$$

$$= \frac{1}{2\sqrt{2}} \frac{1}{\frac{1-\epsilon}{n-1} - \alpha + 1} I_b(E)\, E^{-\frac{1-\epsilon}{n-1} + \alpha - 1} E'^{\frac{1-\epsilon}{n-1} - \alpha + 1} \Big|_{E_0}^{E}$$

$$= \frac{1}{2\sqrt{2}} \frac{1}{\frac{1-\epsilon}{n-1} - \alpha + 1} I_b(E) \left(1 - \left(\frac{E}{E_0}\right)^{-\frac{1-\epsilon}{n-1} + \alpha - 1} \right).$$

So we obtain

$$\frac{I_s(E,0)}{I_b(E)} = \frac{1}{2\sqrt{2}} \frac{1}{\frac{1-\epsilon}{n-1} - \alpha + 1} \left(1 - \left(\frac{E}{E_0}\right)^{-\frac{1-\epsilon}{n-1} + \alpha - 1} \right).$$

How this result depends on the various variables is shown in Figure 5.

MAGNETIC FIELD AND PITCH ANGLE

We have neglected the magnetic field in the solar wind in all the above discussions. Actually the reflection efficiency depends on the angles the magnetic field lines in front of and behind the shock wave make with the shock wave and on the pitch angle and the phase of the particle when it hits the shock wave (Hudson, 1965). In the following analysis we will make some very rough approximations to find the general shape of the pitch angle distribution in the enhanced region.

If there is no correlation between the shock wave normal and the magnetic field direction, the average angle between the two is 1 radian. Looking at the computer calculated graphs of Hudson we see that for this angle, particles with pitch angle $70^{o} < \nu < 110^{o}$ are all reflected while those with smaller or greater pitch angle are less efficiently reflected. In an isotropic flux, particles in the above mentioned pitch angle range comprise about 1/3 of the total number of particles. If the intensity of these particles is enhanced at the shock by a factor of 20 over the undisturbed isotropic intensity and particles with other pitch angles are enhanced by a factor of 5, the enhanced intensity averaged over pitch angle would be increased a factor of 10 over background. The intensity of particles with pitch angle $\approx 90^{o}$ would be several times the intensity of particles with velocity more aligned with the field lines.

The above numbers have not been rigorously calculated; pitch angle data is not yet available making comparisons between theory and experiment impossible. When pitch angle measurements become available, a more exact analysis might be useful. However, the calculations performed in this paper along with Hudson's analysis (1965) predict the general features of the pitch angle distribution described here.

COMPARISON WITH DATA

$I_s(E,x)$ is very sensitive to α, ϵ, and E/E_o which perhaps accounts for the wide range of peak heights we see in the data mentioned in the introduction. In the solar wind rest frame a 200 eV proton has roughly the same velocity as a 200 $\frac{km}{sec}$ shock wave, which would thus place an approximate lower limit on E_o. The approximation for $R(E, E')$ is very poor for large E/E_o, however it can be seen from Figure 5 that ϵ must be in the neighborhood of 0.9 for

$$\frac{E}{E_o} \geq 10,$$

$$\alpha = 3$$

and

$$\frac{I_s(E, 0)}{I_b(E)} \approx 10,$$

which are reasonable values for 1 MeV protons (Ogilvie and Arens, 1970).

Using the lower x-scale in Figure 1, a plot of $\psi(x,0)$ with

$$D = 6.0 \times 10^{16} \frac{cm^2}{sec}$$

is shown superposed on data from the November 29 shock wave. The x-scale was obtained by knowing that the data points were taken at 82 second intervals, the shock wave speed v was 52 $\frac{km}{sec}$ relative to the solar wind, the solar wind speed v_{sw} was 386 $\frac{km}{sec}$, and the angle between v and v_{sw} was 37° (Ogilvie and Burlaga, 1969. N.B. Corrections have been made in this paper).

