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A "SOURCE SURFACE THEORY" COROLLARY: THE MEAN SOLAR FIELD-INTERPLANETARY FIELD CORRELATION

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Abstract

The correlation of the mean solar magnetic field and the interplanetary magnetic field reported by Wilcox et al., (1969) and Severny et al., (1970) has been interpreted by comparing the relationship of the measurement of the mean solar field with the physics involved in the formation of the interplanetary field. The high correlation observed is thus interpreted as a fortuitous correspondance between two integrals. The high correlation thus provides further support for the source surface model involved in these calculations. A new method is then suggested for observing the "mean solar field" that might improve the correlations slightly.

Recently Wilcox et al. (1969) have reported a correlation of the daily mean magnetic field of the sun (seen as a star) with the polarity of the interplanetary magnetic field observed 4½ days later near the earth for the interval March-June 1968. They report an almost complete agreement. These observations are quite surprising since the mean solar field is an average of the sun's field over a entire hemisphere (13½ days by solar rotation) whereas the interplanetary field is thought to be a direct extension of the solar field, thus not correlating with the mean field from an entire solar hemisphere. This paper attempts to explain these startling observations using the "source surface" model of Schatten et al. (1969).

In this model a potential field exists close to the sun. Beyond some distance, about 0.6 solar radii above the photosphere, the solar wind plasma begins to convect the magnetic field outward. This model has been tested by comparisons of solar eclipse structure from 1-3 solar radii, of Faraday rotation measurements of the coronal field from 4-12 solar radii, and of interplanetary magnetic field observations near the earth at 1 AU with computations from photospheric field observations; see Schatten et al. (1969), Stelzried et al. (1970), Schatten (1969, 1970) and Smith and Schatten (1970).

Figure 1 illustrates the manner in which the source surface model suggests the mean solar field - interplanetary field correlation. The mean solar field represents an average of the photospheric field over the solar disk with an appropriate weighting factor. This factor is a function of the spherical angle from that portion of the photosphere to the subsolar point. The main contributions to this factor are an area projection factor due to the difference between the magnetograph measuring the line-of-sight magnetic field and the angular distribution of the direction of the photospheric field (perhaps radial on the average). Limb darkening and effects of sunspots, not seen by the magnetograph, are also contributing factors.

The "source surface" model states that the interplanetary field near the earth results from the "source surface" field convected by the solar wind outward in about $4\frac{1}{2}$ days. Thus the field at the earth is the extended field from position A in Figure 1. The field at position A may be computed in this model as an integral of the photospheric field. This integral also has a weighting factor as a function of angle from the subsolar point and is quite similar to the mean solar field integral. The similarity of these two integrals results in the surprising correlation reported by Wilcox et al.(1969). Note that it is not important wher the footpoint in the photosphere is, for the particular field line, but only that it's direction be determined by the weighted photospheric field.

The similarity between these integrals will now be demonstrated. From Schatten et al. (1969) the "source surface" field is given by the following expression:

$$\vec{B}_n\,(\theta,\varphi;R_s) = \int \vec{B}_n(\theta,\varphi,\theta',\varphi';\,R_s)\cdot M\,(\theta',\varphi')d\Omega'$$

where $$\vec{B}_n = -\frac{M\,\hat{r}}{R_s^{\,2}}\;\frac{R_s}{R_\odot}\left(1-\frac{R_s^{\,2}}{R_\odot^{\,2}}\right)\Big/\left(1+\frac{R_s^{\,2}}{R_\odot^{\,2}}-\frac{2R_s}{R_\odot}\cos\gamma\right)^{3/2}$$

γ is the angle from any point in the photosphere to the subsolar point; R_s is the source surface radius and M is a flux source in the photosphere.

