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**OPTIMIZATION OF SELF-ACTING STEP THRUST
BEARINGS FOR LOAD CAPACITY AND STIFFNESS**

by Bernard J. Hamrock
Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER proposed for presentation
at Lubrication Conference sponsored by the
American Society of Lubrication Engineers and
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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LOAD CAPACITY AND STIFFNESS

by Bernard J. Hamrock

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ABSTRACT

A linearized analysis of a finite width rectangular step thrust bearing was performed. Dimensionless load capacity and stiffness are expressed in terms of a Fourier cosine series. The dimensionless load capacity and stiffness were found to be a function of the dimensionless bearing number Λ , the pad length-to-width ratio λ , the film thickness ratio k , the step location parameter ψ , and the feed groove parameter η .

The equations obtained in the analysis were verified. The assumptions imposed were substantiated by comparison of the results with an existing exact solution for the infinite width bearing. A digital computer program was developed which determines optimal bearing configuration for maximum load capacity or stiffness. Simple design curves are presented. Results are shown for both compressible and incompressible lubrication. Through a parameter transformation the results are directly usable in designing optimal step sector thrust bearings.

SYMBOLS

A, B, D, E	integration constants
AA, BB, CC, DD, EE	constants defined in body of report

b	width of rectangular thrust bearing
C	film thickness in the ridge region
h	film thickness
I_m	Fourier coefficient
J	separation constant
K	dimensionless stiffness, $-C(\partial W/\partial C)$
k	film thickness ratio, $(C + \Delta)/C$
l	length of pad region
M	last odd positive integer used in evaluation of Fourier cosine series
m	odd positive integers
N	number of pads placed in the overall length
P	dimensionless pressure, $(p - p_a)/p_a$
p	pressure
p_a	ambient pressure
Q	mass flow rate
R_i	inner radius of a sector thrust bearing
R_o	outer radius of a sector thrust bearing
U	velocity of bearing surface
W	dimensionless load capacity of finite width bearing, $w/[\bar{p}_a b(l_r + l_s + l_g)]$
w	load capacity
X	dimensionless width coordinate, x/b
x	coordinate in direction of motion
Y	dimensionless width coordinate, y/b
y	coordinate in the direction of the width of the bearing

β dimensionless bearing number used by Kocki (ref. 1),

$$\beta = \frac{3\mu U(l_s + l_r)}{p_a \Delta^2} = \frac{\Lambda}{(k-1)^2}$$

Δ depth of step

η feed groove parameter, $\frac{l_r + l_s}{l_r + l_s + l_g}$

Λ dimensionless bearing number, $6\mu Ub/p_a C^2$

λ ratio of length-to-width of pad, $(l_r + l_s + l_g)/b$

μ viscosity of fluid

$$\xi_r \sqrt{\left(\frac{\Lambda}{2}\right)^2 + m^2 \pi^2}$$

$$\xi_s \sqrt{\left(\frac{\Lambda}{2k^2}\right)^2 + m^2 \pi^2}$$

ρ mass density of lubricant

ψ step location parameter, $\frac{l_s}{l_r + l_s + l_g}$

ω angular velocity

Subscripts:

g denotes feed groove region

r denotes ridge region

s denotes step region

INTRODUCTION

One of the first to apply the step film to a gas-lubricated bearing was Kochi (ref. 1). For the infinitely wide single thrust bearing Kochi obtained an exact numerical solution. The expressions for the pressure were found to be contained in a set of transcendental equations. Graphical methods were used to obtain the results.

In practical applications one must use finite width bearings. The finite width step thrust bearing can appear in the shape of a rectangular pad or as a sector. For both the rectangular and the sector step thrust bearings there is a definite need to know the optimal step configurations for maximum load capacity or maximum stiffness.

Ausman (ref. 2) in 1961 analyzed the gas lubricated step sector thrust bearing. He applied linearization assumptions to the Reynolds equation thereby enabling the pressure to be determined. Knowing the pressure, the load capacity was obtained. The expression for the load capacity appeared in terms of eigenvalues and Bessel functions. Ausman's results do not lend themselves readily to obtaining optimal step configurations for maximum load capacity or maximum stiffness when various bearing operating conditions are considered. The reason for this is the way in which parameters were made dimensionless and the nature of the resulting equations.

In this paper a rectangular step thrust bearing is analyzed. Linearization assumptions comparable to those imposed by Ausman (ref. 2) are used. The sector bearing results are obtained directly from the rectangular step bearing results since curvature effects are shown to be very small. Because of the simplified nature of the resulting equa-

tions a computer program was developed which optimized step parameters for maximum load capacity or maximum stiffness for a wide range of bearing operating conditions. Results are shown for both compressible and incompressible lubrication. Therefore, the objective of this paper is to present easily usable design information for finding optimal step bearings of rectangular or sector shape bearings. The results are to be valid for a wide range of operating conditions.

BEARING DESCRIPTION

Sketch 1 shows the bearing to be studied. In this sketch the ridge region is where the film thickness is C and the step region is where the film thickness is $C + \Delta$. The feed groove is the deep groove separating the end of the ridge region and the beginning of the next step region. Although not shown in this figure, the depth of the feed groove is orders of magnitude deeper than the film thickness C . A "pad" is defined as the section which includes a ridge, step, and feed groove regions. The length of the feed groove is small relative to the length of the pad. It should be noted that each pad acts independently since the pressure profile is broken at the lubrication feed groove.

