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WIND-TUNNEL LIFT INTERFERENCE ON SWEPTBACK WINGS IN RECTANGULAR TEST SECTIONS WITH SLOTTED SIDE WALLS

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16. Abstract

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SUMMARY

A theory is presented for the boundary-induced upwash interference on a sweptback wing mounted in a rectangular wind-tunnel test section with closed top and bottom walls and slotted side walls. The interference factor can be computed at any point in the test section. In an example, the interference with slotted side walls is compared with that with slotted top and bottom walls.

INTRODUCTION

In reference 1 a theory was derived for the subsonic wind-tunnel-boundary lift interference on sweptback wings mounted in the horizontal center plane of rectangular test sections with slots in the top and bottom walls, where the slots were used to reduce the interference effects of the boundaries. However, it is sometimes convenient in making yaw tests to rotate the model through a right angle about the longitudinal axis of the test section. If the direction of positive lift is designated "up," the slots are then in the sides of the test section rather than in the top and bottom walls. In general, the change in configuration may be expected to produce a change in both magnitude and distribution of the boundary-induced upwash. The theory of reference 1 has been modified to apply to a sweptback wing in a test section with slotted side walls and solid top and bottom walls. The theory herein presented has been left in more general form than that of reference 1 in that the upwash interference factor can be computed for any point in the test section and the model may be mounted anywhere in the test section, as long as the lift may be taken as (positive or negative) up.

SYMBOLS

a	segment of airfoil span included in a discrete-point representation
$B(\omega,f),C(\omega,f)$	parameters in solution of transformed Laplace equation

b width of test section

C cross-sectional area of test section

C_L lift coefficient of model

d distance between two adjacent slots

F exponential Fourier transform of φ on $x - x_1$ and $z - z_1$

F_d' exponential Fourier transform of Ω_d' on $z - z_1$

F_1' exponential Fourier transform on $x - x_1$ and $z - z_1$ of an interference potential corresponding to F_d'

f variable of transformation on $z - z_1$

h height of test section

$i = \sqrt{-1}$

j summation index

K_1 modified Bessel function of the second kind

k odd summation index

L total lift of model

ΔL element of lift

l restriction constant of slotted walls

M Mach number

$n \ldots N$ even summation indices

$p = h\omega$

Q_{ij} function of q defined by equation (26)

$q = hf$	
r_o	ratio of slot width to distance between slot centers
S	area on which C_L is based
V	tunnel stream velocity
v	upwash velocity
X,Y,Z	Cartesian coordinate axes
x,y,z	Cartesian coordinates
x_1,y_1,z_1	coordinates of image of a lifting element
α	a positive real parameter
Γ	circulation
δ	upwash interference factor
$\delta(\omega)$	Dirac delta
ξ,η,ζ	coordinates of a lifting element
φ	velocity potential
φ_1	interference potential
φ_d	velocity potential of a semi-infinite line doublet
φ_d'	modified velocity potential defined by equation (3)
Ω_d'	exponential Fourier transform of φ_d' on $x - x_1$
ω	variable of transformation on $x - x_1$

Subscripts:

i,j position identifications

ANALYSIS

The lifting wing is represented by a distribution of semi-infinite doublet lines starting at discrete points of lift application on the sweptback wing and extending downstream toward infinity. The total interference is the sum of the interferences due to the interaction of the boundaries on the individual doublet lines. The doublets are oriented as indicated in figure 1, which also shows the coordinate system relative to the test section of height h and width b. With the origin at the center of the test section, the coordinate x is in the stream direction, y is normal to the slotted side walls, and z is vertical, positive upward. Positive lift is taken in the positive z-direction. The boundary-induced upwash velocity, positive in the positive z-direction, due to a single lifting element located at some point (ξ,η,ζ) in the test section is now derived.

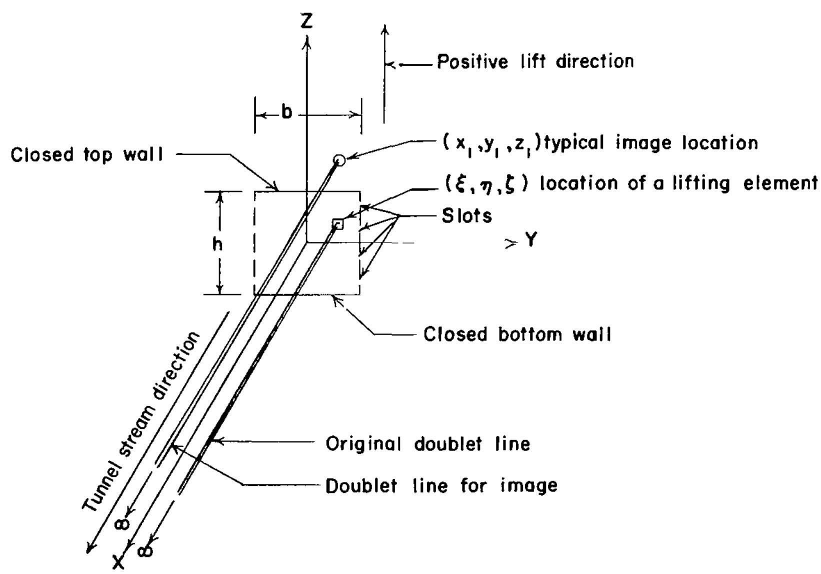

Figure 1.- Schematic drawing of test section and doublet configuration showing coordinate designation.

