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THERMAL CONTRACTION OF MERCURY

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16. Abstract

The cooling of the planet Mercury would produce a secular decrease of the moment of inertia and accelerate its rotation. It is shown that the initiation of the tidal evolution on Mercury was balanced with the effect of the thermal contraction during the earlier epoch of cooling in a period of about 48 million years. The theory also serves to explain the surface features of Mercury.

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CONTENTS

	Page
INTRODUCTION	1
COOLING	1
SECULAR DECREASE IN MOMENT OF INERTIA	2
AXIAL ROTATION	4
OROGENETIC SURFACE FEATURES	5
CONCLUSIONS	8
ACKNOWLEDGMENT	8
References	8
Appendix A–List of Symbols	9

THERMAL CONTRACTION OF MERCURY

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INTRODUCTION

Liu (1968) has suggested that the trapping of the rotation of the planet Mercury was originally effected by thermal expansion or contraction of its configuration during solidification. According to this hypothesis, Mercury was orignally at a very high temperature, and different parts have since cooled by different amounts. This suggestion was based on results of quantitative calculations. In this paper, the derivation of the results on the underlying physical problem is given. On the basis of the analysis made in the course of this study, it is shown that the thermal process on the planet Mercury during the earlier epoch of cooling is quite adequate to account for the origin of trapping. The theory also predicts the existence of folded and thrusted mountain systems on the surface of Mercury.

COOLING

The thermal history of Mercury, after solidification, can be determined from the laws of thermal conduction. The equation of conduction is

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(k\, r^2 \frac{\partial T}{\partial r} \right) + A\,(r,t), \tag{1}$$

where ρ is the density, c is the specific heat, T is the temperature, t is the time since solidification, r is the radial distance from the center of the planet, k is the thermal conductivity, and A is the rate of generation of heat per unit volume.

The general solution of Equation 1 may be separated into two parts. One part concerns a nonradioactive Mercury that cools by conduction from the actual initial state; the other concerns a Mercury initially at zero temperature which is warmed by radioactivity and loses heat by conduction to the surface. Numerous solutions can be given. For the present purposes it is convenient to make certain approximations which cannot introduce much error and which lead to an easily intelligible result. One may assume that (1) ρ, c, and k are the same at all depths; (2) cooling is appreciable only at depths which are a small fraction of the radius of Mercury; (3) A is independent of time and is appreciable only in a layer whose thickness is very small compared with the depth to which cooling has extended. With these simplifications, the temperature below the radioactive layer at a depth x is (Jeffreys, 1959)

$$T = \mu x + \left(S - \frac{Al^2}{2k}\right) \operatorname{erf}\left(\frac{x}{2ht^{1/2}}\right) + \frac{Al^2}{2k}, \tag{2}$$

where μ is the initial temperature gradient, S is the initial surface temperature, l is the depth of radioactive layer, $h^2 = k/\rho c$ is the thermometric conductivity, t is the time since solidification, and the error function is defined by

$$\operatorname{erf} \theta = \frac{2}{\pi^{1/2}} \int_0^\theta \exp(-y^2)\, dy.$$

For

$A = 7.3 \times 10^{-13}$ cal cm^{-3} sec^{-1}
$c = 0.2$ cal g^{-1} deg^{-1}
$k = 0.5 \times 10^{-2}$ cal cm^{-1} sec^{-1} deg^{-1}
$l = 1.2 \times 10^6$ cm
$S = 1400°$ K
$\rho = 5$ g cm^{-3}

it is found that

$$T = \mu x + L \operatorname{erf}\left(\frac{x}{2ht^{1/2}}\right) + Q \tag{3}$$

and the rate of cooling is

$$\frac{\partial T}{\partial t} = -\frac{L}{\pi^{1/2}} \frac{x}{2ht^{3/2}} \exp\left(-\frac{x^2}{4h^2 t}\right), \tag{4}$$

where

$h^2 = 5 \times 10^{-3}$ cm^2 sec^{-1}
$L = 1295°$ K
$Q = 105°$ K
$\mu = 4 \times 10^{-5}$ deg cm^{-1}.

