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TITLE– Use of Magnetic Torque for CMG Momentum Management

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AUTHOR(S)– W. Levidow

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Control Moment Gyroscopes
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ABSTRACT

This study shows that an on-board magnetic dipole reacting with the earth's magnetic field can be used for dumping CMG bias momentum. System implementation on a Second AAP Workshop would require only the addition of a magnetometer to measure the earth's magnetic field, an electromagnet to produce the magnetic torque, a control amplifier to supply the proper current to the magnet coils, and a small amount of digital computer software. Operation is automatic and momentum dump attitude maneuvers are eliminated.

A magnetic moment control law is devised which closely approximates the one that requires minimum electric energy from the power supply. The control law is evaluated for three attitude modes-POP (long axis perpendicular to the orbital plane), IOP (long axis in the orbital plane), and 45° OP (long axis 45° to the orbital plane). The 45° OP mode, although undesirable for the Second Workshop, is considered only to establish an upper bound for the magnetic torque requirement.

Minimum weight magnetic design equations are developed and actual air coil and iron core coil magnet designs are presented for a Second Workshop. Using 20 ft diameter air coils, the total required coil and power supply weights are 134 lbs for the POP mode and 366 lbs for the IOP mode. For the IOP mode the maximum magnetic field produced at the center of the coils is only 2-1/2 times that of the earth's field. The coil field decreases rapidly with distance from the coil, allowing the magnetometer to be so located that its readings are not adversely affected.

A method is also presented for quickly estimating magnet designs for variations of Workshop inertia properties.

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955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

SUBJECT: Use of Magnetic Torque for CMG Momentum Management

DATE: December 29, 1969

FROM: W. Levidow

TECHNICAL MEMORANDUM

1.0 INTRODUCTION

Gravity gradient and aerodynamic torques acting on the AAP Workshop are counteracted by the momentum exchange action of the control moment gyros (CMGs) used for attitude control. However, the momentum accumulation due to the bias components of these torques must be dumped before CMG saturation is reached.

For a Second AAP Workshop, for which stellar astronomy is being considered, it would be preferable to dump this bias momentum with a scheme that does not require attitude maneuvers. This memorandum investigates such a dump scheme, using an on-board magnetic dipole interacting with the earth's magnetic field. A magnetic moment control law is suggested and minimum weight designs presented for both air and iron core coils.

2.0 MAGNETIC DUMP CONCEPT

In order to prevent momentum saturation, it is necessary to establish an external torque on the vehicle directed opposite to the bias momentum. If a magnet of magnetic moment $\underline{M}$ is placed in the earth's magnetic field $\underline{B}$, a torque $\underline{T}$ acts on the magnet such that

$$\underline{T} = \underline{M} \times \underline{B} \qquad (1)$$

If the magnet is fixed to a vehicle located in the earth's magnetic field, this torque is transmitted to the vehicle.

The momentum change produced by this magnetic torque is given by

$$\Delta\underline{H} = \int_0^t T dt = \int_0^t (\underline{M}\times\underline{B}) dt$$

If, as shown in Fig. 1, $\underline{B}$ is the earth's magnetic field and $\underline{H}_b$ is the bias component of the CMG momentum, dumping requires establishing a magnetic torque opposite to $\underline{H}_b$.

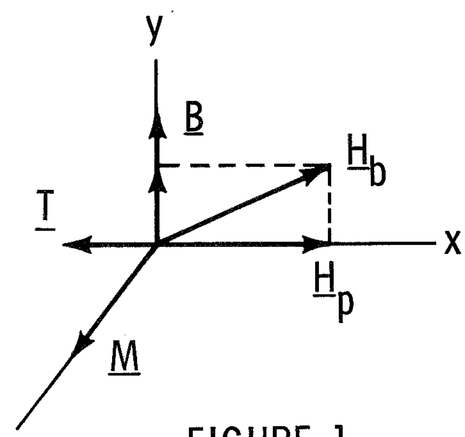

FIGURE 1

However, equation (1) indicates that a magnetic torque can be developed only perpendicular to $\underline{B}$. At best then, a torque $\underline{T}$ can be developed opposite to $\underline{H}_p$, the component of $\underline{H}_b$ perpendicular to $\underline{B}$. It can be developed by providing a magnetic moment $\underline{M}$ perpendicular to $\underline{H}_p$. Its effect in time is to cancel or dump $\underline{H}_p$.

If $\underline{B}$ maintained a fixed orientation over an orbit, it would be impossible to dump the component of the bias momentum parallel to $\underline{B}$. But except for the orbit lying in the magnetic equatorial plane, the direction of $\underline{B}$ relative to an inertially stabilized vehicle varies with orbital position and earth rotation (Appendix A). As $\underline{B}$ changes direction, it is possible to continually dump the component of $\underline{H}_b$ that is perpendicular to $\underline{B}$. This action over an orbit can dump all components of $\underline{H}_b$.

3.0 MAGNETIC MOMENT CONTROL LAW

To determine the required magnetic moment $\underline{M}(t)$ for a particular orbit, one can solve the momentum equation

$$\int_0^T (\underline{M}(t) \times \underline{B}(t))dt = -\underline{H}_b \quad (2)$$

where $\underline{B}(t)$ is the earth's magnetic field variation for that orbit and $\underline{H}_b$ is the orbital bias momentum due to gravity gradient and aerodynamic torques. There is not a unique solution unless a constraint is imposed on $\underline{M}$, for any number of profiles of $\underline{M}$ can satisfy the above equation. It will be shown later that for a given magnet coil resistance, the power required to produce $\underline{M}$ is proportional to the square of the magnitude of $\underline{M}$. A reasonable constraint then is one which minimizes $\int_0^T |\underline{M}|^2 dt$, resulting in minimum electrical energy for that orbit. The solution (Appendix B) is

$$\underline{M}(t) = [(-\int_0^T \underline{\tilde{B}}^2(t)dt)^{-1}\underline{H}_b] \times \underline{B}(t) \qquad (3)$$

where $\tilde{B}$ is the matrix equivalent to the vector cross product operation $\underline{B}\times$.

The quantity within the bracket is a constant vector for the orbit. $\underline{M}(t)$ varies over the orbit due to variations in $\underline{B}(t)$. This solution for $\underline{M}(t)$ could be implemented but it requires knowledge of both $\underline{\tilde{B}}(t)$ and $\underline{H}_b$ before the orbit is flown. Because these quantities are not precisely known in advance, and because of the computational requirement involved, the solution is not as attractive as the following control law which it suggests.

Let $\underline{M}(t)$ be defined over an orbit as

$$\underline{M}(t) = k\underline{H}_s \times \underline{B}(t)$$

where $\underline{H}_s$ is the CMG momentum (computed from gimbal angles) sampled at the beginning of the orbit. $\underline{M}(t)$ varies over the orbit due to variations in $\underline{B}(t)$. If the vector $k\underline{H}_s$ is set equal to the vector in the brackets of equation (3), then the two solutions are identical. This can be accomplished by both adjustment of the CMG gimbal angles at the initial sampling and selection of the scale factor k. With this initialization, exactly $\underline{H}_b$ is dumped on the following orbit.

If the variation of $\underline{B}(t)$ from orbit to orbit were identical, then $\underline{H}_b$ would be dumped each orbit and $\underline{H}_s$ would be identical at each sampling (the cyclic component of the CMG momentum does not affect the once per orbit sample value). However, $\underline{B}(t)$ is not identical from orbit to orbit because the earth's rotation varies the direction of the earth's magnetic dipole relative to the orbital plane. As a result, a momentum somewhat different from $\underline{H}_b$ is dumped for some orbits. $\underline{H}_s$ then varies within a small bounded region determined by the scale factor k. Furthermore, simulation of this control law shows that even if $\underline{H}_s$ is not initialized properly, it converges to within the same region in a few orbits; it automatically adjusts to provide the proper $\underline{M}(t)$ for dumping. For example, if $\underline{B}(t)$ and $\underline{H}_b$ during a particular orbit are such that the control law provides insufficient dumping, $\underline{H}_s$ at the next sampling is changed.

$\underline{M}(t)$ is then different in the following orbit resulting in greater dumping action. Thus $\underline{H}_s$ automatically adjusts for orbit by orbit changes in $\underline{B}(t)$ and $\underline{H}_b$.

$\underline{H}_s$ can be made to converge to a different region by setting $\underline{M}(t) = k(\underline{H}_s - \underline{H}_o) \times \underline{B}(t)$ where $\underline{H}_o$ is the desired shift in the region.

Except for the change in $\underline{B}(t)$ from orbit to orbit resulting from the earth's rotation, this $\underline{M} = k\underline{H}_s \times \underline{B}$ control law requires minimum energy from the power supply.

4.0 SYSTEM IMPLEMENTATION

Figure 2 is a simplified block diagram of the magnetic dump system. The additional equipment over that required by the present Attitude Control System are the magnetometer (approximate weight = 10 lbs) to measure the earth's magnetic field, the control amplifier to control the currents to the magnets, and the magnet coils.