In a development similar to that in reference 1, the velocity potential at (x,y,z) due to a lifting semi-infinite line doublet starting at (x_1,y_1,z_1) is found to be

$$\varphi_d = \frac{\Gamma a}{4\pi} \frac{z - z_1}{(y - y_1)^2 + (z - z_1)^2}\left[1 + \frac{x - x_1}{\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2}}\right] \tag{1}$$

where Γ is circulation and a is the segment of airfoil span over which the element of lift is carried.

In order to facilitate the required integral transforms, let a modified potential be

$$\varphi_d' = \frac{\Gamma a}{4\pi} \frac{z - z_1}{(y - y_1)^2 + (z - z_1)^2} \left[1 - \frac{1}{\alpha} \frac{\partial e^{-\alpha\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}}}{\partial(x - x_1)}\right] \tag{2}$$

where $\alpha > 0$. Then

$$\varphi_d = \lim_{\alpha \to 0} \varphi_d' \tag{3}$$

and as in reference 1, the exponential integral transform of φ_d' on $x - x_1$ with variable of transformation ω is

$$\Omega_d'(\omega,y,z) = \frac{\Gamma a}{2\pi} \frac{z - z_1}{(y - y_1)^2 + (z - z_1)^2}\left(\pi\delta(\omega) - \frac{i\omega\sqrt{(y - y_1)^2 + (z - z_1)^2}}{\sqrt{\omega^2 + \alpha^2}}\right.$$

$$\left.\times K_1\left\{\sqrt{\left[(y - y_1)^2 + (z - z_1)^2\right](\omega^2 + \alpha^2)}\right\}\right) \tag{4}$$

for $\alpha > 0$ and $\sqrt{(y - y_1)^2 + (z - z_1)^2} > 0$ where K_1 is the modified Bessel function of the second kind.

From reference 2 (formula 15, p. 65 and formula 43, p. 112), the exponential transform of Ω_d' on $z - z_1$ with variable of transformation f is

$$F_d'(\omega,y,f) = -\frac{\Gamma a}{2}\left[i\pi\delta(\omega) \frac{f}{|f|} e^{-|y-y_1||f|} + \frac{\omega f}{\omega^2 + \alpha^2} \frac{e^{-|y-y_1|\sqrt{\omega^2+f^2+\alpha^2}}}{\sqrt{\omega^2 + f^2 + \alpha^2}}\right] \tag{5}$$

With transformations on $x - x_1$ and $z - z_1$, the Laplace equation $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$ transforms to

$$\frac{\partial^2 F}{\partial y^2} = \left(\omega^2 + f^2\right)F \tag{6}$$

A solution of this equation is

$$F_1'(\omega,y,f) = B(\omega,f) \sinh\left(y\sqrt{\omega^2 + f^2}\right) + C(\omega,f) \cosh\left(y\sqrt{\omega^2 + f^2}\right) \tag{7}$$

As in reference 1, but with y taking the place of z since the side walls rather than the top and bottom walls are slotted, the equivalent homogeneous wall boundary condition (originally derived in ref. 3) is given by

$$\varphi \pm l \frac{\partial \varphi}{\partial y} = 0 \tag{8}$$

where the positive and negative signs apply at $y = \frac{b}{2}$ and $y = -\frac{b}{2}$, respectively. The restriction constant l is given by

$$l = \frac{d}{\pi} \log_e \csc \frac{\pi r_o}{2} \tag{9}$$

where d is the distance between the centers of two adjacent slots and r_o is the ratio of slot width to the distance d. The boundary condition (8) transforms to

$$F \pm l \frac{\partial F}{\partial y} = 0 \tag{10}$$

With F_d' the transform of the modified potential of the doublet line, the transform F_1' of the corresponding interference potential is to be so determined that $F = F_d' + F_1'$ satisfies the boundary condition (10). For this purpose the partial derivatives of F_d' and F_1' at the slotted boundaries are required.

From equations (5) and (7) the partial derivatives with respect to y for positive $y - y_1$ are found to be

$$\left(\frac{\partial F_d'}{\partial y}\right)_{y-y_1>0} = \frac{\Gamma a}{2}\left[i\pi\delta(\omega)\, f e^{-(y-y_1)|f|} + \frac{\omega f}{\omega^2 + \alpha^2} e^{-(y-y_1)\sqrt{\omega^2+f^2+\alpha^2}}\right] \tag{11}$$

and

$$\frac{\partial F_1'}{\partial y} = B(\omega,f) \sqrt{\omega^2 + f^2} \cosh y\sqrt{\omega^2 + f^2} + C(\omega,f) \sqrt{\omega^2 + f^2} \sinh y\sqrt{\omega^2 + f^2} \tag{12}$$

Similarly, for negative $y - y_1$

$$\left(\frac{\partial F_d'}{\partial y}\right)_{y-y_1<0} = -\frac{\Gamma a}{2}\left[i\pi\delta(\omega)\, f e^{(y-y_1)|f|} + \frac{\omega f}{\omega^2 + \alpha^2} e^{(y-y_1)\sqrt{\omega^2+f^2+\alpha^2}}\right] \tag{13}$$

and equation (12) again applies. Since y_1 lies between the boundaries, equation (11) applies at the boundary $y = \frac{b}{2}$ and equation (13) applies at the boundary $y = -\frac{b}{2}$. Insertion of $F_d' + F_1'$ for F in equation (10) at the two boundaries $y = \frac{b}{2}$ and $y = -\frac{b}{2}$ gives, respectively,

$$B(\omega,f)\left[\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)+\imath\sqrt{\omega^2+f^2}\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)\right]+C(\omega,f)\left[\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)+\imath\sqrt{\omega^2+f^2}\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)\right]$$

