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ELECTRON SCATTERING FROM LIGHT DEFORMED NUCLEI

by Richard C. Braley and William F. Ford
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TECHNICAL PAPER proposed for presentation at
American Physical Society Meeting
Los Angeles, California, December 29-31, 1969

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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ABSTRACT

The elastic scattering of high-energy electrons from several 2s-1d shell nuclei is studied. The method used here is based on the solution of the Dirac equation in the Glauber approximation, and was originally developed by Baker.[1] The initial and final states of the s-d shell nuclei are obtained by projecting states of good angular momentum from intrinsic Hartree-Fock states. The results are compared with Born approximation and with experiment.

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INTRODUCTION

The scattering of high-energy electrons from nuclei provides a very powerful tool for use in the study of nuclear structure. As a means by which nuclear models may be tested, electron scattering has the attractive feature that the electron-nucleus interaction is well understood. This is in contrast to the situation in nucleon scattering where the uncertainties in the nuclear model are further complicated by the lack of detailed knowledge of the nucleon-nucleus force.

The analysis of high-energy electron scattering is carried out most easily in first Born approximation. However, it is well known that this approximation is inadequate except for the very lightest nuclei, and that zeros occur at the diffraction minima. An alternative approach to the problem is the method of phase shift analysis which has been successfully applied over a wide range of nuclei. Unfortunately this method is very complicated and would prove to be impractical for later investigations of inelastic scattering. Baker has developed an approximation for high-energy electron scattering based on the approach which Glauber developed

for high-energy nucleon scattering.[1,2] Baker's high-energy approximation washes out the zeros of the diffraction minima which are given by the Born approximation, and agrees well with the results of phase shift analysis. To our knowledge, however, this high-energy approximation has only been tested for the very simplest nuclear models.

In this paper, the method of Baker is used to study the elastic scattering of high-energy electrons from several light deformed nuclei. The major objective of this investigation is to test the microscopic projected Hartree-Fock wavefunctions for the nuclei of interest.

THEORY

The high-energy approximation for electron scattering consists of using the approach of Glauber to solve the Dirac equation for an electron in the presence of a scalar potential. Baker shows that, in this approximation, the matrix element which must be evaluated is:

$$M = \frac{k}{i}\int_0^\infty J_0(qb)\left[e^{i\chi(b)} - 1\right] b\, db \tag{1}$$

where

$$\chi(b) = -\frac{E}{k}\int_{-\infty}^{\infty} V(b,z)\, dz \tag{2}$$

$$V(b,z) = V(r) = \int \rho(\vec{R}) V_c(|\vec{R} - \vec{r}|)\, d\vec{R} \tag{3}$$

$\rho(R)$ is the nuclear density function and $V_c(|\vec{R} - \vec{r}|)$ is the screened coulomb potential.

$$V_c(\chi) = -\frac{\alpha Z}{\chi} f\left(\frac{\chi}{a}\right)$$

where α is the fine structure constant, Z is the nuclear charge, a is the screening radius, and the screening function used here is

$$f\left(\frac{x}{a}\right) = 1 - \frac{x}{\sqrt{x^2 + 4a^2}}$$

Assuming that the nuclear density is spherically symmetric, it can be shown that

$$\chi(b) = 8\pi\eta \int_0^\infty \rho(R) \int_0^\infty v_0(r,R)\, dz\, R^2\, dR \tag{4}$$

where $r = \sqrt{b^2 + z^2}$, $v_0(r,R)$ is the zero order multipole of V_c and $\eta = \alpha Z \frac{E}{k}$. Now $v_0(r,R)$ must be known only for values of $R \leq R_0$ under the assumption that $\rho(R)$ is negligibly small for $R > R_0$. Based on this assumption, it can be shown that

$$v_0(r,R) = \begin{cases} \frac{1}{r} f\left(\frac{r}{a}\right), & r \geq R \\ R^{-1}, & r \leq R \end{cases} . \tag{5}$$

