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REAL SURFACE EFFECTS ON RADIATIVE HEAT TRANSFER

by
A. F. HOUCHENS
R. G. HERING

Technical Report No. ME-TR-661-1
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Heat Transfer Laboratory

Department of Mechanical and Industrial Engineering

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FORWORD

Except for minor changes, this report constitutes the doctoral dissertation of Mr. A. F. Houchens which was submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering in the Graduate College of the University of Illinois at Urbana-Champaign.

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ABSTRACT

An analytical study of steady radiative heat transfer for a simple system of radiatively interacting opaque surfaces separated by a radiatively transparent medium is presented. Basic principles of radiative transfer are employed to formulate radiative heat transfer for identical, uniform property adjoint plates uniformly irradiated by a collimated radiative flux. The formulation accounts for directional and wavelength dependence of surface properties. The governing equations simplify when bidirectional reflectance may be expressed as the sum of direction dependent specular and scattered components and/or the spectral variation of properties may be approximated by a semi-gray model. Numerical results for local and overall heat transfer were obtained in the absence of external radiative flux and temperature distributions were evaluated for radiatively adiabatic surfaces in the presence of a solar flux. Heat transfer and equilibrium temperature calculations employed bidirectional reflectance models representative of rough metal surfaces and coated surfaces as well as property models incorporating much less detail.

Local and overall heat transfer results for equal temperature, identical property adjoint plates in the absence of an external radiation flux support the following conclusions. First, the directional dependence of surface properties significantly influences local radiant heat transfer for low and high emittance materials at surface elements where radiant interaction is high and major contributions to local irradiation are from energy incident at large angle of incidence. These effects are essentially independent of the spatial distribution

of reflected radiation and are unimportant insofar as overall heat transfer is concerned. Second, radiant heat transfer between rough surfaces differs only slightly from that predicted by a directional property analysis employing a specular reflection model. Third, heat transfer results using an approximate rough surface property model were correlated with those calculated using directional property analysis by basing the apportionment of reflected radiation between specular and diffuse components on optical roughness of the surface. Fourth, a dielectric coating on a diffusely reflecting substrate can significantly alter radiant heat transfer. Analysis using an approximate reflectance model for the coated surface which accurately accounts for the directional dependence of specularly reflected radiation gave excellent agreement with the heat transfer results using the more detailed coated surface bidirectional reflectance model.

Equilibrium temperature distributions for identical property, perpendicular adjoint plates which are specularly reflecting to surface emitted radiation demonstrated that the spatial distribution of reflected solar radiation strongly affects local temperature. Surface roughness on materials of low to intermediate values of solar absorptance significantly influenced local temperature, particularly at surface elements where radiant interaction was weak. For the values selected for the surface roughness parameters, analysis neglecting directional property effects and employing a diffuse reflection model for solar energy generally gave acceptable temperature results.

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NOMENCLATURE

ΔA	element of surface area, ft^2
dA	differential element of surface area, ft^2
a	correlation length, μm
a, b	dimensionless distances on plates
$B, \bar{B}, G, H, \bar{H}$	geometrical functions, dimensionless
E	emissive power, Btu/hr-ft^2
FF	diffuse angle factor, dimensionless
$f_{sp, \lambda}$	monochromatic specular reflectance for a perfectly conducting randomly rough material, dimensionless
$f_{bd, s, \lambda}$	scattered component of spectral bidirectional reflectance for a perfectly conducting randomly rough material, $(\text{steradians})^{-1}$
$\bar{f}_{bd, s}$	scattered component of bidirectional reflectance for a perfectly conducting randomly rough material of large optical roughness, $(\text{steradians})^{-1}$
$F_{s, \lambda}$	fraction of incident energy which is scattered by a perfectly conducting material, dimensionless
$\mathcal{F}_1, \mathcal{F}_2$	functions defined by Eqs. (4.3.4.6) and (4.3.4.7)
$H_\lambda(\eta, \theta, \phi)$	dimensionless spectral scattered radiant intensity, $H_\lambda(\eta, \theta, \phi) = [I_{s, \lambda}^+(\eta, \theta, \phi)]/I_{b, \lambda}$
$H(\eta, \theta, \phi)$	dimensionless scattered radiant intensity, $H(\eta, \theta, \phi) = [\pi I_s^+(\eta, \theta, \phi)]/\bar{\sigma}T^4$
$H_t(\eta, \theta, \phi)$	dimensionless scattered intensity of thermal radiation, $H_t(\eta, \theta, \phi) = [I_{t, s}^+(\eta, \theta, \phi)]/S$
$H^*(\eta, \theta, \phi)$	dimensionless scattered intensity of solar radiation, $H^*(\eta, \theta, \phi) = [I_{so, s}^+(\eta, \theta, \phi)]/S$
I^+, I^-	intensity of emergent and incident radiation, $\text{Btu/hr-ft}^2\text{-steradians}$
$I_{r, sp, \lambda}^+$	spectral radiant intensity in a specular reflection, $\text{Btu/hr-ft}^2\text{-}\mu\text{m}\text{-steradians}$
k	absorption index, dimensionless

K, K_Y, K_{2Y}, K_{3Y}	geometry kernels, ft^{-2}
L	plate length, ft
m	rms surface profile slope, dimensionless
M	summation index, dimensionless
n	refractive index, dimensionless
$\vec{n}$	outward unit normal vector, dimensionless
$P(x), Q(y, z)$	points on the lower and upper plates, dimensionless
q	local radiant heat flux, Btu/hr-ft^2
$q_{a,so}$	local absorbed solar flux, Btu/hr-ft^2
Q	overall radiant heat transfer per unit width, Btu/hr-ft
$\mathcal{Q}$	dimensionless local absorbed solar flux, $\mathcal{Q} = (q_{a,so})/S$
r_{sp}	specularity ratio, dimensionless
R_{sp}	ratio of energy reflected into specular solid angle to that reflected throughout hemispherical space, dimensionless
$\mathcal{R}$	general radiation property function
$\bar{\mathcal{R}}$	geometry averaged general radiation property function
S	solar flux, Btu/hr-ft^2
$\mathcal{I}_{sp}$	solar intensity in a specular reflection, $\text{Btu/hr-ft}^2\text{-steradians}$
T	absolute temperature, $^{\circ}\text{R}$
$x, \bar{x}, y, z, \bar{z}$	spatial coordinates on plate surfaces, ft
α	absorptance, dimensionless
$\alpha_1, \alpha_2, \alpha_3$	angles defined by Eqs. (4.3.4.3)-(4.3.4.5), radians
γ	included angle between plates, degrees
δ	Dirac delta function, dimensionless
ϵ	emittance, dimensionless
$\zeta, \bar{\zeta}, \eta, \bar{\eta}, \xi$	dimensionless distances on plate, $\zeta = z/L$, $\bar{\zeta} = \bar{z}/\bar{L}$, $\eta = x/L$, $\bar{\eta} = \bar{x}/\bar{L}$, $\xi = y/L$
θ, θ'	polar angles of emergent and incident radiation, radians

$\theta'_{p,\gamma}; \theta'_{p,2\gamma}; \theta'_{p,3\gamma}$	polar angles for incident radiation defined by Eq. (A-4), radians
$\theta_{Q,\gamma}; \theta_{Q,2\gamma}; \theta_{Q,3\gamma}$	polar angles for emergent radiation defined by Eq. (A-5), radians
$\theta'_{p,sp}$	polar angle of incidence for energy specularly reflected at angle θ_p , radians
θ'_{so}	polar angle of directly incident solar radiation, radians
$\theta'_{11}, \theta'_{12}, \theta'_{22}$	polar angles for specular reflections, radians
θ	dimensionless equilibrium temperature, $\theta = (\bar{\sigma}T^4)/S$
λ	radiation wavelength, μm
λ_c	characteristic radiation wavelength, μm
ρ	reflectance, dimensionless
ρ_{bd}	bidirectional reflectance, $(\text{steradians})^{-1}$
$\bar{\rho}_{hd,s,\lambda}$	scattered component of spectral bidirectional reflectance for large optical roughness, $(\text{steradians})^{-1}$
$\rho_{F,c}, \bar{\rho}_{F,c}$	Fresnel reflectance of air-coating interface for radiation incident from the exterior and interior of the coating, dimensionless
ρ_{sp}	specular reflectance, dimensionless
σ	rms roughness height, μm
$\bar{\sigma}$	Stefan-Boltzmann constant, $\text{Btu/hr-ft}^2\text{-}^\circ\text{R}^4$
τ_o	optical thickness, dimensionless
ϕ, ϕ'	azimuthal angle of emergent and incident radiation, radians
$\phi_{Q,\gamma}; \phi_{Q,2\gamma}; \phi_{Q,3\gamma}$	azimuthal angles of emergent radiation defined by Eq. (A-7), radians
$\phi'_{p,\gamma}; \phi'_{p,2\gamma}; \phi'_{p,3\gamma}$	azimuthal angles of incident radiation defined by Eq. (A-12), radians
$\phi'_{p,sp}$	azimuthal angle of incidence for energy specularly reflected with azimuthal angle ϕ_p , radians
$\Delta\omega, \Delta\omega'$	small solid angles for emergent and incident radiation, steradians
$d\omega, d\omega'$	differential solid angles for emergent and incident radiation, steradians

Subscripts

b	black body
d	directional
D	diffuse
e	emitted
H	hemispherical
i	incident
P	point $P(x, z = 0)$
$\bar{P}$	point $\bar{P}(\bar{x}, \bar{z})$
Q	point $Q(y, z)$
r	reflected
s	scattered component
so	solar
t	thermal
λ	spectral quantity

Superscripts

*	solar
+	emergent intensity
-	incident intensity

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1. INTRODUCTION

The transfer of energy by thermal radiation is an important mechanism in a growing number of contemporary engineering systems. Among them are devices for infrared surveillance, solar energy collectors, storage systems for liquids at cryogenic temperatures, and spacecraft. Spacecraft thermal design requires accurate control of internal vehicle temperatures to insure efficient operation of sensitive electronic equipment. For manned spacecraft reliable and accurate temperature control can be even more critical. Internal space vehicle temperatures are determined by internal energy dissipation rates, external surface temperatures, and the interchange of thermal radiation between external surfaces and the space environment. Hence, there is considerable interest in evaluating and improving analytical methods for predicting radiant heat exchange between engineering materials in a space environment.

Radiative energy transfer between opaque materials depends on the external environment as well as the geometrical arrangement and the thermal radiation properties of the interacting surfaces. The surface property information required for radiative transfer analysis is determined by the level of detail employed in analysis. Until recently, radiant exchange calculations were made using surface property models which essentially specify only the magnitude of radiant energy emitted, absorbed, and reflected by a surface. To achieve more accurate prediction of radiant heat transfer, more detailed knowledge of the emission and reflection characteristics of the participating surfaces is required. In particular, the spatial and spectral distribution, as well as the state of polarization of emitted radiation and the dependence of the magnitude and spatial distribution of reflected energy on

wavelength, polarization and direction of incident radiation, can each influence radiative transfer. Unfortunately, such complete property information is generally not available and is expensive to obtain. Wavelength, polarization and directional characteristics of a perfect surface; that is, a surface which is plane, homogeneous, and physically, as well as chemically, uncontaminated, are completely determined by the optical properties refractive index and absorption index of the material. However, engineering materials generally cannot be considered perfect surfaces, and it is necessary to also consider topographical, chemical and physical surface characteristics. Each of these characteristics influences the wavelength, polarization, and directional dependence of surface radiation properties. Comparatively little work has been reported to evaluate the importance of these real surface characteristics on radiant heat transfer, and this is the general objective of this research. Simultaneous investigation of all real surface characteristics is extremely complicated, and it is more practical to isolate an individual factor and study its effect.

The primary objective of this research is evaluation of surface topography effects on radiant heat transfer. In addition, the influence of direction dependent properties due to a dielectric coating on a thick substrate is investigated. A further goal is an assessment of the accuracy attainable with analysis using simple radiation property models.

To achieve the stated objectives the fundamental equations of radiative transfer are applied to a simple system of radiatively interacting surfaces. Using detailed direction and wavelength dependent property models representative of engineering materials, the governing equations are evaluated to obtain numerical results for radiative transfer rates.

These results are then compared to those obtained with analysis using less detailed property models. Interpretation of the results is directed to establishing the magnitude of real surface property effects on radiant transfer and the accuracy attainable with simplified property models. This information should provide valuable direction for radiation property measurement programs which must supply the necessary data for thermal design calculations.

In Chapter 2, literature relevant to this research is reviewed. The system of interacting surfaces and the external environment selected for study are described in Chapter 3. In Chapter 4, radiation property models used in this study are discussed. Radiant heat transfer and equilibrium temperature distributions are presented and interpreted in Chapters 5 and 6, respectively. A summary of important conclusions and a brief discussion of recommended extensions of this study are given in Chapter 7. Finally, in APPENDICES A and B, respectively, details of the mathematical formulation and numerical methods are discussed.

2. LITERATURE SURVEY

2.1 INTRODUCTION

Literature relevant to this research is reviewed in two main parts. In Section 2.2, surface property models commonly used in radiative heat transfer are critically examined in view of experimental and theoretical property studies. Available information concerning the influence of surface roughness on radiative properties is also examined. In Section 2.3, the observed influence of spectral, directional and polarization property dependence on radiative heat transfer is summarized.

Throughout this survey radiation properties are considered total properties (integrated over all wavelengths) unless otherwise designated. For brevity, the abbreviation BDR is used for bidirectional reflectance.

2.2 THERMAL RADIATION SURFACE PROPERTIES

2.2.1 Common Property Models

Most radiative heat transfer analysis is based on the assumption that surface properties are independent of polarization, wavelength, and direction. In particular, emission is usually considered diffuse and reflection either diffuse or specular. The use of these simple property models is motivated by several factors of which the following are probably the most important. First, precise accounting for directional, spectral and polarization dependence of surface properties severely complicates radiative heat transfer analysis. Second, the detailed property information required to refine calculation

procedures has only recently started to become available. Several property models have been developed which essentially retain the computational simplicity provided by simple property models and partially compensate for observed characteristics of engineering surfaces.

Radiation properties of many materials are very different for radiation in the visible and infrared portions of the spectrum. This fact seriously impairs the accuracy of gray (wavelength independent) property radiative transfer analysis when solar radiation is present. Bobco [1][†] proposed a semi-gray property model to overcome this difficulty. It was suggested that radiative energy transfer in the infrared and solar spectral bands be considered separately, and within each spectral interval gray property models be used. Discussion by Hering [2] provided justification for this semi-gray model.

Seban, in discussion appended to [3], suggested a reflection model which expressed hemispherical reflectance as the sum of direction independent specular and diffuse components. Sparrow and Lin [4] developed methods of analysis incorporating this model for diffusely emitting, gray materials. Hering [2] generalized this analysis to include the semi-gray spectral model and extended its application to systems irradiated by an external radiant source. The semi-gray, diffuse emission, specular plus diffuse reflection model is particularly attractive because it yields relatively simple and general expressions for radiant heat transfer and includes both diffuse and specular reflection as limiting cases.

Ample experimental and theoretical evidence is available which demonstrates that the radiation properties of metallic engineering

[†]Numbers in brackets refer to entries in REFERENCES.

surfaces are not accurately represented by the simple models mentioned above. Some of this evidence is summarized in Section 2.2.2.

2.2.2 Surface Roughness Effects on Radiation Properties

2.2.2.1 Introduction

In this section literature concerned with surface roughness effects on thermal radiation properties of surfaces is reviewed. It will become evident that present knowledge of roughness effects is qualitative in many respects. A major reason for this situation is that important topographical characteristics of engineering surfaces have not yet been clearly established. Early radiation property data were, at most, accompanied by concise descriptions of sample surface characteristics based simply on visual observation. Recently, attempts have been made to report some quantitative measures of surface condition. Frequently, rms (root-mean-square) roughness height σ measured with a mechanical stylus is reported. Few investigators have reported any additional measure of surface topography [5,6].

For convenience, the term optical roughness is introduced to denote the ratio σ/λ , where λ is a characteristic radiation wavelength. Data concerned with surface roughness effects on radiation properties are conveniently organized in terms of two major optical roughness ranges. The two are termed small and large optical roughness, and denote values of optical roughness much less and considerably greater than unity, respectively. The designations small and large optical roughness are used in this context throughout this research.

2.2.2.2 Reflectance

The dependence of reflectance of a perfect surface on direction, wavelength, and polarization of incident energy is described by electromagnetic theory in terms of wavelength dependent material optical indices. A pencil of radiant energy within a small solid angle incident on a perfect surface is specularly reflected; that is, the reflected energy lies in the plane of incidence with identical polar angles of incidence and reflection as well as solid angles of transfer.

Roughening of an initially smooth surface is known to reduce reflectance. The decrease in reflectance is attributed to changes in the optical indices of the surface material (surface damage) and to multiple reflections (cavity effect) within surface asperities. Quantitative investigation of these phenomena was the objective of several recent publications [7-14].

Edwards and Catton [7] measured the near-normal reflectance of aluminum, titanium, and stainless steel samples which had undergone common surface finishing operations. They concluded that surface damage was the major cause of the reduction in reflectance at small optical roughness values. Topography effects were of major importance only for sandblasted surfaces. Bennett [8] also demonstrated the influence of surface damage on reflectance. Normal reflectance of electropolished and mechanically polished copper samples was measured. The mechanically polished sample exhibited lower reflectance values than that of the undamaged electropolished sample in the 0.3-32 micron (μm) spectral range. The largest reductions occurred at short wavelengths (0.9-0.6 at $\lambda = 0.6 \mu\text{m}$) with smaller but measurable differences observed in the

infrared (0.990-0.985 at $\lambda = 10.0 \mu\text{m}$). A summary of literature relevant to surface damage effects on optical indices is available [12].

Data which contradict the conclusion that topographical effects on reflectance are unimportant for small optical roughness were reported by Birkebak, et al. [9]. Four roughened glass substrates with rms roughness height values between 0.58 and 2.0 μm were successively coated with vacuum-deposited, opaque layers of aluminum, gold, and platinum. After each coating was applied, spectral reflectance for near-normal illumination was measured. Data for the three materials were correlated within a few percent by plotting the ratio of rough sample reflectance to the reflectance of an optically smooth sample of the same material versus optical roughness. The reflectance ratio decreased rapidly with increasing optical roughness between 0.0 and 0.2 indicating that for small optical roughness surface topography is an important factor. Reflectance data for nickel, however, were in poor agreement with those for the roughened surfaces coated with aluminum, gold, and platinum. This was attributed to a condition of unusually high surface stress in the rough nickel samples.

Ample evidence of the importance of topographical effects on reflectance for large optical roughness is available [15,16] and is not discussed here.

In summary, detailed knowledge of roughness effects on reflectance is lacking. Much of the available data are not accompanied by quantitative description of surface condition. Surface topography has been established as an important factor for large optical roughness. For small optical roughness, however, its importance relative to that of surface damage is uncertain due to contradictory data.

2.2.2.3 Emittance

The state of polarization and spatial distribution of radiation emitted by a perfect surface are available from the Fresnel expressions in terms of the wavelength dependent material optical indices. Surface roughness, however, is one of the characteristics which contributes to discrepancies between the emittance of a perfect and an engineering surface.

The influence of surface roughness on emittance was the subject of several investigations [5,7,8,17,18,19]. Rolling [5] reported that roughness did not produce an appreciable increase in hemispherical emittance of annealed platinum samples with rms roughness heights between 0.6 and 2.4 μm for sample temperatures between 865°K and 1840°K. Although hemispherical emittance was not significantly increased, even slightly rough surfaces exhibited significant changes in the spatial distribution of emitted energy. Several investigators [5,7,8] reported that normal emittance is only slightly increased by roughness until the cavity effect becomes important. However, measurements by Rolling [5] as well as Edwards and Catton [7] demonstrated that directional emittance of rough metal surfaces increases more rapidly with increasing polar angle of emission up to about 70° than that for a perfect surface of the same material. For larger emission angles, roughness tended to reduce emittance in comparison to that of a perfect surface. Thus, a slow transition toward diffuse emission was observed as surface roughness increased. This trend was more pronounced at short wavelengths.

Rolling's comprehensive study [5] on platinum also provided information concerning the influence of roughness on the spectral and

temperature dependence of the polarized components of directional emittance. Roughness reduced spectral dependence of emittance, although this factor remained significant for the roughness range considered. The dependence of emittance on temperature was essentially eliminated by roughening the surface. The parallel polarized component of emitted energy was dominant, especially for moderate to large emission angles. In addition, the influence of roughness on directional emittance was due primarily to changes in the parallel polarized component of emitted energy.

The complex analysis required to evaluate emittance of rough metal surfaces makes theoretical prediction difficult. Limited success has been achieved, however, in predicting hemispherical emittance of very rough surfaces [20-23] with simple surface contours. Experimental studies have established that, if surface topography does not yield appreciable multiple reflections within roughness asperities, and if the material optical indices are not significantly changed by surface damage, then hemispherical emittance of a rough sample does not differ appreciably from that for a perfect surface of the same material. On the other hand, directional emittance, especially the parallel polarized component of emitted energy, is sensitive even to slight surface roughness. With increasing optical roughness, surfaces tend toward diffuse emitters. When roughness increases to the point where multiple reflections are significant, the cavity effect increases hemispherical emittance in comparison to a perfect surface with identical material properties.

2.2.2.4 Bidirectional Reflectance

The influence of surface topography on the spatial distribution of radiation reflected by a material is complicated, and experimental studies require an enormous amount of data. Most investigators reduced the experimental task by observing energy reflected in a single direction which was usually the specular direction [8,10,24-29]. However, in the literature dealing with radar transmission and communication, it is common to measure radiation reflected back to the transmitter, that is, in the backscattering direction. Several investigators [13,30-39] reported BDR data for rough metal samples. Of these studies, only a few [13,30,32,38] reported data out of the plane of incidence.

Many of the trends exhibited by detailed measurements of the spatial distribution of reflected radiation can be inferred from specular reflectance measurements to which attention is now directed. For small optical roughness, measurements [24,25,27,28,30,38] support several conclusions. First, the specular reflectance of a surface with prescribed rms roughness height increases when the polar angle of incidence or the wavelength of incident radiation is increased. Furthermore, increasing rms roughness height, for fixed direction and wavelength of incident radiation, reduces specular reflectance. For prescribed direction of illumination, the specular reflectance for several materials was correlated [30] with a single curve by plotting the ratio of rough surface specular reflectance to that of a sample of the same material with an optically smooth surface versus optical roughness. It should be noted that this correlation has been substantiated only for near-normal and near-grazing illumination.

Roughness effects on specular reflectance for large optical roughness has not yet been clearly established because of the lack of sufficient data. Toperets' [26] specular reflectance data for two aluminized ground glass samples indicated that specular reflectance decreased with increasing wavelength and angle of illumination. In addition, specular reflectance increased with increasing roughness for fixed direction and wavelength of incident radiation. These trends are contrary to those observed for small optical roughness values. Toperets qualitatively explained these results in terms of geometrical optics and diffraction effects.

Torrance, Sparrow, and Birkebak [31] measured the polarization of energy reflected in the specular direction for several aluminized ground glass samples illuminated with unpolarized light. They concluded that for large optical roughness the perpendicularly polarized component of reflected radiation was slightly larger than the parallel component. The degree of polarization varied with rms roughness height and polar angle of incidence. Surface roughness increased the degree of polarization compared to that for a smooth sample, but the increase was not monotonic with increasing rms roughness height. Finally, the degree of polarization for each sample was greatest when the angle of incidence was near the Brewster angle.

Detailed bidirectional reflectance data were reported by Birkebak [30,38] for nickel and aluminized ground glass samples illuminated at a 10° angle of incidence. Several general conclusions were reported. As optical roughness approached zero the reflected radiation concentrated about the specular direction and approached the distribution peculiar to the specular reflection limit. For optical roughness values near

unity and near-normal illumination, the spatial distribution of reflected radiation approached the diffuse limit. The transition from specular to diffuse reflection occurred rapidly in the range of optical roughness values between approximately 0.05 and 0.4. The data of Torrance and Sparrow [36], Herold and Edwards [13], and Love and Francis [32] support these conclusions for small optical roughness values and near-normal illumination. In addition, these studies provided information concerning the influence of incidence angle. For small optical roughness values, an increase in the incidence angle increased the specularity of the surface. For intermediate values of optical roughness and a limited range of directions for incident energy (45-60 degrees) the reflected radiant intensity did not peak in the specular direction, but attained a maximum at reflection angles greater than the specular angle. This phenomenon was extensively studied by Torrance and Sparrow [36] and called off-specular peaking. Further increase in angle of incidence gave nearly specular reflection. For large optical roughness and near-normal illumination, the spatial distribution of reflected intensity was nearly diffuse and insensitive to further increases in optical roughness [13,30,32,36]. With increasing incidence angle, however, off-specular peaks were observed. For prescribed optical roughness, the ratio of maximum reflected radiant intensity to reflected intensity in the specular direction increased with increasing incidence angle up to 75 degrees. Similarly, the angular shift from the specular direction of the maximum intensity increased with increasing angle of incidence. Torrance, Sparrow, and Birkebak [31] investigated this phenomenon by measuring the degree of polarization of radiation reflected in the plane of incidence for aluminized ground glass samples

illuminated with unpolarized light at selected angles of incidence between 10 and 87 degrees. They concluded that reflected radiation was significantly polarized only for polar angles of reflection near to and greater than the angle of illumination, and polarization generally increased with increasing angle of incidence. The perpendicularly polarized component of reflected intensity was larger than the parallel component, and both were characterized by off-specular peaks. Furthermore, both polarized components were influenced by surface roughness. In contrast, it is interesting to note that Rolling [5] reported a dominance of the parallel polarized component of radiation for emitted energy, and that the primary influence of roughness was to change this component.

Attempts to predict the spatial distribution of radiation reflected by rough engineering materials have had limited success. The physical factors that must be considered generally may be characterized according to the magnitude of optical roughness. For small and intermediate optical roughness values, diffraction and interference effects strongly influence scattering of incident radiation. Theoretical analysis of reflection for this range of optical roughness requires consideration of such phenomena. On the other hand, for large optical roughness values diffraction and interference effects are small and the methods of geometrical optics are employed to predict reflectance.

Many investigators have studied diffraction effects on the spatial distribution of reflected radiation. An excellent summary of relevant literature was given by Beckmann and Spizzichino [40]. Many diffraction theory analyses of reflection are limited to simple surface contours, such as periodically-rough surfaces. Only a few theoretical investigations have been conducted for surface contours as complicated as

those commonly found on engineering materials. Models developed by Beckmann [40] and Davies [41] were evaluated by Houchens and Hering [42] to determine whether these BDR expressions were applicable to common engineering materials. It was shown that Beckmann's model provided reasonably good agreement with limited available BDR data for metal samples and small optical roughness values. The salient features and limitations of Beckmann's results are briefly discussed here, deferring details of the model to later.

Beckmann applied scalar Helmholtz-Kirchhoff diffraction theory [43] to examine reflection of plane, monochromatic waves from a perfect electrical conductor. The limitation to infinitely conducting media eliminated energy absorption. Basically, the model describes the influence of surface topography on the spatial distribution of reflected radiation. The surface contour was considered isotropic and generated by a continuous, stationary random process. The two-dimensional distribution function was Gaussian with Gaussian autocorrelation coefficient. Surface contour was further restricted to conditions for which shadowing by and multiple reflections between neighboring roughness elements were negligible. Polarization of incident and reflected radiation was not considered. The model described BDR in terms of the characteristic roughness parameters, optical roughness σ/λ and rms surface profile slope m .

Torrance [44] proposed a BDR model based on geometrical optics. He considered an isotropic surface constructed of randomly oriented mirror-like facets, which were large compared to the wavelength of incident radiation. Angles between local surface normals and the mean surface normal were normally distributed. Reflected radiation leaving the

surface was described by the sum of a specular and a diffuse component. Furthermore, any ray incurring more than one reflection within a surface cavity was included in the diffuse BDR component. Hence, the specular BDR component accounted only for once-reflected rays. Torrance demonstrated qualitative agreement of theory with experiment for BDR data for very rough samples of both conducting and nonconducting materials.

2.2.2.5 Summary of Radiation Property Investigations

The investigations of radiation properties reviewed in this section reveal the following important points:

- (1) The thermal radiation properties of metal surfaces are strongly influenced by surface roughness.
- (2) Experimental and theoretical information is generally insufficient to completely define the influence of surface roughness on directional emittance, absorptance, and reflectance. However, for small optical roughness, these properties are influenced less by roughness than is the spatial distribution of reflected energy (BDR).
- (3) For large optical roughness values, diffuse reflection is observed only for near normal incidence. Energy incident at moderate to large angles of incidence exhibits off-specular peaks in the intensity of reflected energy.
- (4) Some theoretical BDR models have been examined for their applicability to engineering materials and found representative of available data.

2.3 RADIATIVE HEAT TRANSFER

2.3.1 Introduction

In this section results illustrating the influence of spectral, directional, and polarization dependence of radiation properties on radiative heat transfer are reviewed. It should be emphasized that radiation properties exhibit directional dependence in two forms. First,

emittance varies with the direction of emitted radiation. Similarly, absorptance and directional reflectance depend on the direction of incident radiation. This variation of emittance, absorptance and reflectance with direction is referred to as angular property dependence. Second, reflectance exhibits directional dependence due to the variation of the spatial distribution of reflected radiation with direction of incident energy.

Many of the studies reviewed share common characteristics; thus, it is convenient to enumerate these here with the understanding that exceptions will be noted. Reported results are for systems of plane surfaces with uniform properties and specified uniform temperatures. Generally, properties are considered wavelength independent (gray) and polarization effects are ignored.

2.3.2 Spectral Effects

Errors associated with the gray property model have been illustrated by several authors. Goodman [45] computed net radiant heat exchange over a wide range of temperature and temperature difference for infinite parallel plates of Inconel and aluminum for which spectral property data was available. Emission and reflection were considered diffuse. Comparison of heat transfer results including detailed spectral dependence with similar results employing gray analysis revealed that the gray property model introduced errors dependent on material and surface temperature. Errors in net heat exchange of 29 and 17 percent of the non-gray values were observed for Inconel and aluminum, respectively. Branstetter [46] studied the same system with polished molybdenum, tantalum and tungsten plates and reported that errors using

gray analysis for these materials were very large. Holt, Grosh and Geynet [47] also evaluated net heat exchange between non-gray, infinite, parallel plates. Plate materials were considered optically smooth with spectral and angular dependent radiation properties calculated from electromagnetic theory. Net radiant exchange values obtained using variable property analysis were in all cases greater than results from gray, angular independent property analysis. Discrepancies in net radiant exchange as large as a factor of two were observed between the two methods of analysis. Since both spectral and angular property dependencies were included, it was not possible to assess the influence of each factor separately.

Plamondon and Landram [48] investigated the accuracy of analysis employing the semi-gray property model for predicting the equilibrium temperature distribution on semi-infinite, adjoint plates exposed to a collimated solar source. Plate materials considered were polished aluminum and PV-100 white + black paint. All radiation properties were considered diffuse. For the materials and geometry investigated, semi-gray analysis gave equilibrium temperatures in excellent agreement with those in which detailed spectral property dependence was included.

2.3.3 Angular Dependent Property Effects

Hering [49] provided specific information concerning the influence of angular dependent properties on radiant heat transfer. Local and overall heat loss was evaluated for semi-infinite, specularly reflecting, adjoint plates with angular dependent properties evaluated from the expressions of electromagnetic theory. Three angular dependent property models were considered by selecting three distinct optical

index pairs (n,k) which corresponded to hemispherical emittance values of 0.1, 0.5, and 0.9. Various plate material combinations, surface temperatures, and included angles between the plates were considered. The influence of angular dependent properties on heat transfer was a function of all these parameters, but in each case the major effect was on local heat flux at the apex. Hering concluded that for engineering purposes the effect on overall heat transfer of the angular property dependence of specularly reflecting surfaces was unimportant for the conditions and system investigated. However, discrepancies of greater than a factor of two were observed between local heat transfer rates calculated employing angular dependent and angular independent property models.

Toor and Viskanta [50] used the Monte Carlo method to evaluate radiant exchange between simply arranged specularly reflecting surfaces with angular dependent properties identical to those selected by Hering. The effect of angular dependent properties for the selected configurations was essentially the same as observed by Hering for the adjoint plate system. The angular dependence of emittance was suggested as more important than that of reflectance.

Schornhorst and Viskanta [51] examined the influence of angular dependent properties as described by electromagnetic theory for smooth electrical conductors. The system studied was semi-infinite parallel plates, and the spatial distribution of reflected energy was taken as either specular or diffuse. It was concluded that the influence of angular property dependence was much greater for specular reflection than for diffuse reflection.

Bevans and Edwards [52] investigated the influence of directional.

emittance as predicted by electromagnetic theory on overall heat loss from semi-infinite parallel metal plates which were specular reflectors with normal emittance of 0.1. It was demonstrated that if the surfaces were considered diffuse emitters, predicted heat loss was 35 percent lower than that calculated using the directional property values.

Treat [53] evaluated overall heat loss for pairs of finite, square surfaces arranged in either parallel or oblique configurations. Plates were considered to have angular dependent properties characteristic of very rough nonconductors. For the open configurations considered, reflected radiation had virtually no influence on overall heat loss. The author concluded that in a few situations the small amount of energy emitted by one surface and absorbed by the second was strongly influenced by the angular dependence of emittance.

Edwards [54] demonstrated the importance of considering angular property dependence for systems exposed to solar radiation. To isolate the influence of directional absorptance, absorbed incident solar radiation for several convex configurations exposed to a collimated solar flux was evaluated. The system selected for study eliminated interreflections; thus, absorbed energy was determined by the directional dependence of absorptance. Using spectral directional absorptance data, the total (integrated over solar spectral interval) near-normal absorptance as well as a total geometry-averaged absorptance were calculated. The latter quantity accounted for the directional variation of irradiation over the extent of the system. The author concluded that errors of 30 percent in temperature can be introduced into temperature control equipment design when the angular variation of solar absorptance is ignored.

2.3.4 Spatial Distribution of Reflected Energy

The influence of the spatial distribution of reflected energy on heat transfer can be estimated to some extent by comparing available results obtained using specular and diffuse reflection analyses. For semi-infinite adjoint plates exposed to a collimated solar field [2], factors of 2-3 in local heat transfer, and temperature differences as large as 150°R were observed to occur as surfaces of fixed emittance and absorptance were varied between the extreme combinations of diffuse and specular reflection for surface and solar radiation. Many more example comparisons of radiant exchange for systems of surfaces with diffuse and specular reflection characteristics are available in the literature [55]. These results are well known and are not reviewed here except to note that reported differences between results obtained from diffuse and specular reflection analyses are dependent on factors such as geometry, thermal environment, and material properties.

Information concerning the transition of heat transfer results between those for diffuse and specular reflection is provided by results obtained from analysis employing the simple direction independent specular plus diffuse reflection model. Several authors [2,4,51,56] applied this analysis to evaluate local heat transfer for simple systems. Generally these results show that for high emittance materials the transition of heat flux from that for diffuse to that for specular reflection is nearly proportional to the ratio of the specular reflectance component to the hemispherical reflectance. For low emittance materials, however, the transition is quite geometry dependent.

2.3.5 Polarization Effects

The influence of polarization on radiative heat transfer was investigated by Edwards and Bevens [57]. Radiant transfer within a general enclosure with direction and polarization dependent radiation properties was formulated. Illustrative numerical results were given for radiant transfer between smooth surfaces with properties described by electromagnetic theory. Errors in the value of once reflected then absorbed solar radiation as large as 92 percent were observed when polarization effects were ignored. It was concluded that polarization is important when solar radiation is reflected from one surface to another, particularly for angles of incidence greater than 60 degrees when the reflector is a dielectric or semiconductor and the absorber is a metal.

Edwards and Tobin [58] studied polarization and directional effects on the transmittance of square and semi-infinite slot passages with properties described by electromagnetic theory for smooth surfaces. Results were given for both isothermal and radiatively adiabatic passage walls. Polarization effects were more important for the semi-infinite slot passage than for the passage with square cross section, and in each case the influence increased with increasing passage length. Errors in transmittance due to neglecting polarization effects of 50 and 30 percent were observed for slot and square passages, respectively. For long passages, directional property dependence strongly influenced transmittance. In one example, in which both polarization and directional property dependence was ignored, the transmittance values showed a 93 percent deviation from the true value.

2.3.6 Combined Effects

Research reviewed in Sections 2.3.2-2.3.5 provide insight into the influence of directional, spectral, and polarization property dependence on radiant exchange. The conclusions drawn from these investigations were generally based on results for either perfect surfaces or surfaces with properties represented by simple models. In this section experimental studies of radiative heat transfer between engineering surfaces and analytical heat transfer results for surfaces with theoretical property models representative of rough engineering materials are reviewed.

Several investigators measured radiant heat transfer for systems of simply arranged surfaces and compared measurements to predicted values. Bevans, et al. [59], investigated radiant heat exchange between six-inch squares of aluminum coated with flat black paint, aluminum paint, vacuum deposited aluminum, and anodized aluminum. Samples were arranged with different coating material combinations in one of three simple configurations. Net heat input to the heated surface and surface temperatures were measured. Measurements were compared with values calculated using three radiation property models. The simple, gray, direction-independent property model with reflection considered either diffuse or specular was used as well as the more detailed gray, angular dependent model with reflection taken as the sum of direction dependent specular and diffuse components. In the latter case directional properties for the selected materials were measured, and the second approximate method of analysis proposed by Bevans and Edwards [52] was used to estimate surface temperature and overall heat loss. The more realistic property model did not

yield results in significantly better agreement with experiment than the results employing angular independent properties with either diffuse or specular reflection. However, the authors emphasized that the selected materials and simple surface configurations did not provide a critical test for the different models. Furthermore, approximations inherent in the analysis employed [52] may have counterbalanced the use of a more realistic property model.

Gilbert [60] measured net overall radiant exchange between twelve-inch square parallel plates constructed of sandblasted 302 stainless steel. For the temperatures and system considered, analysis using gray, diffuse properties provided results in good agreement with measurements.

A more comprehensive experimental investigation of radiant exchange between surfaces was reported recently by Schornhorst and Viskanta [61]. Local irradiation was measured at several locations on two long, narrow plates which were either parallel or perpendicular to each other. In all cases the plates were maintained at uniform, but not always equal, temperatures. Several plate materials and surface conditions were studied including smooth gold, rough gold, polished 303 selenium stainless steel, sandblasted stainless steel, 3M Black Velvet Paint, Pyromark Black Paint, and PV-100 White paint. Surfaces were further characterized by profilometer measurements of rms roughness height and experimentally determined hemispherical emittance values. Plate temperatures between approximately 50°F and 800°F were used. Measured local irradiation values were compared with those calculated using three simple property models. Emittance, absorptance, and directional reflectance were considered gray and their values estimated from the measured hemispherical emittance value. Emission was considered diffuse and the spatial

distribution of reflected radiation was either diffuse, specular, or the sum of direction independent specular and diffuse components. The authors concluded that for the ranges of system parameters and materials considered, local irradiation measurements were predicted remarkably well with gray diffuse analysis. However, differences of 20 percent between measured and calculated irradiation values were observed. The apparent accuracy attained with gray diffuse analysis was surprising since some metal samples appeared to reflect specularly. It was not possible to accurately assess the separate effects of directional and spectral property dependence since spectral irradiation values were not measured. The individual influence of spectral and directional property dependencies may have been partially compensating in such a manner that gray diffuse analysis provided excellent results.

Schornhorst and Viskanta [51] also considered a theoretical approach for evaluating the accuracy of simple radiation property models. Radiant exchange between semi-infinite parallel plates with plate spacing-to-width ratios between 0.25 and 2.0 was calculated. Local spectral heat loss and irradiation were evaluated employing a detailed property model and compared to values obtained with six less detailed property models. The detailed property model included angular dependent emittance, absorptance, and directional reflectance as predicted by electromagnetic theory for optically smooth electrical conductors. Spectral dependence of the material optical indices was described with Drude's theory [62] using an electrical resistivity representative of high alloy steel (actual value of resistivity was not reported). The spatial distribution of reflected radiation was described by the Beckmann bidirectional reflectance model. Local irradiation predicted with the detailed property model was

consistently lower than that evaluated with the six less detailed models. Discrepancies of a factor greater than three were observed between Beckmann model irradiation results and those calculated with some simple property models. A possible explanation for the consistently low irradiation values obtained with the Beckmann model was proposed to the authors in a personal communication. The Beckmann bidirectional reflection model does not satisfy energy conservation for all choices of the governing characteristic roughness parameters [42]. Calculations by this author indicated that for some of the parameter choices of Schornhorst and Viskanta, 75 percent of the reflected radiation was not accounted for by the bidirectional reflectance model. This situation should result in low irradiation values.

Schornhorst and Viskanta [61] also compared irradiation values calculated using detailed property models with measurements for the parallel plate configuration. Comparison of calculated and measured results was limited by several factors. All calculations were made considering radiation properties gray which necessitated selection of appropriate gray property values. Directional properties were not measured, and electromagnetic theory expressions were used to estimate them. All properties were evaluated at the wavelength corresponding to the spectral black body peak for selected plate temperatures. Furthermore, characteristic roughness parameters in the theoretical bidirectional reflectance expression were estimated. The rms roughness height was taken as the value measured with a mechanical stylus. Neither the value selected nor the method of selection were reported for rms roughness slope. Hence, the extent to which the theoretical property model corresponded to the actual properties of the surfaces used in the experimental program is

unknown. The authors concluded that an angular dependent property model with reflection taken as the sum of direction dependent specular and diffuse components provided an excellent compromise between realistic level of property detail and computational effort. Furthermore, for the cases considered selection of a property model was more critical for evaluating local irradiation than local heat flux.

Toor and Viskanta [50] used the Monte Carlo method to calculate radiant exchange employing a detailed property model. Local and overall spectral absorption factors were determined for each of four simple geometrical configurations. Overall radiant interchange, as well as local and overall spectral radiant heat transfer were determined for various material property and surface temperature combinations. The detailed property model consisted of angular dependent emittance, absorptance, and directional reflectance as predicted by electromagnetic theory for smooth materials. Three combinations of material optical indices (n, k) were selected (values reported by Hering [49] were used) to yield spectral, hemispherical emittance values 0.1, 0.5, and 0.9, and the spatial distribution of reflected radiation was described by Beckmann's bidirectional reflectance model. A single value for rms slope m was employed in the local radiant exchange calculations. For comparison, absorption factors and radiant exchange values were determined from four simple property models. The authors concluded that differences between results obtained with individual property models were larger when calculating radiant interchange between surfaces than when determining radiant heat transfer from a surface. With all other conditions fixed, the choice of property model was most important when determining the radiant interchange between highly reflecting surfaces arranged in an open

configuration. On the other hand, closed systems with highly reflecting surfaces showed the largest variation of radiant heat transfer for individual property models. Finally, simple constant property models with either diffuse or specular reflection sometimes gave significantly different results than those obtained with the detailed property model.

Analytical studies by Treat as well as Wilden were reported in a joint publication [53]. Each author determined radiant exchange between surfaces with properties characteristic of very rough nonconductors. The property model developed by Torrance [44] was adjusted to fit BDR data for magnesium-oxide ceramic and potassium silicate-zinc oxide. In addition, properties of a fictitious, high emittance material were created by judicious selection of adjustable constants in Torrance's model. It should be noted that the selected materials exhibited nearly diffuse reflection, except for large angles of incidence. Radiant exchange results for the detailed property models were compared to those for simple, constant property models with reflection taken either specular or diffuse. Treat considered overall heat exchange for finite surfaces. The selected geometries were so open that reflected radiation had little influence on net radiant heat loss. However, it was suggested that for open configurations directional dependence of emission can be an important factor when determining radiant energy interchange between surfaces. Wilden calculated radiant exchange between semi-infinite parallel plates. Local and overall heat loss values were evaluated for plates with properties characterized by Torrance's model for each of three very rough nonconductors. The results were intermediate to those for angular independent plate properties with reflection considered either specular or diffuse. For given plate material, radiant heat loss values determined using

detailed and simple property models did not vary by more than ten percent. Furthermore, results for detailed property models were, in all cases, within one percent of those obtained with the simple constant property diffuse reflection model.

2.3.7 Summary of Heat Transfer Investigations

This review of available heat transfer results reveals the following important points:

- (1) The directional, spectral, and polarization dependence of radiation properties can significantly influence radiative heat transfer.
- (2) The importance of variable property effects, particularly directional effects, depends strongly on material properties and geometry (degree of interaction between surfaces). The choice of property model is especially critical when evaluating radiant interchange between low emittance materials arranged in open configurations, and when calculating radiant heat transfer from low emittance materials arranged in closed configurations. Furthermore, radiative transfer within systems exposed to external sources of radiation appears to be particularly sensitive to variable surface properties.
- (3) The majority of available results evaluating angular property effects are for smooth, specularly reflecting materials for which the radiation properties are adequately described by electromagnetic theory.
- (4) Few results are available to illustrate the influence of surface roughness on radiant exchange between surfaces of prescribed temperature, and apparently no results are available for radiant transfer between rough surfaces exposed to an external radiation source.
- (5) It has been suggested that a model consisting of direction dependent emission and direction dependent specular plus diffuse reflectance components may provide an excellent compromise between property detail and computational effort; however, very few results are available to evaluate the proposed model.

3. RADIATIVE TRANSFER FORMULATION FOR ADJOINT PLATE SYSTEM

3.1 INTRODUCTION

In this section the system of radiatively interacting surfaces selected for study is defined. Fundamental principles of radiative transfer are applied to the system and the equations governing local heat flux and temperature are developed. Certain simplifications are introduced to eliminate parameters of secondary importance to the objectives of this research. Both directional and spectral dependence of surface properties are accounted for in the analysis. Although the expressions developed strictly apply to the chosen system, there is no fundamental difficulty in extending the analysis to other systems of interest.

3.2 ASSUMPTIONS AND SYSTEM DESCRIPTION

Certain simplifying assumptions are employed in the analysis. These are not expected to seriously impair the relevancy of the results to the objectives of this research. It is assumed that surface heat exchange occurs only by radiant energy transport, and except for the presence of a collimated radiation field, external sources of thermal radiation are negligible. The interacting surfaces are opaque to thermal radiation, and all surface radiation properties are temperature independent and uniform over the extent of each surface. The intervening medium is radiatively transparent with unit refractive index. Only steady transfer is considered, and polarization effects are ignored.

The system selected for study is illustrated in Fig. 3.1. Identical, semi-infinite, adjoint planes of length L and including an angle γ are uniformly irradiated by a collimated radiation field directed along the

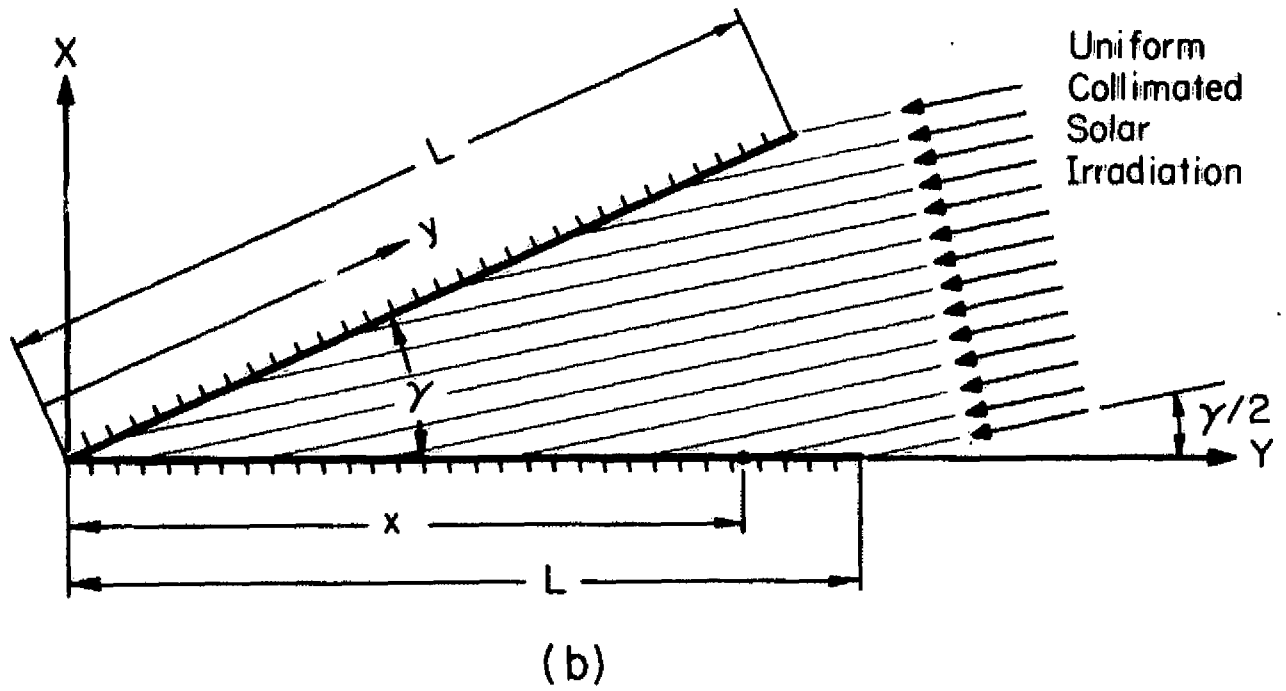
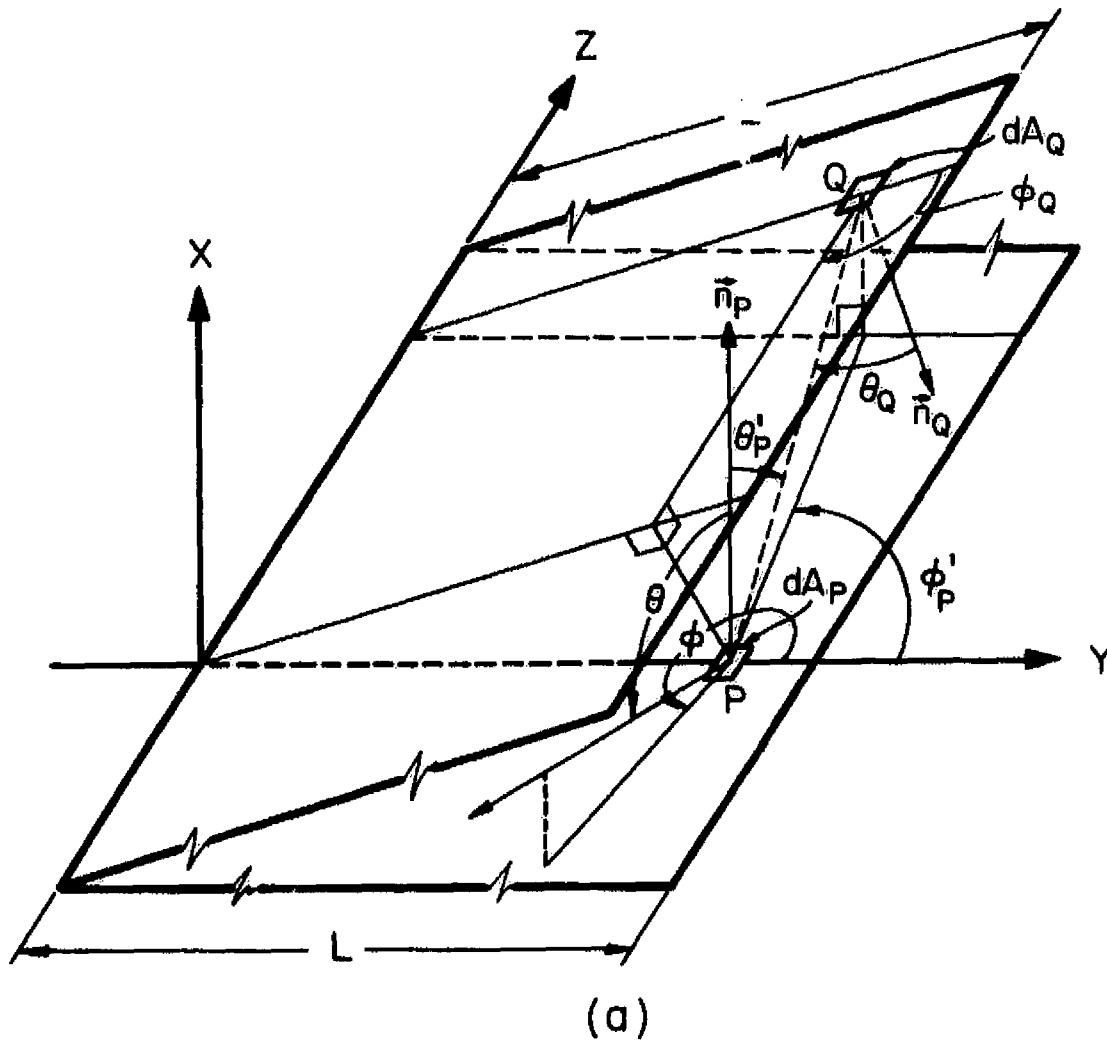


Figure 3.1 Adjoint plate system

bisector of the included angle and in planes normal to the common edge of the plates. This system was chosen for study because of its geometrical simplicity, the availability of heat transfer results for certain simple surface property models, and the important features of common temperature control components present. Among the latter are interreflection phenomena and external radiation fields.

