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SITE ACCESSIBILITY AND CHARACTERISTIC VELOCITY REQUIREMENTS FOR DIRECT-DESCENT LUNAR LANDINGS

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16. Abstract

A direct descent is one in which the main descent propulsion system burns continuously from lunar approach to touchdown. The characteristic velocity requirement for direct lunar descents is presented as a function of the landing site location relative to the normal impact point. Results are included for translunar trip times of 60, 75, and 90 hours, for specific impulses representative of both Earth storable and cryogenic propulsion systems, for landing sites located anywhere on the lunar surface, and for ignition thrust-to-Earth weight ratios between 0.12 and 10.0. The data presented are useful in determining approximate performance capability and in evaluating tradeoffs during preliminary mission planning studies.

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SITE ACCESSIBILITY AND CHARACTERISTIC VELOCITY REQUIREMENTS FOR DIRECT-DESCENT LUNAR LANDINGS

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SUMMARY

Direct-descent lunar landings are shown to be possible for landing sites located at any point on the lunar surface. The direct descent characteristic velocity requirement is shown as a function of landing site location for various combinations of descent engine specific impulse, translunar trip time, and initial thrust to Earth-weight ratio values.

The landing site location is referenced to the normal impact point of the incoming trajectory. A means of estimating the normal impact point is provided for cases in which it is not known.

A number of typical descent trajectories are depicted graphically. The time histories of several important trajectory parameters are presented and discussed.

INTRODUCTION

Evaluation of the performance capability of a specific vehicle for a lunar landing mission requires the selection of a basic descent mode. Possible descent modes can be grouped into three general categories: (1) multiple-orbit descents, (2) single-orbit descents, and (3) direct descents. In the multiple-orbit descent mode there is an initial high altitude orbit, at least one intermediate orbit at a lower altitude, and a main powered descent to touchdown on the lunar surface. This mode is used in the Apollo manned lunar landing program. If all but the final orbit are eliminated, the result is a single-orbit descent. The vehicle brakes directly into a low altitude orbit from which the main powered descent to the surface is initiated. The single-orbit mode is proposed for use in a lunar logistics vehicle program in reference 1. A direct descent is one in which the spacecraft does not enter lunar orbit prior to touchdown. The direct-descent mode was used in the Surveyor program and is the mode of interest in this report.

The main advantage of a direct descent is the simplicity associated with requiring only one burn of the main descent propulsion system. The main disadvantage of this mode is that (since the perilune distance is usually less than the lunar radius) the vehicle approaches the moon on a collision course and will impact the lunar surface if the descent engine fails. For this reason the direct-descent mode is not usually considered for manned missions. Because of its simplicity, however, this mode is usually given some consideration during the preliminary planning phase of any unmanned soft lunar landing program. An evaluation of the vehicle's direct-descent performance capability then becomes necessary.

The characteristic velocity ΔV (all symbols are defined in appendix A) required to accomplish a direct-descent soft lunar landing at a prescribed landing site is a function of (1) Earth, moon, and transfer-orbit geometrical characteristics such as the transfer orbit vis viva energy $C_{3,e}$, the location of the landing site, the current Earth-Moon geometry and (2) vehicle propulsion characteristics such as the specific impulse I_{sp} of the descent propulsion system and the ratio of thrust-to-vehicle weight at descent engine ignition $(F/W)_i$ (All thrust-to-weight ratio values are based on equivalent Earth weight). In the preliminary planning phase of a program, the values of some of these variables are not well known: estimates change frequently and over relatively wide ranges of values, and performance analysis is time consuming. The results included herein are intended to facilitate the estimation of performance capability and to permit an evaluation of the effect of changes in many of the above variables.

The ΔV requirement for direct descents is shown as a function of landing site location for lunar approach trajectory vis viva energy levels of 1.78, 1.09, and 0.82 kilometers squared per second squared (translunar trip times of approximately 60, 75, and 90 hr, respectively), descent engine I_{sp} values of 300 and 440 seconds (typical of Earth storable and cryogenic propellants, respectively), and $(F/W)_i$ values of 0.12, 0.15, 0.25, 0.50, 1.00, and 10.0.

ANALYSIS

Procedure

The mathematical problem is to determine the optimum constant thrust direct-descent maneuver from a lunar approach trajectory of known energy with a specified normal impact point (NIP) to a specified landing site on the lunar surface. The perilune distance of the approach trajectory and the point at which the powered descent is initiated are not specified and are available for optimization. The thrust direction during the powered descent is also unspecified and may be optimized.

Consider the case of the specified landing site being at the NIP. Then the translunar

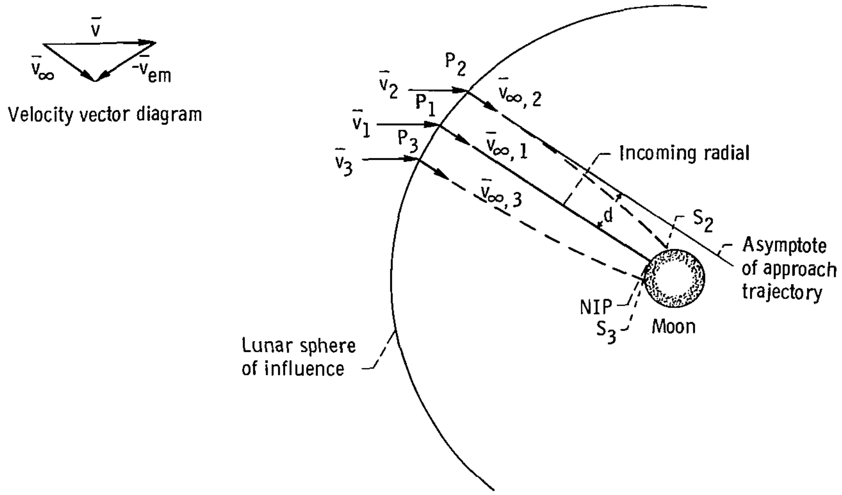


Figure 1. - Schematic of lunar approach trajectory geometry. Velocity of spacecraft with respect to Earth at lunar sphere of influence, $\bar{v}$; velocity of Moon with respect to Earth, v_{em}; velocity of spacecraft with respect to Moon, $\bar{v}_\infty$.

trajectory would be shaped as shown by trajectory 1 in figure 1. The spacecraft reaches the lunar sphere of influence with a velocity $\bar{v}_1$ at the point P_1. The velocity of the Moon relative to Earth $\bar{v}_{em}$ is subtracted vectorially from $\bar{v}_1$ to give

$$\bar{v}_{\infty,1} = \bar{v}_1 - \bar{v}_{em}$$

which is the spacecraft velocity relative to the moon. As shown in figure 1, $\bar{v}_{\infty,1}$ is in the incoming radial direction. If the constant thrust force of the vehicle is applied at the appropriate time and in the radial direction the vehicle will decelerate to a soft landing at the NIP. The perilune distance of the approach trajectory is zero in this case.