The source surface in 1965 was set at 1.6 solar radii. If we choose this value: $\vec{B}_n = M\,\hat{r}\;\;0.975\;(3.56-3.2\,\cos\gamma)^{-3/2}$. A source surface radius of 2.0 solar radii, used more recently in comparison with eclipse structures, results in the equation:

$$\vec{B}_n = M\,\hat{r}\;\;1.5\;(\,5-4\,\cos\gamma)^{-3/2}$$

The interplanetary field $4\frac{1}{2}$ days later is the extended source surface field diminished in intensity by approximately $\sqrt{2}\, R_s{}^2/(215\, R_\odot)^2$. This allows for a radial expansion of the source surface field and an enhancement by the square root of two to account for the azimuthal component of the interplanetary field due to solar rotation.

Thus we obtain

$$(1) \quad \vec{B}_{INT} = \vec{B}_n \sqrt{2}\, \frac{R_s{}^2}{(215\, R_\odot)^2} = \frac{\sqrt{2}}{(215)^2}\, \frac{\int_{sol.\ surf.} \vec{B}_n\, (R_s/R_\odot)^2\, d\Omega}{\int_{sol.\ surf.} d\Omega}$$

$$= \frac{\sqrt{2}}{4\pi(215)^2} \int_0^\pi \vec{B}_n \left(\frac{R_s}{R_\odot}\right)^2 2\pi \sin\gamma\, d\gamma$$

$$= \frac{\sqrt{2}}{2(215)^2} \int_0^\pi \vec{B}_{sf}\ (\text{weighting factor})\, d\gamma$$

where (weighting factor) = $-\sin\gamma (R_s/R_\odot)\ (1-(\frac{R_s}{R_\odot})^2)/(1+\frac{R_s^2}{R_\odot^2}-\frac{2R_s}{R_\odot}\cos\gamma)^{3/2}$

The weighting factor is shown in Figure 2 for R_s=1.6 and 2.0 $R_\odot$. The half widths of a bipolar magnetic region (BMR) and a unipolar magnetic region (UMR) are shown on this graph. Bumba and Howard (1965 and 1966) discuss the development of solar fields. A BMR has two roughly equal and opposite flux sources and thus does not contribute much to the total integral. A UMR contributes substantially to the total integral.

The association made by Wilcox and Ness (1965) of 'UMR's with interplanetary sectors thus seems valid. The integral thus becomes approximately $2.3\langle\vec{B}_{sf}\rangle$ for the 1.6 solar radii source surface and $2.0 \langle \vec{B}_{sf} \rangle$ for the 2 solar radii source surface.

$\langle \vec{B}_{sf} \rangle$ refers to the solar field weighted by these integrals. Thus

$$\vec{B}_{INT} = \frac{\sqrt{2}}{2(215)^2} \; 2.0 \langle \vec{B}_{sf}\rangle$$

$$= 3.0 \times 10^{-5} \; \langle\vec{B}_{sf}\rangle \text{ for } 1.6\ R_\odot$$

and $\vec{B}_{INT} = 2.4 \times 10^{-5} \langle \vec{B}_{sf} \rangle$ for $2.0\ R_\odot$

Evaluating the mean solar field integral we obtain

$$\text{(2)} \qquad \vec{B}_{MSF} = \frac{\int_{\text{sol. disk}} \vec{B}_{sf} \cdot (I) \cdot (\text{line of sight})\, dA}{\int_{\text{sol.disk}} (I)\, dA}$$

$$= \frac{\bar{I} \int_{\frac{1}{2}\text{ sol.surf.}} \vec{B}_{sf} \; \cos\gamma \;\; 2\pi\, r \;\; dr}{\bar{I} \;\; 2\pi\, R_\odot^{\;2}}$$

$$= \frac{1}{2\pi R_\odot^{\;2}} \int_0^{\pi/2} \vec{B}_{sf} \cos\gamma \; 2\pi\, R_\odot \sin\gamma \; R_\odot \cos\gamma \; d\gamma$$