LINEARIZATION ASSUMPTIONS

The Reynolds equation for the steady state isothermal gas-lubricated thrust bearing can be written as

$$\frac{\partial}{\partial x} \left(p \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(p \frac{\partial p}{\partial y} \right) = \frac{6 \mu U}{h^2} \frac{\partial p}{\partial x} \quad (1)$$

Expanding and rearranging terms, the above equation becomes

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \frac{6 \mu U}{p h^2} \frac{\partial p}{\partial x} = -\frac{1}{p} \left[\left(\frac{\partial p}{\partial x} \right)^2 + \left(\frac{\partial p}{\partial y} \right)^2 \right] \quad (2)$$

In order to get an analytic solution to the above equation linearization assumptions will be imposed. The first linearization assumption states that the right side of equation (2) is zero. A second and final linearization assumption which is required is that p , where it appears as a coefficient, be replaced by the ambient pressure, p_a . Applying these assumptions, equation (2) becomes

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{6\mu U}{p_a h^2} \frac{\partial p}{\partial x} \quad (3)$$

PRESSURE ANALYSIS

From equation (3) we can write the linearized Reynolds equation separately for the ridge and step regions of the finite step thrust bearing as:

$$\begin{aligned} \frac{\partial^2 p_r}{\partial x^2} + \frac{\partial^2 p_r}{\partial y^2} &= \frac{6\mu U}{p_a C^2} \frac{\partial p_r}{\partial x} \\ \frac{\partial^2 p_s}{\partial x^2} + \frac{\partial^2 p_s}{\partial y^2} &= \frac{6\mu U}{p_a (C + \Delta)^2} \frac{\partial p_s}{\partial x} \end{aligned}$$

Subscript r refers to the ridge region (see sketch 1), subscript s refers to the step region, and subscript g refers to the feed groove region. Letting $x = bX$, $y = bY$, $p_r = p_a(P_r + 1)$, and $p_s = p_a(P_s + 1)$ the above equations become

$$\frac{\partial^2 P_r}{\partial X^2} + \frac{\partial^2 P_r}{\partial Y^2} = \Lambda \frac{\partial P_r}{\partial X} \quad (4)$$

$$\frac{\partial^2 P_s}{\partial X^2} + \frac{\partial^2 P_s}{\partial Y^2} = \frac{\Lambda}{k^2} \frac{\partial P_s}{\partial X} \quad (5)$$

where

$$\Lambda = \frac{6\mu Ub}{p_a C^2}$$

and

$$k = \frac{C + \Delta}{C}$$

Using a separation of variables technique on equations (4) and (5) gives the following

$$P_r = e^{\Lambda X/2} \left(A_r e^{X \sqrt{\frac{\Lambda^2}{4} + J_r^2}} + B_r e^{-X \sqrt{\frac{\Lambda^2}{4} + J_r^2}} \right) \left[D_r \sin(J_r Y) + E_r \cos(J_r Y) \right] \quad (6)$$

$$P_s = e^{\frac{\Lambda X}{2k^2}} \left(A_s e^{X \sqrt{\frac{\Lambda^2}{4k^4} + J_s^2}} + B_s e^{-X \sqrt{\frac{\Lambda^2}{4k^4} + J_s^2}} \right) \left[D_s \sin(J_s Y) + E_s \cos(J_s Y) \right] \quad (7)$$

The boundary conditions are

$$(1) P_s = 0 \text{ when } X = 0$$

$$(2) P_r = 0 \text{ when } X = \frac{l_s + l_r}{b} = \left(\frac{l_s + l_r}{l_s + l_r + l_g} \right) \left(\frac{l_s + l_r + l_g}{b} \right) = \eta \lambda$$

$$(3) P_r = P_s = \sum_{m=1, 3, \dots}^{\infty} I_m \cos(m\pi Y) \text{ when } X = \frac{l_s}{b} = \psi \lambda$$

where I_m is a Fourier coefficient

$$(4) \frac{\partial P_r}{\partial Y} = \frac{\partial P_s}{\partial Y} = 0 \text{ when } Y = 0$$

$$(5) P_r = P_s = 0 \text{ when } Y = 1/2$$

$$(6) Q_r = Q_s \text{ when } X = \psi\lambda$$

Making use of boundary conditions 1 through 5, equations (6) and (7) become

$$P_r = \sum_{m=1, 3, \dots}^{\infty} \left[\frac{I_m \cos(m\pi Y) e^{\frac{\Lambda}{2}(X - \psi\lambda)}}{e^{-\lambda\psi\xi_r} - e^{-\lambda\xi_r(2\eta - \psi)}} \right] \left[e^{-X\xi_r} - e^{-\xi_r(2\lambda\eta - X)} \right] \quad (8)$$

$$P_s = \sum_{m=1, 3, \dots}^{\infty} \left[\frac{I_m \cos(m\pi Y) e^{\frac{\Lambda}{2k^2}(X - \psi\lambda)}}{e^{-\psi\lambda\xi_s} - e^{\psi\lambda\xi_s}} \right] \left[e^{-X\xi_s} - e^{X\xi_s} \right] \quad (9)$$

where

$$\xi_r = \sqrt{\left(\frac{\Lambda}{2}\right)^2 + m^2\pi^2}$$

and

$$\xi_s = \sqrt{\left(\frac{\Lambda}{2k^2}\right)^2 + m^2\pi^2}$$

The linearized equations describing the mass flow across the ridge and step regions can be written as

$$Q_r = \frac{\rho_a}{p_a} \left(\frac{p_r UC}{2} - \frac{p_a C^3}{12\mu} \frac{\partial p_r}{\partial x} \right)$$

$$Q_s = \frac{\rho_a}{p_a} \left[\frac{p_s U(C + \Delta)}{2} - \frac{p_a (C + \Delta)^3}{12 \mu} \frac{\partial p_s}{\partial x} \right]$$

These equations may be made dimensionless by letting $p_r = p_a(P_r + 1)$, $p_s = p_a(P_s + 1)$ and $x = bX$ as was done for the Reynolds equations.

$$Q_r = \frac{\rho_a p_a C^3}{12 \mu b} \left[\Lambda(P_r + 1) - \frac{\partial P_r}{\partial X} \right]$$

$$Q_s = \frac{\rho_a p_a (C + \Delta)^3}{12 \mu b} \left[\frac{\Lambda}{k^2} (P_s + 1) - \frac{\partial P_s}{\partial X} \right]$$

Making use of boundary conditions 3 and 6 gives the following

$$k^3 \left(\frac{\partial P_s}{\partial X} \right)_{X=\lambda\psi} - \left(\frac{\partial P_r}{\partial X} \right)_{X=\lambda\psi} = \Lambda(k-1) \left[1 + \sum_{m=1, 3, \dots}^{\infty} I_m \cos(m\pi Y) \right] \quad (10)$$