Suppose now that the desired landing site is at the point S_2 of figure 1 instead of at the NIP. The trajectory could again be shaped to arrive at the sphere of influence at P_1, approach the moon along the incoming radial until descent engine ignition, and then employ a thrust direction profile which would allow a landing at S_2. However, it is less costly, from a ΔV standpoint, to increase the perilune distance by reaching the sphere of influence at P_2 and following the dashed approach path to S_2. Since the NIP is specified, the incoming radial of trajectory 2 is the same as that of trajectory 1. The asymptote of trajectory 2 is parallel to the incoming radial, and corresponds to an asymptote translation of magnitude d. The translunar trajectory can be designed to intersect the sphere of influence at P_2 instead of at P_1 with a velocity $\bar{v}_2$ such that

$$\bar{v}_{\infty,2} = \bar{v}_2 - \bar{v}_{em} = \bar{v}_{\infty,1}$$

by making slight adjustments in the Earth injection conditions and injection time. Lunar approach trajectories 1 and 2 have the same energy and the same NIP but the descent ΔV requirement for landing at S_2 is much lower for trajectory 2. The perilune distance of trajectory 2 is greater than zero. The change in perilune distance is equivalent to an asymptote translation of magnitude d in the direction of S_2 from the NIP. The distance d is optimized in order to minimize the required descent ΔV.

Now consider the problem of landing at the site S_3 of figure 1. Site S_3 is the same distance as S_2 from the NIP but in a different (not necessarily opposite) direction. The minimum ΔV descent to S_3 obviously requires a lunar approach path having an asymptote translated by the amount d from the incoming radial in the direction of S_3. The characteristics of the optimum descent from trajectory 3 to landing site S_3 are identical to those of the optimum descent from trajectory 2 to site S_2 (assuming a spherical, nonrotating moon). The minimum descent ΔV requirement is a function of the distance but not of the direction of the specified landing site from the NIP. The mathematical problem is therefore two-dimensional with all motion confined to the plane formed by the incoming radial and the approach trajectory asymptote. The descent ΔV requirements are shown as a function of the distance of the landing site from the NIP in this report.

The descent paths are obtained by numerical integration of the two-dimensional descent trajectory equations of motion. For convenience, the integration starts on the lunar surface and terminates on an approach trajectory having the specified energy and NIP location. Consequently, the initial integration conditions correspond to trajectory conditions at touchdown, and the final integration conditions are those of the approach trajectory. The optimum instantaneous thrust direction ψ and its time rate of change $\dot{\psi}$ are determined by Lagrange multiplier calculus of variations techniques as in reference 3. The required optimum solution and minimum ΔV requirement are obtained by solving the resulting two point boundary value problem by finite-difference Newton-Raphson iteration techniques. The mathematical details of the boundary value problem are presented in appendix B.

The location of the NIP itself is a function of the Earth-Moon geometry, the energy of the translunar trajectory $C_{3,e}$ and the relative inclination Δi between the Moon's orbit plane and the trajectory plane. It is assumed that the reader will have independent knowledge of the location of the NIP before attempting to use the results of this report for performance evaluations. A procedure for estimating the NIP location follows.

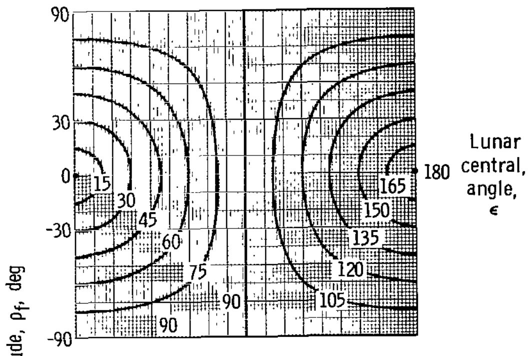
Normal Impact Point

The ΔV required for a direct-descent lunar landing depends on the angular distance ϵ from the NIP to the landing site and is shown as a function of ϵ in this report. If the locations of both the NIP and the desired landing site are known, ϵ can be calculated as

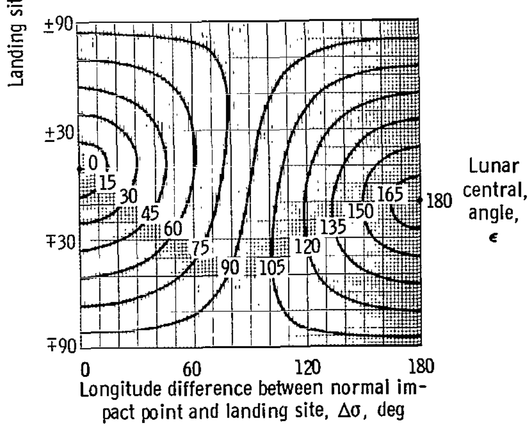
$$\cos \epsilon = \sin \rho_n \sin \rho_f + \cos \rho_n \cos \rho_f \cos \Delta\sigma$$

where ρ_n and ρ_f are the latitudes of the NIP and the landing site, respectively, and $\Delta\sigma$ is the longitudinal difference between the NIP and the landing site. For convenience, the value of ϵ is shown for all combinations of ρ_f and $\Delta\sigma$ in figure 2 for $\rho_n = 0^o$ and $\pm 10^o$.

If the NIP location is not known explicitly, it can be estimated by assuming that (1) the Moon moves about the Earth in a circular orbit, (2) the lunar equator is in the orbit plane of the Moon, (3) the Earth-Moon line intersects the lunar surface at a lunar latitude ρ and longitude σ of zero, and (4) the mass of the Moon does not have a first-order effect on the NIP location. These are minor approximations and provide adequate ac-



(a) Normal impact point latitude, 0.



(b) Normal impact point latitude, $\pm 10^{\circ}$.

Figure 2. - Angular distance of landing site from normal impact point for all combinations of landing site latitude and longitude.

curacy for the type of planning estimates for which these data are intended. The NIP location can then be determined analytically. Details of the calculation are included as appendix C. The longitude and latitude of the NIP are shown in figures 3(a) and (b), respectively, as a function of $C_{3,e}$ and Δi for several values of r_{em}. The vis-viva energy of the lunar approach trajectory ($C_{3,m} = v_\infty^2$) is given in figure 3(c) as a function of $C_{3,e}$ for the same r_{em} values.

Trip Time

Analysis of the boundary value problem and conditions shows that, for a vehicle with specified I_{sp} and fixed $(F/W)_i$, the descent ΔV requirement depends only on the landing site location and the energy of the lunar approach trajectory. As discussed previously, the landing site location is specified with respect to the NIP. The energy $C_{3,m}$ can be related to the translunar trip time. The actual trip time depends on the specific trajectory, Earth-Moon geometry, etc. A good approximation of the actual trip time is obtained by simply calculating the two-body trip time t_c which is the time from Earth perigee passage to a radius equal to r_{em}. The value of t_c is shown as a function of $C_{3,e}$ for representative values of r_{em} in figure 3(d). Two-body trip time values of 60,

