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TABLE OF VALUES OF INTERGALS FOR THE LONGITUDINAL AND LATERAL VON KARMAN TURBULENCE SPECTRA

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June 15, 1970

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16. Abstract

The longitudinal and lateral von Karman turbulence spectra are integrated over the domain $L\Omega_c < L\Omega < \infty$, where L is the integral scale of turbulence, Ω is a wave number and Ω_c is a cut-off wave number which defines the lower bound of integration. The integration procedure consists of expanding the spectra in Maclaurin series and integrating the resulting series term by term to yield tables of σ_u/σ and σ_w/σ as functions of $L\Omega_c$, where σ_u and σ_w are the contributions to the longitudinal and lateral deviations of turbulence from the Fourier components in the wave number domain $\Omega_c \leq \Omega \leq \infty$. The total longitudinal and lateral standard deviations are both equal to σ which is obtained by setting $\Omega_c = 0$. The tables provide values of σ_u/σ and σ_w/σ for $10^{-3} \leq L\Omega_c \leq 10^6$. A sample calculation is provided to aid the user.

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DEFINITION OF SYMBOLS

Symbol	Definition
a	a constant, $\Gamma(1/3)/\Gamma(1/2)\ \Gamma(5/6)$
k	summation index
K	number of terms required in summations
L	integral scale of turbulence
x	dimensionless wave number, $L\Omega$
x_c	cut-off dimensionless wave number, $L\Omega_c$
$\Gamma(\ )$	gamma function
σ	standard deviation of turbulence
σ_u, σ_w	contributions to the longitudinal (u) and lateral (w) standard deviation of turbulence from the Fourier components in the interval
Φ_u, Φ_w	longitudinal (u) and lateral (w) spectra of turbulence; the units being velocity squared per radian per unit length
Ω	wave number; the units being radians per unit length
Ω_c	cut-off wave number

TECHNICAL MEMORANDUM X-64529

TABLE OF VALUES OF INTEGRALS FOR THE LONGITUDINAL AND LATERAL VON KARMAN TURBULENCE SPECTRA

SUMMARY

The longitudinal and lateral von Karman turbulence spectra are integrated over the domain $L\Omega_c < L\Omega < \infty$, where L is the integral scale of turbulence, Ω is a wave number and Ω_c is a cut-off wave number which defines the lower bound of integration. The integration procedure consists of expanding the spectra in Maclaurin series and integrating the resulting series term by term to yield tables of σ_u/σ and σ_w/σ as functions of $L\Omega_c$, where σ_u and σ_w are the contributions to the longitudinal and lateral deviations of turbulence from the Fourier components in the wave number domain $\Omega_c \leq \Omega \leq \infty$. The total longitudinal and lateral standard deviations are both equal to σ which is obtained by setting $\Omega_c = 0$. The tables provide values of σ_u/σ and σ_w/σ for $10^{-3} \leq L\Omega_c \leq 10^6$. A sample calculation is provided to aid the user.

I. INTRODUCTION

Various investigators have found that the von Karman spectrum of turbulence appears to be an adequate representation of clear air turbulence [1].

The longitudinal and lateral spectra, $\Phi_u(\Omega,L)$ and $\Phi_w(\Omega,L)$ at wave number Ω, are

$$\Phi_u(\Omega,L) = \sigma^2 \frac{2L}{\pi} \frac{1}{(1 + (aL\Omega)^2)^{5/6}} \tag{1}$$

and

$$\Phi_w(\Omega,L) = \sigma^2 \frac{L}{\pi} \frac{1 + \frac{8}{3}(aL\Omega)^2}{(1 + (aL\Omega)^2)^{11/6}} \tag{2}$$

where L is the integral scale of turbulence and σ^2 is the variance of turbulence. Integration of (1) and (2) over the domain $0 \leq \Omega < \infty$ will yield σ^2. The constant a is defined as*

$$a = \frac{\Gamma(1/3)}{\Gamma(1/2)\ \Gamma(5/6)} \tag{3}$$

The contributions to the longitudinal and lateral variances σ_u^2 and σ_w^2 for $\Omega > \Omega_c$ are given by

$$\sigma_u^2 = \int_{\Omega_c}^{\infty} \Phi_u(\Omega, L)\ d\Omega \tag{4}$$

and

$$\sigma_w^2 = \int_{\Omega_c}^{\infty} \Phi_w(\Omega, L)\ d\Omega, \tag{5}$$

where Ω_c is a cut-off wave number. Often the meteorologist and engineer require values of these integrals for various values of Ω_c. The purpose of this report is to provide tabular values of σ_u and σ_w for various values of Ω_c.

I would like to acknowledge the help of Dr. George H. Fichtl of the Aerospace Environment Division at the George C. Marshall Space Flight Center in formulating the idea for this report and thank him for his numerous suggestions and our frequent discussions. I would also like to thank Mrs. Ella Mae McAllister of the Marshall Center's Computation Laboratory for her help in "debugging" the computer program.

*The established value of the parameter a is 1.339 while the value of a to the nine-decimal place accuracy used in this table is 1.338985279.

II. MATHEMATICAL CONSIDERATIONS

It is useful to define the new variable

$$x = \Omega L. \tag{6}$$

This transformation permits the expression of (4) and (5) as

$$\frac{\sigma_u^2}{\sigma^2} = \frac{2}{\pi} \int_{x_c}^{\infty} \frac{dx}{(1 + (ax)^2)^{5/6}} \tag{7}$$

and

$$\frac{\sigma_w^2}{\sigma^2} = \frac{1}{\pi} \int_{x_c}^{\infty} \frac{1 + \frac{8}{3}(ax)^2}{(1 + (ax)^2)^{11/6}} dx \tag{8}$$

where

$$x_c = \Omega_c L. \tag{9}$$

It is not possible to evaluate the integrals (7) and (8) in closed form. Accordingly, these integrals were evaluated by series expansion.

When $ax_c > 1$ the following expansions are valid:

$$\frac{\sigma_u^2}{\sigma^2} = \frac{3}{\pi a} \sum_{k=0}^{\infty} (-1)^k \frac{\Gamma(\frac{5}{6} + k)}{\Gamma(\frac{5}{6})} \frac{(ax_c)^{1+2k}}{(2k + 1)k!} \tag{10}$$

and

$$\frac{\sigma_w^2}{\sigma^2} = \frac{4}{\pi a} \sum_{k=o}^{\infty} (-1)^k \frac{\Gamma(\frac{5}{6} + k)}{\Gamma(\frac{5}{6})} \frac{(ax_c)^{1+2k}}{(2k + 1)k!} - \frac{5}{2\pi a} \sum_{k=0}^{\infty} (-1)^k \frac{\Gamma(\frac{11}{6} + k)}{\Gamma(\frac{11}{6})} \frac{(ax_c)^{1+2k}}{(2k + 1)k!}. \tag{11}$$

The series were obtained by expanding the denominators of the integrals in (7) and (8) with Maclaurin series in terms of the variable $(ax)^{-2}$ and then integrating term by term.

If $ax_c < 1$, we have the expansions

$$\frac{\sigma_u^2}{\sigma^2} = 1 - \frac{2}{\pi a} \sum_{k=0}^{\infty} (-1)^k \frac{\Gamma(\frac{5}{6} + k)}{\Gamma(\frac{5}{6})} \frac{(ax_c)^{1+2k}}{(2k + 1)k!} \tag{12}$$

and

$$\frac{\sigma_w^2}{\sigma^2} = 1 + \frac{5}{3\pi a} \sum_{k=0}^{\infty} (-1)^k \frac{\Gamma(\frac{11}{6} + k)}{\Gamma(\frac{11}{6})} \frac{(ax_c)^{1+2k}}{(2k + 1)k!}$$

$$+ \frac{8}{3\pi a} \sum_{k=0}^{\infty} (-1)^k \frac{\Gamma(\frac{5}{6} + k)}{\Gamma(\frac{5}{6})} \frac{(ax_c)^{1+2k}}{(2k + 1)k!} . \tag{13}$$

In this case the series were obtained by expanding the denominators of the integrands in (7) and (8) with Maclaurin series in terms of the variable $(ax)^2$ and again integrating term by term.

The series given by (10) and (11) are absolutely convergent for $ax_c > 1$, while the series given by (12) and (13) are absolutely convergent for $ax_c < 1$. In both cases the series are divergent at $ax_c = 1$.

III. THE TABLES

The tables that follow give numerical values of the normalized standard deviations σ_u/σ and σ_w/σ as functions of $L\Omega_c$. In addition, the number of terms K that were used in the truncated version of (10) through (13) to give eight place accuracies in the normalized standard deviations are given in the tables. In the neighborhood of $x_c = a^{-1} \simeq 0.747$ a relatively large number of terms are needed to produce eight place accuracies in σ_u/σ and σ_w/σ. For $L\Omega_c \geq 2.5 \times 10^4$, only one term is required, while for $L\Omega_c \leq 4.9 \times 10^{-3}$, three terms are required.

The quantities σ_u/σ, σ_w/σ, and $L\Omega_c$ are dimensionless with the restriction that Ω_c has the units of radians per unit length. Thus, the user can enter the tables with either English or metric units. Plots of σ_u/σ and σ_w/σ as functions of $L\Omega_c$ are given in Figure 1.