In the analysis to follow, symbols x and y denote distances measured normal to the common edge of the plates along the lower and upper surfaces, respectively, while z denotes distance measured parallel to the common edge from an arbitrary plane normal to the common plate axis. Local spherical coordinates (θ', ϕ') and (θ, ϕ) are used to define the direction of incident (prime superscript) and leaving (no superscript) energy. The spherical coordinates can be expressed as functions of the position coordinates x, y, z . These relations are given in Appendix A.

The complexity of the analysis is significantly reduced because of the infinite width of the plates, the geometrical symmetry, and the uniformity of the surface properties and external irradiation. The radiant intensity leaving an element of either surface in a prescribed direction defined by polar and azimuthal angles (θ, ϕ) depends only on the distance of the element measured normal to the common axis of the plates. Furthermore, the radiant intensity distribution is identical on the two surfaces at equal distances ($x = y$) from the apex.

3.3 LOCAL HEAT TRANSFER

Local monochromatic heat transfer per unit time and per unit surface area $q_\lambda(x)$ at a typical element of the lower surface located at point $P(x, z = 0)$ is evaluated as the difference between the rate of emission

and the rate of absorption of radiant energy with wavelengths within an infinitesimal spectral interval $d\lambda$ around wavelength λ .

$$q_{\lambda}(x) = \epsilon_{H,\lambda} \pi I_{b,\lambda}(x) - \int_{y=0}^L \int_{z=-\infty}^{\infty} \alpha_{d,\lambda}(\theta'_p) I_{\lambda}^{-}(x, \theta'_p, \phi'_p) K(x, y, z) dz dy - \alpha_{d,\lambda}(\theta'_{so}) S_{\lambda} \sin \frac{\gamma}{2} \quad (3.3.1)$$

In Eq. (3.3.1)

$$\theta'_{so} = \frac{\pi - \gamma}{2} \quad (3.3.2)$$

and the geometrical quantity $K(x, y, z)$ is given by the following relation

$$K(x, y, z) = \frac{xy \sin^2 \gamma}{(x^2 + y^2 - 2xy \cos \gamma + z^2)^2} \cdot \quad (3.3.3)$$

Spectral directional absorptance for polar angle of incidence θ' and spectral hemispherical emittance are denoted by the symbols $\alpha_{d,\lambda}(\theta')$ and $\epsilon_{H,\lambda}$, respectively. Directional absorptance is taken azimuthally independent. Superscript minus (-) is used to designate spectral radiant intensity $I_{\lambda}^{-}(x, \theta'_p, \phi'_p)$ incident at x from the (θ'_p, ϕ'_p) direction, while later a superscript plus (+) will signify radiant intensity leaving a surface. The spectral black body radiant intensity $I_{b,\lambda}(x)$ for absolute temperature $T(x)$ is available from Planck's law. The energy of the external radiation field is expressed in terms of the spectral solar constant S_{λ} . This quantity represents solar energy in the spectral interval $d\lambda$ around wavelength λ transported per unit time and per unit area normal to the direction of energy propagation.

Local total heat transfer per unit time and per unit surface area $q(x)$ is obtained by integrating monochromatic heat transfer over the

entire spectrum.

$$q(x) = \int_{\lambda=0}^{\infty} q_{\lambda}(x) d\lambda \quad (3.3.4)$$

Inserting the spectral flux expression from Eq. (3.3.1) yields

$$\begin{aligned} q(x) = & \epsilon_H \bar{\sigma} T^4(x) - \int_{y=0}^L \int_{z=-\infty}^{\infty} K(x,y,z) \int_{\lambda=0}^{\infty} \alpha_{d,\lambda}(\theta'_P) I_{\lambda}^{-}(x, \theta'_P, \phi'_P) d\lambda dz dy \\ & - \sin \frac{Y}{2} \int_{\lambda=0}^{\infty} \alpha_{d,\lambda}(\theta'_{so}) S_{\lambda} d\lambda \end{aligned} \quad (3.3.5)$$

The symbol ϵ_H denotes total hemispherical emittance, and $\bar{\sigma}$ is the Stefan-Boltzmann constant.

Under the assumptions introduced, the intensity incident at x from the (θ'_P, ϕ'_P) direction $I_{\lambda}^{-}(x, \theta'_P, \phi'_P)$ equals the intensity leaving a point $Q(y, z)$ on the adjacent plate in the (θ_Q, ϕ_Q) direction $I_{\lambda}^{+}(y, z, \theta_Q, \phi_Q)$. Q is the point of intersection of a line from P directed along the (θ'_P, ϕ'_P) direction and the adjacent surface. Hence, Eq. (3.3.5) can be written

$$\begin{aligned} q(x) = & \epsilon_H \bar{\sigma} T^4(x) - \int_{y=0}^L \int_{z=-\infty}^{\infty} K(x,y,z) \int_{\lambda=0}^{\infty} \alpha_{d,\lambda}(\theta'_P) I_{\lambda}^{+}(y, z, \theta_Q, \phi_Q) d\lambda dz dy \\ & - \sin \frac{Y}{2} \int_{\lambda=0}^{\infty} \alpha_{d,\lambda}(\theta'_{so}) S_{\lambda} d\lambda \end{aligned} \quad (3.3.6)$$

The terms on the right-hand side of Eq. (3.3.6) represent energy emitted and absorbed at x per unit time and per unit surface area. The first term is the rate of emission from point x . The second term is irradiation

from the adjacent surface which is absorbed at x , and the last term is directly incident solar irradiation which is absorbed at x .

Solutions to Eq. (3.3.6) are commonly sought for two thermal conditions. When the temperature distribution is specified, local radiant heat transfer is evaluated. When local total heat transfer is specified, local surface temperature is calculated. The condition of zero local radiant flux is a situation of particular interest. The resulting plate temperature distribution is called the equilibrium temperature distribution. From Eq. (3.3.6) it follows that to evaluate either $q(x)$ or $T(x)$ the local spectral radiant intensity must be available.

3.4 LOCAL SPECTRAL RADIANT INTENSITY

For the selected system of interacting surfaces with specified radiation properties and known temperature (flux) irradiated by an external radiation field of known spectral composition, the fundamental quantity required to evaluate local heat flux (temperature) is local, spectral radiant intensity $I_{\lambda}^{+}(x, \theta_P, \phi_P)$. The spectral intensity of leaving energy at x in the (θ_P, ϕ_P) direction consists of the sum of emitted energy and of radiation incident from the surroundings which is reflected into the (θ_P, ϕ_P) direction. It satisfies the following integral equation [63]

$$\begin{aligned}
 I_{\lambda}^{+}(x, \theta_P, \phi_P) = & \epsilon_{d, \lambda}(\theta_P) I_{b, \lambda} \\
 & + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd, \lambda}(\theta'_P, \phi'_P; \theta_P, \phi_P) I_{\lambda}^{+}(y, z, \theta_Q, \phi_Q) K(x, y, z) dz dy \\
 & + \rho_{bd, \lambda}(\theta'_{so}, 0; \theta_P, \phi_P) S_{\lambda} \sin \frac{\gamma}{2}
 \end{aligned} \tag{3.4.1}$$

The first term on the right-hand side of Eq. (3.4.1) represents the intensity of monochromatic energy emitted at x in the (θ_P, ϕ_P) direction. The second term is the contribution to the intensity leaving x due to energy arriving from the adjacent surface and undergoing reflection at x into the (θ_P, ϕ_P) direction. The final term represents directly incident energy from the collimated source which is reflected at x into the (θ_P, ϕ_P) direction. The symbol $\varepsilon_{d,\lambda}(\theta)$ denotes the azimuthally independent spectral directional, emittance for polar emission angle θ . The remaining surface property in Eq. (3.4.1) is the spectral bidirectional reflectance $\rho_{bd,\lambda}(\theta', \phi'; \theta, \phi)$. It is defined as the ratio of the intensity of reflected energy in the (θ, ϕ) direction due to energy incident from the (θ', ϕ') direction within the elemental solid angle $d\omega'$, $dI_{r,\lambda}^+(\theta', \phi'; \theta, \phi)$, to the radiant power per unit surface area incident from the (θ', ϕ') direction.

$$\rho_{bd,\lambda}(\theta', \phi'; \theta, \phi) \equiv \frac{dI_{r,\lambda}^+(\theta', \phi'; \theta, \phi)}{I_{\lambda}^-(\theta', \phi') \cos \theta' d\omega'} \quad (3.4.2)$$

3.5 DISCUSSION OF GOVERNING EQUATIONS

Consider a given system with specified surface properties which is irradiated by an external source of known spectral composition. When local surface temperature is specified the direction independent but temperature dependent black body intensity $I_{b,\lambda}(x)$ can be evaluated with Planck's law. Thus, Eq. (3.4.1) is a linear integral equation from which local spectral intensity can be evaluated, and, in turn, radiant heat transfer can be calculated directly from Eq. (3.3.6). The procedure is not as straight forward when total heat transfer is specified. Since the black body intensity cannot be evaluated until surface temperature is specified, local spectral intensity cannot be directly evaluated with

Eq. (3.4.1). If local spectral heat flux were known, this difficulty could be overcome by combining Eqs. (3.3.6) and (3.4.1) to eliminate the unknown black body spectral intensity. Local spectral intensity could then be evaluated with the resulting integral equation. However, the common situation is for local total heat flux to be known, and direct elimination of the unknown surface temperature from Eqs. (3.3.6) and (3.4.1) does not appear possible except for certain property models. Notable exceptions occur for the gray and semi-gray models for spectral property dependence. For arbitrary spectral property dependence, Eqs. (3.3.6) and (3.4.1) represent simultaneous integral equations in I_{λ}^+ and T . One method for attacking this problem is to determine the unknown surface temperature distribution by successive iterations. A reasonable estimate of surface temperature distribution is made, and local spectral intensity is evaluated with Eq. (3.4.1). Substitution in Eq. (3.3.6) then yields local total heat transfer corresponding to the assumed temperature distribution. Should the calculated heat transfer values not agree with those originally specified, the iterative procedure may be repeated until the temperature distribution which yields the prescribed flux values is obtained.

The governing equations for radiative transfer including direction and wavelength dependent properties have been applied to the adjoint plate system of radiatively interacting surfaces, and the general procedure necessary for evaluation of local flux or surface temperature briefly discussed. Simplifications to Eqs. (3.3.6) and (3.4.1) occur for certain radiation property models. Two such approximations are discussed in Sections 3.6 and 3.7.

3.6 SEMI-GRAY APPROXIMATION

The semi-gray model [1,2,48] for spectral property dependence is employed in this investigation for surfaces exposed to the collimated solar source. The spectral interval which encompasses the significant portion of surface emitted thermal radiation is assumed sufficiently far removed from that for solar radiation that there is negligible spectral overlap of radiation from the two sources. Within each spectral interval, surface properties are taken independent of wavelength which will be denoted by deleting the λ subscript. There are two important consequences of this spectral separation of thermal and solar radiation. First, distinct surface properties can be assigned to the thermal and to the solar spectral intervals. Second, for temperature independent properties, the local solar intensity and absorbed solar energy can be evaluated independent of the surface temperature or surface properties for the thermal spectral interval. Under the above restrictions, the equations governing radiant exchange simplify considerably. The local intensity of surface emitted radiation (thermal intensity) I_t^+ can be written

$$I_t^+(x, \theta_P, \phi_P) = \epsilon_d(\theta_P) \frac{\bar{\sigma} T^4(x)}{\pi} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd}(\theta'_P, \phi'_P; \theta_P, \phi_P) I_t^+(y, z, \theta_Q, \phi_Q) K(x, y, z) dz dy \quad (3.6.1)$$

and the local intensity of solar radiation (solar intensity) I_{so}^+ can be written

$$I_{so}^+(x, \theta_p, \phi_p) = \rho_{bd}^*(\theta'_{so}, 0; \theta_p, \phi_p) S \sin \frac{\gamma}{2} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd}^*(\theta'_p, \phi'_p; \theta_p, \phi_p) \cdot I_{so}^+(y, z, \theta_Q, \phi_Q) K(x, y, z) dz dy \quad (3.6.2)$$

The asterisk superscript distinguishes surface properties for short wavelength solar radiation from those for long wavelength radiation. It should be noted that the radiant intensities I_t^+ and I_{so}^+ appearing in Eqs. (3.6.1) and (3.6.2) are not monochromatic quantities but represent the total intensity of energy in the thermal spectral interval and solar spectral interval, respectively. The symbol S denotes the solar constant, which is the total solar energy of the external field transferred per unit time and per unit area normal to the direction of energy propagation. The local absorbed solar irradiation $q_{a,so}(x)$ can be expressed in terms of the local solar intensity, the solar constant, and the surface properties for solar wavelengths as

$$q_{a,so}(x) = \alpha_d^*(\theta'_{so}) S \sin \frac{\gamma}{2} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \alpha_d^*(\theta'_p) I_{so}^+(y, z, \theta_Q, \phi_Q) K(x, y, z) dz dy \quad (3.6.3)$$

Local total radiant heat transfer follows as

$$q(x) = \epsilon_H \bar{\sigma} T^4(x) - \int_{y=0}^L \int_{z=-\infty}^{\infty} \alpha_d(\theta'_p) I_t^+(y, z, \theta_Q, \phi_Q) K(x, y, z) dz dy - q_{a,so}(x) . \quad (3.6.4)$$

Consider a system of surfaces with known radiation properties for the solar spectral interval irradiated by a specified collimated source. The local solar intensity can be evaluated with Eq. (3.6.2), and, in turn, local absorbed solar irradiation follows from Eq. (3.6.3). For specified surface properties for the thermal spectral interval and surfaces of prescribed temperature, local thermal intensity may be calculated from Eq. (3.6.1), and local total heat transfer follows from Eq. (3.6.4).

For radiatively adiabatic surfaces [$q(x) \equiv 0$] it is convenient to rearrange the above equations and evaluate directly local equilibrium temperature. Using Eq. (3.6.4) with $q(x) = 0$ to eliminate the unknown temperature from Eq. (3.6.1) yields

$$I_t^+(x, \theta_P, \phi_P) = \frac{\epsilon_d(\theta_P)}{\pi \epsilon_H} q_{a,so}(x) + \int_{y=0}^L \int_{z=-\infty}^{\infty} \left[\rho_{bd}(\theta'_P, \phi'_P; \theta_P, \phi_P) + \frac{\epsilon_d(\theta_P) \alpha_d(\theta'_P)}{\pi \epsilon_H} \right] I_t^+(y, z, \theta_Q, \phi_Q) dz dy \quad (3.6.5)$$

Equation (3.6.5) constitutes a linear integral equation for the local thermal intensity. Local equilibrium temperature follows from Eq. (3.6.4) as

$$T(x)^4 = \frac{q_{a,so}(x)}{\epsilon_H \bar{\sigma}} + \frac{1}{\epsilon_H \bar{\sigma}} \int_{y=0}^L \int_{z=-\infty}^{\infty} \alpha_d(\theta'_P) I_t^+(y, z, \theta_Q, \phi_Q) K(x, y, z) dz dy \quad (3.6.6)$$

3.7 SUBDIVISION OF BDR INTO SPECULAR AND SCATTERED COMPONENTS

Many theoretical bidirectional reflectance models are expressed as the sum of specular and scattered components. In this section such a model is defined and the integral equation for spectral radiant intensity is rewritten.

With reference to the spherical coordinates shown in Fig. 3.2, consider the BDR expression

$$\begin{aligned} \rho_{bd,\lambda}(\theta', \phi'; \theta, \phi) = & \rho_{bd,s,\lambda}(\theta', \phi'; \theta, \phi) \\ & + 2\rho_{sp,\lambda}(\theta') \delta[\sin^2\theta' - \sin^2\theta] \delta[\phi' - (\phi \pm \pi)] \end{aligned} \quad (3.7.1)$$

where $\delta[\eta]$ is the Dirac delta function with argument η . The two BDR components are called the scattered component $\rho_{bd,s,\lambda}(\theta', \phi'; \theta, \phi)$ and the specular component $2\rho_{sp,\lambda}(\theta') \delta[\sin^2\theta' - \sin^2\theta] \delta[\phi' - (\phi \pm \pi)]$, respectively. The former, accounts for radiation incident within solid angle $d\omega'$ around the (θ', ϕ') direction which is scattered by the surface into direction (θ, ϕ) of hemispherical space. The latter, represents incident energy which undergoes a specular reflection. In a specular reflection the reflected beam is contained within a solid angle $d\omega$ equal to the solid angle $d\omega'$ of the incident beam and is located in the plane of incidence with polar angle of reflection equal to the polar angle of incidence. Symbol $\rho_{sp,\lambda}(\theta')$ denotes the monochromatic specular reflectance which is defined as the ratio of reflected radiant intensity in a specular reflection $I_{r,sp,\lambda}^+$ to the intensity of incident energy.

$$\rho_{sp,\lambda}(\theta') \equiv \frac{I_{r,sp,\lambda}^+}{I_{\lambda}^-(\theta', \phi')} \quad (3.7.2)$$

All BDR models used in this research are conveniently expressed in terms of the subdivision indicated in Eq. (3.7.1), and characteristics of the specular and the scattered components of selected BDR models are discussed in Chapter 4.

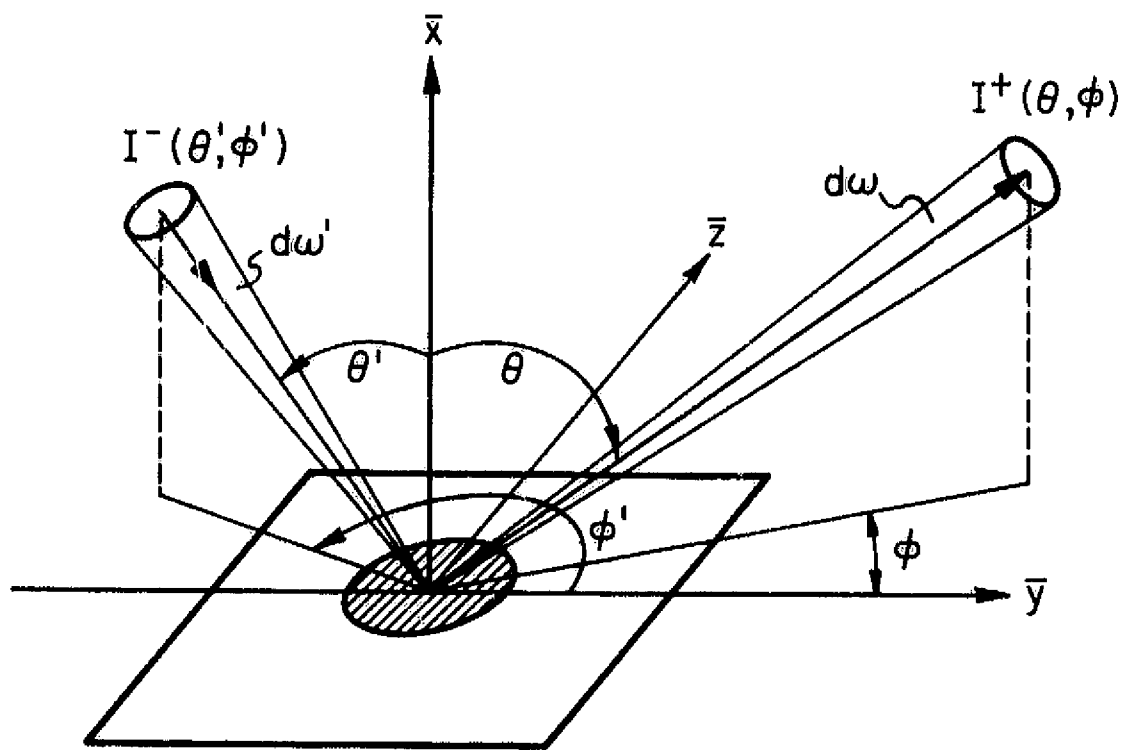


Figure 3.2 Local spherical coordinate system

Substitution of Eq. (3.7.1) into Eq. (3.4.1) and subsequent simplification yields the following expression for spectral radiant intensity:

$$\begin{aligned}
 I_{\lambda}^{+}(x; \theta_P, \phi_P) &= \varepsilon_{d, \lambda}(\theta_P) I_{b, \lambda}(x) \\
 &+ \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd, s, \lambda}(\theta'_P, \phi'_P; \theta_P, \phi_P) I_{\lambda}^{+}(y, z; \theta_Q, \phi_Q) K(x, y, z) dz dy \\
 &+ \rho_{bd, s, \lambda}(\theta'_{so}, 0; \theta_P, \phi_P) S_{\lambda} \sin \frac{\gamma}{2} + \rho_{sp, \lambda}(\theta'_{P, sp}) \\
 &\cdot I_{\lambda}^{-}(x, \theta'_{P, sp}, \phi'_{P, sp}) + \mathcal{J}_{sp, \lambda}(\theta_P, \phi_P)
 \end{aligned} \tag{3.7.3}$$

where

$$\theta'_{P, sp} = \theta_P$$

$$\phi'_{P, sp} = \phi_P \pm \pi$$

and

$$\mathcal{J}_{sp, \lambda}(\theta_P, \phi_P) = \rho_{sp, \lambda}(\theta'_{so}) S_{\lambda} \sin \frac{\gamma}{2} \tag{3.7.4}$$

if $\theta_P = \frac{\pi - \gamma}{2}$ and $\phi_P = \pi$ and zero otherwise.

The first term on the right-hand side of Eq. (3.7.3) is the intensity of emitted radiation leaving x in the (θ_P, ϕ_P) direction. The second term is the contribution to the intensity leaving x in the (θ_P, ϕ_P) direction

due to radiation issuing from the adjacent plate which is incident on and scattered (reflected and accounted for by the scattered component of BDR) by the surface at x into the (θ_p, ϕ_p) direction. The third term represents reflected intensity leaving x in the (θ_p, ϕ_p) direction due to directly incident solar energy which is scattered. The final two terms represent reflected intensity leaving the point x in the (θ_p, ϕ_p) direction due to incident energy which undergoes a specular reflection. The first of the two terms is due to energy incident from the adjacent surface, and the second is due to directly incident solar energy.

Separation of the scattered and specular contributions of leaving intensity as represented by Eq. (3.7.3) offers several advantages. Probably the most important is that the specularly reflected energy can now be treated with the familiar image methods [64]. More detailed discussion of this procedure is given in Appendix A.

The integral equations for local thermal intensity and local solar intensity presented in Section 3.6 for the semi-gray spectral model can also be rewritten for specular plus scattered subdivision of BDR. Substitution of Eq. (3.7.1) in Eq. (3.6.1) and (3.6.2) yields the following result for local thermal intensity

$$I_t^+(x, \theta_p, \phi_p) = \frac{\epsilon_d(\theta_p) \bar{\sigma} T^4(x)}{\pi} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s}(\theta'_p, \phi'_p; \theta_p, \phi_p) I_t^+(y, z, \theta_Q, \phi_Q) \\ \cdot K(x, y, z) dz dy + \rho_{sp}(\theta'_p, \phi'_p) I_t^-(x, \theta'_p, \phi'_p) \quad (3.7.5)$$

and the following expression for local solar intensity.

$$I_{so}^+(x, \theta_P, \phi_P) = \rho_{bd,s}^*(\theta'_{so}, 0; \theta_P, \phi_P) S \sin \frac{\gamma}{2}$$

$$+ \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s}^*(\theta'_P, \phi'_P; \theta_P, \phi_P) I_{so}^+(y, z, \theta_Q, \phi_Q)$$

$$\cdot K(x, y, z) dz dy + \rho_{sp}^*(\theta'_{P,sp}) I_{sc}^-(x, \theta'_{P,sp}, \phi'_{P,sp})$$

$$+ \mathcal{L}_{sp}(\theta_P, \phi_P) \quad (3.7.6)$$

4. RADIATION PROPERTIES

4.1 INTRODUCTION

The surface properties introduced in the analysis of Chapter 3 are all related to spectral bidirectional reflectance. Section 4.2 is devoted to demonstrating this radiation property interdependence. In Section 4.3 property models employed in this research are presented and characteristics of the models important to this study are discussed. Throughout this section subscripts i, r, and e denote quantities associated with incident, reflected and emitted radiation, respectively.

4.2 PROPERTY RELATIONSHIPS

Spectral, bidirectional reflectance for radiation incident from the (θ', ϕ') direction within solid angle increment $d\omega'$ and reflected into solid angle increment $d\omega$ about the (θ, ϕ) direction was defined in Eq. (3.4.2) and is rewritten here for convenience.

$$\rho_{bd,\lambda}(\theta', \phi'; \theta, \phi) = \frac{dI_{r,\lambda}^+(\theta', \phi'; \theta, \phi)}{I_{i,\lambda}^-(\theta', \phi') \cos \theta' d\omega'} \quad (4.2.1)$$

Spectral, directional (directional-hemispherical) reflectance $\rho_{d,\lambda}(\theta')$ is defined as the fraction of monochromatic energy incident within solid angle $d\omega'$ about direction θ' that is reflected throughout hemispherical space. Spectral directional reflectance is obtained by multiplying spectral bidirectional reflectance with $\cos \theta \sin \theta d\theta d\phi$ and integrating over all directions for reflected energy.

$$\rho_{d,\lambda}(\theta') = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \rho_{bd,\lambda}(\theta', \phi'; \theta, \phi) \cos \theta \sin \theta d\phi d\theta \quad (4.2.2)$$

Spectral directional absorptance $\alpha_{d,\lambda}(\theta')$ is defined as the fraction of monochromatic energy incident at polar angle θ' within solid angle $d\omega'$ that is absorbed. For opaque materials, a radiant energy balance at the material surface relates directional absorptance and reflectance.

$$\alpha_{d,\lambda}(\theta') = 1 - \rho_{d,\lambda}(\theta') \quad (4.2.3)$$

Spectral directional emittance $\epsilon_{d,\lambda}(\theta)$ is defined as the ratio of the spectral intensity of emission of a surface at polar angle θ , $I_{e,\lambda}^+(\theta)$, to the direction independent spectral intensity of a black surface at the same temperature $I_{b,\lambda}$.

$$\epsilon_{d,\lambda}(\theta) = \frac{I_{\lambda,e}^+(\theta)}{I_{b,\lambda}} \quad (4.2.4)$$

In Eqs. (4.2.2)-(4.2.4) directional reflectance, absorptance, and emittance were assumed to be azimuthally symmetric.

Spectral hemispherical emittance $\epsilon_{H,\lambda}$ is defined as the ratio of monochromatic energy emitted by a surface per unit time, per unit area, and per unit wavelength interval into hemispherical space $E_{e,\lambda}$ to that emitted by a black surface at the same temperature $E_{b,\lambda} (= \pi I_{b,\lambda})$. It is obtained by multiplying $\epsilon_{d,\lambda}(\theta)$ with $(\cos \theta \sin \theta d\theta d\phi)/\pi$ and integrating over all directions of hemispherical space.

$$\begin{aligned}
\epsilon_{H,\lambda} &= \frac{E_{e,\lambda}}{\pi I_{b,\lambda}} = \frac{1}{\pi} \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \epsilon_{d,\lambda}(\theta) \sin \theta \cos \theta \, d\phi \, d\theta \\
&= 2 \int_{\theta=0}^{\pi/2} \epsilon_{d,\lambda}(\theta) \sin \theta \cos \theta \, d\theta \quad (4.2.5)
\end{aligned}$$

Total hemispherical emittance ϵ_H is the ratio of total energy emitted per unit time and per unit area by a surface throughout hemispherical space E_e to that emitted by a black surface at the same temperature

$$E_b \left(= \int_{\lambda=0}^{\infty} \pi I_{b,\lambda} \, d\lambda = \sigma T^4 \right).$$

$$\epsilon_H = \frac{E_e}{E_b} \quad (4.2.6)$$

In terms of spectral hemispherical emittance, ϵ_H can be expressed as

$$\epsilon_H = \frac{\pi}{\sigma T^4} \int_{\lambda=0}^{\infty} \epsilon_{H,\lambda} I_{b,\lambda} \, d\lambda \quad (4.2.7)$$

Finally, when polarization is neglected, Kirchhoff's law equates spectral directional emittance and absorptance,

$$\epsilon_{d,\lambda}(\theta = \theta') = \alpha_{d,\lambda}(\theta') \quad (4.2.8)$$

Thus, Eqs. (4.2.1)-(4.2.8) relate all surface properties used in Chapter 3 to spectral bidirectional reflectance.

4.3 SELECTED BDR MODELS

4.3.1 Introduction

BDR models selected for this study are presented in this section. A detailed model for rough surface radiation properties is discussed in Section 4.3.2, followed in Section 4.3.3 by a simplified rough surface property model. In Section 4.3.4 a model with directional dependence considerably different from that exhibited by the rough surface models is discussed. This property model applies to a composite surface consisting of a thin dielectric coating on a diffusely reflecting substrate. An approximate model for coated surfaces is discussed in Section 4.3.5. Finally, in Section 4.3.6, common simple property models are briefly discussed. The latter are included to facilitate study of the level of surface property detail required for precise radiative heat transfer calculations.

4.3.2 Rough Surface Bidirectional Reflectance

Major assumptions employed in the Beckmann analysis [40] of rough surface reflectance were summarized in Chapter 2, and details of certain theoretical aspects of the BDR model as well as its ability to represent available BDR measurements were previously reported [42]. For brevity, only BDR expressions and a summary of important model characteristics are presented here.

For a perfect electrical conductor with a randomly rough surface, Beckmann's analysis yields the following expression for monochromatic specular reflectance and scattered component of BDR in terms of optical roughness σ/λ and rms roughness profile slope m .

$$f_{sp,\lambda}(\theta') = \exp \{- [4\pi(\sigma/\lambda) \cos \theta']^2\} \quad (4.3.2.1)$$

$$f_{bd,s,\lambda}(\theta', \phi'; \theta, \phi) = \frac{2\pi(\sigma/\lambda)^2 B \exp \{- [(\sigma/\lambda)G]^2\}}{\cos \theta \cos \theta' m^2} \cdot \sum_{M=1}^{\infty} \frac{[(\sigma/\lambda)G]^{2M}}{M(M!)} \exp \left[- \frac{2\pi^2}{M} \frac{(\sigma/\lambda)^2 H}{m^2} \right] \quad (4.3.2.2)$$

The symbol ρ has been replaced with f to emphasize that Eqs. (4.3.2.1) and (4.3.2.2) apply to perfectly conducting material. The symbols B , G , and H denote purely geometrical quantities and are given by

$$B = \left[\frac{1 + \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\phi' - \phi)}{\cos \theta + \cos \theta'} \right]^2 \quad (4.3.2.3)$$

$$G = 2\pi(\cos \theta + \cos \theta') \quad (4.3.2.4)$$

$$H = \sin^2 \theta + \sin^2 \theta' + 2 \sin \theta \sin \theta' \cos (\phi' - \phi) \quad (4.3.2.5)$$

while rms roughness slope m is related to correlation length a by the relation

$$m = \sqrt{2} (\sigma/a) \quad (4.3.2.6)$$

For large values of optical roughness ($\sigma/\lambda \gg 1$), the specular reflectance $f_{sp,\lambda}(\theta')$ is nearly zero for all but near-grazing directions of incidence, and a useful approximation to Eq. (4.3.2.2) is

$$\bar{f}_{bd,s}(\theta', \phi'; \theta, \phi) = \frac{\bar{B}}{2\pi m^2 \cos \theta \cos \theta'} \exp [-\bar{H}/(2m^2)] \quad (4.3.2.7)$$

where

$$\overline{B} = \frac{B}{(\cos \theta + \cos \theta')^2} \quad (4.3.2.8)$$

and

$$\overline{H} = \frac{H}{(\cos \theta + \cos \theta')^2} \quad (4.3.2.9)$$

The bar notation is used to differentiate Eq. (4.3.2.2) from Eq. (4.3.2.7) and B and H are defined by Eqs. (4.3.2.3) and (4.3.2.5), respectively.

This approximate expression is hereafter designated the large optical roughness (LOR) approximation. Note that the LOR BDR expression is independent of wavelength and depends on the single surface roughness parameter rms slope m . It should be emphasized that the LOR BDR approximation is not an additional BDR model, but merely a useful approximation for the more complicated expression of Eq. (4.3.2.2) for large optical roughness.

Since the Beckmann BDR model strictly applies to perfectly conducting material, all energy of an incident beam must be accounted for in the reflected energy leaving the surface. According to the model, the fraction of the energy of an incident beam which undergoes a specular reflection is equal to the specular reflectance $f_{sp,\lambda}(\theta')$. The remaining energy is scattered throughout hemispherical space with a distribution given by the scattered BDR component $f_{bd,s,\lambda}(\theta',\phi';\theta,\phi)$. The fraction of the energy incident from the (θ',ϕ') direction which is scattered $F_{s,\lambda}(\theta')$ may be evaluated by integrating the product of $f_{bd,s,\lambda}$ and the cosine of the polar angle of reflection over all solid angles of reflection.

$$F_{s,\lambda}(\theta') = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} f_{bd,s,\lambda}(\theta', \phi'; \theta, \phi) \cos \theta \sin \theta d\phi d\theta \quad (4.3.2.10)$$

The sum of the fraction of incident energy from the (θ', ϕ') direction which is scattered and that which undergoes a specular reflection should be unity.

$$f_{sp,\lambda}(\theta') + F_{s,\lambda}(\theta') = 1 \quad (4.3.2.11)$$

It has been shown [42,65] that the conservation of energy requirement expressed by Eq. (4.3.2.11) is usually satisfied within a few percent if rms slope is restricted to values less than about 0.14. However, the magnitude of the discrepancy depends on the value of optical roughness, rms slope, and the polar angle of incidence.

Typical results for specular reflectance are illustrated in Fig. 4.1 where $f_{sp,\lambda}(\theta')$ distributions are shown for selected values of optical roughness. Since, according to Eq. (4.3.2.11), the factor $[1 - f_{sp,\lambda}(\theta')]$ represents the fraction of an incident beam of energy which is scattered $F_{s,\lambda}(\theta')$ the right-hand ordinate scale may be used to evaluate this quantity for a surface of prescribed optical roughness illuminated from a specified direction. Apportionment of reflected energy between specular and scattered energy components is determined by the polar angle of incidence θ' and optical roughness σ/λ . For prescribed direction of incidence, specular reflectance decreases with increasing optical roughness. With optical roughness prescribed, the specular reflectance increases with increasing angle of incidence.

With the direction of incident energy and value of optical roughness

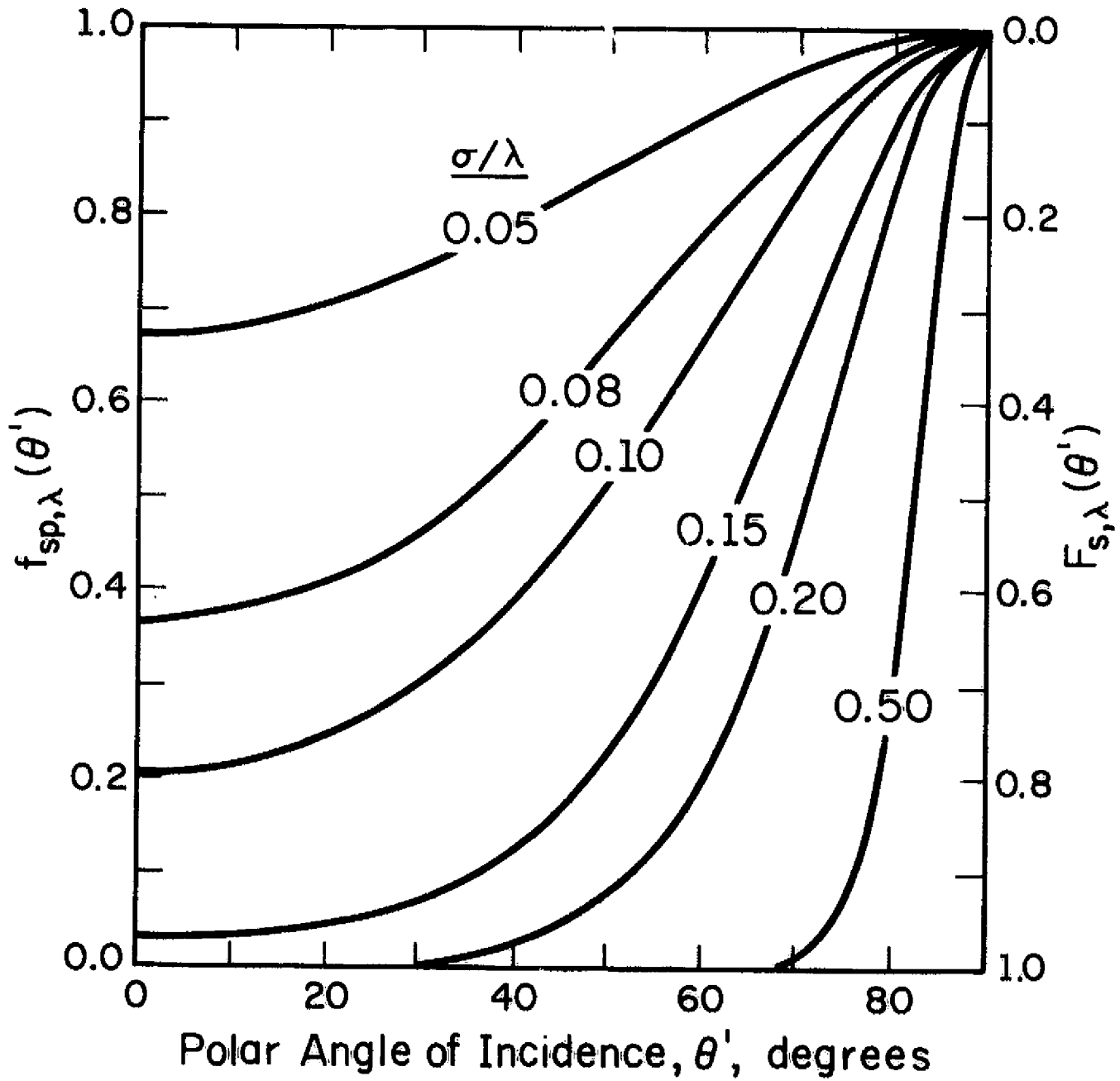


Figure 4.1 Specular and scattered energy components of reflected energy from Beckmann BDR model

specified, the magnitude of the scattered energy is fixed. Its spatial distribution is uniquely determined by rms slope. The general influence of roughness slope on scattered BDR is similar for all angles of incidence and values of optical roughness. Some representative results are illustrated in Fig. 4.2. The product of LOR scattered BDR and cosine of the polar angle of reflection is shown versus polar angle of reflection for normal illumination and rms slope values of 0.14, 0.094, 0.057, and 0.040. The product of BDR and $\cos \theta$ is the fraction of incident energy reflected per unit time and per unit area into a unit solid angle around the (θ, ϕ) direction. No azimuthal angles are indicated on the figure since for normal illumination BDR is azimuthally symmetric. A diffusely reflecting surface is represented in the figure by the dashed curve. The angular spread about the specular direction within which the scattered energy is significant increases with increasing roughness slope. The increasing angular spread of the scattered energy is accompanied with a reduction of BDR in the specular direction. Thus, the surface tends toward a diffusely reflecting surface as roughness slope increases. Even for the largest value of rms slope ($m = 0.14$), however, the spatial distribution of reflected energy differs significantly from that for diffuse reflection.

For a fixed value of rms slope, the dependence of the spatial distribution of scattered energy on direction of incidence is markedly different for large and small values of optical roughness. The product of scattered BDR and $\cos \theta$ for a surface with rms slope equal to 0.14 is shown in Figs. 4.3a,b versus polar angle of reflection for large optical roughness ($\sigma/\lambda \gg 1$) and a small value of optical roughness ($\sigma/\lambda = 0.1$), respectively. For each value of optical roughness, distributions are

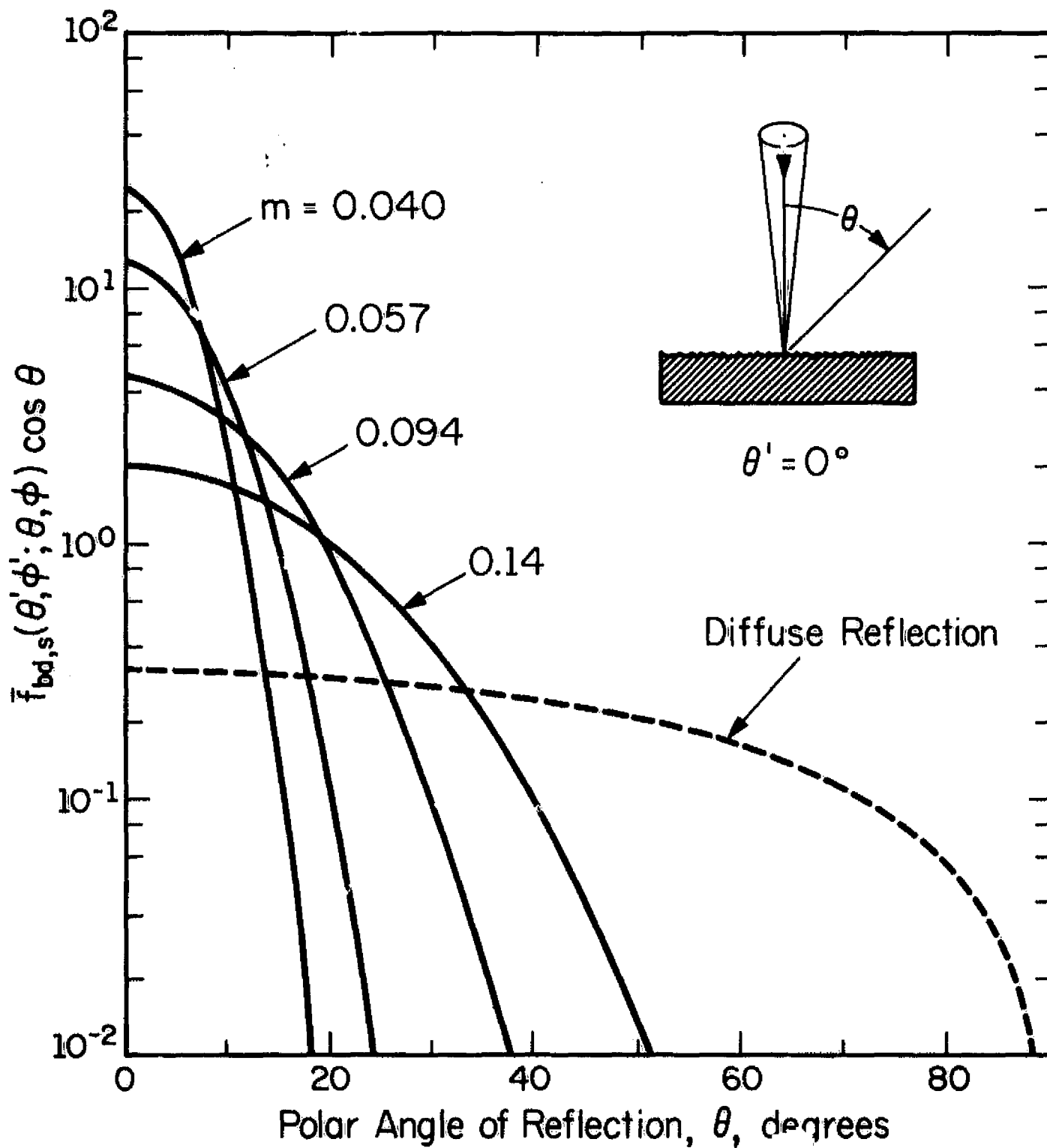
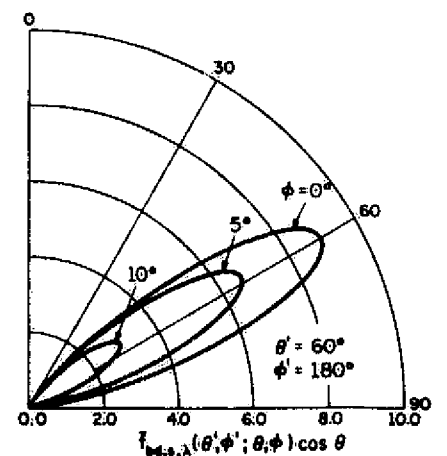
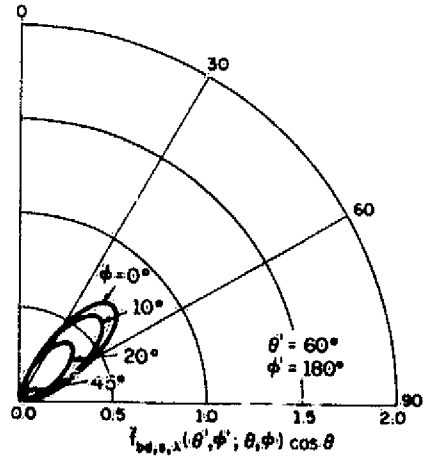
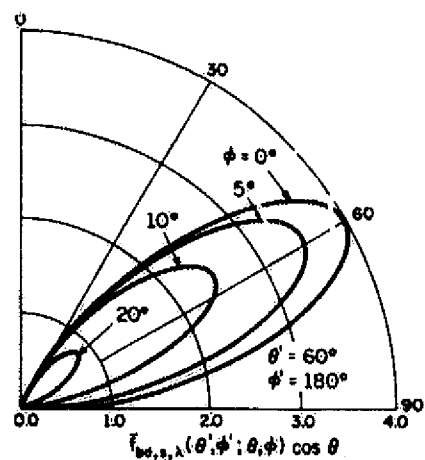
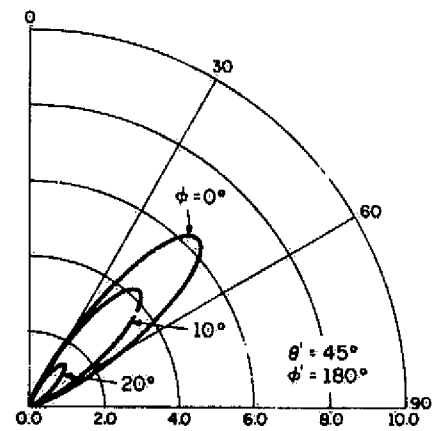
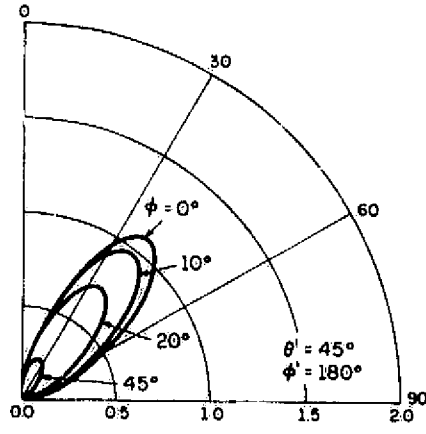
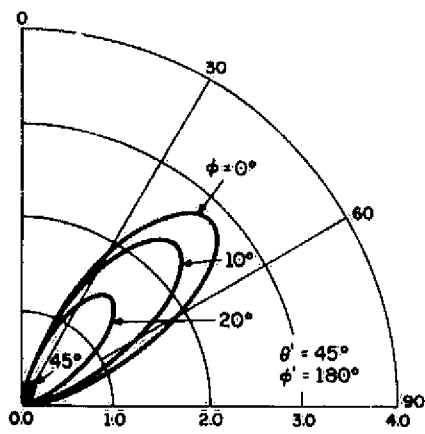
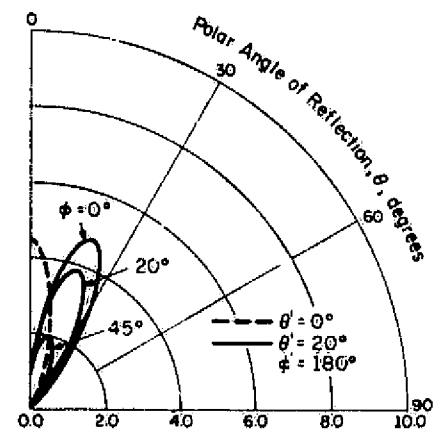
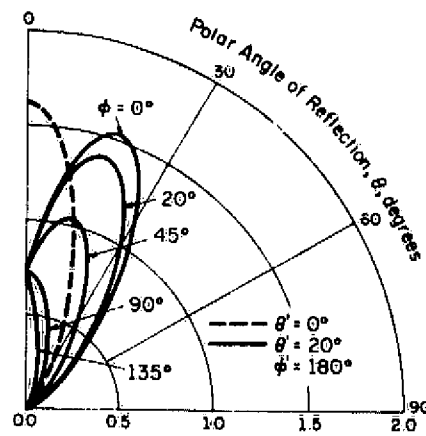
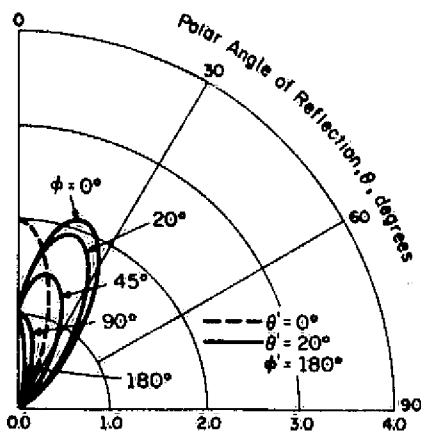
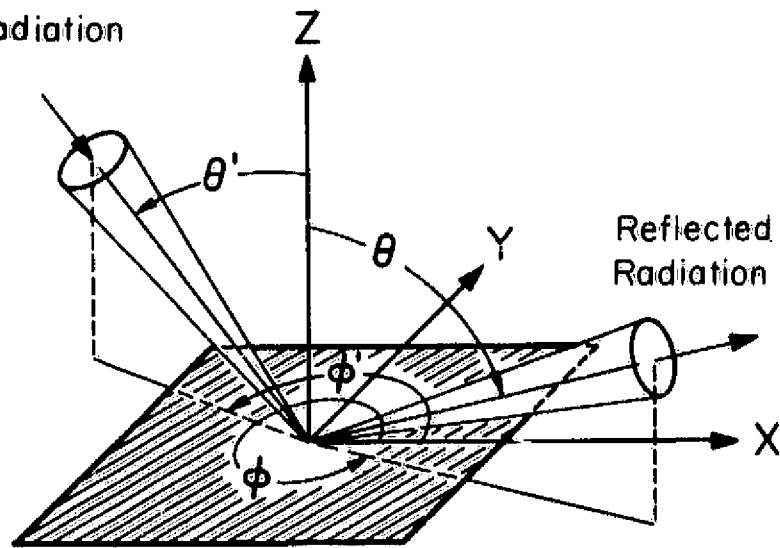


Figure 4.2 Scattered energy component from Beckmann LOR BDR model

Incident
Radiation

(a) Large optical
roughness
($m = 0.14$)

(b) $\sigma/\lambda = 0.1$
 $m = 0.14$

(c) large optical
roughness
($m = 0.094$)

Figure 4.3 BDR model

shown for polar angle of illumination equal to 0, 20, 45, and 60 degrees, and for each angle of illumination results are shown for selected values of azimuthal angle ϕ . The distributions shown in Fig. 4.3c for rms slope equal to 0.094 and large optical roughness are included for later reference.