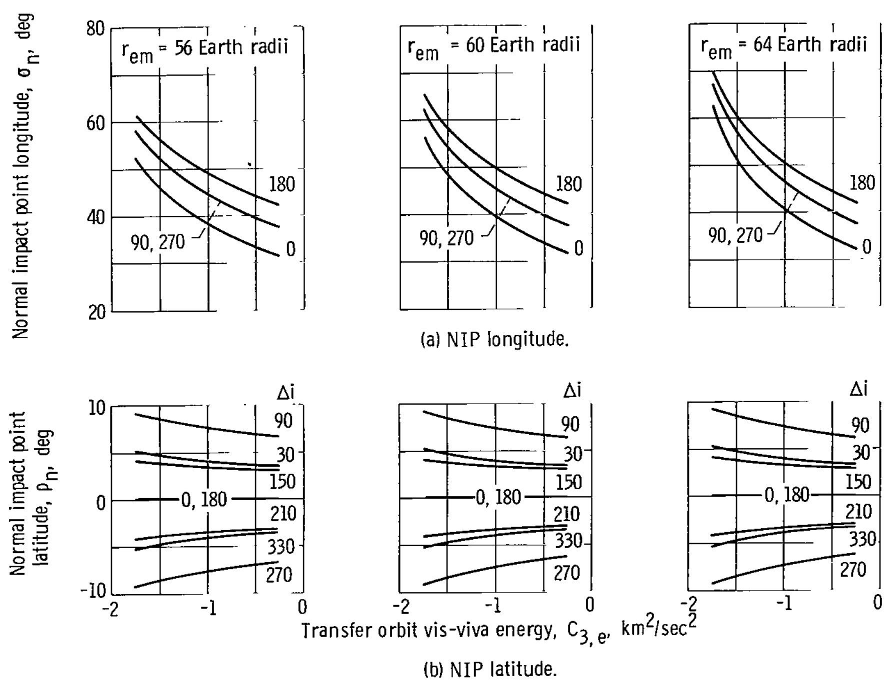
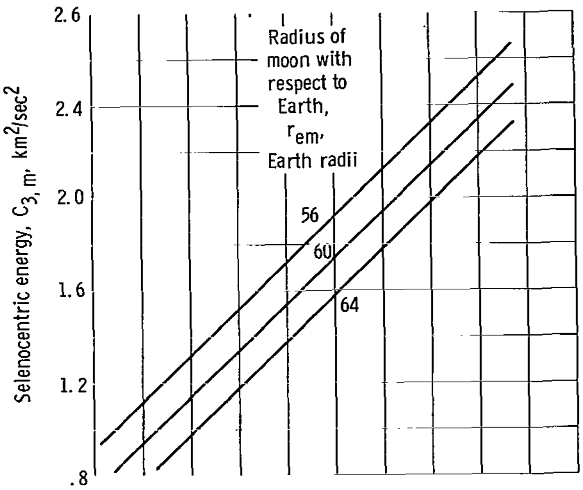
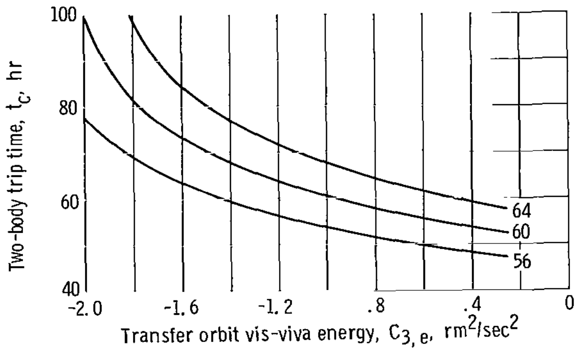


Figure 3. - Normal impact point longitude and latitude, selenocentric energy and two-body trip time as function of transfer orbit energy for several values of Earth-Moon distance.



(c) Selenocentric energy.



(d) Two-body trip time.

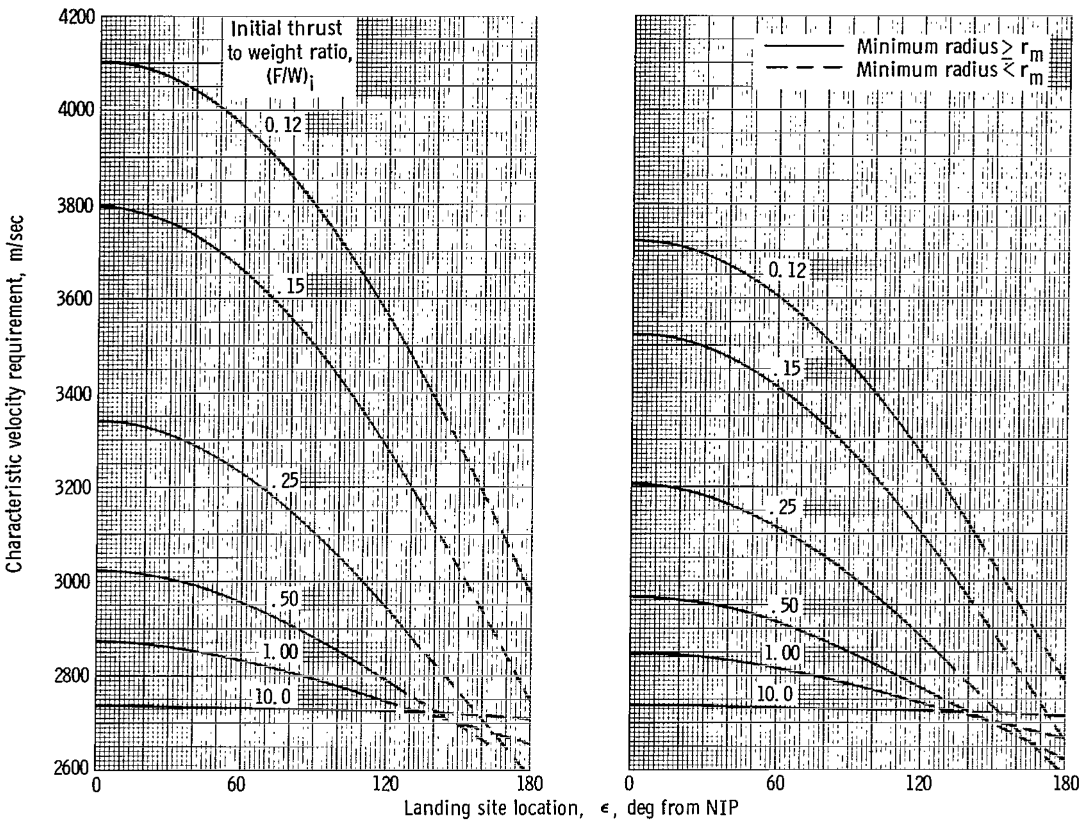
Figure 3. - Concluded.

75, and 90 hours and a lunar distance of 60 Earth radii are considered representative of the range of interest and correspond to the values of $C_{3,m}$ used as parameters in the RESULTS section.

RESULTS

Descent Characteristic Velocity Requirements

The ΔV requirement for direct-descent lunar landings for $C_{3,m} = 1.78$ kilometers squared per second squared (60-hr translunar trip time with $r_{em} = 60$ Earth radii) and a descent engine I_{sp} of 440 seconds is shown as a function of ϵ in figure 4(a). Results are included for $(F/W)_i$ values of 0.12, 0.15, 0.25, 0.50, 1.0, and 10.0. The



(a) Vis-viva energy ($C_{3,m}$), 1.78 kilometers squared per second squared; specific impulse I_{sp}, 440 seconds.