SECULAR DECREASE IN MOMENT OF INERTIA

Throughout the process of cooling, a certain amount of mechanical adjustment must have occurred in the shape of Mercury. Consider a shell of internal radius r and external radius $r + dr$. The rise of temperature of the shell in a definite time is of course a very small negative value. The density of the shell will change at the same time from the initial value ρ to $\rho(1 - 3\alpha T)$ where α is the coefficient of linear thermal expansion. If the inner radius changes to $r(1 + \beta)$, the external radius becomes

$$r(1 + \beta) + dr\left[1 + \frac{\partial}{\partial r}(r\beta)\right].$$

Hence the mass of the shell after the change of temperature is

$$4\pi r^2 \rho \mathrm{dr}\left[1 + 2\beta + \frac{\partial}{\partial r}(r\beta) - 3aT\right].$$

But the mass is unaltered. Thus the equation of continuity is

$$2\beta + \frac{\partial}{\partial r}(r\beta) - 3aT = 0. \tag{5}$$

For a given cooling, this is the equation for determining the value of β subject to the boundary condition that β is zero at the center of Mercury. Therefore, for any shell, β is determined by the changes of temperature within that shell. If the shell simply expands independently of the interior, the radius will increase by aTr instead of by $r\beta$, so that the excess

$$r(\beta - aT) = \delta r$$

is due to stretching. The stretching is given by (Jeffreys, 1959)

$$-\frac{\mathrm{d}}{\mathrm{dr}}(\delta r^3) = r^3 \frac{\mathrm{d}(aT)}{\mathrm{dr}}.$$

The rate of stretching below the radioactive layer at a depth x is then

$$\frac{\partial\delta}{\partial t} = -a\frac{\partial T}{\partial t} + \frac{3}{r_0}\int_x^{\infty} a\frac{\partial T}{\partial t}\,\mathrm{dx}, \tag{6}$$

where r_0 is the radius of Mercury. Now, the value a can be expressed as

$$a = \epsilon_1 + \epsilon_2 T,$$

where ϵ_1 and ϵ_2 are constants. From Equations 3, 4, 5, and 6, one obtains the amount of stretching at the surface

$$\delta = -\frac{6Lh}{\pi r_0}\left\{\epsilon_1 (\pi t)^{1/2} + \epsilon_2 \left[(Q + 2^{1/2}L)(\pi t)^{1/2} + \mu h\pi t\right]\right\}. \tag{7}$$

For

$$\epsilon_1 = 7 \times 10^{-6}\ \mathrm{deg}^{-1},\ \epsilon_2 = 2.4 \times 10^{-8}\ \mathrm{deg}^{-2},\ \text{and } t = 1.5 \times 10^{15}\ \mathrm{sec},$$

Equation 7 gives

$$\delta = -2.8 \times 10^{-3}.$$

From this the increase of the radius of Mercury by thermal expansion during the earlier stage of cooling of the shell with a depth of 300 km, is at once found to be about −7 km. The surface of Mercury, by crumpling, has therefore been diminished by about 0.2×10^6 km^2. The rate of increase of the radius is, then, about -4.7×10^{-10} cm sec^{-1}. Therefore the average rate of change of the moment of inertia about the axis of rotation during the cooling period of about 48 million years is approximately

$$\frac{dC_{(t)}}{dt} = -3.1 \times 10^{-10} M_m r_0 \text{ cm}^2 \text{ g sec}^{-1}, \tag{8}$$

where M_m is the mass of Mercury.

AXIAL ROTATION

During the earlier epoch of cooling, Mercury would rotate about its polar axis with a time-dependent inertial tensor. The rotation of Mercury is described by

$$\frac{d}{dt}\left[C_{(t)} \frac{d(f+\phi)}{dt}\right] = -N \tag{9}$$

where f is the true anomaly, ϕ is the angle of orientation of Mercury relative to the Sun, and N is the couple due to solar tides, Equation 9 can be rewritten in the following form

$$\frac{d^2(f+\phi)}{dt^2} = -\frac{N}{C_{(t)}} - \frac{H}{C_{(t)}}, \tag{10}$$

where

$$N = \frac{18}{5} k_2 \pi G \rho \frac{M_s{}^2}{M_m{}^2} \cdot \frac{r_0{}^6 (1 + e \cos f)^6}{a^6 (1 - e^2)^6} \cdot C_{(t)} \cdot \sin 2\epsilon$$

and

$$H = \frac{d(f+\phi)}{dt} \cdot \frac{dC_{(t)}}{dt}.$$

In Equation 10, a is the major semiaxis, e is the orbital eccentricity, G is the gravitational constant, k_2 is the Love number, ϵ is the phase lag of the tides, and M_s is the mass of the Sun.

For $e = 0.2$; $G = 6.7 \times 10^{-8}$ dyne cm^2 g^{-2}; $k_2 = 0.02$; $2\epsilon = 0.005$; $\rho = 5.0$ g cm^{-3}; $M_s/M_m = 6.0 \times 10^6$; $r_0/a = 4.1 \times 10^{-5}$; and $r_0 = 2.42 \times 10^8$ cm, the average value of $N/C_{(t)}$ over a period of revolution is

$$\frac{N}{C_{(t)}} = O\,(10^{-23}) \text{ rad sec}^{-2}.$$

The calculation also includes the reversal of the phase lag during perihelion passage* (Peale and Gold, 1965; Liu, 1966).