$\underline{H}_s$ is obtained by sampling the CMG momentum which is already available in the digital computer. The impact on the computer is of a software nature and the computer time requirement is small.

5.0 $\underline{M} = k\underline{H}_s \times \underline{B}$ SIMULATION

The control law was simulated for a Second Dry Workshop with a stellar ATM located on the side of the MDA. The CSM is axially docked. The Workshop flies in a 220 nm, 35° inclination orbit.

Three orientation modes (Fig. 3) were considered - POP (long axis perpendicular to the orbital plane), IOP (long axis in the orbital plane) and 45° OP* (long axis 45° to the orbital plane).

For the POP mode, the bias momentum is due primarily to the bias aerodynamic torque resulting from the effect of the diurnal bulge. The aerodynamic torque model at Bellcomm produces an aerodynamic bias momentum of 225 ft-lb-sec with the sun in the orbital plane. The gravity gradient bias momentum due to misalignment of the principal and geometric axes can be reduced to 30 ft-lb-sec by a small Z axis maneuver.[3] Both bias momentum vectors lie in the orbital plane.

For the IOP mode, the sun is considered to lie in the orbital plane and the telescope points 45° out of the orbital

*The 45° OP mode is undesirable for the Second Workshop. It is considered only to establish an upper bound on the magnetic torque.

MAGNETIC DUMP SYSTEM

$\underline{M} = k\ \underline{H}_s \times \underline{B}$ CONTROL LAW

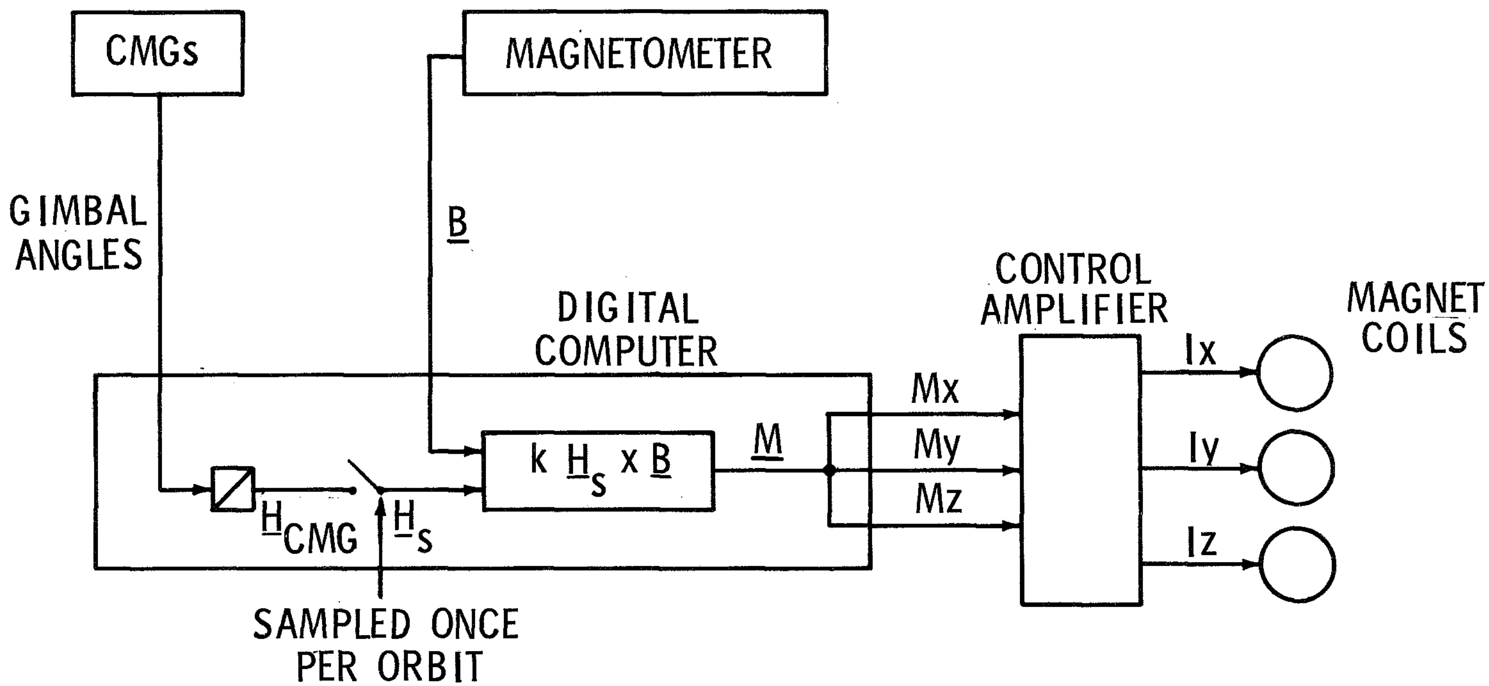

FIGURE 2

plane to a star. This orientation maximizes both the aerodynamic and gravity gradient bias momentum vectors. The former lies perpendicular to the orbital plane and the latter lies in the orbital plane.

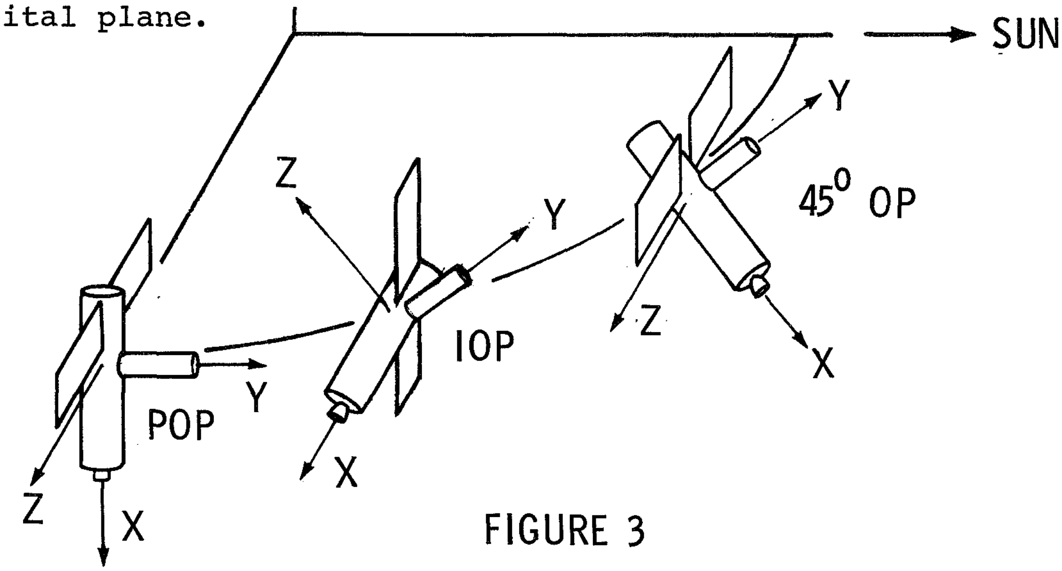

FIGURE 3

For the 45° OP mode, the Z axis lies in the orbital plane and the X axis is rotated 45° out of the orbital plane. This 45° OP orientation is considered here because it represents the upper limit for gravity gradient bias momentum. This vector lies in the orbital plane. The aerodynamic bias momentum is comparatively negligible in this mode.

The estimated principal moments of inertia (slug-ft^2) for the Second Workshop are:* I_x = 634,400, I_y = 4,112,900, I_z = 4,220,600, resulting in per orbit total aerodynamic and gravity gradient bias momenta along the vehicle coordinate axes of

Mode	$\underline{H}_b$ (ft-lb-sec)
POP	(0, 0, -255)
IOP	(575, 160, -160)
45° OP	(0, 0, 19000)

For each mode, POP, IOP, and 45° OP, the simulation consisted of setting $k\underline{H}_s$ for the first orbit equal to the value within the brackets of equation (3). At each successive sampling, $\underline{H}_s$ was incremented by $\Delta\underline{H}_s = \underline{H}_b + \int_0^T \underline{M} \times \underline{B}dt$. Each simulation was run for 16 orbits during which time the earth makes approximately a full rotation.

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*Calculated by W. W. Hough of Bellcomm, Inc. from MSFC data on the first Workshop.

As expected from the previous analysis, the value of k had little effect on $\underline{M}(t)$ when $k\underline{H}_s$ was properly initialized. The simulation also demonstrated that even if $\underline{H}_s$ were initialized to the null vector, a value of k ranging from 3×10^6 to 5×10^6 produced, after a few orbits, the same $\underline{M}(t)$ as when $\underline{H}_s$ had been properly initialized. All data in this report are based on using $k = 4 \times 10^6$, with $\underline{H}_s$ given in ft-lb-sec, $\underline{B}$ in webers per meter^2, and $\underline{M}$ in amp-turn-meter^2. The magnitudes of the variation of $\underline{H}_s$ over 16 orbits were as follows.