IV. SAMPLE CALCULATION

Let us suppose that we desire the contributions to the longitudinal and lateral variances of turbulence from the Fourier components with wavelengths λ in the interval 0.01π km $\leq \lambda \leq \pi$ km in a turbulent flow with L = 1 km and $\sigma = 0.5$ m sec^{-1}. Now $\Omega = 2\pi/\lambda$, so that $2 \leq L\Omega \leq 200$. The normalized variances in this interval are

$$\frac{\sigma_u^2}{\sigma^2} = \int_2^{200} \frac{\Phi_u(x)}{L\sigma^2}\,dx = \int_2^{\infty} \frac{\Phi_u(x)}{L\sigma^2}\,dx - \int_{200}^{\infty} \frac{\Phi_u(x)dx}{L\sigma^2}$$

$$\frac{\sigma_w^2}{\sigma} = \int_2^{200} \frac{\Phi_w(x)}{L\sigma^2}\,dx = \int_2^{\infty} \frac{\Phi_w(x)dx}{L\sigma^2} - \int_{200}^{\infty} \frac{\Phi_w(x)dx}{L\sigma^2}\,.$$

The square roots of integrals on the right-hand side of these equations are given in the tables (see eqs. (7) and (8)), so that the normalized variances are

$$\frac{\sigma_u^2}{\sigma^2} = (0.599825366)^2 - (0.131016074)^2$$

and

$$\frac{\sigma_w^2}{\sigma^2} = (0.685825187)^2 - (0.151284166)^2,$$

or

$$\frac{\sigma_u^2}{\sigma^2} = 0.342625258 \quad \text{and} \quad \frac{\sigma_w^2}{\sigma^2} = 0.447469288.$$

Multiplication of these quantities by $\sigma^2 = 0.25$ m sec^{-1} yields the desired contributions to the longitudinal and lateral variances, namely,

$$\sigma_u^2 = 0.085656315$$

$$\sigma_w^2 = 0.111867322.$$

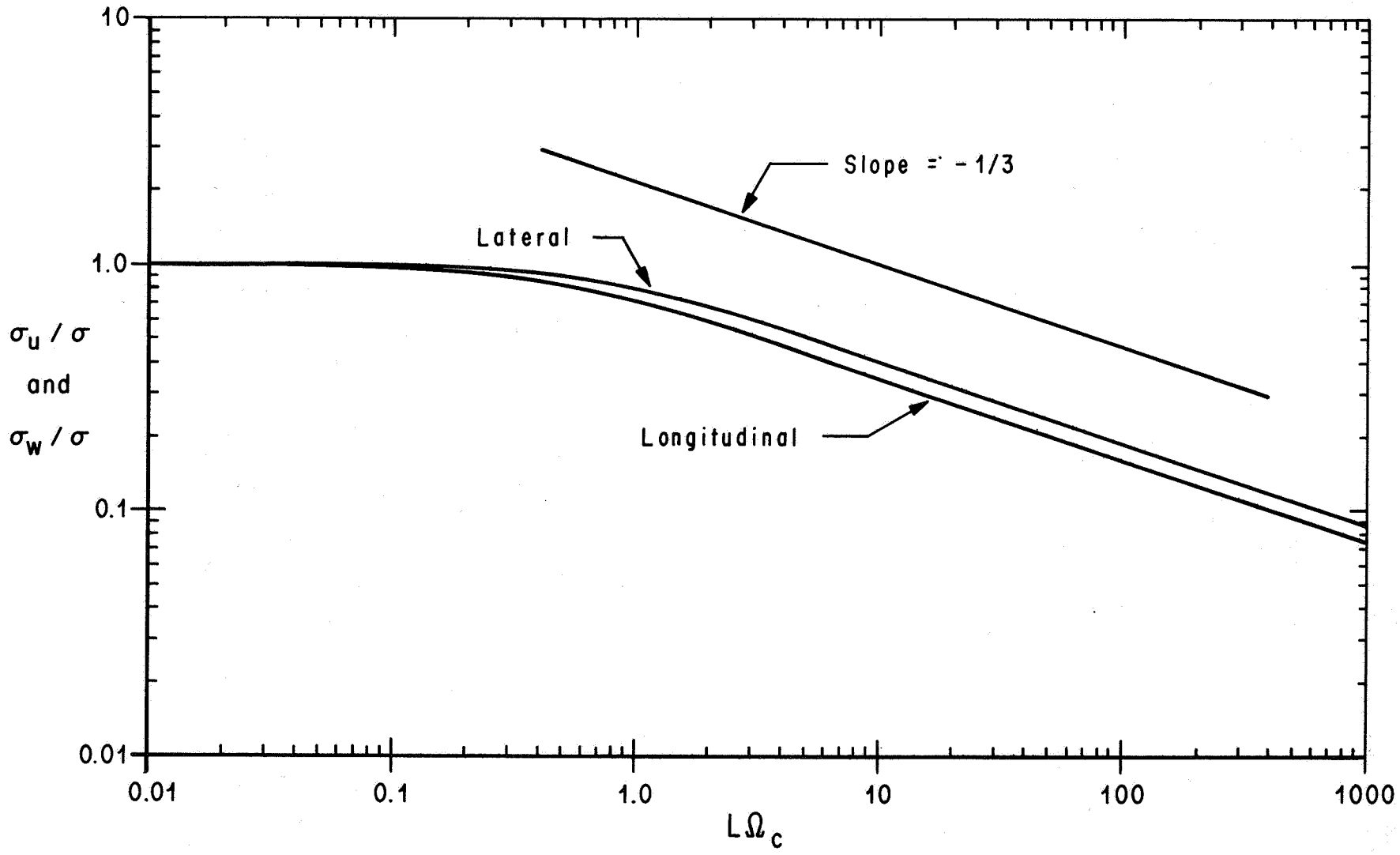

Figure 1. The Dimensionless Longitudinal and Lateral Standard Deviations as Functions of $L\Omega_c$

$L\Omega_c$	σ_w/σ	σ_u/σ	K
1.00000000E-03	9.99840832E-01	9.99681640E-01	2
1.10000000E-03	9.99824914E-01	9.99649798E-01	2
1.20000000E-03	9.99808996E-01	9.99617955E-01	2
1.30000000E-03	9.99793077E-01	9.99586112E-01	2
1.40000000E-03	9.99777158E-01	9.99554267E-01	2
1.50000000E-03	9.99761239E-01	9.99522422E-01	2
1.60000000E-03	9.99745319E-01	9.99490575E-01	2
1.70000000E-03	9.99729400E-01	9.99458727E-01	2
1.80000000E-03	9.99713480E-01	9.99426879E-01	2
1.90000000E-03	9.99697559E-01	9.99395029E-01	2
2.00000000E-03	9.99681639E-01	9.99363179E-01	2
2.10000000E-03	9.99665718E-01	9.99331327E-01	2
2.20000000E-03	9.99649797E-01	9.99299475E-01	2
2.30000000E-03	9.99633876E-01	9.99267621E-01	2
2.40000000E-03	9.99617954E-01	9.99235766E-01	2
2.50000000E-03	9.99602032E-01	9.99203911E-01	2
2.60000000E-03	9.99586110E-01	9.99172054E-01	2
2.70000000E-03	9.99570188E-01	9.99140197E-01	2
2.80000000E-03	9.99554265E-01	9.99108338E-01	2
2.90000000E-03	9.99538342E-01	9.99076479E-01	2
3.00000000E-03	9.99522419E-01	9.99044618E-01	2
3.10000000E-03	9.99506496E-01	9.99012757E-01	2
3.20000000E-03	9.99490572E-01	9.98980894E-01	2
3.30000000E-03	9.99474648E-01	9.98949031E-01	2
3.40000000E-03	9.99458724E-01	9.98917166E-01	2
3.50000000E-03	9.99442799E-01	9.98885301E-01	2
3.60000000E-03	9.99426874E-01	9.98853434E-01	2
3.70000000E-03	9.99410949E-01	9.98821567E-01	2
3.80000000E-03	9.99395024E-01	9.98789699E-01	2
3.90000000E-03	9.99379098E-01	9.98757829E-01	2
4.00000000E-03	9.99363172E-01	9.98725959E-01	2
4.10000000E-03	9.99347246E-01	9.98694088E-01	2
4.20000000E-03	9.99331320E-01	9.98662215E-01	2
4.30000000E-03	9.99315393E-01	9.98630342E-01	2
4.40000000E-03	9.99299466E-01	9.98598468E-01	2
4.50000000E-03	9.99283539E-01	9.98566593E-01	2
4.60000000E-03	9.99267611E-01	9.98534716E-01	2
4.70000000E-03	9.99251684E-01	9.98502839E-01	2
4.80000000E-03	9.99235755E-01	9.98470961E-01	2
4.90000000E-03	9.99219827E-01	9.98439082E-01	2
5.00000000E-03	9.99203898E-01	9.98407202E-01	3
5.10000000E-03	9.99187970E-01	9.98375321E-01	3
5.20000000E-03	9.99172040E-01	9.98343439E-01	3
5.30000000E-03	9.99156111E-01	9.98311556E-01	3
5.40000000E-03	9.99140181E-01	9.98279672E-01	3
5.50000000E-03	9.99124251E-01	9.98247787E-01	3