$$=\frac{\Gamma a}{2}\left[i\pi\delta(\omega)\,e^{-\left(\frac{b}{2}-y_1\right)|f|}\left(\frac{f}{|f|}-\imath f\right)+\frac{\omega f}{\omega^2+\alpha^2}\,e^{-\left(\frac{b}{2}-y_1\right)\sqrt{\omega^2+f^2+\alpha^2}}\left(\frac{1}{\sqrt{\omega^2+f^2+\alpha^2}}-\imath\right)\right] \tag{14}$$

and

$$B(\omega,f)\left[\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)+\imath\sqrt{\omega^2+f^2}\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)\right]-C(\omega,f)\left[\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)+\imath\sqrt{\omega^2+f^2}\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)\right]$$

$$=\frac{\Gamma a}{2}\left[i\pi\delta(\omega)\,e^{-\left(\frac{b}{2}+y_1\right)|f|}\left(\imath f-\frac{f}{|f|}\right)+\frac{\omega f}{\omega^2+\alpha^2}\,e^{-\left(\frac{b}{2}+y_1\right)\sqrt{\omega^2+f^2+\alpha^2}}\left(\imath-\frac{1}{\sqrt{\omega^2+f^2+\alpha^2}}\right)\right] \tag{15}$$

Addition and subtraction of equations (14) and (15) yield, respectively,

$$B(\omega,f)\left[\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)+\imath\sqrt{\omega^2+f^2}\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)\right]$$

$$=\frac{\Gamma a}{2}\left[i\pi\delta(\omega)\,e^{-\frac{b}{2}|f|}\sinh\left(y_1|f|\right)\left(\frac{f}{|f|}-\imath f\right)+\frac{\omega f}{\omega^2+\alpha^2}\,e^{-\frac{b}{2}\sqrt{\omega^2+f^2+\alpha^2}}\sinh\left(y_1\sqrt{\omega^2+f^2+\alpha^2}\right)\left(\frac{1}{\sqrt{\omega^2+f^2+\alpha^2}}-\imath\right)\right] \tag{16}$$

and

$$C(\omega,f)\left[\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)+\imath\sqrt{\omega^2+f^2}\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)\right]$$

$$=\frac{\Gamma a}{2}\left[i\pi\delta(\omega)\,e^{-\frac{b}{2}|f|}\cosh\left(y_1|f|\right)\left(\frac{f}{|f|}-\imath f\right)+\frac{\omega f}{\omega^2+\alpha^2}\,e^{-\frac{b}{2}\sqrt{\omega^2+f^2+\alpha^2}}\cosh\left(y_1\sqrt{\omega^2+f^2+\alpha^2}\right)\left(\frac{1}{\sqrt{\omega^2+f^2+\alpha^2}}-\imath\right)\right] \tag{17}$$

Solution of equations (16) and (17) for $B(\omega,f)$ and $C(\omega,f)$, respectively, and substitution of the expressions so obtained into equation (7) gives the transform of the modified interference potential as

$$F_1' = \frac{\Gamma a}{2}\left\{\frac{\left[i\pi\delta(\omega)\, e^{-\frac{b}{2}|f|}\sinh\left(y_1|f|\right)\left(\frac{f}{|f|} - lf\right) + \frac{\omega f e^{-\frac{b}{2}\sqrt{\omega^2+f^2+\alpha^2}}}{\omega^2+\alpha^2}\sinh\left(y_1\sqrt{\omega^2+f^2+\alpha^2}\right)\left(\frac{1}{\sqrt{\omega^2+f^2+\alpha^2}} - l\right)\right]\sinh\left(y\sqrt{\omega^2+f^2}\right)}{\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right) + l\sqrt{\omega^2+f^2}\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)}\right.$$

$$\left. + \frac{\left[i\pi\delta(\omega)\, e^{-\frac{b}{2}|f|}\cosh\left(y_1|f|\right)\left(\frac{f}{|f|} - lf\right) + \frac{\omega f e^{-\frac{b}{2}\sqrt{\omega^2+f^2+\alpha^2}}}{\omega^2+\alpha^2}\cosh\left(y_1\sqrt{\omega^2+f^2+\alpha^2}\right)\left(\frac{1}{\sqrt{\omega^2+f^2+\alpha^2}} - l\right)\right]\cosh\left(y\sqrt{\omega^2+f^2}\right)}{\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right) + l\sqrt{\omega^2+f^2}\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)}\right\} \tag{18}$$

The interference potential due to the slotted side walls in the presence of the potential φ_d of the lifting doublet line is now obtained by taking the limit as α approaches zero of the inverse transform of F_1'.

$$\varphi_1 = \lim_{\alpha \to 0} \frac{\Gamma a}{8\pi^2}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} e^{i\left[(x-x_1)\omega + (z-z_1)f\right]} F_1'\, df \tag{19}$$

and the upwash velocity is therefore

$$\frac{\partial \varphi_1}{\partial z} = \lim_{\alpha \to 0} \frac{\Gamma a}{8\pi^2}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} e^{i\left[(x-x_1)\omega + (z-z_1)f\right]}\, if F_1'\, df \tag{20}$$