	POP	IOP	45°OP	
$\lvert \Delta \underline{H}_s \rvert$*	48	362	4000	ft-lb-sec

Between samplings the variation of CMG momentum is greater due to the cyclic components of the aerodynamic and gravity gradient torques.

The bias momentum vector $\underline{H}_b$ for any mode can lie in a variety of spatial directions depending upon vehicle attitude. Since the $\underline{M}(t)$ profile required to dump $\underline{H}_b$ depends upon the angular relationship with time between $\underline{B}$ and $\underline{H}_b$, $\underline{M}(t)$ depends upon vehicle attitude. The simulations in this study are for the solar array and telescope orientation within each mode that requires the largest magnetic moment.

It will be shown later that the wave shape of $|\underline{M}|$ vs time, in particular its maximum value M_m and its root mean square values M_n, M_d, M_o during orbital night, day, and entire orbit determine the weight penalty of magnetic dumping. The greatest values encountered over 16 orbits are given below in Table 1.

Table 1

Mode	Magnetic Moments, amp-turn-meter^2			
	M_n	M_d	M_o	M_m
POP	2320	2600	2500	3180
IOP	6210	6870	6620	9160
45°OP	173,000	193,000	186,000	236,000

*The present CMG system has a $|\Delta \underline{H}_{CMG}|$ capacity of nearly 12000 ft-lb-sec.

It is also interesting to compare the results in Table 1 with the minimum energy solution. Table 2 below is the result of evaluating the minimum energy solution, Equation (3), for each of the 16 orbits and noting the largest values of M_o. M_o is a measure of the energy requirement since $M_o = (\frac{1}{T}\int_0^T |\underline{M}|^2 dt)^{1/2}$ and, as will be shown later, $|\underline{M}|^2$ is proportional to the electric power required to produce $\underline{M}$.

Table 2

Mode	Mo (amp-turn-meter^2)
POP	2380
IOP	6040
45°OP	184,000

As expected, the results of Tables 1 and 2 are not strikingly different because the $\underline{M} = k\underline{H}_s \times \underline{B}$ control law approximates the minimum energy solution.

6.0 MAGNET STRUCTURES

Evaluation of any proposed magnetic dumping control law should include the effect of the resulting $\underline{M}$ vs time wave shape on the weight of the required magnetic structure and the weight of the power supply for energizing the magnet. The magnetic designs considered in this memorandum are:

1) Gimbaled circular air coil

2) Three fixed orthogonal circular air coils

3) Gimbaled iron core coil

4) Three fixed orthogonal iron core coils

For all designs, the magnitude of the magnetic moment of each coil is directly proportional to its current. For the gimbaled designs, the direction of the magnetic moment $\underline{M}$ is controlled by orienting the magnetic structure by means of a two degree of freedom mechanism.

For the three orthogonal coil designs, the direction of $\underline{M}$ is controlled by properly apportioning the energizing current among the three coils.

If I_i is the current through the i^{th} coil and if K is the constant relating the magnetic moment M_i of the coil to the current, then,

$$I_i = \frac{M_i}{K}$$

The total power to all three coils, each of resistance R, is

$$\begin{aligned} P &= I_1^2 R + I_2^2 R + I_3^2 R \\ &= \frac{R}{K^2} (M_1^2 + M_2^2 + M_3^2) \\ &= \frac{R}{K^2} |\underline{M}|^2 \end{aligned} \qquad (5)$$

Equation (5) shows that for three identical orthogonal coils, the power required is proportional to the square of the magnitude of $\underline{M}$ and hence is independent of its direction. For a given coil design, high peak magnitudes of $\underline{M}$ require large maximum powers. Also, there is no power demand advantage if only one of these coils is used in a gimbaled design instead of three in an orthogonal design, although only one third of the coil weight is required. However, as will be shown later, for minimum system weight the two designs should differ.

7.0 MINIMUM WEIGHT MAGNETIC DESIGN EQUATIONS, DAY AND NIGHT DUMP

If magnetic dumping is done continuously, battery power is required during orbital night. During orbital day, however, solar cells can power the magnet and also recharge the batteries.

Appendix C shows that for day and night dumping the weight of the power supply (solar array, batteries and control amplifier) is given by

$$W_p = K_n P_n + K_d P_d + K_a P_m$$

where

P_m = Maximum power delivered to the magnetic load

P_n, P_d = Average power delivered to the magnetic load during orbital night and day

K_n, K_d = Constants dependent upon the solar array and battery characteristics. K_n also depends upon the length of orbital night and day

K_a = Pounds per watt of the control amplifier. K_a for a three-magnet design is approximately three times that for a gimbaled magnet design because three amplifier channels are required.

Appendix D shows that for all four magnetic designs, the minimum total weight of power supply and conductor (total weight of three coils for the three-magnet design) occurs if the power supply weight equals the conductor weight. Magnetic design equations for this criterion are (Appendix D) as follows.

1) <u>Gimbaled Circular Air Coil</u>

$$I_m N = 1970\, M_m/d^2 \quad \text{amp-turns} \tag{7}$$

$$Na = 1970 \sqrt{\rho/\sigma}\, M_e/d^2 \quad \text{turn-in.}^2 \tag{8}$$

$$P_m = \frac{6200 \sqrt{\rho\sigma}\, M_m^2}{d \quad M_e} \quad \text{watts} \tag{9}$$

$$\text{Conductor Wt*} = \frac{6200 \sqrt{\rho\sigma}\, M_e}{d} \quad \text{lbs} \tag{10}$$

Total Weight = Conductor Weight + Power Supply Weight

= 2 × Conductor Weight

I_m = Maximum Coil Current, amperes

N = Number of turns per coil

M_m = Maximum magnetic moment, amp-turn-meter^2

M_n = Root mean square value of magnetic moment during orbital night

M_d = Root mean square value of magnetic moment during orbital day

$$M_e = \sqrt{K_n M_n^2 + K_d M_d^2 + K_a M_m^2}$$

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*Does not include insulation and winding form weight.

d = Diameter of coil, in.

a = Area of conductor per turn, in.^{2}

ρ = Conductor resistivity, ohms per in.^{3}

σ = Conductor specific weight, lbs per in.^{3}

P_m = Maximum power, watts

2) <u>Three Fixed Orthogonal Circular Air Coils</u>

$$I_m N = 1970\ M_m/d^2 \text{ amp-turns (per coil)} \quad (11)$$

$$Na = 1140\ \sqrt{\rho/\sigma}\ \ M_e/d^2 \text{ turn-in.}^2 \text{ (per coil)} \quad (12)$$

$$P_m = \frac{10700\sqrt{\rho\sigma}}{d}\frac{M_m^{\ 2}}{M_e} \text{ watts (total three coils)} \quad (13)$$

$$\text{Conductor Weight} = \frac{10700\ \sqrt{\rho\sigma}\ M_e}{d} \text{ lbs (total three coils)} \quad (14)$$

Total Weight = Conductor Weight + Power Supply Weight

= 2 × Conductor Weight

For the air coil designs:

1) The weights and maximum powers are inversely proportional to the chosen coil diameter.

2) Except for the slight effect of K_a, which differs for the two designs, the total weights and maximum powers of the three-coil design are $\sqrt{3}$ times those of the gimbaled coil design.

3) The weights of conductor and power supply depend upon the root mean square values of $|\underline{M}|$ during both night and day and also upon the maximum over the orbit.

3) <u>Gimbaled Iron Core Coil</u>

$$\text{Iron Core Length} = 0.461\left(\frac{Q^2}{\alpha B}\right)^{1/3} M_m^{\ 1/3} \quad \text{in.} \qquad (15)$$

$$\text{Iron Core Diameter} = \text{Core length}/Q \quad \text{in.} \qquad (16)$$

$$\text{Iron Core Weight} = \frac{0.0767\ \sigma_i\ M_m}{\alpha B} \quad \text{lbs.} \qquad (17)$$

$$I_m N = 9310 \left(\frac{D^3 Q^2 B^2}{\alpha}\right)^{1/3} M_m^{\ 1/3} \quad \text{amp-turns} \qquad (18)$$

$$Na = 9310 \left(\frac{D^3 Q^2 B^2}{\alpha}\right)^{1/3} \frac{\sqrt{\rho/\sigma}\ M_e}{M_m^{\ 2/3}} \quad \text{turn-in.}^2 \qquad (19)$$

$$P_m = 13500 \left(\frac{D^3 QB}{\alpha^2}\right)^{1/3} \frac{\sqrt{\rho\ \sigma}\ M_m^{\ 5/3}}{M_e} \quad \text{watts} \qquad (20)$$

$$\text{Conductor Weight*} = 13500 \left(\frac{D^3 QB}{\alpha^2}\right)^{1/3} \frac{\sqrt{\rho\sigma}\ M_e}{M_m^{\ 1/3}} \quad \text{lbs} \qquad (21)$$

Total Weight = Conductor Weight + Power Supply Weight
+ Iron Core Weight
= 2 × Conductor Weight + Iron Core Weight

Q = Length/diameter of iron core

α, D = Emperical constants dependent upon choice of Q

B = Maximum flux density established in iron core, webers per meter^2

σ_i = Specific weight of core material, lbs per in.^{3}

*Does not include insulation or winding form weight.