$$D^2 \simeq D_{\parallel}^2 \cos^2 \zeta + D_{\perp}^2 \sin^2 \zeta$$

where $D_{\parallel}$ and $D_{\perp}$ are the parallel and perpendicular diffusion coefficients and ζ is the angle between the magnetic field and the normal to the shock wave. ζ was 24° for the November 29 event and if $D_{\parallel}$ is much larger than $D_{\perp}$,

$$D_{\parallel} \simeq 6.6 \times 10^{16} \frac{cm^2}{sec} .$$

If the enhanced intensity is largely made up of particles with pitch angle

$$80^o < \upsilon < 100^o,$$

the mean free path along the field line is approximately

$$\ell_{\parallel} = \frac{2 D_{\parallel}}{V_{\parallel}} \approx \frac{2 D_{\parallel}}{V \sin 10^o} \approx 5500 \text{ km}.$$

The gyroradius of a 1 MeV proton in a 5 gamma field is 3×10^4 km. The result for the mean free path implies that a proton will scatter through an angle $\approx 20^o$ once every gyrorevolution.

The mean free path and the diffusion coefficient found here are much smaller than the values found by considering the time needed for particles to propagate from a solar flare to the vicinity of the earth (Bryant et al, 1962; Hofmann et al, 1963; Krimigis, 1965; McCracken et al, 1967). Our mean free path denotes a distance traveled along a field line by a particle near the earth with pitch angle $\approx 90^o$ before it scatters through an angle $\approx 20^o$. The other determinations of the mean free path are averages of the mean free path between the sun and the earth rather than the mean free path near the earth. Also, in those paper particles with velocities more aligned along the field lines are included and the scattering angle is much larger, being $\approx 180^o$.

Similar calculations for other shock waves are difficult to make because the events are complicated or, more often, some necessary measurements are lacking.

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REFERENCES

Armstrong, T. P., S. M. Krimigis and K. W. Behannon, Proton Fluxes at 300 kev Associated with Propagating Interplanetary Shock Waves, J. Geophys. Res., 75, companion paper, 1970.

Axford, W. I. and G. G. Reid, Increases in Intensity of Solar Cosmic Rays before Sudden Commencements of Geomagnetic Storms, J. Geophys. Res., 68, 1793-1803, 1963.

Bryant, D. A., T. L. Cline, V. D. Desai and F. B. McDonald, Explorer 12 observations of solar cosmic rays and energetic storm particles after the solar flare of September 28, 1961, J. Geophys. Res., 67, 4983-5000, 1962.

Chandrasekhar, S., Stochastic Problems in Physics and Astronomy, Rev. Mod. Phys., 15, 1-89, 1943.

Fisk, L., Increases in the low energy cosmic ray intensity at the front of propagating interplanetary shock waves, J. Geophys. Res., 75, companion paper, 1970.

Hofmann, D. J. and J. R. Winckler, Simultaneous balloon observations at Fort Churchill and Minneapolis during the solar cosmic ray events of July 1961, J. Geophys. Res., 68, 2067-2098, 1963.

Hudson, P. D., Reflection of charged particles by plasma shocks, Mon. Not. R. Astr. Soc., 131, 23-50, 1965.

Krimigis, S. M., Interplanetary diffusion model for the time behavior of intensity in a solar cosmic ray event, J. Geophys. Res., 70, 2943-2960, 1965.

McCracken, K. G., U. R. Rao and R.P. Bukata, Cosmic ray propagation processes, J. Geophys. Res., 72, 4293-4324, 1967.

Ogilvie, K. W. and J. F. Arens, Acceleration of Protons by Interplanetary Shocks, J. Geophys. Res., 75, companion paper, 1970.

Ogilvie, K. W. and L. F. Burlaga, Hydromagnetic shocks in the solar wind, Solar Physics, 8, 422-434, 1969.