By taking the limit in equation (20), integrating the term in F_1' containing $\delta(\omega)$, and noting the evenness in f, the upwash velocity v is found to be

$$v = \frac{\partial \varphi_1}{\partial z} = \frac{\Gamma a}{8\pi^2}\int_{-\infty}^{\infty}\left\{-\pi\left(|f| - lf^2\right)e^{-\frac{b}{2}|f|}\left[\frac{\sinh\left(y_1|f|\right)\sinh(y|f|)}{\sinh\left(\frac{b}{2}|f|\right) + l|f|\cosh\left(\frac{b}{2}f\right)} + \frac{\cosh\left(y_1 f\right)\cosh(yf)}{\cosh\left(\frac{b}{2}f\right) + l|f|\sinh\left(\frac{b}{2}|f|\right)}\right]\right.$$

$$- \int_{-\infty}^{\infty} f^2\left(\frac{1}{\sqrt{\omega^2+f^2}} - l\right)e^{-\frac{b}{2}\sqrt{\omega^2+f^2}}\,\frac{\sin\left[(x-x_1)\omega\right]}{\omega}\left[\frac{\sinh\left(y_1\sqrt{\omega^2+f^2}\right)\sinh\left(y\sqrt{\omega^2+f^2}\right)}{\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right) + l\sqrt{\omega^2+f^2}\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)}\right.$$

$$\left.\left. + \frac{\cosh\left(y_1\sqrt{\omega^2+f^2}\right)\cosh\left(y\sqrt{\omega^2+f^2}\right)}{\cosh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right) + l\sqrt{\omega^2+f^2}\sinh\left(\frac{b}{2}\sqrt{\omega^2+f^2}\right)}\right]d\omega\right\}\cos\left[(z-z_1)f\right]\, df \tag{21}$$

The upwash interference factor is defined as

$$\delta = \frac{Cv}{SVC_L} \tag{22}$$

where

v upwash velocity

V tunnel stream velocity

C cross-sectional area of test section

C_L lift coefficient of model

S area on which C_L is based

As in reference 1,

$$\frac{C}{SVC_L}\frac{\Gamma a}{8\pi^2} = \frac{C}{16\pi^2}\frac{\Delta L}{L} \tag{23}$$

where ΔL is the element of lift represented by Γa and L is the total lift of the model. Then with $C = hb$ and with change of variables to $p = h\omega$ and $q = hf$, the contribution to the upwash interference factor at a point (x,y,z) due to the slotted-wall influence associated with a lifting element at $\left(x_1,y_1,z_1\right)$ is found from equations (21), (22), and (23) to be

$$\begin{aligned}\Delta\delta = \frac{1}{8\pi^2}\frac{b}{h}\frac{\Delta L}{L}\int_0^\infty \Bigg\{ & \pi\left(\frac{l}{h}q^2 - q\right)e^{-\frac{1}{2}\frac{b}{h}q}\left[\frac{\sinh\left(\frac{y_1}{h}q\right)\sinh\left(\frac{y}{h}q\right)}{\sinh\left(\frac{1}{2}\frac{b}{h}q\right) + \frac{l}{h}q\cosh\left(\frac{1}{2}\frac{b}{h}q\right)} + \frac{\cosh\left(\frac{y_1}{h}q\right)\cosh\left(\frac{y}{h}q\right)}{\cosh\left(\frac{1}{2}\frac{b}{h}q\right) + \frac{l}{h}q\sinh\left(\frac{1}{2}\frac{b}{h}q\right)}\right] \\ & + 2\int_0^\infty \left(\frac{l}{h}q^2 - \frac{q^2}{\sqrt{p^2 + q^2}}\right)e^{-\frac{1}{2}\frac{b}{h}\sqrt{p^2+q^2}}\frac{\sin\left[\left(\frac{x}{h} - \frac{x_1}{h}\right)p\right]}{p}\left[\frac{\sinh\left(\frac{y_1}{h}\sqrt{p^2 + q^2}\right)\sinh\left(\frac{y}{h}\sqrt{p^2 + q^2}\right)}{\sinh\left(\frac{1}{2}\frac{b}{h}\sqrt{p^2 + q^2}\right) + \frac{l}{h}\sqrt{p^2 + q^2}\cosh\left(\frac{1}{2}\frac{b}{h}\sqrt{p^2 + q^2}\right)}\right. \\ & \left. + \frac{\cosh\left(\frac{y_1}{h}\sqrt{p^2 + q^2}\right)\cosh\left(\frac{y}{h}\sqrt{p^2 + q^2}\right)}{\cosh\left(\frac{1}{2}\frac{b}{h}\sqrt{p^2 + q^2}\right) + \frac{l}{h}\sqrt{p^2 + q^2}\sinh\left(\frac{1}{2}\frac{b}{h}\sqrt{p^2 + q^2}\right)}\right]dp\Bigg\}\cos\left[\left(\frac{z}{h} - \frac{z_1}{h}\right)q\right]dq\end{aligned} \tag{24}$$

In equation (24) $\lim_{p \to 0} \dfrac{\sin\left[\left(\frac{x}{h} - \frac{x_1}{h}\right)p\right]}{p} = \frac{x}{h} - \frac{x_1}{h}$. The limits as q or q and p approach zero of all other expressions with zeros in the denominator are zero. The integrals should be convergent in the neighborhood of $p = 10$ and $q = 20$.