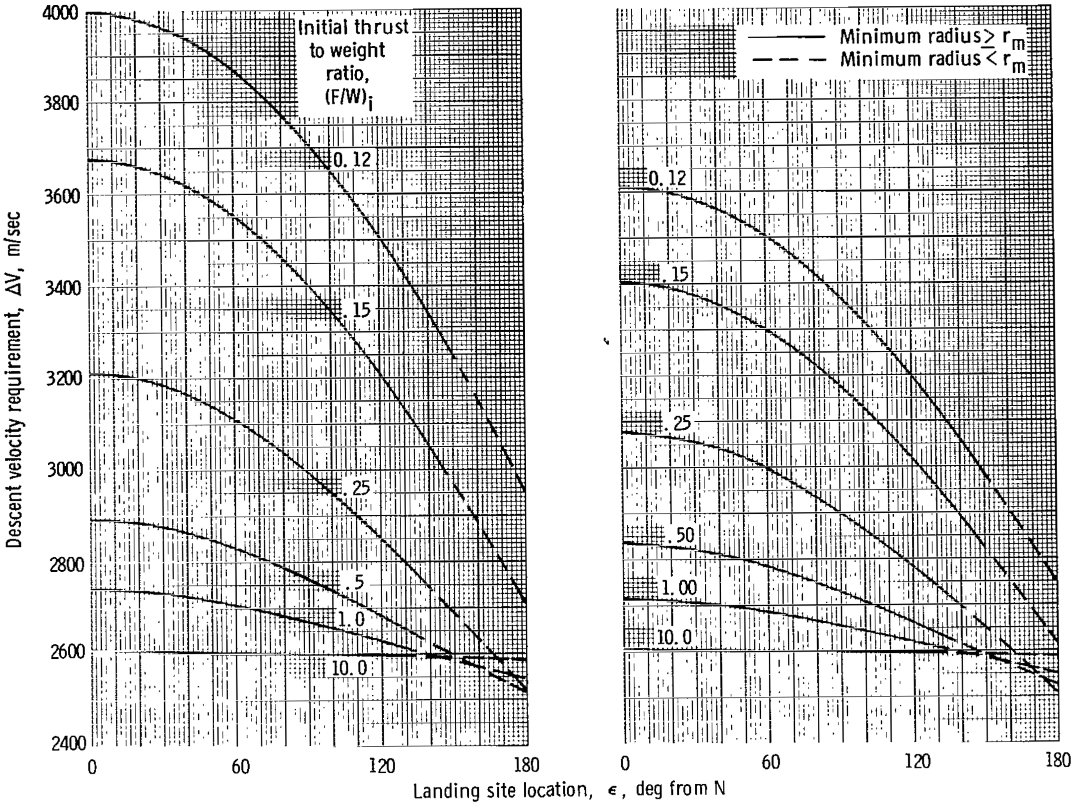
(b) Vis-viva energy $C_{3,m}$, 1.78 kilometers squared per second squared; specific impulse I_{sp}, 300 seconds.

Figure 4. - Descent characteristic velocity requirement as function of landing site location for various $(F/W)_i$ values.

ΔV requirement at a given landing site increases rapidly with decreasing $(F/W)_i$, especially for $(F/W)_i$ values below about 0.5. This is a result of the increased gravity losses corresponding to the longer burn times for the low $(F/W)_i$ values.

For a fixed $(F/W)_i$ value the maximum ΔV requirement is encountered when landing at the NIP ($\epsilon = 0$). For this case the vehicle descends vertically along the incoming radial all the way from descent engine ignition to touchdown on the lunar surface. The thrust vector is therefore directly opposite the lunar gravity vector for the duration of the powered descent phase, and gravity losses are maximized. As ϵ increases, the gravity losses and total ΔV requirement decrease. This behavior will be mentioned again later when the trajectory profiles are discussed.

The ΔV requirement for the same trip time (60 hr) but for a descent engine I_{sp} of 300 seconds is shown in figure 4(b) as a function of ϵ for the same $(F/W)_i$ values as in figure 4(a). The characteristics of the curves in figure 4(b) are directly comparable to those of figure 4(a) but the actual ΔV requirement is lower for corresponding cases.



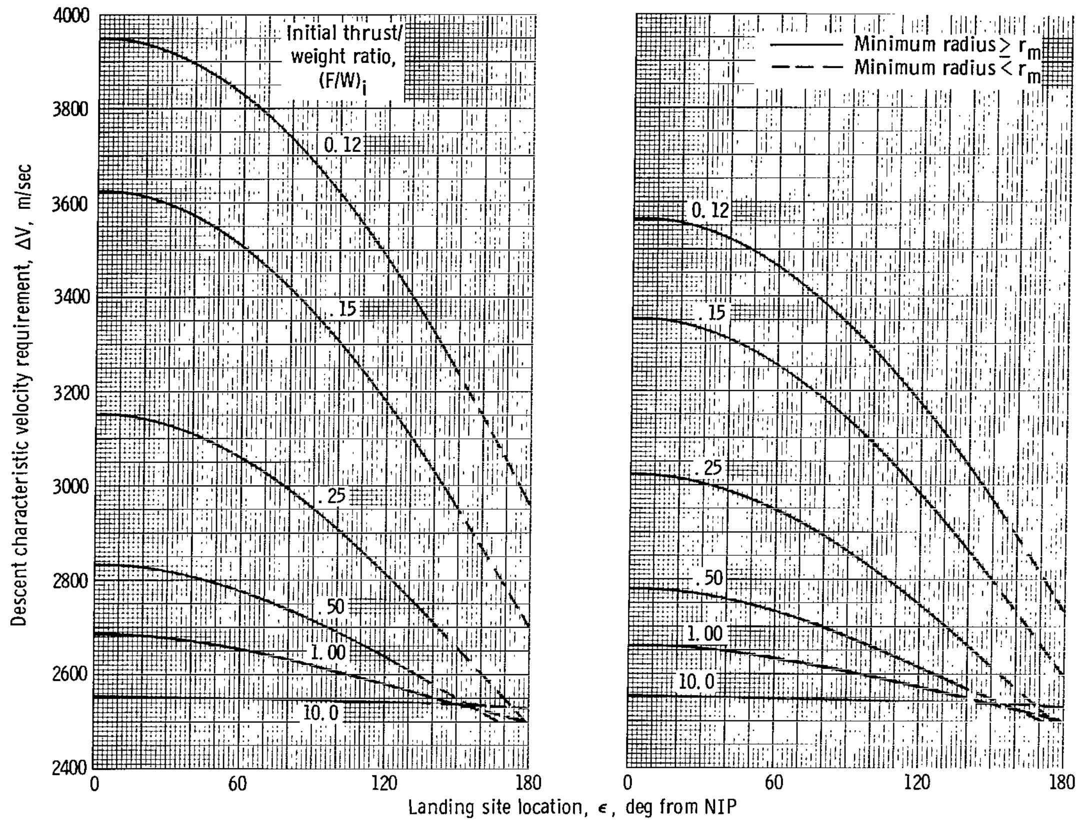
(c) Vis-viva energy $C_{3,m}$, 1.09 kilometers squared per second squared; specific impulse I_{sp}, 440 seconds.

(d) Vis-viva energy $C_{3,m}$, 1.09 kilometers squared per second squared; specific impulse I_{sp}, 300 second.

Figure 4. - Continued.

This difference is greater at the lower $(F/W)_i$ values. The reason is that the $I_{sp} = 300$ case requires a higher propellant consumption rate than the $I_{sp} = 440$ case for the same thrust and weight at descent engine ignition. As a result the vehicle weight decreases more rapidly and the average (F/W) during the descent is higher. At the low $(F/W)_i$ values where the gravity losses are high, the increase in average (F/W) is important and results in reduced gravity losses. For the higher $(F/W)_i$ values gravity losses are small to start with and the difference in ΔV requirement between the 300 and 440 I_{sp} cases is also small.