Consider the distributions for rms slope equal to 0.14. It is important to note the strong reflection of energy into the plane containing the specular direction (strong forward scattering). For large optical roughness, (Fig. 4.3a), the maximum value of $\bar{f}_{bd,s} \cos \theta$ increases sharply, and the angular spread of the scattered energy around the specular direction decreases for increasing polar angle of illumination. Of particular note is the rapid decrease of energy scattered out of the plane of incidence ($\phi = 0$) with increasing angle of illumination. For small optical roughness, (Fig. 4.3b), the maximum value of $f_{bd,\lambda,s} \cos \theta$ decreases with increasing angle of illumination due to the increasing magnitude of the incident energy which undergoes specular reflection. In contrast to the distributions for large optical roughness, the lobes of scattered energy in the plane of incidence for small optical roughness and moderate to large angles of illumination are not centered about the specular direction but are shifted toward the surface normal.

It should be noted that the Beckmann scattered BDR contains a singularity for polar angle of reflection equal to 90 degrees. This accounts for the generally small, but finite, value of the product $f_{bd,s,\lambda} \cos \theta$ for grazing reflection. This error results from neglecting shadowing effects in the derivation of BDR, but is not expected to significantly influence radiant heat transfer for the selected system of interacting surfaces.

Limitation of the Beckmann analysis to perfectly conducting materials is particularly restrictive and the model requires some modification to adapt it to engineering materials. In view of the difficulty of including finite conductivity effects in the development [40], BDR of engineering materials is approximated by multiplying the result for perfect conductors with spectral directional reflectance. The latter must be provided as additional information. Hence, approximate specular reflectance and scattered BDR of engineering materials are written

$$\rho_{sp,\lambda}(\theta') = \rho_{d,\lambda}(\theta') f_{sp,\lambda}(\theta') \quad (4.3.2.12)$$

and

$$\rho_{bd,s,\lambda}(\theta', \phi'; \theta, \phi) = \rho_{d,\lambda}(\theta') f_{bd,s,\lambda}(\theta', \phi'; \theta, \phi) \quad (4.3.2.13)$$

while for large optical roughness

$$\bar{\rho}_{bd,s,\lambda}(\theta', \phi'; \theta, \phi) = \rho_{d,\lambda}(\theta') \bar{f}_{bd,s}(\theta', \phi'; \theta, \phi) \quad (4.3.2.14)$$

The scattered BDR expressions do not satisfy the reciprocity requirement [42], but this is not expected to introduce appreciable error into heat transfer results for the selected system.

In the absence of a more precise model for directional reflectance and, consequently, directional emittance of rough engineering materials, the Fresnel relations [66] for smooth surface reflectance are used. Furthermore, in an attempt to isolate roughness effects on heat transfer, material optical indices n and k are taken wavelength independent. It should be noted that, although the material optical indices are taken

wavelength independent, BDR remains spectrally dependent because the spatial distribution of reflected radiation is a function of optical roughness. To facilitate direct comparison of heat transfer results with reported results the directional emittance (reflectance) distributions which were selected by Hering [49] are used. Directional emittance distributions for the selected optical indices are shown in Fig. 4.4. Values of optical indices were selected to provide hemispherical emittance values and directional emittance distributions characteristic of metals ($\epsilon_H = 0.1$; $n = 23.452$, $k = 1.0$) and dielectrics [$(\epsilon_H = 0.5$; $n = 6.1038$, $k = 0.0)$, $(\epsilon_H = 0.9$; $n = 1.5565$, $k = 0.0)$].

In summary, the proposed model for rough surface radiation properties consists of the following:

- (1) Directional reflectance, emittance and absorptance given by the Fresnel relations with wavelength independent material optical indices.
- (2) Spatial distribution of reflected energy given by Beckmann's BDR model in terms of optical roughness σ/λ and rms roughness profile slope m .

Realizing that approximations have been introduced, it is proposed that this model is sufficiently realistic to permit a useful investigation of surface roughness effects on heat transfer while still incorporating a tractable number of characteristic parameters.

4.3.3 Simplified Rough Surface BDR Model

A simplified model for rough surface BDR is discussed in this section. The simplified model retains all characteristics of the Beckmann BDR model with the exception that the spatial distribution of scattered energy is diffusely distributed. Therefore, the time consuming calculation required to evaluate the Beckmann scattered BDR expression is avoided. The simplified rough surface model and results derived using it are hereafter designated BDS (Beckmann Diffuse Scattered). BDS

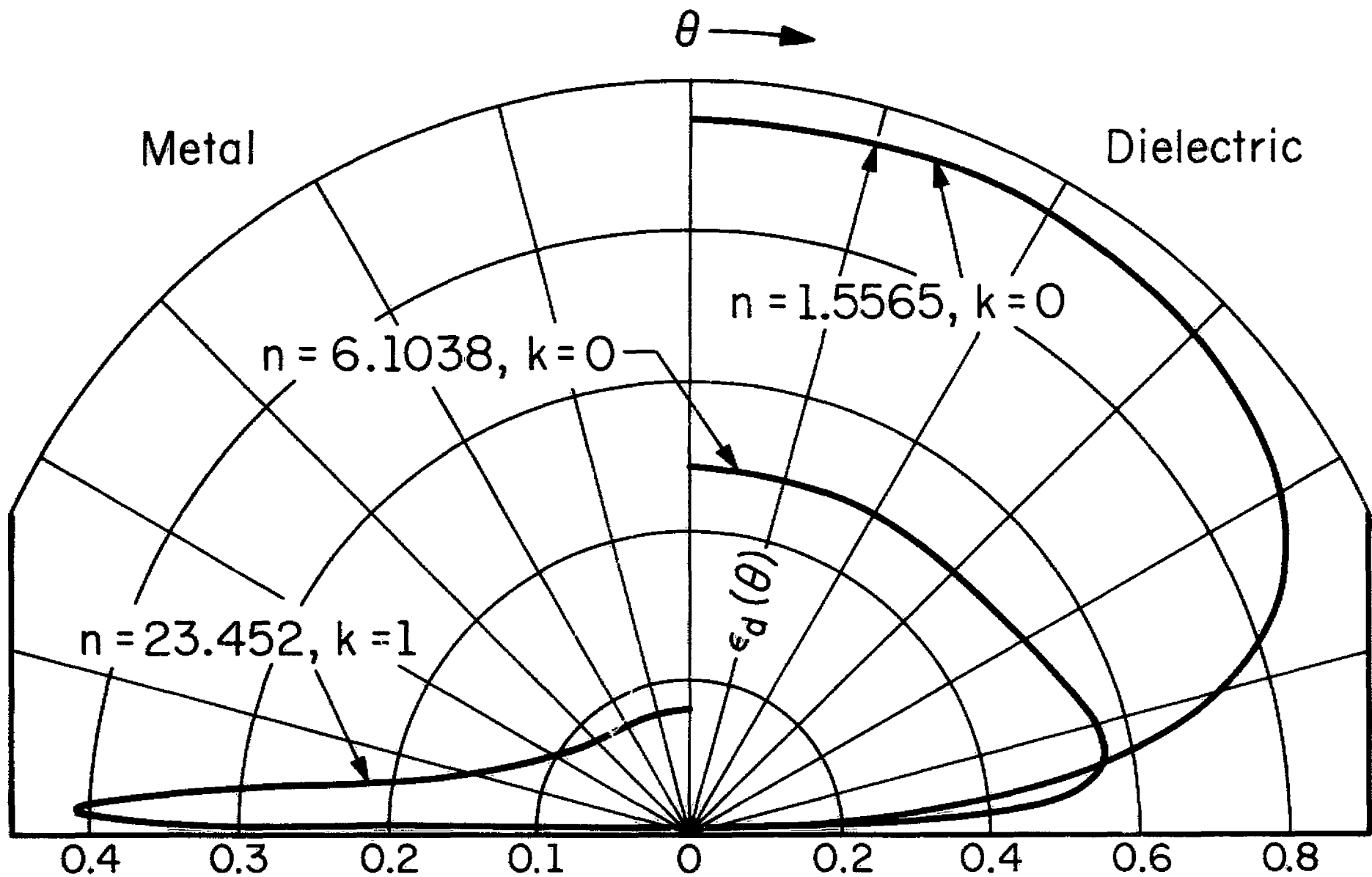


Figure 4.4 Directional emittance distributions

specular reflectance and scattered BDR are, respectively,

$$\rho_{sp,\lambda}(\theta') = \rho_d(\theta') f_{sp,\lambda}(\theta') \quad (4.3.3.1)$$

and

$$\rho_{bd,s,\lambda}(\theta') = \frac{\rho_d(\theta')[1 - f_{sp,\lambda}(\theta')]}{\pi} \quad (4.3.3.2)$$

Equation (4.3.2.1) shows that the apportionment of reflected energy to specular and diffuse components depends on optical roughness and direction of illumination. This direction and wavelength dependent specular plus diffuse reflection model has been proposed [50,51,59] as a compromise between realistic property representation and computational effort required for heat transfer calculations. Recently, Toor, Viskanta, and Winter [67] reported that heat transfer evaluated using the BDS model was in some cases in better agreement with experiment than that calculated from simple constant property specular plus diffuse reflection analysis. However, results were insufficient to allow any general conclusions to be drawn.

4.3.4 Coated Surface BDR Model

To extend the scope of this study, a surface property model developed by Love and Francis [68] was investigated. This model gives the BDR of a composite surface consisting of a dielectric coating with refractive index n on an opaque metal substrate. The air-dielectric interface is optically smooth, and the substrate surface in contact with the coating is diffusely reflecting with direction independent reflectance ρ_D . Polarization effects are ignored. The expressions for specular

reflectance and scattered BDR of the coated surface are

$$\rho_{sp}(\theta') = \rho_{F,c}(\theta') \quad (4.3.4.1)$$

and

$$\rho_{bd,s}(\theta', \theta) = \frac{\rho_D \left[1 - \rho_{F,c}(\theta') \right] \left[1 - \bar{\rho}_{F,c}(\alpha_1) \right] \exp \left[\frac{-\tau_o}{\cos \alpha_2} \right] \exp \left[\frac{-\tau_o}{\cos \alpha_1} \right]}{\pi n^2 \left[1 - 2\rho_D \left\{ \mathcal{J}_1 + \mathcal{J}_2 \right\} \right]} \quad (4.3.4.2)$$

The symbols $\rho_{F,c}(\theta')$ and $\bar{\rho}_{F,c}(\theta)$ denote Fresnel reflectance of the air-coating interface for radiation incident from the exterior and interior of the coating, respectively. The optical thickness of the coating, which is the product of the absorption coefficient and the thickness of the coating is denoted τ_o . Furthermore,

$$\cos \alpha_1 = \left[1 - \frac{\sin^2 \theta}{n^2} \right]^{1/2} \quad (4.3.4.3)$$

$$\cos \alpha_2 = \left[1 - \frac{\sin^2 \theta'}{n^2} \right]^{1/2} \quad (4.3.4.4)$$

$$\cos \alpha_3 = \frac{1}{n} \left[n^2 - 1 \right]^{1/2} \quad (4.3.4.5)$$

$$\mathcal{J}_1 = \int_0^{\cos \alpha_3} \exp(-2\tau_o / \cos \theta) \cos \theta d(\cos \theta) \quad (4.3.4.6)$$

$$\mathcal{J}_2 = \int_{\cos \alpha_3}^1 \bar{\rho}_{F,c}(\theta) \exp(-2\tau_o / \cos \theta) \cos \theta d(\cos \theta) \quad (4.3.4.7)$$

The specular reflectance $\rho_{sp}(\theta')$ is the Fresnel reflectance of the air-coating interface for wavelength independent refractive index n .

The scattered BDR is independent of both the azimuthal angle of incidence

and reflection.

Directional reflectance, which for this model is wavelength independent, is evaluated using Eq. (4.2.2)

$$\rho_d(\theta') = \rho_{sp}(\theta') + 2\pi \int_{\theta=0}^{\pi/2} \rho_{bd,s}(\theta', \theta) \sin \theta \cos \theta d\theta \quad (4.3.4.8)$$

Directional absorptance, hemispherical emittance, directional emittance follow from Eqs. (4.2.3), (4.2.7), and (4.2.8), respectively.

In [68], specular reflectance, scattered BDR, and directional reflectance were illustrated for selected values of coating refractive index, optical thickness, substrate reflectance, and direction of illumination. The authors concluded that, for other parameters fixed, the magnitude of reflected energy accounted for by the scattered BDR decreases with increasing optical thickness of the coating. As the optical thickness of the coating approaches zero, the magnitude of reflected energy approaches that which is reflected by the substrate in the absence of a coating. Hence, the specular reflectance is zero, and the scattered BDR is direction independent and equal to ρ_D/π . For finite optical thickness, the energy reflected by the coated surface consists of two components: First, incident energy which undergoes specular reflection at the air-coating interface, and second, energy which is refracted into the coating and re-emerges from the air-coating interface after undergoing one or more intervening diffuse reflections at the substrate-coating interface. As optical thickness becomes large, the scattered BDR approaches zero, and the reflectance of the coated surface nearly equals the Fresnel reflectance of the dielectric coating material.

Specular reflectance, directional reflectance, scattered BDR, and

directional emittance are illustrated in Figs. 4.5-4.7 for a selected set of parameters. The values for refractive index, optical thickness, and substrate reflectance selected are representative of thermoplastic coatings on bright metal substrates [68]. The values employed were 1.5565, 0.1, and 0.75, respectively.

Specular reflectance $\rho_{sp}(\theta')$ and directional reflectance $\rho_d(\theta')$ are shown in Fig. 4.5 versus the polar angle of illumination θ' . Both the specular and directional reflectance are nearly constant for angle of incidence between 0 and 50 degrees, and then increase rapidly to unity for angle of incidence equal to 90 degrees. For prescribed polar angle of incidence the difference between the directional reflectance and specular reflectance is equal to the magnitude of the energy accounted for by the scattered BDR.

Scattered BDR $\rho_{bd,s}(\theta',\theta)$ versus the polar angle of reflection θ with polar angle of illumination θ' as a parameter is shown in Fig. 4.6. For purposes of comparison, BDR for a diffusely reflecting surface is shown as a dashed curve for angle of illumination equal to 0, 70, and 85 degrees. The magnitude of scattered BDR is a weak function of the angle of incidence from 0 to about 60 degrees, but then decreases rapidly for greater angles of incidence. This is explained by noting that according to Fig. 4.5, for angle of incidence less than 60 degrees the fraction of incident energy refracted into the coating material $[\rho_d(\theta') - \rho_{sp}(\theta')]$ is nearly constant. With further increase of the angle of incidence, the magnitude of energy refracted into the coating decreases sharply due to the rapid increase of the specular reflectance of the coating material. The shapes of the scattered BDR distributions are similar for each of the selected angles of illumination. For θ less than 60 degrees, scattered

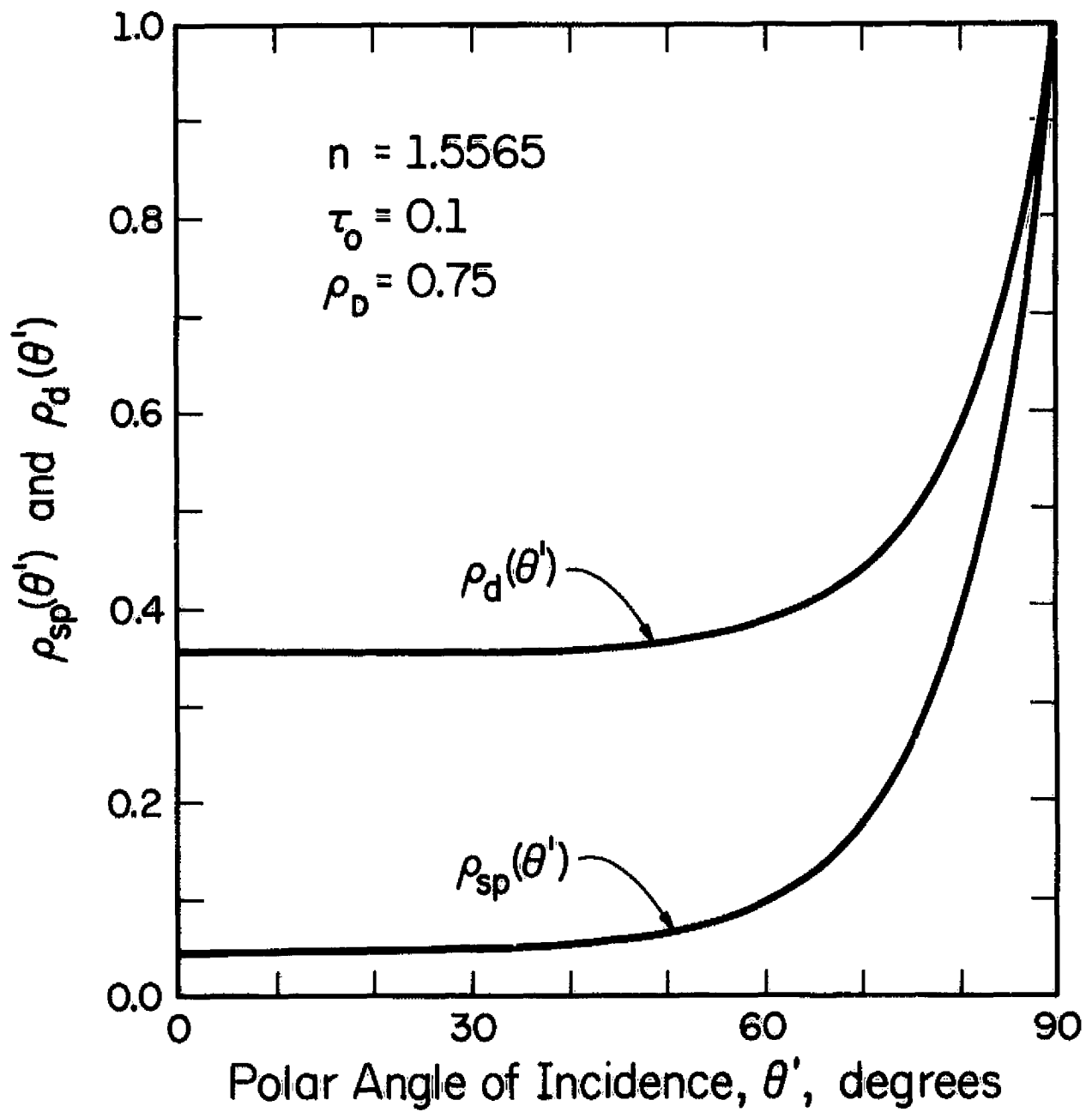


Figure 4.5 Specular reflectance and directional reflectance for coated surface

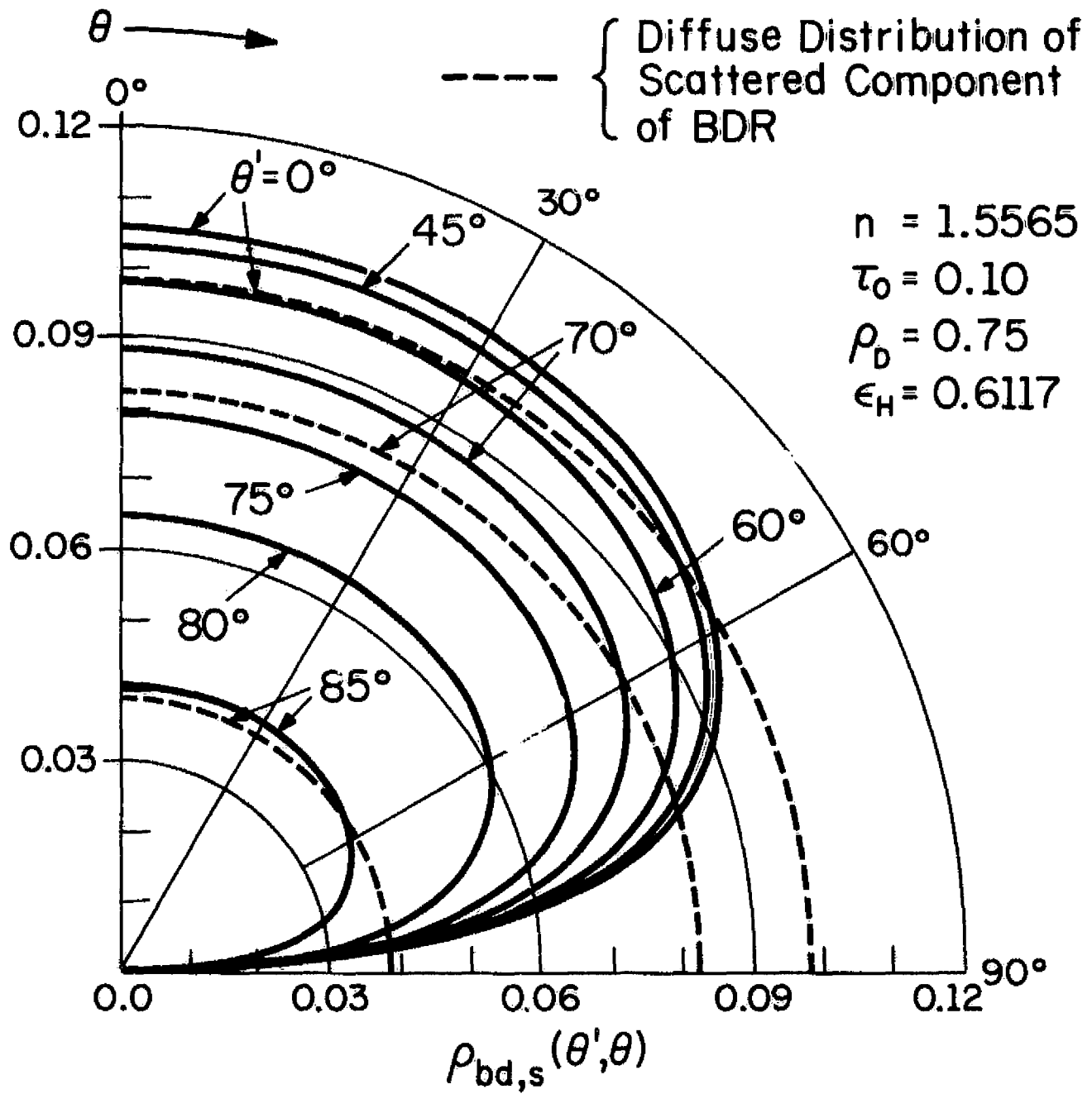


Figure 4.6 Scattered BDR component for coated surface

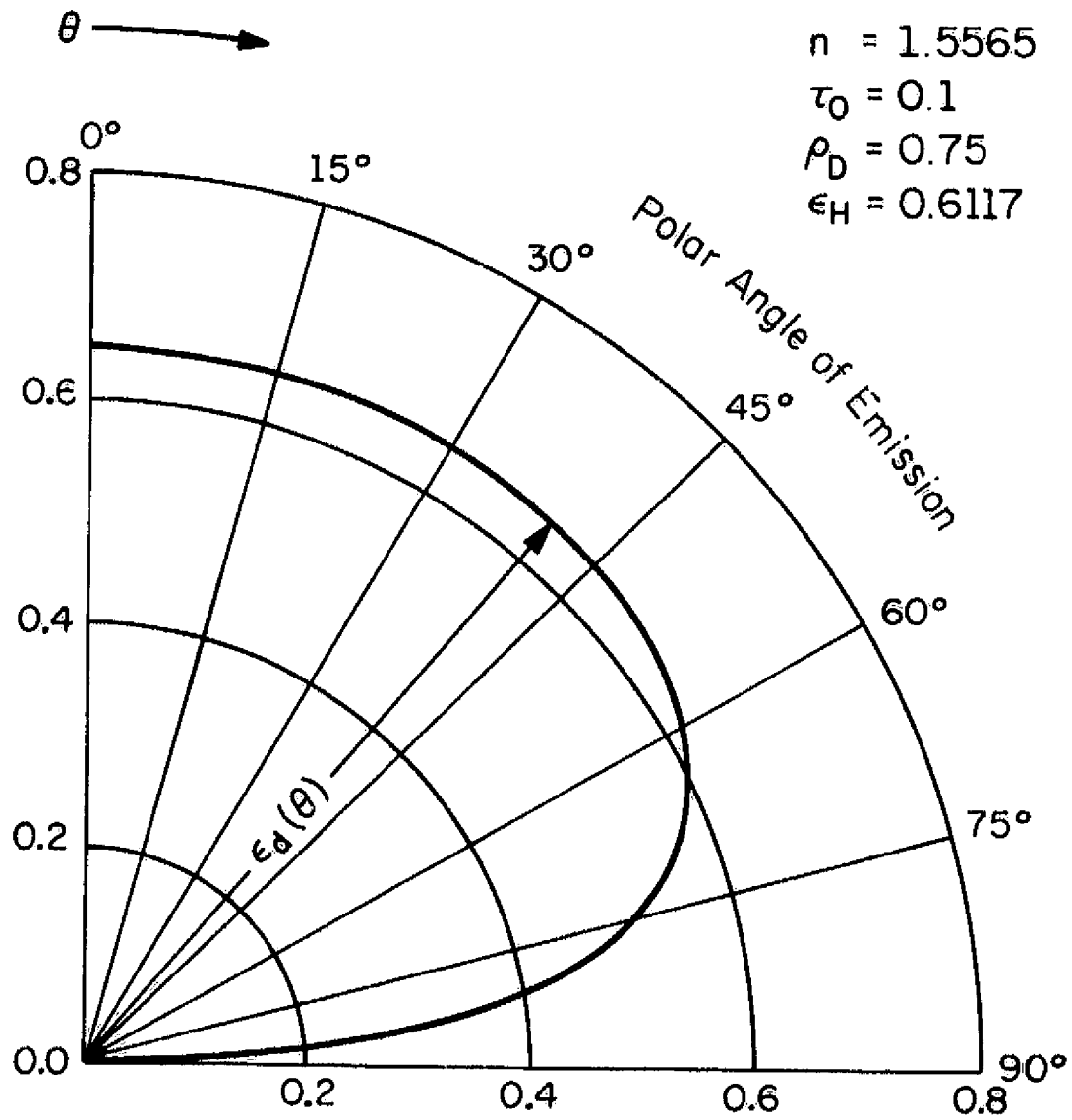


Figure 4.7 Directional emittance of coated surface

BDR is nearly constant and approximately 7 percent larger than the corresponding diffuse reflection results. For greater angles of reflection, BDR reduces rapidly to zero at $\theta = 90$ degrees.

It should be noted that the coated surface BDR model differs significantly from the rough surface Beckmann model. Although both models have direction dependent specular components which increase with increasing angle of illumination, their scattered BDR components are very different. The Beckmann model predicts strong forward scattering of energy with little energy scattered out of the plane of incidence. The coated surface model, however, predicts an azimuthally symmetric spatial distribution of scattered energy which, for the selected material properties, is nearly diffuse except for angles of reflection greater than 60 degrees. According to the Beckmann model, hemispherical emittance is not altered by surface roughness. Hence, the rough surface exhibits the hemispherical emittance of the constituent material. The coated surface, however, exhibits hemispherical emittance intermediate to those for the coating and substrate materials. Use of these two different models in heat transfer calculations facilitates a more comprehensive evaluation of directional property effects.

Directional emittance $\epsilon_d(\theta)$ for the coated surface is shown in Fig. 4.7. The characteristic shape of the directional emittance curve is similar to that exhibited by optically smooth dielectrics as predicted by electromagnetic theory. Integration of directional emittance over half-space yields a hemispherical emittance value for the composite surface of 0.6117. In the absence of a coating and for large coating thickness, the surface hemispherical emittance is 0.25 (substrate emittance) and 0.9 (coating emittance), respectively.

4.3.5 Simplified Coated Surface BDR Model

It was shown in Section 4.3.4 that the scattered component of BDR for the composite surface differs significantly from that for diffuse reflection only for large polar angles of reflection. To investigate the importance of this difference, heat transfer results were obtained for surface properties identical to those for the composite surface but with the spatial distribution of scattered energy taken diffuse. This model and results obtained with it are designated CSDS (Coated Surface Diffuse Scattered BDR). The specular reflectance and scattered component of BDR are, respectively,

$$\rho_{sp}(\theta') = \rho_{F,c}(\theta') \quad (4.3.5.1)$$

and

$$\rho_{bd,s}(\theta') = \frac{\rho_d(\theta') - \rho_{sp}(\theta')}{\pi} \quad (4.3.5.2)$$

The variables appearing on the right-hand side of Eqs. (4.3.5.1) and (4.3.5.2) were discussed in Section 4.3.4.

4.3.6 Diffuse Plus Specular Approximate BDR Models

To investigate the surface property detail required for accurate prediction of radiant transfer, other approximate reflection models were considered. These models represent the distribution of reflected energy in terms of specular and diffuse reflectance components with apportionment of reflected energy to each component independent of

the direction of incident energy. Directional reflectance is written as the sum of a specular component $\rho_{sp}(\theta')$ and a diffuse component $\rho_d(\theta')$.

$$\rho_d(\theta') = \rho_{sp}(\theta') + \rho_D(\theta') \quad (4.3.6.1)$$

Furthermore, the specularity ratio r_{sp} defined as

$$r_{sp} = \frac{\rho_{sp}(\theta')}{\rho_d(\theta')} \quad (4.3.6.2)$$

is independent of the angle of illumination θ' .

Specular reflectance and scattered BDR are given as

$$\rho_{sp}(\theta') = \rho_d(\theta') r_{sp} \quad (4.3.6.3)$$

and

$$\rho_{bd,s}(\theta') = \frac{\rho_d(\theta')}{\pi} (1 - r_{sp}) \quad (4.3.6.4)$$

This model and results derived from its use are denoted DSD (Directional property Specular plus Diffuse). When directional reflectance is taken independent of θ' , Eqs. (4.3.6.1)-(4.3.6.4) reduce to well known expressions for the gray property diffuse emission, direction independent specular plus diffuse reflection model. Results derived using this constant property model are designated CSD (Constant property Specular plus Diffuse). The CSD model includes as limiting cases direction independent specular and diffuse reflection.

5. RADIANT HEAT TRANSFER RESULTS AND DISCUSSION

5.1 INTRODUCTION

In this chapter radiant heat transfer results are presented for uniform temperature adjoint plates with direction and wavelength dependent surface properties.

In Section 5.2 local and overall heat transfer rates are presented for surfaces with properties represented by the DSD property model. The influence of angular property dependence on heat transfer for markedly different spatial distributions of reflected energy is assessed.

Local and overall heat transfer for surfaces with properties represented by the approximate rough surface (BDS) model are investigated in Section 5.3. The influence of optical roughness on heat transfer is illustrated, and the results are compared to those obtained using the less detailed DSD property model. The heat transfer results obtained using the BDS and DSD property models are used to relate the surface specular parameter r_{sp} to the optical roughness of a surface.

In Section 5.4, local and overall heat transfer results obtained using the detailed Beckmann rough surface property model are presented, and the importance of surface roughness on heat transfer is assessed. Rough surface results are compared to those obtained for less detailed property models, and the accuracy attainable with the simpler models is evaluated.

Finally, in Section 5.5 local and overall heat transfer results for coated surfaces are presented. The influence of directional property dependence is established, and the accuracy of simple property model heat transfer predictions is investigated.

The general dependence of heat transfer on position, included angle, hemispherical emittance, and reflection model for adjoint plates with direction independent properties has been fully discussed elsewhere [2] and, therefore, is only briefly summarized here. For prescribed uniform temperature plates, radiant heat flux is minimum at the apex and continuously increases to the outer edge. The level of the heat transfer increases as either hemispherical emittance or included angle is increased. For all other conditions fixed, local heat transfer is greater for specularly reflecting surfaces than for diffusely reflecting surfaces. Discussion of these general trends is not repeated in this chapter. Instead, attention is directed to evaluation of variable property effects.

Most of the constant property analysis results used for comparison purposes in this chapter were reported earlier [2,64.69].

5.2 DIRECTIONAL PROPERTY EFFECTS FOR SIMPLE REFLECTION MODELS

The influence on heat transfer of directional emission, absorption, and reflection for specularly reflecting surfaces has received considerable attention in the literature (see Chapter 2). Although engineering surfaces exhibit direction dependent emittance, absorptance, and reflectance, they do not generally reflect energy specularly. The radiant heat flux results presented and discussed in this section provide insight into the importance of directional property effects for surfaces which are not specularly reflecting.

5.2.1 Local Heat Transfer

Dimensionless local heat flux distributions are shown in

Figs. 5.1-5.4 for surfaces with properties represented by the DSD and CSD models. Results are shown for specularity ratio values of 0.0, 0.5, and 1.0. Figures 5.1-5.3 are for included angle $\gamma = 45^\circ$ and surfaces with hemispherical emittance values equal to 0.1, 0.5, and 0.9, respectively. Figure 5.4 illustrates results for $\gamma = 90^\circ$ for each of the three emittance values but only for the limiting values for specularity parameter. Following Hering [49], results obtained using the DSD and CSD property models are referred to as directional property (DP) and constant property (CP) results, respectively. Hering [49] compared results from DP and CP analysis for specularly reflecting surfaces. The objective here is to investigate directional property effects when the participating surfaces are not specular reflectors. It is important to note the magnitude of the differences between results obtained with DP specular and DP diffuse reflection analyses. The largest differences occur at the apex. For $\gamma = 45^\circ$ and hemispherical emittance values of 0.1, 0.5, and 0.9, DP specular reflection results are higher than those obtained from DP diffuse reflection analysis by approximately 32, 67, and 14 percent, respectively. For $\gamma = 90^\circ$ the respective differences are reduced by a factor of five. These differences are nearly identical to those previously reported for CP analysis. It is clear that for systems in which multiple reflections are important, knowledge of the spatial distribution of reflected energy is essential for applications requiring accurate heat transfer predictions even when the directional dependence of emittance and absorptance are accounted for.

Several trends are common to results for the two included angles. For the intermediate emittance value ($\epsilon_H = 0.5$), CP results are in excellent agreement with DP results along the entire plate length for all

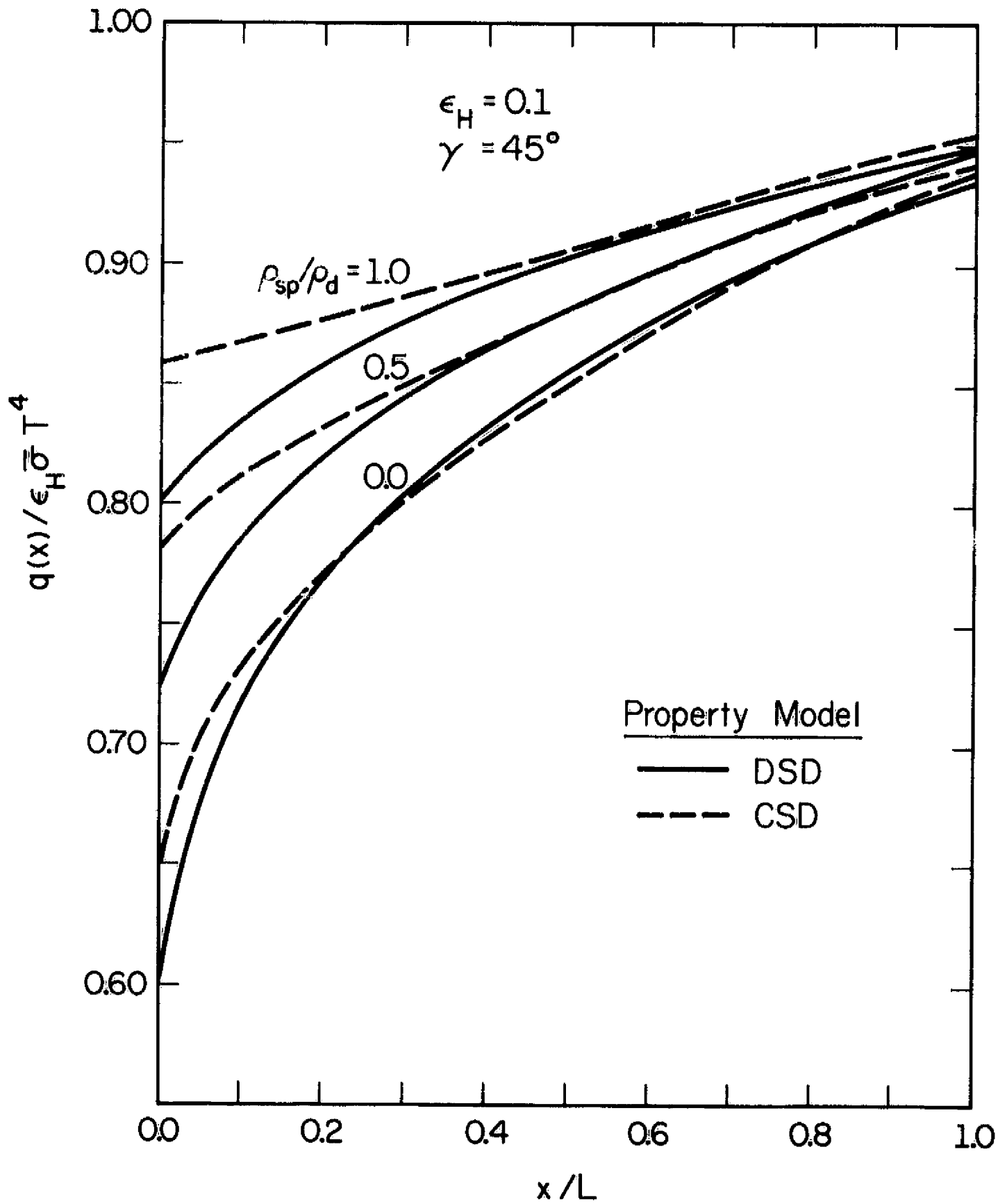


Figure 5.1 Influence of angular dependent properties on local radiant heat flux ($\epsilon_H = 0.1$, $\gamma = 45^\circ$)

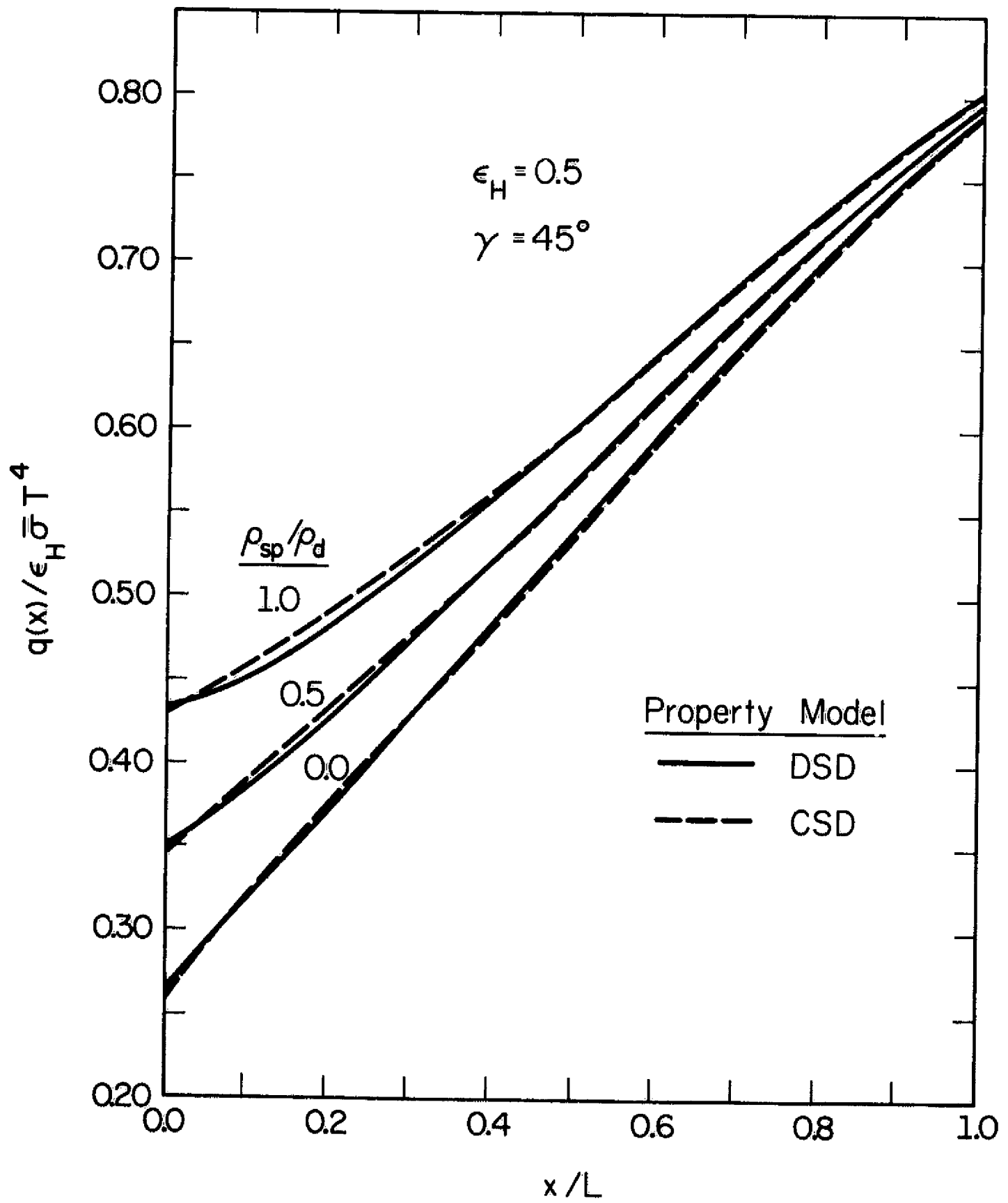


Figure 5.2 Influence of angular dependent properties on local radiant heat flux ($\epsilon_H = 0.5$, $\gamma = 45^\circ$)

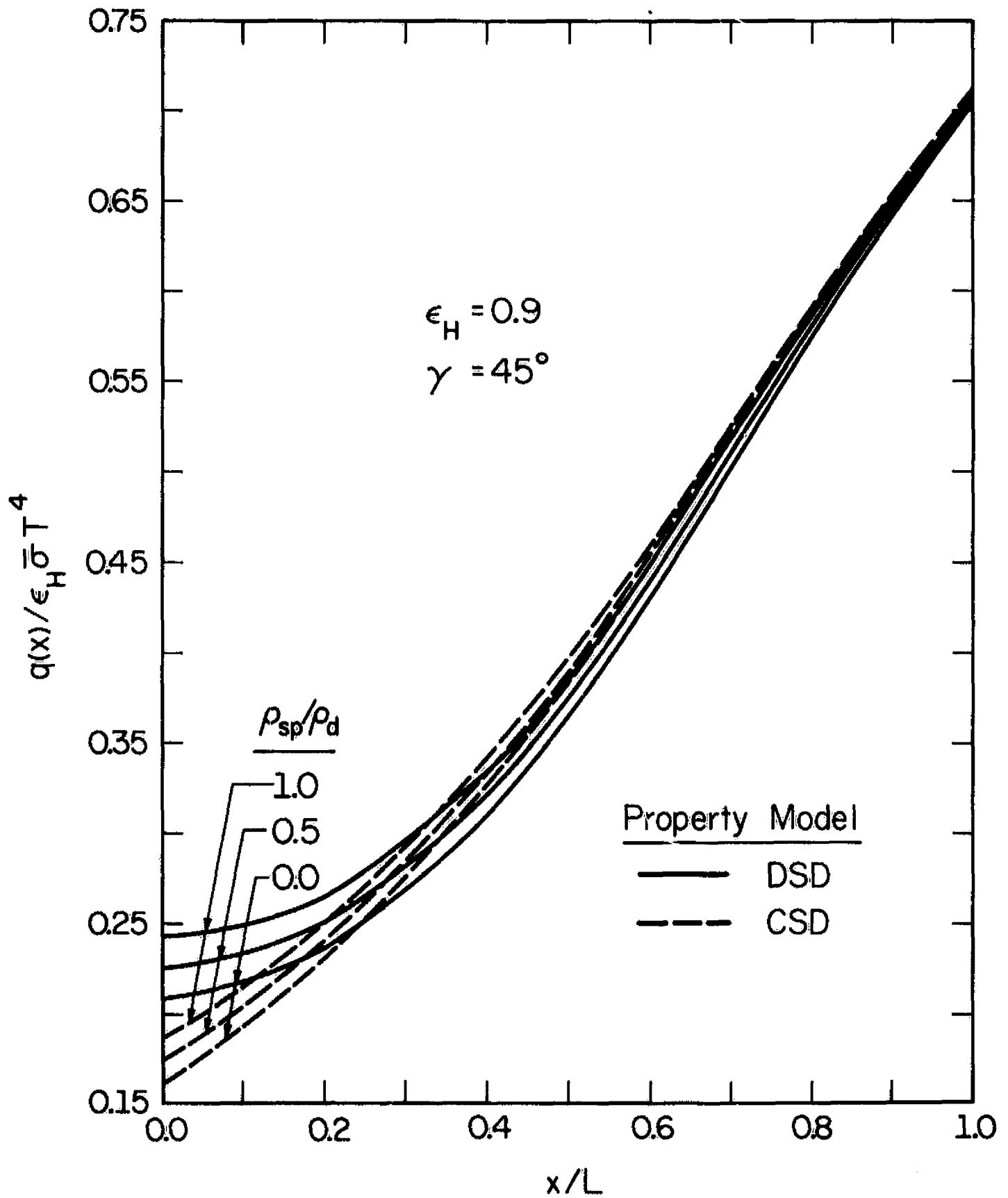


Figure 5.3 Influence of angular dependent properties on local radiant heat flux ($\epsilon_H = 0.9$, $\gamma = 45^\circ$)

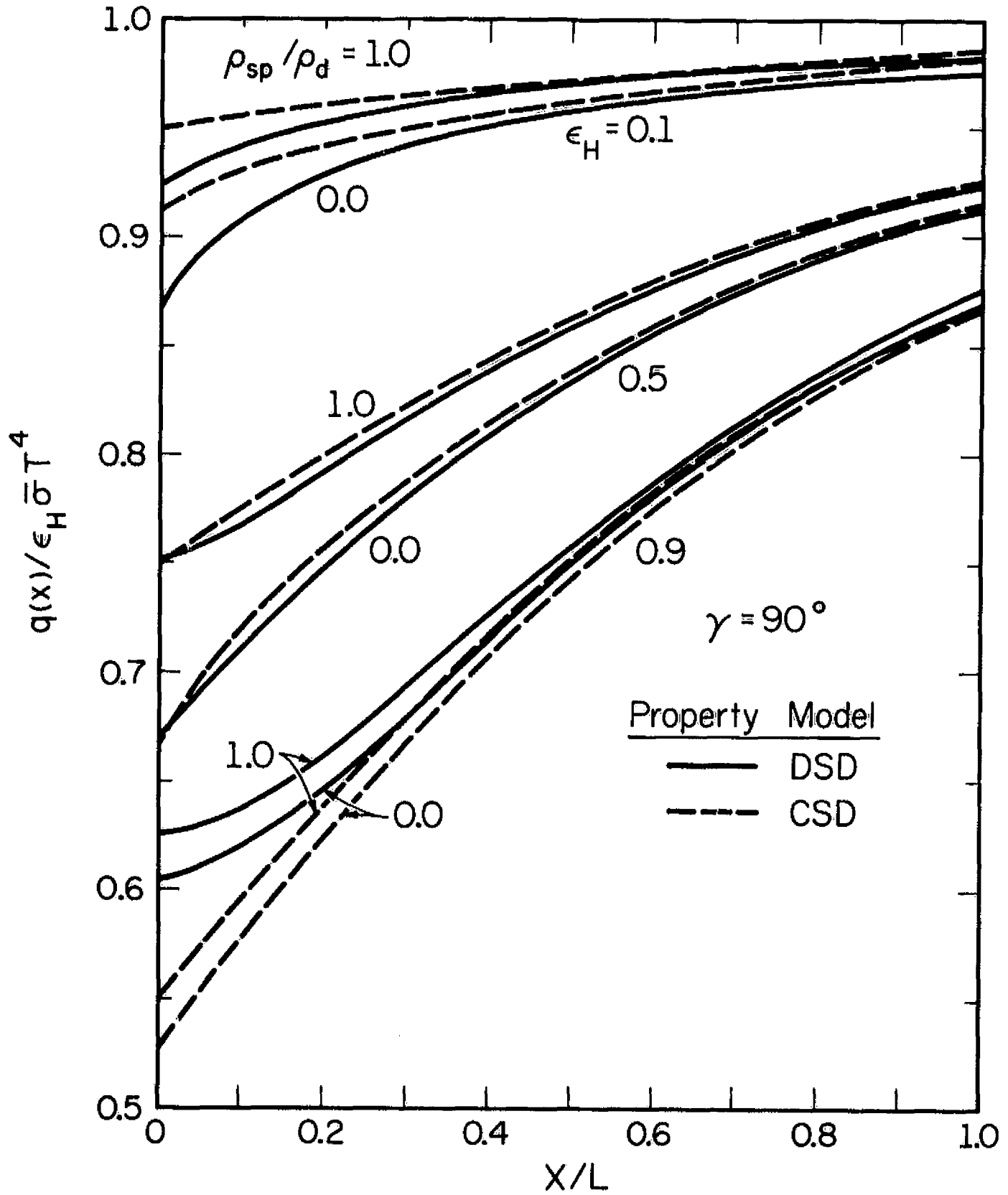


Figure 5.4 Influence of angular dependent properties on local radiant heat flux ($\gamma = 90^\circ$)

values of the specularity parameter and both values of included angle. The largest difference between DP and CP local heat flux values is approximately 2 percent. Consequently, attention is directed to the results for low ($\epsilon_H = 0.1$) and high ($\epsilon_H = 0.9$) emittance surfaces. For each value of the specularity parameter, the largest difference between results obtained using DP and CP analysis occurs for surface elements near the apex. This is attributed to the fact that these elements receive major contributions to their irradiation from energy incident at large angles of illumination. Furthermore, a significant portion of the irradiation of these elements is due to energy which is emitted by the adjacent surface at large angles of emission. These factors are important because the largest differences between direction dependent and direction independent properties occur for polar angles greater than about 60 degrees (see Fig. 4.4).

