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EXPERIMENTAL DETERMINATION OF APPARENT MASS AND MOMENT OF INERTIA OF A LARGE DISK SUSPENDED AS A PENDULUM AT DIFFERENT AIR DENSITIES

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16. Abstract

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EXPERIMENTAL DETERMINATION OF APPARENT MASS AND MOMENT OF INERTIA OF A LARGE DISK SUSPENDED AS A PENDULUM AT DIFFERENT AIR DENSITIES

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SUMMARY

An experiment has been performed to measure the apparent mass of air and added moment of inertia of the air associated with a freely swinging disk pendulum. These results were then used to determine the mass coefficient associated with rotation about an axis of a disk and with translation. The results were found to be substantially in agreement with Lamb's theoretical values. Error studies were made which indicate accuracy within 10 percent for the best method tried. A linear increase in added mass and moment of inertia with increasing air density was experimentally verified from essentially zero air density up to 1 atm (101 325 N/m^2).

INTRODUCTION

When an object is set in motion, the surrounding air is also set in motion. Therefore, the effective mass and inertia of an object are greater in air than in a vacuum. This phenomenon is generally referred to as apparent mass or inertia and can be important for lightweight objects because the added effects of the surrounding air can be of the same order of magnitude as the structural mass and inertia. There is currently much interest in lightweight aerodynamic structures; therefore, interest in the determination of apparent mass and inertia has been renewed. In the past the added effects generally have been measured on small models (ref. 1) and extrapolated to large scale as required. Large vacuum spheres are now available which can be used for the measurement of apparent mass and inertia of structures approaching flight-hardware size. The purpose of the present study was to examine apparent mass and inertia effects for such a structure, specifically the apparent mass and moment of inertia associated with a large disk suspended as a pendulum at different air densities.

SYMBOLS

A	variation of P^2 with ρ based on least-squares fit of data to a straight line
C_D	drag coefficient
d,d_1,d_2	length of pendulum extensions, meters
g	acceleration due to gravity, meters/second^2
I	total moment of inertia (structural + apparent inertia), kilogram-meter^2
K_1	nondimensional coefficient of apparent mass of air due to rotation of a flat disk about a diameter
K_2	nondimensional coefficient of apparent mass of air due to translation of a flat disk in a direction normal to a plane surface
k	radius of gyration, meters
l_{cg}	distance from pivot point to center of disk, meters
l_{aero}	distance from pivot point to center of aerodynamic pressure, meters
l_{cg}	distance from pivot point to center of gravity of structure, meters
$(l_{cg})_a$	distance from pivot point to center of gravity of apparent mass for basic configuration, meters
m	mass, kilograms
n	number of data points
P	period, seconds
p	atmospheric pressure, newtons/meter^2
r	radius of disk, meters

S reference area, meters^2

V velocity, meters/second

x,y,z dummy variables representing pertinent parameters in a propagation-of-error expression

δ deviation from mean

θ angular displacement, radians

ρ air density, kilograms/meter^3

σ standard deviation

Subscripts:

a apparent

cg center of gravity

i the ith measurement of a data point

s structural

vac vacuum

0 zero-length configuration

1 basic pendulum

2 basic pendulum with extension

3 basic pendulum with longer extension

Dots over symbols denote differentiation with respect to time.

ANALYSIS

Pendulum With Damping Proportional to Velocity Squared

A common method of determining the effective moment of inertia of a body is the use of an oscillation test. A suitable oscillation test to determine the moment of inertia of a disk requires that the disk be suspended in a test chamber so that it can swing as a simple pendulum. It is, of course, necessary to maintain a rigid structural shape.

If the motion of the pendulum is undamped, as it would be if the pendulum were suspended in a vacuum, and frictional losses at the support point are small enough to be ignored, then the equation of motion for small-amplitude oscillations is

$$I\ddot{\theta} + mgl_{cg}\theta = 0 \tag{1}$$

The period of oscillation is given by the well-known formula

$$P = 2\pi\sqrt{\frac{I}{mgl_{cg}}} \tag{2}$$

In contrast, the damping moment acting on a pendulum oscillating in still air is proportional to the drag (fig. 1) which, in turn, is proportional to the square of the velocity. In this case, the drag force is given by the product of the aerodynamic pressure $\frac{1}{2}\rho V^2$, the surface area of the pendulum on which this pressure acts S, and the drag coefficient of the pendulum C_D. The damping moment is the product of the drag force and the moment arm. Hence, for small-amplitude oscillations, the equation of motion with drag included is given by

$$I\ddot{\theta} + \frac{1}{2}\rho V|V|SC_D l_{aero} + m_s g l_{cg}\theta = 0 \tag{3}$$

The moment of inertia I includes the structural and apparent moments of inertia. However, the restoring moment is related to the structural mass m_s only. The apparent mass produces no restoring force since the buoyant force balances the gravity force. The buoyancy of the air displaced by the volume of the pendulum is assumed to be negligible.

The velocity term involved in equation (3) is given by

$$V = l_{aero}\dot{\theta}$$

Therefore, equation (3) can be written as

$$I\ddot{\theta} + \frac{1}{2}\rho\left(l_{aero}\right)^3 SC_D\dot{\theta}\left|\dot{\theta}\right| + m_s g l_{cg}\theta = 0 \tag{4}$$

This equation differs from the more commonly encountered equation of damped oscillatory motion in which the damping is proportioned to the first power of the velocity.

Equation (4) is a nonlinear, second-order, differential equation which does not have a known closed-form solution. However, the behavior of the solution was examined numerically for a wide range of constant damping coefficients. A typical result is shown in figure 2. After one or two complete oscillations, the period remained nearly constant, was independent of the damping, and was equal to the period which would be predicted for the undamped system if the initial amplitude is small. The same result was obtained in the approximate solutions (ref. 2) of equation (4). The results, therefore, indicate that if damping is proportional to the square of the velocity, the period is essentially the same as that for the undamped situation. Hence, the moment of inertia is given to sufficient accuracy by

$$I = \frac{P^2 m_s g l_{cg}}{4\pi^2} \tag{5}$$

provided that the period is measured after the first few oscillations. The period will, however, still differ with density because I is a function of the amount of air dragged with the pendulum.