At the 3:2 resonance condition, $d(f+\phi)/dt = (3/2)n = 1.8 \times 10^{-6}$ rad sec^{-1}, where n is the mean orbital motion. From Equation 8 one obtains

*Liu, H-S., unpublished results, 1966.

$$\frac{H}{C_{(t)}} = -\,O\,(10^{-23})\text{ rad sec}^{-2}.$$

Therefore, it is seen that the initiation of the tidal evolution was completely balanced by the effect of the thermal contraction during the epoch of cooling lasting about 48 million years.

In applying Darwin's hypothesis of the formation of the Earth, it should be noted that in the initial stage the thermal contraction of Mercury may be more effective in determining its rotation than the solar tides because the loss of heat at that stage was exceptionally great. Therefore the estimate of $H/C_{(t)}$ may be regarded as a lower limit. Since the calculation for this quantity is of the same order of magnitude as that for $N/C_{(t)}$, it seems reasonable to conclude that the influence of thermal contraction provided the conditions for the rotation of Mercury to be trapped into the 3:2 resonance state.

OROGENETIC SURFACE FEATURES

By performing the integrations on the right-hand side of Equation 6, one finds

$$\begin{aligned}\frac{\partial\delta}{\partial t} &= (\epsilon_1 + \epsilon_2\, Q + 2\epsilon_2\, \mu h t^{1/2}\, q + \epsilon_2\, L \operatorname{erf} q)\,\frac{Lq}{\pi^{1/2} t}\exp(-\,q^2) \\ &- \frac{3Lh}{(\pi t)^{1/2} r_0}\Bigg\{(\epsilon_1 + \epsilon_2 Q)\exp(-q^2) \\ &+ \epsilon_2 \mu h t^{1/2}\left[2q\exp(-\,q^2) + \pi^{1/2}(1-\operatorname{erf} q)\right] \\ &+ \epsilon_2\, L\left[\exp(-q^2)\operatorname{erf} q + 2^{1/2}(1-\operatorname{erf} 2^{1/2} q)\right]\Bigg\},\end{aligned} \tag{11}$$

where

$$q = \frac{x}{2ht^{1/2}}.$$

The value of $\partial\delta/\partial t$ is negative when q is zero, but when q is large enough, $\partial\delta/\partial t$ is positive. For one value of q, $\partial\delta/\partial t$ is zero, and the material is being neither crumpled nor stretched. The level with this value of q is defined as the level of no strain. For $\partial\delta/\partial t = 0$, Equation 11 gives $q = 0.1$. Hence, the depth of the level of no strain is about 40 km below the surface at the present time.

The internal states of strain, which were developed by thermal contraction, are illustrated in Figure 1. The mass from the center of Mercury to within about 300 km below its surface has not been affected by any appreciable cooling. Within the shell from 300 to 40 km below the surface, cooling by conduction is taking place, and this shell is contracting about the unchanging interior. Hence, it is in a state of internal tension. Above the level of no strain, the layers have already cooled and therefore are under a state of internal compression with a horizontal crushing stress. Actually, in a solid material with a finite strength, differences of cooling would produce stress–stresses which would increase until they reached the strength of the material and then be mainly relieved by fracture. The power freed by the fracture of the contracting solid shell is considered here to be the cause of Mercurian orogenesis.