4) Three Fixed Orthogonal Iron Core Coils

The design of each iron core is the same as for the gimbaled design, but three are now required. Except for the slight effect of K_a, which differs for the two designs, the total maximum power and total weight of conductor and power supply for the three-iron core design equals $\sqrt{3}$ times that for the gimbaled iron core design. However, since the core weight far exceeds these weights, the total weight of a three-iron core design is three times that of a gimbaled iron core design.

For the iron core designs:

1) As the chosen value of Q (length/ diameter of iron core) increases, α increases slightly and D decreases rapidly. The effect is illustrated by the following chart.[1]

Q	α	D	$\left(\frac{D^3Q^2}{\alpha}\right)^{1/3}$	$\left(\frac{D^3Q}{\alpha^2}\right)^{1/3}$
20	0.76	0.0060	0.049	0.020
30	0.77	0.0030	0.031	0.011
40	0.78	0.0020	0.025	0.0082

Thus the iron core weight is rather insensitive to Q, and, for a particular core material, depends upon the maximum value of the magnetic moment. No optimization of this weight is possible.

2) The conductor and power supply weights drop rapidly as Q increases, but large iron core lengths result. These weights can be optimized for a given Q. They depend upon both the maximum and the root mean square values of the magnetic moment.

8.0 MAGNET DESIGNS

These night and day dump minimum weight equations applied to the control law data of Table 1 result in the designs shown in Figures 4 and 5. Fig. 4 are air coil designs using twenty foot diameter coils of copper conductor. For other diameters the maximum powers and weights can be determined by noting that they are inversely proportional to the coil diameter. However, these are the minimum weights for a twenty foot diameter

Figure 4 - Air Coil Magnet Design

Continuous dumping, $\underline{M} = k\underline{M}_s \times \underline{B}$

20 ft diameter coil, copper conductor

	Gimbaled Coil			Three Identical Orthogonal Coils*		
	POP	IOP	45°OP	POP	IOP	45°OP†
Maximum Amp-turns	109	314	8090	109	314	8090
Turns × Conductor Area, $in.^2$	.141	.381	10.4	.092	.251	6.8
Maximum Power, watts	47	144	3480	72	217	5330
Copper Weight, lbs	34	92	2540	67	183	4960
Power Supply Weight, lbs	34	92	2540	67	183	4960
Total Weight, lbs (Copper + Power Supply)	68	184	5080	134	366	9920

*Power and copper weight are total for three coils.
†The 45° OP mode is undesirable for the Second Workshop. It is listed here only to show the upper bound on the magnet design.

Figure 5 - Gimbaled Iron Core Magnet Design

Continuous dumping, $\underline{M} = k\underline{H}_s \times \underline{B}$

core length/diameter = 30

	POP	IOP	45°OP†
Iron* Core			
Length, ft	6.0	8.5	25.2
Diameter, in.	2.4	3.4	10.1
Weight, lbs	95	274	7050
Maximum Amp-Turns	4360	6190	18,300
Turns × Conductor Area, in.^{2}	5.62	7.56	23.7
Maximum Power, watts	18	40	328
Copper Weight, lbs	14	26	240
Power Supply Weight, lbs	14	26	240
Total Weight,** lbs (Copper + Power Supply + Core)	123	326	7530

*45% Nickel Permalloy

**Total weight of a three orthogonal iron core design equals approximately 3 × total weight of a gimbaled design.

†The 45° OP mode is undesirable for the Second Workshop. It is listed here only to show the upper bound on the magnet design.

coil design. The power supply weight includes batteries, solar array, and control amplifier. The copper weight does not include insulation nor winding forms. A twenty foot coil diameter has been chosen in this example because it fits within the Workshop skin. Air coils of a slightly different shape and diameter could be located within the empty LOX tank which is to be used as a refuse container. The total copper and power supply weights for the three-coil designs are almost twice that for the single gimbaled coil designs. However, the additional weight and complexity of the gimbal mechanism must be considered in comparing the two designs. Also, a large volume of space is required to accomodate the movement of the gimbaled coil whereas for the fixed coils this space can be used for storage or as a passageway.

Fig. 5 shows single gimbaled iron core designs for a chosen core length/diameter ratio of 30. The core material is 45 Nickel Permalloy operating at a maximum flux density of 1 weber/meter^2. The weights for the three-magnet designs (not shown) are approximately three times that for the single magnet designs because three iron cores are required.

The three orthogonal air coil designs appear more attractive than either the gimbaled air coil or the iron core designs.

For both figures, either the maximum current, turns, or conductor area can be chosen, consistent with a reasonable physical design. This choice determines the remaining two and fixes the current density at about 800 amps/in.^{2} for the gimbaled designs and 1200 amps/in.^{2} for the three-magnet designs. From a temperature rise standpoint, these current densities are conservative.

A typical three-air coil magnet design for the IOP mode might be a tape-wrapped bundle of 31 turns of #10 copper wire (conductor area = 0.008155 in.^{2}) for each coil. Maximum current and voltage to any coil is 10.1 amperes and 21.5 volts. The additional weight due to wire insulation is estimated at 8% of the copper weight.

A 30% savings in coil conductor and power supply weight can be realized if the coil conductors are made of aluminum instead of copper.

The effect of the required magnetic moment on the magnetic field in the vicinity of the Workshop must be considered in locating the magnetometer to measure the earth's magnetic field. For the twenty foot diameter air coils, the following table gives the maximum field strength (B) along the coil axis in per cent of the earth's magnetic field (B_o) at the magnetic equator.

Coil Diameters From Coil Center	B/B_o,% 0	2	3	4
POP	86	1.2	.38	.17
IOP	249	3.6	1.1	.48
45°OP	6400	91	29	12

If the coils are located at the aft end of the Workshop, the magnetometer can be located in the MDA at a distance of 3 coil diameters. For the IOP mode, the maximum field strength produced there by the magnetic moment is 1.1% of the earth's magnetic field. This will have little adverse effect on the dump operation.

Although the design equations would have to be revised, it is not necessary that the three-magnet design comprise three identical magnets. It may be more economical to construct larger magnets on the vehicle axes of greatest required magnetic moment. This may, in turn, place a constraint on the vehicle mode and attitude.

It should be remembered that the values in Figs. 4 and 5 are conservative in that the sun is considered to lie in the orbital plane. As the sun moves out of the orbital plane the solar panels are no longer perpendicular to the orbital plane and the aerodynamic torque decreases. Also, less battery capacity and solar cell peak power is required because of the lengthened orbital daylight. Hence over a mission the battery and solar array weight penalty is less than indicated.

9.0 NIGHT AND DAY DUMP, LIMITED M_m

Noting that the iron core weight (Equation 17) is directly proportional to the maximum magnitude (M_m) of $\underline{M}$, the control law was simulated with an upper limit placed on M_m. This resulted in a more nearly constant $|\underline{M}|$ since greater dumping action was required during periods when $|\underline{M}|$ previously was small. It was found that adequate dumping still occurred if M_m was limited to 85% of its unconstrained value. This allows a 15%

reduction in iron core weight, which is the major weight in the iron core magnet designs, and a 15% reduction in weight of the control amplifier in all designs. Although M_m decreased, the root mean square values remained roughly the same. Hence there is little weight savings if air coils are used.

10.0 DAY DUMP - CONSTANT $|\underline{M}|$

If magnetic dumping is done only during orbital daylight, batteries are not needed if the solar cells are sized to supply peak power. In this case it may be advantageous to select a control law which results in a constant magnitude of $\underline{M}$, with only its direction changing.

To study day dump with constant $|\underline{M}|$, the $\underline{M} = k\underline{H}_s \times \underline{B}$ control law with a limited maximum magnitude M_m was applied only during the 220° orbital day. A constant $|\underline{M}|$ 30% higher than the M_m for day and night unconstrained dumping was found adequate.

The minimum weight design equations for day dump-constant $|\underline{M}|$ are developed in Appendix E. When applied to the above results, the total weight of the solar array, control amplifier and copper coils for the three twenty-foot diameter air coil designs are:

	POP	IOP	45°OP*
Total Weight, lbs	153	441	11,400

The portion of these weights attributable to the solar array is conservative since as the sun moves out of the orbital plane the dump interval increases, requiring a smaller constant $|\underline{M}|$ and a smaller solar cell power load. However, there is no weight advantage over continuous day and night dumping since the system must be sized for the minimum daylight orbit.

For the iron core designs, the 30% increase in weight of the iron core clearly offsets any saving in battery weight.