Then it follows that

$$\chi(b) = \chi_0(b) + \chi_1(b) \tag{6a}$$

where

$$\chi_0(b) = \eta \log (1 + 4a^2/b^2) \tag{6b}$$

and

$$\chi_1(b) = 8\pi\eta \int_0^\infty dz \int_r^\infty \left(\frac{1}{R} - \frac{1}{r}\right) \rho(R) R^2 \, dR \tag{6c}$$

We have found it convenient to integrate Eq. (1) by parts obtaining

$$M = \frac{k}{q} \int_0^\infty J_1(qb) \chi'(b) \, e^{i\chi(b)} b \, db \tag{7}$$

where $\chi'(b) = d\chi(b)/db$. The evaluation of $\chi_0'(b)$ is simple, and it can be shown that

$$\chi_1'(b) = \frac{8\pi}{b} \eta \int_0^\infty \rho\left(\sqrt{b^2 + r^2}\right) r^2 \, dr \tag{8}$$

This can then be integrated to obtain $\chi_1(b)$. Now, if we assume that $\rho(r)$ can be represented by a series of Gauss-Laguerre functions[3]

$$\rho(R) = \sum_{n=0}^{\infty} A_n F_n(\alpha, \beta r),$$
$$F_n(\alpha, \beta r) = e^{-\alpha\beta^2 r^2} L_n^{1/2}(\beta^2 r^2). \tag{9}$$

and then make use of the addition theorem for Laguerre polynomials it can be shown that

$$\rho\left(\sqrt{b^2 + r^2}\right) = e^{-\alpha\beta^2 b^2} \left\{ \rho(r) - (\beta b)^2 \sum_{m=1}^{\infty} \frac{1}{m} L_{m-1}^1(\beta^2 b^2) \right.$$
$$\left. \times \sum_{n=0}^{\infty} A_{m+n} F_n(\alpha, \beta r) \right\}$$

From this we obtain

$$\chi_1'(b) = \frac{2}{b}\,\eta\, e^{-\alpha\beta^2 b^2}\left\{1 - 4\pi(\beta b)^2 \sum_{m=1}^{\infty} \frac{1}{m} L_{m-1}^{1}(\beta^2 b^2)\, B_m\right\}$$

where

$$B_m \equiv \sum_{n=0}^{\infty} A_{m+n} D_n$$

and

$$D_n = \int_0^{\infty} \mathcal{F}_n(\alpha, \beta r) r^2\, dr$$

Equation ( ) may be integrated to yield

$$\chi_1(b) = -2\eta \int_0^{\infty} e^{-\alpha\beta^2 R^2} \frac{dR}{R} + 4\pi\eta \sum_{m=1}^{\infty} I_{m-1}(\beta^2 b^2)\, B_m,$$

$$I_m(\chi) = \int_{\chi}^{\infty} e^{-\alpha y}\, L_m^1(y)\, \alpha y.$$

The $\chi_0(b)$ and $\chi_1(b)$ and their derivatives are then used in Eq. (7) to obtain M. The final evaluation of the integral is carried out using the same general procedure as was used by Baker. The electron scattering form factor is related to the square of M divided by the point cross section. The change in the electron wavelength as the electron approaches

the scattering center must be accounted for in the calculation of the form factor. The correction factor suggested by Ravenhall has been used here.[4] A gaussian form factor has been used in this work to correct for the finite size of the proton.

The nuclear structure calculations (for Mg^{24}, Si^{28}, S^{32}) were made using projected Hartree-Fock wavefunctions. The basis spaces used in the HF calculations span the first five major shells (1s, 1p, 2s-1d, 2p-1f, 3s-2d-1g) and have harmonic oscillator radial dependence. The oscillator parameter was chosen by reproducing, as closely as possible, the RMS radius for the nuclei be considered; this essentially introduces a second constraint into the Hartree-Fock problem.