$$= \int_0^{\pi/2} \vec{B}_{sf} \cos^2\gamma \sin\gamma \; d\gamma$$

$$\cong 1/3 \langle \vec{B}_{sf} \rangle$$

The solar field weighted by this integral is shown in Figure 2 also. It resembles the 2.0 solar radii integral more closely than the 1.6 solar radii integral although it is a good approximation for either in the region where the solar fields themselves are well correlated (up to 45°

in γ). The effect of limb darkening is negligible. The influence of a nonradial photospheric magnetic field has not been included. An isotropic photospheric field would only slightly modify the shape of the weighting integral because only one power of $\cos\gamma$ would result. The value of the integral would increase to $0.5\langle B_{sf}\rangle$. Combining these two equations we obtain:

$$\vec{B}_{INT} = 3.0 \times 10^{-5}\,(3)\,\vec{B}_{MSF}$$
$$\cong 9 \times 10^{-5}\ \vec{B}_{MSF} \quad \text{for } 1.6\ R_\odot$$

and

$$\vec{B}_{INT} = 7.2 \times 10^{-5}\ \vec{B}_{MSF} \quad \text{for } 2.0\ R_\odot$$

Thus there is a very direct relationship in polarity and in magnitude between the mean solar field and the observed interplanetary field with a $4\frac{1}{2}$ day delay. Recently Severny et al. (1970) has shown a further correlation between the magnitude of the two fields with an 8×10^{-5} Gauss interplanetary field comparing with a 1 Gauss photospheric field. This fits in very well with the above analysis. The startling agreement between the interplanetary field and the mean photospheric field thus represents a fortunate coincidence between the source surface weighting factor and the integrated line-of-sight disk factor.

The author suggests that if a radial density filter were employed in observing the "mean solar field", that would allow the photospheric observations to resemble more closely the source surface curves in Figure 2, the agreement between interplanetary and solar field comparisons would improve. Figure 3 shows the shape of transmission for such filters corresponding to 1.6 $R_\odot$ and 2.0 $R_\odot$ source surfaces.

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FIGURE CAPTIONS

Figure 1 Relationship between mean solar field, source surface field and interplanetary field. The mean solar field is a weighted average of the disk field (indicated by the shading). The source surface field is the magnetic field on the source surface, position A. This is computed from a weighted average of the photospheric field, quite similar to the mean solar field. The solar wind convects this field to the earth in about $4\frac{1}{2}$ days while solar rotation twists the field to approximate an archimedes spiral as shown.

Figure 2 Weighting factor for source surface integrals and mean solar field integral. Note that the shape of the mean solar field weighting factor is very similar to the 2.0 solar radii source surface factor. The half width of a bipolar magnetic region and unipolar magnetic region are shown to indicate the scales over which the photospheric fields are well correlated.

Figure 3 Transmission functions for filters that would allow the mean solar field observations to more closely correspond to source surface integrals. Use of such filters might improve correlations with interplanetary field observations.