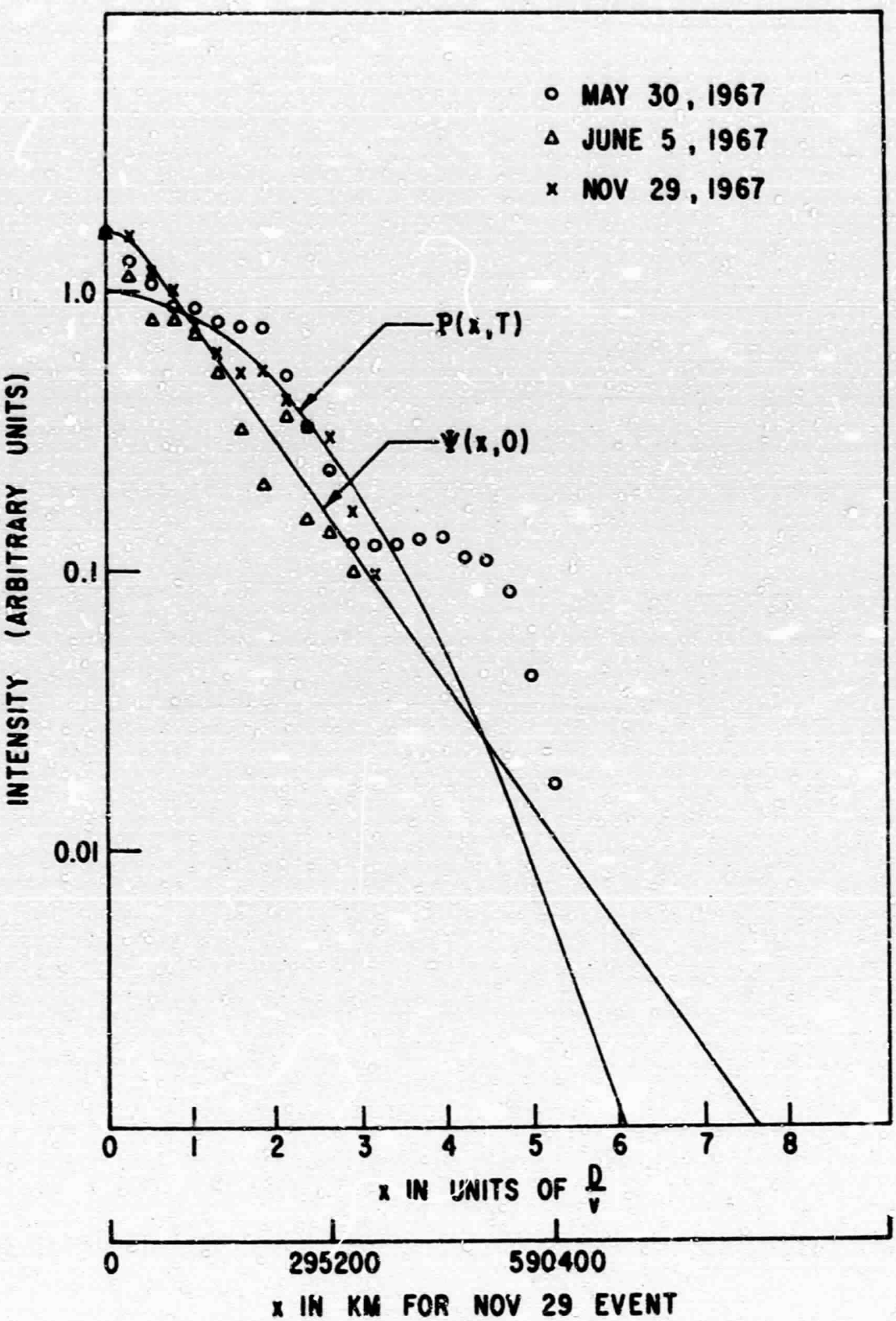
Singer, S., The association of energetic storm particles with interplanetary shock waves, LASL LA-DC-11014, 1970.