RESULTS AND DISCUSSION

The failure of the Born approximation to yield reliable results in the region of diffraction minima is demonstrated in figure 1 for electron scattering from Ne^{20}. Unfortunately there is little experimental data available on Ne^{20}, so comparison is not made with experiment; however, within the range of validity of the nuclear model, it has been demonstrated by Baker that the high-energy approximation is in excellent agreement with phase shift analysis. Figure 1 demonstrates rather clearly that Baker's approximation fills in the diffraction minima.

In figure 2 is presented the result of our calculation for S^{32}. These results are unnormalized, and over the entire range of q are quite good except for the slight displacement of the second minimum. In figures 3 and 4, the results are presented for Mg^{24} and Si^{28}. These results are generally not in very good agreement with experiment. The first minima are displaced considerably and the first maximum is quite low in both cases.

In an attempt to understand this discrepancy, the data of Helm (from which the RMS radii were determined) were re-examined. In figure 5 we show Helm's results. The results for S^{32} are quite reliable for the determination of an RMS radius; however, for Mg^{24} and Si^{28} the minima were not determined accurately. We feel, therefore, that it is not surprising that the RMS radii determined by Helm yield poor structure re-

sults and therefore poor elastic form factors for electron scattering. The nuclear structure results have been adjusted through the parameters appearing in Eq. (9) so as to yield the results appearing in figures 6 and 7. These are considerable improvements over the previous results although the first maximum is still low in both cases.

It is gratifying that one can obtain the results appearing in figures 2, 6, and 7 using fundamental wavefunctions in the description of the nuclear structure. This is particularly true in the case of S^{32} where the wavefunction needs to be improved only slightly in order to obtain excellent agreement with experiment. The results for Mg^{24} and Si^{28} should be improved beyond the first diffraction minimum when a more detailed study is made of the wavefunctions for these nuclei.

REFERENCES

1. A. Baker, Phys. Rev. 134 B, 252 (1964).

2. R. J. Glauber, Lectures in theoretical Physics (Interscience Publ., Inc., New York, 1959).

3. W. F. Ford and R. C. Braley, "A Systematic Method for Calculating Reduced Matrix Elements," presented at Am. Phys. Soc. Meeting, Boulder, Colo., Oct. 30 - Nov. 1, 1969.

4. R. H. Helm, Phys. Rev. 104, 1466 (1956).

5. J. Bar-Touv, R. Braley, and W. Ford, to be published.

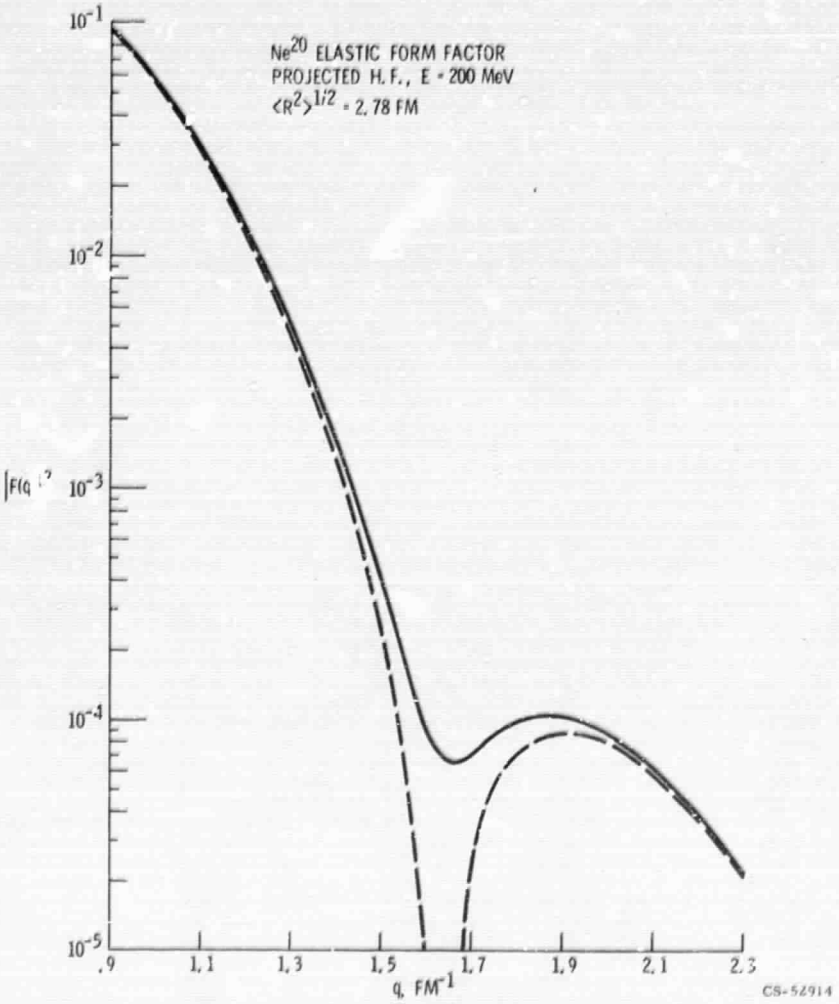
Ne20 ELASTIC FORM FACTOR
PROJECTED H.F., E = 200 MeV
<R2>1/2 = 2.78 FM
10-1
10-2
10-3
10-4
10-5
.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
q, FM-1
CS-52914