11.0 APPROXIMATE MAGNETIC DESIGN FOR DAY AND NIGHT DUMP

Except for the slight effect of the rotating earth, the orbiting vehicle experiences two cyclic variations of the earth's magnetic field $\underline{B}$ in one orbit. $\underline{M}$, as defined by $\underline{M} = k\underline{H}_s \times \underline{B}$, also has two cyclic variations in one orbit. If orbital daylight time is not much greater than orbital night time, the root mean square values of $|\underline{M}|$ during night and day are similar.

*See footnotes on Fig. 4 and 5 for this mode.

Then, $M_n \simeq M_d \simeq M_o$

It was noticed from a number of simulations that the ratio M_o/M_m varied between 0.7 and 0.8, with a mean of 0.78. With these approximations the expression

$$M_e = \sqrt{K_n M_n^2 + K_d M_d^2 + K_a M_m^2}$$

becomes

$M_e = .87M_m$ for the gimbaled magnet designs.

$M_e = .98M_m$ for the three-magnet designs.

Substituting these into the minimum weight air coil design equations (7 to 14) results in the following approximations for the 220 nm, 220° daylight orbit.

1. Gimbaled Circular Air Coil

$$P_m = \frac{3.5M_m}{d} \quad \text{watts}$$

$$\text{Copper Weight} = \frac{2.6M_m}{d} \quad \text{lbs}$$

2. Three Orthogonal Circular Air Coils

$$P_m = \frac{5.3M_m}{d} \quad \text{watts}$$

$$\text{Copper Weight} = \frac{5.1M_m}{d} \quad \text{lbs}$$

Given a required maximum magnetic moment M_m, one can use these formulas to estimate the maximum power and weight. The total weight of copper and power supply equals twice the copper weight.

Using the same approximations, Fig. 6 shows the gimbaled iron core designs for three different l/d ratios. The iron core weight, which predominates, is practically independent of the l/d ratio.

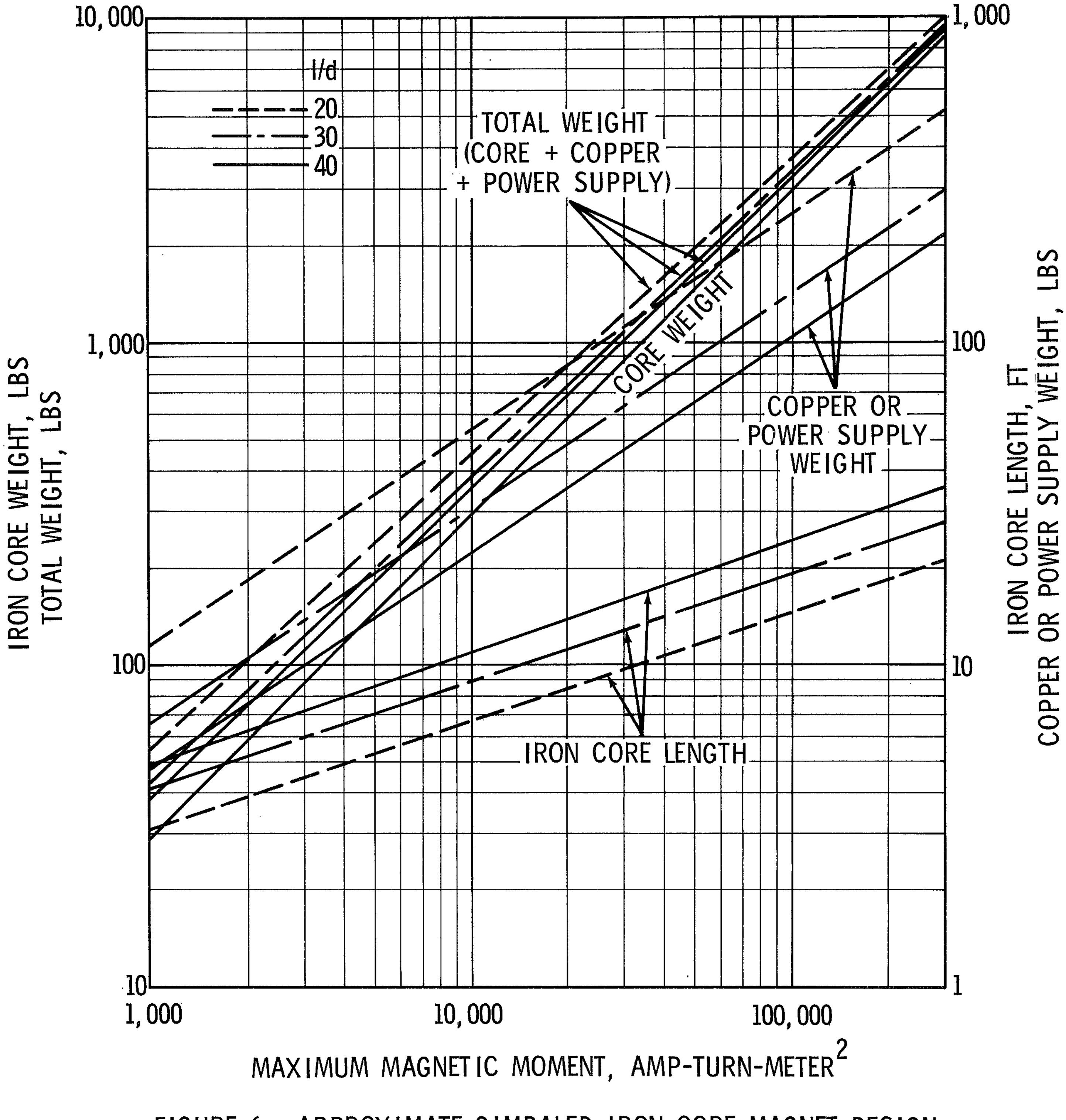

FIGURE 6 - APPROXIMATE GIMBALED IRON CORE MAGNET DESIGN

The total weight of a three-iron core design is approximately three times that of the gimbaled coil design.

12.0 M_m ESTIMATION

Since estimates of the properties of a Second Workshop are subject to change, it would be convenient to have a quick method of estimating the magnetic dumping weight penalty for a variety of vehicle configurations. For this reason the control law was simulated for a range of bias momenta. The results suggest the following.

For the POP mode the aerodynamic and gravity gradient bias momentum vectors lie in the orbital plane. For this case, M_m can be estimated by

$$M_m = 12.5\ H_b \quad \text{amp-turn-meter}^2$$

where H_b(ft-lb-sec) is the magnitude of the sum of the bias momentum vectors.

For the 45°OP mode, the gravity gradient bias momentum vector lies in the orbital plane. M_m can be estimated by

$$M_m = 12.5\ H_b = 0.0665\ (I_z - I_x)\ \text{amp-turn-meter}^2$$

where I_x, I_z are the vehicle X and Z axes principal moments of inertia (slug ft^2).

For the IOP mode, for which the aerodynamic bias momentum vector is perpendicular to the orbital plane and the gravity gradient bias momentum vector lies in the orbital plane, M_m is plotted in Fig. 7 as a function of $(I_y - I_z)$. Similar curves can be drawn for other values of aerodynamic bias.

From these estimates of M_m, and from the equations and Fig. 7 of the previous section, a quick estimate can be made of the power supply and magnet weights.

13.0 SUMMARY

CMG bias momentum dumping can be accomplished without attitude maneuvers by means of an on-board electromagnet reacting with the earth's magnetic field to produce torque. A control law has been devised for determining the required magnetic moment profile over the orbit for proper dumping. Except for the small