Making use of equations (8), (9), and (10) the Fourier coefficient I_m can be solved

$$I_m = \frac{4(k-1) \sin\left(\frac{m\pi}{2}\right)}{m\pi \left\{ \frac{1-k}{2} + \frac{\xi_s k^3}{\Lambda} \left(\frac{1 + e^{-2\xi_s \lambda \psi}}{1 - e^{-2\xi_s \lambda \psi}} \right) + \frac{\xi_r}{\Lambda} \left[\frac{1 + e^{-2\lambda \xi_r (\eta - \psi)}}{1 - e^{-2\lambda \xi_r (\eta - \psi)}} \right] \right\}} \quad (11)$$

LOAD ANALYSIS

The dimensionless load capacity for the ridge and step region can be written as

$$W_r = \frac{w_r}{p_a b L} = \frac{2}{\lambda} \int_0^{1/2} \int_{\lambda\psi}^{\lambda\eta} P_r dX dY$$

$$W_s = \frac{w_s}{p_a b L} = \frac{2}{\lambda} \int_0^{1/2} \int_0^{\lambda \psi} P_s dX dY$$

Substituting equations (8) and (9) into the above equations and integrating gives the following

$$W_r = \sum_{m=1, 3, \dots}^{\infty} \frac{2I_m \sin\left(\frac{m\pi}{2}\right)}{m^3 \pi^3 \lambda} \left\{ \frac{\Lambda}{2} + \frac{\xi_r \left[1 - 2e^{-\lambda(\eta-\psi)} \left(\xi_r - \frac{\Lambda}{2} \right) + e^{-2\lambda\xi_r(\eta-\psi)} \right]}{1 - e^{-2\lambda\xi_r(\eta-\psi)}} \right\} \quad (12)$$

$$W_s = \sum_{m=1, 3, \dots}^{\infty} \frac{2I_m \sin\left(\frac{m\pi}{2}\right)}{m^3 \pi^3 \lambda} \left\{ -\frac{\Lambda}{2k^2} + \frac{\xi_s \left[1 - 2e^{-\lambda\psi} \left(\xi_s + \frac{\Lambda}{2k^2} \right) + e^{-2\lambda\psi\xi_s} \right]}{1 - e^{-2\lambda\psi\xi_s}} \right\} \quad (13)$$

The total dimensionless load supported by the rectangular step slider bearing can be written as

$$W = \frac{w_r + w_s}{p_a L b} = W_r + W_s \quad (14)$$

STIFFNESS ANALYSIS

The equation for the dimensionless stiffness can be written as

$$K = -C \frac{\partial W}{\partial C}$$

Making use of equations (12), (13), and (14), the above equation becomes

$$K = \sum_{m=1, 3, \dots}^{\infty} \left\{ W(AA) - \frac{2I_m \sin\left(\frac{m\pi}{2}\right)}{m^3 \pi^3 \lambda} \left[\Lambda \left(1 - \frac{1}{k^3}\right) - \xi_r(BB) - \xi_s(CC) + \frac{\Lambda^2}{2\xi_r} (DD) + \frac{\Lambda^2}{2\xi_s k^5} (EE) \right] \right\} \quad (15)$$

where AA, BB, CC, DD, and EE are constants defined in the Appendix.

Therefore, with equations (11) through (15) the dimensionless load capacity and stiffness for a self-acting gas-lubricated finite width step thrust bearing is completely defined. From these equations it is evident that the dimensionless load capacity and stiffness are functions of the following five parameters:

- (1) $\Lambda = 6 \mu U b / p_a C^2$, the dimensionless bearing number
- (2) $\lambda = \frac{l_s + l_r + l_g}{b}$, the length-to-width ratio of a pad
- (3) $k = \frac{C + \Delta}{C}$, film thickness ratio
- (4) $\psi = \frac{l_s}{l_s + l_r + l_g}$, step location parameter
- (5) $\eta = \frac{l_s + l_r}{l_s + l_r + l_g}$, feed groove parameter

VERIFICATION OF EQUATIONS

The equations for the dimensionless load capacity and stiffness were programmed on a digital computer. It should be recalled that linearization assumptions were imposed in order to obtain simplified results.

Figure 1 shows that these assumptions are generally valid for the infinite

width bearing or a finite analysis where $\lambda \rightarrow 0$. This figure compares the results from the present work with Kochi's (ref. 1) exact solution. The agreement is good. Comparing equations (2) and (3) with and without the width coordinates (y), one could further conclude that for any finite width bearing the linearized analysis should be in good agreement with the exact results.

Table I shows that the solutions for the finite and infinite analyses approach each other when the length-to-width ratio approaches zero. The infinite width solution was obtained from reference 3. The results when five hundred terms ($M = 1001$) are used in the Fourier cosine series approach the infinite width analysis much closer than when only fifty terms ($M = 101$) are used. Furthermore, the rate of convergence is much slower at large dimensionless bearing numbers ($\Lambda \rightarrow 500$) than at smaller values of Λ . Note the decrease in dimensionless load capacity (W) when going from $\Lambda = 100$ to $\Lambda = 500$. This is due to the fact that the step parameters are held constant. That is, the step parameters chosen happen to be closer to the optimal for $\Lambda = 100$ than for $\Lambda = 500$.

Table II compares the dimensionless load capacity obtained from Ausman (ref. 2) with the present work for various dimensionless bearing numbers (Λ) and inner to outer radius ratios R_i/R_o . Ausman (ref. 2) considers curvature effects whereas the present work does not. The equivalent length of a sector pad is assumed to be the arc length along the average radius. The width is the difference in inner and outer radii. For all inner to outer radius ratios, there is close agreement between the two analyses. Curvature effects are small. Therefore,

the simplified equations of the present analysis are valid for evaluating the circular sector thrust bearing.

OPTIMIZING PROCEDURE

The problem as defined in the introduction is to find the optimal step bearing for maximum load capacity or stiffness for various bearing numbers. This means, given the dimensionless bearing number Λ , finding the optimal length-to-width ratio λ , optimal film thickness ratio k , and optimal step location parameter ψ . The significance of the feed groove parameter η is much less than that of the other parameters. Therefore, for all evaluations the feed groove parameter η will be set equal to 0.97.

The basic problem in optimizing λ , k , and ψ for maximum load and stiffness is essentially that of finding values of λ , k , and ψ which satisfy the following equations:

$$\frac{\partial W}{\partial \lambda} = \frac{\partial W}{\partial k} = \frac{\partial W}{\partial \psi} = 0 \quad (16)$$

$$\frac{\partial K}{\partial \lambda} = \frac{\partial K}{\partial k} = \frac{\partial K}{\partial \psi} = 0 \quad (17)$$

The method used in solving the above equations is the Newton-Raphson method for solving simultaneous equations. This method is described in Scarborough (ref. 4) or most other texts on numerical analysis.