Figure 1

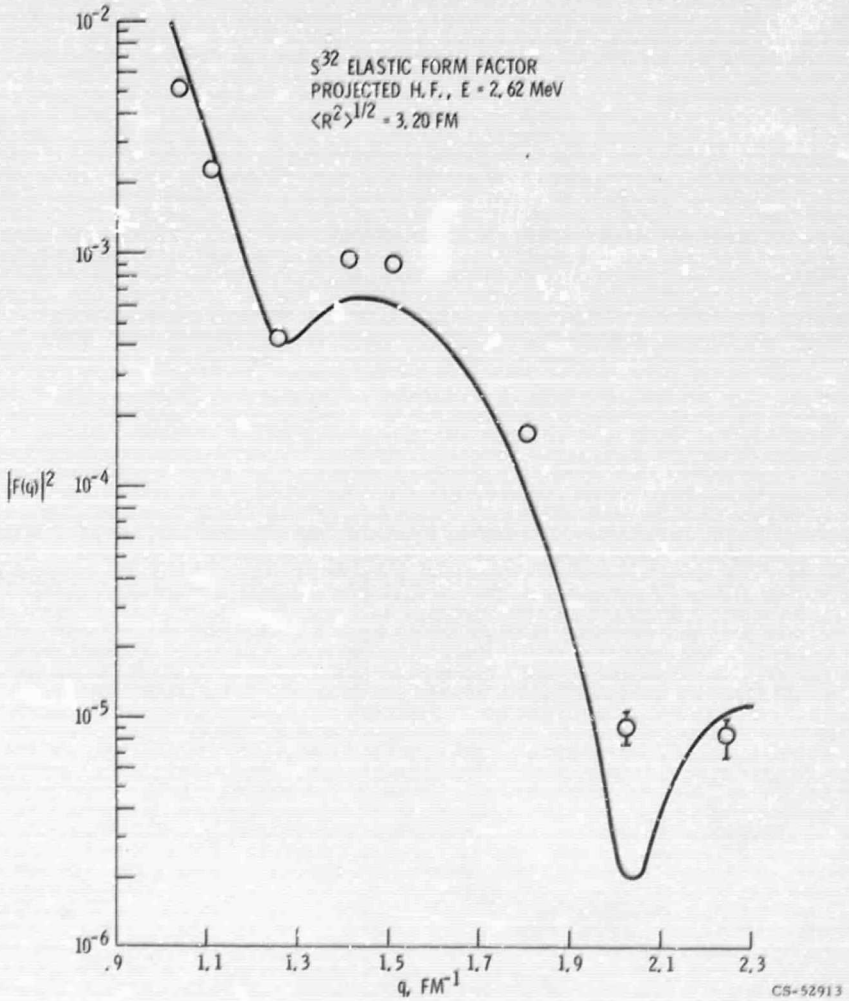
S^{32} ELASTIC FORM FACTOR
PROJECTED H. F., E = 2.62 MeV
$\langle R^2 \rangle^{1/2}$ = 3.20 FM
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$|F(q)|^2$
.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
q, FM^{-1}
CS-52913