Sonnerup, B. U. O., Acceleration of Particles Reflected at a Shock Front, J. Geophys. Res., 74, 1301-1304, 1969.

Van Allen, J. A. and N. F. Ness, Observed particle effects of an interplanetary shock wave on July 8, 1966, J. Geophys. Res., 72, 935-942, 1967.

FIGURE CAPTIONS

1. Approximate $(P(x,T))$ and exact $(I_s(x))$ intensity distributions in front of the shock wave. Data from three shock waves (Ogilvie and Arens, 1970) are also plotted with the x-scale spacing adjusted to make reasonable agreement with the theoretical curves. Data points have the undisturbed intensity, I_b, subtracted and are normalized to the origin. Data points were taken at 82 second intervals, which is the sampling rate of the satellite experiment.
2. Unchanging intensity distribution, $\psi(x)$, as shock wave moves.
3. Paths of shock wave and typical particle.
4. Intensity distribution of a batch of particles after successive reflections off the shock wave.
5. Intensity at the shock wave is very sensitive to α, ϵ and E/E_o.



MAY 30, 1967
JUNE 5, 1967
NOV 29, 1967
INTENSITY (ARBITRARY UNITS)
1.0
0.1
0.01
Φ(x,T)
Ψ(x,0)
0
1
2
3
4
5
6
7
8
x IN UNITS OF D/v
0
295200
590400
x IN KM FOR NOV 29 EVENT