Near the apex, heat flux values obtained from CP analysis are greater than those from DP analysis for low emittance surfaces and less for high emittance surfaces. For $\gamma = 90^\circ$ these relationships exist along the entire plate length. The discrepancy between DP and CP results is nearly independent of surface specularity for both low and high emittance surfaces. For low emittance surfaces the maximum differences are approximately 8 and 5 percent for $\gamma = 45^\circ$ and 90° , respectively. For high emittance surfaces the respective maximum discrepancies are 25 and 12 percent.

The major difference between results for $\gamma = 45^\circ$ and $\gamma = 90^\circ$ is that the discrepancy between DP and CP results is generally larger for the former. This is evidence of the greater importance of interreflection phenomena and of irradiation at large angles of illumination for $\gamma = 45^\circ$.

An interesting comparison may be made between present results and those reported by Schoenhorst and Viskanta [51] for local heat flux on identical, equal temperature, semi-infinite, parallel plates. For emittance values less than 0.34 and a plate spacing-to-width ratio value of 0.5, these investigators reported discrepancies between DP and CP heat flux results which were much larger for specular than for diffuse reflection (in some cases more than ten times larger). For the adjoint plate system local heat transfer is much less sensitive to the specularity of the surfaces.

5.2.2 Overall Heat Transfer

Dimensionless overall heat transfer per unit width is presented in Table 5.1. The influence of the reflectance model on total heat transfer in the adjoint plate system is not very great. This is attributed to the dominance of emission over absorbed incident radiation for a major portion of the plate surface. For the smaller included angle, total heat transfer evaluated using directional property analysis and a specular reflection model is greater than that calculated with a diffuse reflection model by 13 percent for the intermediate emittance surface. The same comparison for low and high emittance surfaces reveals that the total heat transfer differences are reduced to 7 and 5 percent, respectively. For the larger included angle the aforementioned differences are reduced by approximately a factor of three. The influence of the reflection model on total heat transfer is not significantly different from that reported for constant property analysis.

Inspection of the total heat transfer results presented in Table 5.1 shows that constant property analysis yields results which are

γ	ϵ_H	Specularity Parameter ρ_{sp}/ρ_d					
		1.0		0.5		0.0	
		DP	CP	DP	CP	DP	CP
45°	0.1	0.0895	0.0906	0.0869	0.0877	0.0835	0.0838
	0.5	0.304	0.304	0.285	0.286	0.268	0.267
	0.9	0.378	0.376	0.369	0.370	0.359	0.364
90°	0.1	0.0965	0.0971	—	—	0.0947	0.0958
	0.5	0.425	0.427	—	—	0.410	0.412
	0.9	0.677	0.663	—	—	0.668	0.655

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TABLE 5.1

Overall heat transfer $(Q/L)/\bar{\sigma}T^4$ from DP and CP property models

in excellent agreement with those obtained using directional property analysis. The largest difference which is less than 3 percent is observed for specularly reflecting surfaces of high emittance with the larger value for included angle. Thus, constant property analysis accurately predicts overall heat transfer in the adjoint plate system.

5.2.3 Conclusions

Radiant heat flux distributions evaluated for equal temperature identical adjoint plates demonstrate that the spatial distribution of reflected energy can significantly influence local flux values. The use of a representative reflectance model for a given surface is particularly important for surface elements of intermediate and low emittance surfaces which experience large radiant interaction with other elements and receive major contributions to their irradiation from energy incident at large angles of incidence. For the system studied, the directional dependence of surface properties was unimportant for surfaces of intermediate emittance and analysis which completely ignores directional property dependence is exceptionally accurate. For the critical surface elements mentioned above, disregard of directional property dependence yields high flux values for low emittance surfaces and low flux values for high emittance surfaces. Directional property effects were most pronounced for the high emittance surface where the lack of accounting for surface property detail gave a 25 percent error in local flux. The error incurred by analysis which ignores directional property dependence was nearly independent of the specularity of the surface. Directional dependence of surface properties is not important for evaluating overall heat transfer and constant property analysis gave

accurate overall heat transfer results.

5.3 BDS PROPERTY MODEL RESULTS

In the previous section significant differences in local heat flux values were observed between the results for markedly different spatial distributions of reflected energy. For applications requiring precise heat transfer calculations neither the specular nor diffuse reflectance model may adequately describe reflection from engineering surfaces. A more detailed model for the spatial distribution of reflected energy may be required. The direction independent specular plus diffuse reflectance model is attractive because its use yields a tractable analysis for radiant heat transfer. No information is available, however, to guide selection of the apportionment of reflected energy to the specular and diffuse reflectance components for engineering materials.

In Section 5.3.1 heat flux distributions calculated using the BDS property model are presented. According to this rough surface reflectance model, the specularity of a rough surface depends both on the direction and wavelength of incident energy as well as the rms surface roughness height σ . It is convenient to interpret heat transfer results as total values. For this purpose, optical roughness σ/λ is replaced by a characteristic optical roughness σ/λ_c where λ_c is a characteristic wavelength of emitted radiation. The heat transfer results can also be viewed as spectral values. General characteristics of the heat transfer results are discussed, the influence of σ/λ_c is illustrated, and the important range of optical roughness insofar as heat transfer is concerned is defined.

The BDS model and DSD model heat flux distributions are used

to develop a correlation between the specularity parameter ρ_{sp}/ρ_d and the optical roughness parameter σ/λ_c in Section 5.3.2. The application of the correlation is extended in Section 5.3.3 to other geometrical arrangements of interacting surfaces, and overall heat transfer results are presented and discussed in Section 5.3.4. Finally, the results are summarized in Section 5.3.5.

5.3.1 Local Heat Transfer

Dimensionless local heat transfer results for $\gamma = 45^\circ$ and hemispherical emittance values of 0.1, 0.5, and 0.9 are presented in Figs. 5.5, 5.6, and 5.7, respectively. In each figure results are shown for selected values for optical roughness from zero to very large values. The results labeled $\sigma/\lambda_c = 0.0$ and $\sigma/\lambda_c \rightarrow \infty$ are those obtained using the DSD property model with specular reflection and diffuse reflection, respectively. As expected, for all emittance values local heat flux increases monotonically from a minimum value for diffuse reflection to a maximum for specular reflection. More important, however, is the observation that the heat flux distributions corresponding to optical roughness values of 0.01 and 1.0 are indistinguishable from those obtained using a specular reflection model and a diffuse reflection model, respectively. Furthermore, even for $\sigma/\lambda_c = 0.05$ and $\sigma/\lambda_c = 0.5$ the maximum discrepancies between BDS model heat flux results and those for the simpler reflection models are less than 6, 8, and 2 percent for emittance values of 0.1, 0.5, and 0.9, respectively. Hence, local heat flux may be calculated to engineering accuracy using a specular reflection model for $\sigma/\lambda_c \leq 0.05$ and employing a diffuse reflection model for $\sigma/\lambda_c \geq 0.5$. In the intermediate range of optical roughness

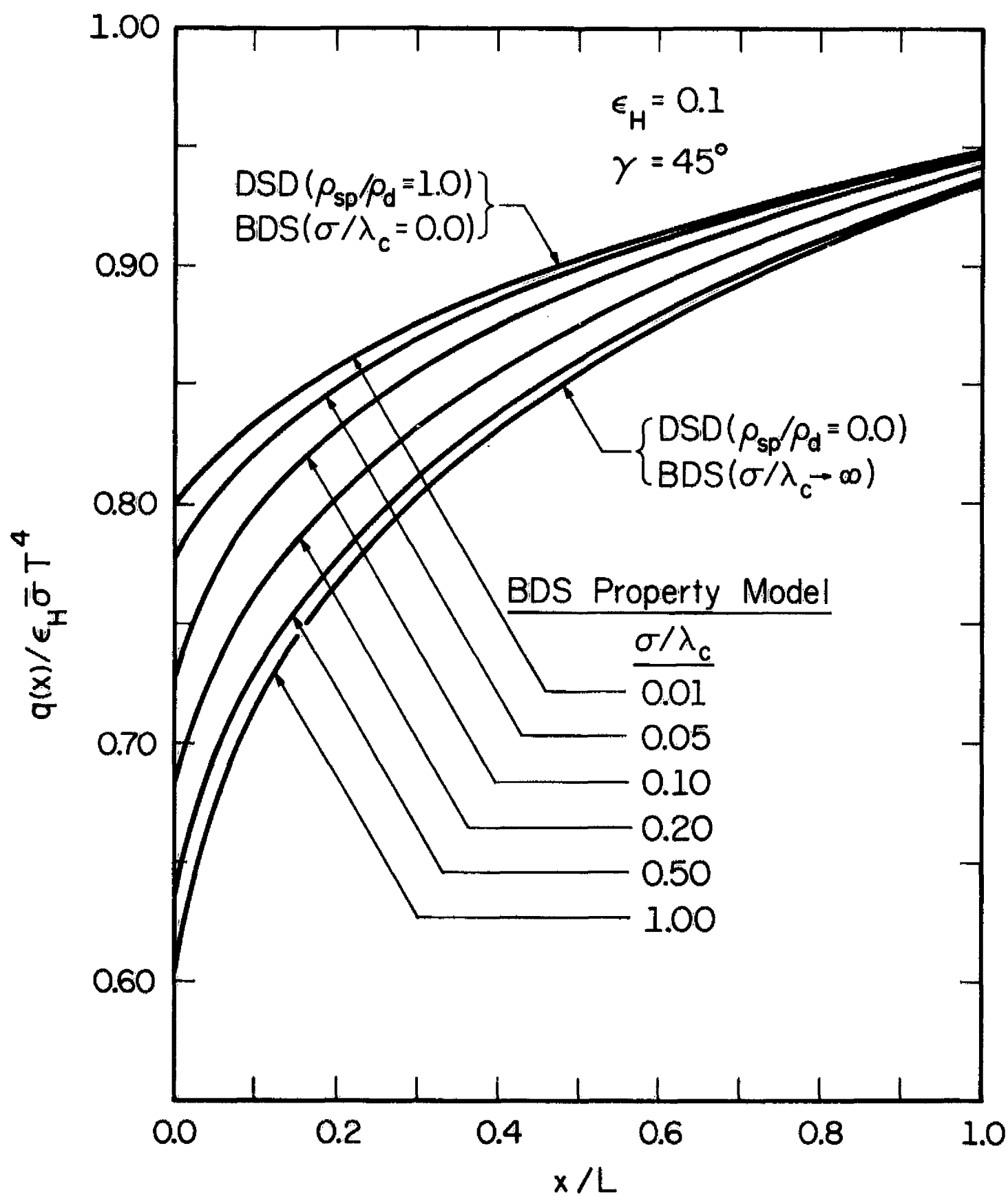


Figure 5.5 Local radiant heat transfer from
BDS property model ($\epsilon_H = 0.1$)

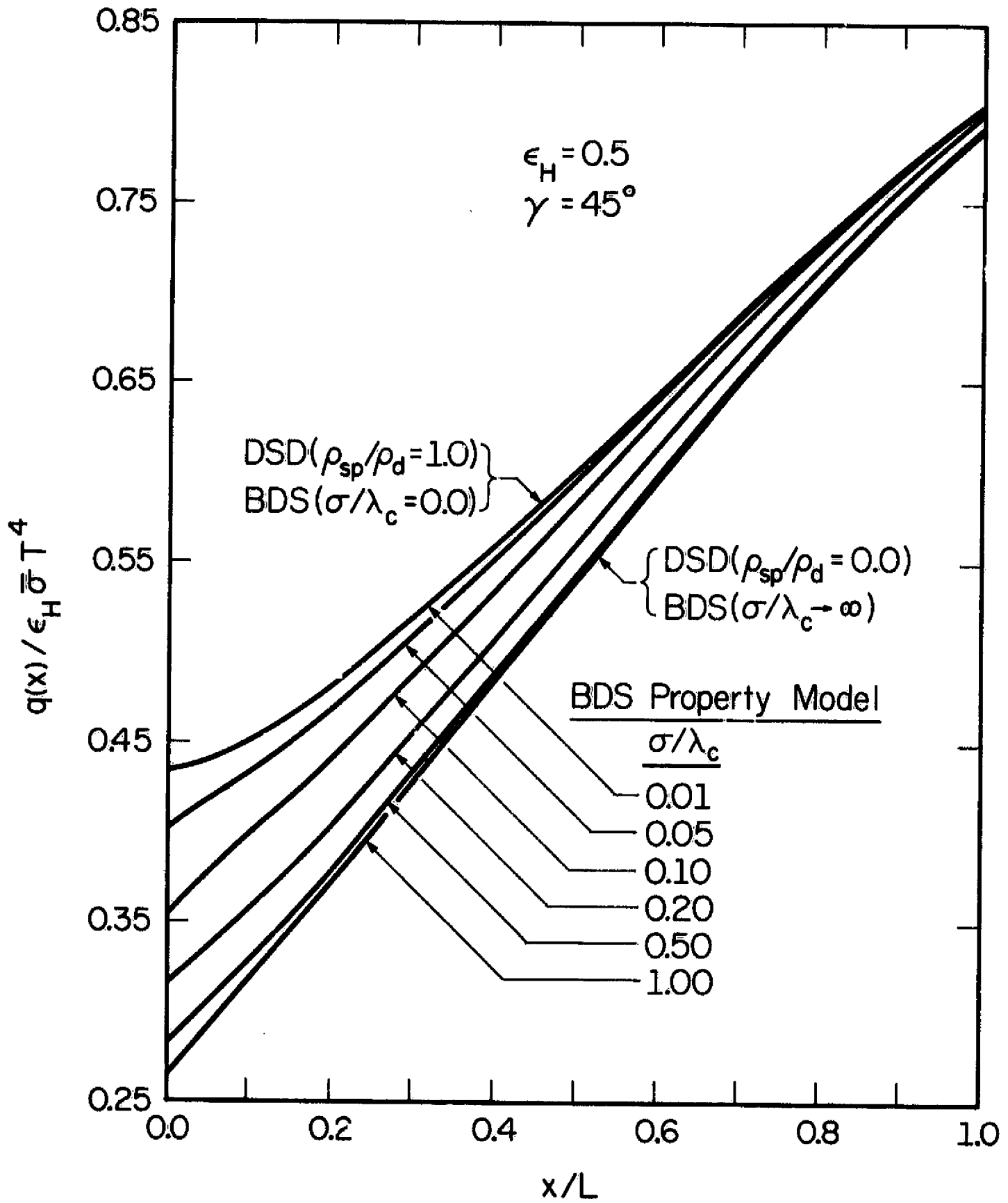


Figure 5.6 Local radiant heat transfer from BDS property model ($\epsilon_H = 0.5$)

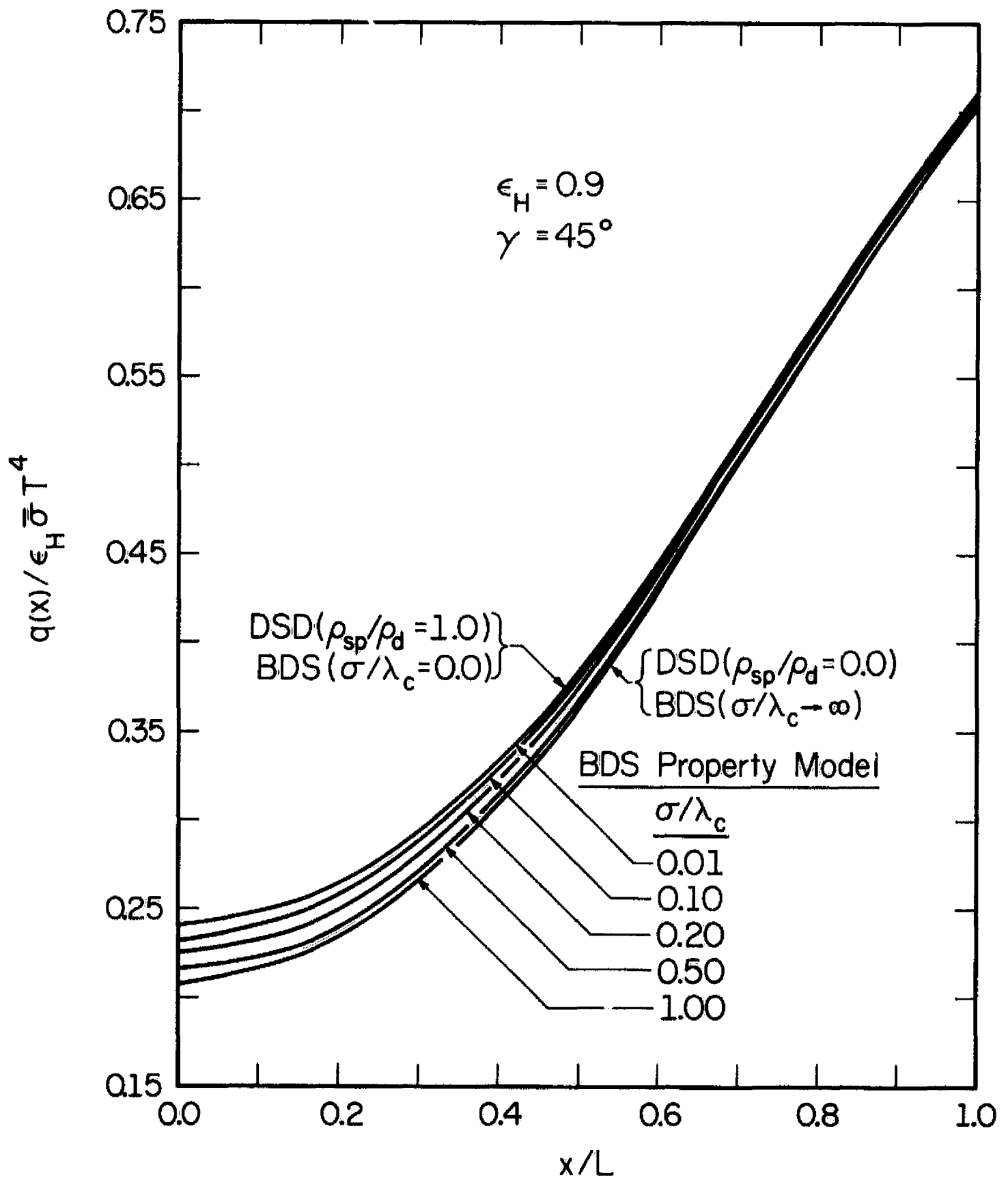


Figure 5.7 Local radiant heat transfer from BDS property model ($\epsilon_H = 0.9$)

($0.05 < \sigma/\lambda_c < 0.5$) a specular plus diffuse reflectance model should be used to evaluate local radiant flux.

As pointed out above, the transition in local heat flux from that corresponding to a specular reflection model to that corresponding to a diffuse reflection model is confined to the range of optical roughness values between 0.01 and 1.0. The characteristics of this transition can be observed from the results presented in Fig. 5.8. There is shown the ratio of the difference between local flux for specular reflection and that for arbitrary optical roughness to the difference between local flux for specular and diffuse reflection at $x/L = 0.05$. Although the results shown are for a position near the apex where interreflections are important, they are representative of those at other locations on the plate. The transition from specular to diffuse reflection heat transfer results occurs most rapidly for the intermediate emittance surface and least rapidly for the large emittance surface. The results for the low emittance surface lie between those for intermediate and large emittance.

5.3.2 Correlation of Specularity Parameter with Optical Roughness

A detailed comparison of local heat flux calculated with the DSD and the BDS property models was made for $\gamma = 45^\circ$. The purpose of this study was to determine whether the simpler DSD property model could satisfactorily predict radiant heat transfer between rough surfaces with properties represented by the more complex BDS property model. For a rough surface with a characteristic optical roughness σ/λ_c , the problem essentially reduces to relating optical roughness of the BDS model to the specularity parameter ρ_{sp}/ρ_d of the DSD model, that is, to the

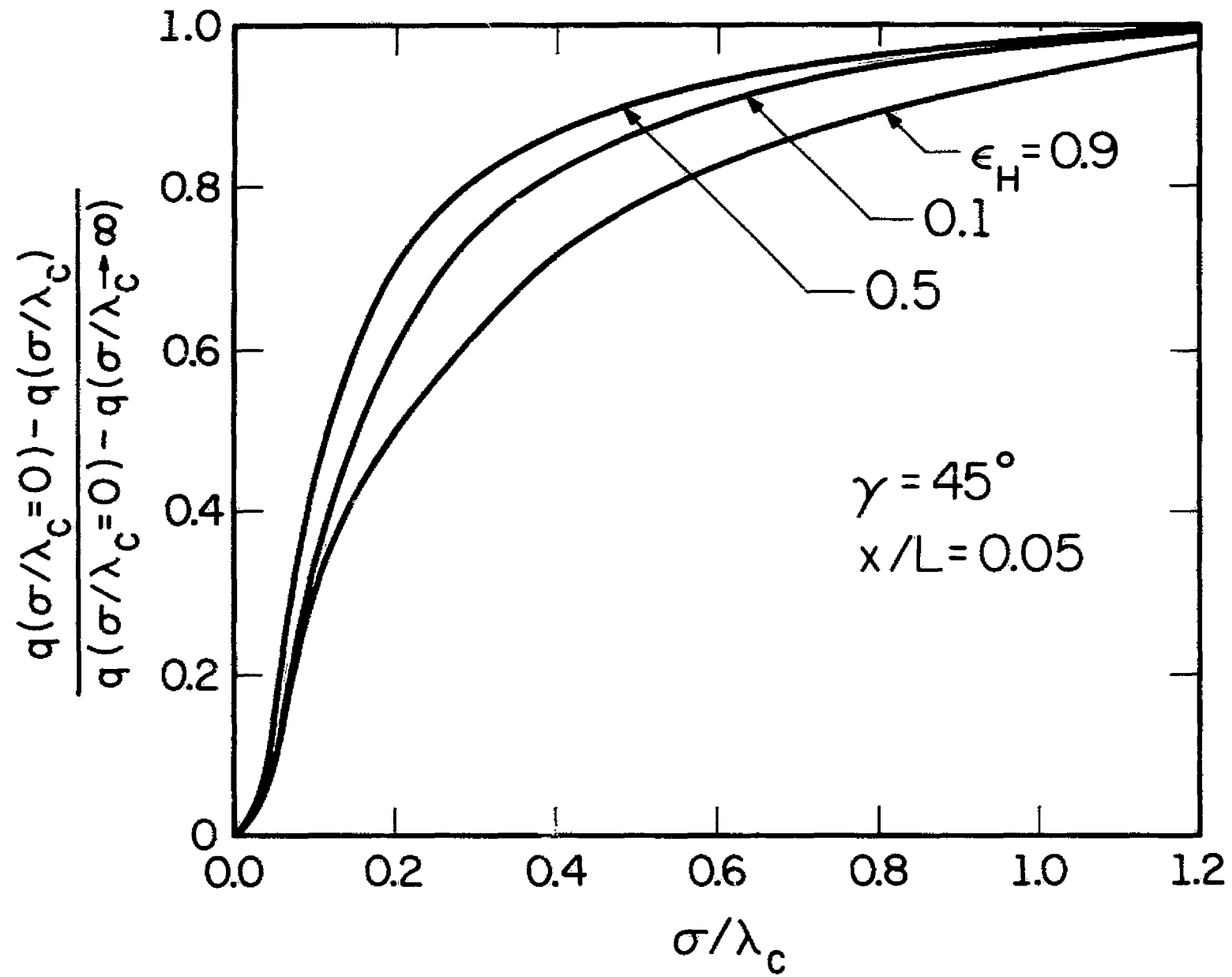


Figure 5.8 Transition from specular to diffuse reflection local heat flux values from BDS property model ($x/L = 0.05$)

apportionment of reflected energy between the specular and diffuse reflectance components of the DSD model. Since local radiant flux is most sensitive to the spatial distribution of reflected energy at locations near the apex, initial comparison of flux results was limited to this region. To facilitate this study the heat flux results obtained at $x/L = 0.05$ with the BDS and DSD models were used to construct Figs. 5.8 and 5.9. The former illustration was discussed previously. In Fig. 5.9 the ratio of the difference between local flux for specular reflection ($\rho_{sp}/\rho_d = 1.0$) and that for arbitrary ρ_{sp}/ρ_d to the difference between local flux for specular reflection and that for diffuse reflection ($\rho_{sp}/\rho_d = 0.0$) is shown. For each emittance a value of optical roughness was selected and the corresponding relative heat flux difference evaluated from Fig. 5.8. Entering Fig. 5.9 with the relative heat flux difference just determined, the corresponding value of the specularity parameter was evaluated from the curve appropriate to the chosen surface emittance. This procedure was repeated for a number of σ/λ_c values for each emittance and Fig. 5.10 was constructed from the results. Since the relationship between the specularity parameter and optical roughness illustrated in Fig. 5.10 was established for only a single position, it is of interest to examine the validity of the correlation at other plate locations. Local heat transfer calculated with the DSD model analysis employing the specularity parameter-optical roughness correlation developed is compared to the results of BDS analysis in Figs. 5.11-5.13 for emittance values of 0.1, 0.5, and 0.9, respectively. BDS model results are shown as solid curves and DSD model results as dashed curves. Agreement of BDS and DSD model heat flux distribution is excellent over the entire plate length for each emittance value. The maximum discrepancy

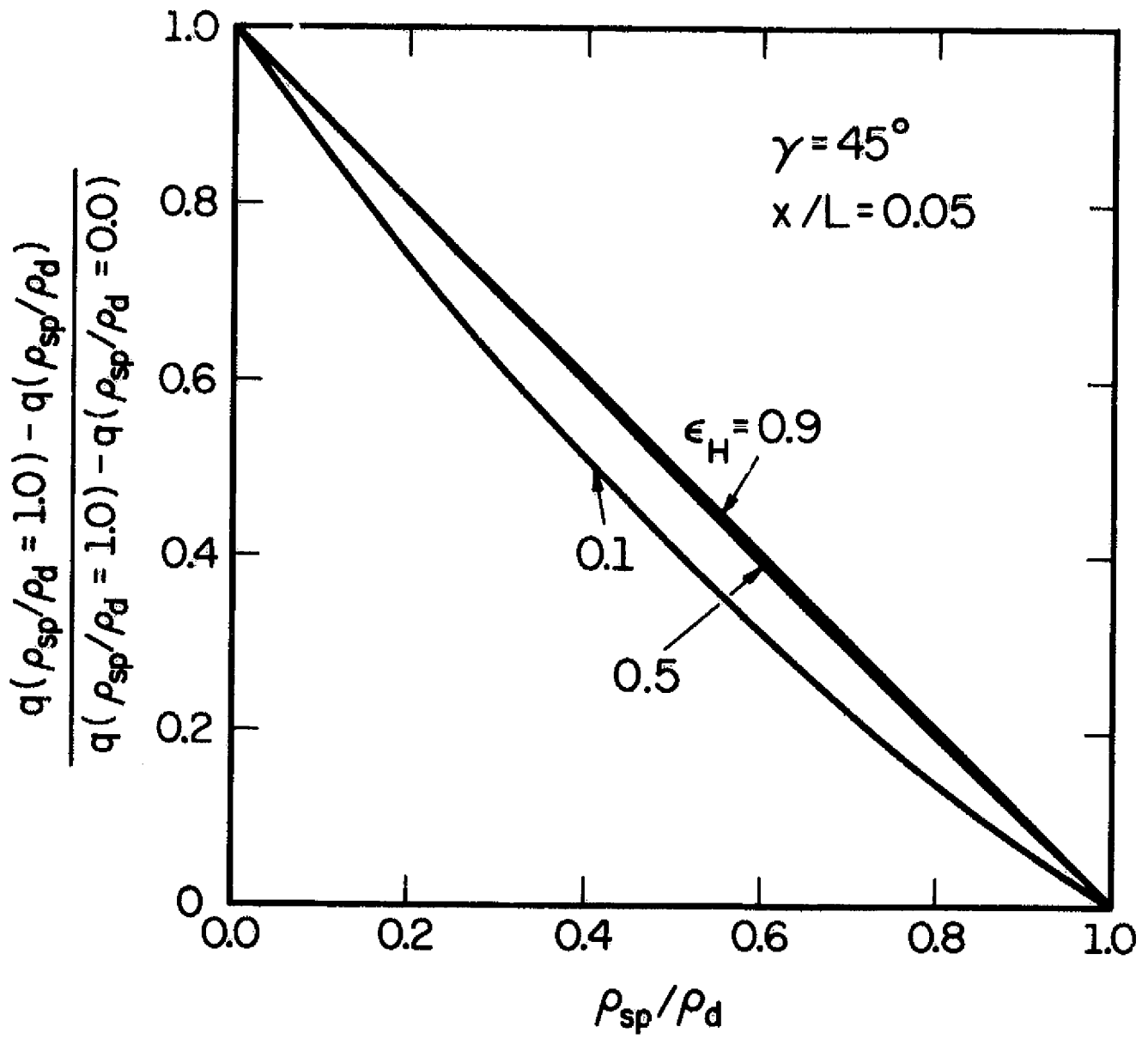


Figure 5.9 Transition from diffuse to specular reflection local heat flux values from DSD property model ($x/L = 0.05$)

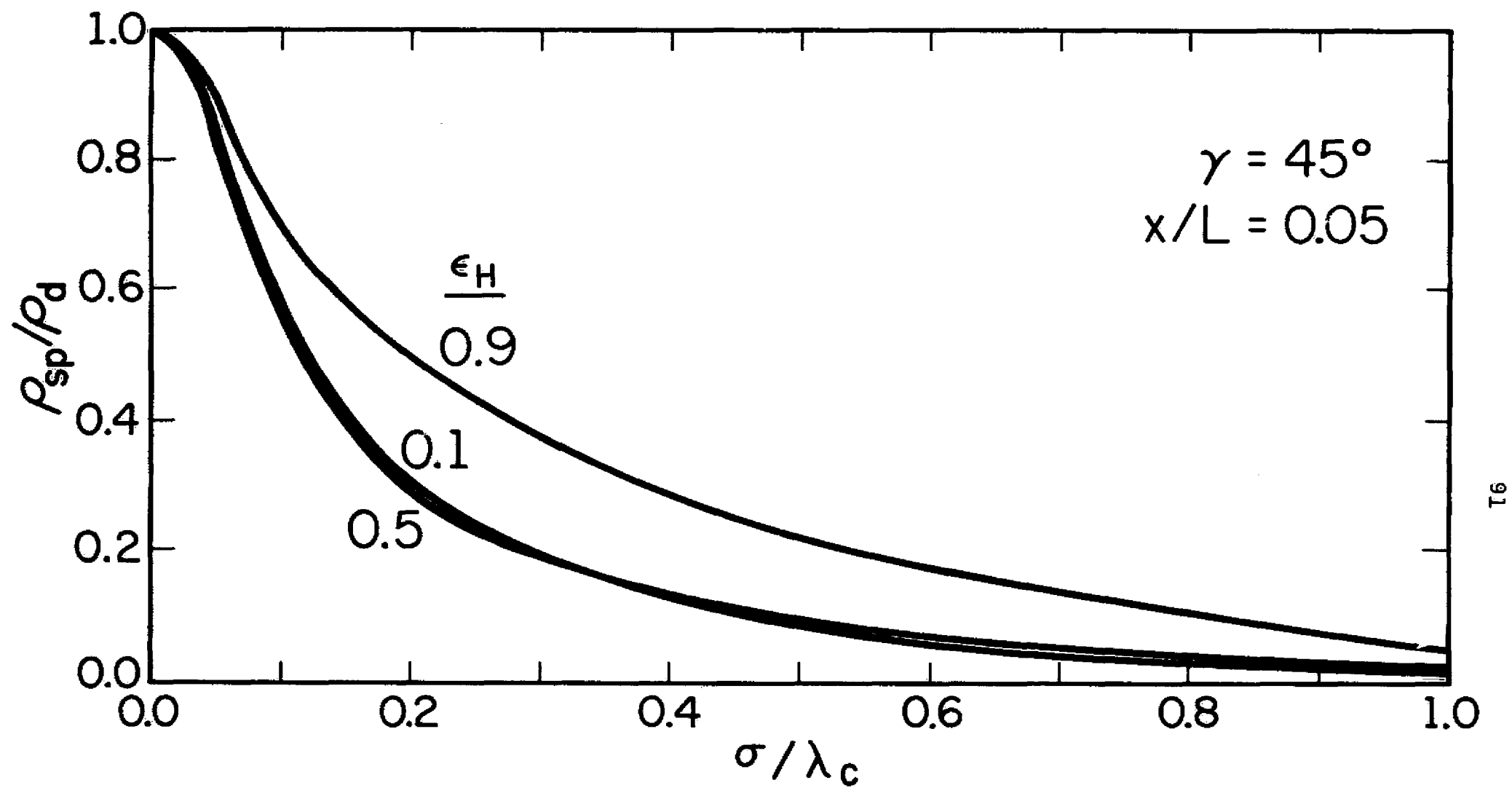


Figure 5.10 Correlation of specularity parameter to optical roughness ($\gamma = 45^\circ$, $x/L = 0.05$)

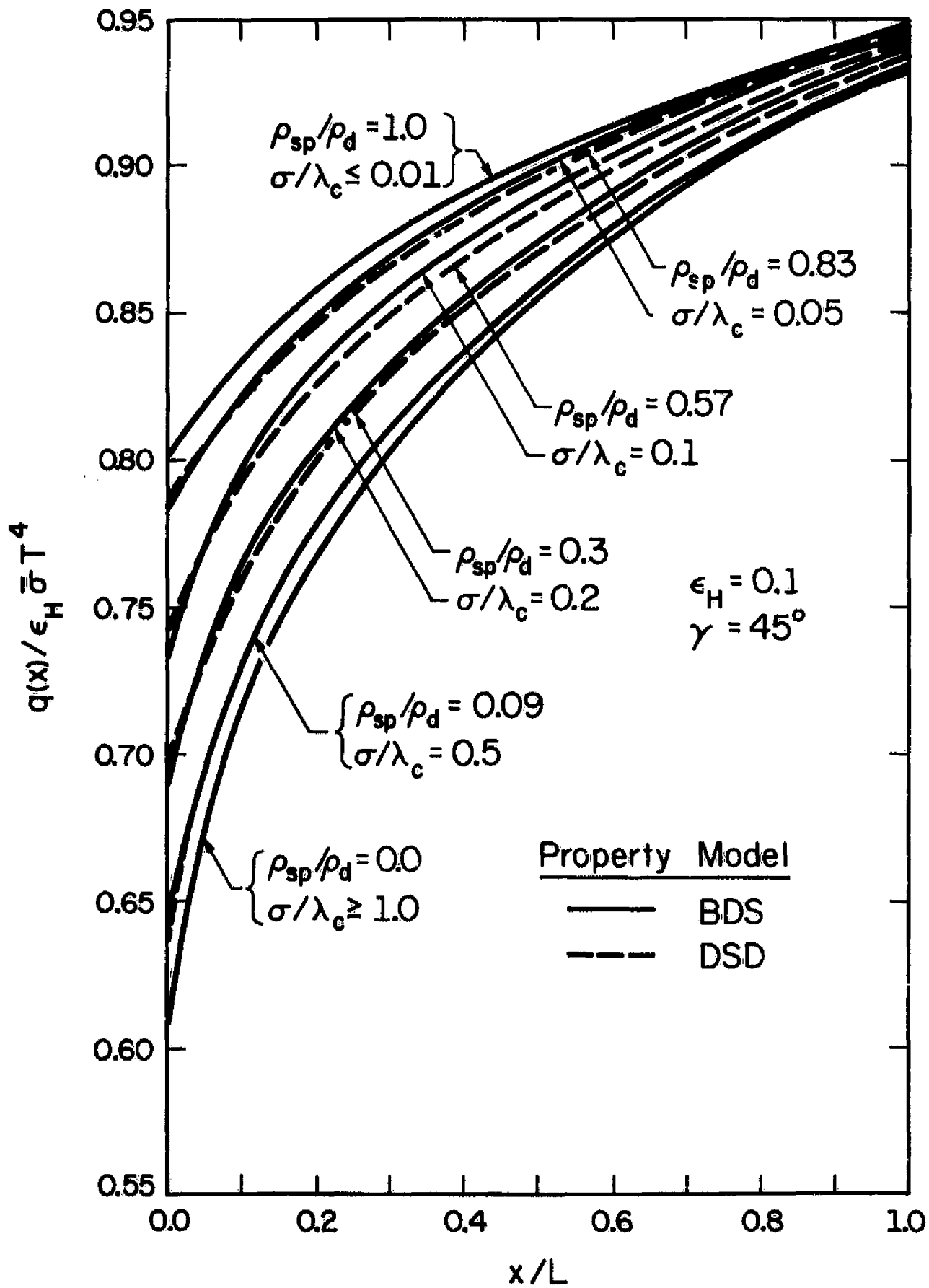


Figure 5.11 Comparison of local heat transfer from BDS and DSD property models ($\gamma = 45^\circ$, $\epsilon_H = 0.1$)

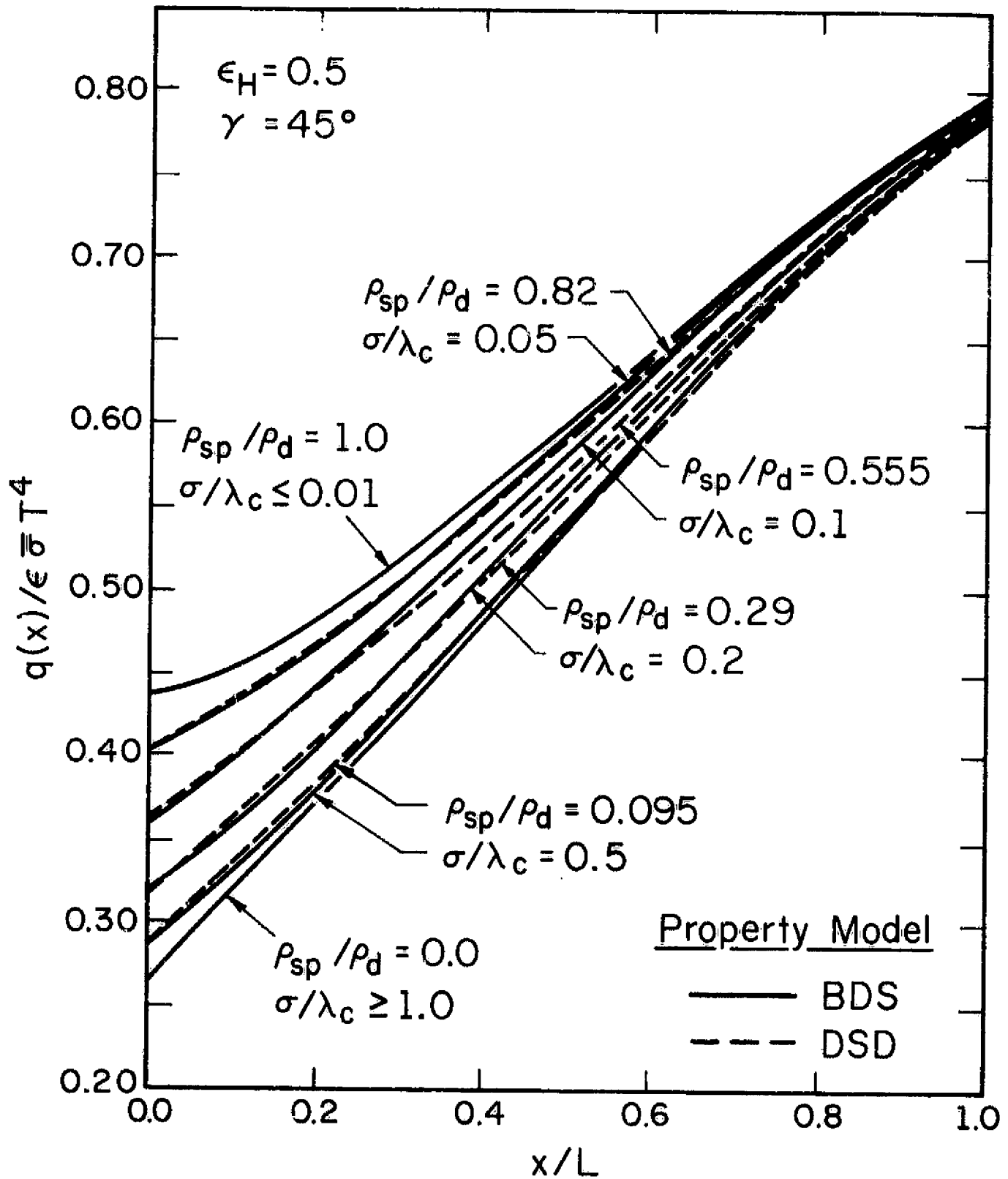


Figure 5.12 Comparison of local heat transfer from BDS and DSD property models ($\gamma = 45^\circ$, $\epsilon_H = 0.5$)

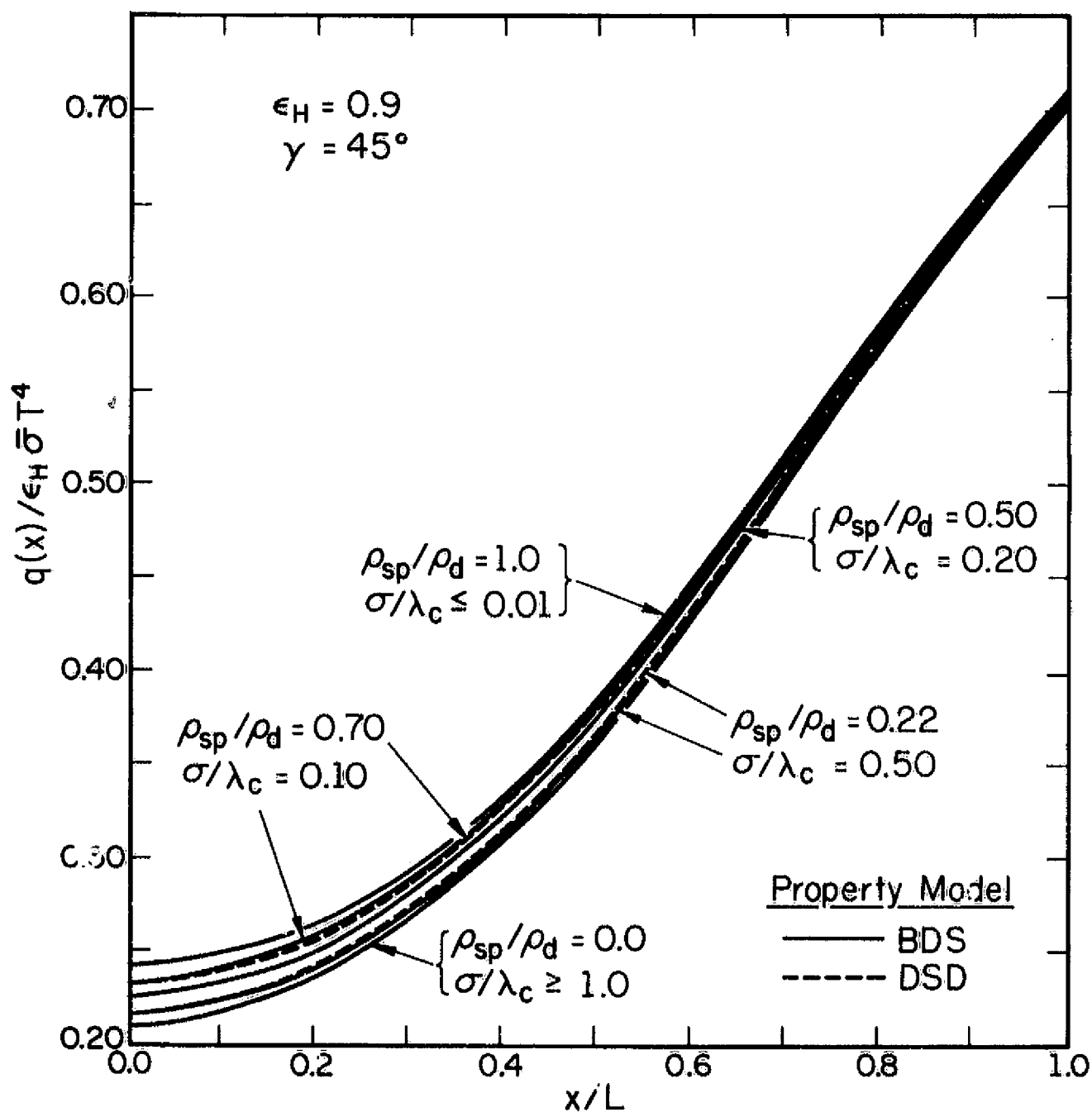


Figure 5.13 Comparison of local heat transfer from BDS and DSD property models ($\gamma = 45$, $\epsilon_H = 0.9$)

is less than 2 percent. The following general characteristics are apparent, however. For low emittance and optical roughness less than about 0.2, DSD model results are low for all x/L values greater than 0.05. For high emittance, DSD model results are low for optical roughness less than 0.2 and high for optical roughness greater than 0.2.

5.3.3 Extension of Specularity Parameter-Optical Roughness Correlation

Since no other information is apparently available to guide selection of the specularity parameter, an attempt is made to extend the specularity parameter-optical roughness correlation of the previous section. The purpose of this extension is to provide thermal designers with a means of approximately accounting for surface roughness effects in heat transfer calculations for systems other than adjoint plates until more complete results are available.

The procedure used to develop the correlation of Fig. 5.10 was repeated for other representative plate locations and the results are shown in Figs. 5.14-5.16 for emittance values of 0.1, 0.5, and 0.9, respectively. It is apparent that significant differences from the curves developed for $x/L = 0.05$ exist at other plate locations. These differences, however, are not important insofar as local radiant flux values in the adjoint plate system are concerned since, as shown earlier, the flux distributions from DSD analysis obtained using the correlation of Fig. 5.10 were in excellent agreement with BDS results (maximum error 2 percent). They may be important for other geometrical arrangement of surfaces. With this possibility in mind appropriate alterations of the results in Fig. 5.10 are recommended.