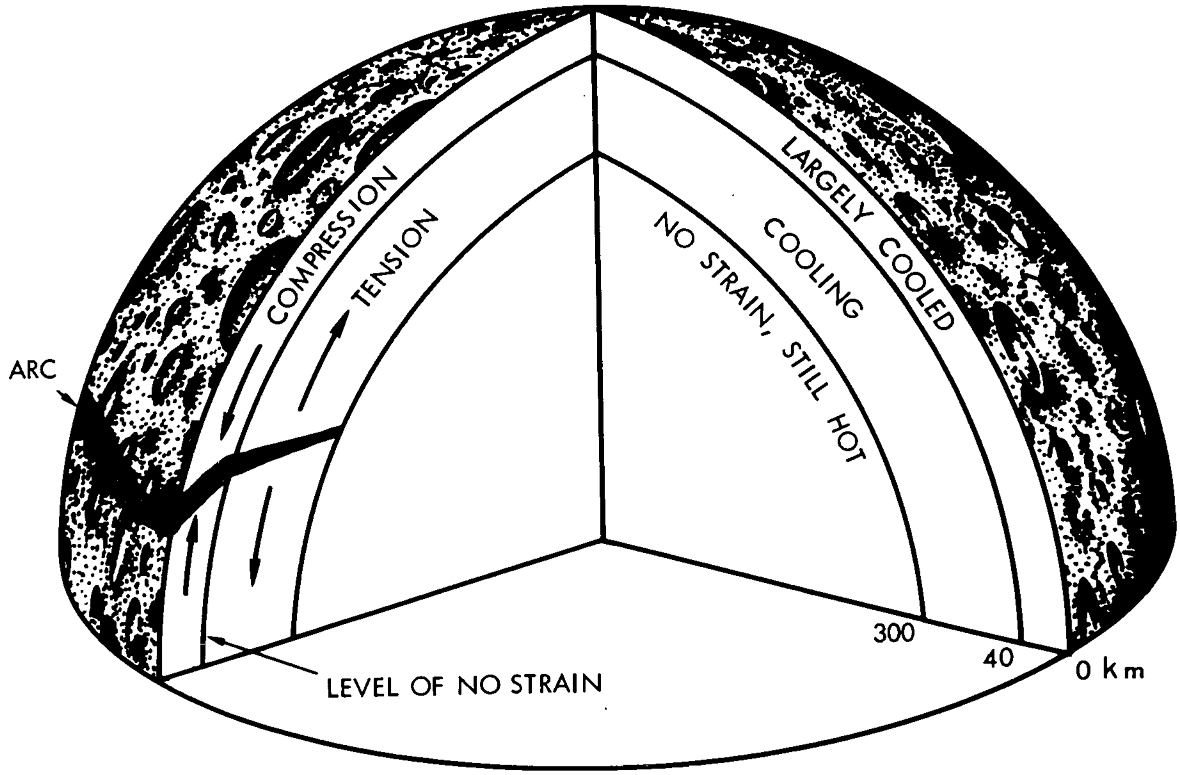

Figure 1 – States of thermal strain of Mercury.

Since the solid contracting shell has to fit the unchanging interior, it is necessary to determine whether a readjustment that will reduce all great circles of the sphere by different amounts is possible. This can be done. For simplicity, suppose that there is crumpling across the meridian of the thermal bulge and also across the equator from 90° E through 180° E to 90° W. Every great circle of the sphere would intersect both of these arcs. Because of the solar heating after the rotation becomes commensurable with the mean motion (Liu, 1968), the shortenings of different great circles would vary at different positions. Such a distribution is remarkably like that shown in Schiaparelli's map of Mercury (Figure 2). (Schiaparelli, 1889).

What is likely to happen when a fracture is formed at the surface by contraction? The stress differences arise because the outer shell is too large to fit the interior; they can be relieved only by a displacement that reduces the outer surface. A vertical fracture or a horizontal split will not do. There must be an oblique fracture with sliding along the thrust plane. Relief by thrusting would be at an angle of somewhat less than 45° to the horizontal because the outer shell is in a state of compression (Figure 3). In this way the crust of Mercury to a depth of about 40 km would be overthrusted, leaving the upper edge overhanging by a similar height. Therefore the surface features of Mercury (Figure 2) are probably folded and thrusted mountain systems that were formed as a natural consequence of thermal contraction.

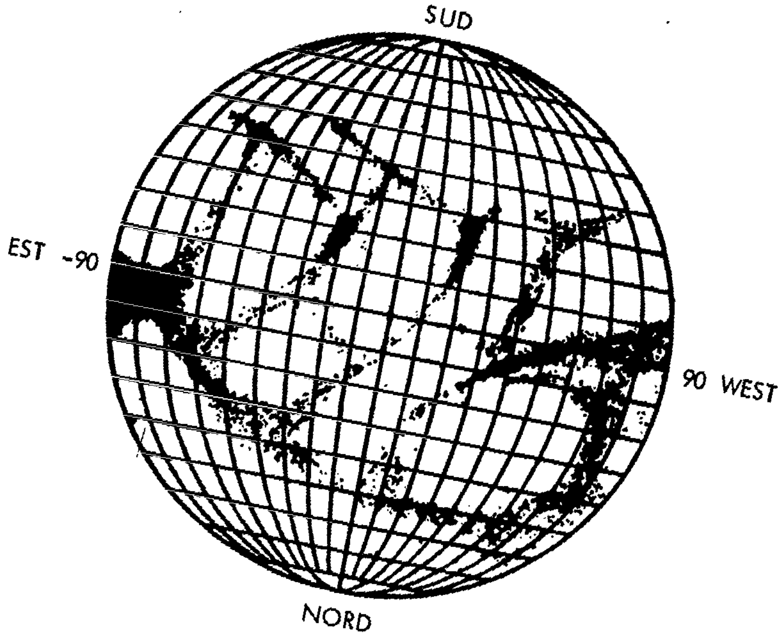

Figure 2 – Schiaparelli's map of Mercury.