Therefore, given the dimensionless bearing number Λ , the optimization computer program obtains optimum values of λ , k , and ψ for maximum dimensionless load capacity or stiffness. As a check on the optimization procedure the following case was considered $\Lambda = \lambda = 1 \times 10^{-5}$. This case approaches an infinitely wide incompressibility lubricated step

bearing for which the results are known. For this case the computer program indicated that $k = 1.866$ and $\psi = 0.718$ were optimal for maximum load capacity. These results are in exact agreement with Archibald (ref. 5).

STEP SECTOR THRUST BEARING

For optimization of a step sector thrust bearing, parameters for the sector must be found that are analogous to those for the rectangular step bearing. The following substitutions accomplish this transformation.

$$\begin{aligned} b &\rightarrow R_o - R_i \\ N(l_s + l_r + l_g) &\rightarrow \pi(R_o + R_i) \\ U &\rightarrow \frac{\omega}{2}(R_o + R_i) \end{aligned}$$

Where N is the number of pads placed in the step sector. Making use of the above equations, the dimensionless bearing number can be rewritten as

$$\Lambda \rightarrow \frac{3\mu\omega(R_o^2 - R_i^2)}{p_a C^2} \quad (18)$$

The optimal number of pads to be placed in the sector is obtained from the following formula:

$$N = \frac{\pi(R_o + R_i)}{(\lambda)_{\text{opt}}(R_o - R_i)} \quad (19)$$

In the above equation $(\lambda)_{\text{opt}}$ is the optimal value for the length-to-width ratio. The way $(\lambda)_{\text{opt}}$ is obtained will be discussed in the next section.

Since N will not be an integer normally, rounding it to the nearest integer is required.

DISCUSSION OF RESULTS

Tables III and IV give optimal step parameters (ψ , λ , and k) for resulting maximum load capacity and stiffness. The differences between these tables are: Table III optimizes with respect to load capacity, whereas table IV optimizes with respect to stiffness. The following observations can be made about both tables III and IV as Λ (or bearing speed) is increased:

- (1) The length-to-width ratio (λ) increases. That is, the length of the pad increases relative to its width.
- (2) The step location parameter (ψ) decreases. This means that the length of the step region decreases relative to the length of the pad.
- (3) The film thickness ratio (k) increases. That is, the step depth increases relative to the clearance.

Figures 2, 3, 4(a) and 4(b) are obtained directly from the data presented in tables III and IV. Figure 2 shows the effect of Λ on λ , k , and ψ for maximum load capacity condition for a range of Λ from 0 to 410. The optimal step parameters (λ , k , and ψ) are seen to approach an asymptote as the dimensionless bearing number (Λ) becomes small. That is, for small Λ ($\Lambda \leq 0.1$), the optimal step parameters are not a function of Λ . In the incompressible solution of a step bearing the right hand side of equations (4) and (5) are zero. Therefore, it must be concluded that the asymptotic values which the step parameters approach in figure 2 correspond to the incompressible solution. These asymptotes are $\lambda = 0.918$, $\psi = 0.555$, and $k = 1.693$.

Figure 3 shows the effect of Λ on λ , k , and ψ for maximum stiffness condition for a range of Λ from 0 to 410. As in figure 2 the optimal step parameters are seen to approach asymptotes as the incompressible solution is reached. The asymptotes are $\lambda = 0.915$, $\psi = 0.557$, and $k = 1.470$. Note that there is a difference in the asymptote for the film thickness ratio but virtually no change in λ and ψ when compared to that obtained from figure 2.

Figure 4(a) and 4(b) show the effect of dimensionless bearing number (Λ) on dimensionless load capacity and stiffness. The difference in these figures is that the optimal step parameters are obtained in figure 4(a) for maximum load capacity and in figure 4(b) for maximum stiffness. Also shown in these figures are values of K and W when the step parameters are held fixed as the optimal values obtained for the incompressible solution. The significant decrease between the solid and dash lines in W or K does not occur until $\Lambda > 8$.

Figures 3, 3, 4(a), and 4(b) contain all the necessary information for the design of an optimal rectangular step thrust bearing for maximum load capacity or stiffness. With the dimensionless bearing number Λ given the optimal values of λ , ψ , and k can be obtained from figures 2 or 3, depending if maximum load or stiffness is major consideration. From figures 4(a) or 4(b) the resulting values for the dimensionless load and stiffness can be obtained. Furthermore from figures 2, 3, 4(a), and 4(b) and equations (18) and (19), the optimal step sector thrust bearing considering load capacity or stiffness can be obtained. The dimensionless bearing number (Λ) is obtained from equation (18). Knowing Λ the optimal values of λ , ψ , and k can be obtained from figures 2 or 3 de-

pending whether maximum load or stiffness is a major consideration. From equation (19) the optimal number of pads placed in a sector can be determined. Finally, from figures 4(a) or 4(b) the resulting values for the dimensionless load capacity and stiffness can be obtained.

CONCLUSION

A linearized analysis of a rectangular step thrust bearing was performed. Dimensionless load capacity and stiffness are expressed in terms of a Fourier cosine series. The equations obtained in the analysis were verified. The assumptions imposed were substantiated by comparison of the results with an existing exact solution for the infinite width bearing. A digital computer program was developed which determines optimal bearing configuration for maximum load capacity or stiffness. Simple design curves are presented. Results are shown for both compressible and incompressible lubrication. Through a parameter transformation the results are directly usable in designing an optimal step sector thrust bearing.