Because the period is approximately independent of damping, experimental determination of the apparent moment of inertia and of the apparent mass is relatively simple. If the experiment is performed in a vacuum facility, the period can be determined as a function of air density. From oscillation tests made at a high-vacuum condition and at various degrees of vacuum, the apparent inertia is obtained as a function of air density by using the equation

$$I_a = I - I_{vac} = \left(P^2 - P_{vac}^2\right)\frac{m_s g l_{cg}}{4\pi^2} \tag{6}$$

The apparent mass, which is also of interest, can be found by using the following form of the parallel-axis theorem about the point of support:

$$I_a = \left(I_a\right)_{cg} + m_a\left(l_{cg}\right)_a^2 \tag{7}$$

In a general case, equation (7) contains three unknowns: m_a, $(l_{cg})_a$, and $(I_a)_{cg}$. In order to eliminate the unknown quantities, the experiment can be repeated with the pendulum length extended by some known amount d_1, and then d_2. Results from these experiments provide the second and third of three equations in three unknowns, which can then be solved for m_a. If the weight and inertia of the extensions are neglected, the equations which result are

$$\left(I_a\right)_1 = \left(I_a\right)_{cg} + m_a\left(l_{cg}\right)_a^2 \tag{7a}$$

$$\left(I_a\right)_2 = \left(I_a\right)_{cg} + m_a\left[\left(l_{cg}\right)_a + d_1\right]^2 \tag{7b}$$

$$\left(I_a\right)_3 = \left(I_a\right)_{cg} + m_a\left[\left(l_{cg}\right)_a + d_2\right]^2 \tag{7c}$$

Note that equation (7) expresses the apparent inertia about a point of support as the sum of two quantities:

(1) The apparent moment of inertia associated with rotation about the center of gravity

(2) The apparent moment of inertia associated with rectilinear motion $m_a\left(l_{cg}\right)_a^2$.

From equation (7) it can be shown that

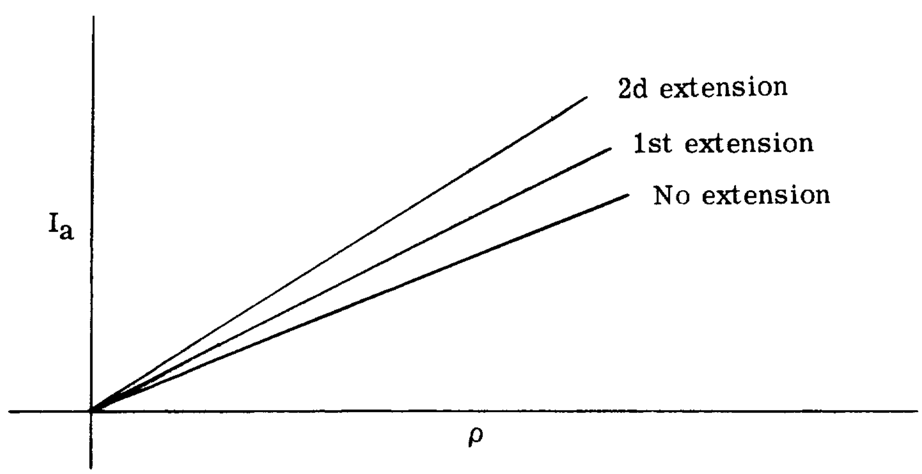
$$m_a = \frac{1}{d_2 - d_1}\left[\frac{\left(I_a\right)_3 - \left(I_a\right)_1}{d_2} - \frac{\left(I_a\right)_2 - \left(I_a\right)_1}{d_1}\right] \tag{8}$$

If the incremental lengths are chosen so that $d = d_1 = \frac{1}{2} d_2$, equation (8) reduces to the simpler form

$$m_a = \frac{\left(I_a\right)_1 - 2\left(I_a\right)_2 + \left(I_a\right)_3}{2d^2} \tag{8a}$$

The term l_{cg} and m_s of equation (6) will depend to some extent on the weight of the extensions and their lengths d_1 and d_2 for a general pendulum configuration. However, m_s and $\left(l_{cg}\right)_s$ are measurable quantities.

Both I_a and m_a are hypothetically linear functions of the air density. If $\left(I_a\right)_{cg}$ and m_a exist, the experimental curves as shown in sketch (a) would be expected. The difference in the slopes of the curves would be related to the apparent mass.



Sketch (a)

Although the foregoing analysis has been general and applies to any pendulum, it is now convenient to consider the special case of a disk pendulum. Equations (7) and (8) were derived for a general case in which $\left(l_{cg}\right)_a$, m_a, and $\left(I_a\right)_{cg}$ are known. The $\left(l_{cg}\right)_a$ can be assumed to be the distance from the pivot point to the geometric center for some simple configurations. A disk with string supports is such a configuration, and if the strings are very light, the center of apparent mass is at the geometric center of the disk, so that $\left(l_{cg}\right)_a = \left(l_{cg}\right)_s = l_{cg}$. In such a case, the total moment of inertia may be written as

$$I = I_s + I_a$$

Expressing the variables of this equation in terms of the parallel-axis theorem gives

$$I = \left[\left(I_s\right)_{cg} + m_s l_{cg}^2\right] + \left[K_1 \rho r^5 + K_2 \rho r^3 l_{cg}^2\right] \tag{9}$$

where the terms on the right have been chosen by analogy to the inertias of a solid disk and the constants K_1 and K_2 are left as unknown coefficients determined by the configuration of the apparent air mass (appendix A).

$$K_1 \rho r^5 = \left(I_a\right)_{cg} \tag{9a}$$

$$K_2 \rho r^3 = m_a \tag{9b}$$

These assumptions are justified if the experimental results do indeed assume the linear form shown in sketch (a) since the term on the right in brackets is linear in ρ. Substituting I from equation (9) into equation (2) and squaring results in the following equation:

$$P^2 = \frac{4\pi^2}{m_s g l_{cg}} \left\{\left[\left(I_s\right)_{cg} + m_s l_{cg}^2\right] + \left[K_1 \rho r^5 + K_2 \rho r^3 l_{cg}^2\right]\right\} \tag{10}$$

For $\rho = 0$,

$$P_{vac}^2 = \frac{4\pi^2}{m_s g l_{cg}}\left[\left(I_s\right)_{cg} + m_s l_{cg}^2\right] \tag{11}$$

and for the initial pendulum length,

$$P_1^2 = \left(P_1^2\right)_{vac} + \frac{4\pi^2}{m_s g l_{cg}}\left(K_1 \rho r^5 + K_2 \rho r^3 l_{cg}^2\right) \tag{12}$$

Since K_1 and K_2 are unknowns, results from two series of oscillations with different pendulum lengths must be used. If l_{cg} is the basic length, then the period for the pendulum extended by length d is

$$P_2^2 = \left(P_2^2\right)_{vac} + \frac{4\pi^2}{m_s g\left(l_{cg} + d\right)}\left[K_1\rho r^5 + K_2\rho r^3\left(l_{cg} + d\right)^2\right] \tag{13}$$

In slope-intercept form, the periods are given by

$$P_1^2 = \left(P_1^2\right)_{vac} + A_1\rho \tag{14}$$

$$P_2^2 = \left(P_2^2\right)_{vac} + A_2\rho \tag{15}$$

where

$$A_1 = \frac{4\pi^2}{m_s g l_{cg}}\left[K_1 r^5 + K_2 r^3 l_{cg}^2\right] = \frac{P_1^2 - \left(P_1^2\right)_{vac}}{\rho} \tag{16}$$

$$A_2 = \frac{4\pi^2}{m_s g\left(l_{cg} + d\right)}\left[K_1 r^5 + K_2 r^3\left(l_{cg} + d\right)^2\right] = \frac{P_2^2 - \left(P_2^2\right)_{vac}}{\rho} \tag{17}$$