Recall that the non-uniformity of local heat flux in the adjoint

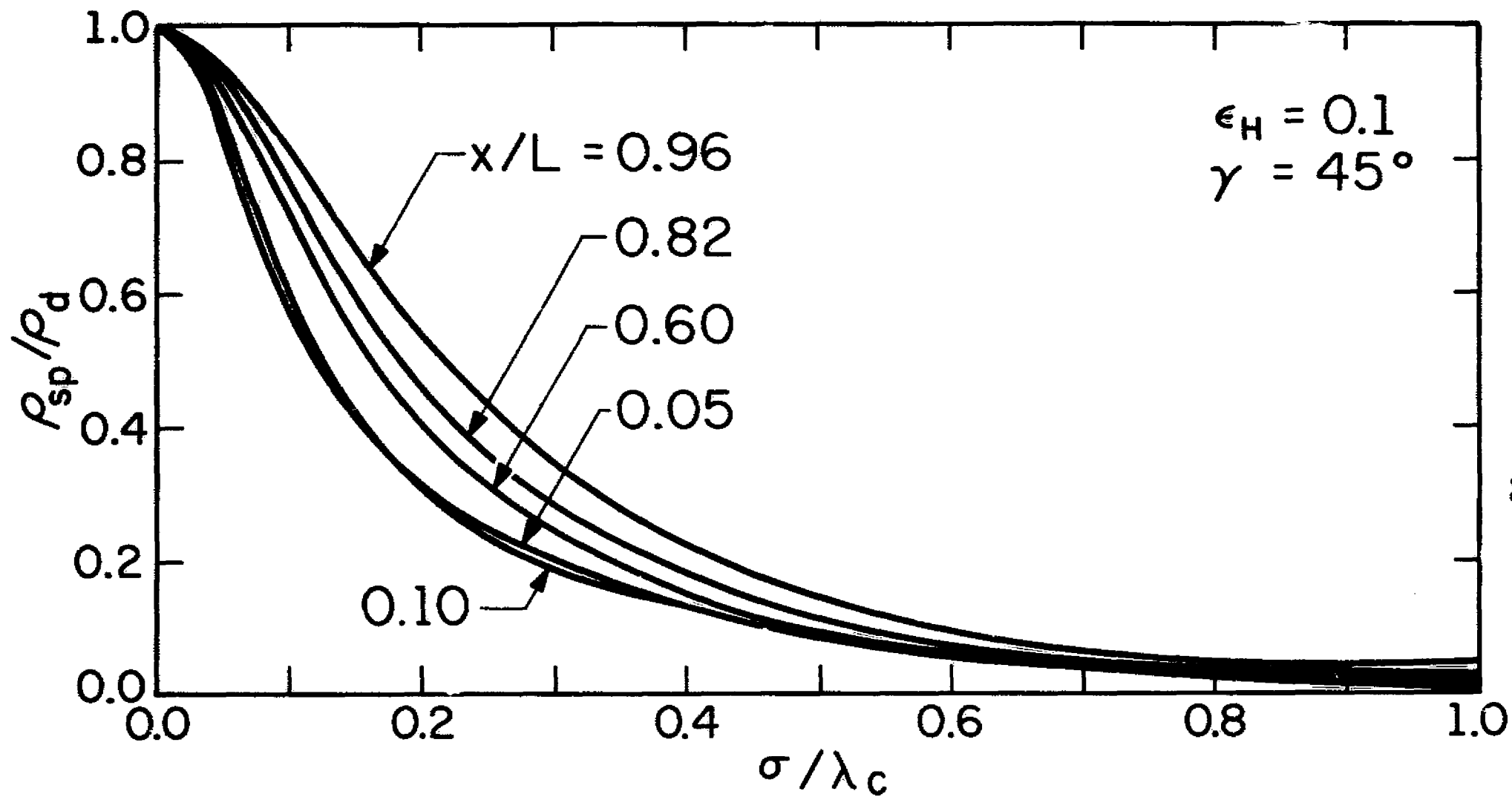


Figure 5.14 Correlation of specularity parameter to optical roughness ($\gamma = 45^\circ$, $\epsilon_H = 0.1$)

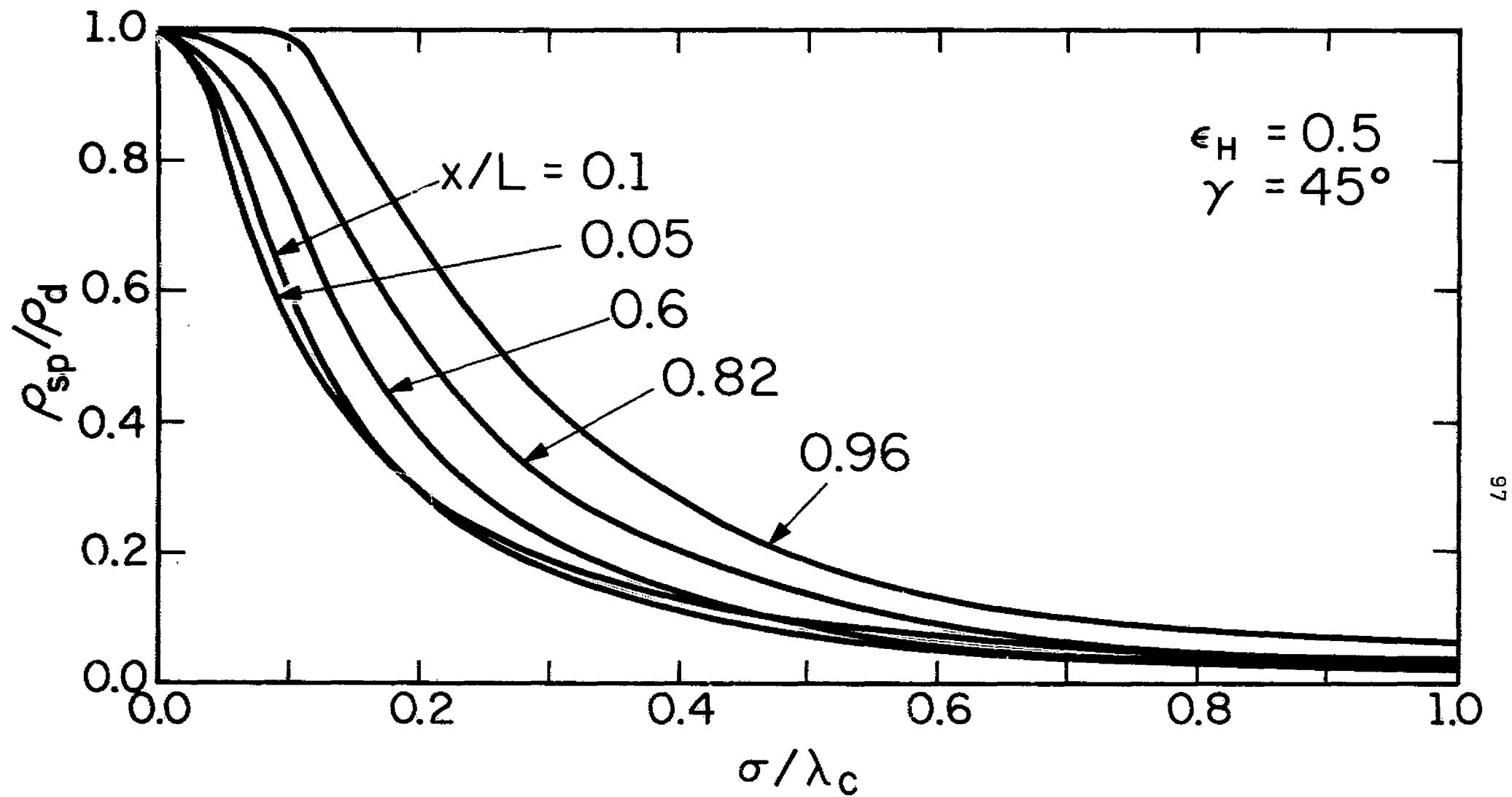


Figure 5.15 Correlation of specularity parameter to optical roughness ($\gamma = 45^\circ$, $\epsilon_H = 0.5$)

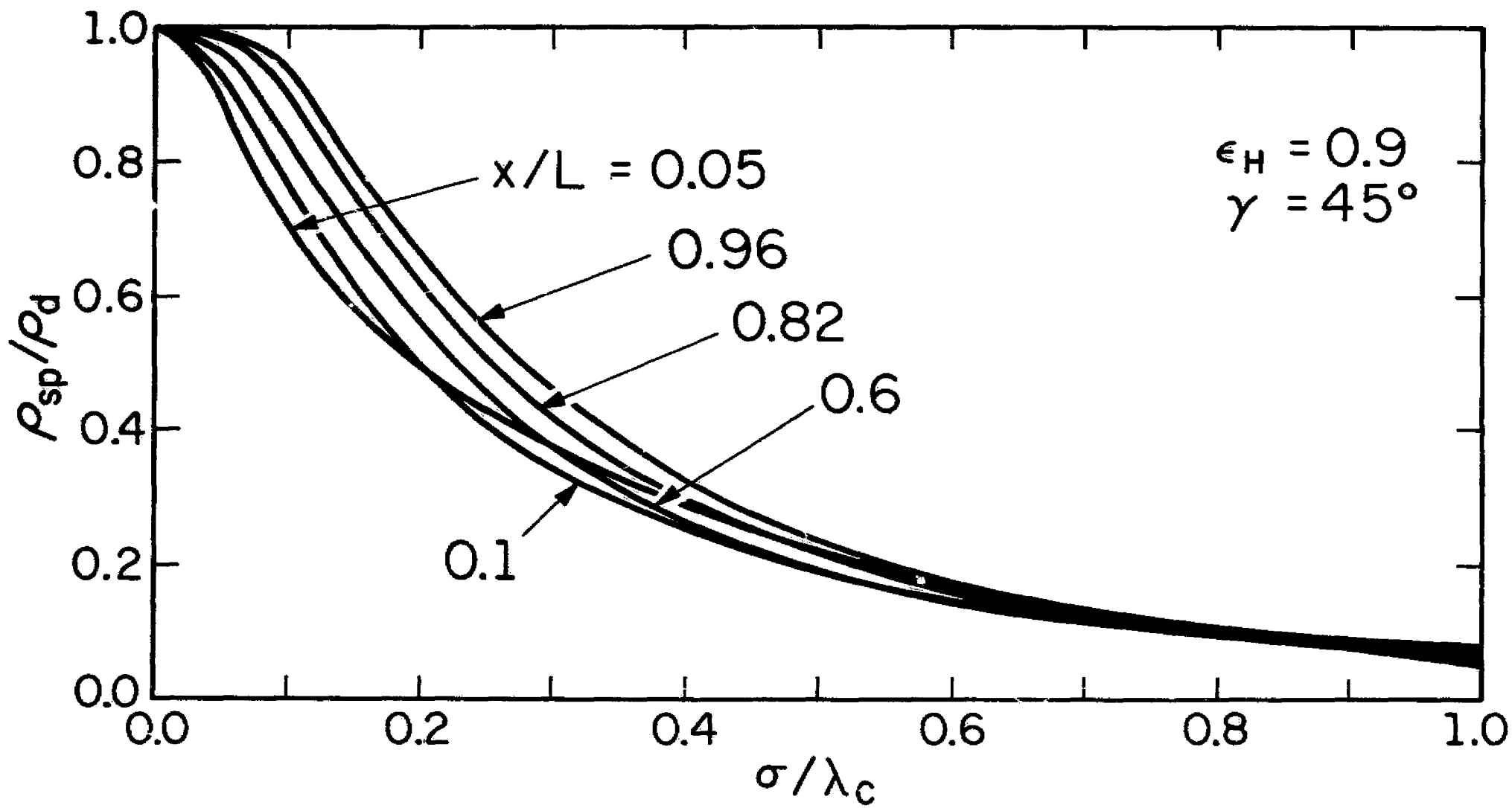


Figure 5.16 Correlation of specularity parameter to optical roughness ($\gamma = 45^\circ$, $\epsilon_H = 0.9$)

plate system is attributed to the diminishing importance of interreflected energy (radiant interaction with the adjacent surface) with increasing distance from the apex. The x/L regions from 0.0 to 0.1, 0.1 to 0.6, and 0.6 to 1.0 may be considered regions of large, moderate, and low radiant interaction. Interpreting the correlations of Figs. 5.14-5.16 for the various x/L values leads to the following recommendations for the evaluation of heat transfer in systems of rough surfaces:

- (1) For optical roughness values equal to and greater than 1.0, use the directional diffuse property model.
- (2) For optical roughness values equal to and less than 0.01, use the directional specular property model.
- (3) When the radiant interaction between the surfaces of the system is large and $0.01 < \sigma/\lambda_c < 1.0$, use the directional specular plus diffuse property model with the specular property parameter evaluated from Fig. 5.10. For moderate radiant interaction between surfaces increase the specular property parameter value determined from Fig. 5.10 by 20 to 30 percent. In situations where radiant interaction is small increase the ρ_{sp}/ρ_d value read from Fig. 5.10 by 80 to 100 percent as emittance increases from low to intermediate values. For larger emittance values the percentage increase in ρ_{sp}/ρ_d is less attaining a value of no more than 35 percent for large ϵ .

5.3.4 Overall Heat Transfer

Dimensionless overall heat transfer rates per unit width obtained with the BDS property model for 45 degree included angle are presented in Fig. 5.17 for hemispherical emittance values 0.1, 0.5, and 0.9. Results for selected values of optical roughness are tabulated for convenience in Table 5.2.

Overall heat transfer is less sensitive to surface roughness than is local heat transfer. As discussed in Section 5.2.2, overall heat transfer for specular ($\sigma/\lambda_c = 0.0$) reflection is larger than that for diffuse ($\sigma/\lambda_c \rightarrow \infty$) reflection by 13 percent for the intermediate

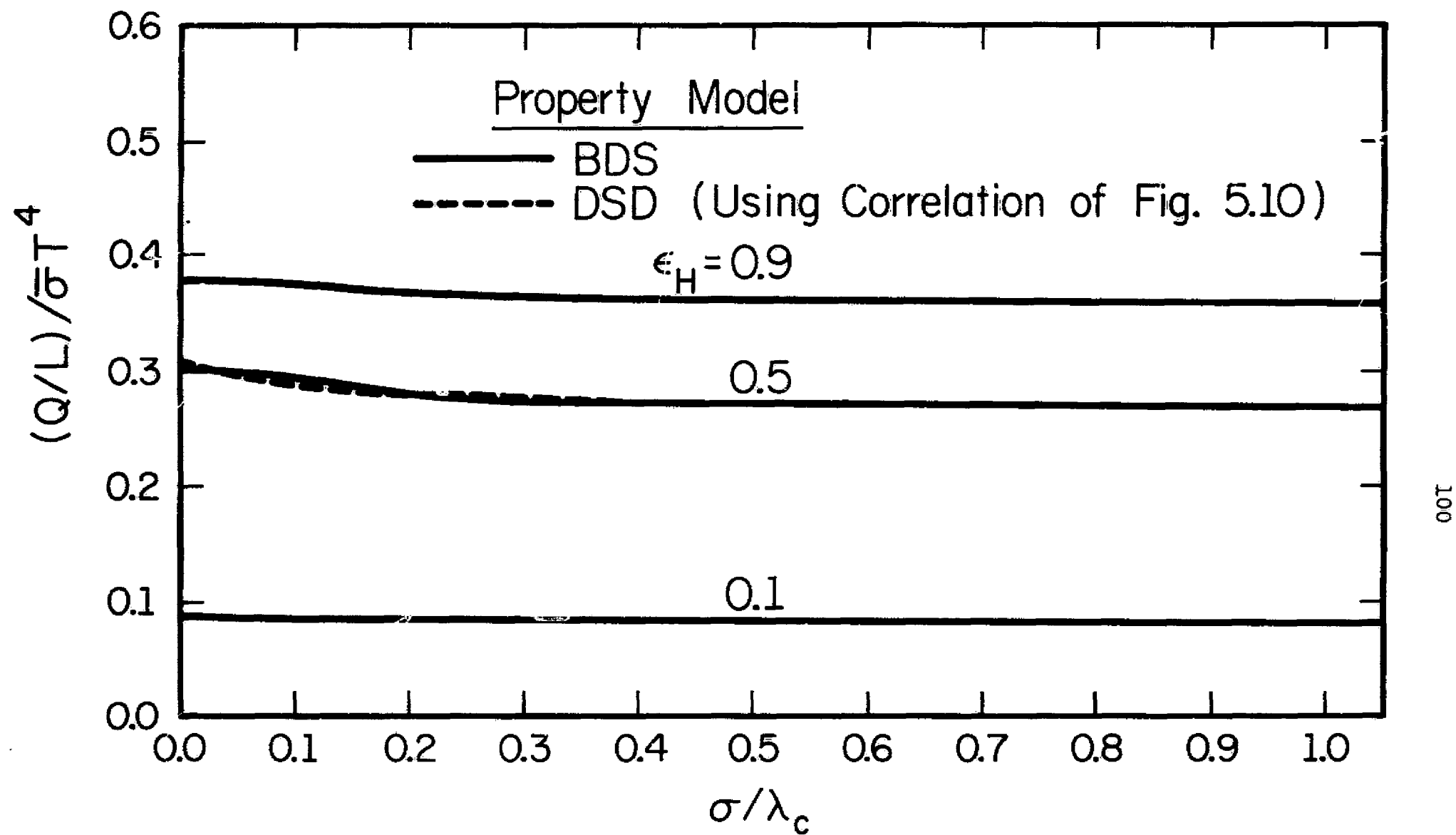


Figure 5.17 Overall heat transfer from BDS and DSD property models ($\gamma = 45^\circ$)

ϵ_H	σ/λ_c							
	0.0	0.01	0.05	0.1	0.2	0.5	1.0	$\rightarrow \infty$
0.1	0.0895	0.0894	0.0889	0.0878	0.0860	0.0842	0.0837	0.0835
0.5	0.304	0.303	0.299	0.292	0.280	0.271	0.269	0.268
0.9	0.378	0.378	0.377	0.374	0.369	0.363	0.360	0.359

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TABLE 5.2

Overall heat transfer $(Q/L)/\overline{\sigma T^4}$ from BDS property model ($\gamma = 45^\circ$)

emittance surfaces. For the low and high emittance surfaces, specular analysis results are high by 7 and 5 percent, respectively.

For intermediate emittance surfaces and $\sigma/\lambda_c \leq 0.05$, overall heat transfer predicted by the simpler specular reflection analysis is high by less than 2 percent. Furthermore, for high and low emittance surfaces the specular analysis results are high by no more than 1 percent. The diffuse reflection overall heat transfer results for each emittance value are lower than the BDS results by less than 1.5 percent for $\sigma/\lambda_c \geq 0.5$. In the range of optical roughness $0.05 \leq \sigma/\lambda_c \leq 0.5$ the DSD model can be used to reduce uncertainty in overall heat transfer predictions for rough surfaces. In this regard the $\rho_{sp}/\rho_d - \sigma/\lambda_c$ correlation of Fig. 5.10 was investigated to determine whether it yields acceptable overall heat transfer results. Using the specularity-roughness correlation of Fig. 5.10, overall heat transfer results were obtained with the DSD property model and are compared to BDS model results in Fig. 5.17. Dashed lines representing DSD model results are distinguishable from the more detailed BDS model results (shown as solid lines) only for the intermediate emittance surfaces. However, in this case the maximum error is less than 1.5 percent.

5.3.5 Conclusions

The results reported in this section for radiant heat transfer in the adjoint plate system support the following conclusions. First the transition from specular to diffuse reflection model heat transfer occurs in the optical roughness range $0.01 < \sigma/\lambda_c < 1.0$. For $\sigma/\lambda_c \leq 0.01$ a specular reflection model accurately predicts local flux distributions while for $\sigma/\lambda_c \geq 1.0$ a diffuse reflection model is

appropriate. In the transition region a specular plus diffuse reflectance model should be employed. Second, a correlation of the specularity parameter value required for DSD analysis was developed in terms of the optical roughness of the surface. The use of this correlation gave flux distributions which were in excellent agreement along the entire plate length. Third, recommendations are suggested for the application of the correlation to other systems of different geometrical arrangement. Fourth, roughness effects on overall heat transfer are less important than for local flux. For $\sigma/\lambda_c < 0.05$ specular reflection analysis gives accurate results while for $\sigma/\lambda_c > 0.5$ diffuse reflection analysis is adequate. In the transition region the specularity parameter-optical roughness correlation obtained for local flux in conjunction with the DSD property analysis yields excellent overall heat transfer results.

5.4 BECKMANN PROPERTY MODEL RESULTS

In this section local and overall radiant heat transfer results are presented for adjoint plates with 45 and 90 degree included angle. Surface radiation properties are represented by the Beckmann BDR model. Radiant flux distributions are reported for surfaces with low to high emittance and selected values for the roughness parameters. The influence of the surface roughness parameters optical roughness (σ/λ) and rms roughness slope (m) on radiant flux is established. Although the heat flux distributions may be considered spectral values, it is convenient to interpret them as total values by replacing σ/λ with a constant characteristic optical roughness σ/λ_c where λ_c is a characteristic wavelength of emitted energy. Finally, the heat transfer results obtained using the Beckmann BDR model are compared to those calculated

with less detailed property models. For brevity, heat transfer for surfaces with properties represented by the Beckmann model are referred to as rough surface results.

5.4.1 Large Optical Roughness (LOR) Beckmann BDR Model Results

5.4.1.1 Local Heat Transfer

Dimensionless local heat transfer for rough surfaces with properties represented by the large optical roughness (LOR) Beckmann BDR model with rms roughness slope (m) equal to 0.14 is shown in Figs. 5.18 and 5.19 for $\gamma = 45^\circ$ and 90° , respectively. In each figure, results are presented for emittance values of 0.1, 0.5, and 0.9. Also included in each figure are flux distributions evaluated with simpler surface property models. Distributions obtained using constant property (CP) analysis which ignores all directional dependence of surface properties are illustrated for both diffuse reflection (curves denoted CSD with $\rho_{sp}/\rho_d = 0.0$) and specular reflection (curves denoted CSD with $\rho_{sp}/\rho_d = 1.0$). Results evaluated employing directional property (DP) analysis which accounts for the directional dependence of surface properties but approximates the spatial distribution of reflected energy in terms of a specular component and a diffuse component are also included for the limiting cases of diffuse reflection (curves denoted DSD with $\rho_{sp}/\rho_d = 0.0$) and specular reflection (curves denoted DSD with $\rho_{sp}/\rho_d = 1.0$).

For both included angles, the rough surface heat flux distribution for low emittance ($\epsilon_H = 0.1$) and intermediate emittance ($\epsilon_H = 0.5$) surfaces lies between those evaluated from CP analysis using diffuse and specular reflection models. This is not the case, however, for the high

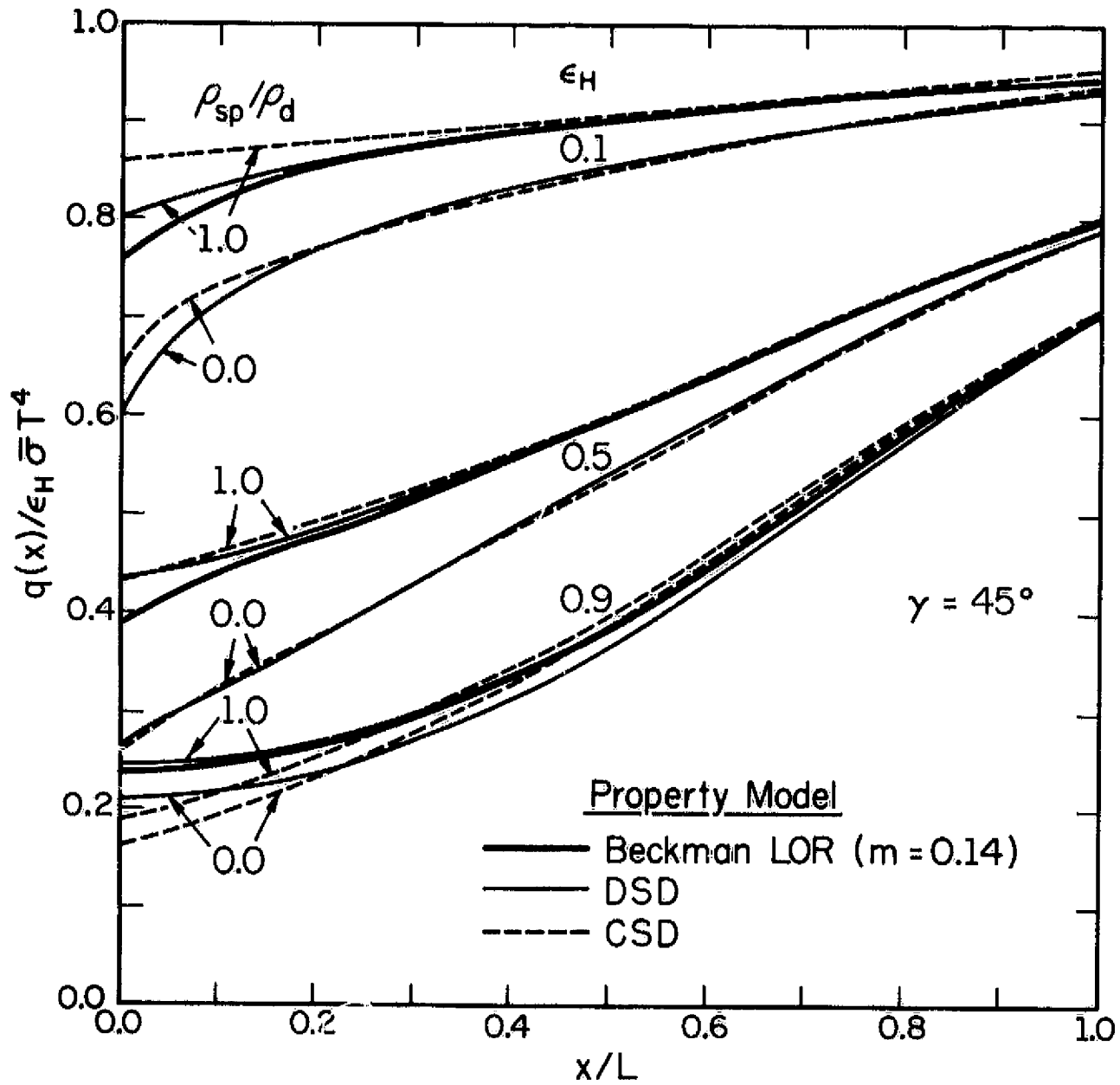


Figure 5.18 Rough surface local radiant heat transfer for large optical roughness ($\gamma = 45^\circ$)

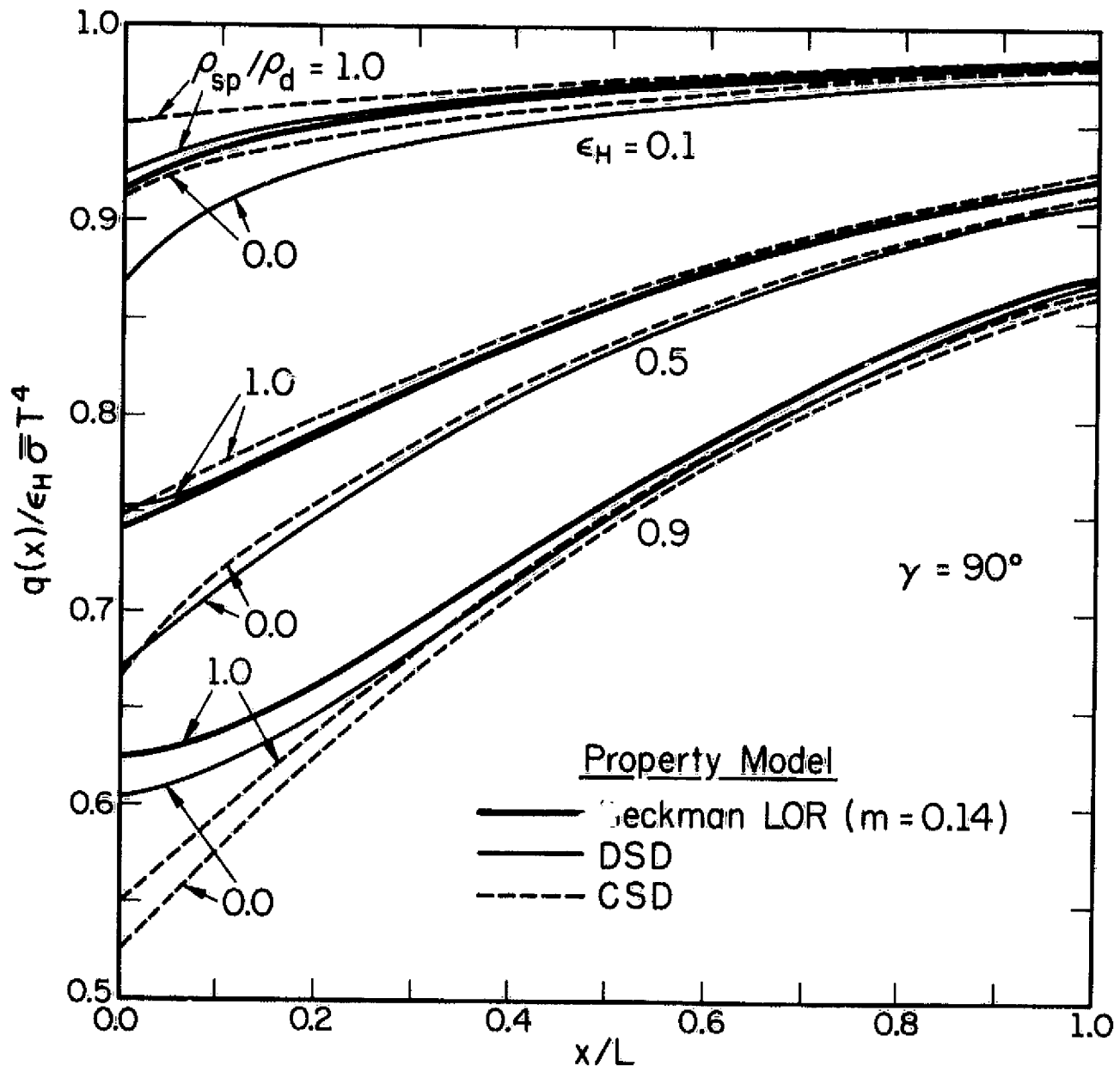


Figure 5.19 Rough surface local radiant heat transfer for large optical roughness ($\gamma = 90^\circ$)

emittance ($\epsilon_H = 0.9$) surface. Near the apex where the largest difference between CP and rough surface results is evident, rough surface flux is greater than that calculated from CP analysis. This clearly contradicts the often stated idea that real surface heat flux is bounded by that calculated using a diffuse and a specular reflection model. Except near the apex of low emittance perpendicular plates, CP specular reflection analysis yields results which are in closer agreement with the rough surface flux distributions than does CP analysis employing a diffuse reflection model. For $\gamma = 45^\circ$, CP specular reflection results deviate from rough surface heat fluxes by a maximum of 13, 12, and 22 percent for emittance values of 0.1, 0.5, and 0.9, respectively. For perpendicular plates, the respective errors are lower by a factor of three to four.

The flux distributions evaluated with DP analysis and a specular reflection model are generally in closest agreement with the rough surface results with the largest discrepancies found at the apex. DP analysis with a specular reflection model yields results for both included angles which are nearly indistinguishable from the rough surface results for high emittance surfaces and within 1 percent for all emittance values when $\gamma = 90^\circ$. This exceptional agreement deteriorates somewhat for the smaller value for included angle and yields flux errors which are no larger than 6 and 13 percent for the low and intermediate emittance surfaces, respectively.

5.4.1.2 Overall Heat Transfer

Dimensionless overall heat transfer rates per unit width obtained with the Beckmann LOR property model for rms

roughness slope (m) equal to 0.14 are tabulated in Table 5.3 for $\gamma = 45^\circ$ and 90° . For both included angles, results are presented for emittance values of 0.1, 0.5, and 0.9. Also included for comparison are results obtained with the simpler CSD and DSD property models for both specular and diffuse reflection. Directional property analysis with a specular reflection model yields overall heat transfer results in excellent agreement with rough surface results. In each case the DP specular reflection results are high, but the maximum error is less than 1.5 percent. Constant property specular reflection analysis also yields excellent results with the maximum error less than 3 percent.

5.4.2 Influence of Surface Roughness on Heat Transfer

5.4.2.1 Local Heat Transfer

The generally excellent agreement of local heat flux distribution evaluated using DP specular reflection analysis with the results calculated using the large optical roughness Beckmann BDR model for $m = 0.14$ is particularly significant. Recall that the aforementioned rough surface property model corresponds to the maximum degree of scattering of reflected energy for which the Beckmann model satisfies the conservation of energy requirement. For optical roughness values which may not be considered large and for roughness slope values equal to or less than $m = 0.14$, the spatial distribution of reflected energy more closely approximates that peculiar to a specularly reflecting surface. Hence, rough surface flux distributions obtained using finite values for optical roughness and $m \leq 0.14$ should lie intermediate to those obtained using the large optical roughness Beckmann BDR model and those evaluated with the directional specular property model. It

γ	ϵ_H	CSD		DSD		Beckmann LOR
		$\rho_{sp}/\rho_d = 1.0$	$\rho_{sp}/\rho_d = 0.0$	$\rho_{sp}/\rho_d = 1.0$	$\rho_{sp}/\rho_d = 0.0$	m=0.14
45°	0.1	0.0906	0.0838	0.0895	0.0835	0.0858
	0.5	0.304	0.267	0.304	0.268	0.300
	0.9	0.376	0.364	0.378	0.359	0.376
90°	0.1	0.0971	0.0958	0.0965	0.0947	0.0963
	0.5	0.427	0.412	0.425	0.410	0.424
	0.9	0.663	0.655	0.677	0.668	0.677

TABLE 5.3

Overall heat transfer $(Q/L)/\bar{\sigma}T^4$ from various property models

follows that, for finite optical roughness and $m \leq 0.14$, rough surface flux distributions will be in even closer agreement with those calculated using DP specular reflection analysis.

Dimensionless radiant heat flux distributions calculated with the LOR BDR model and $m = 0.094$ were found almost indistinguishable from those presented earlier in Figs. 5.18 and 5.19 for $m = 0.14$. To illustrate the influence of decreasing rms roughness slope, Fig. 5.20 was constructed utilizing a greatly expanded ordinate scale. There is shown the local flux distributions evaluated using the LOR Beckmann model with $m = 0.14$ and $m = 0.094$ as well as that calculated with the DSD model for $\gamma = 90^\circ$ and $\epsilon_H = 0.1$. As expected, local rough surface heat flux approaches the results of DP specular reflection analysis as rms surface roughness slope diminishes. Local heat transfer results are presented for $\gamma = 45^\circ$ in Figs. 5.21 and 5.22 for emittance values of 0.1 and 0.5, respectively, and for $\gamma = 90^\circ$ in Fig. 5.23 for a low emittance surface. The flux distributions calculated using a number of different surface property models are included in each figure. Results evaluated with the general Beckmann BDR model are shown for values of the characteristic optical roughness (σ/λ_c) and rms roughness slope (m) of 0.1 and 0.14, respectively. Also illustrated are the results evaluated using the large optical roughness Beckmann BDR expression employing an identical value of roughness slope as that used with the more general Beckmann BDR model. The distributions obtained using the less detailed DSD and CSD property models are included for comparison purposes as are those obtained with the approximate rough surface BDS model with $\sigma/\lambda = 0.1$. The flux distributions evaluated employing the general Beckmann BDR property model confirm the arguments cited earlier. Local flux results for $\sigma/\lambda_c = 0.1$

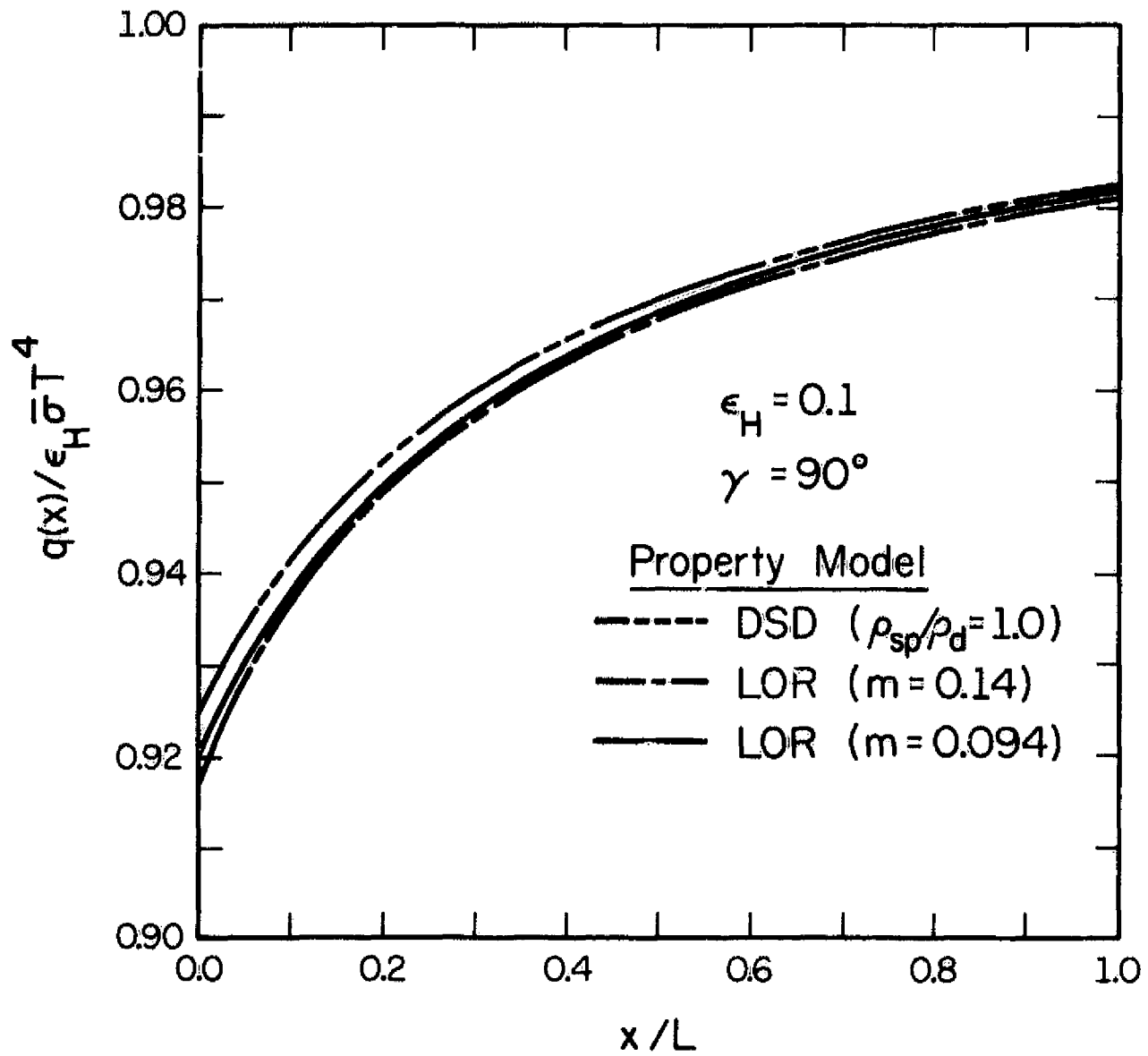


Figure 5.20 Influence of roughness slope on local radiant heat transfer for large optical roughness ($\gamma = 90^\circ$, $\epsilon_H = 0.1$)

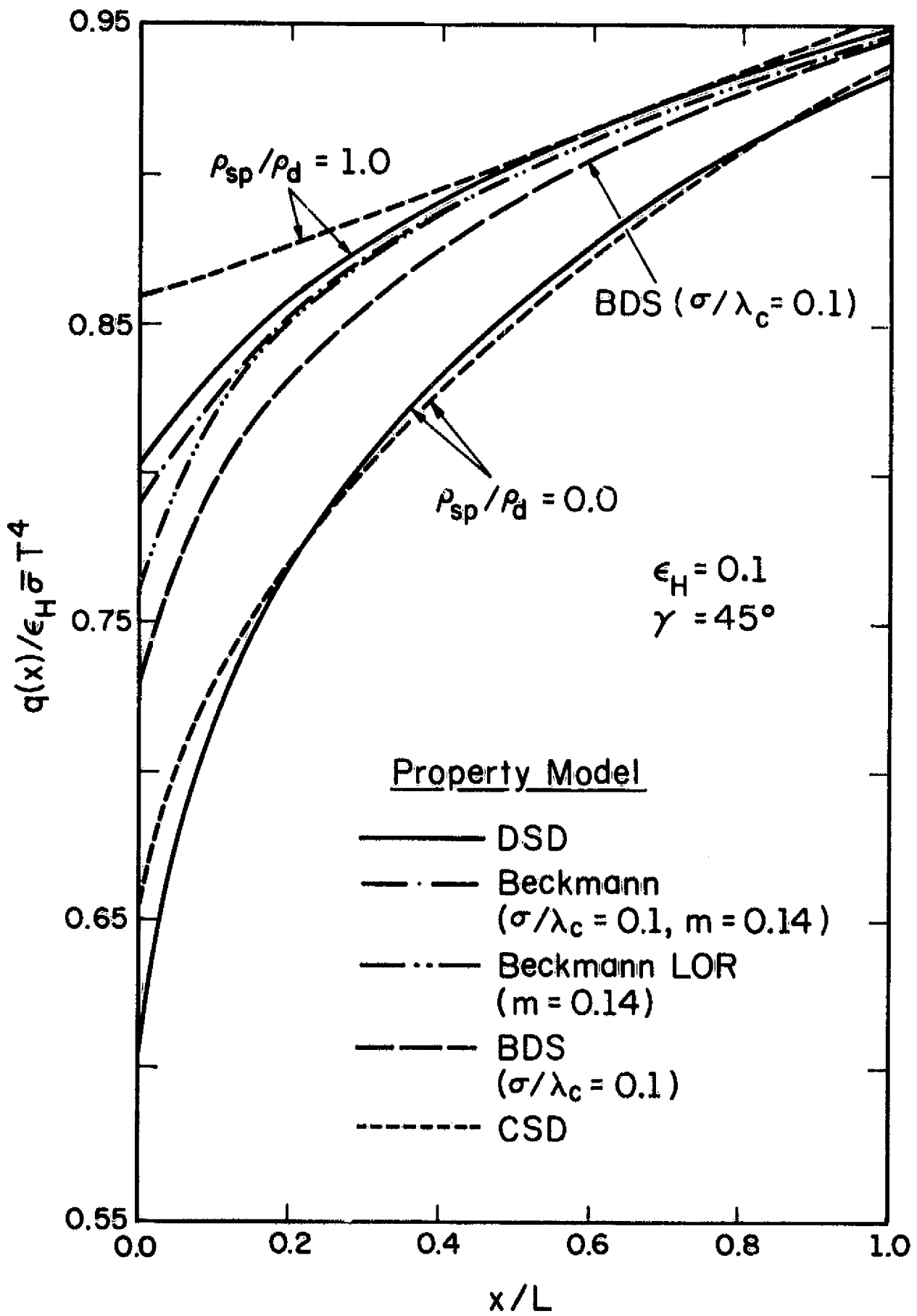


Figure 5.21 Rough surface local radiant heat transfer ($\gamma = 45^\circ$, $\epsilon_H = 0.1$)

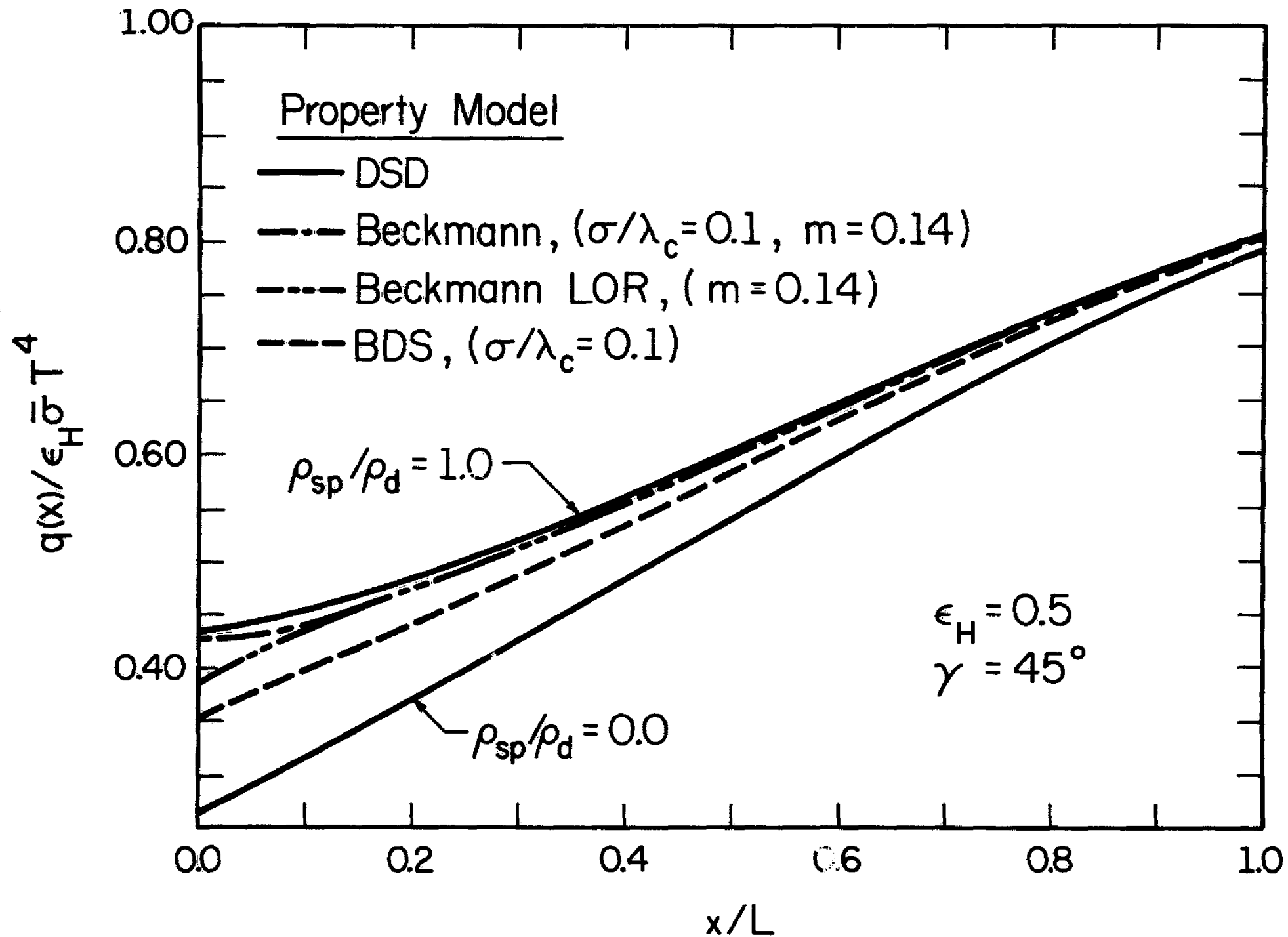


Figure 5.22 Rough surface local radiant heat transfer ($\gamma = 45^\circ$, $\epsilon_H = 0.5$)

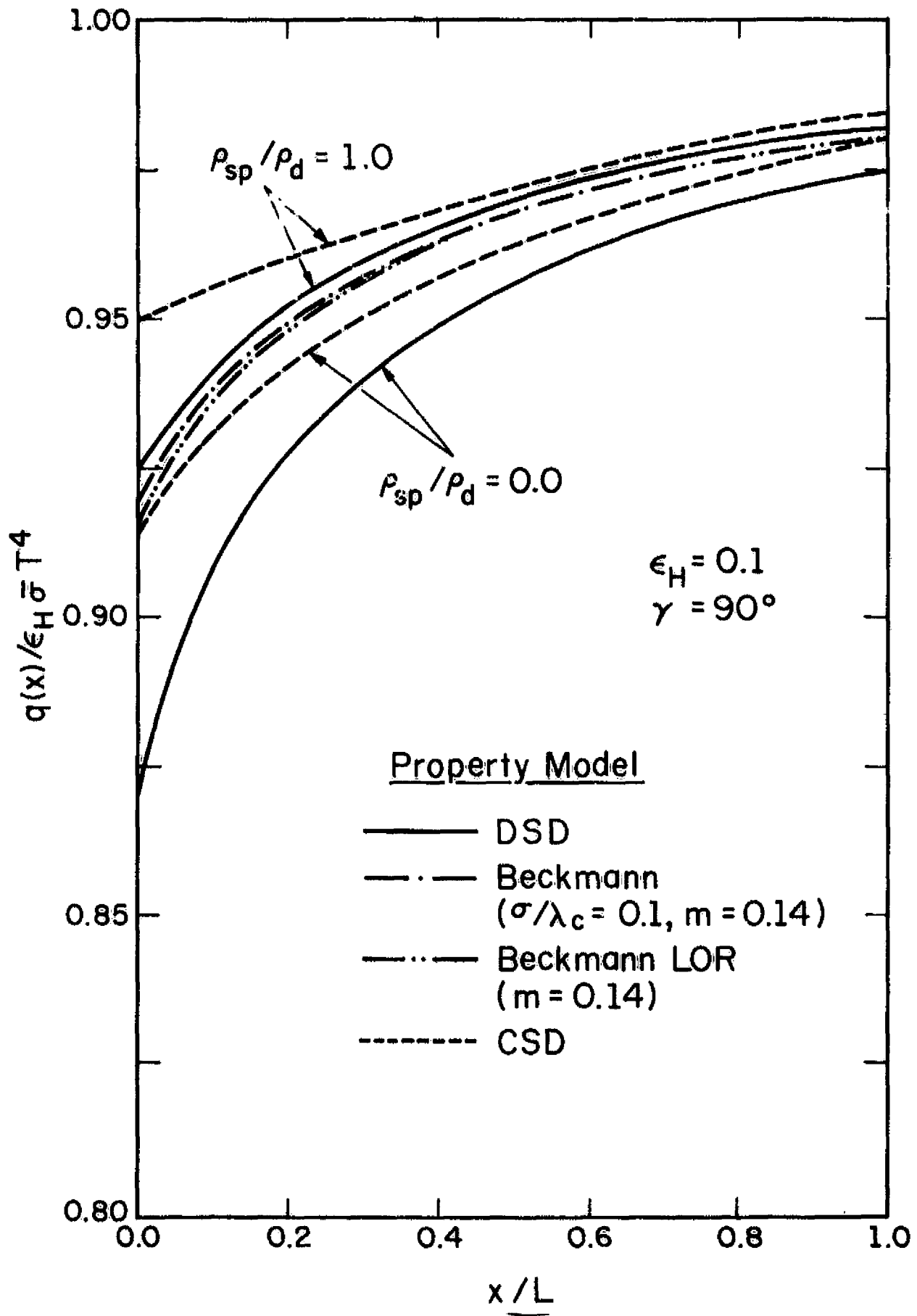


Figure 5.23 Rough surface local radiant heat transfer ($\gamma = 90^\circ, \epsilon_H = 0.1$)

and $m = 0.14$ lie between those calculated with the LOR Beckmann model for an identical value of roughness slope and those calculated with the DP specular reflection model and are in even closer agreement with those evaluated with DP specular reflection analysis.