Let the original lifting element be located at (ξ,η,ζ); then the element and its images in the top and bottom walls are alternately positive and negative and are located at $(\xi,\eta,kh - \zeta)$ and at $(\xi,\eta,nh + \zeta)$, where

$$k = \pm 1, \pm 3, \pm 5 \ . \ . \ .$$

and

$$n = 0, \pm 2, \pm 4, \pm 6 \ . \ . \ .$$

To every such element there corresponds a contribution $\Delta\delta$ to the upwash interference factor arising from the slotted side walls and having the form of equation (24). Further contributions to the downwash interference factor are made by the images in the solid top and bottom walls. From differentiation of equation (1) the upwash velocity produced at (x,y,z) by a lifting element located at (x_1,y_1,z_1) is

$$\frac{\partial\varphi_d}{\partial z} = \frac{\Gamma a}{4\pi}\left(\frac{(y - y_1)^2 - (z - z_1)^2}{\left[(y - y_1)^2 + (z - z_1)^2\right]^2} + \frac{(x - x_1)\left\{\left[(y - y_1)^2 + (z - z_1)^2\right]\left[(x - x_1)^2 + (y - y_1)^2 - 2(z - z_1)^2\right] - 2(x - x_1)^2(z - z_1)^2\right\}}{\left[(y - y_1)^2 + (z - z_1)^2\right]^2\left[(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2\right]^{3/2}}\right)$$

By equations (22) and (23), the corresponding contribution to the upwash interference factor is

$$\Delta\delta = \frac{1}{8\pi}\frac{b}{h}\frac{\Delta L}{L}\left(\frac{\left(\frac{y}{h} - \frac{y_1}{h}\right)^2 - \left(\frac{z}{h} - \frac{z_1}{h}\right)^2}{\left[\left(\frac{y}{h} - \frac{y_1}{h}\right)^2 + \left(\frac{z}{h} - \frac{z_1}{h}\right)^2\right]^2} + \frac{\left(\frac{x}{h} - \frac{x_1}{h}\right)\left\{\left[\left(\frac{y}{h} - \frac{y_1}{h}\right)^2 + \left(\frac{z}{h} - \frac{z_1}{h}\right)^2\right]\left[\left(\frac{x}{h} - \frac{x_1}{h}\right)^2 + \left(\frac{y}{h} - \frac{y_1}{h}\right)^2 - 2\left(\frac{z}{h} - \frac{z_1}{h}\right)^2\right] - 2\left(\frac{x}{h} - \frac{x_1}{h}\right)^2\left(\frac{z}{h} - \frac{z_1}{h}\right)^2\right\}}{\left[\left(\frac{y}{h} - \frac{y_1}{h}\right)^2 + \left(\frac{z}{h} - \frac{z_1}{h}\right)^2\right]^2\left[\left(\frac{x}{h} - \frac{x_1}{h}\right)^2 + \left(\frac{y}{h} + \frac{y_1}{h}\right)^2 + \left(\frac{z}{h} - \frac{z_1}{h}\right)^2\right]^{3/2}}\right) \quad (25)$$

Let $(\Delta\delta)_{ij}$ be the contribution to the upwash interference factor at a point (x_i, y_i, z_i) corresponding to a lifting element located at a point (ξ_j, η_j, ζ_j). Then by summing all the contributions of the form of equation (24) due to the slotted side walls and all contributions of the form of equation (25) due to the images in the top and bottom walls, the total contribution to the upwash interference factor at (x_i, y_i, z_i) due to the lifting element at (ξ_j, η_j, ζ_j) is obtained. For convenience in the summation, let

$$
\begin{aligned}
Q_{ij} = {} & \pi\left(\frac{l}{h}q^2 - q\right)e^{-\frac{1}{2}\frac{b}{h}q}\left[\frac{\sinh\left(\frac{\eta_j}{h}q\right)\sinh\left(\frac{y_i}{h}q\right)}{\sinh\left(\frac{1}{2}\frac{b}{h}q\right) + \frac{l}{h}q\cosh\left(\frac{1}{2}\frac{b}{h}q\right)} + \frac{\cosh\left(\frac{\eta_j}{h}q\right)\cosh\left(\frac{y_i}{h}q\right)}{\cosh\left(\frac{1}{2}\frac{b}{h}q\right) + \frac{l}{h}q\sinh\left(\frac{1}{2}\frac{b}{h}q\right)}\right] \\
& + 2\int_0^\infty \left(\frac{l}{h}q^2 - \frac{q^2}{\sqrt{p^2+q^2}}\right)e^{-\frac{1}{2}\frac{b}{h}\sqrt{p^2+q^2}}\,\frac{\sin\left[\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)p\right]}{p}\left[\frac{\sinh\left(\frac{\eta_j}{h}\sqrt{p^2+q^2}\right)\sinh\left(\frac{y_i}{h}\sqrt{p^2+q^2}\right)}{\sinh\left(\frac{1}{2}\frac{b}{h}\sqrt{p^2+q^2}\right) + \frac{l}{h}\sqrt{p^2+q^2}\cosh\left(\frac{1}{2}\frac{b}{h}\sqrt{p^2+q^2}\right)}\right. \\
& \left. + \frac{\cosh\left(\frac{\eta_j}{h}\sqrt{p^2+q^2}\right)\cosh\left(\frac{y_i}{h}\sqrt{p^2+q^2}\right)}{\cosh\left(\frac{1}{2}\frac{b}{h}\sqrt{p^2+q^2}\right) + \frac{l}{h}\sqrt{p^2+q^2}\sinh\left(\frac{1}{2}\frac{b}{h}\sqrt{p^2+q^2}\right)}\right]dp
\end{aligned}
\tag{26}
$$