Figure 2

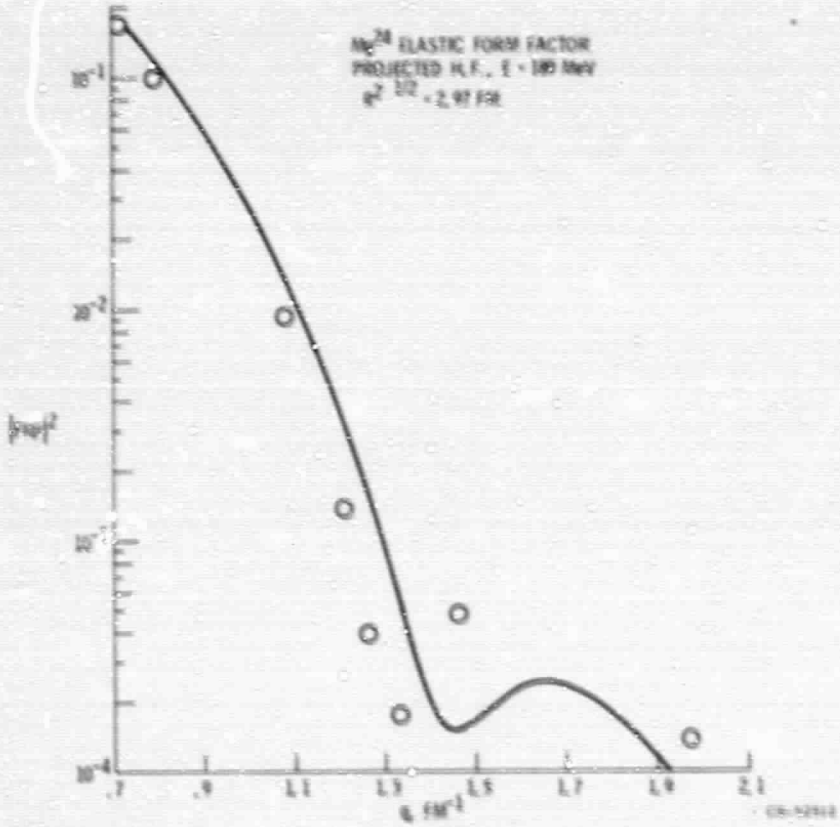
Mg^{24} ELASTIC FORM FACTOR
PROJECTED H.F., E = 189 MeV
q^2 1/2 < 2.97 FM
10^{-1}
10^{-2}
10^{-3}
10^{-4}
.7
.9
1.1
1.3
1.5
1.7
1.9
2.1
q, FM^{-1}