$L\Omega_c$	σ_w/σ	σ_u/σ	K
5.60000000E-03	9.99108321E-01	9.98215901E-01	3
5.70000000E-03	9.99092390E-01	9.98184014E-01	3
5.80000000E-03	9.99076459E-01	9.98152126E-01	3
5.90000000E-03	9.99060528E-01	9.98120237E-01	3
6.00000000E-03	9.99044597E-01	9.98088348E-01	3
6.10000000E-03	9.99028665E-01	9.98056457E-01	3
6.20000000E-03	9.99012733E-01	9.98024565E-01	3
6.30000000E-03	9.98996801E-01	9.97992673E-01	3
6.40000000E-03	9.98980868E-01	9.97960779E-01	3
6.50000000E-03	9.98964935E-01	9.97928884E-01	3
6.60000000E-03	9.98949002E-01	9.97896989E-01	3
6.70000000E-03	9.98933069E-01	9.97865092E-01	3
6.80000000E-03	9.98917135E-01	9.97833195E-01	3
6.90000000E-03	9.98901201E-01	9.97801297E-01	3
7.00000000E-03	9.98885267E-01	9.97769397E-01	3
7.10000000E-03	9.98869332E-01	9.97737497E-01	3
7.20000000E-03	9.98853397E-01	9.97705596E-01	3
7.30000000E-03	9.98837462E-01	9.97673694E-01	3
7.40000000E-03	9.98821527E-01	9.97641790E-01	3
7.50000000E-03	9.98805591E-01	9.97609886E-01	3
7.60000000E-03	9.98789655E-01	9.97577981E-01	3
7.70000000E-03	9.98773719E-01	9.97546075E-01	3
7.80000000E-03	9.98757782E-01	9.97514168E-01	3
7.90000000E-03	9.98741845E-01	9.97482261E-01	3
8.00000000E-03	9.98725908E-01	9.97450352E-01	3
8.10000000E-03	9.98709971E-01	9.97418442E-01	3
8.20000000E-03	9.98694033E-01	9.97386531E-01	3
8.30000000E-03	9.98678095E-01	9.97354620E-01	3
8.40000000E-03	9.98662157E-01	9.97322707E-01	3
8.50000000E-03	9.98646218E-01	9.97290793E-01	3
8.60000000E-03	9.98630279E-01	9.97258879E-01	3
8.70000000E-03	9.98614340E-01	9.97226963E-01	3
8.80000000E-03	9.98598400E-01	9.97195047E-01	3
8.90000000E-03	9.98582460E-01	9.97163130E-01	3
9.00000000E-03	9.98566520E-01	9.97131212E-01	3
9.10000000E-03	9.98550580E-01	9.97099292E-01	3
9.20000000E-03	9.98534639E-01	9.97067372E-01	3
9.30000000E-03	9.98518698E-01	9.97035451E-01	3
9.40000000E-03	9.98502757E-01	9.97003529E-01	3
9.50000000E-03	9.98486815E-01	9.96971606E-01	3
9.60000000E-03	9.98470873E-01	9.96939683E-01	3
9.70000000E-03	9.98454931E-01	9.96907758E-01	3
9.80000000E-03	9.98438989E-01	9.96875832E-01	3
9.90000000E-03	9.98423046E-01	9.96843905E-01	3
1.00000000E-02	9.98407103E-01	9.96811978E-01	3

$L\Omega_c$	σ_w/σ	σ_u/σ	K
1.00000000E-02	9.98407103E-01	9.96811978E-01	3
1.10000000E-02	9.98247655E-01	9.96492651E-01	3
1.20000000E-02	9.98088176E-01	9.96173233E-01	3
1.30000000E-02	9.97928666E-01	9.95853724E-01	3
1.40000000E-02	9.97769125E-01	9.95534124E-01	3
1.50000000E-02	9.97609551E-01	9.95214436E-01	3
1.60000000E-02	9.97449945E-01	9.94894659E-01	3
1.70000000E-02	9.97290306E-01	9.94574794E-01	3
1.80000000E-02	9.97130633E-01	9.94254843E-01	3
1.90000000E-02	9.96970925E-01	9.93934806E-01	3
2.00000000E-02	9.96811184E-01	9.93614684E-01	3
2.10000000E-02	9.96651407E-01	9.93294478E-01	3
2.20000000E-02	9.96491594E-01	9.92974189E-01	3
2.30000000E-02	9.96331745E-01	9.92653817E-01	3
2.40000000E-02	9.96171860E-01	9.92333364E-01	3
2.50000000E-02	9.96011938E-01	9.92012831E-01	3
2.60000000E-02	9.95851978E-01	9.91692218E-01	3
2.70000000E-02	9.95691980E-01	9.91371526E-01	4
2.80000000E-02	9.95531944E-01	9.91050756E-01	4
2.90000000E-02	9.95371869E-01	9.90729909E-01	4
3.00000000E-02	9.95211754E-01	9.90408986E-01	4
3.10000000E-02	9.95051599E-01	9.90087988E-01	4
3.20000000E-02	9.94891403E-01	9.89766915E-01	4
3.30000000E-02	9.94731167E-01	9.89445769E-01	4
3.40000000E-02	9.94570890E-01	9.89124550E-01	4
3.50000000E-02	9.94410570E-01	9.88803259E-01	4
3.60000000E-02	9.94250208E-01	9.88481897E-01	4
3.70000000E-02	9.94089803E-01	9.88160466E-01	4
3.80000000E-02	9.93929355E-01	9.87838965E-01	4
3.90000000E-02	9.93768863E-01	9.87517396E-01	4
4.00000000E-02	9.93608327E-01	9.87195760E-01	4
4.10000000E-02	9.93447745E-01	9.86874057E-01	4
4.20000000E-02	9.93287119E-01	9.86552289E-01	4
4.30000000E-02	9.93126447E-01	9.86230456E-01	4
4.40000000E-02	9.92965729E-01	9.85908559E-01	4
4.50000000E-02	9.92804964E-01	9.85586599E-01	4
4.60000000E-02	9.92644151E-01	9.85264577E-01	4
4.70000000E-02	9.92483291E-01	9.84942494E-01	4
4.80000000E-02	9.92322383E-01	9.84620351E-01	4
4.90000000E-02	9.92161427E-01	9.84298148E-01	4
5.00000000E-02	9.92000421E-01	9.83975887E-01	4
5.10000000E-02	9.91839366E-01	9.83653568E-01	4
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5.30000000E-02	9.91517105E-01	9.83008761E-01	4
5.40000000E-02	9.91355899E-01	9.82686274E-01	4
5.50000000E-02	9.91194641E-01	9.82363734E-01	4

$L\Omega_c$	σ_w/σ	σ_u/σ	K
5.60000000E-02	9.91033331E-01	9.82041140E-01	4
5.70000000E-02	9.90871969E-01	9.81718494E-01	4
5.80000000E-02	9.90710554E-01	9.81395796E-01	4
5.90000000E-02	9.90549086E-01	9.81073048E-01	4
6.00000000E-02	9.90387564E-01	9.80750251E-01	4
6.10000000E-02	9.90225988E-01	9.80427405E-01	4
6.20000000E-02	9.90064358E-01	9.80104510E-01	5
6.30000000E-02	9.89902672E-01	9.79781569E-01	5
6.40000000E-02	9.89740931E-01	9.79458582E-01	5
6.50000000E-02	9.89579134E-01	9.79135549E-01	5
6.60000000E-02	9.89417280E-01	9.78812473E-01	5
6.70000000E-02	9.89255370E-01	9.78489352E-01	5
6.80000000E-02	9.89093402E-01	9.78166190E-01	5
6.90000000E-02	9.88931376E-01	9.77842985E-01	5
7.00000000E-02	9.88769293E-01	9.77519740E-01	5
7.10000000E-02	9.88607150E-01	9.77196455E-01	5
7.20000000E-02	9.88444949E-01	9.76873131E-01	5
7.30000000E-02	9.88282688E-01	9.76549769E-01	5
7.40000000E-02	9.88120367E-01	9.76226369E-01	5
7.50000000E-02	9.87957985E-01	9.75902934E-01	5
7.60000000E-02	9.87795543E-01	9.75579463E-01	5
7.70000000E-02	9.87633040E-01	9.75255957E-01	5
7.80000000E-02	9.87470474E-01	9.74932418E-01	5
7.90000000E-02	9.87307847E-01	9.74608845E-01	5
8.00000000E-02	9.87145157E-01	9.74285241E-01	5
8.10000000E-02	9.86982404E-01	9.73961606E-01	5
8.20000000E-02	9.86819588E-01	9.73637941E-01	5
8.30000000E-02	9.86656708E-01	9.73314247E-01	5
8.40000000E-02	9.86493763E-01	9.72990524E-01	5
8.50000000E-02	9.86330754E-01	9.72666774E-01	5
8.60000000E-02	9.86167680E-01	9.72342997E-01	5
8.70000000E-02	9.86004540E-01	9.72019195E-01	5
8.80000000E-02	9.85841334E-01	9.71695367E-01	5
8.90000000E-02	9.85678062E-01	9.71371516E-01	5
9.00000000E-02	9.85514723E-01	9.71047642E-01	5
9.10000000E-02	9.85351317E-01	9.70723745E-01	5
9.20000000E-02	9.85187844E-01	9.70399827E-01	5
9.30000000E-02	9.85024302E-01	9.70075889E-01	5
9.40000000E-02	9.84860692E-01	9.69751932E-01	5
9.50000000E-02	9.84697014E-01	9.69427955E-01	5
9.60000000E-02	9.84533266E-01	9.69103961E-01	5
9.70000000E-02	9.84369448E-01	9.68779950E-01	5
9.80000000E-02	9.84205561E-01	9.68455923E-01	5
9.90000000E-02	9.84041603E-01	9.68131881E-01	5
1.00000000E-01	9.83877575E-01	9.67807824E-01	5