Then

$$
\begin{aligned}
(\Delta\delta)_{ij} = {} & \frac{1}{8\pi^2}\frac{b}{h}\left(\frac{\Delta L}{L}\right)_j\Bigg[\sum_n \int_0^\infty Q_{ij}\cos\left[\left(\frac{z_i}{h} - \frac{\zeta_j}{h} - n\right)q\right]dq - \sum_k \int_0^\infty Q_{ij}\cos\left[\left(\frac{z_i}{h} + \frac{\zeta_j}{h} - k\right)q\right]dq \\
& + \pi\sum_{\substack{n \\ n\neq 0}}\left(\frac{\left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 - \left(\frac{z_i}{h} - \frac{\zeta_j}{h} - n\right)^2}{\left[\left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 + \left(\frac{z_i}{h} - \frac{\zeta_j}{h} - n\right)^2\right]^2}\right. \\
& \left. + \frac{\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)\left\{\left[\left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 + \left(\frac{z_i}{h} - \frac{\zeta_j}{h} - n\right)^2\right]\left[\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)^2 + \left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 - 2\left(\frac{z_i}{h} - \frac{\zeta_j}{h} - n\right)^2\right] - 2\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)^2\left(\frac{z_i}{h} - \frac{\zeta_j}{h} - n\right)^2\right\}}{\left[\left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 + \left(\frac{z_i}{h} - \frac{\zeta_j}{h} - n\right)^2\right]^2\left[\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)^2 + \left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 + \left(\frac{z_i}{h} - \frac{\zeta_j}{h} - n\right)^2\right]^{3/2}}\right) \\
& - \pi\sum_k\left(\frac{\left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 - \left(\frac{z_i}{h} + \frac{\zeta_j}{h} - k\right)^2}{\left[\left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 + \left(\frac{z_i}{h} + \frac{\zeta_j}{h} - k\right)^2\right]^2}\right. \\
& \left. + \frac{\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)\left\{\left[\left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 + \left(\frac{z_i}{h} + \frac{\zeta_j}{h} - k\right)^2\right]\left[\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)^2 + \left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 - 2\left(\frac{z_i}{h} + \frac{\zeta_j}{h} - k\right)^2\right] - 2\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)^2\left(\frac{z_i}{h} + \frac{\zeta_j}{h} - k\right)^2\right\}}{\left[\left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 + \left(\frac{z_i}{h} + \frac{\zeta_j}{h} - k\right)^2\right]^2\left[\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)^2 + \left(\frac{y_i}{h} - \frac{\eta_j}{h}\right)^2 + \left(\frac{z_i}{h} + \frac{\zeta_j}{h} - k\right)^2\right]^{3/2}}\right)\Bigg]
\end{aligned}
\tag{27}
$$

where

$$k = \pm 1, \pm 3, \pm 5 \ldots$$

$$n = 0, \pm 2, \pm 4, \pm 6 \ldots$$

The total upwash interference factor at (x_i,y_i,z_i) due to all lifting elements in the presence of the tunnel boundaries is

$$\delta_i = \sum_j (\Delta\delta)_{ij} \tag{28}$$

If the lift distribution of the model lies symmetric with respect to the XZ-plane, the terms of Q_{ij} containing $\sinh\left(\frac{y_i}{h}q\right)$ and $\sinh\left(\frac{y_i}{h}\sqrt{p^2+q^2}\right)$ can be ignored, since their effects will add to zero in the summation of equation (28). Also with this type of symmetry, values of y_i can be restricted to positive or to negative values, since the interference is likewise symmetrical relative to the XZ-plane.

If the lift distribution may be considered to lie in the XY-plane, the summations on n and k can be combined, because ζ_j is zero.

SAMPLE CALCULATION

Let a wing swept back 35° and spanning 0.7 of the test section width be mounted at the center of a square, slotted wind-tunnel test section with four symmetrically spaced slots in each of the side walls and with solid top and bottom walls as shown in figure 2.

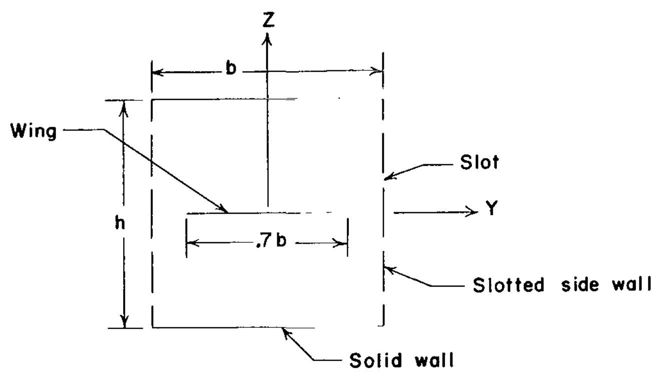

Figure 2.- Schematic drawing of cross section of wing and test section. For sample calculation, sum of slot widths in boundary of height h is 0.06h.

Let the slots occupy 6 percent of the area of each slotted boundary. Then by application of equation (9), the restriction constant l is given by

$$\frac{l}{h} = \frac{d}{\pi h}\log_e \csc\frac{\pi r_o}{2} = \frac{b/4}{\pi h}\log_e \csc\frac{0.06\pi}{2} = \frac{1}{4\pi}\frac{b}{h}\log_e \csc(0.03\pi) \approx 0.18$$

Let the lifting wing be represented by lifting elements located at points P_1, P_2, . . ., P_{10} on lines of 35° sweep as shown in figure 3.

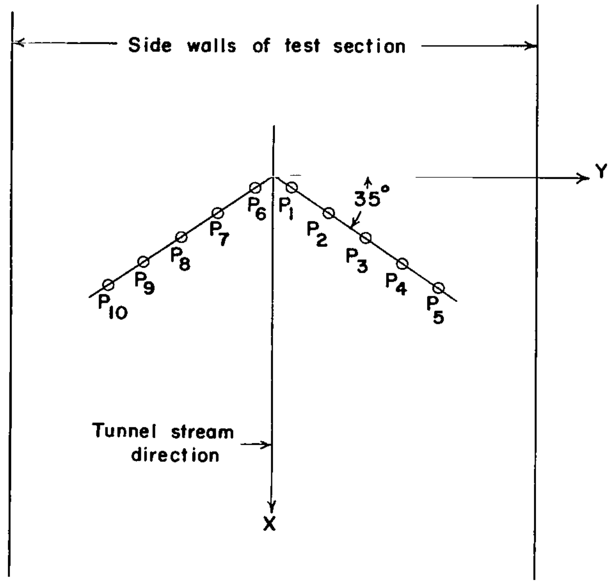

Figure 3.- Representation of sweptback wing for sample calculation.