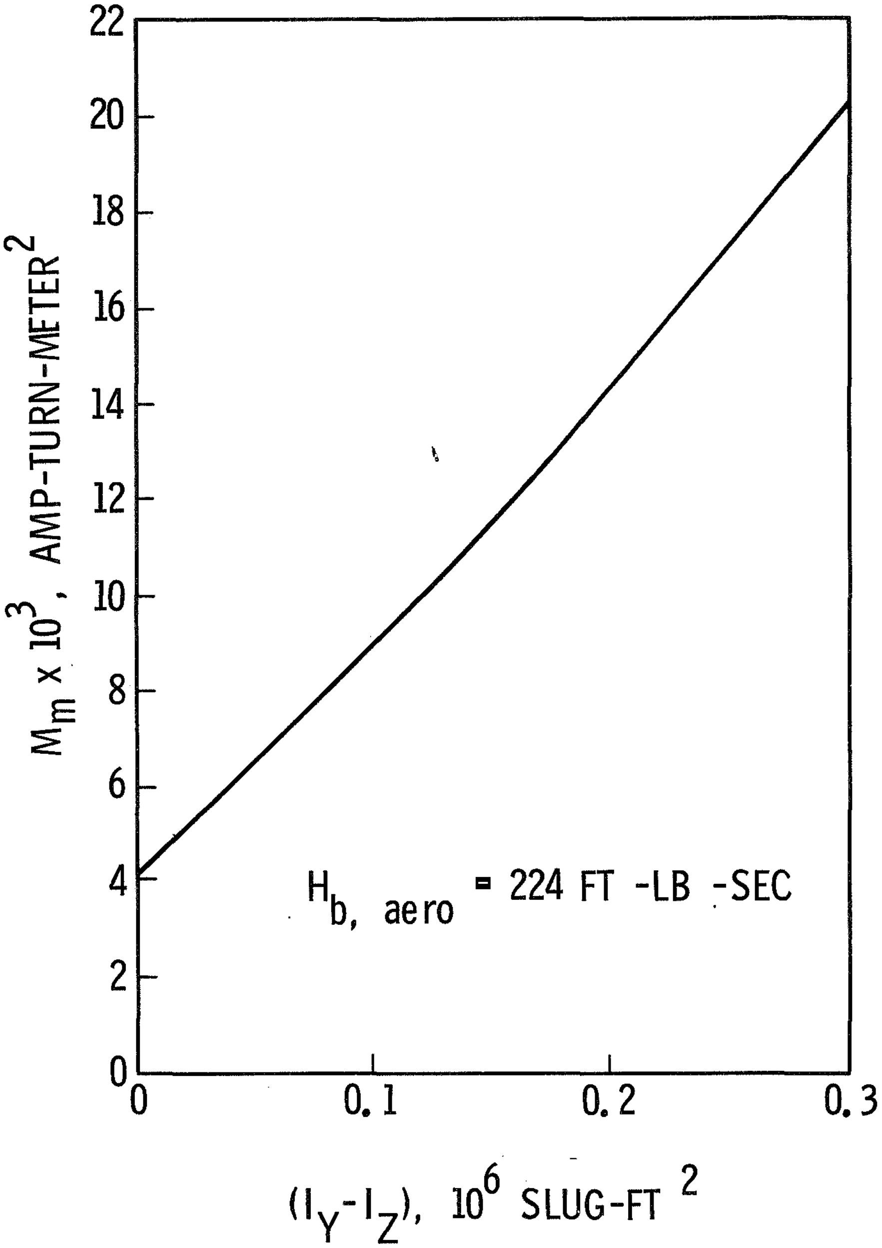

FIGURE 7 - ESTIMATION OF M_m, IOP MODE

effect of orbit-to-orbit variations in the earth's magnetic field, this control law minimizes the electric energy drawn from the power supply.

System implementation for a Second AAP Workshop requires only the addition of a magnetometer, a control amplifier, and the electromagnet.

The magnet structure may consist of a single gimbaled magnet or three fixed magnets. The magnet itself can be of air coil or iron core construction.

In all cases, the total weight of conductor (total of three coils for the three-magnet design) and power supply (solar array, batteries, and control amplifier) is a minimum if the conductor weight equals the power supply weight. Design equations for this criterion have been developed which give coil design, maximum power, and weights.

For air coils, the total weight of a three-magnet design is approximately $\sqrt{3}$ times that of a gimbaled magnet design. For iron core coils, the total weight of a three-magnet design is approximately $\sqrt{3}$ times the weight of the power supply and conductor of a gimbaled magnet design plus three times its iron core weight.

Although the total weight of magnet and power supply is smaller for a gimbaled design, the additional weight and complexity of the gimbal mechanism and the space required to accomodate the moving magnet makes the three-magnet design, in particular the air coils, more attractive. With three fixed twenty foot diameter copper air coils, the total of power supply and conductor weight is 134 lbs for the POP mode and 366 lbs for the IOP mode. With a gimbaled iron core magnet, the coil and power supply weights are less but the iron core weight makes the total weight of the two designs comparable. The addition of a gimbaled mechanism makes its system weight greater.

For the IOP mode, the maximum field produced at the center of the twenty foot diameter air coils is 2-1/2 times the earth's magnetic field. At three coil diameters distance it is 0.011 times the earth's field. The magnetometer can be placed at this distance for satisfactory operation.

Although the magnet designs presented in this memorandum are for a particular Workshop geometry, a method has been developed for quickly estimating magnet designs for variations in Workshop inertia properties.

Acknowledgement

This study stems from the interest of O. K. Garriott and O. G. Smith of Manned Spacecraft Center in the possible application of magnetic torque for attitude control of the wet Workshop.

The helpful suggestions of J. Kranton during the course of this study and the computer programming assistance of Mrs. Nancy I. Kirkendall and Mrs. Barbara W. Lab are gratefully acknowledged.

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1022-WL-cf W. Levidow

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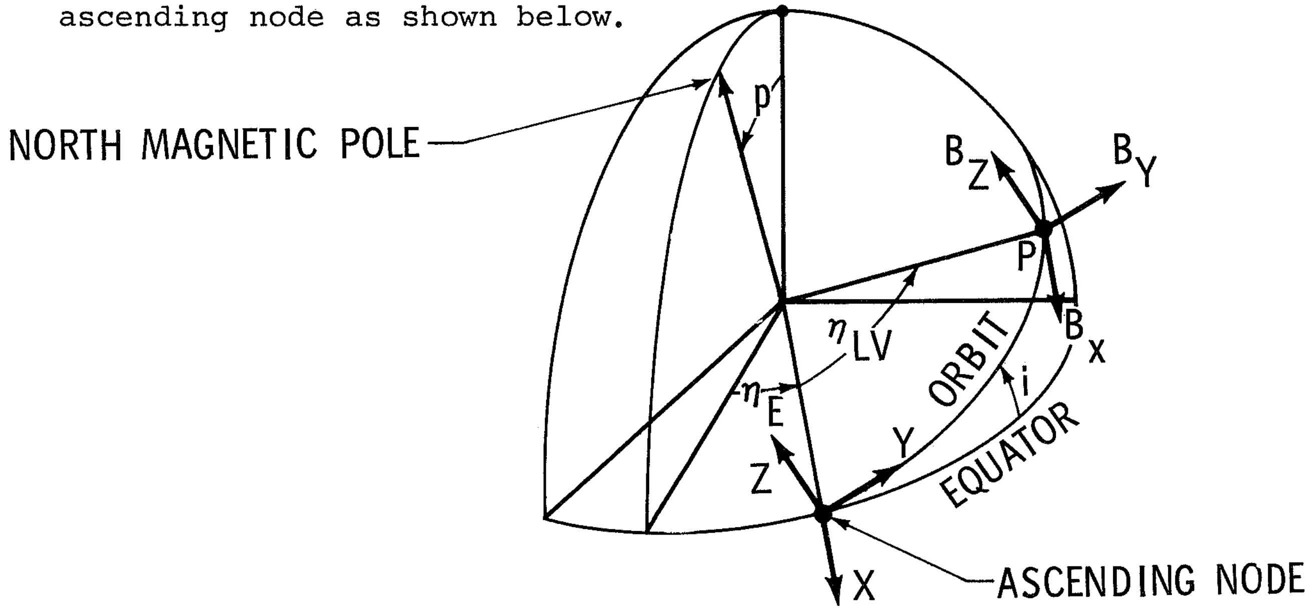
APPENDIX A

Earth's Magnetic Field

The earth's magnetic field $\underline{B}$ at any point P in orbit can be expressed in a nodal coordinate system defined at the ascending node as shown below.



The x axis is directed from earth center to the ascending node, the Y axis is directed along the orbital velocity vector and the Z axis completes a right-hand orthogonal system. For this system,

$$\frac{1}{B_o}\underline{B} = \begin{bmatrix} -a/2 \\ -b/2 \\ c \end{bmatrix} - \frac{3}{2} A \begin{bmatrix} \cos(2\eta_{LV}-\phi) \\ \sin(2\eta_{LV}-\phi) \\ 0 \end{bmatrix}^{*} \qquad \text{(A-1)}$$

B_o = magnetic field strength magnitude at the magnetic equator at the orbital altitude. (0.26×10^{-4} webers per meter^2 at 220nm).

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*Representation of the earth's magnetic field in various coordinate systems can be found in the literature (TRW Space Data, Third Edition).

$$a = \cos \eta_E \sin p$$

$$b = \sin i \cos p - \cos i \sin p \sin \eta_E$$

$$c = \sqrt{1 - (a^2+b^2)}$$

$$A = \sqrt{1-c^2}$$

$$\phi = \tan^{-1} \frac{b}{a}$$

η_E = angle from the magnetic pole meridian plane to the line of nodes.

η_{LV} = angle from the line of nodes to the point P in orbit.

i = orbital inclination.

p = colatitude of the north magnetic pole.

The first term of (A-1) varies slowly, depending upon the variation of η_E due to the earth's rotation and orbital regression. The second term varies more quickly due to the change in orbital position η_{LV}. Since $\dot{\eta}_{LV} >> \dot{\eta}_E$, the X and Y components of $\underline{B}$ are approximately periodic with a frequency twice that of orbital frequency.