Equations (16) and (17) can be used to obtain

$$K_2 = \frac{m_s g}{4\pi^2 r^3}\left[\frac{l_{cg}\left(A_2 - A_1\right) + A_2 d}{d\left(2l_{cg} + d\right)}\right] \tag{18}$$

$$K_1 = \frac{m_s g l_{cg} A_1}{4\pi^2 r^5} - K_2\frac{l_{cg}^2}{r^2} \tag{19}$$

In pendulum oscillation tests, l_{cg} is usually made much greater than r; therefore, small errors in K_2 will be amplified in the calculation of K_1 by means of the $\left(\frac{l_{cg}}{r}\right)^2$ term of equation (19). Also, K_1 and K_2 are found to be the small differences of large numbers, which may result in poor accuracy. Values of K_1 are best calculated from a special experiment in which the disk is oscillated about a diameter. The restoring moment can be provided by adding a weight inside the disk at a known distance from the axis of rotation. Under these conditions it can be shown in a manner analogous to the derivation of equation (19) that

$$K_1 = \frac{m_0 g l_0}{4\pi^2 r^5}\left[\frac{P_0^2 - \left(P_0^2\right)_{vac}}{\rho}\right] \tag{20}$$

where m_0 is the concentrated mass used to provide restoring moment to the disk and l_0 is the distance of this mass from the center of the disk. This statement means that errors in K_2 will not affect the value of K_1. If K_1 is obtained as indicated, then K_2 can be obtained more easily than in equation (18) by rearranging equation (12) to give

$$K_2 = \frac{m_s g}{4\pi^2 r^3 l_{cg}} \left[\frac{P_1^2 - \left(P_1^2\right)_{vac}}{\rho} \right] - K_1 \frac{r^2}{l_{cg}^2} \tag{21}$$

For $r << l_{cg}$ the value of K_1 will not have an appreciable effect on the value of K_2 computed by using this equation.

Experimental Procedure

The tests for the experimental determination of apparent mass and moment of inertia of a disk were conducted in a 15-meter-diameter vacuum sphere at the Langley Research Center at nominal air densities from 0.00154 to 1.225 kg/m^3. The corresponding air pressures were from 1 to 760 mm of mercury (1 mm Hg = 133.3 N/m^2). The basic disk configuration is shown in figure 3. The disk, which had a 0.9525-meter radius, consisted of a lightweight, balsa-wood frame, both faces of which were covered by sheets of polyethylene. The disk was supported from the ceiling by means of two inextensible strings (dacron 20 kg fishing line) clamped at the top of the sphere. Figure 4 is a photograph of the disk as tested in the zero-length configuration and suspended by ball-bearing supports, which increased the mass somewhat. The restoring mass was located at the bottom of the disk as indicated. For each test, the disk was displaced approximately 10° and permitted to oscillate freely as a simple pendulum.

Early in the experiment, the period was obtained visually by using a manually operated stop watch; later in the experiment, the timing data were supplemented by results from an electronic-photocell-actuated timer and counter. The timer was accurate within 0.1 second and the stop watch was inherently accurate to 0.01 second. However, a certain amount of error was present in the visual timing because the observer viewed the test from outside the sphere through observation ports, approximately 10 meters from the disk. For purposes of this experiment, both timer and observer data are taken as accurate to 0.1 second. Comparison runs using both timer and stop-watch data yielded no detectable difference.

The pendulum was allowed ten or more cycles for each test, except at the two highest air-density values for which the rapid damping forced a reduction to five cycles. Ten or more tests were conducted at each of the six values of air density selected for testing and for each pendulum length. The nominal pressures were 1, 21, 179, 375, 540, and 760 mm of mercury. Pressures were recorded before and after each test and all discrepancies were less than 6 mm even at the highest pressures, as ample time was allowed to attain temperature stability before each series of tests at a given pressure. Air densities were computed from pressure measurements by using the perfect gas law

$$\rho = \frac{p}{RT}$$

where R is the universal gas constant and T is the uncontrolled ambient air temperature. Both p and T were measured at the test conditions. Density measurements were accurate within about 1 percent, the error being due mainly to changes in air temperature during the course of an experiment. Length measurements were accurate to 0.01 meter. During a given evacuation sequence, string length changes were found to be less than 0.007 meter. Mass was measured to an accuracy of 0.03 kg.

Statistical Treatment of Data

The time for each series of swings was divided by the number of swings to obtain the average measured period. Each set of periods (at a given pressure) was then subjected to a computer program for statistical analysis in which the mean, standard deviation, and individual deviations were calculated. The basic equations are given in appendix B. The individual deviations were then compared with the standard deviation and all data points which showed an individual deviation greater than twice the standard deviation were rejected. To determine the validity of this data-rejection scheme, the rejection criterion was tightened to 1.5 times the standard deviation and a second comparison of the data points was made. Few additional data points were rejected and the result of the experiment was not seriously affected; therefore, the data probably do represent a true picture of the mean pendulum periods at the air densities of this investigation.

The surviving period and density data were then adjusted to a best, straight-line fit, in a least-squares sense (ref. 3), to a plot of period squared against air density. The periods, taken from the straight line at selected density values, were used in subsequent calculations of the moment of inertia and apparent mass.

RESULTS AND DISCUSSION

Five pendulum lengths were tested, counting the zero-length pendulum which was weighted with a point mass at the bottom. A summary of the pertinent results is presented as table I for two data-rejection criteria, a deviation of twice the standard deviation and of 1.5 times the standard deviation. Figure 5, which shows period squared as a function of density, includes data points from table I along with the least-squares best-fit straight lines. Figure 6 shows the total moments of inertia obtained from the periods of figure 5 (by use of eq. (5)) as a function of air density. Figure 7 shows the moments of inertia (from eq. (6)) due to the apparent mass only, that is, with the structural inertia subtracted.

The value of K_1 as calculated for the zero-length pendulum (eq. (20)) was found to be

$$K_1 = 0.3933 \pm 0.0127$$

Lamb's theoretical value (ref. 4) is

$$K_1 = 0.3555$$

The average value of K_2 as calculated from equation (21) was found to be

$$K_2 = 2.9269 \pm 0.1195$$

Lamb's value is

$$K_2 = 2.667$$

It can be seen that the experimental values are approximately 10 percent higher than the theoretical values, which is considered good agreement within the experimental framework.