The coordinates of these points and the lift distribution assumed are given in the following table:

Point	$\frac{\xi_j}{h}$ or $\frac{x_i}{h}$	$\frac{\eta_j}{h}$ or $\frac{y_i}{h}$	$\frac{\zeta_j}{h}$ or $\frac{z_i}{h}$	$\left(\frac{\Delta L}{L}\right)_j$
P_1	0.0246	0.0351	0	0.1342
P_2	.0738	.1054	0	.1334
P_3	.1229	.1756	0	.1118
P_4	.1721	.2458	0	.0769
P_5	.2212	.3160	0	.0437
P_6	.0246	-.0351	0	.1342
P_7	.0738	-.1054	0	.1334
P_8	.1229	-.1756	0	.1118
P_9	.1721	-.2458	0	.0769
P_{10}	.2212	-.3160	0	.0437

The values in this table are the same as those used for the sample calculation in reference 1. The lift distribution given here approximates that on a model which was of interest when the original calculation was performed.

With substitution of $\frac{b}{h} = 1$ and $\frac{l}{h} = 0.18$ and with use of the coordinates $\frac{\xi_j}{h}$, $\frac{x_i}{h}$, $\frac{\eta_j}{h}$, $\frac{y_i}{h}$, $\frac{\zeta_j}{h}$, and $\frac{z_i}{h}$ and of the lift distribution $\left(\frac{\Delta L}{L}\right)_j$ given in the table, the upwash interference factor $(\Delta\delta)_{ij}$ at any point P_i corresponding to a lifting element at any point P_j can be computed by use of equation (27) where p and q must be carried to values large enough to insure convergence of the integrals with infinite upper limits and k and n must be carried to values large enough to insure convergence of the summations. The total upwash interference factor at point P_i is the sum of contributions from all points P_j and is given by equation (28) as

$$\delta_i = \sum_j (\Delta\delta)_{ij}$$

Since the wing is symmetrical about its midspan, it was necessary to compute δ_i for points on only one side of the midspan. The points P_1, P_2, P_3, P_4, and P_5 were chosen. Computation of δ_i was made also for the points $P_0 = \left(\frac{x_i}{h}, \frac{y_i}{h}\right) = (0, 0)$ and $P_{11} = \left(\frac{x_i}{h}, \frac{y_i}{h}\right) = (0.2451, 0.350)$. (Note that P_0 and P_{11} are i-points, but not j-points, whereas all other points are both i-points and j-points.) The calculation was performed on a Control Data 6600 digital computer.

The calculated values of δ_i are shown in figure 4 as a function of spanwise position $\frac{2}{0.7}\frac{y_i}{h}$ on units of the semispan. For comparison, the upwash interference factor for the same wing and lift distribution in the same test section, but rotated 90° about the streamwise X-axis (or, otherwise, slots in top and bottom walls), is also shown from reference 1.

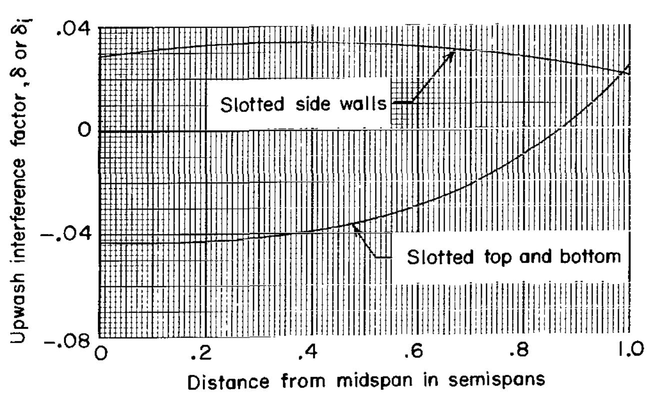

Figure 4.- Upwash interference factor for wing swept back 35° and spanning 0.7 of width of a square test section with four slots in each of its slotted walls, slot opening being 6 percent of each slotted wall.

DISCUSSION

The effects of rotating the lift distribution given in the sample calculation through a right angle about the X-axis are seen to be:

1. With the slots on the sides instead of in the top and bottom walls, the sign of the lift interference was reversed over most of the span. The average interference over the span was only slightly increased.

2. With slots in the sides, the variation of lift interference over the span was much less than that with the slots in the top and bottom walls.

It should not necessarily be inferred from these results that for testing wings, slotted sides are preferable to slotted top and bottom walls. The variation of interference over the span must be balanced against the average value. If a choice of wind-tunnel geometries were available, it might be preferable to increase the width and decrease the height of the test section so that the tip of large-span wings would not be so near to the walls. To attain zero average lift interference in a square test section with solid top and bottom walls would require that the side walls be effectively almost open.