Comparison of the radiant flux distributions calculated with the Beckmann model and those evaluated with the BDS reflectance model using an identical optical roughness value provides insight into the influence of the spatial distribution of reflected energy on local heat flux. Recall that the only difference between these surface property models is that the BDS model assumes scattered energy is diffusely distributed rather than spatially distributed according to the complex expression for the Beckmann model scattered energy component. It is evident from a comparison of the flux distributions presented in Figs. 5.21 and 5.22 that the error introduced by using a diffuse distribution of reflected energy is significant and local flux errors of approximately 8 and 18 percent for emittance values 0.1 and 0.5, respectively, are introduced. Although these discrepancies are not small, they are considerably less than those incurred when surface roughness is completely disregarded and directional property analysis employing a diffuse reflection model is used. Thus the complex calculations required to fully account for roughness effects may be avoided and the rough surface flux distribution bracketed by the results of directional property analysis with a specular reflection model and the results of BDS analysis employing the optical roughness of the rough surface. This procedure significantly reduces the region of uncertainty for the rough surface local flux from that which would exist if directional property analyses with a specular and a diffuse reflection model were employed to bound rough surface

flux. As the optical roughness of the surface increases, however, the worth of this procedure diminishes since, as shown earlier, the results of BDS analysis for $\sigma/\lambda_c = 1.0$ are identical to those evaluated with DSD analysis using a diffuse reflection model.

5.4.2.2 Overall Heat Transfer

Dimensionless overall heat transfer rates per unit width obtained with the Beckmann BDR model for the roughness parameter values discussed in Section 5.4.2.1 and results obtained with several simpler property models are summarized in Table 5.4. The results omitted from the table were not calculated. It is evident that for large optical roughness overall heat flux approaches the results of directional property specular reflection analysis as rms slope diminishes. Furthermore, an identical trend occurs for fixed rms roughness slope as optical roughness diminishes. As discussed in the previous section, the BDS results for the rough surface optical roughness may be used to reduce the uncertainty of rough surface overall heat transfer from that which would exist if directional property analyses with a specular and a diffuse reflection model were employed to bound rough surface heat flux. For example, the Beckmann results for $\sigma/\lambda_c = 0.1$, $m = 0.14$, and $\gamma = 45^\circ$ are bounded by the directional property specular reflection results and BDS model results for $\sigma/\lambda_c = 0.1$ with uncertainty of 2 and 4 percent for emittance values 0.1 and 0.5, respectively. However, the corresponding uncertainties of rough surface results when bounded by directional property specular and diffuse reflection results are 7 and 12 percent, respectively.

γ	ϵ_H	Property Model								
		CSD $\rho_{sp}/\rho_d=1.0$	DSD $\rho_{sp}/\rho_d=1.0$	Beckmann			BDS $\sigma/\lambda_c=0.1$	CSD $\rho_{sp}/\rho_d=0.0$	DSD $\rho_{sp}/\rho_d=0.0$	
				SOR	BDR	LOR				BDR
				$\sigma/\lambda_c=0.1$ $m=0.14$	$m=0.094$	$m=0.14$				
45°	0.1	0.0906	0.0895	0.0890	0.0888	0.0888	0.0878	0.0838	0.0835	
	0.5	0.304	0.304	0.300	0.300	0.300	0.292	0.267	0.268	
	0.9	0.376	0.378	—	0.377	0.376	0.374	0.364	0.359	
90°	0.1	0.0971	0.0965	0.0963	0.0964	0.0963	—	0.0958	0.0947	
	0.5	0.427	0.425	—	0.425	0.424	—	0.412	0.410	
	0.9	0.663	0.677	—	0.677	0.677	—	0.655	0.668	

TABLE 5.4

Rough surface overall heat transfer $(Q/L)/\bar{\sigma}T^4$
from various property models

5.4.3 A Thermal Design Consideration

In most engineering applications, it is impractical to incorporate the detailed rough surface radiation properties, particularly bidirectional reflectance, into design calculations. For such situations, it would be useful to have available an easily determined measure of surface specularity to enable design engineers to choose between a specular and a diffuse reflection model for the representation of surface properties in design calculations. One such measure is briefly discussed here.

An experimental technique which has been suggested for obtaining a measure of the specularity of a surface may be summarized as follows: Illuminate a surface at polar angle θ' with a beam of radiant energy of known solid angle, $\Delta\omega_i$, and measure both the energy reflected into an equal size solid angle, $\Delta\omega_r$, about the specular direction and the energy reflected throughout entire hemispherical space. The ratio of the energy reflected into the specular solid angle to that reflected throughout the hemisphere is denoted by the symbol $R_{sp}(\theta', \Delta\omega_i)$ and is represented in terms of bidirectional reflectance by the following expression

$$R_{sp}(\theta', \Delta\omega_i) = \frac{1}{\rho_d(\theta')} \int_{\Delta\omega_r = \Delta\omega_i} \rho_{bd}(\theta', \phi'; \theta, \phi) \cos \theta d\omega_r$$

For surfaces which are specular reflectors R_{sp} is equal to unity. For other surfaces, R_{sp} is less than unity and depends on direction and solid angle of illumination. As an illustration, consider illumination of a sample at normal incidence with a beam solid angle of 0.003 steradians. If the sample were a diffusely reflecting surface, the value for R_{sp} is approximately 0.0009. If the sample BDR was well represented by the LOR Beckmann model, the value for R_{sp} is about 0.006. Although the R_{sp}

value for the rough surface is more than a factor of 6 larger than that for a diffusely reflecting surface, it is indeed small in comparison to that for a specularly reflecting surface. One could infer from this measurement then that a diffuse reflection model is more appropriate for analysis than a specular reflection model. However, in view of the generally excellent agreement of the rough surface heat transfer results with those employing a specular reflection model, the above inference from the cited measurement procedure is obviously incorrect. The quantity R_{sp} apparently is not a sufficient measure of surface specularity. A more detailed measure of the spatial distribution of reflected energy is needed and the determination of such a measure merits further study.

5.4.4 Conclusions

Rough surface heat transfer calculations for the adjoint plate system were confined to values of the rough surface parameters, namely, optical roughness (σ/λ_c) and roughness slope (m), for which the Beckmann BDR satisfied the conservation of energy requirement. The limitation restricted surface roughness slope to values equal to and less than 0.14. The following conclusions may be drawn from the results.

- (1) Local radiant flux evaluated with the LOR Beckmann model for the maximum degree of scattering of incident energy was only slightly less than that calculated with a specular reflection model which accounted for the directional dependence of surface properties. Excellent agreement was observed between the local flux calculated using the aforementioned property models except at the apex for the smaller included angle and surfaces of intermediate and low emittance where the maximum differences were 13 and 6 percent, respectively. Significantly poorer agreement is found between rough surface radiant flux and that evaluated with a specular reflection model which ignores the directional dependence of surface properties. The error, however, is limited to less than 25 percent.

- (2) As roughness slope decreases, local radiant flux evaluated with the LOR Beckmann model increases with the flux values confined to the small region between those calculated with $m = 0.14$ and those calculated for direction dependent surface properties employing a specular reflection model. Thus, as roughness slope decreases, the discrepancy between rough surface local flux and that from DP analysis with a specular reflection model diminishes. The same trend is observed when optical roughness is reduced while other parameters are fixed.
- (3) The difficult calculations associated with the use of the Beckmann scattered energy expression may be avoided and the low and high flux limit for the rough surface flux bracketed by the values calculated using the BDS property model with rough surface optical roughness and the DSD model with specular reflection, respectively. Even the BDS model calculations may be avoided by employing DSD analysis with the specularly parameter evaluated from surface optical roughness and the ρ_{sp}/ρ_d correlation with σ/λ_c presented earlier. Note, however, that the region of uncertainty for the rough surface flux increases as optical roughness increases.
- (4) The shortcomings of a proposed technique for estimating specularity of a surface have been illustrated.
- (5) Overall heat transfer obtained for the LOR Beckmann property model with $m = 0.14$ was only slightly less than that obtained with a directional property specular reflection model. The maximum discrepancy was less than 1.5 percent. Furthermore, constant property specular reflection analysis gave results within 3 percent of the rough surface values.

5.5 COATED SURFACE PROPERTY MODEL RESULTS

In this section, dimensionless local and overall radiant heat transfer results are presented for the adjoint plate system of included angle $\gamma = 45^\circ$ with surface properties represented by the coated surface BDR model. The major reason that this study was undertaken in an investigation almost entirely devoted to surface roughness effects on radiant heat transfer, was to provide further insight into the influence of the spatial distribution of scattered energy on local radiant flux. Recall that the greatest difference between the Beckmann rough surface BDR model and the coated surface BDR model lies in the spatial distribution

of scattered energy. According to the Beckmann BDR model, scattered energy is concentrated into directions not far removed from the specular direction. The coated surface BDR model on the other hand, possesses a distribution of scattered energy which is more representative of a diffusely reflecting surface. Before passing on to these results, however, heat transfer for the coated surface is compared to that evaluated using analysis in which either the coating or the substrate properties are ignored. For the former situation, surface properties are taken as those of the substrate. All properties are wavelength and direction independent with reflection diffuse and hemispherical emittance equal to 0.25. For the latter situation, surface properties are taken as those of an infinitely thick slab of coating material with an optically smooth surface. Angular dependent properties were calculated from the Fresnel relations for wavelength independent optical indices n and k equal to 1.5565 and 0.0, respectively. Reflection is specular with hemispherical emittance equal to 0.9. Finally, the coated surface heat flux distribution is compared to that predicted by several less detailed surface property models which employ the apparent hemispherical emittance of the coated surface.

5.5.1 Coating and Substrate Property Effects on Heat Transfer

5.5.1.1 Local Heat Transfer

Dimensionless radiant heat flux distributions are presented in Fig. 5.24 for the coated surface as well as for the two limiting situations of negligible coating thickness in which substrate properties are employed and of large coating thickness in which coating properties are used. Note that the dimensionless flux used in the

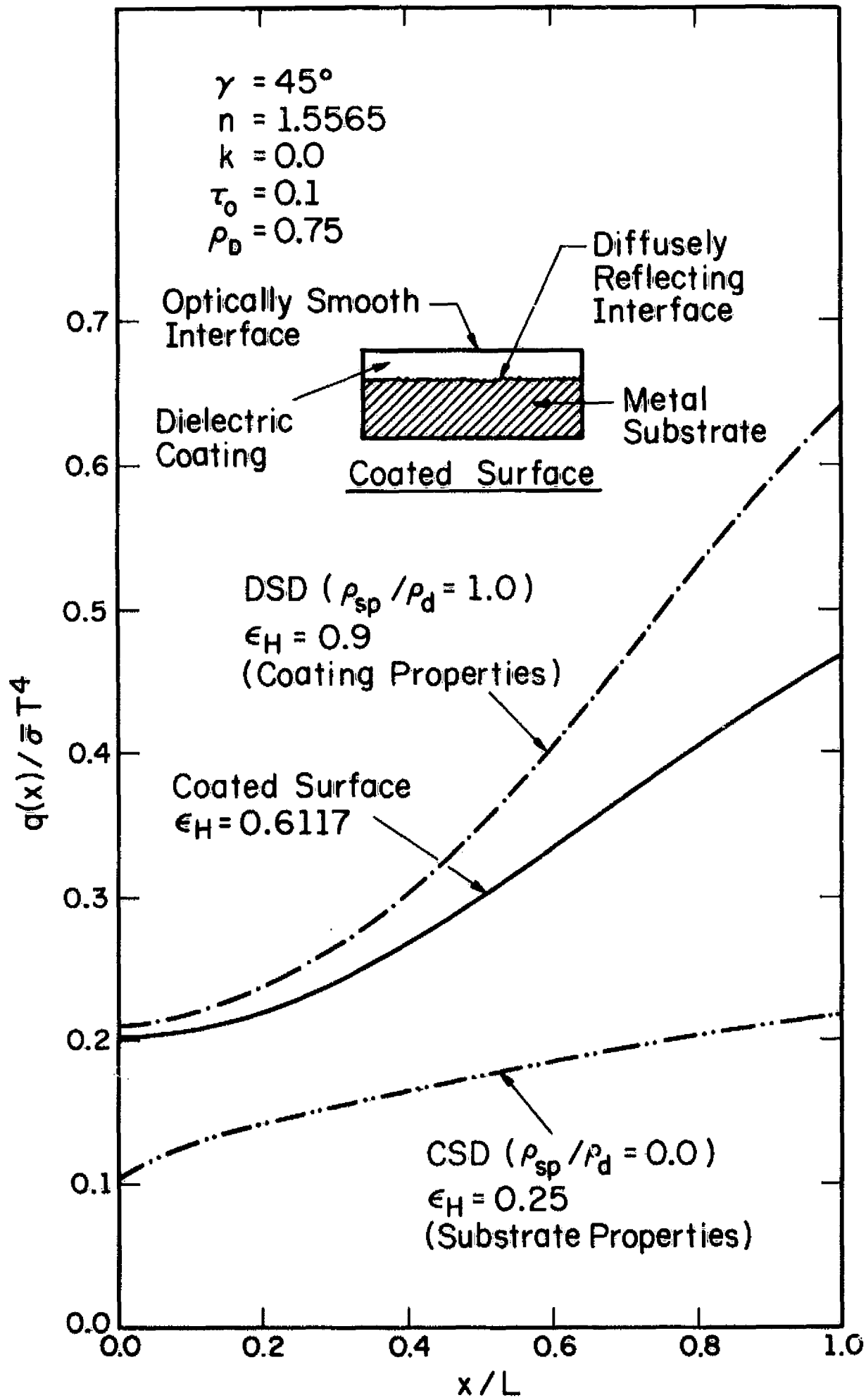


Figure 5.24 Local radiant heat transfer for coated surface ($\gamma = 45^\circ$)

presentation of these results differs from that employed earlier by the absence of the hemispherical emittance factor.

Ignoring either the coating or the substrate effects on surface properties introduces large errors in local heat transfer. When the coating is neglected and substrate properties are used, local flux along the plate is almost uniformly low by a factor of two. When the substrate is ignored and coating properties employed, the local flux error increases from a value 5 percent high at the apex to almost 40 percent too large at the outer edge. These large errors in local flux are primarily attributed to the marked differences in hemispherical emittance values for the coated surface, the coating material, and the substrate. For all other conditions fixed, local heat transfer increases with increasing hemispherical emittance with the largest increase in the adjoint plate system occurring at the outer plate edge. It is evident that at the very least, heat transfer calculations must employ apparent hemispherical emittance of the composite surface.

5.5.1.2 Overall Heat Transfer

Dimensionless overall heat transfer rates per unit width for the coated surface, substrate, and coating material properties are included in Table 5.5. Overall heat transfer obtained using the coating material properties is 21 percent high while that obtained using the substrate properties is 45 percent low. It is evident that ignoring either the coating or substrate effects on surface properties yields unacceptable errors in overall heat transfer.

Property Model			
Coating Prop. DSD $\rho_{sp}/\rho_d = 1.0$ $\epsilon_H = 0.9$	Substrate Prop. CSD $\rho_{sp}/\rho_d = 0.0$ $\epsilon_H = 0.25$	Coated Surface $\epsilon_H = 0.6117$	CSDS $\epsilon_H = 0.6117$
0.378	0.171	0.311	0.312

Property Model			
DSD		CSD	
$\rho_{sp}/\rho_d = 1.0$ $\epsilon_H = 0.6117$	$\rho_{sp}/\rho_d = 0.0$ $\epsilon_H = 0.6117$	$\rho_{sp}/\rho_d = 1.0$ $\epsilon_H = 0.6117$	$\rho_{sp}/\rho_d = 0.0$ $\epsilon_H = 0.6117$
0.338	0.297	0.334	0.300

TABLE 5.5
Overall heat transfer $(Q/L)/\bar{\sigma}T^4$ for coated
surface $\gamma = 45^\circ$ ($\gamma = 45^\circ$)

5.5.2 Directional Property Effects

5.5.2.1 Local Heat Transfer

The importance of directional dependence of surface properties on local radiant flux is evaluated by comparing the heat transfer distribution obtained using the detailed coated surface BDR model to those evaluated with property models of less detail. Dimensionless heat flux results calculated using the coated surface BDR model are shown in Fig. 5.25. Also illustrated are the distributions evaluated with constant property (CP) analysis (curves labeled CSD) which neglects all directional dependence of surface properties as well as those obtained using directional property (DP) analysis (curves labeled DSD). The latter analysis employed the directional properties of the coated surface (see Figs. 4.5 and 4.7). Both a specular and a diffuse reflection model were considered for CP and DP analysis and the results are differentiated by the value for the specularity parameter--zero for a diffusely reflecting surface model ($\rho_{sp}/\rho_d = 0.0$), and unity for a specularly reflecting surface model ($\rho_{sp}/\rho_d = 1.0$). Also shown in Fig. 5.25 is the flux distribution obtained using the CSDS model. This model is identical to the detailed coated surface BDR model with the exception that the spatial distribution of reflected energy which is not specularly reflected is diffusely distributed. All analyses employed the apparent hemispherical emittance of the coated surface, that is, $\epsilon_H = 0.6117$. The importance of an accurate representation of the spatial distribution of reflected energy for local flux calculations is clearly established by the heat flux differences at the apex. The results of both CP and DP analyses demonstrate that local flux evaluated with a specular reflection model

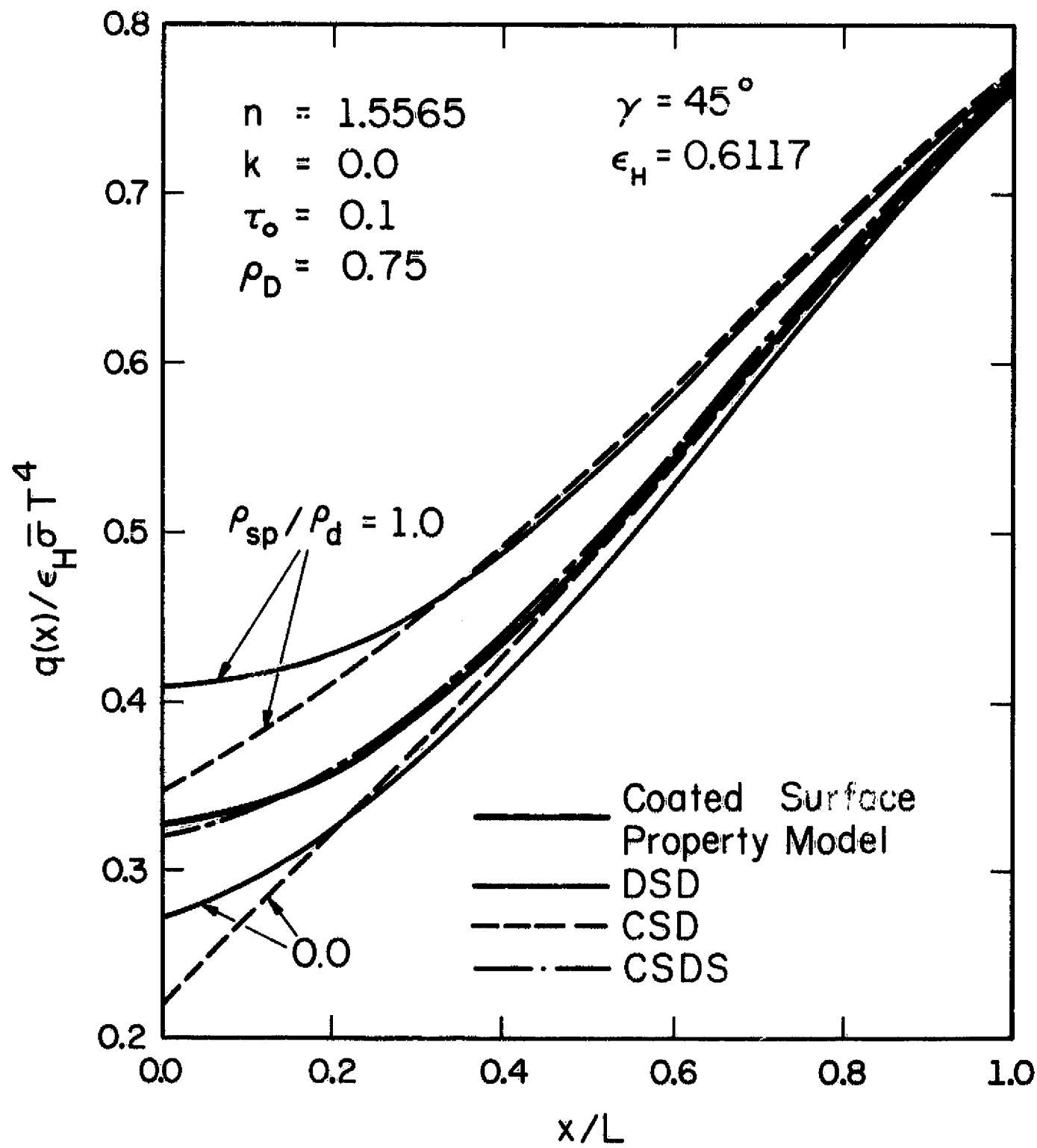


Figure 5.25 Comparison of heat flux distributions for coated surface

is larger than that calculated with a diffuse reflection model by a factor of 1.5. Furthermore, analysis which neglects the directional dependence of surface properties yields flux values at the apex which are about 20 percent less than those evaluated with directional property analysis for both the specular and the diffuse reflection analysis. As expected, the influence of the spatial distribution of reflected energy and of the directional dependence of properties on local flux is less near the outer plate edge where, because interreflection phenomena are less important, local flux evaluated with different property models differ by less than 5 percent.

The flux distribution evaluated using the detailed coated surface BDR model is intermediate to that calculated employing either CP or DP analysis with a specular and a diffuse reflection model. However, analysis using a diffuse reflection model clearly gives a flux distribution in closer agreement with the coated surface results than that employing a specular reflection model. The largest flux errors are evident at the apex where DP analysis with a diffuse reflection model is 15 percent low and CP analysis with the same reflection model is 30 percent low. At x/L values greater than 0.2 heat transfer from CP analysis is in closer agreement with the coated surface results although the error associated with DP analysis is less than 10 percent and diminishes with distance from the apex.

The flux distribution evaluated with the CSDS model is in excellent agreement with the coated surface results. The CSDS model results can be distinguished from the coated surface flux near the apex. Thus, local heat transfer is predicted accurately by a reflection model which approximates the spatial distribution of scattered energy for the coated

surface with the simpler diffuse distribution.

5.5.2.2 Overall Heat Transfer

Dimensionless overall heat transfer rates per unit width are presented in Table 5.5 for the coated surface and several simpler property models. Results of both CP and DP analyses yield overall heat transfer approximately 12 percent larger for specular reflection than for diffuse reflection. For both specular and diffuse reflection, the results of CP and DP analyses are in excellent agreement. The maximum error is less than 2 percent. Both DP and CP diffuse reflection analyses yield overall heat flux in good agreement with the coated surface result. The corresponding errors are 5 and 4 percent, respectively. As expected from the discussion of the previous section, the CSDS model yields an overall heat flux value in excellent agreement with the coated surface result.

5.5.3 Conclusions

The heat transfer results for the adjoint plate system of equal temperature dielectric coated surfaces demonstrate that it is important to account for the influence of the coating and substrate properties on the apparent emittance of the composite surface. An accurate representation of the directional dependence of the radiation properties, however, is just as important for the calculation of local flux at surface elements which experience large radiant interaction with other surfaces. Local flux values may be changed by a factor of two by each of the aforementioned influences. For the system studied, constant property analysis gave flux values at the apex which were lower by 20 percent than

those evaluated with directional property analysis. Of the flux distributions evaluated with the simpler property models (CP and DP analysis with specular or diffuse reflection), that calculated employing a diffuse reflection model is in closer agreement with the coated surface BDR model results. Excellent agreement with the coated surface results was observed when the approximate coated surface model was employed to evaluate heat transfer. The approximate model replaces the detailed distribution of scattered energy of the coated surface BDR model with a diffuse distribution.

Overall heat flux is less sensitive to the directional dependence of surface properties than is local heat flux. Diffuse reflection analysis yields results closer to the coated surface results than does specular reflection analysis. Finally, the CSDS model provides an excellent approximation to the detailed coated surface model for purposes of determining overall heat transfer.

5.6 SUMMARY

The local and overall radiant heat transfer results for equal temperature identical adjoint plates in the absence of an external radiation source suggest the following general conclusions. First, ignoring the directional dependence of emittance, absorptance, and reflectance can result in significant errors in heat flux predictions for low and high emittance surfaces in regions of large radiant interaction and for elements which receive major contributions to their irradiation at near-grazing incidence. Such errors are essentially independent of whether the surface is specularly or diffusely reflecting. On the other hand, the aforementioned directional dependence of properties is relatively

unimportant in overall heat transfer calculations. Second, directional property analysis with a reflection model consisting of a specular and diffuse reflectance component can provide excellent heat flux predictions for surfaces with more complicated reflectance characteristics provided the apportionment of reflected energy between the reflectance components is carefully selected. Such an apportionment may be based on optical roughness when surface roughness is the dominant factor influencing the spatial distribution of reflected energy. Third, radiant heat transfer between rough surfaces for which diffraction and interference effects are the primary cause for the scattering of reflected energy into directions other than the specular direction differs only slightly from that predicted with directional property analysis employing a specular reflection model. Fourth, the radiant heat transfer between rough surfaces with spatial distributions of reflected energy described by the Beckmann BDR model lies within a generally small region bounded by the results of directional property specular reflection analysis and the BDS model analysis using rough surface optical roughness. Finally, when the spatial distribution of scattered energy possesses a distinctly diffuse character such as for the coated surface studied here, excellent heat transfer predictions are obtained using a simpler directional property model which accurately accounts for the directional dependence of the specularly reflected energy and assumes the remaining energy to be diffusely distributed.

6. EQUILIBRIUM TEMPERATURE RESULTS AND DISCUSSION

6.1 INTRODUCTION

In this chapter equilibrium temperature distributions are presented for perpendicular adjoint plates illuminated by a uniform collimated solar field directed along the bisector of the included angle. Direction dependent, semi-gray surface property models are employed in the computations. It was shown in Chapter 5 that many engineering surfaces can be considered specular reflectors for long wavelength thermal radiation without introducing appreciable error into radiant heat transfer results. For short wavelength solar radiation, engineering surfaces are not as well represented by the specular reflection model. Equilibrium temperature results are presented in this chapter for surfaces which are specularly reflecting to surface emitted thermal radiation. Surface properties for solar radiation are represented by several models including the simple CSD model as well as the more detailed DSD and Beckmann property models.

In Section 6.2 equilibrium temperature distributions are presented for surfaces with properties represented by the CSD and DSD property models. The influence of angular property dependence on equilibrium temperature is assessed for markedly different spatial distributions of reflected solar radiation.

Equilibrium temperature distributions are presented in Section 6.3 for surfaces with solar properties represented by the Beckmann BDR model, and the influence of surface roughness on equilibrium temperature distribution is assessed. Rough surface results are compared to those evaluated with less detailed property models, and the accuracy

attainable with the simpler models is evaluated.

Section 6.4 summarizes conclusions concerning the influence of the directional dependence of properties on equilibrium temperature distribution.

The general dependence of local equilibrium temperature in the adjacent plate system on position, included angle, emittance, solar absorptance, and surface specularity for surface and solar radiation was thoroughly investigated by Hering [2] for direction independent surface property models. These general trends are only briefly summarized here. For uniformly irradiated perpendicular plates with semi-gray, direction independent surface properties, local equilibrium temperature is maximum at the common edge and decreases continuously to the outer edge. With all other parameters fixed, equilibrium temperature increases as solar absorptance becomes larger or emittance takes on smaller values. Surface specularity for solar radiation has a major effect on equilibrium temperature causing it to decrease with decreasing specularity. Discussion of these trends is not repeated in this chapter. Instead, attention is directed to the influence of directional property dependence and surface roughness on equilibrium temperature distribution.

6.2 DIRECTIONAL PROPERTY EFFECTS FOR SIMPLE REFLECTION MODELS

The influence on equilibrium temperature distribution of angle dependent surface properties for surface emitted radiation is investigated in Section 6.2.1. Surface properties for solar radiation are represented by the simple direction independent CSD property model.

To illustrate the effect of direction dependent absorptivity and reflectivity for solar radiation, equilibrium temperature distributions

are presented in Section 6.2.2 for solar properties represented by the DSD property model. The temperature error associated with neglecting directional property dependence is evaluated.

Section 6.2.3 summarizes the conclusions concerning directional property effects on equilibrium temperature.

6.2.1 Direction Dependent Thermal Properties

Dimensionless equilibrium temperature distributions are presented in Figs. 6.1-6.3 for surfaces with hemispherical emittance (ϵ_H) equal to 0.1 and solar absorptance values (α^*) of 0.1, 0.5, and 0.9, respectively. Results for a hemispherical emittance value of 0.9 and solar absorptance values 0.1 and 0.5 are shown in Figs. 6.4 and 6.5, respectively. Surface properties for long wavelength thermal radiation are represented by the DSD model for the limiting case of specular reflection. Surface reflection for solar radiation is represented by the direction independent CSD model (designated by broken lines) and the direction dependent DSD model (designated by solid lines) for specularity ratio values 0.0, 0.5, and 1.0. For purposes of comparison, results of analysis using direction independent thermal and solar properties represented by the CSD model are included and shown as dashed lines.

The effect of direction dependent thermal properties is observed by comparing temperature distributions obtained with DSD (broken lines) and CSD (dashed lines) thermal property models when solar properties are represented by the CSD model. For reasons which were discussed in Section 5.2.1, the influence of direction dependent thermal properties is most pronounced near the common edge, and continuously decreases with distance from the apex. For low emittance ($\epsilon_H = 0.1$), constant property

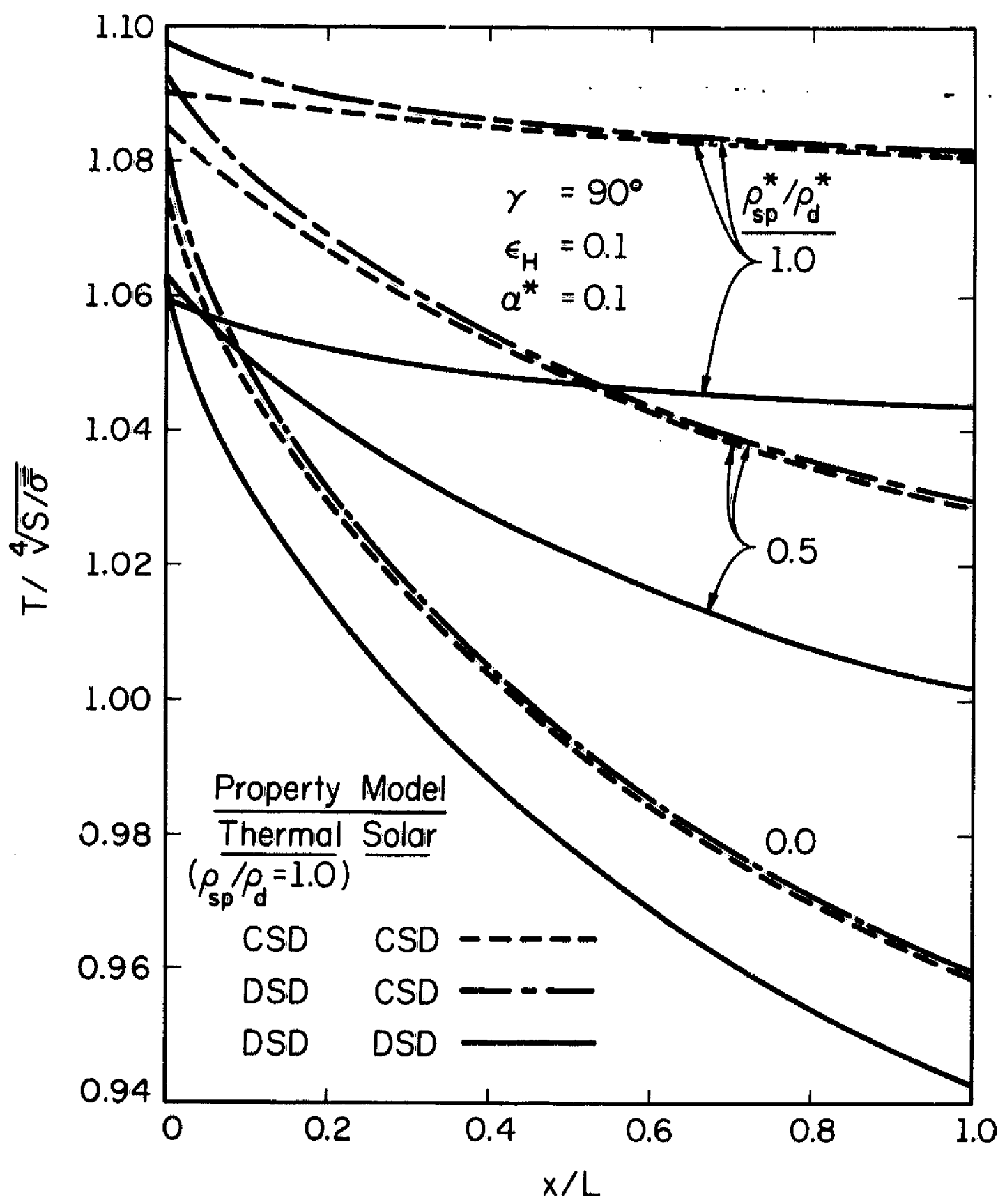


Figure 6.1 Angular dependent property effects on local equilibrium temperature ($\epsilon_H = 0.1$, $\alpha^* = 0.1$)

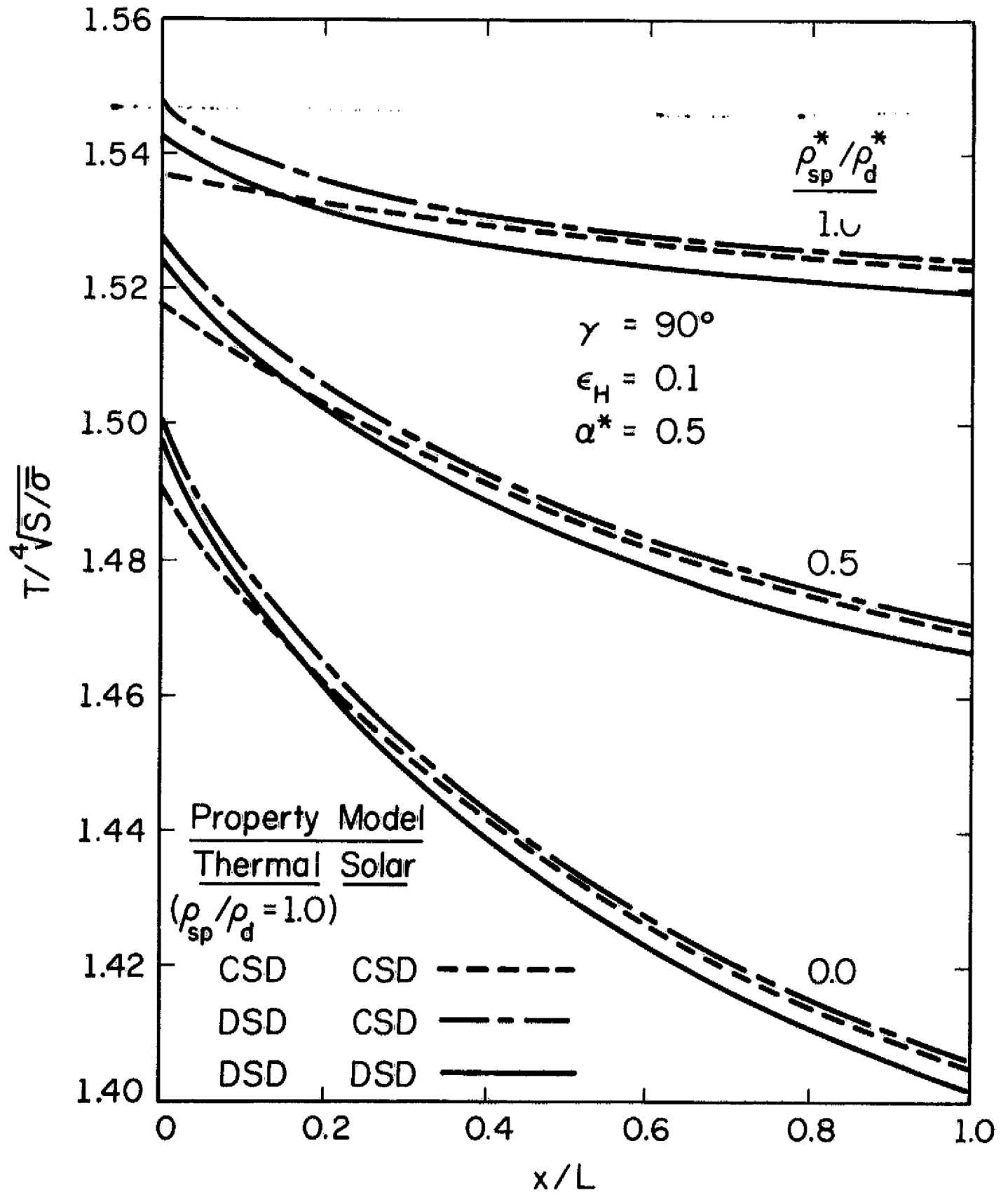


Figure 6.2 Angular dependent property effects on local equilibrium temperature ($\epsilon_H = 0.1$, $\alpha^* = 0.5$)

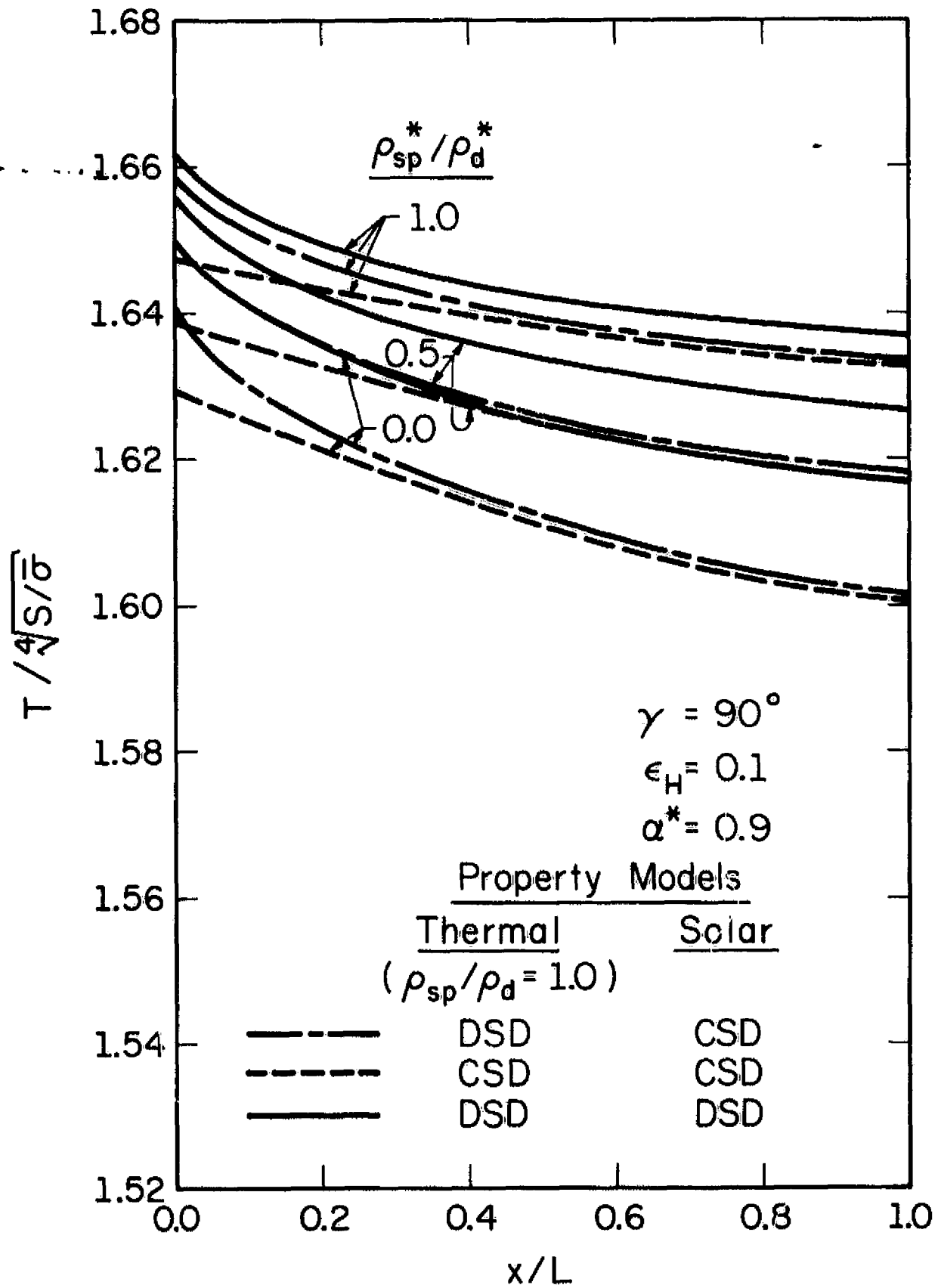


Figure 6.3 Angular dependent property effects on local equilibrium temperature ($\epsilon_H = 0.1$, $\alpha^* = 0.9$)

model results yield lower temperatures than those evaluated with analysis which includes angular dependent thermal properties. However, the discrepancy is less than 1 percent, and it is insensitive to the value for solar absorptance and solar specularity. For the high emittance surface ($\epsilon_H = 0.9$), local temperature is higher from the constant property model than that evaluated with direction dependent thermal properties, but the difference is less than 3 percent. Again, the error is insensitive to the value of solar absorptance and solar specularity.

It is interesting to note that the percentage error in local dimensionless equilibrium temperature associated with ignoring directional dependence of thermal properties is less than that for local radiant heat transfer discussed in Section 5.2.1 for the same emittance value by a factor of 4 to 5.

6.2.2 Direction Dependent Solar Properties

The effect of angle dependent solar properties may be observed by comparing temperature distributions in Figs. 6.1-6.5 for solar properties represented by the direction dependent DSD model (solid lines) and the direction independent CSD model (broken lines). Thermal properties are represented by the DSD model for specular reflection.

The difference between results for direction dependent and independent solar properties is nearly uniform over the entire plate length. For both the low ($\epsilon_H = 0.1$) and the high ($\epsilon_H = 0.9$) emittance surfaces, local temperature is higher from analysis with direction independent solar properties for low ($\alpha^* = 0.1$) and intermediate ($\alpha^* = 0.5$) solar absorptance values. However, for surfaces of high solar absorptance ($\alpha^* = 0.9$), constant solar property analysis yields low temperatures. For

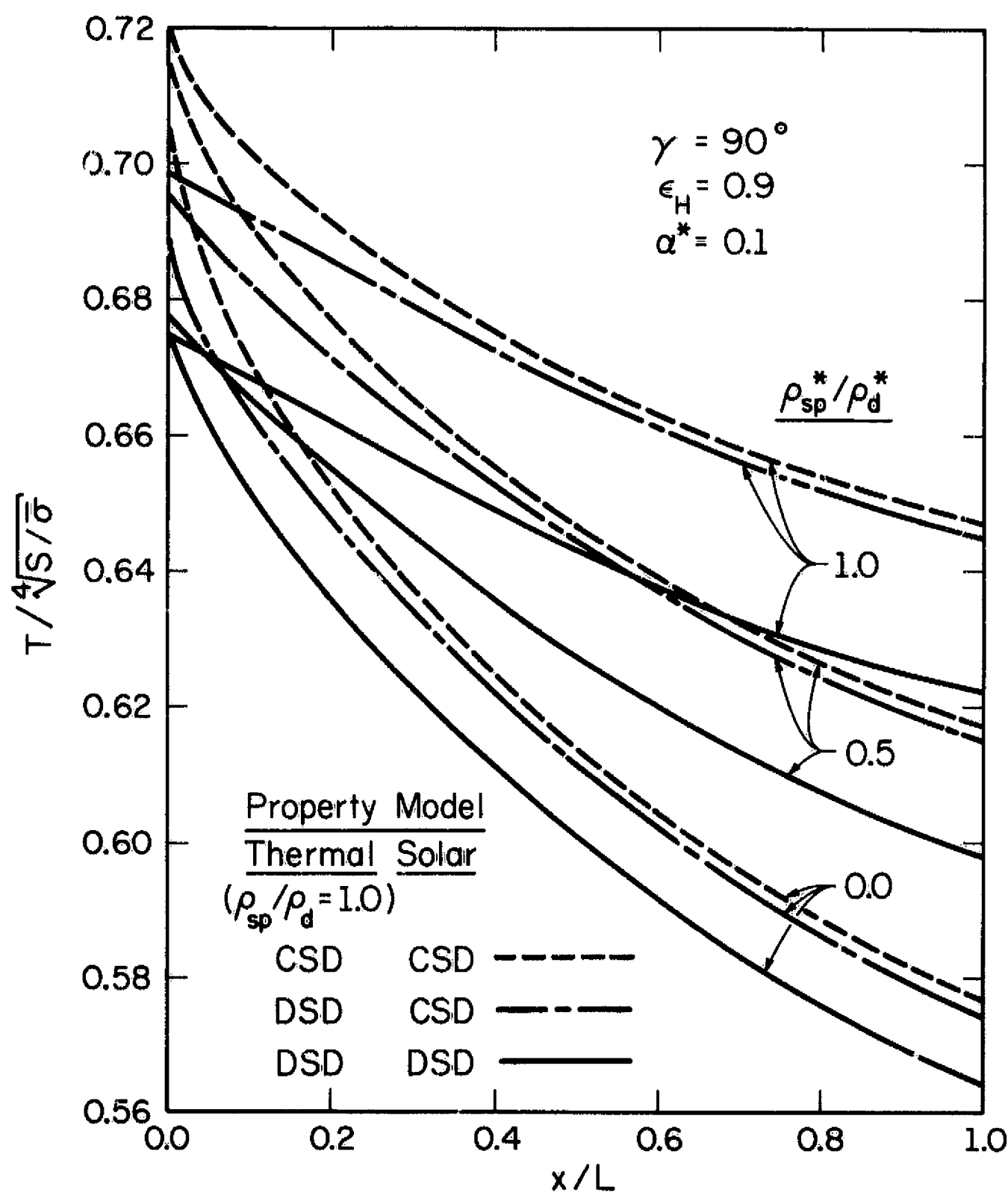


Figure 6.4 Angular dependent property effects on local equilibrium temperature ($\epsilon_H = 0.9$, $\alpha^* = 0.1$)

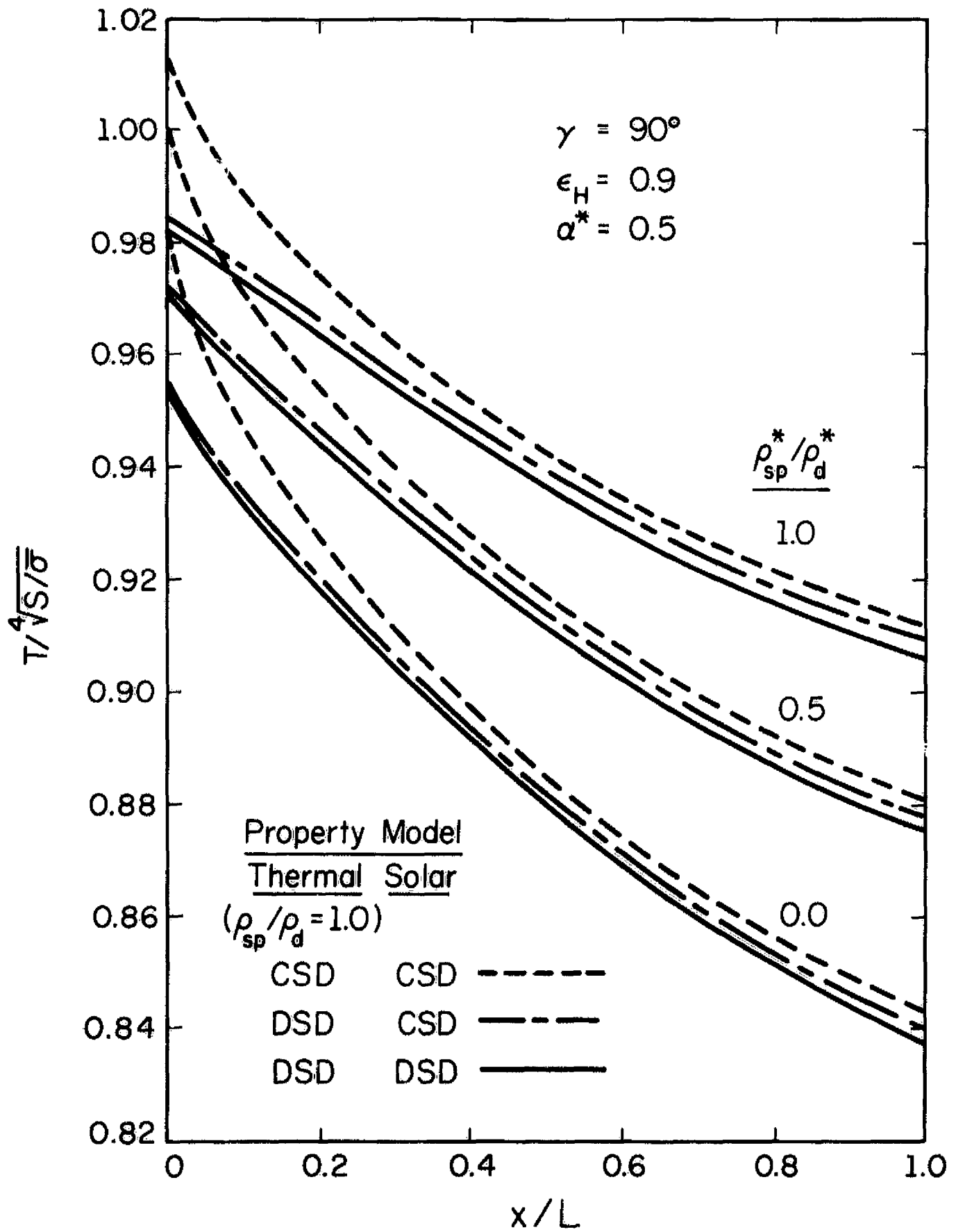


Figure 6.5 Angular dependent property effects on local equilibrium temperature ($\epsilon_H = 0.9$, $\alpha^* = 0.5$)

intermediate and high solar absorptance values, the difference between local temperature values for direction dependent and independent solar property models is less than 1 percent. For low solar absorptance the error is less than 4 percent and decreases as the surface changes from specularly reflecting to diffusely reflecting to solar energy.