APPENDIX B

Minimum Energy Magnetic Moment

Problem: Minimize $\int_0^t \underline{M}^T \underline{M} dt$

Subject to the constraint

$$\int_0^t \underline{M} \times \underline{B} dt = -\underline{H}_b \qquad \text{(B-1)}$$

Solution:

Let $G = \int_0^t (\underline{M}^T \underline{M} + (\tilde{\underline{B}}\ \underline{M})^T \underline{\lambda}) dt$

Where $\tilde{\underline{B}}$ is the matrix equivalent to the vector cross product operation $\underline{B}\ \times$.

$\underline{\lambda}$ is a vector of Lagrange Multipliers.

Minimizing G by differentiating the integrand with respect to $\underline{M}$ and equating the resulting expression to zero, yields

$$2\underline{M} + \tilde{\underline{B}}^T \underline{\lambda} = 0$$

$$M = -\frac{\tilde{\underline{B}}^T \underline{\lambda}}{2} \qquad \text{(B-2)}$$

Substituting into (B-1)

$$\underline{\lambda} = -2 \left(\int_0^t \tilde{\underline{B}} \tilde{\underline{B}}^T dt \right)^{-1} \underline{H}_b = 2 \left(\int_0^t \tilde{\underline{B}}^2 dt \right)^{-1} \underline{H}_b$$

Substituting $\underline{\lambda}$ into (B-2),

$$\underline{M} = \left(-\int_0^t \tilde{\underline{B}}^2 dt \right)^{-1} \underline{H}_b \times \underline{B} \qquad \text{(B-3)}$$

It will now be shown that the matrix within the parentheses is positive definite and hence non-singular.* Let $\underline{u}$ be an arbitrary time invarient vector. Then,

$$\underline{u}^T(-\int_0^t \tilde{\underline{B}}^2 dt)\underline{u} = \int_0^t \underline{u}^T[\underline{B} \times (\underline{u}\times\underline{B})]dt$$

$$= \int_0^t |\underline{B}|^2 |\underline{u}|^2 \sin^2\phi dt \qquad \text{(B-4)}$$

where ϕ is the angle between $\underline{B}$ and $\underline{u}$. Except for the orbit coincident with the earth's magnetic equatorial plane, the direction of $\underline{B}$ is not time invarient. Hence for any fixed $\underline{u}$, $\sin \phi$ cannot be zero over the entire interval [0,t]. In general, then, the integral (B-4) is positive for any $\underline{u}$ and the matrix is positive definite.

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*This proof contributed by J. Kranton.

APPENDIX C

Power Supply Weight

Symbols

(The numbers in parentheses are the estimated values used in calculations in this memorandum.)

P_n	= Nighttime average load power, watts	
P_d	= Daytime average load power, watts	
P_m	= Maximum load power, watts	
P_s	= Solar Array power rating, watts	
T_n	= Orbital nighttime period, hours	
T_d	= Orbital daytime period, hours	
E_{rd}	= Regulator efficiency × distribution efficiency	(.82)
E_b	= Battery efficiency (output energy/input energy)	(.68)
E_{ch}	= Battery Charger efficiency	(.96)
D	= Allowable battery depth of discharge	(.3)
K_b	= Battery watt-hours per lb	(10)
K_s	= Solar array watts per lb	(3)
K_a	= Control Amplifier lbs per watt	
	Single Channel	(.12)
	Three Channel	(.32)

For continuous dumping, the batteries power the electrical load during the night. The solar cells recharge the batteries and power the load during the day.

Battery nighttime energy output $\frac{P_n T_n}{E_{rd}}$ watt-hours

Battery rating $= \frac{P_n T_n}{E_{rd} D}$ watt-hours

Battery weight $= \frac{P_n T_n}{E_{rd} D K_b}$ lbs

Battery daytime recharge energy input $= \frac{P_n T_n}{E_{rd} E_b}$ watt-hours

Solar Array required orbital energy output = daytime energy to battery and charger + daytime energy to regulator, distribution, and load

$$P_s T_d = \frac{P_n T_n}{E_{rd} E_b E_{ch}} + \frac{P_d T_d}{E_{rd}} \quad \text{watt-hours}$$

Solar Array weight $= \frac{P_s}{K_s} = \frac{1}{E_{rd} K_s} \left[\frac{P_n T_n}{E_b E_{ch} T_d} + P_d \right]$ lbs

Control amplifier weight $= K_a P_m$ lbs

Total Power Supply Weight (solar array, battery and control amplifier) $= K_n P_n + K_d P_d + K_a P_m$ lbs

where $$K_n \equiv \frac{T_n}{E_{rd}} \left[\frac{1}{D K_b} + \frac{1}{K_s E_b E_{ch} T_d} \right]$$

$$K_d = \frac{1}{E_{rd} K_s}$$

APPENDIX D

Minimum Weight Magnetic Design Equations - Day and Night Dump

Symbols

I = Current through coil, amps

M = Magnetic moment, amp-turn-meter^2

M_m = Maximum orbital magnetic moment

N = Number of turns per coil

A = Area of coil, inches^2

R = Resistance per coil, ohms

P_m = Maximum orbital load power, watts

ℓ = Length per turn of coil, inches

ρ = Conductor resistivity, ohms per inch^3

a = Conductor area per turn, inches^2

W_p = Power Supply weight, lbs

M_n = Root mean square of magnetic moment during orbital night

M_d = Root mean square of magnetic moment during orbital day

$M_e = (K_n M_n^2 + K_d M_d^2 + K_a M_m^2)^{1/2}$

σ = Conductor specific weight, lbs per inch^3

d = Air coil diameter, inches

B = Maximum flux density established in iron core, webers per meter^2

α, D = Emperical constants dependent upon choice of Q. Sample values are listed in body of memorandum.

$Q = \ell_i/d_i$ = length/diameter of iron core

σ_i = Core material specific weight, lbs per inch^3

D 1 Gimbaled Circular Air Coil

The coil current required to produce a magnetic moment M is given by

$$I = \frac{1550M}{NA}$$

The average coil power during an interval t is

$$P = \frac{1}{t}\int_0^t I^2 R dt = \frac{(1550)^2 R}{N^2 A^2}\frac{1}{t}\int_0^t M^2 dt = \frac{(1550)^2 R}{N^2 A^2} M_{rms}^2 \qquad (D1)$$

The maximum power is

$$P_m = \frac{(1550)^2 R\ M_m^2}{N^2 A^2} \qquad (D2)$$

The coil resistance is given by $R = \frac{N\ell\rho}{a}$ (D3)

From Appendix C the power supply weight is related to its load power by

$$W_p = K_n P_n + K_d P_d + K_a P_m \qquad (D4)$$

where P_n, P_d, and P_m are the average nighttime load power, average daytime load power, and maximum load power during the orbit.

Substituting (D1), (D2), and (D3) into (D4) yields

$$W_p = \frac{(1550)^2\ \ell\rho}{A^2 Na}\left[K_n M_n^2 + K_d M_d^2 + K_a M_m^2\right] = \frac{(1550)^2\ \ell\rho\ M_e^2}{A^2 Na} \qquad (D5)$$

where

$$M_e = \left[K_n M_n^2 + K_d M_d^2 + K_a M_m^2\right]^{1/2}$$

The coil conductor weight is

$$W_c = N\ell a\sigma \tag{D6}$$

The total weight of conductor and power supply is

$$W_t = N\ell a\sigma + \frac{(1550)^2 \ \ell\rho \ M_e^2}{A^2 Na} \tag{D7}$$

Minimizing the total weight by differentiating (D7) with respect to (Na) and setting the resulting expression equal to zero yields

$$Na = \frac{1550\sqrt{\rho/\sigma} \ M_e}{A}$$

Substituting this into the expressions for conductor and power supply weight shows that the two should be equal for minimum total weight. For this criterion the following design equations apply for a circular coil.

$$I_m N = \frac{1970 \ M_m}{d^2} \qquad \text{amp-turns}$$

$$Na = \frac{1970\sqrt{\rho/\sigma} \ M_e}{d^2} \qquad \text{turn-in.}^2$$

$$P_m = \frac{6200\sqrt{\rho\sigma} \ M_m^2}{d \quad M_e} \qquad \text{watts}$$

$$\text{Conductor Weight} = \frac{6200\sqrt{\rho\sigma} \ M_e}{d} \quad \text{lbs}$$

Total Weight = Conductor Weight + Power Supply Weight

= 2 × Conductor Weight

D 2 Three Orthogonal Circular Air Coils

If we assume for a moment that the required M is directed along the axis of only one of the three coils, the weight of the power supply required to energize this coil is given by equation (D5). But it has been shown that with three orthogonal identical coils the electric power required to produce a given M is dependent only on its magnitude. Hence the weight of the power supply required to energize all three coils is given by equation (D5) regardless of the direction of M.