As a matter of interest, m_a was computed by using equation (8). Because three pendulum lengths are required for use of this equation and because data were actually taken for four pendulum lengths (excluding the zero-length case), there were four combinations available for computing m_a. The results of this calculation are shown in table II for which the air density was taken at 1 atm. The apparent inertias used in making this calculation are given in table III along with the probable error.

The wide range of the values of m_a determined in this manner indicates that equation (8) is sensitive to error in I_a. Table III shows that errors in I_a were as large as 2.12 percent. These errors in I_a resulted in errors as large as 100 percent in the apparent mass, which would be unacceptable. This sensitivity to I_a, of course, results from the fact that equation (8) involves taking small differences of large numbers. The values of m_a calculated from equation (9b) and experimental values of K_2 are also given at the bottom of table II. The error in m_a is much smaller, around 5 percent.

CONCLUDING REMARKS

An experiment has been performed to measure the apparent mass of air and added moment of inertia of the air associated with a freely swinging disk pendulum. Nondimensional coefficients which determine the apparent inertia about the center of gravity and the apparent mass in translation, K_1 and K_2, respectively, have been determined experimentally for a large disk oscillating in still air at air densities up to 1 atmosphere. Experimental values are approximately 10 percent higher than Lamb's theoretical values.

The experimental apparent inertia I_a is given by the formula

$$I_a = K_1 \rho r^5 + K_2 \rho r^3 l_{cg}^2$$

where ρ is air density, r is the radius of the disk, l_{cg} is the distance from the pivot point to the center of the disk, and

$$K_1 = 0.3933 \pm 0.0127$$

$$K_2 = 2.9269 \pm 0.1195$$

In tests to determine the apparent mass and inertia of a general configuration for which the structural center of mass coincides with the center of apparent mass, the body should be supported from the center of gravity if possible. Using this support point will allow the value of K_1 to be calculated independently of K_2 and will subject K_1 to as few uncertainties as possible. However, if conditions are such that the structural center of mass does not coincide with the center of apparent mass, the general method can be used. Caution should be exercised in using the general method since small errors in measuring the moment of inertia can have serious adverse effects on the resultant apparent mass.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., April 24, 1970.

APPENDIX A

JUSTIFICATION OF EQUATION (9)

The moment of inertia of an object about an axis is given by

$$I = I_{cg} + ml_{cg}^2 \tag{A1}$$

The moment of inertia about the center of gravity can be expressed as

$$I_{cg} = mk^2$$

where k is the radius of gyration and is proportional to r for a disk. Therefore, equation (A1) can be written as

$$I = mk^2 + ml_{cg}^2 \tag{A2}$$

Because mass is proportional to density and the cube of a linear dimension, equation (A2) can be expressed as

$$I = K_1 \rho r^5 + K_2 \rho r^3 l_{cg}^2 \tag{A3}$$

APPENDIX B

STATISTICAL TREATMENT OF DATA

The period data were subjected to two data-rejection tests in which all data points having a deviation greater than twice the standard deviation and 1.5 times the standard deviation were rejected. The two data-rejection criteria yielded essentially the same result. Data were then adjusted in a least-squares best-fit sense to a straight-line plot of P^2 against ρ. The technique employed was a linear regression on the straight-line equation

$$P^2 = \alpha\rho + \beta$$

The desired values of α and β were obtained by using the equations

$$\alpha = \frac{\sum^{n} \rho_i^2 \sum^{n} P_i^2 - \sum^{n} \rho_i \sum^{n} \left(\rho_i P_i^2\right)}{n \sum^{n} \rho_i^2 - \left(\sum^{n} \rho_i\right)^2} \tag{B1}$$

and

$$\beta = \frac{n \sum^{n} \left(\rho_i P_i^2\right) - \sum^{n} \rho_i \sum^{n} P_i^2}{n \sum^{n} \rho_i^2 - \left(\sum^{n} \rho_i\right)^2} \tag{B2}$$

where n is the number of data points. The slope of the resultant straight line was used in subsequent calculations. The standard deviation of the variation of period squared with density was calculated from the sum and difference expression

$$\sigma = \sqrt{\frac{n}{n - 2} \frac{\sum^{n} \left(\delta P_i^2\right)^2}{n \sum^{n} \rho_i^2 - \left(\sum^{n} \rho_i\right)^2}} \tag{B3}$$

Inertias, apparent masses, and values of the Lamb constants K_1 and K_2 were then calculated from the standard propagation-of-error expression for uncorrelated errors by using equation (37) of reference 3

$$\sigma_z = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 \sigma_y^2} \tag{B4}$$

where z is the parameter of interest.

Final results are always presented in terms of the most probable error, under the assumption that the data are representative of a normal distribution. The most probable error is that magnitude of deviation for which the probability of being exceeded is one-half, specifically, the probability is equal to 0.6754 times the standard deviation.

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TABLE I.- SUMMARY OF EXPERIMENTAL DATA USING TWO DATA-REJECTION CRITERIA

[Disk radius = 0.9525 meter]

Mass, m, kg	l_{cg}, m	Density, ρ, kg/m^3	2σ data-rejection criterion (a)		1.5σ data-rejection criterion (b)	
			Mean period, sec	Standard deviation, σ, sec	Mean period, sec	Standard deviation, σ, sec
4.42	0.00	0.00161	3.220	0.000	3.220	0.000
4.42	weighted slug	.0339	3.230	.000	3.230	.000
4.42	of mass	.287	3.297	.005	3.297	.005
4.42	0.850 kg	.609	3.373	.010	3.373	.010
4.42	0.902 m from	.875	3.436	.009	3.440	.000
4.42	c.g.	1.24	3.517	.018	3.517	.018
4.21	4.57	0.00150	4.323	0.005	4.323	0.005
4.21	4.57	.0349	4.375	.005	4.375	.005
4.21	4.57	.287	4.690	.000	4.690	.000
4.21	4.57	.618	5.060	.000	5.060	.000
4.21	4.57	.886	5.307	.010	5.307	.010
4.21	4.57	1.24	5.582	.016	5.582	.016
4.21	6.40	0.00166	5.100	0.000	5.100	0.000
4.21	6.40	.0344	5.168	.008	5.165	.005
4.21	6.40	.290	5.524	.008	5.524	.005
4.21	6.40	.602	5.990	.018	5.987	.010
4.21	6.40	.870	6.285	.015	6.290	.011
4.21	6.40	1.22	6.691	.010	6.691	.010
4.21	8.04	0.00161	5.705	0.005	5.706	0.005
4.21	8.04	.0338	5.775	.016	5.774	.012
4.21	8.04	.289	6.208	.016	6.214	.011
4.21	8.04	.601	6.705	.037	6.705	.037
4.21	8.04	.875	7.140	.016	7.140	.016
4.21	8.04	1.24	7.594	.019	7.600	.000
4.21	9.14	0.00161	6.088	0.008	6.089	0.007
4.21	9.14	.0339	6.159	.008	6.159	.008
4.21	9.14	.287	6.640	.016	6.635	.013
4.21	9.14	.291	6.668	.020	6.655	.005
4.21	9.14	.609	7.208	.018	7.208	.010
4.21	9.14	.614	7.195	.018	7.200	.000
4.21	9.14	.874	7.596	.044	7.608	.010
4.21	9.14	.882	7.602	.016	7.602	.016
4.21	9.14	1.24	8.099	.015	8.099	.015

[a] Data are rejected if the absolute value of the deviation exceeds two times the standard deviation.