$L\Omega_c$	σ_w/σ	σ_u/σ	K
1.00000000E-01	9.83877575E-01	9.67807824E-01	5
1.10000000E-01	9.82233316E-01	9.64566668E-01	6
1.20000000E-01	9.80581573E-01	9.61325046E-01	6
1.30000000E-01	9.78921994E-01	9.58083824E-01	6
1.40000000E-01	9.77254257E-01	9.54843858E-01	6
1.50000000E-01	9.75578062E-01	9.51605994E-01	7
1.60000000E-01	9.73893141E-01	9.48371065E-01	7
1.70000000E-01	9.72199254E-01	9.45139891E-01	7
1.80000000E-01	9.70496191E-01	9.41913280E-01	7
1.90000000E-01	9.68783773E-01	9.38692020E-01	8
2.00000000E-01	9.67061854E-01	9.35476886E-01	8
2.10000000E-01	9.65330317E-01	9.32268634E-01	8
2.20000000E-01	9.63589078E-01	9.29068000E-01	9
2.30000000E-01	9.61838085E-01	9.25875702E-01	9
2.40000000E-01	9.60077317E-01	9.22692438E-01	9
2.50000000E-01	9.58306784E-01	9.19518883E-01	10
2.60000000E-01	9.56526525E-01	9.16355693E-01	10
2.70000000E-01	9.54736610E-01	9.13203501E-01	10
2.80000000E-01	9.52937138E-01	9.10062917E-01	11
2.90000000E-01	9.51128233E-01	9.06934529E-01	11
3.00000000E-01	9.49310049E-01	9.03818901E-01	11
3.10000000E-01	9.47482762E-01	9.00716575E-01	12
3.20000000E-01	9.45646576E-01	8.97628068E-01	12
3.30000000E-01	9.43801713E-01	8.94553876E-01	13
3.40000000E-01	9.41948420E-01	8.91494469E-01	13
3.50000000E-01	9.40086963E-01	8.88450296E-01	14
3.60000000E-01	9.38217626E-01	8.85421781E-01	14
3.70000000E-01	9.36340710E-01	8.82409326E-01	15
3.80000000E-01	9.34456533E-01	8.79413311E-01	15
3.90000000E-01	9.32565424E-01	8.76434092E-01	16
4.00000000E-01	9.30667728E-01	8.73472003E-01	16
4.10000000E-01	9.28763799E-01	8.70527359E-01	17
4.20000000E-01	9.26854002E-01	8.67600449E-01	18
4.30000000E-01	9.24938708E-01	8.64691546E-01	18
4.40000000E-01	9.23018299E-01	8.61800900E-01	19
4.50000000E-01	9.21093160E-01	8.58928741E-01	20
4.60000000E-01	9.19163680E-01	8.56075281E-01	21
4.70000000E-01	9.17230254E-01	8.53240711E-01	22
4.80000000E-01	9.15293275E-01	8.50425207E-01	23
4.90000000E-01	9.13353141E-01	8.47628923E-01	24
5.00000000E-01	9.11410249E-01	8.44852001E-01	25
5.10000000E-01	9.09464993E-01	8.42094561E-01	26
5.20000000E-01	9.07517767E-01	8.39356711E-01	28
5.30000000E-01	9.05568963E-01	8.36638542E-01	29
5.40000000E-01	9.03618968E-01	8.33940130E-01	31
5.50000000E-01	9.01668164E-01	8.31261538E-01	33

$L\Omega_c$	σ_w/σ	σ_u/σ	K
5.60000000E-01	8.99716931E-01	8.28602813E-01	35
5.70000000E-01	8.97765641E-01	8.25963992E-01	37
5.80000000E-01	8.95814661E-01	8.23345097E-01	40
5.90000000E-01	8.93864351E-01	8.20746139E-01	42
6.00000000E-01	8.91915064E-01	8.18167118E-01	46
6.10000000E-01	8.89967146E-01	8.15608021E-01	49
6.20000000E-01	8.88020936E-01	8.13068827E-01	54
6.30000000E-01	8.86076762E-01	8.10549505E-01	59
6.40000000E-01	8.84134947E-01	8.08050013E-01	64
6.50000000E-01	8.82195803E-01	8.05570302E-01	72
6.60000000E-01	8.80259635E-01	8.03110313E-01	80
6.70000000E-01	8.78326738E-01	8.00669980E-01	91
6.80000000E-01	8.76397399E-01	7.98249229E-01	105
6.90000000E-01	8.74471895E-01	7.95847981E-01	125
7.00000000E-01	8.72550493E-01	7.93466147E-01	152
7.10000000E-01	8.70633453E-01	7.91103635E-01	194
7.20000000E-01	8.68721025E-01	7.88760345E-01	268
7.30000000E-01	8.66813449E-01	7.86436172E-01	428
7.40000000E-01	8.64910957E-01	7.84131007E-01	1052
7.50000000E-01	8.63013771E-01	7.81844735E-01	2425
7.60000000E-01	8.61122105E-01	7.79577238E-01	594
7.70000000E-01	8.59236164E-01	7.77328392E-01	341
7.80000000E-01	8.57356143E-01	7.75098072E-01	240
7.90000000E-01	8.55482230E-01	7.72886147E-01	186
8.00000000E-01	8.53614604E-01	7.70692484E-01	152
8.10000000E-01	8.51753436E-01	7.68516946E-01	129
8.20000000E-01	8.49898887E-01	7.66359396E-01	112
8.30000000E-01	8.48051112E-01	7.64219692E-01	100
8.40000000E-01	8.46210258E-01	7.62097691E-01	90
8.50000000E-01	8.44376464E-01	7.59993248E-01	81
8.60000000E-01	8.42549861E-01	7.57906215E-01	75
8.70000000E-01	8.40730573E-01	7.55836445E-01	69
8.80000000E-01	8.38918718E-01	7.53783788E-01	64
8.90000000E-01	8.37114405E-01	7.51748093E-01	60
9.00000000E-01	8.35317738E-01	7.49729208E-01	57
9.10000000E-01	8.33528815E-01	7.47726980E-01	54
9.20000000E-01	8.31747724E-01	7.45741257E-01	51
9.30000000E-01	8.29974552E-01	7.43771883E-01	48
9.40000000E-01	8.28209376E-01	7.41818707E-01	46
9.50000000E-01	8.26452269E-01	7.39881572E-01	44
9.60000000E-01	8.24703298E-01	7.37960325E-01	42
9.70000000E-01	8.22962525E-01	7.36054811E-01	41
9.80000000E-01	8.21230007E-01	7.34164875E-01	39
9.90000000E-01	8.19505795E-01	7.32290364E-01	38
1.00000000E 00	8.17789936E-01	7.30431123E-01	36

$L\Omega_c$	σ_w/σ	σ_u/σ	K
1.00000000E 00	8.19789936E-01	7.30431123E-01	36
1.10000000E 00	8.01097877E-01	7.12645135E-01	27
1.20000000E 00	7.85262660E-01	6.96222036E-01	23
1.30000000E 00	7.72274809E-01	6.81022618E-01	19
1.40000000E 00	7.56105646E-01	6.66920291E-01	17
1.50000000E 00	7.42715388E-01	6.53801697E-01	15
1.60000000E 00	7.30058649E-01	6.41566238E-01	14
1.70000000E 00	7.18088049E-01	6.30125091E-01	13
1.80000000E 00	7.06756489E-01	6.19400008E-01	12
1.90000000E 00	6.96018524E-01	6.09322103E-01	12
2.00000000E 00	6.85831139E-01	5.99830690E-01	11
2.10000000E 00	6.76154128E-01	5.90872235E-01	11
2.20000000E 00	6.66950223E-01	5.82399419E-01	10
2.30000000E 00	6.58185077E-01	5.74370322E-01	10
2.40000000E 00	6.49827145E-01	5.66747714E-01	9
2.50000000E 00	6.41847524E-01	5.59498441E-01	9
2.60000000E 00	6.34219764E-01	5.52592902E-01	9
2.70000000E 00	6.26919669E-01	5.46004596E-01	9
2.80000000E 00	6.19925110E-01	5.39709734E-01	8
2.90000000E 00	6.13215837E-01	5.33686903E-01	8
3.00000000E 00	6.06773305E-01	5.27916781E-01	8
3.10000000E 00	6.00580519E-01	5.22381888E-01	8
3.20000000E 00	5.94621882E-01	5.17066374E-01	8
3.30000000E 00	5.88883070E-01	5.11955829E-01	7
3.40000000E 00	5.83350906E-01	5.07037125E-01	7
3.50000000E 00	5.78013254E-01	5.02298276E-01	7
3.60000000E 00	5.72858923E-01	4.97728313E-01	7
3.70000000E 00	5.67877578E-01	4.93317179E-01	7
3.80000000E 00	5.63059662E-01	4.89055635E-01	7
3.90000000E 00	5.58396325E-01	4.84935177E-01	7
4.00000000E 00	5.53879361E-01	4.80947962E-01	7
4.10000000E 00	5.49501152E-01	4.77086747E-01	7
4.20000000E 00	5.45254615E-01	4.73344827E-01	7
4.30000000E 00	5.41133157E-01	4.69715991E-01	6
4.40000000E 00	5.37130633E-01	4.66194471E-01	6
4.50000000E 00	5.33241308E-01	4.62774905E-01	6
4.60000000E 00	5.29459824E-01	4.59452302E-01	6
4.70000000E 00	5.25781171E-01	4.56222007E-01	6
4.80000000E 00	5.22200655E-01	4.53079674E-01	6
4.90000000E 00	5.18713877E-01	4.50021242E-01	6
5.00000000E 00	5.15316706E-01	4.47042908E-01	6
5.10000000E 00	5.12005265E-01	4.44141109E-01	6
5.20000000E 00	5.08775903E-01	4.41312501E-01	6
5.30000000E 00	5.05625186E-01	4.38553943E-01	6
5.40000000E 00	5.02549878E-01	4.35862483E-01	6
5.50000000E 00	4.99546925E-01	4.33235339E-01	6