The required descent ΔV for $C_{3,m} = 1.09$ square kilometers per second squared (75-hr two-body trip time at $r_{em} = 60$ Earth radii) and for $C_{3,m} = 0.82$ square kilometer per second squared (90-hr two-body trip time at $r_{em} = 60$ Earth radii) is shown in figures 4(c) and 4(e) for $I_{sp} = 440$ seconds and in figures 4(d) and 4(f) for $I_{sp} = 300$ seconds. The descent ΔV requirement for a specified ϵ and a fixed $(F/W)_i$ value is higher for the shorter trip times since these correspond to higher $C_{3,m}$ values.



(e) Trip time, 90 hours; specific impulse, 440 seconds; vis-viva energy $C_{3,m}$, 0.82 kilometer squared per second squared; specific impulse I_{sp}, 440 seconds.

(f) Vis-viva energy $C_{3,m}$, 0.82 kilometers squared per second squared; specific impulse I_{sp}, 300 seconds.

Figure 4. - Concluded.

The vehicle incidence angle at touchdown and the minimum radius prior to touchdown were not restricted in this study. As will be shown later the trajectories become more and more shallow as ϵ increases. As ϵ approaches 180°, the unconstrained variational solution produces trajectories which dip below the lunar surface. In order to obtain physically possible trajectories in this region it would be necessary to reformulate the variational equations to include a minimum radius constraint prior to touchdown. This was not done for this study. The ΔV requirement for those trajectories which include a subsurface dip is shown by the dashed line segments in figure 4. The dashed lines represent a minimum bound since the constrained solution would require higher ΔV values.

An actual direct descent to any landing site will require a higher ΔV than is indicated by the data of figure 4. No provision is included here for possible hover, translation, and terminal descent maneuvers; no flight performance reserve has been incorporated, and no mission constraints or contingencies have been included. These items depend on the particular vehicle and mission being investigated and can be added to the ΔV requirement listed here as required.

Trajectory Characteristics

Descent trajectory profiles for ϵ values between 0 and 170° are shown in polar coordinates in figure 5 for $C_{3,m}$ = 1.09 kilometers squared per second squared (t_c =

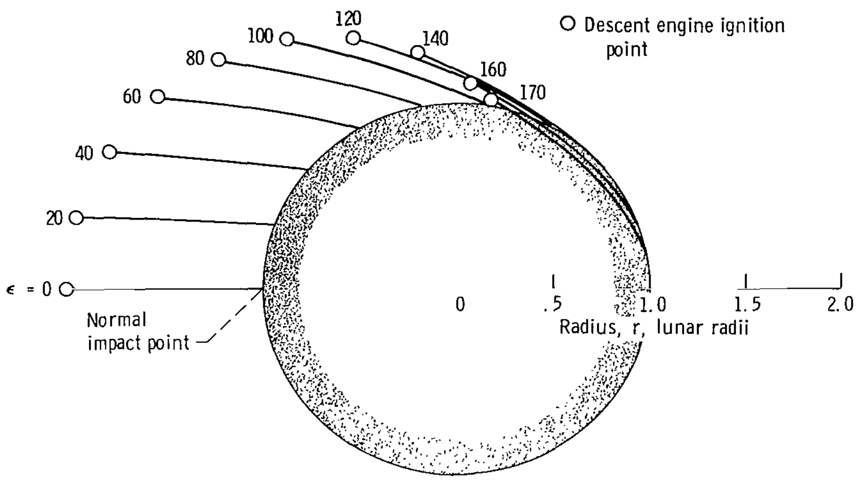


Figure 5. - Descent trajectory profiles. Selenocentric vis-viva energy $C_{3,m}$, 1.09 kilometers squared per second squared; specific impulse I_{sp}, 440 seconds; ignition thrust to weight ratio $(F/W)_i$, 0.15.

75 hr at r_{em} = 60 Earth radii), I_{sp} = 440 seconds and $(F/W)_i$ = 0.15. The $\epsilon = 0$ trajectory is a vertical descent from descent engine ignition to touchdown at the NIP as previously discussed. As ϵ increases the descent engine ignites at lower altitudes, the trajectories become more shallow, the thrust vector is more horizontal, gravity losses are thereby reduced, and the overall descent ΔV requirement decreases as was previously discussed. As indicated in figure 4(c) and shown in figure 5 the trajectories dip below the lunar surface for ϵ values greater than about 145°.

The effect of $(F/W)_i$ is investigated for the $\epsilon = 60$, $I_{sp} = 440$, $C_{3,m} = 1.09$ case in figure 6. The time histories of altitude, velocity, flight path angle, and thrust angle during the descent are shown in figures 6(a) to 6(d), respectively, for various $(F/W)_i$ values. Figure 6(e) shows the relation between horizontal and vertical velocity for the same values of $(F/W)_i$. At the highest $(F/W)_i$ value of 10.0, the altitude and velocity both approach zero in nearly linear fashion and at nearly constant flight path angle and thrust direction values. The plot of vertical velocity as a function of horizontal velocity