[b] Data are rejected if the absolute value of the deviation exceeds 1.5 times the standard deviation.

TABLE II.- APPARENT MASS OF 0.9525-METER-RADIUS DISK AT 1 ATMOSPHERE

[Different suspension lengths taken in groups of three]

l_{cg}, m	$l_{cg} + d_1$, m	$l_{cg} + d_2$, m	m_a, kg (eq. (8))
6.40	8.04	9.14	1.1738 ± 2.555
4.57	6.40	8.04	4.4407 ± 0.9964
4.57	6.40	9.14	3.6535 ± 0.4953
4.57	8.04	9.14	2.4824 ± 1.3912
Mean of the experimental values			2.9376 ± 1.798 kg
Calculated from equation 9(b)			3.1080 ± 0.1425 kg

TABLE III.- APPARENT INERTIA OF 0.9525-METER-RADIUS DISK AT 1 ATMOSPHERE

l_{cg}, m	I_a, kg-m^2	Probable error, percent
4.57	59.356 ± 1.256	2.12
6.40	125.884 ± 1.394	1.01
8.04	210.612 ± 2.745	1.30
9.14	271.096 ± 2.531	.93

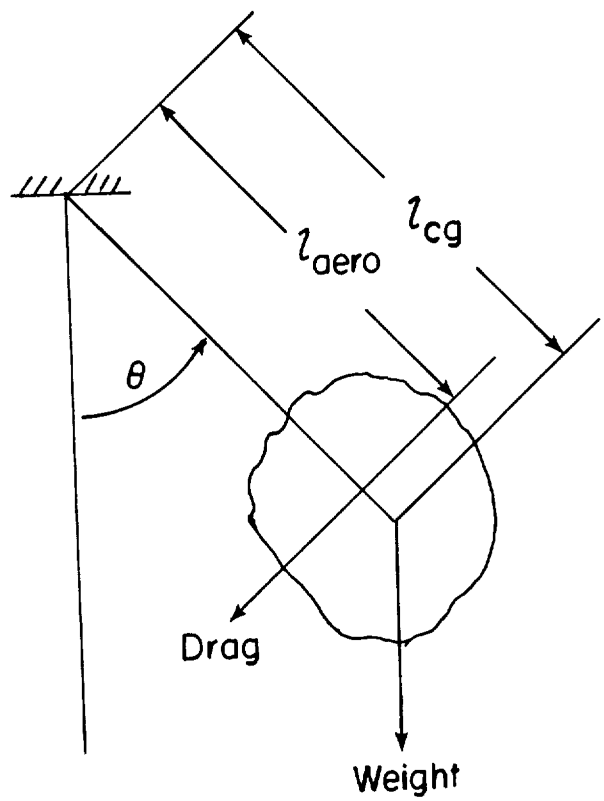

Figure 1.- Physical pendulum showing relative orientation of principal forces and distances.

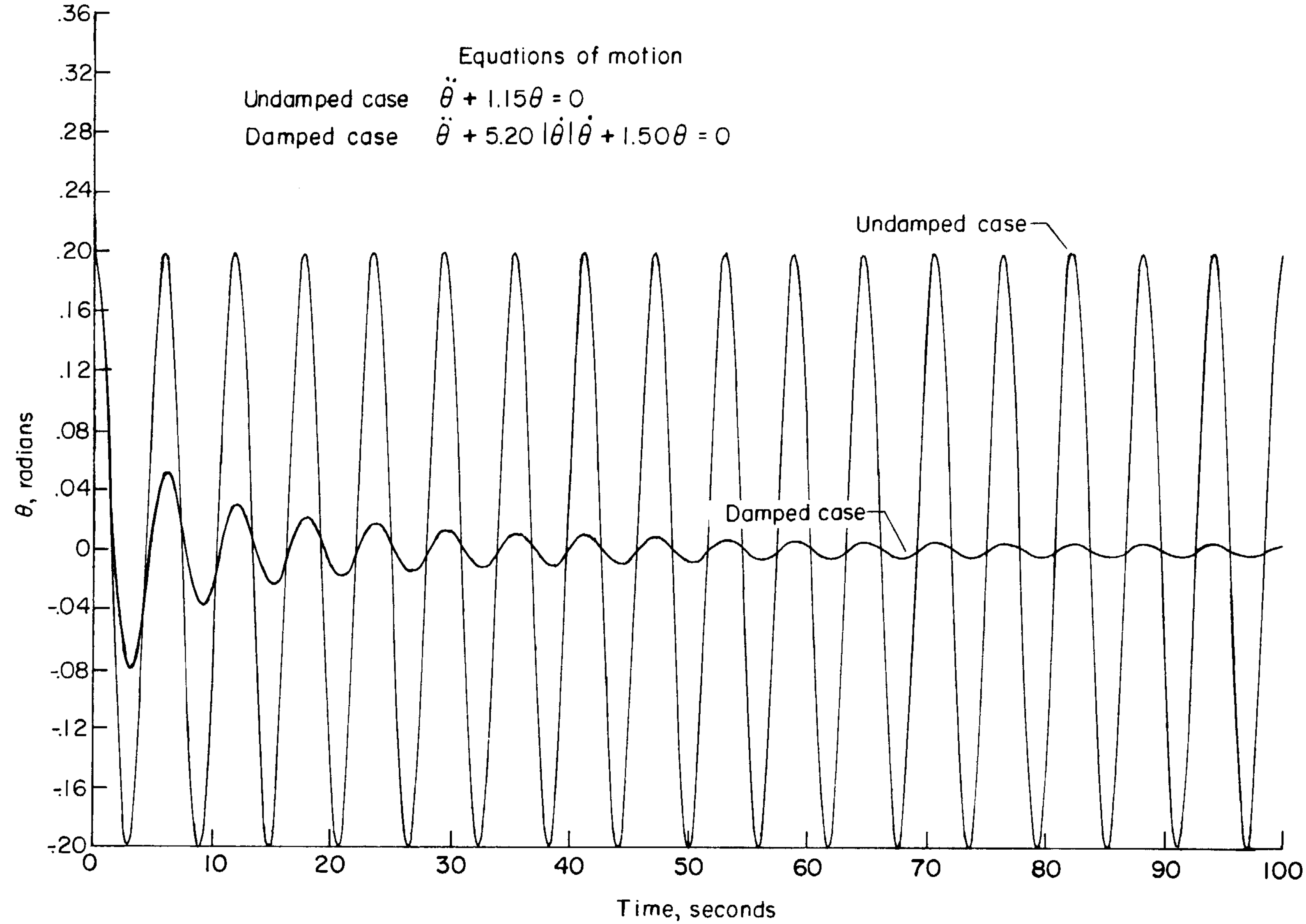

Figure 2.- Typical time histories (eq. (4)) for damped and undamped oscillations. Initial conditions: $\theta_0 = 0.2$ rad, $\dot{\theta}_0 = 0$ rad/sec.