$L\Omega_c$	σ_w/σ	σ_u/σ	K
5.60000000E 00	4.96613445E-01	4.30669893E-01	6
5.70000000E 00	4.93746715E-01	4.28163672E-01	6
5.80000000E 00	4.90944158E-01	4.25714343E-01	6
5.90000000E 00	4.88203336E-01	4.23319701E-01	6
6.00000000E 00	4.85521940E-01	4.20977660E-01	5
6.10000000E 00	4.82897779E-01	4.18686244E-01	5
6.20000000E 00	4.80328774E-01	4.16443583E-01	5
6.30000000E 00	4.77812954E-01	4.14247900E-01	5
6.40000000E 00	4.75348442E-01	4.12097513E-01	5
6.50000000E 00	4.72933455E-01	4.09990820E-01	5
6.60000000E 00	4.70566297E-01	4.07926300E-01	5
6.70000000E 00	4.68245351E-01	4.05902506E-01	5
6.80000000E 00	4.65969077E-01	4.03918062E-01	5
6.90000000E 00	4.63736007E-01	4.01971653E-01	5
7.00000000E 00	4.61544740E-01	4.00062031E-01	5
7.10000000E 00	4.59393937E-01	3.98188000E-01	5
7.20000000E 00	4.57282321E-01	3.96348423E-01	5
7.30000000E 00	4.55208670E-01	3.94542211E-01	5
7.40000000E 00	4.53171816E-01	3.92768324E-01	5
7.50000000E 00	4.51170640E-01	3.91025768E-01	5
7.60000000E 00	4.49204072E-01	3.89313592E-01	5
7.70000000E 00	4.47271086E-01	3.87630885E-01	5
7.80000000E 00	4.45370700E-01	3.85976774E-01	5
7.90000000E 00	4.43501969E-01	3.84350423E-01	5
8.00000000E 00	4.41663989E-01	3.82751031E-01	5
8.10000000E 00	4.39855892E-01	3.81177827E-01	5
8.20000000E 00	4.38076844E-01	3.79630074E-01	5
8.30000000E 00	4.36326042E-01	3.78107063E-01	5
8.40000000E 00	4.34602718E-01	3.76608112E-01	5
8.50000000E 00	4.32906129E-01	3.75132565E-01	5
8.60000000E 00	4.31235563E-01	3.73679794E-01	5
8.70000000E 00	4.29590334E-01	3.72249192E-01	5
8.80000000E 00	4.27969780E-01	3.70840176E-01	5
8.90000000E 00	4.26373265E-01	3.69452184E-01	5
9.00000000E 00	4.24800175E-01	3.68084675E-01	5
9.10000000E 00	4.23249919E-01	3.66737127E-01	5
9.20000000E 00	4.21721926E-01	3.65409038E-01	5
9.30000000E 00	4.20215646E-01	3.64099922E-01	5
9.40000000E 00	4.18730546E-01	3.62809313E-01	5
9.50000000E 00	4.17266115E-01	3.61536757E-01	5
9.60000000E 00	4.15821856E-01	3.60281819E-01	5
9.70000000E 00	4.14397291E-01	3.59044078E-01	5
9.80000000E 00	4.12991956E-01	3.57823126E-01	5
9.90000000E 00	4.11605405E-01	3.56618570E-01	5
1.00000000E 01	4.10237203E-01	3.55430030E-01	4

$L\Omega_c$	σ_w/σ	σ_u/σ	K
1.00000000E 01	4.18237203E-01	3.55430030E-01	4
1.10000000E 01	3.97478532E-01	3.44350060E-01	4
1.20000000E 01	3.86167304E-01	3.34531613E-01	4
1.30000000E 01	3.76039364E-01	3.25743411E-01	4
1.40000000E 01	3.66894288E-01	3.17810269E-01	4
1.50000000E 01	3.58576777E-01	3.10596631E-01	4
1.60000000E 01	3.50964309E-01	3.03995662E-01	4
1.70000000E 01	3.43958694E-01	2.97921818E-01	4
1.80000000E 01	3.37480151E-01	2.92305642E-01	4
1.90000000E 01	3.31463068E-01	2.87090048E-01	4
2.00000000E 01	3.25852897E-01	2.82227598E-01	4
2.10000000E 01	3.20603851E-01	2.77678489E-01	4
2.20000000E 01	3.15677157E-01	2.73409030E-01	4
2.30000000E 01	3.11039723E-01	2.69390473E-01	4
2.40000000E 01	3.06663102E-01	2.65598114E-01	3
2.50000000E 01	3.02522680E-01	2.62010580E-01	3
2.60000000E 01	2.98597030E-01	2.58609273E-01	3
2.70000000E 01	2.94867398E-01	2.55377913E-01	3
2.80000000E 01	2.91317280E-01	2.52302180E-01	3
2.90000000E 01	2.87932092E-01	2.49369420E-01	3
3.00000000E 01	2.84698881E-01	2.46568395E-01	3
3.10000000E 01	2.81606100E-01	2.43889091E-01	3
3.20000000E 01	2.78643418E-01	2.41322543E-01	3
3.30000000E 01	2.75801553E-01	2.38860705E-01	3
3.40000000E 01	2.73072142E-01	2.36496323E-01	3
3.50000000E 01	2.70447625E-01	2.34222842E-01	3
3.60000000E 01	2.67921147E-01	2.32034318E-01	3
3.70000000E 01	2.65486475E-01	2.29925348E-01	3
3.80000000E 01	2.63137929E-01	2.27891006E-01	3
3.90000000E 01	2.60870315E-01	2.25926791E-01	3
4.00000000E 01	2.58678877E-01	2.24028579E-01	3
4.10000000E 01	2.56559249E-01	2.22192585E-01	3
4.20000000E 01	2.54507411E-01	2.20415327E-01	3
4.30000000E 01	2.52519659E-01	2.18693593E-01	3
4.40000000E 01	2.50592572E-01	2.17024417E-01	3
4.50000000E 01	2.48722983E-01	2.15405056E-01	3
4.60000000E 01	2.46907955E-01	2.13832964E-01	3
4.70000000E 01	2.45144762E-01	2.12305779E-01	3
4.80000000E 01	2.43430869E-01	2.10821303E-01	3
4.90000000E 01	2.41763913E-01	2.09377489E-01	3
5.00000000E 01	2.40141690E-01	2.07972428E-01	3
5.10000000E 01	2.38562141E-01	2.06604335E-01	3
5.20000000E 01	2.37023340E-01	2.05271542E-01	3
5.30000000E 01	2.35523483E-01	2.03972484E-01	3
5.40000000E 01	2.34060878E-01	2.02705695E-01	3
5.50000000E 01	2.32633934E-01	2.01469798E-01	3

$L\Omega_c$	σ_w/σ	σ_u/σ	K
5.60000000E 01	2.31241158E-01	2.00263500E-01	3
5.70000000E 01	2.29881144E-01	1.99085580E-01	3
5.80000000E 01	2.28552566E-01	1.97934892E-01	3
5.90000000E 01	2.27254174E-01	1.96810351E-01	3
6.00000000E 01	2.25984790E-01	1.95710937E-01	3
6.10000000E 01	2.24743296E-01	1.94635683E-01	3
6.20000000E 01	2.23528639E-01	1.93583674E-01	3
6.30000000E 01	2.22339820E-01	1.92554046E-01	3
6.40000000E 01	2.21175891E-01	1.91545978E-01	3
6.50000000E 01	2.20035956E-01	1.90558693E-01	3
6.60000000E 01	2.18919161E-01	1.89591452E-01	3
6.70000000E 01	2.17824698E-01	1.88643553E-01	3
6.80000000E 01	2.16751796E-01	1.87714330E-01	3
6.90000000E 01	2.15699722E-01	1.86803149E-01	3
7.00000000E 01	2.14667781E-01	1.85909405E-01	3
7.10000000E 01	2.13655306E-01	1.85032522E-01	3
7.20000000E 01	2.12661664E-01	1.84171951E-01	3
7.30000000E 01	2.11686251E-01	1.83327170E-01	3
7.40000000E 01	2.10728490E-01	1.82497677E-01	3
7.50000000E 01	2.09787828E-01	1.81682996E-01	3
7.60000000E 01	2.08863739E-01	1.80882669E-01	3
7.70000000E 01	2.07955719E-01	1.80096259E-01	3
7.80000000E 01	2.07063285E-01	1.79323349E-01	3
7.90000000E 01	2.06185974E-01	1.78563538E-01	3
8.00000000E 01	2.05323344E-01	1.77816443E-01	3
8.10000000E 01	2.04474971E-01	1.77081695E-01	3
8.20000000E 01	2.03640446E-01	1.76358942E-01	3
8.30000000E 01	2.02819380E-01	1.75647846E-01	3
8.40000000E 01	2.02011396E-01	1.74948082E-01	3
8.50000000E 01	2.01216136E-01	1.74259336E-01	3
8.60000000E 01	2.00433251E-01	1.73581310E-01	3
8.70000000E 01	1.99662409E-01	1.72913714E-01	3
8.80000000E 01	1.98903289E-01	1.72256270E-01	3
8.90000000E 01	1.98155583E-01	1.71608713E-01	3
9.00000000E 01	1.97418995E-01	1.70970784E-01	3
9.10000000E 01	1.96693237E-01	1.70342236E-01	3
9.20000000E 01	1.95978035E-01	1.69722830E-01	3
9.30000000E 01	1.95273122E-01	1.69112336E-01	3
9.40000000E 01	1.94578242E-01	1.68510532E-01	3
9.50000000E 01	1.93893149E-01	1.67917203E-01	3
9.60000000E 01	1.93217602E-01	1.67332143E-01	3
9.70000000E 01	1.92551373E-01	1.66755153E-01	3
9.80000000E 01	1.91894238E-01	1.66186039E-01	3
9.90000000E 01	1.91245982E-01	1.65624615E-01	3
1.00000000E 02	1.90606398E-01	1.65070702E-01	3