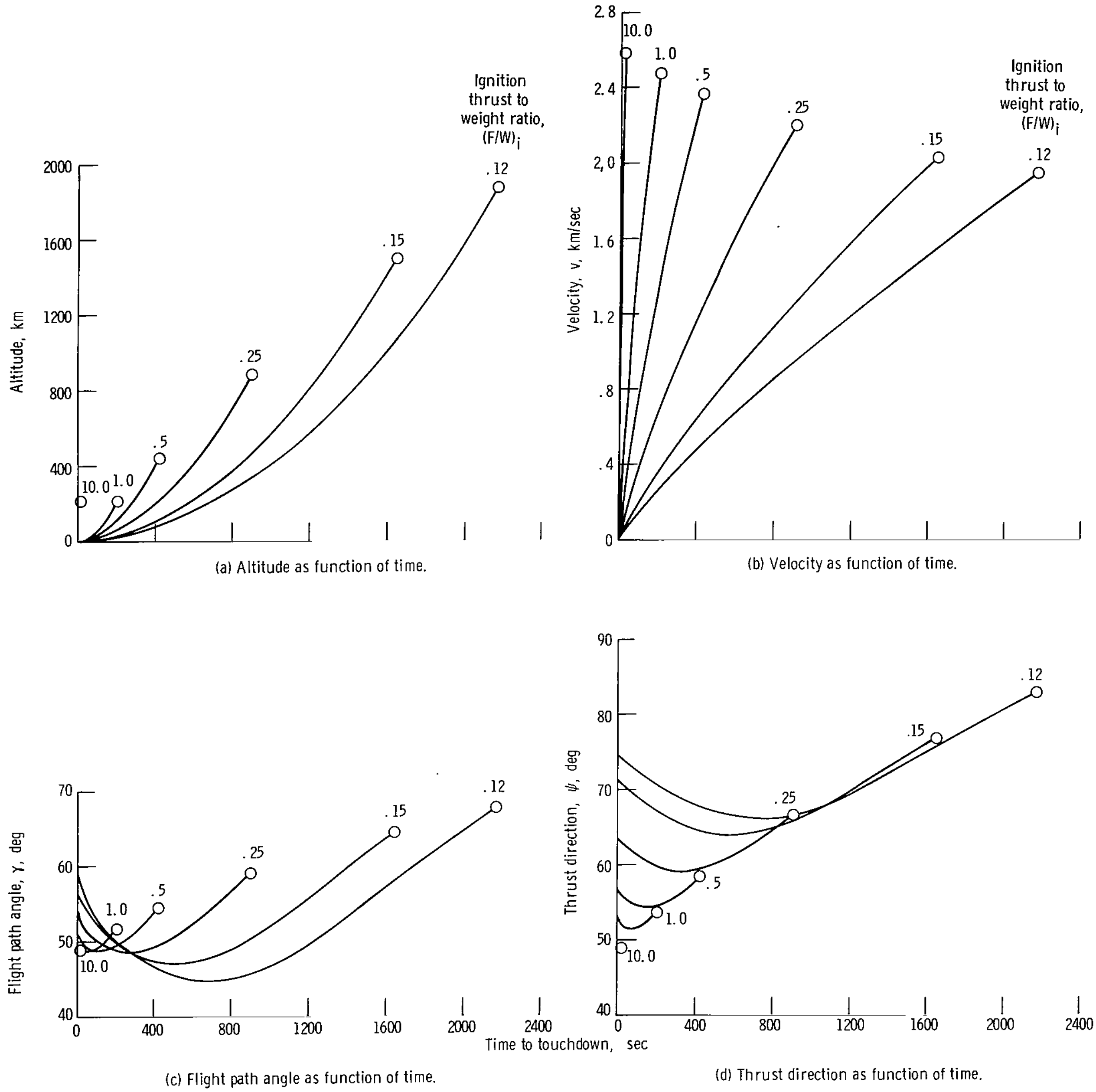
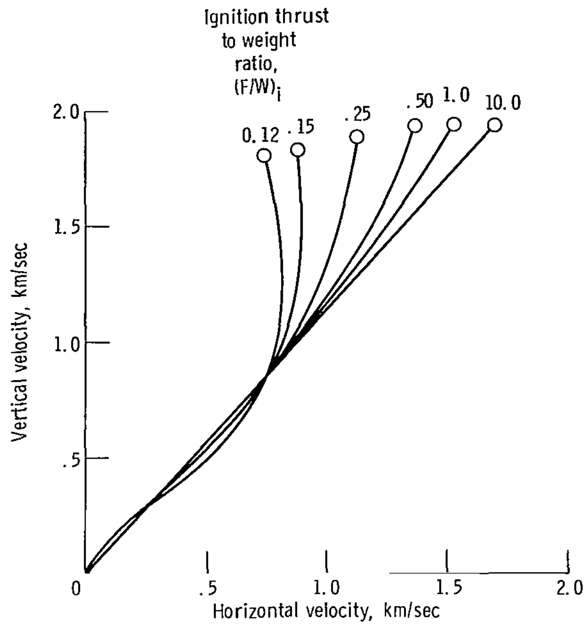


Figure 6. - Time history of descent trajectory parameters. Central angle between landing site and normal impact point (ϵ), 60°; specific impulse I_{sp}, 440 seconds; lunar approach trajectory vis-viva energy $C_{3,m}$, 1.09 kilometers squared per second squared.



(e) Horizontal and vertical velocity component relation.

Figure 6. - Concluded.

is also nearly linear. For the lower $(F/W)_i$ values (longer powered descent burn times) the altitude decreases rapidly at first, since the vertical component of velocity is larger than the horizontal component, then more slowly as the time of touchdown approaches. As the time to touchdown is reduced the instantaneous thrust-to-weight ratio increases and the velocity decreases more and more rapidly. The thrust angle and flight path angle have their maximum value at the time of descent engine ignition, decrease to a minimum as the time to touchdown decreases and then increase again near landing. The flight path angle at time zero is not defined since the velocity is zero at that point. The thrust angle ψ at zero time is a good representation of the vehicle's attitude at touchdown if a tailored final descent phase is not employed.

The effects of variations in $C_{3,m}$ and I_{sp} on the descent profiles and parameter histories are relatively minor for the ranges considered in this study.

CONCLUSIONS

All points on the lunar surface are accessible with direct descent landing maneuvers. The velocity increment (ΔV) requirement is a strong function of the landing site location relative to the normal impact point and of the thrust-to-weight ratio at descent engine ignition. The translunar trip time affects the direct descent ΔV requirement in two ways. First, the lunar approach velocity varies with trip time and this has a direct effect on the ΔV requirement. Secondly, the translunar trip time affects the normal impact point location which changes the angular distance between the normal impact point and the landing site and hence the ΔV requirement. Other factors such as the Earth-Moon distance and the descent engine specific impulse have a smaller effect on the required ΔV.

The results are useful in estimating the ΔV required to land via the direct descent mode at a specified point on the lunar surface. The results can also be applied to determine the effect on performance of changes in such things as launch vehicle capability, translunar injection energy or trip time, descent engine specific impulse and thrust, normal impact point location and desired landing site location.

These results represent the mathematically optimum direct descent ΔV requirements for various combinations of landing site location and lunar approach energy. The actual ΔV requirement for a specific mission must include provisions for mission constraints, performance reserves and final descent and touchdown maneuvers in addition to the requirement shown in this report.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 5, 1970,
731-11.

APPENDIX A

SYMBOLS

A — quantity defined by eq. (B11), m^{-1}

C — constant of integration, dimensionless

C_3 — vis-viva energy, km^2/sec^2

d — magnitude of asymptote translation. See fig. 1.

E — energy, m^2/sec^2

e — orbit eccentricity, dimensionless

(F/W) — ratio of thrust to vehicle equivalent Earth weight, dimensionless

g — quantity to be minimized in eq. (B1), sec

h — vehicle angular momentum, m^2/sec

I_{sp} — specific impulse, sec

Δi — relative inclination between Moon's orbit plane and translunar trajectory plane, deg

m — mass, kg

NIP — normal impact point

$\hat{n}$ — unit normal to Moon's orbit plane

p — semilatus rectum, m

r — radius, m

t — time, sec

ΔV — characteristic velocity increment, m/sec

v — velocity, m/sec

γ — flight path angle, angle between velocity vector and local horizontal in trajectory plane, deg

δx — allowable variation in any variable x

ϵ — lunar central angle from NIP to landing site, deg

η — true anomaly at descent engine ignition, defined in fig. 7, deg

θ — true anomaly of incoming radial, defined in fig. 7, deg

λ_j — Lagrangian multiplier ($j = 1, 2, 3, 4, 5$)

μ — gravitational constant, m^3/sec^2

ρ — latitude, deg

σ — longitude, deg

$\Delta\sigma$ — longitude difference between landing site and NIP, deg

φ — inertial travel angle, defined in fig. 7, deg

ψ — angle between thrust vector and the negative local horizontal, deg

ω — angular velocity, rad/sec

Subscripts:

c — Earth perigee to lunar encounter

e — Earth

em — Earth to Moon or Moon with respect to Earth

f — final or lunar touchdown point

i — initial or descent engine ignition point

m — moon

n	normal impact point
p	Earth pericenter
∞	hyperbolic

Superscripts:

$-$	vector
Λ	unit vector
$\cdot$	time derivative

APPENDIX B

BOUNDARY VALUE PROBLEM

As indicated in the ANALYSIS, the trajectories were integrated starting on the lunar surface and ending in an orbit corresponding to the lunar approach trajectory. In order to completely specify a vehicle's position in a two-dimensional orbit such as this, four independent orbital parameters must be specified. For the case at hand, only two necessary conditions are determined by the problem. The energy of the vehicle at the end of the integration must be equal to the energy of the lunar approach trajectory, and the orientation of the orbit must be such that the vehicle landing point is properly located relative to the NIP. The variational transversality equation can be used to find two auxiliary conditions which, when satisfied, insure that the unspecified orbit parameters assume optimum values.