As a check on the calculated interference for the large-span wing, an additional calculation was performed for a single lift element in the center of a square test section with slotted side walls. This lift element would correspond to a wing which spans only a very small portion of the tunnel width. The upwash interference factor for this case is given by

$$\delta = \frac{1}{8\pi}\left[\sum_{n=-N}^{N} (-1)^n \int_0^\infty \frac{\left(\frac{l}{h} q^2 - q\right) e^{-\frac{q}{2}} \cos nq}{\cosh\left(\frac{q}{2}\right) + \frac{l}{h} q \sinh\left(\frac{q}{2}\right)} dq - \sum_{\substack{n=-N \\ n\neq 0}}^{N} (-1)^n \left(\frac{1}{n^2}\right)\right] \tag{29}$$

This equation was derived independently but can be seen to be equivalent to equations (26) and (27) with x_i, ξ_j, y_i, η_j, z_i, and ζ_j all equal to zero. Equation (29) gives for the upwash interference factor at the center $\delta = 0.0469$. This value is somewhat greater than that calculated near the center of the tunnel (near the midspan in fig. 4) for the large-span wing. Such an effect is to be expected because the interference factor for the large-span wing is a weighted average of contributions from lifting elements located along the span; and those nearer the tips are more affected by the slotted side walls than are those near the midspan position. Figure 5 shows how the upwash interference factor at the midspan position due to a lifting element falls off as the element is moved outward along the span. The interference at the midspan position due to an element located out along the span falls off still further when the wing is swept back.

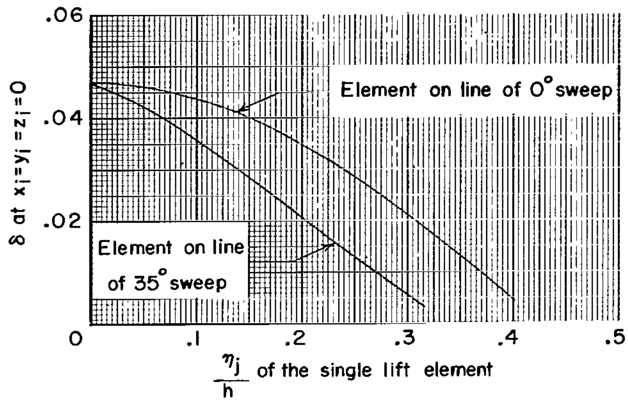

Figure 5.- Upwash interference factor at the center of the test section due to a single lift element in the horizontal center plane.

The theory presented herein for the boundary-induced upwash interference due to lift on swept wings mounted in wind-tunnel test sections with slotted side walls and solid top and bottom walls is exact for the boundary conditions assumed. The accuracy of calculated results depends on accurate representation of the wing and on carrying the integrations and summations far enough to insure covergence. In consideration of the usual uncertainty in the knowledge of the lift distribution and of the approximate knowledge of the boundary conditions, it is believed that representation of the wing by ten lifting elements as in the sample calculation is adequate. The use of homogeneous rather than exact slotted-wall boundary conditions is believed to introduce negligible error if there are several slots rather than only one or two in each slotted wall and if the wing is not too close to a wall. On the other hand, the action of the slots is uncertain and correspondingly so is the true effective restriction constant. The restriction constant used is calculated on the assumption of potential flow outward from the test section through slots with thin edges and no boundary layer. For outflow through coarse slots, this method of calculation should yield approximately correct values for the restriction constant, but for very narrow slots or for strong inflow from the plenum chamber surrounding the slots, the calculated values of the restriction constant may be appreciably in error. If the slots are not of uniform width, an added uncertainty exists.

In this paper, the computing equations are expressed in more general form than are those of reference 1; therefore, the model may be situated anywhere in the test section (provided it is not close enough to a boundary to invalidate the linearized theory assumed) and the upwash interference may be computed anywhere within the test section. The model must be so oriented that the lift vectors of the lifting elements lie approximately parallel to the slotted side walls.

Within the applicability of linearized theory, the upwash interference factor is not affected by compressibility at subsonic speeds, provided the stream boundary, including the slots, is approximately parallel to the tunnel stream direction. The theory presented herein may therefore be applied for subsonic compressible flow as well as for incompressible flow.

The theory can be used to calculate the upwash velocities anywhere in the test section and thus corrections for the moment due to boundary-induced upwash at the tail or for the lift due to boundary-induced curvature of the flow can be applied. A compressibility correction can be made by applying the upwash velocities at $x\sqrt{1 - M^2}$ rather than at x, where x is the distance from the lifting element and M is Mach number. The flow curvature must be multiplied by a compressibility factor $\frac{1}{\sqrt{1 - M^2}}$. For three-dimensional models of a practical size, the lift correction due to flow curvature is commonly assumed to be negligible.

RÉSUMÉ

The upwash interference factor due to wind-tunnel boundary interference on the lift of sweptback wings mounted in test sections with closed top and bottom walls and slotted side walls has been obtained in the form of infinite convergent integrals and summations suitable for calculation by means of a high-speed digital computer. The interference factor can be computed anywhere within the test section for an arbitrary wing mounted anywhere in the test section. In an example, the interference with slotted side walls is compared with that with slotted top and bottom walls.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., March 30, 1970.

REFERENCES

1. Wright, Ray H.; and Barger, Raymond L.: Wind-Tunnel Lift Interference on Sweptback Wings in Rectangular Test Sections With Slotted Top and Bottom Walls. NASA TR R-241, 1966.

2. Staff of the Bateman Manuscript Project, compiler: Tables of Integral Transforms. Vol. I. McGraw-Hill Book Co., Inc., 1954.

3. Davis, Don D., Jr.; and Moore, Dewey: Analytical Study of Blockage- and Lift-Interference Corrections for Slotted Tunnels Obtained by the Substitution of an Equivalent Homogeneous Boundary for the Discrete Slots. NACA RM L53E07b, 1953.

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