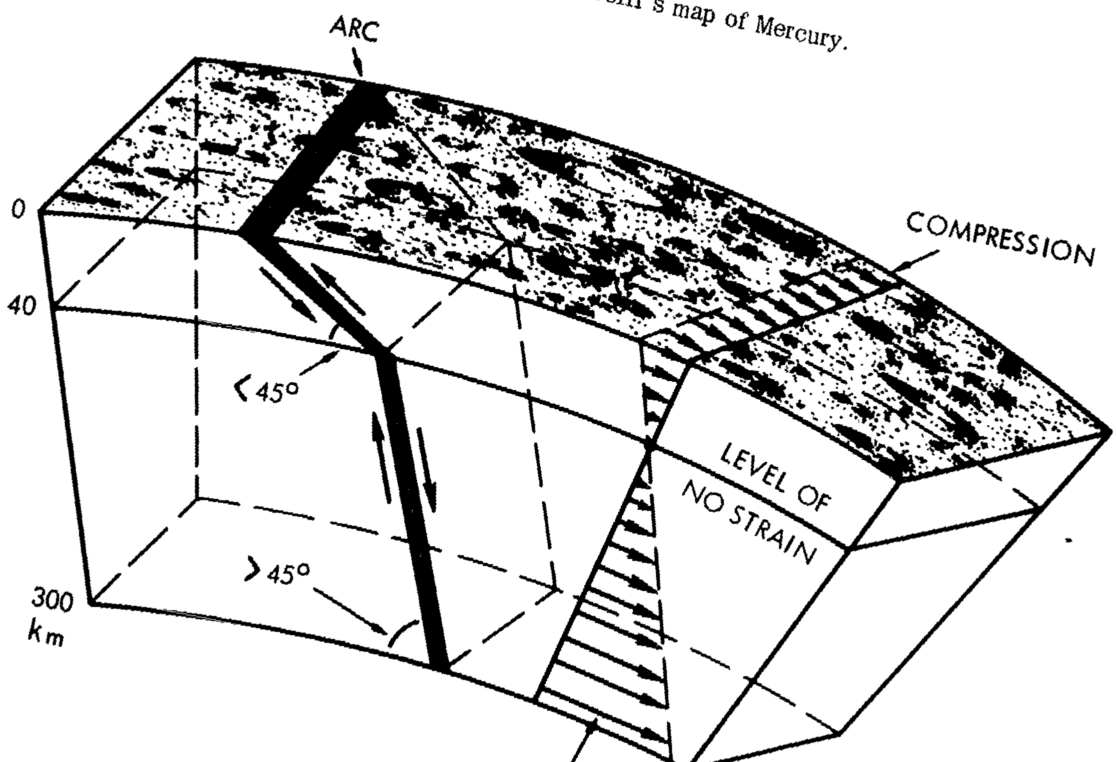

Figure 3 – Thermal stress diagram of Mercury.

CONCLUSIONS

The version of the thermal contraction hypothesis presented in this paper appears to offer a sound physical theory with which predictions for Mercury can be made and checked.

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Appendix A

LIST OF SYMBOLS

a = major semiaxis
A = rate of generation of heat per unit volume
c = specific heat of body
$C_{(t)}$ = moment of inertia about axis of rotation
e = orbital eccentricity
f = true anomaly
G = gravitational constant
h = thermometric conductivity, $\frac{k}{\rho c}$
k = thermal conductivity
k_2 = Love number
l = depth of radioactive layer
L = temperature difference between surfaces of radioactive layers
M_m = mass of Mercury
M_s = mass of Sun
n = mean orbital motion
N = couple due to solar tides
Q = temperature change across radioactive layer
r = internal radial distance of a shell from center of plane at which temperature is determined
r_0 = radius of planet Mercury
S = initial surface temperature
t = time since solidification
T = temperature of body
x = depth below surface of radioactive layer
y = variable of error function
α = coefficient of linear thermal expansion
δ = amount of stretching as a function of time and the depth of the planet
ϵ = phase lag of the tides
ϵ_1 = constant (value of α at $T = 0$)
ϵ_2 = constant (change with T in value of α)
θ = limit of error function
μ = initial temperature gradient
ρ = density of mass
ϕ = angle of orientation of Mercury relative to the Sun

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