The general form of the two-dimensional transversality equation as given in reference 3 and applied to the problem at hand is

$$\left[C\delta t + \lambda_1 \delta \dot{r} + r\lambda_2 \delta\omega + \lambda_3 \delta r + \lambda_4 \delta\varphi + \lambda_5 \delta m\right]_f^i + \delta g = 0 \tag{B1}$$

where points i and f represent the powered descent initial and final points, respectively, and g represents that quantity which is to be minimized (which, in this case, is the powered descent burn time). If the zero time reference is at point f $(t_f = 0)$, then t_i will be negative and

$$g = t_f - t_i = - t_i, \quad \delta g = - \delta t_i \tag{B2}$$

The operator δ represents the allowable variation in the variable on which it operates. The remaining variables in equation (B1) are defined in appendix A and several of them are also indicated in figure 7. For the particular problem at hand (B1) can be simplified by invoking the following considerations.

(1) At point i the mass is specified; therefore,

$$\delta m_i = 0 \tag{B3}$$

(2) At point f $r = r_m$ and $\omega = \dot{r} = t = \varphi = 0$; therefore,

$$\delta r_f = \delta\omega_f = \delta\dot{r}_f = \delta t_f = \delta\varphi_f = 0 \tag{B4}$$

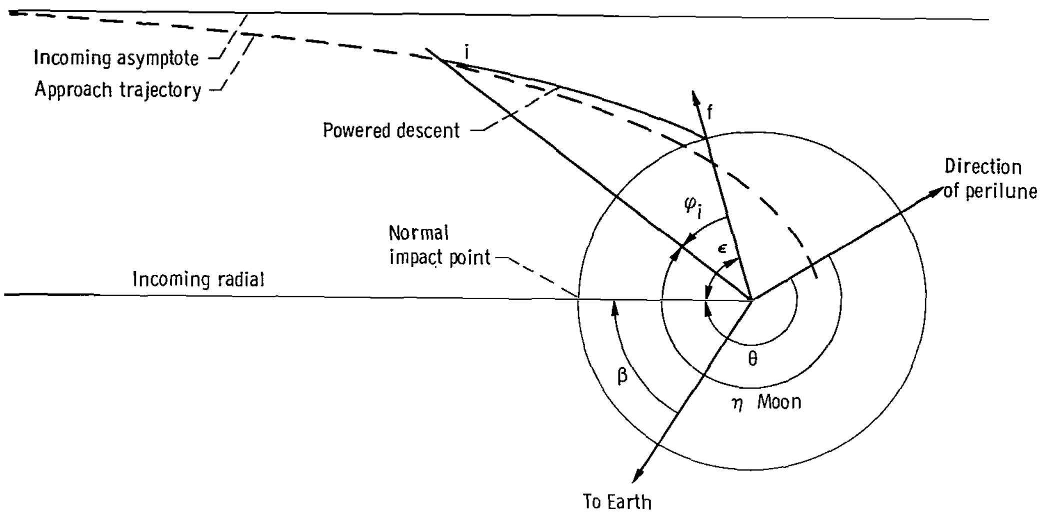


Figure 7. - Geometry and angle definition in vicinity of moon.

(3) The mass at point f is given by

$$m_f = m_i + \dot{m}(t_f - t_i) \tag{B5}$$

where $\dot{m}$ is the rate of change of vehicle mass during the powered descent and is assumed constant. Therefore,

$$\delta m_f = \delta m_i + \dot{m}(\delta t_f - \delta t_i) = -m\delta t_i \tag{B6}$$

Substitution of equations (B2) to (B6) into (B1) and simplification reduce the transversality equation to the form:

$$(C + \dot{m}\lambda_5 - 1)\delta t_i + \lambda_1 \delta\dot{r}_i + r_i\lambda_2 \delta\omega_i + \lambda_3 \delta r_i + \lambda_4 \delta\varphi_i = 0 \tag{B1a}$$

All of the terms in equation (B1a) are evaluated at point i, and the subscripts will now be omitted for convenience.

The two conditions determined by the problem can be used to express $\delta\dot{r}$ and $\delta\varphi$ in terms of δr and $\delta\omega$. The first condition corresponds to the specification of the energy at point i. Since energy is specified, $\delta E = 0$ where

$$E = \frac{1}{2}(\dot{r}^2 + \omega^2 r^2) - \frac{\mu}{r}$$

The variation is

$$\delta E = \dot{r}\delta\dot{r} + \omega r^2\delta\omega + \omega^2 r\delta r + \frac{\mu}{r^2}\delta r = 0$$

which can be solved for $\delta\dot{r}$ as

$$\delta\dot{r} = \frac{-\omega r^2}{\dot{r}}\delta\omega - \left(\frac{\omega^2 r}{\dot{r}} + \frac{\mu}{r^2\dot{r}}\right)\delta r \tag{B7}$$

The second condition which concerns the orientation of the lunar approach trajectory can be expressed with the aid of figure 7 as

$$\epsilon = \theta - \eta + \varphi_i$$

$$\delta\epsilon = \delta\theta - \delta\eta + \delta\varphi_i = 0 \tag{B8}$$

The angle θ is the true anomaly of the incoming radial and is expressible as

$$\theta = \cos^{-1}\frac{-1}{e}$$

It then follows that

$$\delta\theta = \frac{-1}{e\sqrt{e^2 - 1}}\delta e = \frac{-\sqrt{e^2 - 1}}{2e^2 p}\delta p \tag{B9}$$

since, from the relations

$$E = -\frac{\mu}{2p}(1 - e^2) \qquad \text{and} \qquad \delta E = 0$$

we can express δe in terms of δp as

$$\delta e = \frac{e^2 - 1}{2\,ep}\delta p$$

The angle η of figure 7 and equation (B8) is the true anomaly at point i and is given by

$$\eta = \cos^{-1}\left[\frac{1}{e}\left(\frac{p}{r} - 1\right)\right]$$

This can be written as

$$\cos\eta = \frac{1}{e}\left(\frac{p}{r} - 1\right)$$

Taking the differential of both sides yields

$$-\sin\eta\,\delta\eta = -\frac{1}{e^2}\left(\frac{p}{r} - 1\right)\delta e + \frac{1}{e}\left(\frac{\delta p}{r} - \frac{p}{r^2}\,\delta r\right)$$

substituting

$$\sin\eta = \sqrt{\frac{p}{\mu}}\,\frac{\dot{r}}{e}$$

and solving for $\delta\eta$ and simplifying give

$$\delta\eta = \frac{1}{\dot{r}}\sqrt{\frac{\mu}{p}}\left[\frac{-p(e^2 + 1) - r(e^2 - 1)}{2e^2pr}\right]\delta p + \frac{\omega}{\dot{r}}\,\delta r \tag{B10}$$