Since three coils are required, the total conductor weight is given by

$$W_c = 3N\ell a\sigma$$

The total weight of conductor and power supply is

$$W_t = 3N\ell a\sigma + \frac{(1550)^2 \ell\rho M_e{}^2}{A^2 Na}$$

Minimizing this expression results in the following design equations for circular coils.

$$I_m N = \frac{1970\ M_m}{d^2} \qquad \text{amp-turns}$$

$$\underset{\text{(per coil)}}{Na} = \frac{1140\sqrt{\rho/\sigma}\ M_e}{d^2} \qquad \text{turn-in.}^2$$

$$\underset{\text{(total 3 coils)}}{P_m} = \frac{10700\sqrt{\rho\sigma}\ M_m^2}{d\quad M_e} \qquad \text{watts}$$

$$\underset{\text{(total 3 coils)}}{\text{Conductor Weight}} = \frac{10700\sqrt{\rho\sigma}\ M_e}{d} \qquad \text{lbs}$$

Total Weight = Conductor Weight + Power supply Weight

= 2 × Conductor Weight

D 3 Gimbaled Iron Core Coil(1)

The volume of core material required to produce a magnetic moment M_m is

$$\text{Volume} = (39.37)^3 \frac{4\pi 10^{-7} M_m}{\alpha B} \quad \text{in.}^3$$

$$\text{Let } Q = \ell_i / d_i$$

$$\ell_i = 0.461 \left(\frac{Q^2}{\alpha B}\right)^{1/3} M_m^{1/3} \quad \text{in.}$$

$$\text{Core weight} = \text{Volume} \times \sigma_i = \frac{0.0767 \sigma_i M_m}{\alpha B} \quad \text{lbs}$$

The maximum ampere turns required to magnetize the core is

$$I_m N = \frac{B \ell_i D}{(39.37) 4\pi 10^{-7}} = 9310 \left(\frac{D^3 Q^2 B^2}{\alpha}\right)^{1/3} M_m^{1/3} \qquad \text{(D8)}$$

If the core is worked on the straight line portion of the sheared B-H curve, the core flux density and the magnetic moment is proportional to the coil current I.

$$M = KI$$

The maximum coil power is

$$P_m = I_m^2 R = \frac{M_m^2 R}{K^2} \qquad \text{(D9)}$$

The average coil power during an interval t is

$$P = \frac{1}{t} \int_0^t I^2 R dt = \frac{R}{K^2} \frac{1}{t} \int_0^t M_m^2 dt = \frac{R}{K^2} M_{rms}^2$$

or from above

$$P = \frac{M_{rms}^2 \; P_m}{M_m^2} \tag{D10}$$

From Appendix C, the power supply weight is related to the load power by

$$W_p = K_n P_n + K_d P_d + K_a P_m \tag{D11}$$

where P_n, P_d, and P_m are the average nighttime load power, average daytime load power, and maximum load power during the orbit.

Substituting (D10) into (D11) yields

$$W_p = \frac{P_m}{M_m^2} \left[K_n M_n^2 + K_d M_d^2 + K_a \; M_m^2\right] = \frac{P_m}{M_m^2} \; M_e^2 \tag{D12}$$

$$\text{But } P_m = I_m^2 R = \left(\frac{I_m N}{N}\right)^2 \frac{N \ell \rho}{a} = \frac{\left(I_m N\right)^2 \ell \rho}{Na} \tag{D13}$$

Substituting (D8) and (D13) into (D12) yields

$$W_p = (9310)^2 \left(\frac{D^3 Q^2 B^2}{\alpha}\right)^{2/3} \frac{\ell \; \rho \; M_e^2}{Na \; M_m^{4/3}} \tag{D14}$$

The coil conductor weight is

$$W_c = N \ell \sigma a \tag{D15}$$

Summing (D14) and (D15) and minimizing by differentiating with respect to (Na) yields

$$Na = 9310 \left(\frac{D^3 Q^2 B^2}{\alpha}\right)^{1/3} \frac{\sqrt{\rho/\sigma}\ M_e}{M_m^{2/3}} \quad \text{turn-in.}^2 \qquad (D16)$$

Substituting (D16) into (D14) and (D15) shows that for minimum total weight the power supply weight equals the conductor weight.

This criterion yields the following

$$P_m = 13500 \left(\frac{D^3 QB}{\alpha^2}\right)^{1/3} \frac{\sqrt{\rho\sigma}\ M_m^{5/3}}{M_e} \quad \text{watts}$$

$$\text{Conductor Weight} = 13500 \left(\frac{D^3 QB}{\alpha^2}\right)^{1/3} \frac{\sqrt{\rho\sigma}\ M_e}{M_m^{1/3}} \quad \text{lbs}$$

Total Weight = Conductor Weight + Power Supply Weight + Core Weight

= 2 x Conductor Weight + Core Weight

D 4 Three Orthogonal Iron Core Coils

The design of each iron core is the same as for the gimbaled design, but three are required.

The power supply weight is given by (D14) but the total conductor weight of the three coils is

$$W_c = 3N\ell\sigma a$$

Minimizing the total power supply and conductor weight results in

$$I_m N = 9310 \left(\frac{D^3 Q^2 B^2}{\alpha}\right)^{1/3} M_m^{1/3} \quad \text{amp-turns}$$

$$\underset{\text{(per coil)}}{Na} = 5380 \left(\frac{D^3 Q^2 B^2}{\alpha}\right)^{1/3} \frac{\sqrt{\rho/\sigma}\ M_e}{M_m^{2/3}} \quad \text{turn-in.}^2$$

$$P_m \text{ (total 3 coils)} = 23300 \left(\frac{D^3 QB}{\alpha^2} \right)^{1/3} \frac{\sqrt{\rho/\sigma}\, M_m^{5/3}}{M_e} \qquad \text{watts}$$

$$\text{Conductor Weight (total 3 coils)} = 23300 \left(\frac{D^3 QB}{\alpha^2} \right)^{1/3} \frac{\sqrt{\rho/\sigma}\, M_e}{M_m^{1/3}} \qquad \text{lbs}$$

Total Weight = Conductor Weight + Power Supply Weight

+ 3 × Iron Core Weight

= 2 × Conductor Weight + 3 × Iron Core Weight

APPENDIX E

Minimum Weight Magnetic Design Equations
Day Dump - Constant $|\underline{M}|$

(Symbols are the same as in Appendix D)

E 1 Three Orthogonal Circular Air Coils

With a constant magnetic moment magnitude M, the electric power P to the coils is also constant.

Control Amplifier Weight $= K_a P$

Solar Array Weight $= \dfrac{P}{K_s E_{rd}}$

Conductor Weight of Three Coils $= 3N\ell a\sigma$

When $\underline{M}$ is directed along the axis of one of the coils,

Coil Current I $= \dfrac{1550M}{Na}$

Coil Resistance R $= \dfrac{N\ell\rho}{a}$

$\therefore$ Total power for $\underline{M}$ in any direction $= I^2R = \dfrac{(1550)^2 \ell\rho M^2}{A^2 Na}$

$$\text{Total Weight} = 3N\ell a\sigma + \frac{(1550)^2 \ell\rho M^2 \left(K_a + \frac{1}{K_s E_{rd}}\right)}{A^2 Na}$$

For minimum total weight

$$Na = \frac{1{,}140\sqrt{\rho/\sigma}\; M \left(K_a + \frac{1}{K_s E_{rd}}\right)^{1/2}}{d^2} \qquad \text{turn-in.}^2$$

$$\text{Conductor Weight} = \frac{10{,}700\sqrt{\rho\sigma}\, M \left(K_a + \frac{1}{K_s E_{rd}}\right)}{d} \quad \text{lbs}$$

$$\text{Total Weight} = 2 \times \text{Conductor Weight}$$

E 2 Gimbaled Iron Core Coil

The iron core dimensions and weight are calculated as in Appendix D3.

$$\text{Control Amplifier Weight} = K_a P$$

$$\text{Solar Array Weight} = \frac{P}{K_s E_{rd}}$$

$$\text{Conductor Weight} = N\ell a\sigma$$

$$\text{Total Weight} = N\ell a\sigma + P\left(K_a + \frac{1}{K_s E_{rd}}\right)$$

The constant ampere turns required to produce the constant M is

$$IN = 9310\left(\frac{D^3 Q^2 B^2}{\alpha}\right)^{1/3} M^{1/3}$$

$$P = I^2 R = \frac{(IN)^2 \ell\rho}{Na}$$

Substituting this into the total weight expression and minimizing yields

$$Na = 9310\left(\frac{D^3 Q^2 B^2}{\alpha}\right)^{1/3} \frac{\sqrt{\rho/\sigma}\, M^{1/3}}{\left(K_a + \frac{1}{K_s E_{rd}}\right)^{-1/2}} \quad \text{turn-in.}^2$$

$$\text{Conductor Weight} = 13{,}500\left(\frac{D^3 QB}{\alpha^2}\right)^{1/3} \frac{\sqrt{\rho\sigma}\, M^{2/3}}{\left(K_a + \frac{1}{K_s E_{rd}}\right)^{-1/2}} \quad \text{lbs}$$

$$\text{Total Weight} = 2 \times \text{Conductor Weight} + \text{Iron Core Weight}$$