APPENDIX - CONSTANTS OBTAINED IN EVALUATING
DIMENSIONLESS STIFFNESS

$$\begin{aligned}
 AA &= \frac{-m\pi I_m}{4(k-1)\sin\left(\frac{m\pi}{2}\right)} \left(\frac{2\lambda\psi\Lambda}{k^2} \left[\frac{e^{-2\xi_s\lambda\psi}}{\left(1 - e^{-2\lambda\xi_s\psi}\right)^2} \right] \right. \\
 &\quad + \left(\frac{3k^2\xi_s}{\Lambda} - \frac{\Lambda}{2k^2\xi_s} \right) \left(\frac{1 + e^{-2\xi_s\lambda\psi}}{1 - e^{-2\xi_s\lambda\psi}} \right) + \left(\frac{3\xi_r}{\Lambda} - \frac{\Lambda}{2\xi_r} \right) \left[\frac{1 + e^{-2\lambda\xi_r(\eta-\psi)}}{1 - e^{-2\lambda\xi_r(\eta-\psi)}} \right] \\
 &\quad \left. + 2\lambda\Lambda(\eta-\psi) \left\{ \frac{e^{-2\xi_r\lambda(\eta-\psi)}}{\left[1 - e^{-2\xi_r\lambda(\eta-\psi)}\right]^2} \right\} \right) \\
 BB &= \frac{2\lambda\Lambda(\eta-\psi)}{\left[1 - e^{-2\lambda\xi_r(\eta-\psi)}\right]^2} \left(\frac{\Lambda}{2\xi_r} \left\{ 2e^{-2\lambda\xi_r(\eta-\psi)} - e^{-\lambda(\eta-\psi)} \left(\xi_r - \frac{\Lambda}{2} \right) \left[1 + e^{-2\lambda\xi_r(\eta-\psi)} \right] \right\} \right. \\
 &\quad \left. + e^{-\lambda(\eta-\psi)} \left(\xi_r - \frac{\Lambda}{2} \right) \left[1 - e^{-2\lambda\xi_r(\eta-\psi)} \right] \right) \\
 CC &= \frac{2\lambda\psi\Lambda}{k^3 \left(1 - e^{-2\lambda\psi\xi_s}\right)^2} \left\{ \frac{\Lambda}{2\xi_s k^2} \left[2e^{-2\lambda\psi\xi_s} - e^{-\lambda\psi \left(\xi_s + \frac{\Lambda}{2k^2} \right)} \left(1 + e^{-2\lambda\psi\xi_s} \right) \right] \right. \\
 &\quad \left. - e^{-\lambda\psi \left(\xi_s + \frac{\Lambda}{2k^2} \right)} \left[1 - e^{-2\lambda\psi\xi_s} \right] \right\} \\
 DD &= \frac{1 - 2e^{-\lambda(\eta-\psi) \left(\xi_r - \frac{\Lambda}{2} \right)} + e^{-2\lambda\xi_r(\eta-\psi)}}{1 - e^{-2\lambda\xi_r(\eta-\psi)}}
 \end{aligned}$$

$$EE = \frac{1 - 2e^{-\lambda\psi\left(\xi_s + \frac{\Lambda}{2k^2}\right)} + e^{-2\lambda\psi\xi_s}}{1 - e^{-2\lambda\psi\xi_s}}$$

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TABLE I. - COMPARISON OF DIMENSIONLESS LOAD CAPACITY FOR INFINITE-WIDTH SOLUTION AND LIMITING CASE OF
FINITE-WIDTH SOLUTION FOR TWO-SERIES TRUNCATIONS

[Feed groove parameter, $\eta = 1.0$; step location parameter, $\psi = 0.45$; film thickness ratio, $k = 2.0$.]

Dimensionless bearing number, Λ	Dimensionless load capacity of infinite-width bearing, W_{∞}	Limiting case of dimensionless load capacity of finite-width bearing, $\lim W$ $\lambda \rightarrow 0$		Dimensionless stiffness of infinite-width bearing, K_{∞}	Limiting case of dimensionless stiffness of infinite-width bearing, $\lim K$ $\lambda \rightarrow 0$	
		M = 101 (50 terms)	M = 1001 (500 terms)		M = 101. (50 terms)	M = 1001. (500 terms)
10^{-4}	2.5516×10^{-6}	2.5414×10^{-6}	2.5505×10^{-6}	4.1825×10^{-6}	4.1659×10^{-6}	4.1808×10^{-6}
10^{-3}	2.5517×10^{-5}	2.5416×10^{-5}	2.5507×10^{-5}	4.1831×10^{-5}	4.1664×10^{-5}	4.1814×10^{-5}
10^{-2}	2.5533×10^{-4}	2.5431×10^{-4}	2.5522×10^{-4}	4.1887×10^{-4}	4.1720×10^{-4}	4.1870×10^{-4}
10^{-1}	2.5687×10^{-3}	2.5585×10^{-3}	2.5677×10^{-3}	4.2448×10^{-3}	4.2279×10^{-3}	4.2431×10^{-3}
1	2.7185×10^{-2}	2.7077×10^{-2}	2.7174×10^{-2}	4.7869×10^{-2}	4.7678×10^{-2}	4.7849×10^{-2}
10	3.2354×10^{-1}	3.2226×10^{-1}	3.2341×10^{-1}	5.8440×10^{-1}	5.8208×10^{-1}	5.8416×10^{-1}
100	5.7998×10^{-1}	5.7716×10^{-1}	5.7923×10^{-1}	5.6021×10^{-1}	5.5891×10^{-1}	5.6091×10^{-1}
500	5.5600×10^{-1}	5.4967×10^{-1}	5.5165×10^{-1}	5.5200×10^{-1}	5.8944×10^{-1}	5.6141×10^{-1}

TABLE II. - COMPARISON OF DIMENSIONLESS LOAD CAPACITY
OF AUSMAN (REF. 2) WITH PRESENT ANALYSIS

Inner- to outer-radius ratio, R_1/R_0	Dimensionless bearing number, Λ				
	10	20	40	80	160
	Dimensionless load capacity of finite-width bearing, W (a)				
0.2	0.064 .063	0.141 .138	0.286 .280	0.470 .458	0.638 .619
0.3	0.059 .059	0.131 .129	0.270 .267	0.457 .450	0.622 .612
0.4	0.053 .053	0.119 .118	0.248 .246	0.431 .429	0.602 .598
0.5	0.046 .046	0.103 .102	0.219 .218	0.397 .396	0.572 .569
0.6	0.038 .038	0.084 .084	0.184 .183	0.349 .348	0.530 .531
0.7	0.029 .029	0.063 .063	0.141 .140	0.284 .284	0.466 .466
0.8	0.019 .019	0.041 .041	0.091 .091	0.195 .196	0.363 .363

^a First value from ref. 2; second value from present analysis.