$L\sigma_c$	σ_w/σ	σ_u/σ	K
1.00000000E 02	1.98606398E-01	1.65070702E-01	3
1.10000000E 02	1.84646330E-01	1.59908988E-01	3
1.20000000E 02	1.79368045E-01	1.55337754E-01	3
1.30000000E 02	1.74645806E-01	1.51248094E-01	3
1.40000000E 02	1.78384582E-01	1.47557704E-01	2
1.50000000E 02	1.66510967E-01	1.44203006E-01	2
1.60000000E 02	1.62967188E-01	1.41133965E-01	2
1.70000000E 02	1.59707038E-01	1.38310561E-01	2
1.80000000E 02	1.56693035E-01	1.35700331E-01	2
1.90000000E 02	1.53894394E-01	1.33276616E-01	2
2.00000000E 02	1.51285552E-01	1.31017274E-01	2
2.10000000E 02	1.48845071E-01	1.28903740E-01	2
2.20000000E 02	1.46554813E-01	1.26920305E-01	2
2.30000000E 02	1.44399308E-01	1.25053572E-01	2
2.40000000E 02	1.42365266E-01	1.23292030E-01	2
2.50000000E 02	1.40441194E-01	1.21625727E-01	2
2.60000000E 02	1.38617093E-01	1.20046002E-01	2
2.70000000E 02	1.36884213E-01	1.18545277E-01	2
2.80000000E 02	1.35234858E-01	1.17116888E-01	2
2.90000000E 02	1.33662229E-01	1.15754946E-01	2
3.00000000E 02	1.32160290E-01	1.14454224E-01	2
3.10000000E 02	1.30723660E-01	1.13210062E-01	2
3.20000000E 02	1.29347528E-01	1.12018293E-01	2
3.30000000E 02	1.28027571E-01	1.10875173E-01	2
3.40000000E 02	1.26759895E-01	1.09777330E-01	2
3.50000000E 02	1.25540980E-01	1.08721717E-01	2
3.60000000E 02	1.24367637E-01	1.07705569E-01	2
3.70000000E 02	1.23236965E-01	1.06726376E-01	2
3.80000000E 02	1.22146320E-01	1.05781848E-01	2
3.90000000E 02	1.21093286E-01	1.04869892E-01	2
4.00000000E 02	1.20075651E-01	1.03988593E-01	2
4.10000000E 02	1.19091384E-01	1.03136191E-01	2
4.20000000E 02	1.18138615E-01	1.02311067E-01	2
4.30000000E 02	1.17215623E-01	1.01511731E-01	2
4.40000000E 02	1.16320815E-01	1.00736804E-01	2
4.50000000E 02	1.15452720E-01	9.99850097E-02	2
4.60000000E 02	1.14609972E-01	9.92551673E-02	2
4.70000000E 02	1.13791303E-01	9.85461788E-02	2
4.80000000E 02	1.12995536E-01	9.78570232E-02	2
4.90000000E 02	1.12221572E-01	9.71867496E-02	2
5.00000000E 02	1.11468386E-01	9.65344708E-02	2
5.10000000E 02	1.10735022E-01	9.58993580E-02	2
5.20000000E 02	1.10020584E-01	9.52806361E-02	2
5.30000000E 02	1.09324234E-01	9.46775787E-02	2
5.40000000E 02	1.08645186E-01	9.40895047E-02	2
5.50000000E 02	1.07982700E-01	9.35157747E-02	2

$L\Omega_c$	σ_w/σ	σ_u/σ	K
5.600000000E 02	1.07336083E-01	9.29557872E-02	2
5.700000000E 02	1.06704680E-01	9.24089764E-02	2
5.800000000E 02	1.06087878E-01	9.18748091E-02	2
5.900000000E 02	1.05485094E-01	9.13527824E-02	2
6.000000000E 02	1.04895780E-01	9.08424215E-02	2
6.100000000E 02	1.04319419E-01	9.03432777E-02	2
6.200000000E 02	1.03755520E-01	8.98549266E-02	2
6.300000000E 02	1.03203620E-01	8.93769662E-02	2
6.400000000E 02	1.02663277E-01	8.89090158E-02	2
6.500000000E 02	1.02134077E-01	8.84507141E-02	2
6.600000000E 02	1.01615621E-01	8.80017184E-02	2
6.700000000E 02	1.01107536E-01	8.75617028E-02	2
6.800000000E 02	1.00609461E-01	8.71303577E-02	2
6.900000000E 02	1.00121059E-01	8.67073884E-02	2
7.000000000E 02	9.96420041E-02	8.62925145E-02	2
7.100000000E 02	9.91719882E-02	8.58854686E-02	2
7.200000000E 02	9.87107170E-02	8.54859958E-02	2
7.300000000E 02	9.82579098E-02	8.50938529E-02	2
7.400000000E 02	9.78132982E-02	8.47088078E-02	2
7.500000000E 02	9.73766264E-02	8.43306387E-02	2
7.600000000E 02	9.69476494E-02	8.39591335E-02	2
7.700000000E 02	9.65261330E-02	8.35940894E-02	2
7.800000000E 02	9.61118530E-02	8.32353123E-02	2
7.900000000E 02	9.57045949E-02	8.28826163E-02	2
8.000000000E 02	9.53041530E-02	8.25358232E-02	2
8.100000000E 02	9.49103301E-02	8.21947624E-02	2
8.200000000E 02	9.45229370E-02	8.18592700E-02	2
8.300000000E 02	9.41417923E-02	8.15291889E-02	2
8.400000000E 02	9.37667218E-02	8.12043681E-02	2
8.500000000E 02	9.33975580E-02	8.08846627E-02	2
8.600000000E 02	9.30341400E-02	8.05699334E-02	2
8.700000000E 02	9.26763131E-02	8.02600461E-02	2
8.800000000E 02	9.23239286E-02	7.99548720E-02	2
8.900000000E 02	9.19768431E-02	7.96542871E-02	2
9.000000000E 02	9.16349189E-02	7.93581719E-02	2
9.100000000E 02	9.12980230E-02	7.90664114E-02	2
9.200000000E 02	9.09660275E-02	7.87788947E-02	2
9.300000000E 02	9.06388090E-02	7.84955151E-02	2
9.400000000E 02	9.03162484E-02	7.82161694E-02	2
9.500000000E 02	8.99982310E-02	7.79407581E-02	2
9.600000000E 02	8.96846460E-02	7.76691855E-02	2
9.700000000E 02	8.93753864E-02	7.74013587E-02	2
9.800000000E 02	8.90703488E-02	7.71371882E-02	2
9.900000000E 02	8.87694333E-02	7.68765877E-02	2
1.000000000E 03	8.84725435E-02	7.66194736E-02	2