Substituting (B9) and (B10) into (B8) and simplifying with the aid of the relations

$$\mu p = \omega^2 r^4$$

$$\delta p = \frac{1}{\mu}\left(2\omega r^4\delta\omega + 4\omega^2 r^3\delta r\right),$$

letting

$$A = \left\{\frac{\sqrt{e^2 - 1}}{r} - \frac{\omega}{p\dot{r}}\,\frac{\left[p(e^2 + 1) + r(e^2 - 1)\right]}{e^2}\right\} \tag{B11}$$

and solving for $\delta\varphi$ make it possible to write

$$\delta\varphi = \left(2A + \frac{\omega}{\dot{r}}\right)\delta r + \frac{Ar}{\omega}\,\delta\omega \tag{B12}$$

Substituting equations (B7) and (B12) into (B1a) and collecting terms give

$$(C + \dot{m}\lambda_5 - 1)\delta t + \left(r\lambda_2 - \frac{\omega r^2}{\dot{r}}\lambda_1 + \frac{Ar}{\omega}\lambda_4\right)\delta\omega + \left[\frac{-1}{\dot{r}}\left(\omega^2 r + \frac{\mu}{r^2}\right)\lambda_1 + \lambda_3 + \left(2A + \frac{\omega}{\dot{r}}\right)\lambda_4\right]\delta r = 0 \tag{B13}$$

as the final form of the transversality equation. Since δt, $\delta\omega$, and δr are independent of each other, the only way for (B13) to be satisfied is for the coefficients of δt, $\delta\omega$, and δr to all be zero. The coefficient of δt can be zeroed by properly scaling all the λ's and C as explained in reference 3. The auxiliary conditions which must be satisfied for the descent trajectory to be optimum are that the remaining coefficients in equation (B13) be zero. After simplification, the conditions can be written as

$$-\omega r\lambda_1 + \dot{r}\lambda_2 + \frac{A\dot{r}}{\omega}\lambda_4 = 0$$

and

$$\lambda_3 - \frac{1}{\dot{r}}\left(\omega^2 r + \frac{\mu}{r^2}\right)\lambda_1 + \left(2A + \frac{\omega}{\dot{r}}\right)\lambda_4 = 0$$

The boundary value problem consists of finding those values of λ_1 to λ_4 at touchdown, the vehicle weight at touchdown and the duration of the powered descent which result in a trajectory satisfying the required final conditions. As in reference 3 the λ's are scaled and initial values of λ_1, λ_2, and λ_3 are calculated from ψ and $\dot{\psi}$ as an aid in obtaining reasonable initial guesses. Actual boundary conditions to be determined are therefore ψ, $\dot{\psi}$, λ_4, and vehicle weight at touchdown and burn time. The corresponding end conditions which must be satisfied are the specified approach trajectory energy ($C_{3,m}$), the required thrust-to-weight ratio at descent engine ignition $(F/W)_i$, the desired angular distance from the NIP to the landing site (ϵ), and the two auxiliary variational end conditions.

APPENDIX C

NORMAL IMPACT POINT DETERMINATION

If the four assumptions listed in the ANALYSIS are satisfied then the NIP location can be determined analytically as follows. For specified values of $C_{3,e}$ and r_{em}, the velocity v and flight path angle γ of the spacecraft at a radius equal to r_{em} can be calculated as (see ref. 5):

$$v = \left(C_{3,e} + \frac{2\mu}{r_{em}}\right)^{1/2}$$

$$\cos \gamma = \frac{r_p}{r_{em}}\left(\frac{C_{3,e} + \frac{2\mu}{r_p}}{C_{3,e} + \frac{2\mu}{r_{em}}}\right)^{1/2}$$

where μ is the gravitational constant of the Earth and r_p is the Earth perigee radius of the translunar trajectory. The velocity v along with the angles γ and Δi completely specify the vector velocity $\bar{v}$ at lunar intercept as shown in figure 8. The relative inclination Δi is measured from the Moon's velocity vector $\bar{v}_{em}$ to the local horizontal (where the local horizontal is in the trajectory plane) and can have any value from 0 to 360^o.

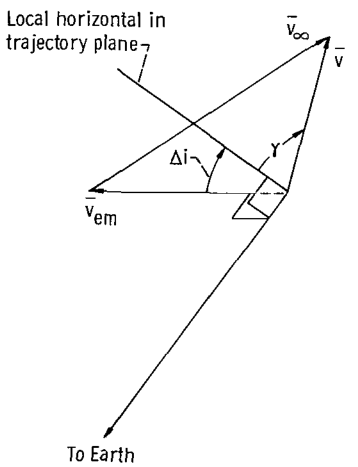


Figure 8. - Velocity vector diagram at lunar orbit intercept.

The Moon's velocity relative to Earth $\overline{v}_{em}$ is orthogonal to the Earth-Moon line because of the circular orbit assumption and has magnitude

$$v_{em} = \left(\frac{\mu}{r_{em}}\right)^{1/2}$$

The spacecraft velocity relative to the Moon is

$$\overline{v}_{\infty} = \overline{v} - \overline{v}_{em}$$

The direction of $\overline{v}_{\infty}$ defines the incoming radial direction. The NIP is the intersection of the incoming radial (or of $-\overline{v}_{\infty}$) and the lunar surface as shown in figure 9.

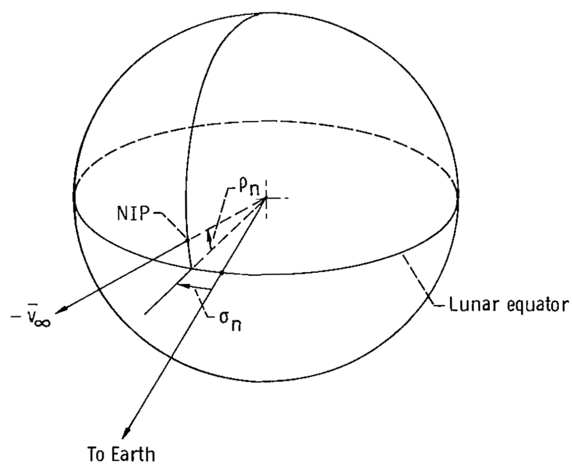


Figure 9. - Normal impact point location.

REFERENCES

1. Anon.: Improve Lunar Cargo and Personnel Delivery System. Rep. LMSC-T-28-68-4, Lockheed Missiles and Space Co. (NASA CR-95234), June 28, 1968.

2. Weber, Richard J.; Pauson, Werner M.; and Burley, Richard P.: Lunar Trajectories. NASA TN D-866, 1961.

3. Teren, Fred; and Spurlock, Omer F.: Payload Optimization of Multistage Launch Vehicles. NASA TN D-3191, 1966.

4. Woolston, Donald S.: Declination, Radial Distance, and Phases of the Moon for the Years 1961 to 1971 for Use in Trajectory Considerations. NASA TN D-911, 1961.

5. Dobson, Wilbur F.; Huff, Vearl N.; and Zimmerman, Arthur V.: Elements and Parameters of the Osculating Orbit and Their Derivatives. NASA TN D-1106, 1962.

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