TABLE III. - OPTIMAL STEP PARAMETERS FOR RESULTING MAXIMUM DIMENSIONLESS LOAD
CAPACITY FOR VARIOUS DIMENSIONLESS BEARING NUMBERS

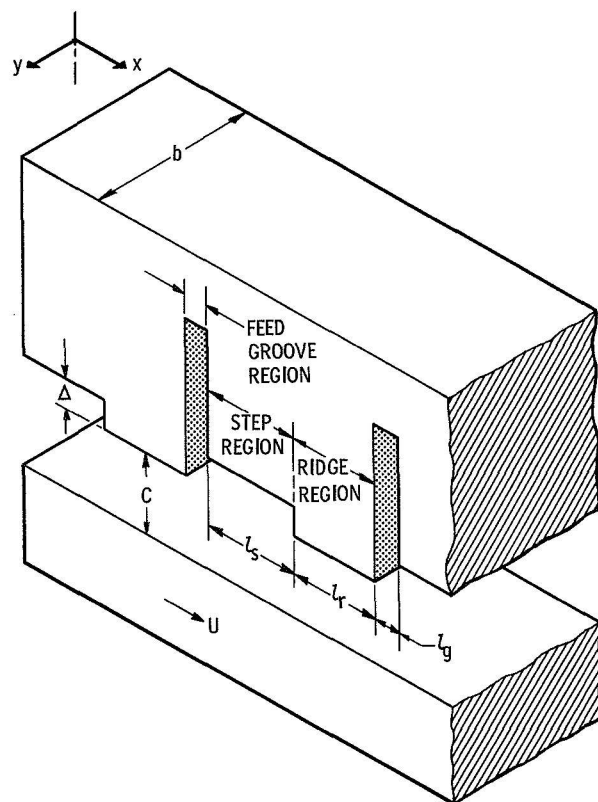
[Resulting dimensionless stiffness also given when optimum step parameters are used.]

Dimensionless bearing number, Λ	Optimal value for length-to-width ratio of pad, $(\lambda)_{opt}$	Optimal value of step location parameter, $(\psi)_{opt}$	Optimal value of film thickness ratio, $(k)_{opt}$	Maximum value of dimensionless load capacity of finite-width bearing, $(W)_{max}$	Dimensionless stiffness, K
10^{-5}	0.918	0.555	1.693	1.181×10^{-7}	2.362×10^{-7}
2.5×10^{-2}	.919	.554	1.693	2.955×10^{-4}	5.913×10^{-4}
5.0×10^{-2}	.920	.553	1.693	5.914×10^{-4}	1.184×10^{-3}
0.1	.922	.552	1.693	1.184×10^{-3}	2.376×10^{-3}
.2	.925	.549	1.693	2.376×10^{-3}	4.779×10^{-3}
.4	.933	.544	1.694	4.779×10^{-3}	9.669×10^{-3}
.8	.948	.533	1.696	9.670×10^{-3}	1.979×10^{-2}
1.6	.980	.511	1.703	1.980×10^{-2}	4.145×10^{-2}
3.2	1.043	.471	1.723	4.144×10^{-2}	9.006×10^{-2}
6.4	1.145	.412	1.790	8.903×10^{-2}	1.974×10^{-1}
12.8	1.294	.344	1.949	1.878×10^{-1}	3.854×10^{-1}
25.6	1.575	.271	2.240	3.651×10^{-1}	6.514×10^{-1}
51.2	2.037	.204	2.698	6.492×10^{-1}	1.003
102.4	2.710	.151	3.359	1.072	1.456
204.8	3.642	.110	4.270	1.674	2.034
409.6	4.901	.080	5.501	2.502	2.770

TABLE IV. - OPTIMAL STEP PARAMETERS FOR RESULTING MAXIMUM DIMENSIONLESS
STIFFNESS FOR VARIOUS DIMENSIONLESS BEARING NUMBERS

[Resulting dimensionless load capacity also given when optimal step parameters are used.]

Dimensionless bearing number, Λ	Optimal value for length-to-width ratio of pad, $(\lambda)_{opt}$	Optimal value of step location parameter, $(\psi)_{opt}$	Optimal value of film thickness ratio, $(k)_{opt}$	Maximum value of dimensionless stiffness, $(K)_{max}$	Dimensionless load capacity of finite-width bearing, W
10^{-5}	0.915	0.557	1.470	2.550×10^{-7}	1.115×10^{-7}
2.5×10^{-2}	.917	.555	1.471	6.334×10^{-4}	2.789×10^{-4}
5.0×10^{-2}	.919	.554	1.471	1.278×10^{-3}	5.582×10^{-4}
0.1	.922	.551	1.471	2.563×10^{-3}	1.118×10^{-3}
.2	.929	.546	1.472	5.152×10^{-3}	2.245×10^{-3}
.4	.943	.535	1.474	1.041×10^{-2}	4.520×10^{-3}
.8	.973	.514	1.479	2.125×10^{-2}	9.164×10^{-3}
1.6	1.035	.474	1.494	4.432×10^{-2}	1.882×10^{-2}
3.2	1.153	.408	1.537	9.561×10^{-2}	3.953×10^{-2}
6.4	1.353	.328	1.642	2.072×10^{-1}	8.518×10^{-2}
12.8	1.863	.232	1.849	4.070×10^{-1}	1.757×10^{-1}
25.6	2.952	.148	2.191	7.036×10^{-1}	3.289×10^{-1}
51.2	5.035	.088	2.687	1.105	5.624×10^{-1}
102.4	9.093	.051	3.368	1.627	8.931×10^{-1}
204.8	17.172	.030	4.274	2.293	1.338



(a)