For a surface of low emittance ($\epsilon_H = 0.1$) and low to intermediate solar absorptance, the effects of direction dependent thermal and solar properties are partially compensating. Therefore, results obtained with analysis for direction independent thermal and solar properties show slightly better agreement with results for direction dependent thermal and solar properties than do those which account only for angular variation of thermal properties. However, for the high emittance surface, results from analysis employing constant thermal and solar properties are too high by 7 and 3 percent for $\alpha^* = 0.1$ and 0.5, respectively.

It is important to note the uncertainty in local equilibrium temperature which results from inadequate knowledge of the spatial distribution of reflected solar radiation. Consider the results for direction dependent thermal and solar properties. The difference between results for specular ($\rho_{sp}^* / \rho_d^* = 1.0$) and diffuse ($\rho_{sp}^* / \rho_d^* = 0.0$) reflection of solar radiation is largest for a surface of low solar absorptance ($\alpha^* = 0.1$) and decreases continuously with increasing values for α^* . The uncertainty is 10, 8, and 1 percent for $\alpha^* = 0.1$, 0.5, and 0.9, respectively. These values are nearly the same as those reported earlier [2] for direction independent surface properties.

6.2.3 Conclusions

The effect of angular dependent surface properties on equilibrium temperature distribution was investigated for perpendicular adjoint plates uniformly irradiated by a collimated solar source. The surfaces were taken to reflect surface emitted thermal radiation specularly, and the semi-gray CSD and DSD property models were used to appraise the importance of the angular dependence of surface properties for both surface emitted and solar radiation.

Direction dependent thermal property effects were determined for solar properties represented by the CSD model, and several conclusions can be drawn. Directional property effects are most important for the critical surface elements near the common edge, and are insensitive to the value of solar absorptivity and solar specularity. Analysis which ignores direction dependence of thermal properties yields equilibrium temperature which is high for high surface emittance and low for low surface emittance. However, the resulting temperature errors are four to five times smaller than the corresponding errors in local heat flux for uniform temperature plates.

Direction dependent solar property effects were determined for thermal properties represented by the DSD model, and they were found to be nearly uniform over the extent of the surface. For both the high and low emittance surfaces, analysis which ignores direction dependence of solar properties yields equilibrium temperature which is high for low and intermediate values of solar absorptivity and temperature which is low for high solar absorptivity. Directional property effects are most important for low solar absorptance values, and with all other parameters fixed they decrease in importance with

decreasing surface specularity for solar radiation. Direction independent thermal and solar property analysis was found to yield largest temperature errors for the high emittance surface. However, the error was limited to less than 7 percent.

Finally, it should be noted that the spatial distribution of reflected solar radiation can have an important effect on equilibrium temperature particularly for surfaces with low and intermediate values of solar absorptivity.

6.3 BECKMANN PROPERTY MODEL EQUILIBRIUM TEMPERATURE RESULTS

In this section surface roughness effects on equilibrium temperature distribution are investigated. Radiation properties for surface emitted thermal radiation are represented by the angle dependent specular reflection model (DSD model with $\rho_{sp}/\rho_d = 1.0$), and those for solar radiation are represented by the Beckmann BDR model. Equilibrium temperature distributions are reported for low and high emittance surfaces with low to high values for solar absorptance and selected values for the roughness parameters. Results for surfaces with large optical roughness for solar radiation are presented in Section 6.3.1. The influence of the surface roughness parameters optical roughness (σ/λ) and rms roughness slope (m) on equilibrium temperature is discussed in Section 6.3.2.

6.3.1 Large Optical Roughness (LOR) Beckmann BDR Model Results

Dimensionless equilibrium temperature distributions for surfaces with properties for solar radiation represented by the LOR Beckmann BDR model with rms roughness slope (m) equal to 0.14 are

shown in Figs. 6.6-6.10. Results for surface hemispherical emittance value 0.1 are shown in Figs. 6.6, 6.7, and 6.8 for hemispherical solar absorptance values 0.1, 0.5, and 0.9, respectively. Results for high surface emittance ($\epsilon_H = 0.9$) are shown in Figs. 6.9 and 6.10 for solar absorptance values 0.1 and 0.5, respectively. Also included in each figure are temperature distributions evaluated with less detailed surface property models. These results were obtained for surfaces which are specularly reflecting for surface emitted radiation and either specularly (curves labeled $\rho_{sp}^*/\rho_d^* = 1.0$) or diffusely (curves labeled $\rho_{sp}^*/\rho_d^* = 0.0$) reflecting for solar radiation. Temperature distributions evaluated using constant property (CP) CSD analysis [2] which ignores all angle dependence of surface properties are shown as dashed lines. Doubly broken curves denote distributions obtained using the directional property (DP) DSD model for thermal radiation while retaining the CP model for solar radiation. Denoted by broken curves are distributions obtained using the DSD model for both thermal and solar radiation surface properties.

Within the limitations of the rough surface property model, the sole influence of surface roughness is to alter the spatial distribution of reflected solar radiation. The effect of this factor on equilibrium temperature can be demonstrated by comparing rough surface results to those obtained using DP models for both thermal and solar radiation surface properties (broken curves). The only difference between the two property models is that the DP model replaces the spatially dependent rough surface solar reflection model with one of the simpler specular or diffuse reflection models. For high solar absorptance the solar reflection model has little influence on equilibrium temperature. For the

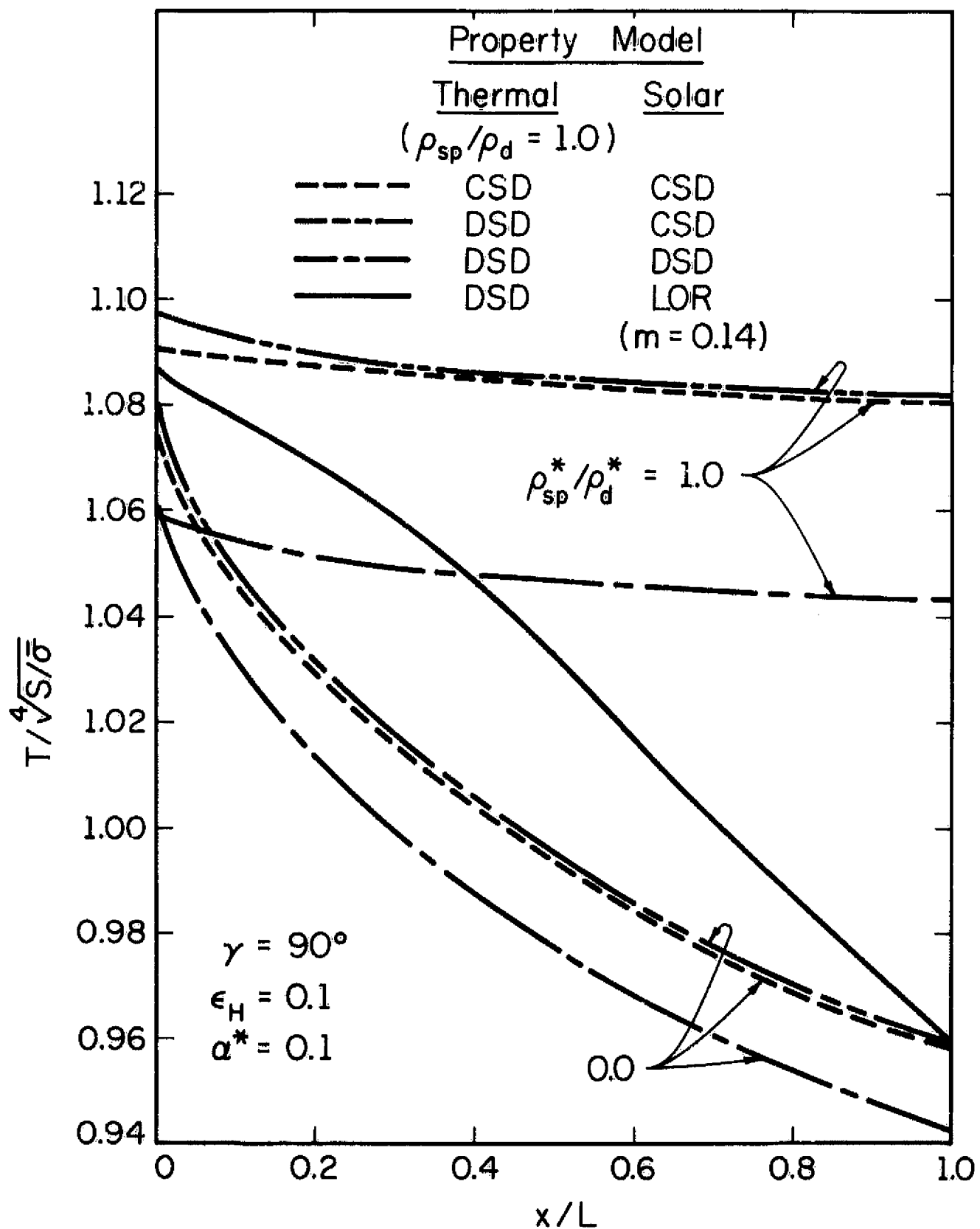


Figure 6.6 Rough surface local equilibrium temperature for large optical roughness ($\epsilon_H = 0.1$, $\alpha^* = 0.1$)

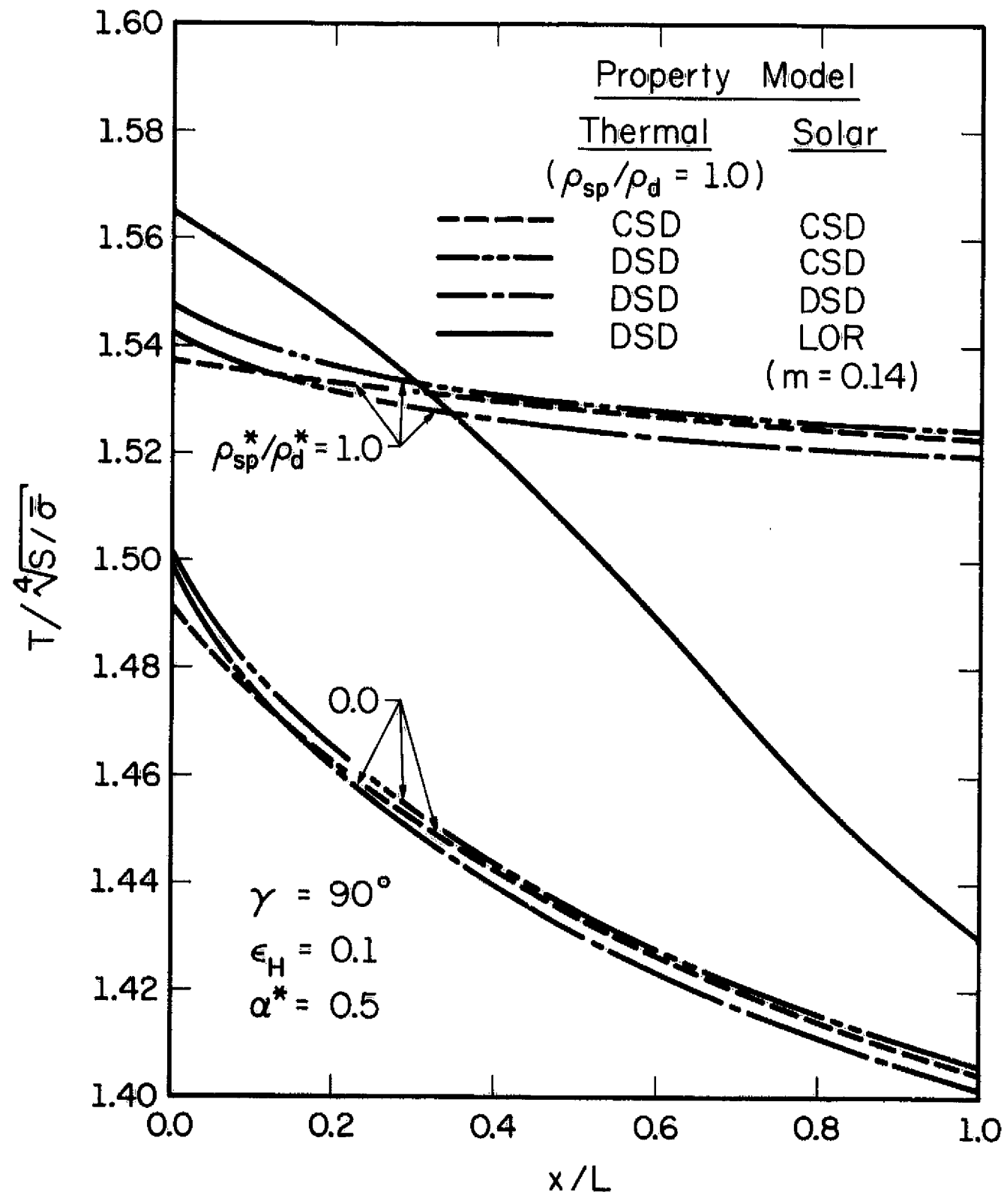


Figure 6.7 Rough surface local equilibrium temperature for large optical roughness ($\epsilon_H = 0.1$, $\alpha^* = 0.5$)

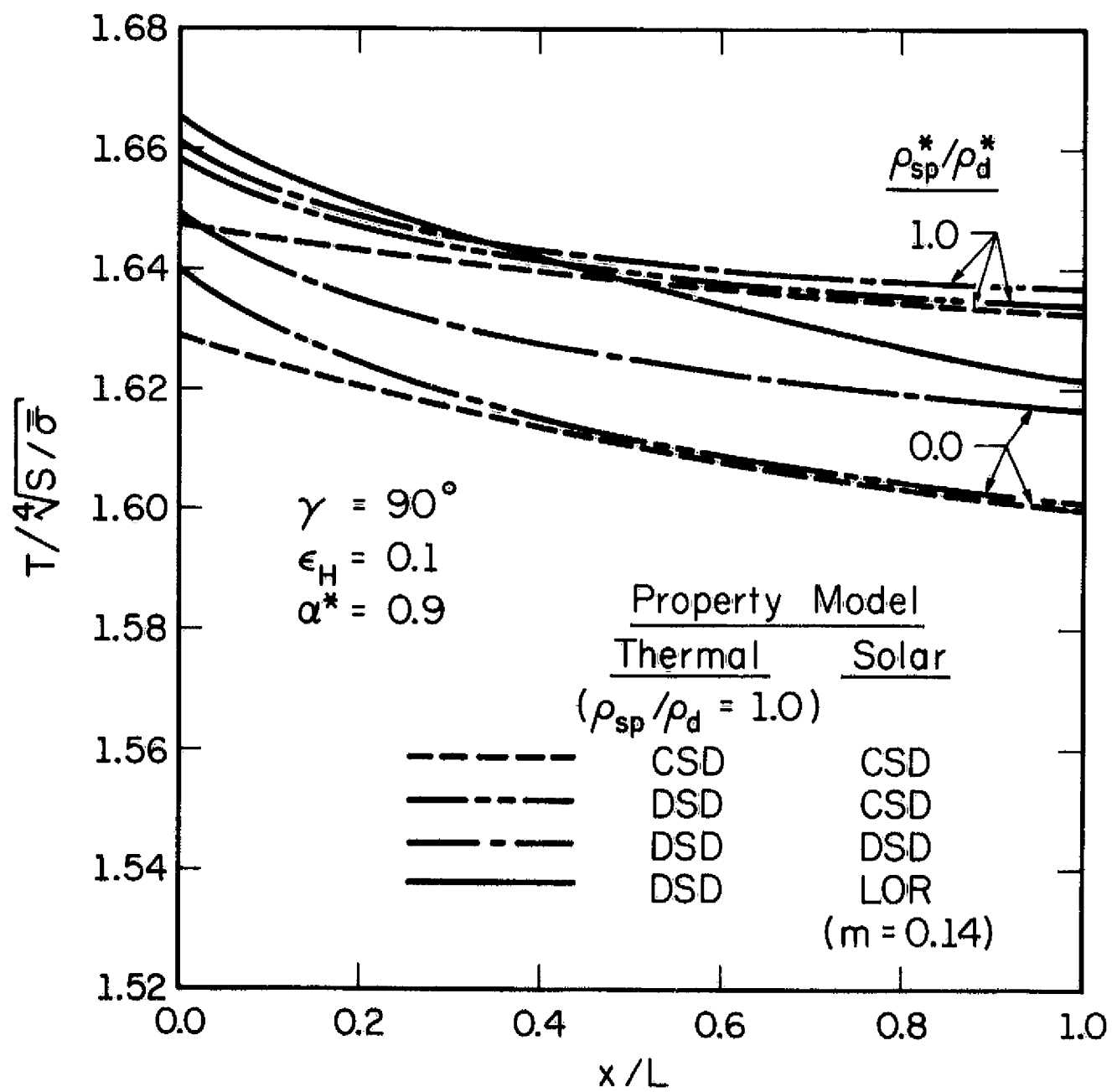


Figure 6.8 Rough surface local equilibrium temperature for large optical roughness ($\epsilon_H = 0.1$, $\alpha^* = 0.9$)

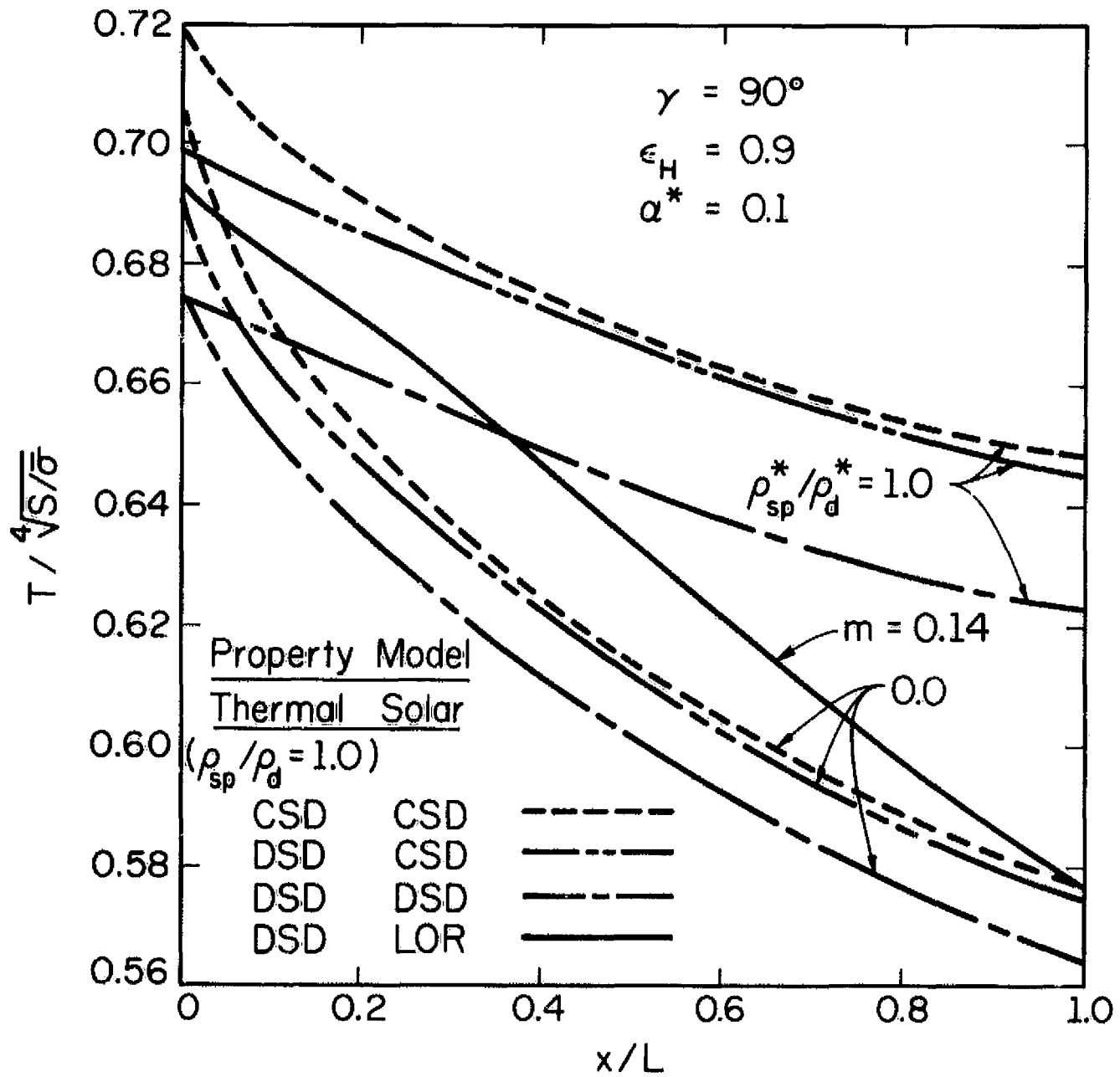


Figure 6.9 Rough surface local equilibrium temperature for large optical roughness ($\epsilon_H = 0.9$, $\alpha^* = 0.1$)

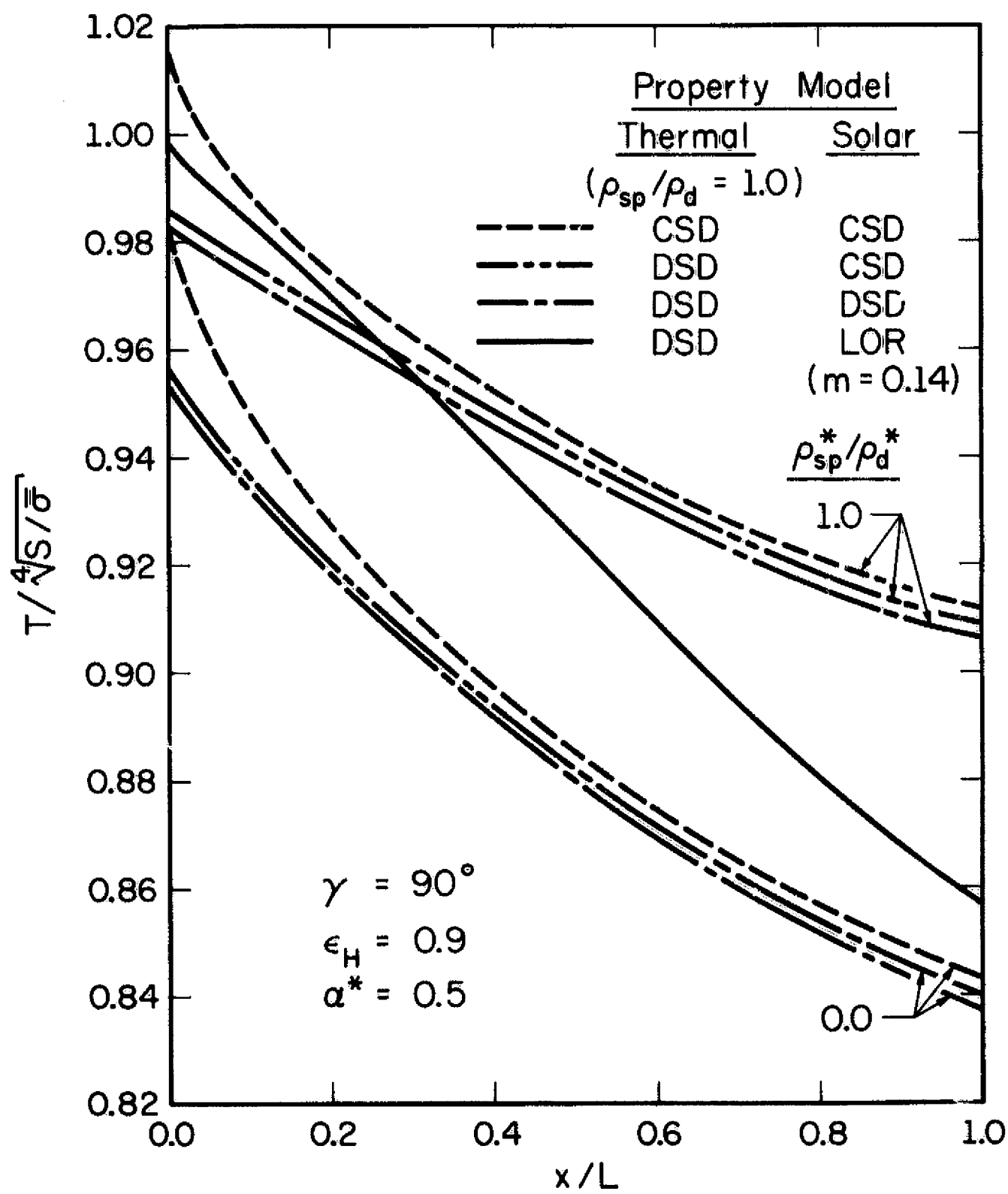


Figure 6.10 Rough surface local equilibrium temperature for large optical roughness ($\epsilon_H = 0.9$, $\alpha^* = 0.5$)

case shown in Fig. 6.8 for $\alpha^* = 0.9$, results obtained with various solar reflection models differ by less than 2 percent. However, for intermediate and low solar absorptance surfaces, solar reflection characteristics become an important influence on equilibrium temperature, particularly at elements near the outer edge. Near the apex rough surface temperature does not lie intermediate to the DP results obtained using diffuse and specular reflection models for solar radiation, but is higher than predicted by either of the two simpler models. This is contrary to conclusions drawn from results for rough surface heat flux for uniform temperature surfaces discussed in Section 5.4.1. For elements near the apex, rough surface temperature is nearer to the predictions of DP analysis employing a specular reflection model for solar radiation. However, near the outer edge, analysis using a diffuse solar reflection model yields more accurate temperature results. In each case considered here, the maximum discrepancy between rough surface temperature and that evaluated with DP analysis is smallest for analysis incorporating the diffuse solar reflection model.

It is evident from the figures that CP analyses with diffuse and specular solar reflection models do not always provide bounds for rough surface temperature. Constant property analysis with specular solar reflection generally yields accurate results near the apex, but this accuracy deteriorates with distance from the apex. For low solar absorptance surfaces, the CP specular reflection analysis gives temperatures which are high by as much as 13 percent at the outer edge. The accuracy of analysis using specular solar reflection is generally improved slightly by accounting for the angle dependencies of solar absorptance and surface properties for thermal radiation. For the low solar absorptance surface, the maximum temperature discrepancy is reduced from 13 to 9 percent. Constant property

analysis using a diffuse solar reflection model yields surprisingly accurate results over the entire plate length. In each case considered, maximum error occurs near the plate midpoint. The largest discrepancy observed is for low emittance ($\epsilon_H = 0.1$) and intermediate solar absorptance ($\alpha^* = 0.5$), but even in this case it is less than 6 percent.

Analysis which accounts for direction dependent thermal properties while retaining the CP diffuse solar reflection model provides little if any improvement over CP analysis. Furthermore, diffuse solar reflection analysis which accounts for direction dependencies of thermal and solar properties does not give appreciably better results than CP analysis. In fact, for some cases CP analysis results are more accurate.

6.3.2 Surface Roughness Effects on Equilibrium Temperature

In the previous section it was shown that analysis employing the CP diffuse reflection model for solar radiation yields relatively accurate predictions for equilibrium temperature of materials with solar properties represented by the LOR BDR model for $m = 0.14$. As discussed in Section 5.4.2.1, this value of roughness slope corresponds to the maximum scattering of reflected solar radiation for which the BDR model satisfies the conservation of energy requirement. For finite values of optical roughness and for roughness slope values less than 0.14, the spatial distribution of reflected solar radiation more nearly corresponds to that peculiar to a specularly reflecting surface. Hence, rough surface equilibrium temperature distributions obtained using a finite value of optical roughness or value of roughness slope less than 0.14 should more nearly agree with that obtained from analysis using the DP

specular reflection model for solar radiation. Furthermore, the accuracy of the CP diffuse reflection model for solar radiation is expected to deteriorate as optical roughness and/or rms roughness slope diminishes.

Dimensionless equilibrium temperature distributions are shown in Figs. 6.11-6.14 for materials with solar properties represented by the Beckmann BDR model for selected values of the roughness parameters. Distributions for low emittance materials ($\epsilon_H = 0.1$) are shown in Figs. 6.11 and 6.12 and those for high emittance ($\epsilon_H = 0.9$) in Figs. 6.13 and 6.14. For each value of surface emittance, results are shown for both a low ($\alpha^* = 0.1$) and an intermediate ($\alpha^* = 0.5$) value of solar absorptance. Temperature distributions for materials with solar properties represented by the LOR BDR model are shown in each figure for roughness slope $m = 0.14$ and 0.094 . Included in Figs. 6.12-6.14 are distributions obtained for solar properties represented by the Beckmann BDR model with rms roughness slope $m = 0.14$ and characteristic optical roughness $\sigma/\lambda_c = 0.1$. For comparison purposes, results obtained using several simpler property models are included. Shown as dashed lines are distributions obtained using constant property CSD analysis with a specular reflection model for surface radiation and both specular and diffuse reflection models for solar radiation. Broken lines denote results obtained for both thermal and solar properties represented by the direction dependent DSD model with a specular reflection model for surface radiation and both specular and diffuse reflection models for solar radiation.

Comparison of temperature distributions obtained for materials with solar properties represented by the LOR BDR model for roughness slope values 0.14 and 0.094 confirms the previously drawn conclusion concerning

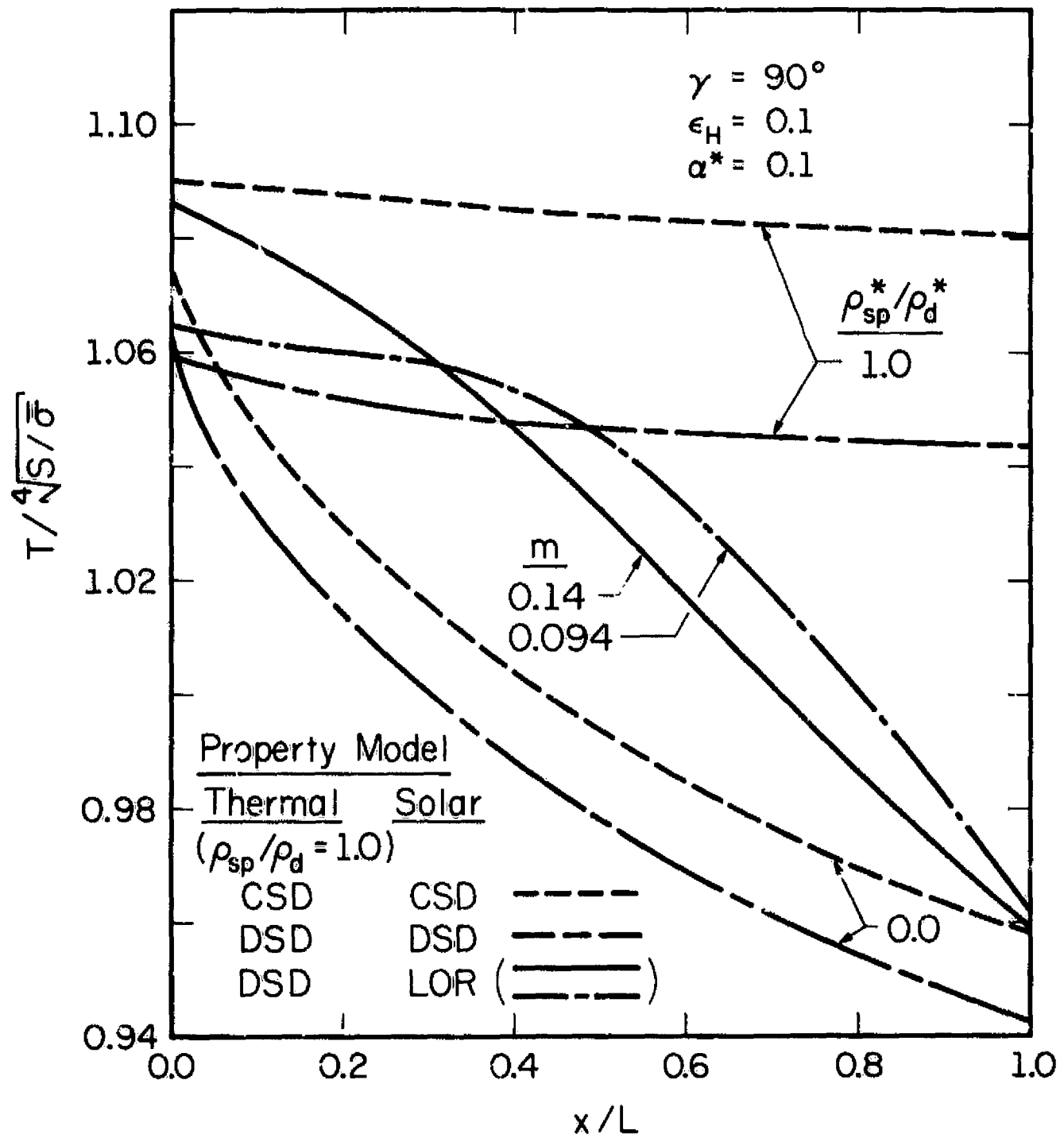


Figure 6.11 Roughness effects on equilibrium temperature ($\epsilon_H = 0.1$, $\alpha^* = 0.1$)

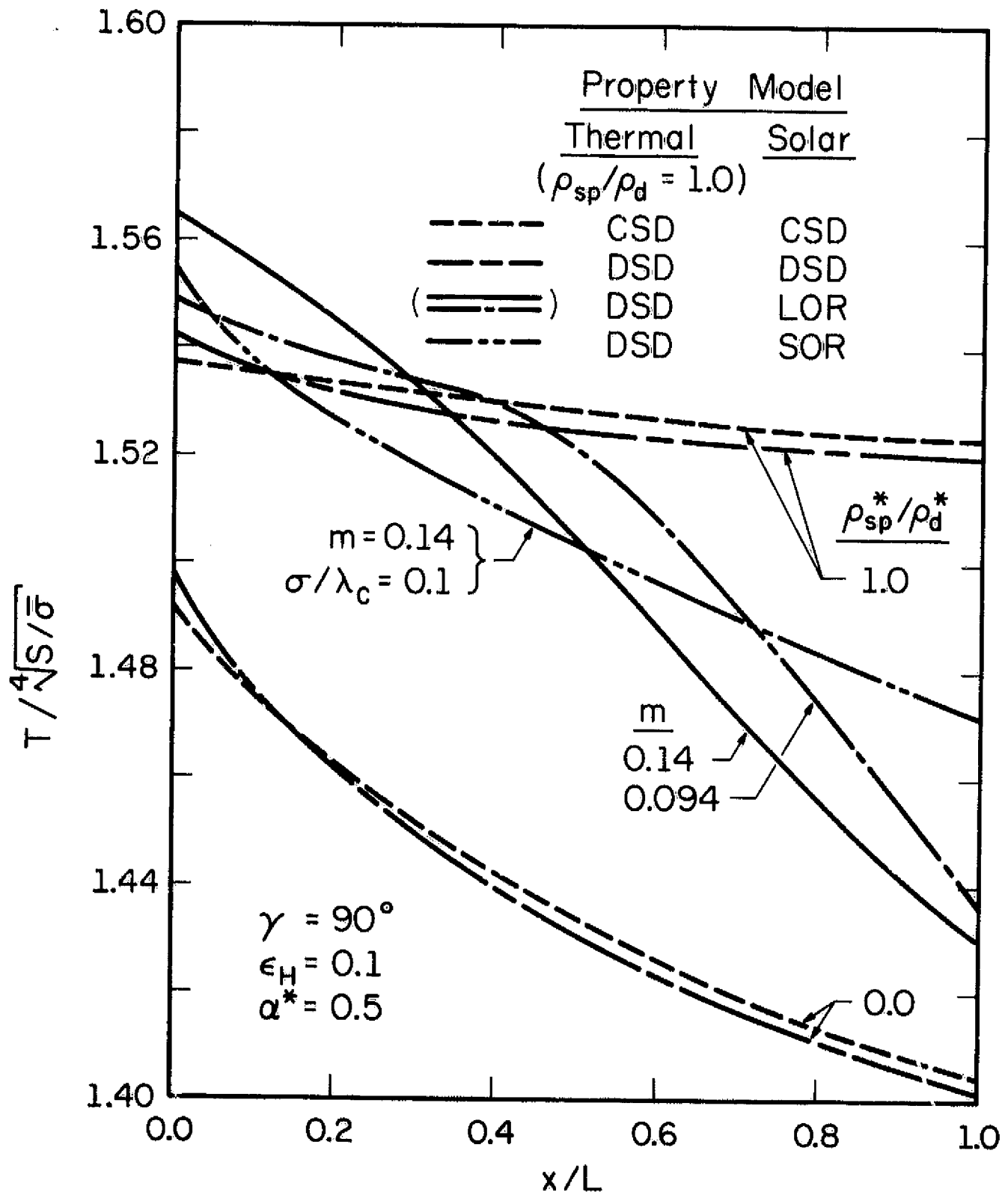


Figure 6.12 Roughness effects on equilibrium temperature ($\epsilon_H = 0.1$, $\alpha^* = 0.5$)

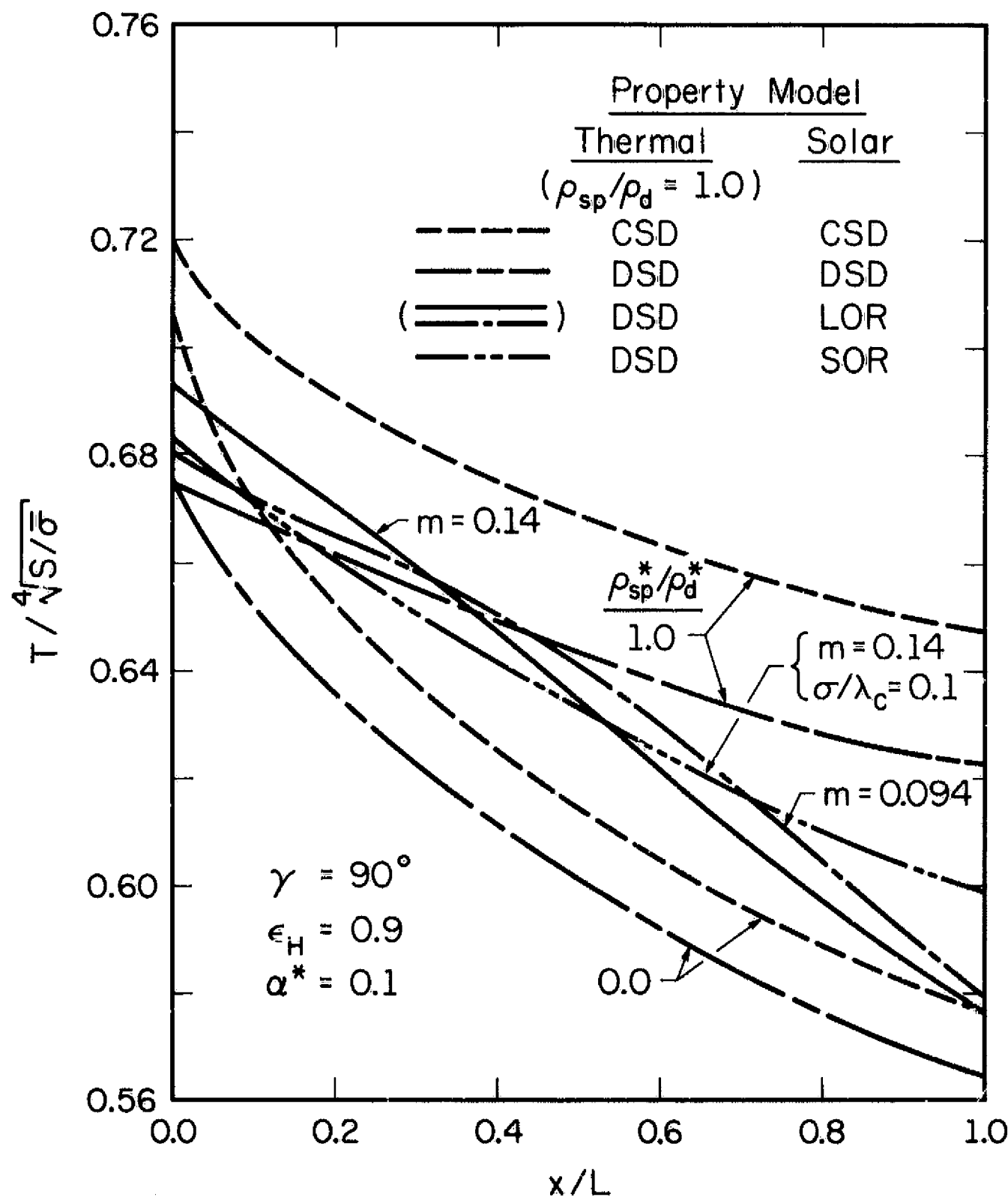


Figure 6.13 Roughness effects on equilibrium temperature ($\epsilon_H = 0.9$, $\alpha^* = 0.1$)

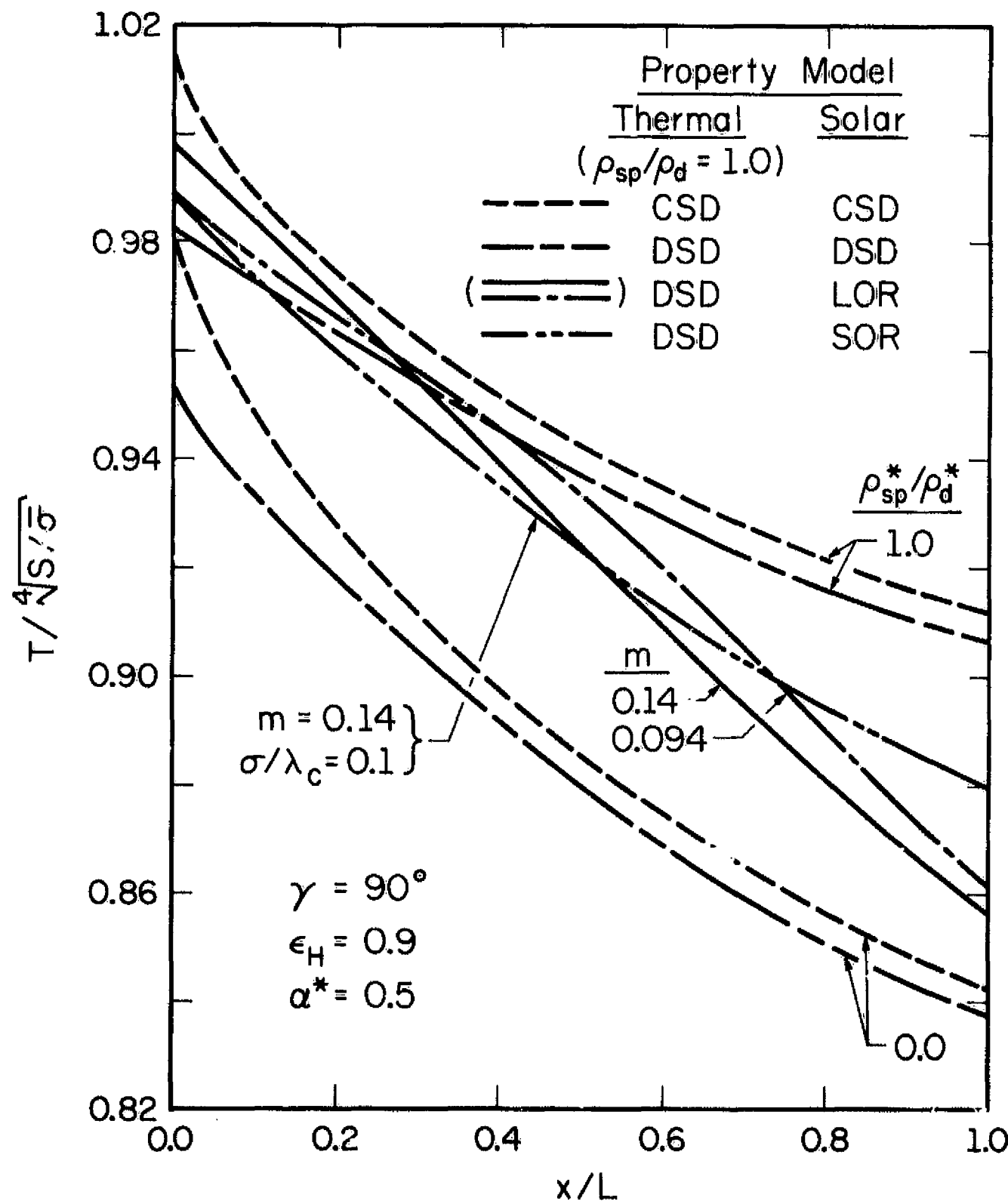


Figure 6.14 Roughness effects on equilibrium temperature ($\epsilon_H = 0.9, \alpha^* = 0.5$)

the influence of roughness slope on equilibrium temperature distribution. Both near the apex and the outer edge, surface temperature for the smaller value of slope is closer to that obtained using the DP specular reflection model for solar radiation. The influence of roughness slope on equilibrium temperature, however, is relatively small even though the selected values for rms slope differ by nearly 33 percent.

The influence of optical roughness on equilibrium temperature for surfaces of fixed roughness slope ($m = 0.14$) is illustrated by comparing results obtained using the LOR property model with those derived using the more general BDR expression for a characteristic optical roughness value of 0.1. At surface elements near the apex as well as near the outer edge, temperature for the smaller value of optical roughness is closer to the result obtained using the DP specular reflection model for solar radiation.

As expected, the accuracy of analysis employing a CP diffuse reflection model for solar radiation generally deteriorates as the material becomes smoother (as optical roughness and roughness slope diminish). This is particularly evident at the outer edge. However, for the selected values for the roughness parameters, a CP diffuse reflection model yields results which deviate by at most 6 percent from the rough surface temperature.

6.3.3 Conclusions

The influence of surface roughness on equilibrium temperature of perpendicular, adjoint, semi-gray plates in a uniform collimated solar field was examined. Radiation properties for surface emitted thermal radiation were represented by the DSD specular reflection model, and those for

solar radiation were represented by the Beckmann BDR model. Accuracy attainable with analysis using a specular reflection model for surface radiation and one of several simple solar property models was also investigated.

For materials with a high value of solar absorptance, the spatial distribution of reflected solar radiation has little influence on equilibrium temperature, and analysis employing either a CP specular or CP diffuse reflection model for solar radiation yields temperatures which differ from rough surface results by no more than 2 percent. For materials with low or intermediate values of solar absorptance, equilibrium temperature evaluated using the LOR BDR model with maximum degree of scattering of reflected solar radiation ($m = 0.14$) differed by no more than 6 percent from results obtained using CP analysis with a diffuse reflection model for solar radiation. Analysis using a CP specular reflection model for solar radiation gave good results near the apex, but predicted temperatures which were high by as much as 13 percent at the outer edge for low solar absorptance materials. Analysis which accounts for angle dependence of thermal and/or solar properties while retaining the specular or diffuse reflection model did not yield results of significantly better accuracy than those obtained with CP analysis.

Equilibrium temperature distribution evaluated using the LOR BDR model approached that obtained using the DP specular reflection model as the value of roughness slope diminished. A similar trend was observed as the value of optical roughness was diminished for a fixed value of roughness slope. The accuracy of results obtained using the CP diffuse reflection model for solar radiation deteriorated as the value of optical roughness or roughness slope was reduced.

For applications requiring precise temperature control, the simple specular or diffuse reflection models do not adequately account for roughness effects for surfaces of low to intermediate values for solar absorptance.

6.4 SUMMARY

Equilibrium temperature results discussed in Chapter 6 reveal several general conclusions concerning direction dependent property effects on equilibrium temperature.

The angle dependence of radiation properties for surface emitted radiation exhibited its strongest influence on equilibrium temperature at surface elements near the apex. The error incurred by ignoring the angle dependence of properties for surface radiation was less than 3 percent, and was nearly independent of material absorptance and specularity for solar radiation.

Angle dependence of solar absorptance (directional hemispherical reflectance) has a nearly uniform effect on equilibrium temperature over the extent of the system. This effect is most important for materials which have low values for solar absorptance. The error due to ignoring direction dependence of solar absorptance, however, was less than 4 percent.

Surface roughness has little influence on equilibrium temperature for materials with high solar absorptance values. However, for materials of low or intermediate solar absorptance, roughness effects on the spatial distribution of reflected solar radiation can significantly alter equilibrium temperature particularly at surface elements where radiant interaction is small. For the selected values of roughness parameters, a CP diffuse reflection model for solar radiation yields

results within 6 percent of the rough surface temperature. However, this accuracy diminishes with increasing smoothness of the interacting materials for solar radiation.

7. SUMMARY AND CONCLUSIONS

An analytical study of steady radiative transfer for a simple system of radiatively interacting opaque surfaces separated by a radiatively transparent medium has been presented. Basic principles of radiative transfer were employed to formulate radiative transfer for identical, uniform property adjoint plates uniformly irradiated by a collimated radiative flux. The formulation accounted for directional and wavelength dependence of surface properties. The governing equations simplify when bidirectional reflectance may be expressed as the sum of direction dependent specular and scattered components and/or the spectral variation of properties may be approximated by a semi-gray model. Numerical results for local and overall heat transfer were obtained in the absence of external radiative flux and equilibrium temperature distributions were evaluated for radiatively adiabatic surfaces in the presence of a solar flux. Heat transfer and equilibrium temperature calculations employed bidirectional reflectance models representative of rough metal surfaces and coated surfaces as well as property models incorporating much less detail.