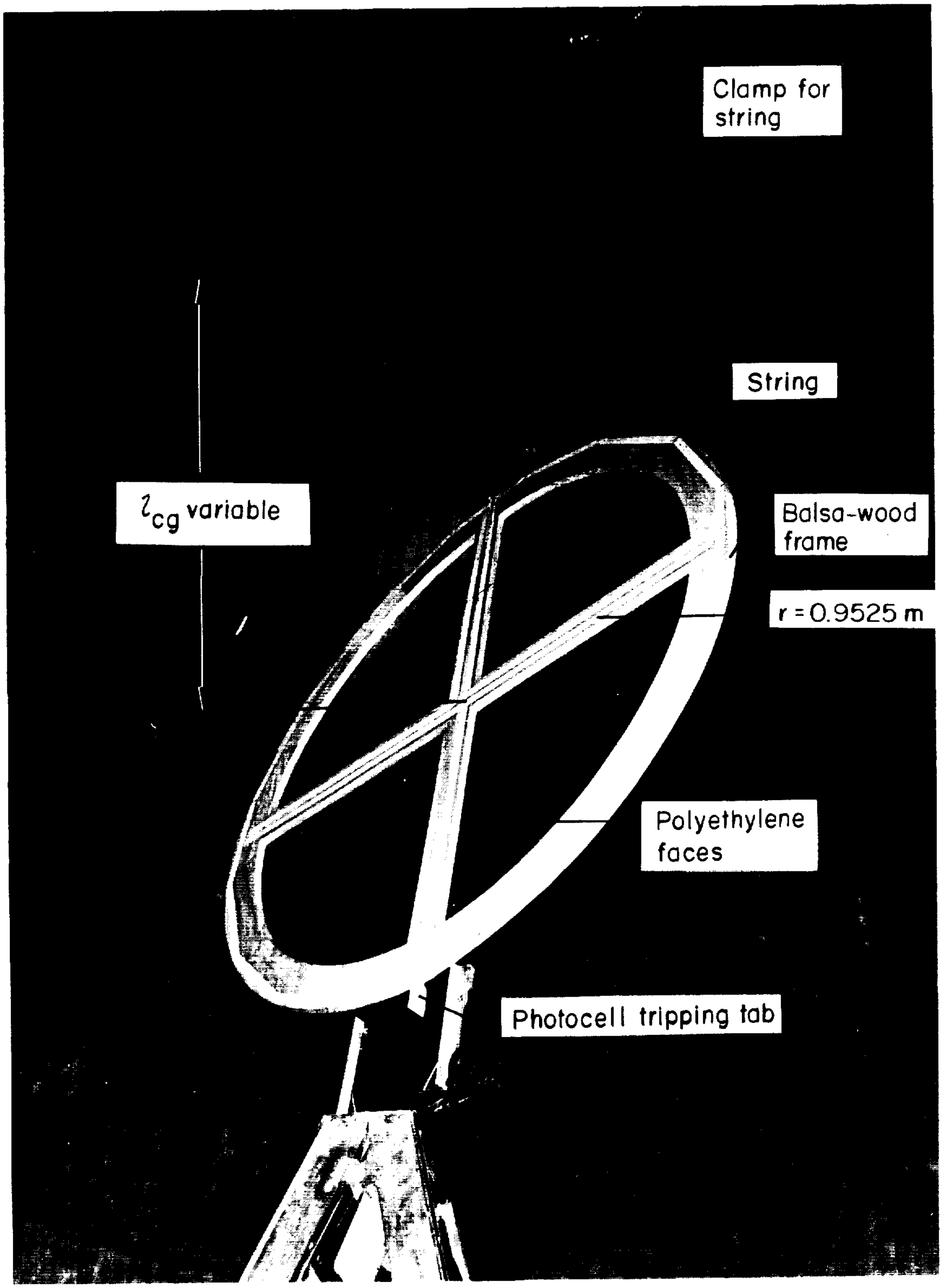

Figure 3.- Photograph of disk in experimental setup. L-69-2068.1

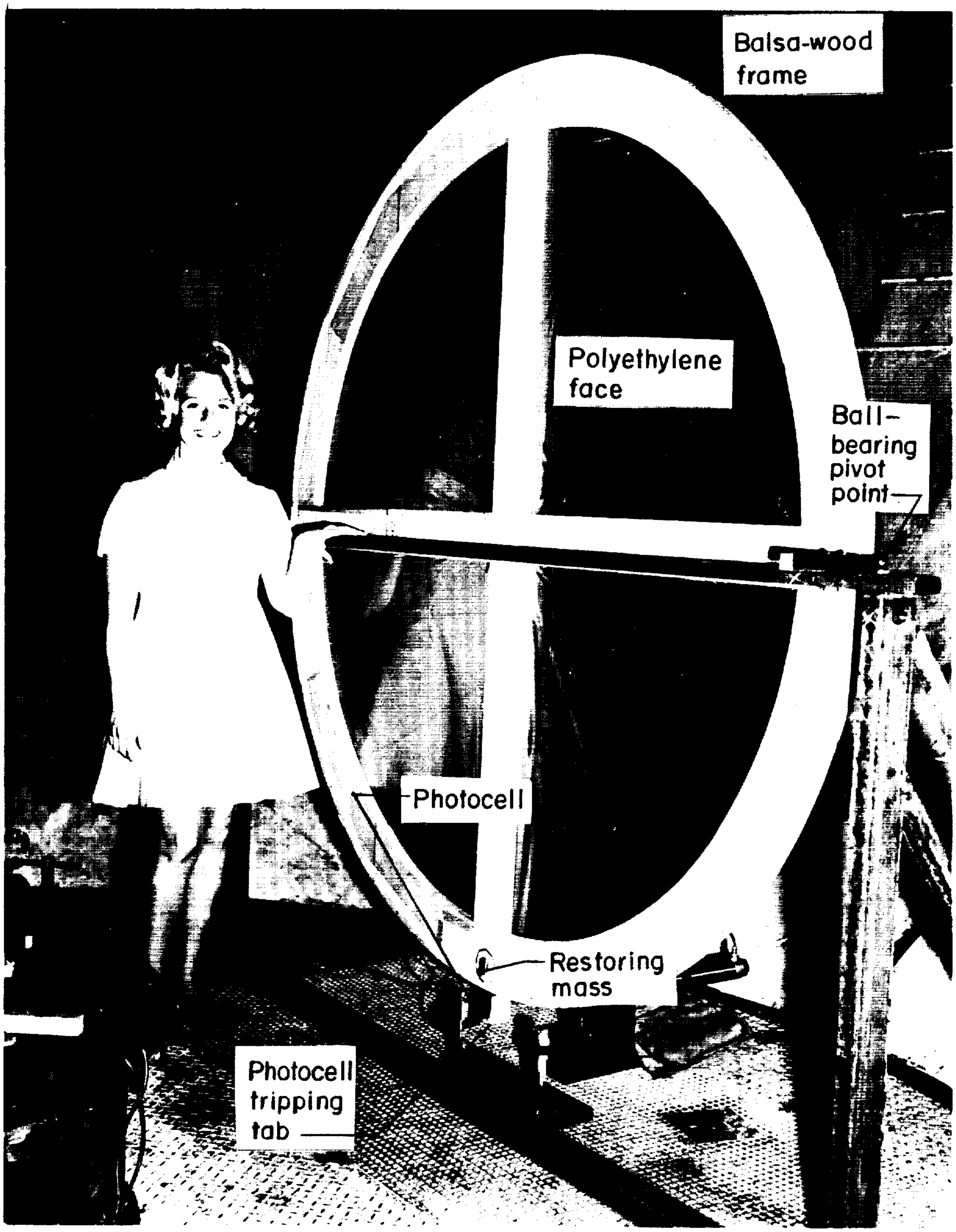

Figure 4.- Disk in zero-length configuration. L-69-3059.1