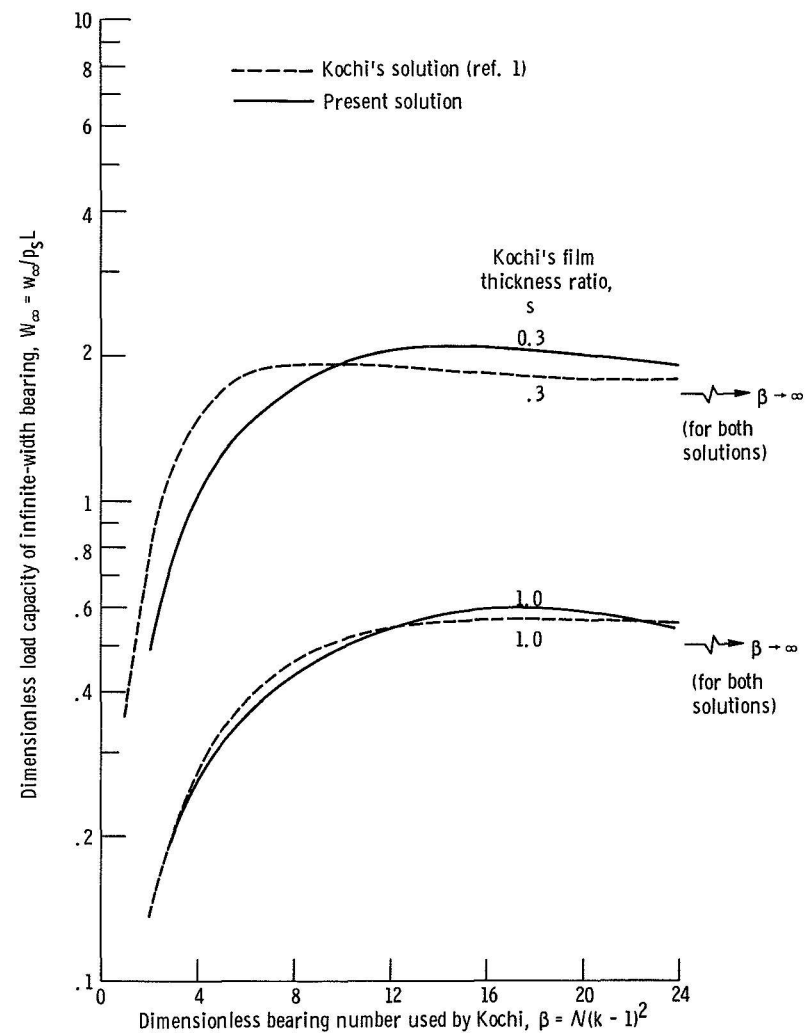


Figure 1. - Comparison of present linearized results with Kochi's exact results for infinite-width step slider bearing when step location parameter is 0.5.

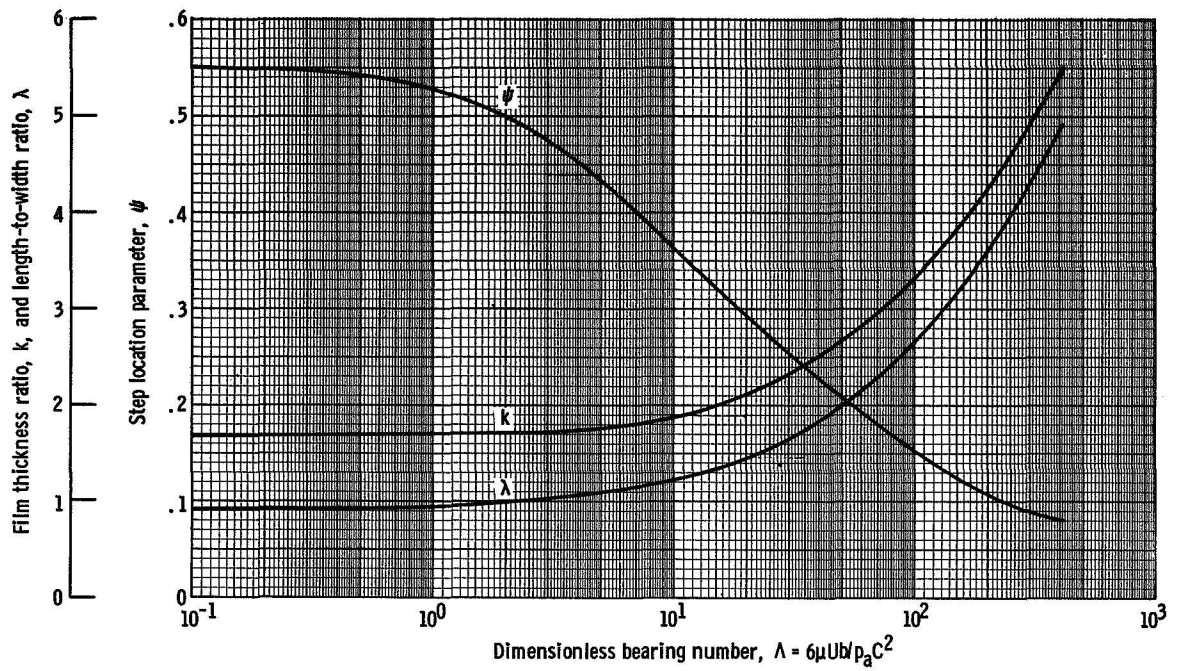


Figure 2 - Effect of dimensionless bearing number on optimum parameters for maximum dimensionless load.