Local and overall heat transfer results for equal temperature, identical property adjoint plates in the absence of an external radiation source suggest several general conclusions. First, the influence of the directional dependence of surface properties on local heat transfer is significant for low and high emittance materials at surface elements where radiant interaction is high and major contributions to local irradiation from energy incident at large angles of incidence. These directional effects are nearly independent of whether the surfaces are specularly or diffusely reflecting. On the other hand, the directional

dependence of surface properties is, for the most part, unimportant in the evaluation of overall heat transfer. Second, the spatial distribution of reflected radiation for surfaces of low to intermediate emittance strongly influences local heat transfer at critical surface elements where radiant interaction is large. For such situations, adequate representation of the spatial distribution of reflected energy can be of greater importance than that for directional emittance, absorptance, and reflectance. Third, directional property analysis using a reflectance model with a direction-independent apportionment of reflected radiation between specular and diffuse components can provide excellent heat transfer predictions for surfaces with more complicated reflectance characteristics provided the apportionment of reflected energy is appropriately selected. A correlation based on surface roughness has been developed for the apportionment of reflected energy in thermal design calculations. Fourth, radiant heat transfer between randomly rough surfaces for which diffraction and interference effects govern the spatial distribution of reflected energy differs only slightly from that predicted by directional property analysis employing a specular reflection model. The largest discrepancies between the results utilizing these two models occur for low to intermediate emittance materials at surface elements where interreflections are dominant. Fifth, heat transfer results obtained with a directional property model and with an approximate rough surface model can be utilized to limit the uncertainty for heat transfer between rough surfaces while avoiding the complex analysis required for the more detailed rough surface model. Sixth, a dielectric coating on a diffusely reflecting substrate significantly affects radiant heat transfer. Since the spatial distribution of the scattered energy from such a surface is nearly diffuse, a direction dependent

property analysis which accurately accounts for the directional dependence of specularly reflected radiation, but assumes the remaining reflected energy to be diffusely distributed, yields excellent local and overall heat transfer results.

Equilibrium temperature distributions evaluated for identical property, perpendicular adjoint plates which are specularly reflecting to surface emitted radiation suggest several general conclusions. First, the spatial distribution of reflected solar radiation influences equilibrium temperature distribution to a greater degree than the directional dependence of either the thermal or solar properties of the surfaces. Furthermore, as one should expect, even the spatial distribution of reflected solar radiation becomes unimportant for surfaces with large values of solar absorptance. Second, for surfaces with low to intermediate values of solar absorptance, surface roughness effects are of the greatest importance insofar as equilibrium temperature is concerned at surface elements where radiant interaction is low. For the selected values for the surface roughness parameters, constant property analysis employing a diffuse reflection model for solar energy generally gave equilibrium temperature results within 6 percent of the rough surface results. As the surfaces became smoother, however, the error of constant property analysis increases.

Several future research topics are suggested by this study. It must be kept in mind that all conclusions drawn from the real surface heat transfer and temperature results were based on the use of certain theoretical models for bidirectional reflectance. Experimental confirmation of these models is lacking. There exists a great need for a sufficient quantity of high resolution, accurate, bidirectional reflectance data

for well-characterized surfaces which can verify existing models, examine the usefulness of the surface parameters suggested by the models, and provide direction to theoretical efforts should available models not prove adequate. Further effort should also be directed to examining and improving the optical roughness-specularity ratio correlation developed herein by extending consideration to other geometries and to equilibrium temperature calculations. Additional attention should be given to the approximate rough surface model which predicts a strong dependence on optical roughness σ/λ . Calculations could be extended to include the variation of optical roughness with wavelength and thereby provide information on the influence of roughness on total radiant transfer. Finally, experimental radiant heat transfer studies would provide much needed data to examine the validity of theoretical heat transfer calculations.

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APPENDIX A

ALTERNATE FORMULATION FOR EQUATIONS GOVERNING RADIANT TRANSFER

A.1 INTRODUCTION

The purpose of this appendix is to express the equations governing radiant transfer in a form which is more amenable to numerical solution. The radiant transfer analysis for diffusely emitting surfaces with both specular and diffuse reflectance components developed by Sparrow and Lin [4] is extended to direction dependent surface properties.

A.2 UNIFORM TEMPERATURE ADJOINT PLATES IN THE ABSENCE OF EXTERNAL FIELDS ($\gamma = 45^\circ$)

In Section 3 the integral equation governing spectral radiant intensity was presented for a BDR model expressed as the sum of a specular and a scattered reflectance component. In the absence of the solar source, the intensity leaving point Q in the direction from Q to P, $I_\lambda^+(y, z, \theta_Q, \phi_Q)$, may be replaced with its equivalent, $I_\lambda^-(x, \theta'_P, \phi'_P)$, and Eq. (3.7.3) written

$$\begin{aligned}
 I_\lambda^+(x, \theta_P, \phi_P) = & \epsilon_{d,\lambda}(\theta_P) I_{b,\lambda} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s,\lambda}(\theta'_P, \phi'_P; \theta_P, \phi_P) \\
 & \cdot I_\lambda^-(x, \theta'_P, \phi'_P) K(x, y, z) dz dy + \rho_{sp,\lambda}(\theta'_P, \phi'_P) \\
 & \cdot I_\lambda^-(x, \theta'_{P,sp}, \phi'_{P,sp})
 \end{aligned} \tag{A-1}$$

where

$$\theta'_{P,sp} = \theta_P$$

$$\phi'_{P,sp} = \phi_P \pm \pi$$

The angles defining the direction of incident energy (θ'_P, ϕ'_P) may be expressed in terms of the space variables (x, y, z) and such relations are presented later.

It is convenient to define a spectral radiant intensity of scattered energy or for brevity, spectral scattered intensity, $I_{s,\lambda}^+$, by the relation

$$I_{s,\lambda}^+(x, \theta_P, \phi_P) = \epsilon_{d,\lambda}(\theta_P) I_{b,\lambda} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s,\lambda}(\theta'_P, \phi'_P; \theta_P, \phi_P) \cdot I_{\lambda}^-(x, \theta'_P, \phi'_P) K(x, y, z) dz dy \quad (A-2)$$

Local spectral radiant intensity can then be written as

$$I_{\lambda}^+(x, \theta_P, \phi_P) = I_{s,\lambda}^+(x, \theta_P, \phi_P) + \rho_{sp,\lambda}(\theta'_{P,sp}) I_{\lambda}^-(x, \theta'_{P,sp}, \phi'_{P,sp})$$

The reason for introducing scattered intensity becomes evident when it is recognized that the incident intensity $I_{\lambda}^-(x, \theta'_P, \phi'_P)$ consists entirely of scattered radiation which emanates from the two surfaces and arrives at P from the (θ'_P, ϕ'_P) direction either by direct transport or by a specular reflection path. For plane surfaces with direction independent properties, Eckert and Sparrow [64] demonstrated that energy which undergoes only specular reflections is conveniently accounted for by the image

method. The basis of this method is the fundamental principle that radiant energy reflected from a plane mirror appears to originate behind the mirror. Hering [49] extended the image method to plane specular surfaces with direction dependent surface properties.

Employing the image method to evaluate specularly reflected energy, local total radiant heat transfer can be expressed as a function of the scattered intensity

$$\begin{aligned}
 q(x) = & \epsilon_H \bar{\sigma} T^4 - \int_{\lambda=0}^{\infty} \int_{y=0}^L \int_{z=-\infty}^{\infty} \alpha_{d,\lambda}(\theta'_{P,\gamma}) I_{s,\lambda}^+(y,z,\theta_{Q,\gamma},\phi_{Q,\gamma}) K_{\gamma}(x,y,z) dz dy d\lambda \\
 & - \int_{\lambda=0}^{\infty} \int_{y=0}^L \int_{z=-\infty}^{\infty} \alpha_{d,\lambda}(\theta'_{P,3\gamma}) \rho_{sp,\lambda}(\theta'_{12}) \rho_{sp,\lambda}(\theta'_{22}) I_{s,\lambda}^+(y,z,\theta_{Q,3\gamma},\phi_{Q,3\gamma}) \\
 & \cdot K_{3\gamma}(x,y,z) dz dy d\lambda - \int_{\lambda=0}^{\infty} \int_{\bar{x}=0}^L \int_{\bar{z}=-\infty}^{\infty} \alpha_{d,\lambda}(\theta'_{P,2\gamma}) \rho_{sp,\lambda}(\theta'_{11}) \\
 & \cdot I_{s,\lambda}^+(\bar{x},\bar{z},\theta_{P,2\gamma},\phi_{P,2\gamma}) K_{2\gamma}(x,\bar{x},\bar{z}) d\bar{z} d\bar{x} d\lambda
 \end{aligned} \tag{A-3}$$

The first term on the right-hand side of Eq. (A-3) represents energy emitted per unit time and per unit area from an element of the lower surface located at the point $P(x,z=0)$.

The second term represents scattered energy which emanates from the upper surface and is directly transported to and absorbed by the lower surface at P per unit time and unit surface area. The polar angles $\theta'_{P,\gamma}$ and $\theta_{Q,\gamma}$, the geometry kernel $K_{\gamma}(x,y,z)$, and azimuthal angle $\phi_{Q,\gamma}$ are evaluated from the following equations:

$$\cos \theta'_{P,\gamma} = \frac{y \sin \gamma}{\sqrt{x^2 + y^2 + z^2 - 2xy \cos \gamma}} \quad (\text{A-4})$$

$$\cos \theta_{Q,\gamma} = \frac{x \sin \gamma}{\sqrt{x^2 + y^2 + z^2 - 2xy \cos \gamma}} \quad (\text{A-5})$$

$$K_\gamma(x,y,z) = \frac{xy \sin^2 \gamma}{(x^2 + y^2 + z^2 - 2xy \cos \gamma)^2} \quad (\text{A-6})$$

$$\tan \phi_{Q,\gamma} = \frac{z}{(x \cos \gamma - y)} \quad (\text{A-7})$$

Since azimuthal angle $\phi_{Q,\gamma}$ can take on values between 0 and 2π radians, the signs of the numerator and denominator of the right-hand side of Eq. (A-7) are used to determine the quadrant in which $\phi_{Q,\gamma}$ lies. Let the numerator and denominator be measured along the ordinate and abscissa, respectively, of a rectangular coordinate system, and measure $\phi_{Q,\gamma}$ counter-clockwise from the positive abscissa sighting toward the surface along the positive surface normal; use the rules of trigonometry and the sign of the numerator and denominator to determine the quadrant in which $\phi_{Q,\gamma}$ lies.

The third term on the right-hand side of Eq. (A-3) represents scattered energy which emanates from the upper surface, and, after undergoing two specular reflections, is incident on and absorbed by the lower surface element at P per unit time and unit surface area. For an observer at P, this energy appears to originate at the second mirror image of the upper surface. This is a fictitious semi-infinite surface of length L adjoint to the lower surface and rotated at an angle of 135 degrees from it. The intensity of the energy leaving the image surface is reduced by

the product of the specular reflectances $\rho_{sp,\lambda}(\theta'_{12})$ and $\rho_{sp,\lambda}(\theta'_{22})$ for the two intervening reflections. The first specular reflection occurs on the lower surface for polar angle of incidence θ'_{12} and the second occurs on the upper surface for polar angle of incidence θ'_{22} . The direction in which scattered energy leaves the upper surface $(\theta_{Q,3\gamma}, \phi_{Q,3\gamma})$, the polar angles θ'_{12} and θ'_{22} for the intervening reflections, and the polar angle of incidence $\theta'_{P,3\gamma}$ are related by the laws governing a specular reflection. Polar angles $\theta'_{P,3\gamma}$ and $\theta_{Q,3\gamma}$, geometry kernel $K_{3\gamma}(x,y,z)$, and azimuthal angle $\phi_{Q,3\gamma}$ are evaluated with Eqs. (A-4), (A-5), (A-6), and (A-7), respectively, by replacing γ in each equation with 3γ . Polar angles θ'_{12} and θ'_{22} are related to the position coordinates by the following equations:

$$\cos \theta'_{12} = \frac{y \sin \gamma + x \sin 2\gamma}{\sqrt{x^2 + y^2 + z^2 - 2xy \cos 3\gamma}} \quad (A-8)$$

$$\cos \theta'_{22} = \frac{x \sin \gamma + y \sin 2\gamma}{\sqrt{x^2 + y^2 + z^2 - 2xy \cos 3\gamma}} \quad (A-9)$$

The fourth term represents scattered energy which emanates from the lower surface, and, after undergoing a specular reflection, is incident on and absorbed by the lower surface at P per unit time and unit surface area. To an observer at P, this energy appears to originate at the first mirror image of the lower surface which is fictitious semi-infinite surface of length L adjoint to the lower surface and rotated at an angle of 90 degrees to it. The intensity of energy leaving the image source is reduced by the specular reflectance $\rho_{sp,\lambda}(\theta'_{11})$ for the intervening reflection.

The point $\bar{P}(\bar{x}, \bar{z})$ is that point on the lower surface at which the scattered energy emerges. The direction

$(\theta_{\bar{P},2\gamma}, \phi_{\bar{P},2\gamma})$ in which energy leaves the lower surface, the polar angle of incidence θ'_{11} for the intervening specular reflection, and the polar angle of incidence $\theta'_{P,2\gamma}$ from which this energy is incident at P are all related by the laws of specular reflection. The polar angle $\theta'_{P,2\gamma}$ can be evaluated from Eq. (A-4) by replacing γ, y , and z with $2\gamma, \bar{x}$, and $\bar{z}$, respectively. For a similar change of variables, the geometry kernel $K_{2\gamma}(x, \bar{x}, \bar{z})$ is obtained from Eq. (A-6). The polar angle $\theta_{\bar{P},2\gamma}$ and azimuthal angle $\phi_{\bar{P},2\gamma}$ are obtained from Eqs. (A-5) and (A-7), respectively, by replacing Q with $\bar{P}$, y with $\bar{x}$, z with $\bar{z}$, and γ with 2γ . The polar angle of incidence for the intervening specular reflection θ'_{11} is evaluated from the equation

$$\cos \theta'_{11} = \frac{(x + \bar{x}) \sin \gamma}{\sqrt{x^2 + \bar{x}^2 + \bar{z}^2 - 2x\bar{x} \cos 2\gamma}} \quad (\text{A-10})$$

Equation (A-3) expresses local total heat transfer as a function of the system geometry, surface properties, and local spectral scattered intensity. Therefore, attention is next directed to evaluation of scattered intensity.

Employing the image method to account for specularly reflected energy which contributes to the irradiation at point $P(x,0)$ on the lower plate, the local spectral scattered radiant intensity in direction (θ_P, ϕ_P) is

$$\begin{aligned}
I_{s,\lambda}^+(x,\theta_P,\phi_P) = & \epsilon_{d,\lambda}(\theta_P) I_{b,\lambda} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s,\lambda}(\theta'_P, \phi'_P; \theta_P, \phi_P) \\
& \cdot I_{s,\lambda}^+(y,z,\theta_{Q,\gamma}, \phi_{Q,\gamma}) K_{\gamma}(x,y,z) dz dy + \int_{y=0}^L \int_{z=-\infty}^{\infty} \\
& \cdot \rho_{bd,s,\lambda}(\theta'_{P,3\gamma}, \phi'_{P,3\gamma}; \theta_P, \phi_P) \rho_{sp,\lambda}(\theta'_{12}) \rho_{sp,\lambda}(\theta'_{22}) \\
& \cdot I_{s,\lambda}^+(y,z,\theta_{Q,3\gamma}, \phi_{Q,3\gamma}) K_{3\gamma}(x,y,z) dz dy + \int_{\bar{x}=0}^L \int_{\bar{z}=-\infty}^{\infty} \\
& \cdot \rho_{bd,s,\lambda}(\theta'_{P,2\gamma}, \phi'_{P,2\gamma}; \theta_P, \phi_P) \rho_{sp,\lambda}(\theta'_{11}) \\
& \cdot I_{s,\lambda}^+(\bar{x}, \bar{z}, \theta_{\bar{P},2\gamma}, \phi_{\bar{P},2\gamma}) K_{2\gamma}(x, \bar{x}, \bar{z}) d\bar{z} d\bar{x} \quad (A-11)
\end{aligned}$$

The first term on the right-hand side of Eq. (A-11) represents the monochromatic intensity of emitted energy at polar angle θ_P from the lower surface at P.

The remaining three terms represent monochromatic intensity of reflected energy leaving P in the (θ_P, ϕ_P) direction due to scattered radiation which emerges from each of the two surfaces and arrives at point P by direct transport and by all possible specular reflection paths. The first of these three terms accounts for scattered energy which emanates from the upper surface and is directly incident on the lower surface. The second accounts for scattered energy which emanates from the upper surface and, after undergoing two intervening specular reflections, is incident on the lower surface. Finally, the last term accounts for scattered energy which emanates from the lower surface and, after one

intervening specular reflection, is incident on the lower surface. The azimuthal angle $\phi'_{P,\gamma}$ is evaluated from the equation

$$\tan \phi'_{P,\gamma} = \frac{z}{y \cos \gamma - x} \quad (\text{A-12})$$

and the sign of the numerator and denominator must be noted to determine the quadrant in which $\phi'_{P,\gamma}$ lies. Azimuthal angles $\phi'_{P,2\gamma}$ and $\phi'_{P,3\gamma}$ can be evaluated with Eq. (A-12) by replacing γ with 2γ and 3γ , respectively. Eqs. (A-3) and (A-11) can be simplified by noting that because of symmetry and the infinite plate width, the scattered intensities on the two surfaces are related as follows

$$I_{s,\lambda}^+(y,z,\theta,\phi) \equiv I_{s,\lambda}^+(y,z=0,\theta,\phi) = I_{s,\lambda}^+(y,\theta,\phi) \quad (\text{A-13})$$

$$I_{s,\lambda}^+(y,\theta,\phi) = I_{s,\lambda}^+(x=y,\theta,\phi) \quad (\text{A-14})$$

$$I_{s,\lambda}^+(x,\theta,\phi) = I_{s,\lambda}^+(x,\theta,2\pi - \phi) \quad (\text{A-15})$$

Accounting for symmetry and introducing the dimensionless space variables $\eta = x/L$, $\bar{\eta} = \bar{x}/L$, $\xi = y/L$, $\zeta = z/L$, $\bar{\zeta} = \bar{z}/L$ and dimensionless intensity H_λ

$$H_\lambda(\eta,\theta,\phi) = \frac{I_{s,\lambda}^+(\eta,\theta,\phi)}{I_{b,\lambda}}$$

yields the following equations for dimensionless local heat transfer, dimensionless local spectral scattered intensity, and the dimensionless geometrical functions:

Dimensionless Local Total Heat Transfer

$$\begin{aligned}
 \frac{q(\eta)}{\epsilon_H \bar{\sigma} T^4} = 1 - \frac{2}{\epsilon_H \bar{\sigma} T^4} \int_{\lambda=0}^{\infty} I_{b,\lambda} & \left\{ \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \alpha_{d,\lambda}(\theta'_{P,\gamma}) H_{\lambda}(\bar{\eta} = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) \right. \\
 & \cdot K_{\gamma}(\eta, \xi, \zeta) d\zeta d\xi + \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \alpha_{d,\lambda}(\theta'_{P,3\gamma}) \rho_{sp,\lambda}(\theta'_{12}) \rho_{sp,\lambda}(\theta'_{22}) \\
 & \cdot H_{\lambda}(\bar{\eta} = \xi, \theta_{Q,3\gamma}, \phi_{Q,3\gamma}) K_{3\gamma}(\eta, \xi, \zeta) d\zeta d\xi \\
 & + \int_{\bar{\eta}=0}^1 \int_{\bar{\zeta}=0}^{\infty} \alpha_{d,\lambda}(\theta'_{P,2\gamma}) \rho_{sp,\lambda}(\theta'_{11}) H_{\lambda}(\bar{\eta}, \theta_{P,2\gamma}, \phi_{P,2\gamma}) \\
 & \cdot K_{2\gamma}(\eta, \bar{\eta}, \bar{\zeta}) d\bar{\zeta} d\bar{\eta} \Big\} d\lambda \quad (A-16)
 \end{aligned}$$

For wavelength independent surface properties (gray surfaces), Eq. (A-16) reduces to

$$\begin{aligned}
 \frac{q(\eta)}{\epsilon_H \bar{\sigma} T^4} = 1 - \frac{2}{\pi \epsilon_H} & \left\{ \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \alpha_d(\theta'_{P,\gamma}) H(\bar{\eta} = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_{\gamma}(\eta, \xi, \zeta) d\zeta d\xi \right. \\
 & + \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \alpha_d(\theta'_{P,3\gamma}) \rho_{sp}(\theta'_{12}) \rho_{sp}(\theta'_{22}) H(\bar{\eta} = \xi, \theta_{Q,3\gamma}, \phi_{Q,3\gamma}) \\
 & \cdot K_{3\gamma}(\eta, \xi, \zeta) d\zeta d\xi + \int_{\bar{\eta}=0}^1 \int_{\bar{\zeta}=0}^{\infty} \alpha_d(\theta'_{P,2\gamma}) \rho_{sp}(\theta'_{11}) \\
 & \cdot H(\bar{\eta}, \theta_{P,2\gamma}, \phi_{P,2\gamma}) K_{2\gamma}(\eta, \bar{\eta}, \bar{\zeta}) d\bar{\zeta} d\bar{\eta} \Big\} \quad (A-17)
 \end{aligned}$$

Dimensionless Local Spectral Scattered Radiant Intensity

$$\begin{aligned}
 H_{\lambda}(\eta, \theta_P, \phi_P) = & \epsilon_{d,\lambda}(\theta_P) + \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \left[\rho_{bd,s,\lambda}(\theta'_{P,\gamma}, \phi'_{P,\gamma}; \theta_P, \phi_P) \right. \\
 & + \left. \rho_{bd,s,\lambda}(\theta'_{P,\gamma}, 2\pi - \phi'_{P,\gamma}; \theta_P, \phi_P) \right] H_{\lambda}(\bar{\eta} = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) \\
 & \cdot K_{\gamma}(\eta, \xi, \zeta) d\zeta d\xi + \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \left[\rho_{bd,s,\lambda}(\theta'_{P,3\gamma}, \phi'_{P,3\gamma}; \theta_P, \phi_P) \right. \\
 & + \left. \rho_{bd,s,\lambda}(\theta'_{P,3\gamma}, 2\pi - \phi'_{P,3\gamma}; \theta_P, \phi_P) \right] H_{\lambda}(\bar{\eta} = \xi, \theta_{Q,3\gamma}, \phi_{Q,3\gamma}) \\
 & \cdot K_{3\gamma}(\eta, \xi, \zeta) d\zeta d\xi + \int_{\bar{\eta}=0}^1 \int_{\bar{\zeta}=0}^{\infty} \left[\rho_{bd,s,\lambda}(\theta'_{P,2\gamma}, \phi'_{P,2\gamma}; \theta_P, \phi_P) \right. \\
 & + \left. \rho_{bd,s,\lambda}(\theta'_{P,2\gamma}, 2\pi - \phi'_{P,2\gamma}; \theta_P, \phi_P) \right] H_{\lambda}(\bar{\eta}, \theta_{P,2\gamma}, \phi_{P,2\gamma}) \\
 & \cdot K_{2\gamma}(\eta, \bar{\eta}, \bar{\zeta}) d\bar{\zeta} d\bar{\eta}
 \end{aligned} \tag{A-18}$$

For gray surfaces, dimensionless local total radiant intensity

$$H(\eta, \theta_P, \phi_P) = \frac{I_s^+(\eta, \theta_P, \phi_P)}{(\sigma T^4/\pi)} \text{ is obtained by deleting subscript } \lambda \text{ in (A-18).}$$

Dimensionless Geometrical Quantities

$$\cos \theta'_{P,\gamma} = \frac{\xi \sin \gamma}{\sqrt{\eta^2 + \xi^2 + \zeta^2 - 2\eta\xi \cos \gamma}} \tag{A-19}$$

$$\cos \theta_{Q,\gamma} = \frac{\eta \sin \gamma}{\sqrt{\eta^2 + \xi^2 + \zeta^2 - 2\eta\xi \cos \gamma}} \tag{A-20}$$

$$\tan \phi_{Q,\gamma} = \frac{\zeta}{\eta \cos \gamma - \xi} \tag{A-21}$$

$$\tan \phi'_{P,\gamma} = \frac{\zeta}{\xi \cos \gamma - \eta} \quad (\text{A-22})$$

$$K_\gamma(\eta, \xi, \zeta) = \frac{\eta \xi \sin^2 \gamma}{(\eta^2 + \xi^2 + \zeta^2 - 2\eta\xi \cos \gamma)^2} \quad (\text{A-23})$$

$$\cos \theta'_{11} = \frac{(\eta + \bar{\eta}) \sin \gamma}{\sqrt{\eta^2 + \bar{\eta}^2 + \zeta^2 - 2\eta\bar{\eta} \cos 2\gamma}} \quad (\text{A-24})$$

$$\cos \theta'_{12} = \frac{\xi \sin \gamma + \eta \sin 2\gamma}{\sqrt{\eta^2 + \xi^2 + \zeta^2 - 2\eta\xi \cos 3\gamma}} \quad (\text{A-25})$$

$$\cos \theta'_{22} = \frac{\eta \sin \gamma + \xi \sin 2\gamma}{\sqrt{\eta^2 + \xi^2 + \zeta^2 - 2\eta\xi \cos 3\gamma}} \quad (\text{A-26})$$

The substitutions cited earlier for evaluating $\theta'_{P,2\gamma}$, $\theta'_{P,3\gamma}$, $\phi'_{P,2\gamma}$, $\phi'_{P,3\gamma}$, $\theta'_{\bar{P},3\gamma}$, $\theta'_{\bar{P},2\gamma}$, $\theta'_{Q,3\gamma}$, $\phi'_{\bar{P},2\gamma}$, $\phi'_{Q,3\gamma}$, $K_{2\gamma}$ and $K_{3\gamma}$ also apply to Eqs. (A-19)-(A-26).

A.3 UNIFORM TEMPERATURE ADJOINT PLATES IN THE ABSENCE OF EXTERNAL FIELDS ($\gamma = 90^\circ$)

For $\gamma = 90^\circ$, energy which emerges from either surface and undergoes a specular reflection on the adjoint plate exits from the system. In view of this observation, dimensionless local heat transfer and dimensionless local spectral scattered intensity for perpendicular plates may be written in the following form:

Dimensionless Local Total Heat Transfer

$$\frac{q(\eta)}{\epsilon_H \bar{\sigma} T^4} = 1 - \frac{2}{\epsilon_H \bar{\sigma} T^4} \int_{\lambda=0}^{\infty} I_{b,\lambda} \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \alpha_{d,\lambda}(\theta'_{P,\gamma}) H_{\lambda}(\eta = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_{\gamma}(\eta, \xi, \zeta) d\zeta d\xi d\lambda \quad (\text{A-27})$$

For gray surfaces, Eq. (A-27) reduces to

$$\frac{q(\eta)}{\epsilon_H \bar{\sigma} T^4} = 1 - \frac{2}{\pi \epsilon_H} \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \alpha_d(\theta'_{P,\gamma}) H(\bar{\eta} = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_{\gamma}(\eta, \xi, \zeta) d\zeta d\xi \quad (\text{A-28})$$

Dimensionless Local Spectral Scattered Intensity

$$\begin{aligned}
 H_{\lambda}(\eta, \theta_P, \phi_P) = & \varepsilon_{d,\lambda}(\theta_P) + \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \left[\rho_{bd,s,\lambda}(\theta'_{P,\gamma}, \phi'_{P,\gamma}; \theta_P, \phi_P) \right. \\
 & \left. + \rho_{bd,s,\lambda}(\theta'_{P,\gamma}, 2\pi - \phi'_{P,\gamma}; \theta_P, \phi_P) \right] \\
 & \cdot H_{\lambda}(\bar{\eta} = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_{\gamma}(\eta, \xi, \zeta) d\zeta d\xi \quad (A-29)
 \end{aligned}$$

For gray surfaces, delete all λ subscripts in Eq. (A-29). The relations presented in Eqs. (A-19)-(A-26) are used to evaluate the geometrical quantities.

A.4. SEMI-GRAY ADJOINT PLATES WITH SPECIFIED RADIANT FLUX ($\gamma = 90^\circ$)

Following the arguments outlined in the previous sections of this appendix, the governing equations for semi-gray surfaces with a specular plus scattered subdivision of BDR can also be recast into a more useful form. Replacing the thermal intensity $I_t^+(y, z, \theta_Q, \phi_Q)$ with $I_t^-(x, \theta'_P, \phi'_P)$ and the solar intensity $I_{so}^+(y, z, \theta_Q, \phi_Q)$ with $I_{so}^-(x, \theta'_P, \phi'_P)$, Eqs. (3.7.5) and (3.7.6) transform to

$$\begin{aligned}
 I_t^+(x, \theta_P, \phi_P) = & \frac{\varepsilon_d(\theta_P) \bar{\sigma} T^4(x)}{\pi} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s}(\theta'_P, \phi'_P; \theta_P, \phi_P) \\
 & \cdot I_t^-(x, \theta'_P, \phi'_P) K_{\gamma}(x, y, z) dz dy \\
 & + \rho_{sp}(\theta'_P, \phi'_P) I_t^-(x, \theta'_{P,sp}, \phi'_{P,sp}) \quad (A-30)
 \end{aligned}$$

and

$$\begin{aligned}
 I_{so}^+(x, \theta_P, \phi_P) &= \rho_{bd,s}^*(\theta'_{so}, 0; \theta_P, \phi_P) S \sin \frac{\gamma}{2} \\
 &+ \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s}^*(\theta'_P, \phi'_P; \theta_P, \phi_P) I_{so}^-(x, \theta'_P, \phi'_P) \\
 &\cdot K_Y(x, y, z) dz dy + \rho_{sp}^*(\theta'_{P,sp}) I_{so}^-(x, \theta'_{P,sp}, \phi'_{P,sp}) \\
 &+ \mathcal{J}_{sp}(\theta_P, \phi_P)
 \end{aligned} \tag{A-31}$$

where

$$\theta'_{P,sp} = \theta_P$$

$$\phi'_{P,sp} = \phi_P \pm \pi$$

$$\theta'_{so} = (\pi - \gamma)/2$$

Define scattered thermal intensity by

$$\begin{aligned}
 I_{t,s}^+(x, \theta_P, \phi_P) &= \frac{\varepsilon_d(\theta_P) \overline{\sigma T^4}(x)}{\pi} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s}(\theta'_P, \phi'_P; \theta_P, \phi_P) \\
 &\cdot I_t^-(x, \theta'_P, \phi'_P) K_Y(x, y, z) dz dy
 \end{aligned} \tag{A-32}$$

and scattered solar intensity by

$$\begin{aligned}
 I_{so,s}^+(x, \theta_P, \phi_P) &= \rho_{bd,s}^*(\theta'_{so}, 0; \theta_P, \phi_P) S \sin \frac{\gamma}{2} \\
 &+ \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s}^*(\theta'_P, \phi'_P; \theta_P, \phi_P) I_{so}^-(x, \theta'_P, \phi'_P) \\
 &\cdot K_Y(x, y, z) dz dy \quad (A-33)
 \end{aligned}$$

Noting that $I_t^-(x, \theta'_P, \phi'_P)$ consists entirely of scattered thermal energy which emerges from either of the two surfaces and arrives at $P(x, 0)$ directly and by all possible specular reflection paths, Eq. (A-32) can be written for $\gamma = 90^\circ$ as

$$\begin{aligned}
 I_{t,s}^+(x, \theta_P, \phi_P) &= \frac{\epsilon_d(\theta_P) \bar{\sigma} T^4(x)}{\pi} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s}(\theta'_P, \gamma, \phi'_P, \gamma; \theta_P, \phi_P) \\
 &\cdot I_{t,s}^+(y, z, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_Y(x, y, z) dz dy \quad (A-34)
 \end{aligned}$$

The incident solar intensity $I_{so}^-(x, \theta'_P, \phi'_P)$ consists of scattered solar energy emanating from each surface and arriving at P directly and by all specular reflection paths, and, in addition, energy of the collimated source which undergoes only specular reflection. Therefore, the scattered solar intensity can be written

$$\begin{aligned}
I_{so,s}^+(x, \theta_P, \phi_P) &= \rho_{bd,s}^*(\theta'_{so}, 0; \theta_P, \phi_P) S \sin \frac{\gamma}{2} \\
&+ \int_{y=0}^L \int_{z=-\infty}^{\infty} \rho_{bd,s}^*(\theta'_{P,\gamma}, \phi'_{P,\gamma}; \theta_P, \phi_P) \\
&\cdot I_{so,s}^+(y, z, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_{\gamma}(x, y, z) dz dy \\
&+ \rho_{bd,s}^*(\theta'_{so}, \pi; \theta_P, \phi_P) \rho_{sp}^*(\theta'_{so}) S \sin \frac{\gamma}{2} \quad (A-35)
\end{aligned}$$

The locally absorbed solar irradiation can be expressed in terms of the solar surface properties, the solar constant, and the scattered solar intensity as

$$\begin{aligned}
q_{a,so}(x) &= \alpha_d^*(\theta'_{so}) S \sin \frac{\gamma}{2} + \alpha_d^*(\theta'_{so}) \rho_{sp}^*(\theta'_{so}) S \sin \frac{\gamma}{2} \\
&+ \int_{y=0}^L \int_{z=-\infty}^{\infty} \alpha_d^*(\theta'_{P,\gamma}) I_{so,s}^+(y, z, \theta_{Q,\gamma}, \phi_{Q,\gamma}) \\
&\cdot K_{\gamma}(x, y, z) dz dy \quad (A-36)
\end{aligned}$$

Finally, local total heat transfer can be expressed in terms of the thermal surface properties, local temperature, scattered thermal intensity, and absorbed solar irradiation as

$$\begin{aligned}
q(x) &= \epsilon_H \bar{\sigma} T^4(x) - \int_{y=0}^L \int_{z=-\infty}^{\infty} \alpha_d(\theta'_{P,\gamma}) I_{t,s}^+(y, z, \theta_{Q,\gamma}, \phi_{Q,\gamma}) \\
&\cdot K_{\gamma}(x, y, z) dz dy - q_{a,so}(x) \quad (A-37)
\end{aligned}$$

For radiatively adiabatic surfaces it is convenient to rearrange the above equations to yield an expression for local equilibrium temperature. Equations (A-35) and (A-36) are unaffected. Setting $q(x) = 0$ in Eq. (A-37) and eliminating temperature between Eqs. (A-37) and (A-34) yields an integral equation for local scattered thermal intensity.

$$\begin{aligned}
 I_{t,s}^+(x, \theta_P, \phi_P) = & \frac{\epsilon_d(\theta_P) q_{a,so}(x)}{\pi \epsilon_H} + \int_{y=0}^L \int_{z=-\infty}^{\infty} \\
 & \cdot \left[\rho_{bd,s}(\theta'_{P,\gamma}, \phi'_{P,\gamma}; \theta_P, \phi_P) \right. \\
 & \left. + \frac{\alpha_d(\theta'_{P,\gamma}) \epsilon_d(\theta_P)}{\pi \epsilon_H} \right] I_{t,s}^+(y, z, \theta_{Q,\gamma}, \phi_{Q,\gamma}) \\
 & \cdot K_\gamma(x, y, z) dz dy
 \end{aligned} \tag{A-38}$$

Local equilibrium temperature follows from Eq. (A-37).

$$\begin{aligned}
 T^4(x) = & \frac{1}{\overline{\sigma} \epsilon_H} \int_{y=0}^L \int_{z=-\infty}^{\infty} \alpha_d(\theta'_{P,\gamma}) I_{t,s}^+(y, z, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_\gamma(x, y, z) dz dy \\
 & + \frac{q_{a,so}(x)}{\overline{\sigma} \epsilon_H}
 \end{aligned} \tag{A-39}$$

Introducing the dimensionless variables

$$H^* = \frac{I_{so,s}^+}{S}, \quad \Theta^4 = \frac{\overline{\sigma} T^4}{S}, \quad H_t = \frac{I_{t,s}^+}{S}, \quad \text{and} \quad \mathcal{Q} = \frac{q_{a,so}}{S},$$

and accounting for symmetry, the following dimensionless equations are obtained:

Dimensionless Absorbed Solar Irradiation

$$\begin{aligned}
 \mathcal{Q}(\eta) = & \alpha_d^*(\theta'_{so}) \sin \frac{\gamma}{2} + \alpha_d^*(\theta'_{so}) \rho_{sp}^*(\theta'_{so}) \sin \frac{\gamma}{2} \\
 & + 2 \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \alpha_d^*(\theta'_{P,\gamma}) H^*(\bar{\eta} = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_{\gamma}(\eta, \xi, \zeta) d\zeta d\xi
 \end{aligned}
 \tag{A-40}$$

Dimensionless Scattered Solar Intensity

$$\begin{aligned}
 H^*(\eta, \theta_P, \phi_P) = & \rho_{bd,s}^*(\theta'_{so}, 0; \theta_P, \phi_P) \sin \frac{\gamma}{2} + \rho_{bd,s}^*(\theta'_{so}, \pi; \theta_P, \phi_P) \\
 & \cdot \rho_{sp}^*(\theta'_{so}) \sin \frac{\gamma}{2} + \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \\
 & \cdot \left[\rho_{bd,s}^*(\theta'_{P,\gamma}, \phi'_{P,\gamma}; \theta_P, \phi_P) \right. \\
 & \left. + \rho_{bd,s}^*(\theta'_{P,\gamma}, 2\pi - \phi'_{P,\gamma}; \theta_P, \phi_P) \right] \\
 & \cdot H^*(\bar{\eta} = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_{\gamma}(\eta, \xi, \zeta) d\zeta d\xi
 \end{aligned}
 \tag{A-41}$$

Dimensionless Scattered Thermal Intensity for Radiatively Adiabatic Surfaces

$$\begin{aligned}
 H_t(\eta, \theta_P, \phi_P) = & \frac{\varepsilon_d(\theta_P) \mathcal{Q}(\eta)}{\pi \varepsilon_H} + \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \left[\rho_{bd,s}^*(\theta'_{P,\gamma}, \phi'_{P,\gamma}; \theta_P, \phi_P) \right. \\
 & + \rho_{bd,s}^*(\theta'_{P,\gamma}, 2\pi - \phi'_{P,\gamma}; \theta_P, \phi_P) \\
 & \left. + \frac{2\alpha_d(\theta'_P) \varepsilon_d(\theta_P)}{\pi \varepsilon_H} \right] H_t(\bar{\eta} = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) K_{\gamma}(\eta, \xi, \zeta) d\zeta d\xi
 \end{aligned}
 \tag{A-42}$$

Dimensionless Equilibrium Temperature

$$\begin{aligned} \Theta^4(\eta) = & \frac{2}{\epsilon_H} \int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \alpha_d(\theta'_P, \gamma) H_t(\bar{\eta} = \xi, \theta_{Q,\gamma}, \phi_{Q,\gamma}) \\ & \cdot K_Y(\eta, \xi, \zeta) d\zeta d\xi + \frac{2(\eta)}{\epsilon_H} \end{aligned} \quad (A-43)$$

The relations presented earlier for K_Y and the directional variables in terms of the coordinates [Eqs. (A-19)-(A-26)] remain applicable.

APPENDIX B

METHOD OF SOLUTION

Results presented in Chapters 5 and 6 were obtained by numerically evaluating expressions derived in APPENDIX A. For uniform temperature surfaces with 45° included angle, dimensionless scattered intensity was evaluated from Eq. (A-18) and dimensionless local heat transfer from Eq. (A-17). For 90° included angle, scattered intensity and local heat transfer were obtained from Eqs. (A-29) and (A-28), respectively. Temperature distributions for radiatively adiabatic, perpendicular surfaces were obtained by evaluating Eqs. (A-41) and (A-40) for scattered solar intensity and local absorbed solar irradiation, respectively. With this information available, scattered thermal intensity was evaluated from Eq. (A-42) and equilibrium temperature from Eq. (A-43).

The integral equations for scattered intensity [Eqs. (A-18), (A-29), (A-41), and (A-42)] were solved using a method of successive approximations. A numerical approach was used because the complexity of the equation appears to preclude analytical methods. The numerical method employed for each of the integral equations was identical and, therefore, is described only for Eq. (A-29). Values for η , θ_p and ϕ_p were selected and the directional emittance evaluated. Next, the integrand was evaluated at a number of specified points (ξ_i, ζ_j) in the region of integration. Since the intensity was initially unknown, the directional emittance for polar angle $\theta_{Q,\gamma}$ was substituted as a first approximation for intensity. Using a quadrature formula to approximate the double integral, the right hand side of Eq. (A-29) was evaluated repeatedly for appropriate values of η , θ_p and ϕ_p until a corrected set of intensity values were obtained. The new values of intensity were then

substituted into the right hand side of the equation and a second set of corrected intensities evaluated. This process was repeated until all values of intensity met a specified convergence requirement. With radiant intensity available, local heat flux follows directly from Eq. (A-28).

Attempts to accurately evaluate the integral term of the governing integral equations by straightforward application of standard quadrature formulae were unsuccessful. Errors of unacceptable magnitude were obtained for surface elements near the common edge. These errors appear to arise because of the very strong dependence of the geometry kernel $K(\eta, \xi, \zeta)$ on the position variables when the points $(\eta, 0)$ and (ξ, ζ) are both near the apex. To avoid these errors it was necessary to develop a special quadrature formula. This formula is conveniently described if it is noted that the integral term in the governing intensity equations is of the general form

$$\int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \mathcal{R}(\eta, \theta_p, \phi_p, \xi, \zeta) H(\eta, \xi, \zeta) K(\eta, \xi, \zeta) d\zeta d\xi \quad (B-1)$$

Dimensionless scattered intensity and dimensionless geometry kernel are denoted by $H(\eta, \xi, \zeta)$ and $K(\eta, \xi, \zeta)$, respectively. The symbol $\mathcal{R}(\eta, \theta_p, \phi_p, \xi, \zeta)$ represents a general radiation property function, and the form of $\mathcal{R}$ for a particular equation may be obtained by inspection. The simplified form of the arguments was chosen to emphasize that H and K depend only on η , ξ , and ζ ; whereas, the property function $\mathcal{R}$ may also depend on θ_p and ϕ_p . After numerous attempts to develop a satisfactory quadrature formula, the following method was found to yield satisfactory results. The region of integration was divided into four sections as shown in Fig. B-1, and the integral written as the sum of four terms.

$$\begin{aligned}
\int_{\xi=0}^1 \int_{\zeta=0}^{\infty} \mathcal{R}(\eta, \theta_p, \phi_p, \xi, \zeta) H(\eta, \xi, \zeta) K(\eta, \xi, \zeta) d\zeta d\xi &= \int_{\xi=0}^a \int_{\zeta=0}^b \mathcal{R} H K d\zeta d\xi \\
&+ \int_{\xi=a}^1 \int_{\zeta=0}^b \mathcal{R} H K d\zeta d\xi + \int_{\xi=a}^1 \int_{\zeta=b}^{\infty} \mathcal{R} H K d\zeta d\xi \\
&+ \int_{\xi=0}^a \int_{\zeta=b}^{\infty} \mathcal{R} H K d\zeta d\xi \quad (B-2)
\end{aligned}$$

Integrand arguments have been excluded for brevity. The terms on the right hand side of Eq. (B-2) represent, in the order in which they appear, the contribution of region one through four. Standard Gaussian quadrature formulae [70] were applied to evaluate the integrals for regions two through four. Five point formulae were used for the integrations in region two. In regions three and four integrations over ξ and ζ were accomplished with five and ten point formulae, respectively. An alternate procedure was used for region one. This region was divided into twenty-five finite surface elements (five rows and five columns). The scattered intensity leaving each of these elements was assumed uniform over the extent of each element, and the double integral was approximated by a double summation

$$\begin{aligned}
\int_{\xi=0}^a \int_{\zeta=0}^b \mathcal{R}(\eta, \theta_p, \phi_p, \xi, \zeta) H(\eta, \xi, \zeta) K(\eta, \xi, \zeta) d\zeta d\xi \\
= \sum_{i=1}^5 \sum_{j=1}^5 \bar{\mathcal{R}}(\eta, \theta_p, \phi_p, \xi_i, \zeta_j) H(\eta, \xi_i, \zeta_j) FF_{dA(\eta)-\Delta A(\xi_i, \zeta_j)}
\end{aligned} \quad (B-3)$$

The symbol $FF_{dA(\eta)-\Delta A(\xi_i, \zeta_j)}$ denotes the diffuse angle factor from a differential surface element $dA(\eta)$ at the point $(\eta, 0)$ to a finite surface

element $\Delta A(\xi_i, \zeta_j)$ at the point (ξ_i, ζ_j) . It is defined as

$$FF_{dA(\eta)-\Delta A(\xi_i, \zeta_j)} = \frac{1}{\pi} \iint_{\Delta A(\xi_i, \zeta_j)} K(\eta, \xi, \zeta) dA(\xi, \zeta) \quad (B-4)$$

The geometry weighted property function $\bar{R}$ in Eq. (B-3) is defined by the equation

$$\bar{R}(\eta, \theta_p, \phi_p, \xi_i, \zeta_j) = \frac{\iint_{\Delta A(\xi_i, \zeta_j)} R(\eta, \theta_p, \phi_p, \xi, \zeta) K(\eta, \xi, \zeta) dA(\xi, \zeta)}{FF_{dA(\eta)-\Delta A(\xi_i, \zeta_j)}} \quad (B-5)$$

Equation (B-4) was analytically evaluated to yield numerical values for the required diffuse angle factors. Hence, Eq. (B-3) represents an approximation to the integral over region one which circumvents the problem associated with numerically integrating the geometry kernel for each iteration.

Even though concerted attempts were made to keep the number of quadrature points to as small a value as possible, several complications arose because of the large number of property values required to evaluate radiant intensity. As an example, for 45° included angle it was necessary to provide the intensity program with approximately 2×10^6 bidirectional reflectance values for each set of surface parameters. To avoid recalculating these BDR values each iteration, they were permanently stored on magnetic tape. Special tape manipulating programs were written to provide efficient access to the stored BDR values by the intensity program. These tape handling programs were designed by Professor L. D. Savage of the Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign.

The numerical procedure was verified by calculating local heat

flux and equilibrium temperature distribution with selected property models and comparing them with results available in the literature. For both values of included angle considered in this study, local heat flux was calculated for materials with properties represented by the CSD model for selected values of surface specularity. Only for a low emittance, diffusely reflecting material and 45° included angle at points near the apex were the results distinguishable from published results. The maximum error then was no more than 2 percent. A more critical test of the numerical method was provided by comparing present results obtained using the DSD model for specularly reflecting surfaces with those reported by Hering [49]. Generally, the results were in excellent agreement. The largest discrepancies occurred for the smaller included angle and high emittance material at surface elements near the apex. The error, however, was less than 3 percent. Equilibrium temperature results for properties represented by the semi-gray CSD property model were compared to those reported by Hering [2]. Again, excellent agreement was obtained with the largest errors occurring near the apex. These errors were limited in magnitude to about 1 percent. As a result of these numerical experiments it is estimated that the results presented are accurate to within at least 3 percent.

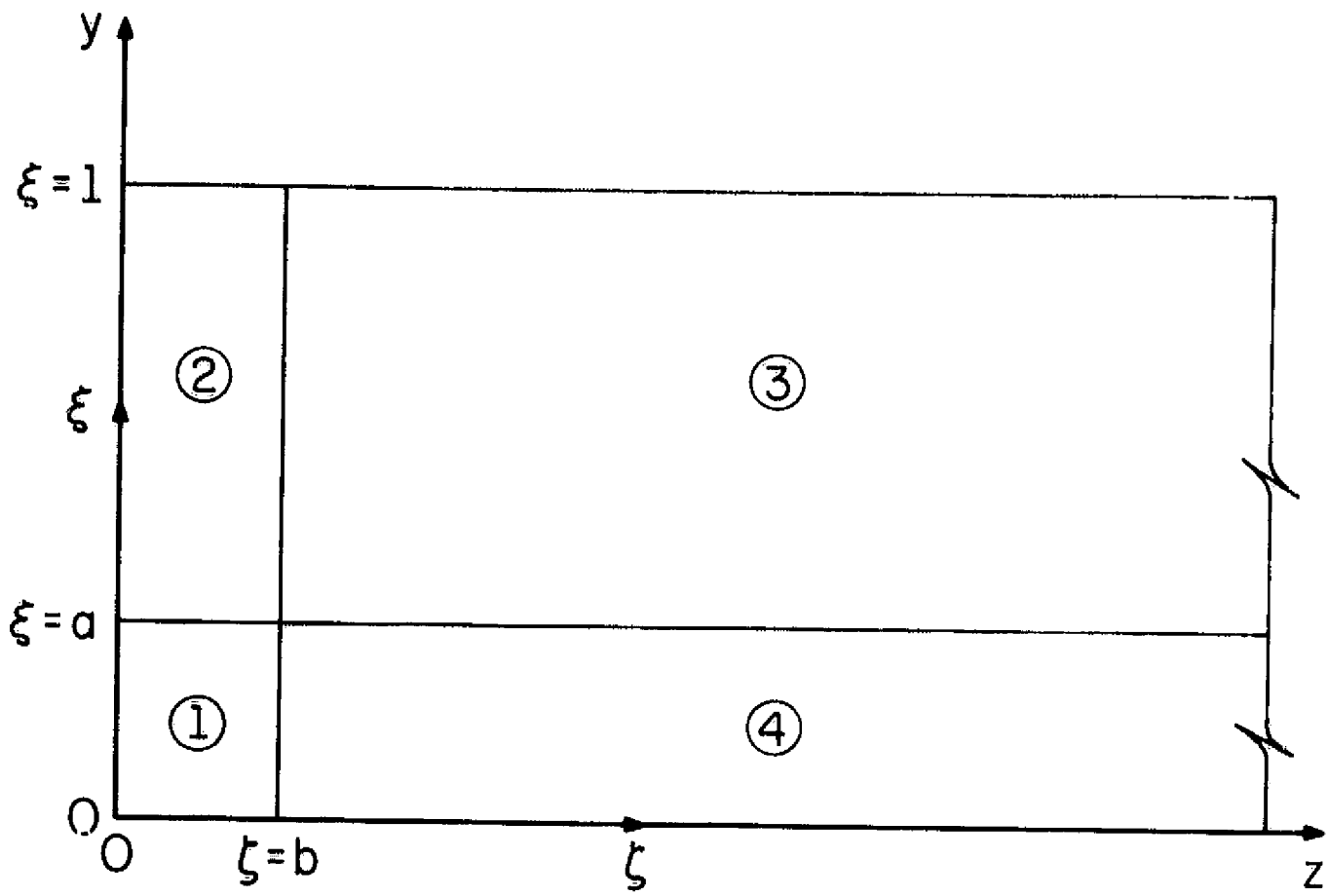


Figure B-1. Subdivision of region of integration