$L\Omega_c$	σ_w/σ	σ_u/σ	K
1.00000000E 03	8.84725435E-02	7.66194736E-02	2
1.10000000E 03	8.57059472E-02	7.42235302E-02	2
1.20000000E 03	8.32558536E-02	7.21016864E-02	2
1.30000000E 03	8.10638850E-02	7.02033856E-02	2
1.40000000E 03	7.90859234E-02	6.84904202E-02	2
1.50000000E 03	7.72878901E-02	6.69332775E-02	2
1.60000000E 03	7.56429654E-02	6.55087308E-02	2
1.70000000E 03	7.41296987E-02	6.41982032E-02	2
1.80000000E 03	7.27306902E-02	6.29866262E-02	2
1.90000000E 03	7.14316494E-02	6.18616237E-02	2
2.00000000E 03	7.02207097E-02	6.08129191E-02	2
2.10000000E 03	6.90879197E-02	5.98318941E-02	2
2.20000000E 03	6.80248597E-02	5.89112571E-02	2
2.30000000E 03	6.70243489E-02	5.80447893E-02	2
2.40000000E 03	6.60802183E-02	5.72271482E-02	2
2.50000000E 03	6.51871330E-02	5.64537136E-02	2
2.60000000E 03	6.43404511E-02	5.57204655E-02	2
2.70000000E 03	6.35361112E-02	5.50238867E-02	2
2.80000000E 03	6.27705414E-02	5.43608838E-02	2
2.90000000E 03	6.20405853E-02	5.37287232E-02	2
3.00000000E 03	6.13434411E-02	5.31249786E-02	2
3.10000000E 03	6.06766118E-02	5.25474874E-02	2
3.20000000E 03	6.00378632E-02	5.19943149E-02	2
3.30000000E 03	5.94251892E-02	5.14637237E-02	2
3.40000000E 03	5.88367824E-02	5.09541484E-02	2
3.50000000E 03	5.82710091E-02	5.04641744E-02	2
3.60000000E 03	5.77263884E-02	4.99925190E-02	2
3.70000000E 03	5.72015740E-02	4.95380164E-02	2
3.80000000E 03	5.66953389E-02	4.90996039E-02	2
3.90000000E 03	5.62065616E-02	4.86763104E-02	2
4.00000000E 03	5.57342153E-02	4.82672464E-02	2
4.10000000E 03	5.52773569E-02	4.78715955E-02	2
4.20000000E 03	5.48351191E-02	4.74886063E-02	2
4.30000000E 03	5.44067023E-02	4.71175864E-02	2
4.40000000E 03	5.39913678E-02	4.67578962E-02	2
4.50000000E 03	5.35884321E-02	4.64089436E-02	2
4.60000000E 03	5.31972617E-02	4.60701802E-02	2
4.70000000E 03	5.28172683E-02	4.57410962E-02	2
4.80000000E 03	5.24479047E-02	4.54212180E-02	2
4.90000000E 03	5.20886612E-02	4.51101040E-02	2
5.00000000E 03	5.17390624E-02	4.48073425E-02	2
5.10000000E 03	5.13986640E-02	4.45125488E-02	2
5.20000000E 03	5.10670505E-02	4.42253631E-02	2
5.30000000E 03	5.07438326E-02	4.39454482E-02	2
5.40000000E 03	5.04286454E-02	4.36724881E-02	2
5.50000000E 03	5.01211460E-02	4.34061858E-02	2

$L\Omega_c$	σ_w/σ	σ_u/σ	K
5.60000000E 03	4.98210122E-02	4.31462623E-02	2
5.70000000E 03	4.95279407E-02	4.28924549E-02	2
5.80000000E 03	4.92416456E-02	4.26445161E-02	2
5.90000000E 03	4.89618575E-02	4.24022125E-02	2
6.00000000E 03	4.86883219E-02	4.21653236E-02	2
6.10000000E 03	4.84207982E-02	4.19336414E-02	2
6.20000000E 03	4.81590591E-02	4.17069686E-02	2
6.30000000E 03	4.79028890E-02	4.14851188E-02	2
6.40000000E 03	4.76520840E-02	4.12679153E-02	2
6.50000000E 03	4.74064503E-02	4.10551903E-02	2
6.60000000E 03	4.71658043E-02	4.08467847E-02	2
6.70000000E 03	4.69299713E-02	4.06425474E-02	2
6.80000000E 03	4.66987854E-02	4.04423346E-02	2
6.90000000E 03	4.64720887E-02	4.02460095E-02	2
7.00000000E 03	4.62497309E-02	4.00534419E-02	2
7.10000000E 03	4.60315685E-02	3.98645077E-02	2
7.20000000E 03	4.58174651E-02	3.96790887E-02	2
7.30000000E 03	4.56072903E-02	3.94970720E-02	2
7.40000000E 03	4.54009196E-02	3.93183498E-02	2
7.50000000E 03	4.51982343E-02	3.91428191E-02	2
7.60000000E 03	4.49991205E-02	3.89703815E-02	2
7.70000000E 03	4.48034697E-02	3.88009430E-02	2
7.80000000E 03	4.46111778E-02	3.86344133E-02	2
7.90000000E 03	4.44221451E-02	3.84707062E-02	2
8.00000000E 03	4.42362762E-02	3.83097390E-02	2
8.10000000E 03	4.40534796E-02	3.81514325E-02	2
8.20000000E 03	4.38736675E-02	3.79957106E-02	2
8.30000000E 03	4.36967556E-02	3.78425004E-02	2
8.40000000E 03	4.35226631E-02	3.76917319E-02	2
8.50000000E 03	4.33513123E-02	3.75433377E-02	2
8.60000000E 03	4.31826284E-02	3.73972532E-02	2
8.70000000E 03	4.30165397E-02	3.72534162E-02	2
8.80000000E 03	4.28529772E-02	3.71117669E-02	2
8.90000000E 03	4.26918742E-02	3.69722476E-02	2
9.00000000E 03	4.25331669E-02	3.68348031E-02	2
9.10000000E 03	4.23767936E-02	3.66993798E-02	2
9.20000000E 03	4.22226948E-02	3.65659263E-02	2
9.30000000E 03	4.20708133E-02	3.64343931E-02	2
9.40000000E 03	4.19210938E-02	3.63047322E-02	2
9.50000000E 03	4.17734831E-02	3.61768976E-02	2
9.60000000E 03	4.16279297E-02	3.60508447E-02	2
9.70000000E 03	4.14843840E-02	3.59265304E-02	2
9.80000000E 03	4.13427980E-02	3.58039133E-02	2
9.90000000E 03	4.12031253E-02	3.56829532E-02	2
1.00000000E 04	4.10653211E-02	3.55636113E-02	2

$L\Omega_c$	σ_w/σ	σ_u/σ	K
1.00000000E 04	4.18653211E-02	3.55636113E-02	2
1.10000000E 04	3.97811801E-02	3.44515126E-02	2
1.20000000E 04	3.86439468E-02	3.34666396E-02	2
1.30000000E 04	3.76265246E-02	3.25855262E-02	2
1.40000000E 04	3.67084358E-02	3.17904379E-02	2
1.50000000E 04	3.58738624E-02	3.10676761E-02	2
1.60000000E 04	3.51103557E-02	3.04064600E-02	2
1.70000000E 04	3.44079594E-02	2.97981669E-02	2
1.80000000E 04	3.37585970E-02	2.92358026E-02	2
1.90000000E 04	3.31556355E-02	2.87136226E-02	2
2.00000000E 04	3.25935670E-02	2.82268570E-02	2
2.10000000E 04	3.20677724E-02	2.77715055E-02	2
2.20000000E 04	3.15743436E-02	2.73441836E-02	2
2.30000000E 04	3.11099475E-02	2.69420049E-02	2
2.40000000E 04	3.06717209E-02	2.65624895E-02	2
2.50000000E 04	3.02571873E-02	2.62034929E-02	1
2.60000000E 04	2.98641924E-02	2.58631493E-02	1
2.70000000E 04	2.94908508E-02	2.55398260E-02	1
2.80000000E 04	2.91355048E-02	2.52320873E-02	1
2.90000000E 04	2.87966891E-02	2.49386643E-02	1
3.00000000E 04	2.84731034E-02	2.46584309E-02	1
3.10000000E 04	2.81635887E-02	2.43903832E-02	1
3.20000000E 04	2.78671078E-02	2.41336233E-02	1
3.30000000E 04	2.75827297E-02	2.38873447E-02	1
3.40000000E 04	2.73096155E-02	2.36508208E-02	1
3.50000000E 04	2.70470067E-02	2.34233949E-02	1
3.60000000E 04	2.67942162E-02	2.32044719E-02	1
3.70000000E 04	2.65506189E-02	2.29935105E-02	1
3.80000000E 04	2.63156454E-02	2.27900174E-02	1
3.90000000E 04	2.60887751E-02	2.25935420E-02	1
4.00000000E 04	2.58695313E-02	2.24036713E-02	1
4.10000000E 04	2.56574764E-02	2.22200264E-02	1
4.20000000E 04	2.54522078E-02	2.20422585E-02	1
4.30000000E 04	2.52533543E-02	2.18700464E-02	1
4.40000000E 04	2.50605731E-02	2.17030929E-02	1
4.50000000E 04	2.48735469E-02	2.15411235E-02	1
4.60000000E 04	2.46919817E-02	2.13838834E-02	1
4.70000000E 04	2.45156044E-02	2.12311362E-02	1
4.80000000E 04	2.43441610E-02	2.10826619E-02	1
4.90000000E 04	2.41774149E-02	2.09382555E-02	1
5.00000000E 04	2.40151455E-02	2.07977261E-02	1
5.10000000E 04	2.38571466E-02	2.06608950E-02	1
5.20000000E 04	2.37032252E-02	2.05275952E-02	1
5.30000000E 04	2.35532008E-02	2.03976702E-02	1
5.40000000E 04	2.34069038E-02	2.02709733E-02	1
5.50000000E 04	2.32641753E-02	2.01473668E-02	1