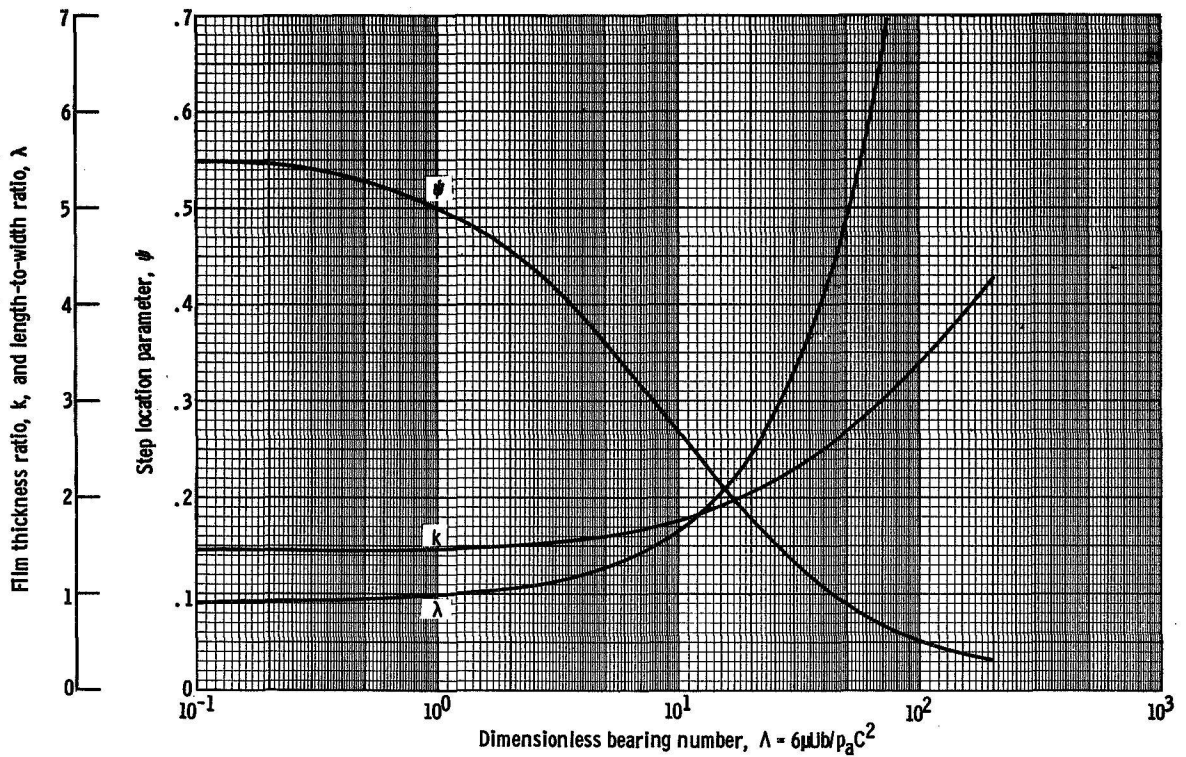
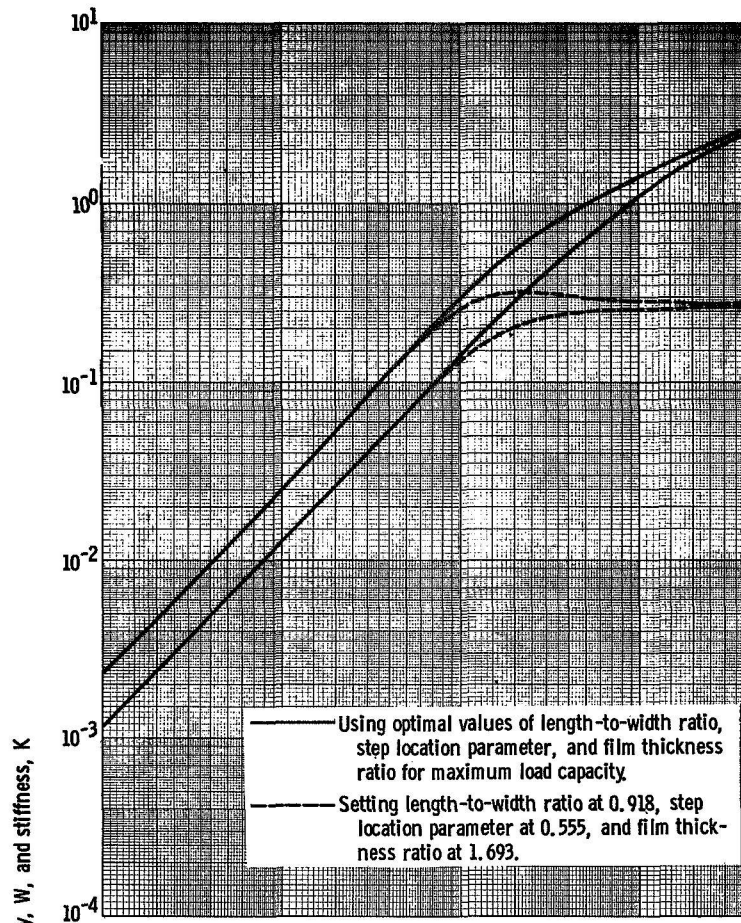
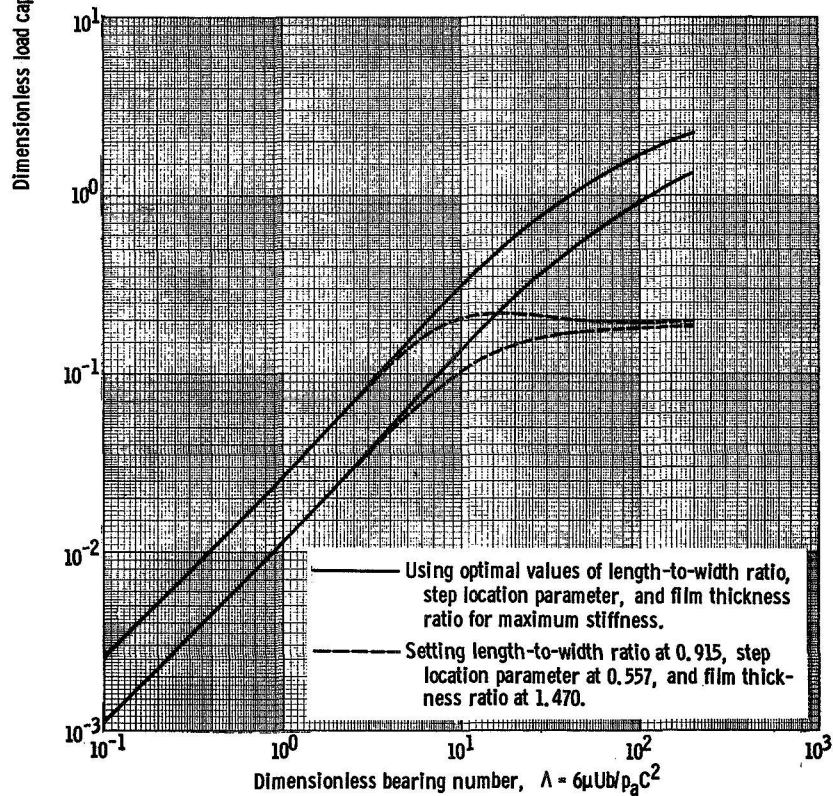


Figure 3 - Effect of dimensionless bearing number on optimal step parameters for maximum dimensionless stiffness.



(a) Maximum load capacity.



(b) Maximum stiffness.

Figure 4. - Effect of dimensionless bearing number on dimensionless load capacity and dimensionless stiffness.