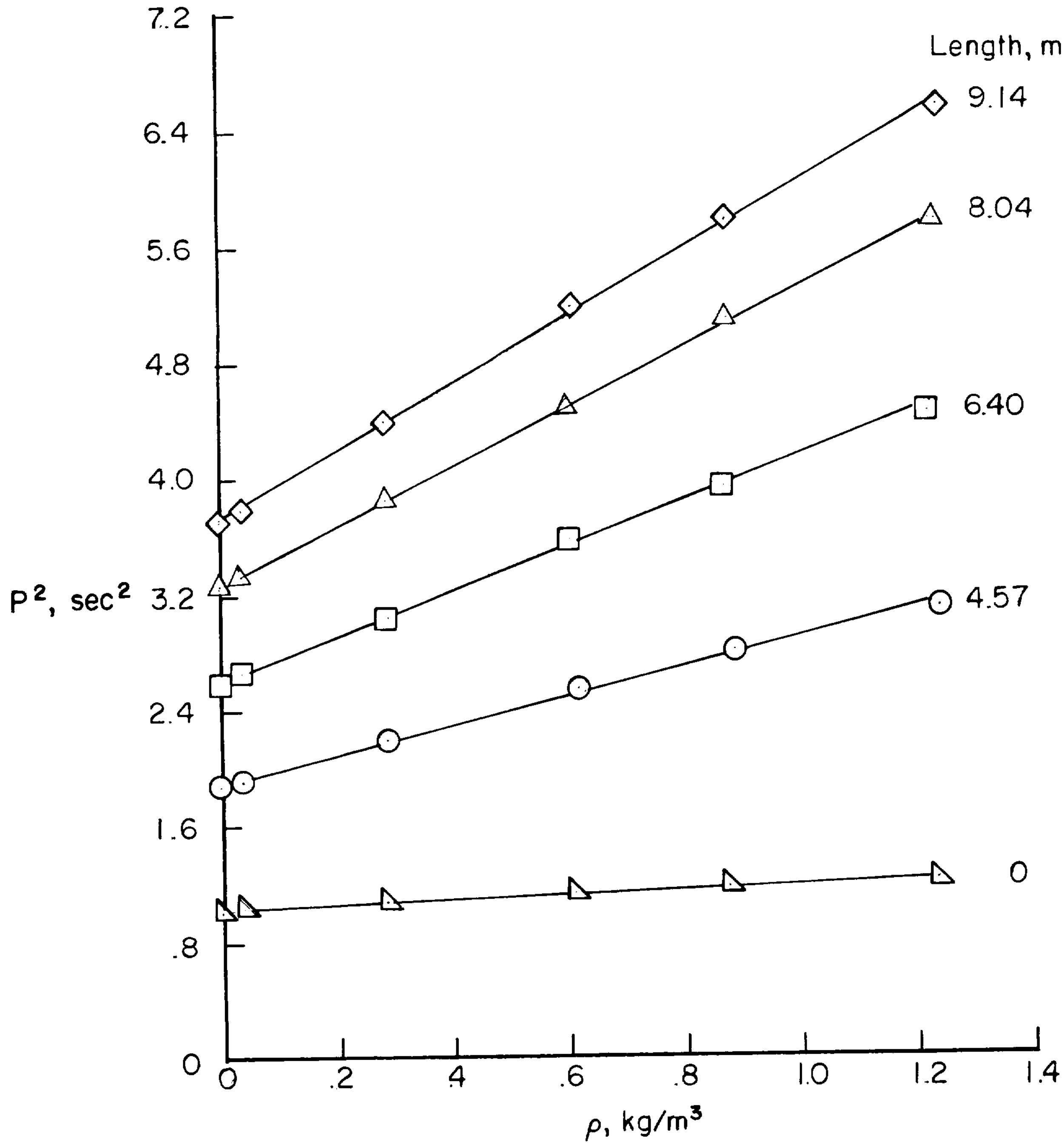

Figure 5.- Period squared as a function of density, data points based on 1.5σ rejection criterion, and least-squares best-fit straight lines.