FIGURE 1

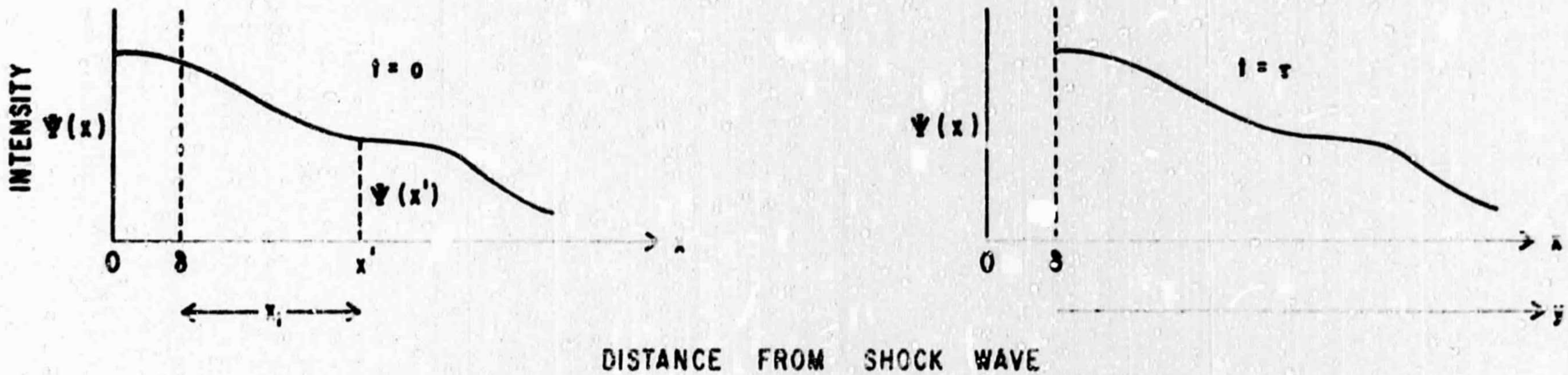


FIGURE 2

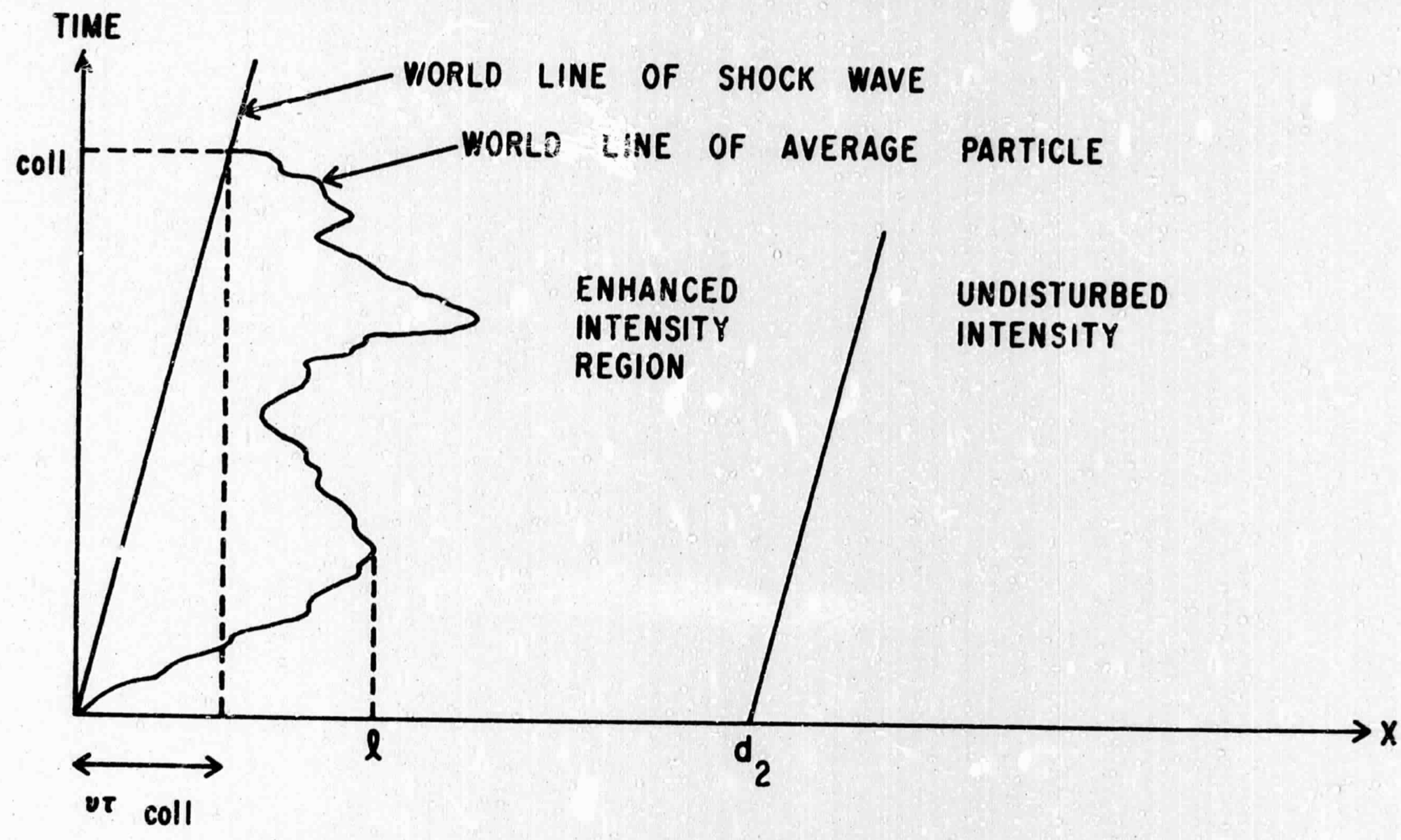
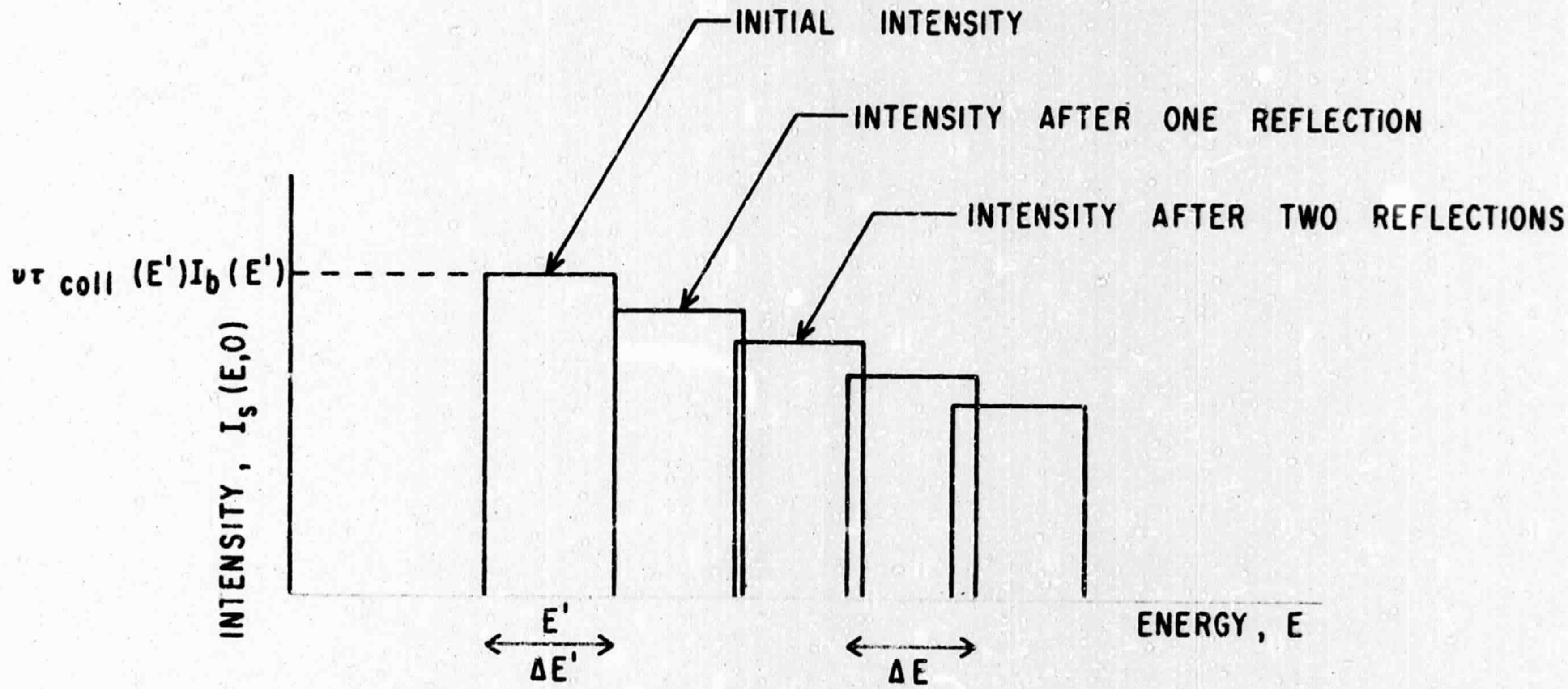
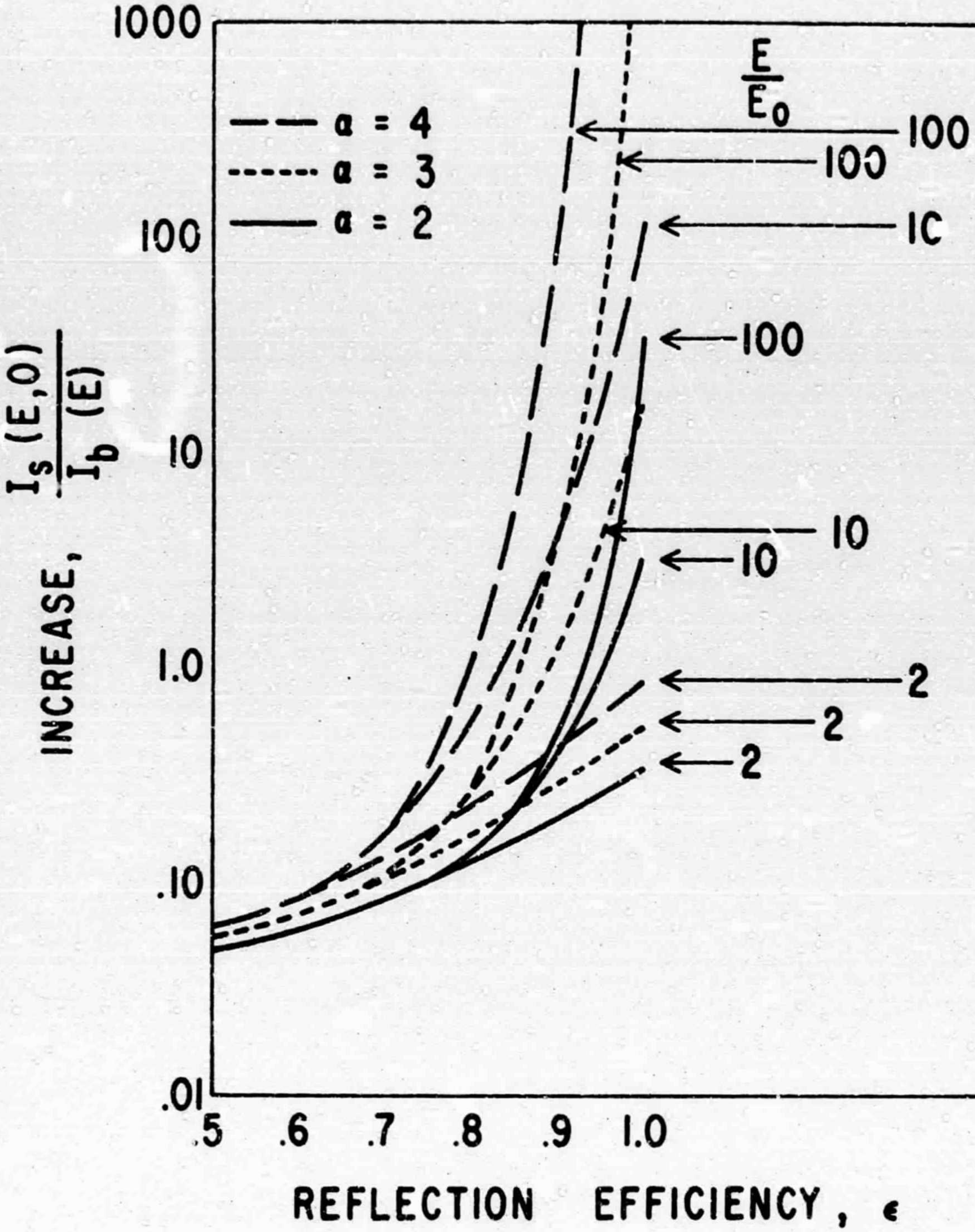

TIME
WORLD LINE OF SHOCK WAVE
τ coll
WORLD LINE OF AVERAGE PARTICLE
ENHANCED
INTENSITY
REGION
UNDISTURBED
INTENSITY
ℓ
d2
X
υτ coll

FIGURE 3



AREAS OF ALL BLOCKS ARE EQUAL: $\frac{I_s(E)}{I_s(E')} = \frac{\Delta E'}{\Delta E} = \frac{E'}{E}$

FIGURE 4



1000
100
10
1.0
.10
.01
.5
.6
.7
.8
.9
1.0
a = 4
a = 3
a = 2
E/E0
100
100
1C
100
10
10
2
2
2
INCREASE, Is (E,0) / Ib (E)
REFLECTION EFFICIENCY, ϵ

FIGURE 5