$L\Omega_c$	σ_w/σ	σ_u/σ	K
5.60000000E 04	2.31248655E-02	2.00267210E-02	1
5.70000000E 04	2.29888337E-02	1.99089140E-02	1
5.80000000E 04	2.28559473E-02	1.97938310E-02	1
5.90000000E 04	2.27260812E-02	1.96813636E-02	1
6.00000000E 04	2.25991172E-02	1.95714096E-02	1
6.10000000E 04	2.24749437E-02	1.94638722E-02	1
6.20000000E 04	2.23534551E-02	1.93586600E-02	1
6.30000000E 04	2.22345515E-02	1.92556865E-02	1
6.40000000E 04	2.21181381E-02	1.91548695E-02	1
6.50000000E 04	2.20041251E-02	1.90561313E-02	1
6.60000000E 04	2.18924271E-02	1.89593980E-02	1
6.70000000E 04	2.17829631E-02	1.88645994E-02	1
6.80000000E 04	2.16756562E-02	1.87716689E-02	1
6.90000000E 04	2.15704329E-02	1.86805428E-02	1
7.00000000E 04	2.14672235E-02	1.85911609E-02	1
7.10000000E 04	2.13659615E-02	1.85034654E-02	1
7.20000000E 04	2.12665835E-02	1.84174015E-02	1
7.30000000E 04	2.11690290E-02	1.83329169E-02	1
7.40000000E 04	2.10732402E-02	1.82499614E-02	1
7.50000000E 04	2.09791620E-02	1.81684872E-02	1
7.60000000E 04	2.08867416E-02	1.80884488E-02	1
7.70000000E 04	2.07959285E-02	1.80098024E-02	1
7.80000000E 04	2.07066745E-02	1.79325061E-02	1
7.90000000E 04	2.06189333E-02	1.78565200E-02	1
8.00000000E 04	2.05326606E-02	1.77818057E-02	1
8.10000000E 04	2.04478139E-02	1.77083263E-02	1
8.20000000E 04	2.03643525E-02	1.76360466E-02	1
8.30000000E 04	2.02822373E-02	1.75649328E-02	1
8.40000000E 04	2.02014307E-02	1.74949522E-02	1
8.50000000E 04	2.01218967E-02	1.74260737E-02	1
8.60000000E 04	2.00436006E-02	1.73582673E-02	1
8.70000000E 04	1.99665091E-02	1.72915041E-02	1
8.80000000E 04	1.98905901E-02	1.72257563E-02	1
8.90000000E 04	1.98158127E-02	1.71609972E-02	1
9.00000000E 04	1.97421473E-02	1.70972011E-02	1
9.10000000E 04	1.96695652E-02	1.70343431E-02	1
9.20000000E 04	1.95980389E-02	1.69723995E-02	1
9.30000000E 04	1.95275417E-02	1.69113472E-02	1
9.40000000E 04	1.94580481E-02	1.68511640E-02	1
9.50000000E 04	1.93895333E-02	1.67918284E-02	1
9.60000000E 04	1.93219734E-02	1.67333198E-02	1
9.70000000E 04	1.92553454E-02	1.66756182E-02	1
9.80000000E 04	1.91896270E-02	1.66187044E-02	1
9.90000000E 04	1.91247966E-02	1.65625597E-02	1
1.00000000E 05	1.90608336E-02	1.65071661E-02	1

$L\Omega_c$	σ_w/σ	σ_u/σ	K
1.00000000E 05	1.98608336E-02	1.65071661E-02	1
1.10000000E 05	1.84647881E-02	1.59909756E-02	1
1.20000000E 05	1.79369312E-02	1.55338381E-02	1
1.30000000E 05	1.74646857E-02	1.51248614E-02	1
1.40000000E 05	1.70385466E-02	1.47558142E-02	1
1.50000000E 05	1.66511719E-02	1.44203379E-02	1
1.60000000E 05	1.62967835E-02	1.41134285E-02	1
1.70000000E 05	1.59707600E-02	1.38310839E-02	1
1.80000000E 05	1.56693527E-02	1.35700575E-02	1
1.90000000E 05	1.53894828E-02	1.33276830E-02	1
2.00000000E 05	1.51285937E-02	1.31017464E-02	1
2.10000000E 05	1.48845414E-02	1.28903910E-02	1
2.20000000E 05	1.46555121E-02	1.26920457E-02	1
2.30000000E 05	1.44399585E-02	1.25053709E-02	1
2.40000000E 05	1.42365517E-02	1.23292154E-02	1
2.50000000E 05	1.40441423E-02	1.21625840E-02	1
2.60000000E 05	1.38617302E-02	1.20046105E-02	1
2.70000000E 05	1.36884404E-02	1.18545371E-02	1
2.80000000E 05	1.35235034E-02	1.17116975E-02	1
2.90000000E 05	1.33662391E-02	1.15755026E-02	1
3.00000000E 05	1.32160439E-02	1.14454298E-02	1
3.10000000E 05	1.30723799E-02	1.13210131E-02	1
3.20000000E 05	1.29347656E-02	1.12018356E-02	1
3.30000000E 05	1.28027690E-02	1.10875232E-02	1
3.40000000E 05	1.26760006E-02	1.09777386E-02	1
3.50000000E 05	1.25541084E-02	1.08721768E-02	1
3.60000000E 05	1.24367735E-02	1.07705618E-02	1
3.70000000E 05	1.23237056E-02	1.06726421E-02	1
3.80000000E 05	1.22146406E-02	1.05781890E-02	1
3.90000000E 05	1.21093367E-02	1.04869932E-02	1
4.00000000E 05	1.20075728E-02	1.03988630E-02	1
4.10000000E 05	1.19091456E-02	1.03136226E-02	1
4.20000000E 05	1.18138684E-02	1.02311101E-02	1
4.30000000E 05	1.17215687E-02	1.01511763E-02	1
4.40000000E 05	1.16320876E-02	1.00736834E-02	1
4.50000000E 05	1.15452778E-02	9.99850384E-03	1
4.60000000E 05	1.14610027E-02	9.92551946E-03	1
4.70000000E 05	1.13791356E-02	9.85462047E-03	1
4.80000000E 05	1.12995586E-02	9.78570479E-03	1
4.90000000E 05	1.12221619E-02	9.71867731E-03	1
5.00000000E 05	1.11468431E-02	9.65344932E-03	1
5.10000000E 05	1.10735065E-02	9.58993795E-03	1
5.20000000E 05	1.10020625E-02	9.52806566E-03	1
5.30000000E 05	1.09324274E-02	9.46775983E-03	1
5.40000000E 05	1.08645223E-02	9.40895235E-03	1
5.50000000E 05	1.07982736E-02	9.35157926E-03	1

$L\Omega_c$	σ_w/σ	σ_u/σ	K
5.60000000E 05	1.07336117E-02	9.29558044E-03	1
5.70000000E 05	1.06704714E-02	9.24089929E-03	1
5.80000000E 05	1.06087910E-02	9.18748249E-03	1
5.90000000E 05	1.05485125E-02	9.13527976E-03	1
6.00000000E 05	1.04895810E-02	9.08424361E-03	1
6.10000000E 05	1.04319448E-02	9.03432918E-03	1
6.20000000E 05	1.03755548E-02	8.98549401E-03	1
6.30000000E 05	1.03203646E-02	8.93769793E-03	1
6.40000000E 05	1.02663303E-02	8.89090284E-03	1
6.50000000E 05	1.02134101E-02	8.84507263E-03	1
6.60000000E 05	1.01615645E-02	8.80017301E-03	1
6.70000000E 05	1.01107558E-02	8.75617141E-03	1
6.80000000E 05	1.00609484E-02	8.71303686E-03	1
6.90000000E 05	1.00121080E-02	8.67073990E-03	1
7.00000000E 05	9.96420248E-03	8.62925247E-03	1
7.10000000E 05	9.91720082E-03	8.58854785E-03	1
7.20000000E 05	9.87107364E-03	8.54860054E-03	1
7.30000000E 05	9.82579285E-03	8.50938622E-03	1
7.40000000E 05	9.78133164E-03	8.47088168E-03	1
7.50000000E 05	9.73766440E-03	8.43306474E-03	1
7.60000000E 05	9.69476664E-03	8.39591420E-03	1
7.70000000E 05	9.65261495E-03	8.35940976E-03	1
7.80000000E 05	9.61118691E-03	8.32353202E-03	1
7.90000000E 05	9.57046105E-03	8.28826240E-03	1
8.00000000E 05	9.53041681E-03	8.25358307E-03	1
8.10000000E 05	9.49103448E-03	8.21947697E-03	1
8.20000000E 05	9.45229513E-03	8.18592771E-03	1
8.30000000E 05	9.41418062E-03	8.15291957E-03	1
8.40000000E 05	9.37667353E-03	8.12043748E-03	1
8.50000000E 05	9.33975711E-03	8.08846692E-03	1
8.60000000E 05	9.30341527E-03	8.05699397E-03	1
8.70000000E 05	9.26763255E-03	8.02600523E-03	1
8.80000000E 05	9.23239407E-03	7.99548780E-03	1
8.90000000E 05	9.19768549E-03	7.96542929E-03	1
9.00000000E 05	9.16349304E-03	7.93581776E-03	1
9.10000000E 05	9.12980342E-03	7.90664169E-03	1
9.20000000E 05	9.09660384E-03	7.87789001E-03	1
9.30000000E 05	9.06388196E-03	7.84955204E-03	1
9.40000000E 05	9.03162588E-03	7.82161745E-03	1
9.50000000E 05	8.99982412E-03	7.79407632E-03	1
9.60000000E 05	8.96846559E-03	7.76691904E-03	1
9.70000000E 05	8.93753960E-03	7.74013634E-03	1
9.80000000E 05	8.90703582E-03	7.71371929E-03	1
9.90000000E 05	8.87694425E-03	7.68765923E-03	1
1.00000000E 06	8.84725525E-03	7.66194780E-03	1

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TABLE OF VALUES OF INTEGRALS FOR THE LONGITUDINAL AND LATERAL VON KARMAN TURBULENCE SPECTRA

by Douglas D. Mackiernan

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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