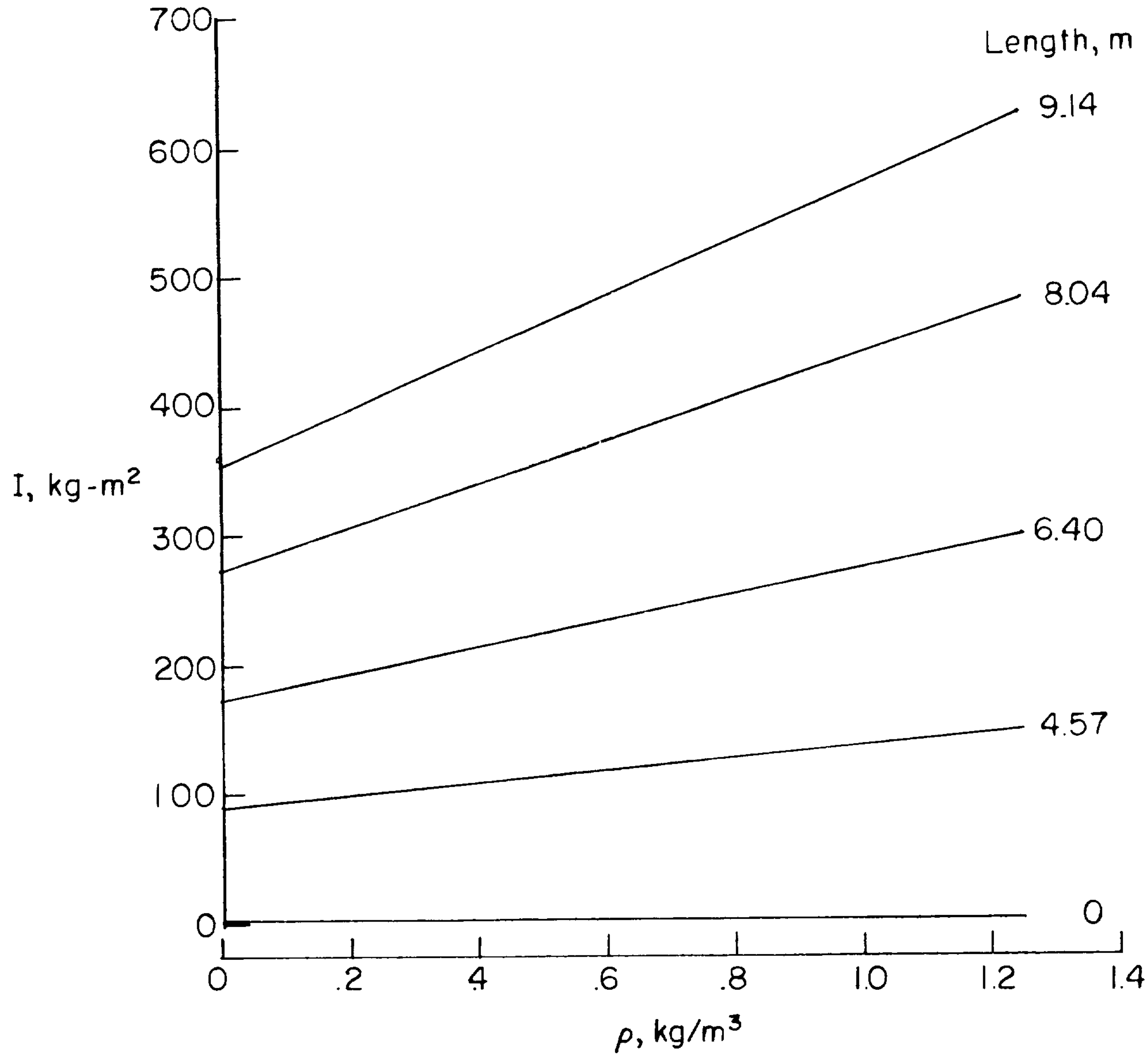

Figure 6.- Moment of inertia of structural and apparent mass as a function of air density.

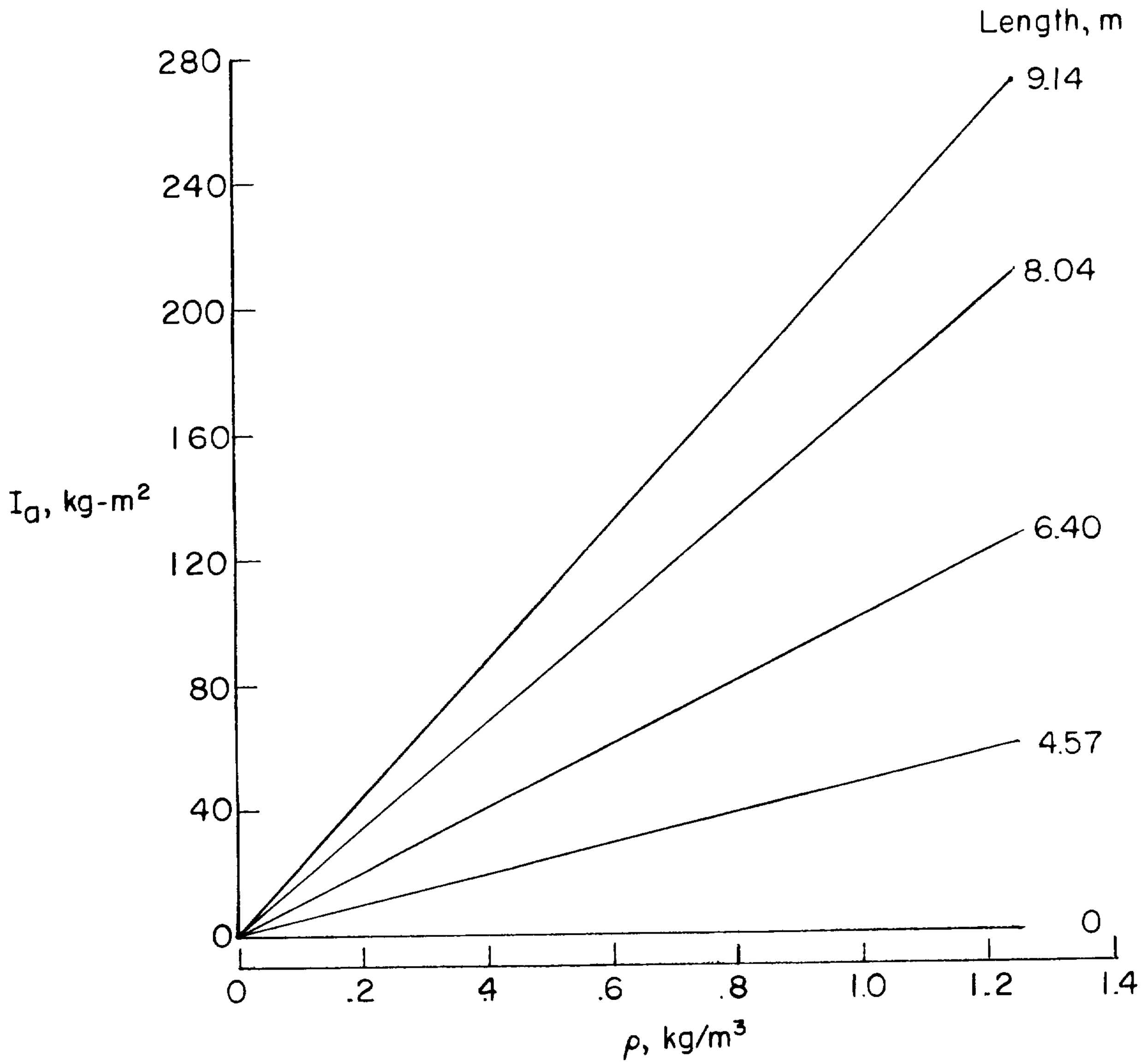

Figure 7.- Moment of inertia of apparent mass as a function of air density.