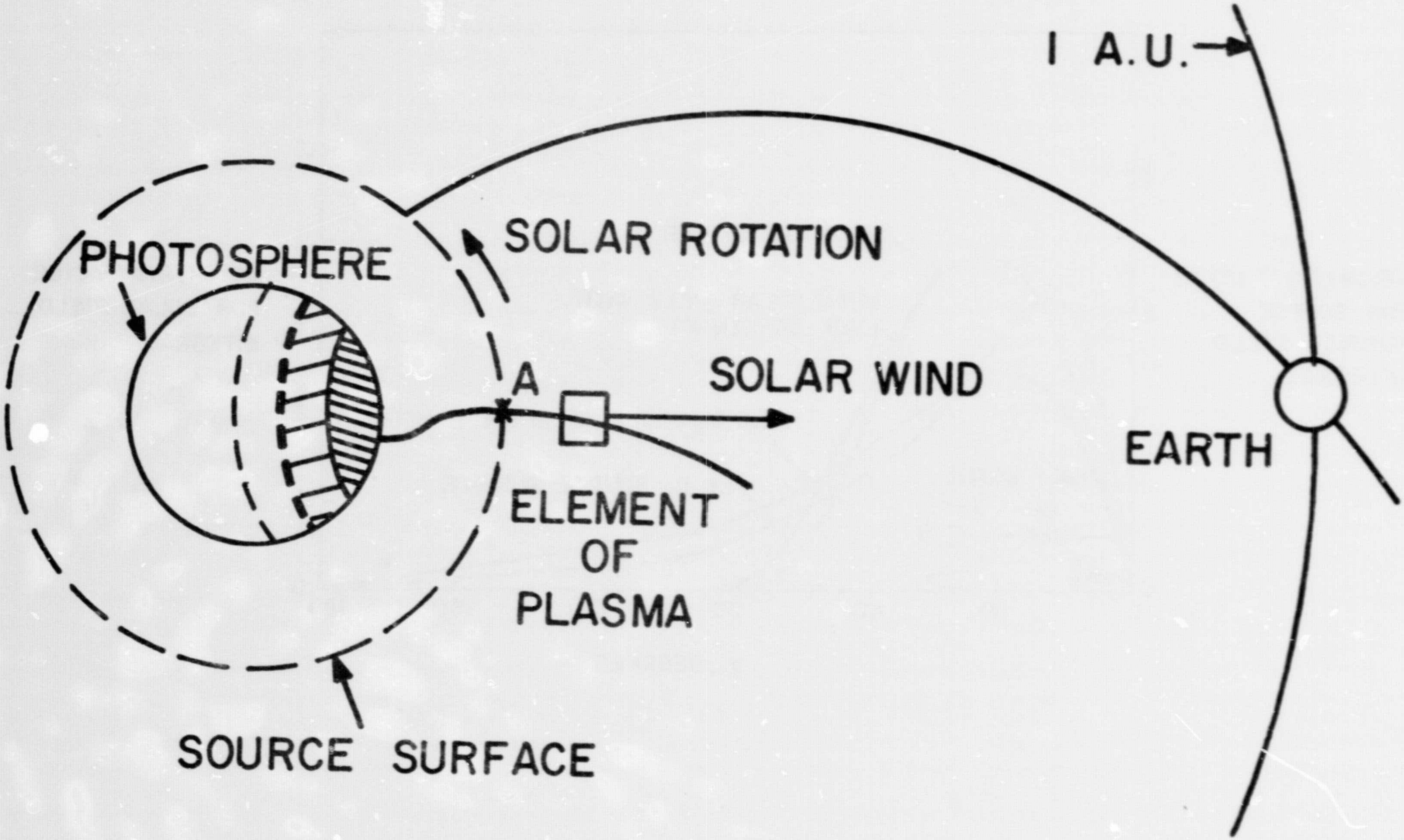

I A.U.
SOLAR ROTATION
PHOTOSPHERE
A
SOLAR WIND
EARTH
ELEMENT
OF
PLASMA
SOURCE SURFACE

FIGURE 1

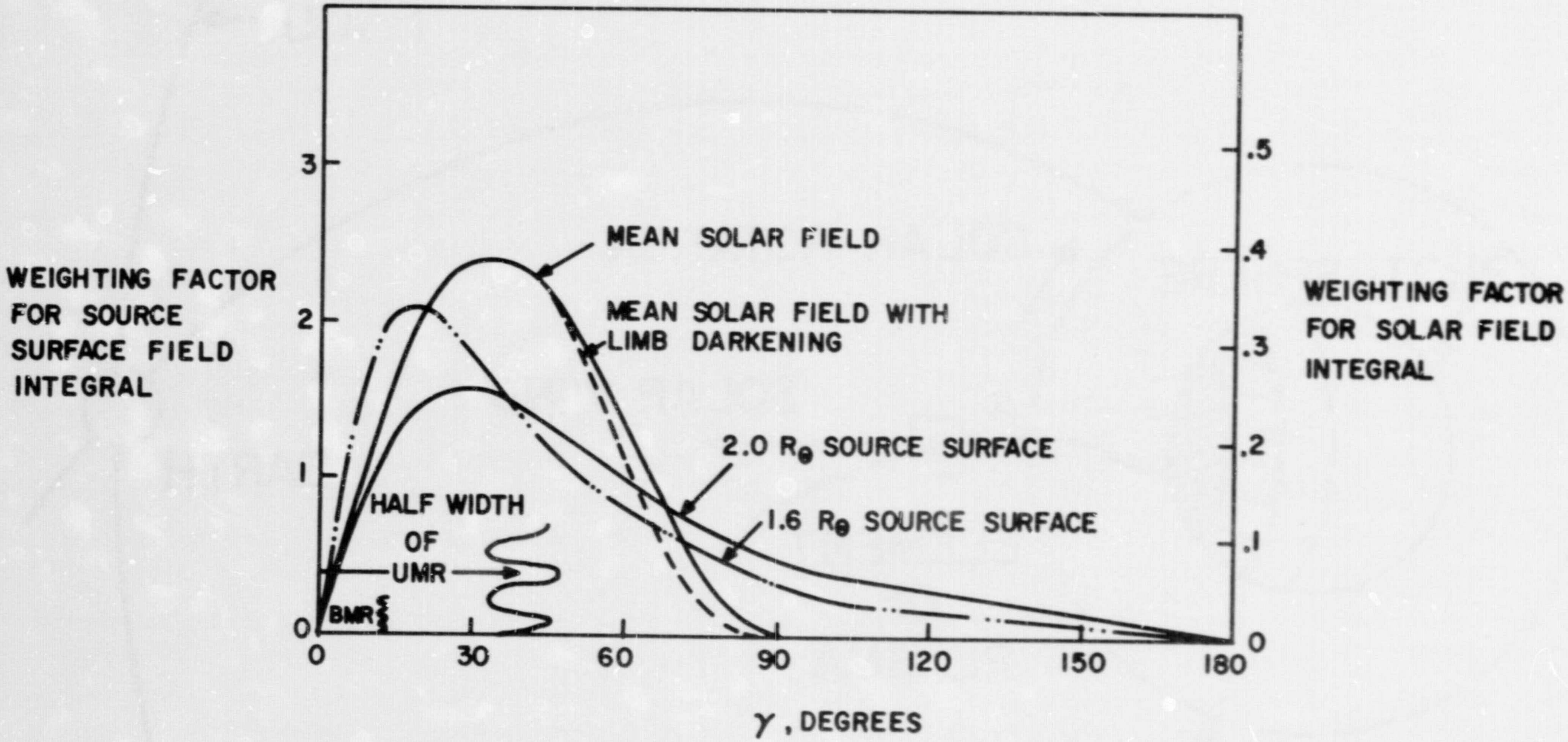
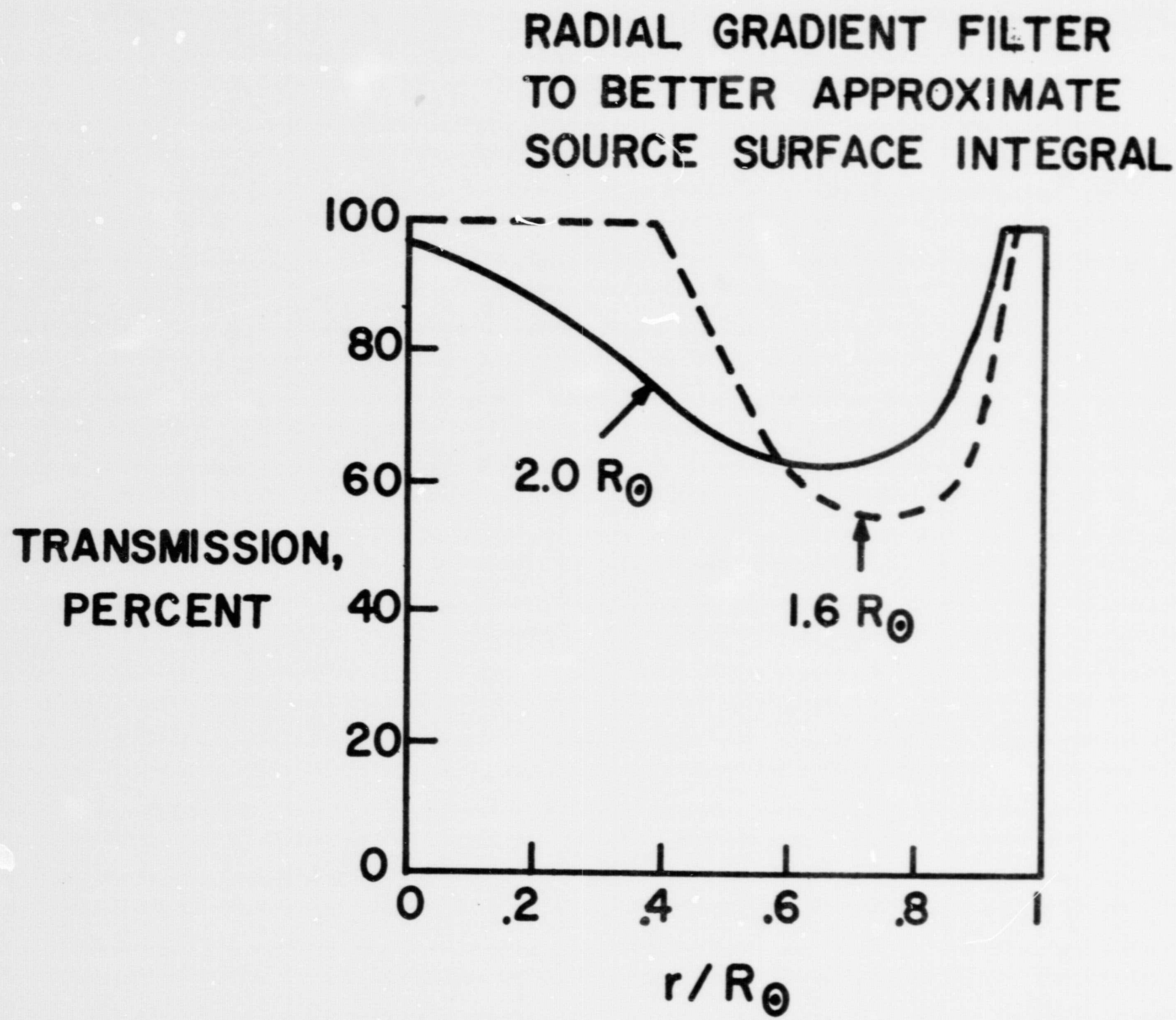

WEIGHTING FACTOR FOR SOURCE SURFACE FIELD INTEGRAL
WEIGHTING FACTOR FOR SOLAR FIELD INTEGRAL
MEAN SOLAR FIELD
MEAN SOLAR FIELD WITH LIMB DARKENING
2.0 R⊙ SOURCE SURFACE
1.6 R⊙ SOURCE SURFACE
HALF WIDTH OF UMR
BMR
γ, DEGREES
0
30
60
90
120
150
180
1
2
3
.1
.2
.3
.4
.5

FIGURE 2


RADIAL GRADIENT FILTER
TO BETTER APPROXIMATE
SOURCE SURFACE INTEGRAL
TRANSMISSION,
PERCENT
100
80
60
40
20
0
.2
.4
.6
.8
1
2.0 $R_\odot$
1.6 $R_\odot$
r/$R_\odot$