Figure 3

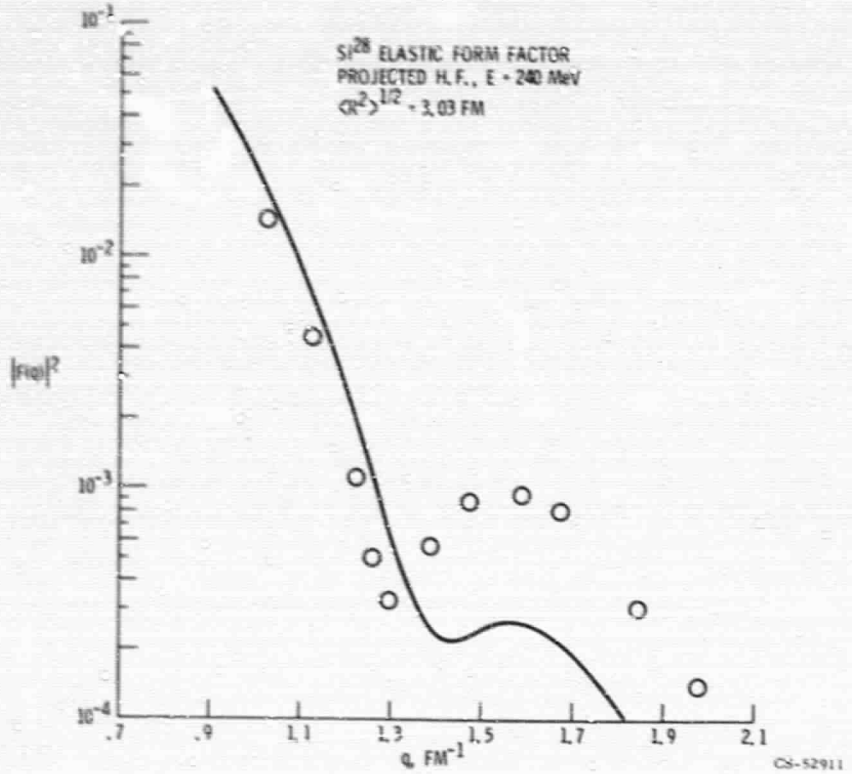
Si^{28} ELASTIC FORM FACTOR
PROJECTED H.F., E = 240 MeV
$\langle R^2 \rangle^{1/2}$ = 3.03 FM
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$|F(q)|^2$
.7
.9
1.1
1.3
1.5
1.7
1.9
2.1
q, FM^{-1}
CS-52911

Figure 4

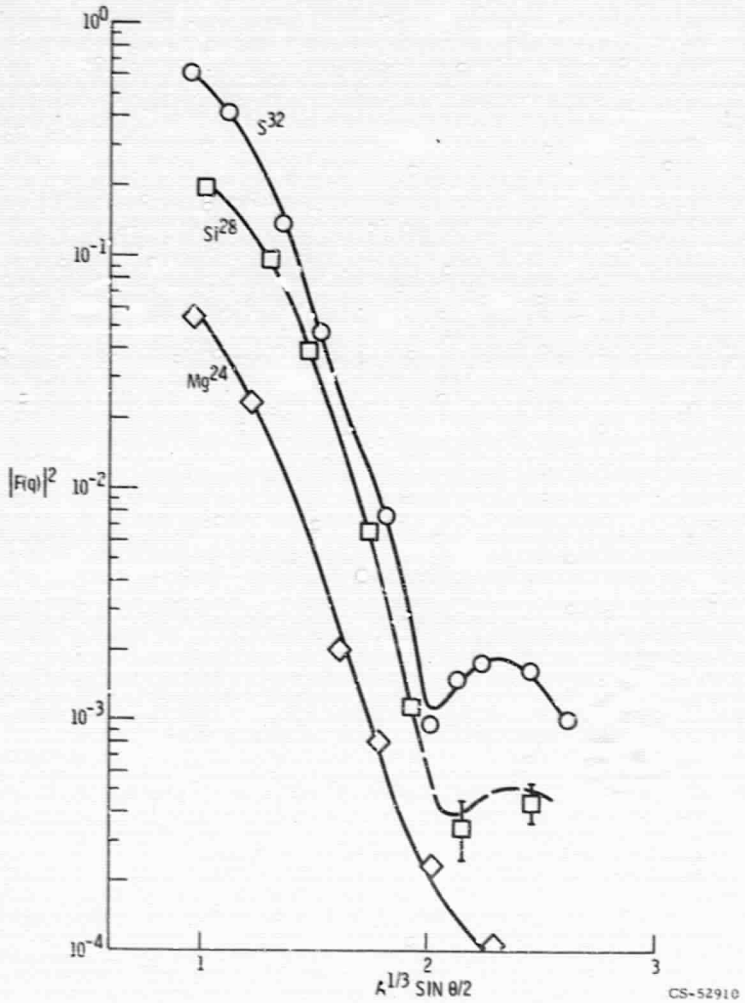

Figure 5

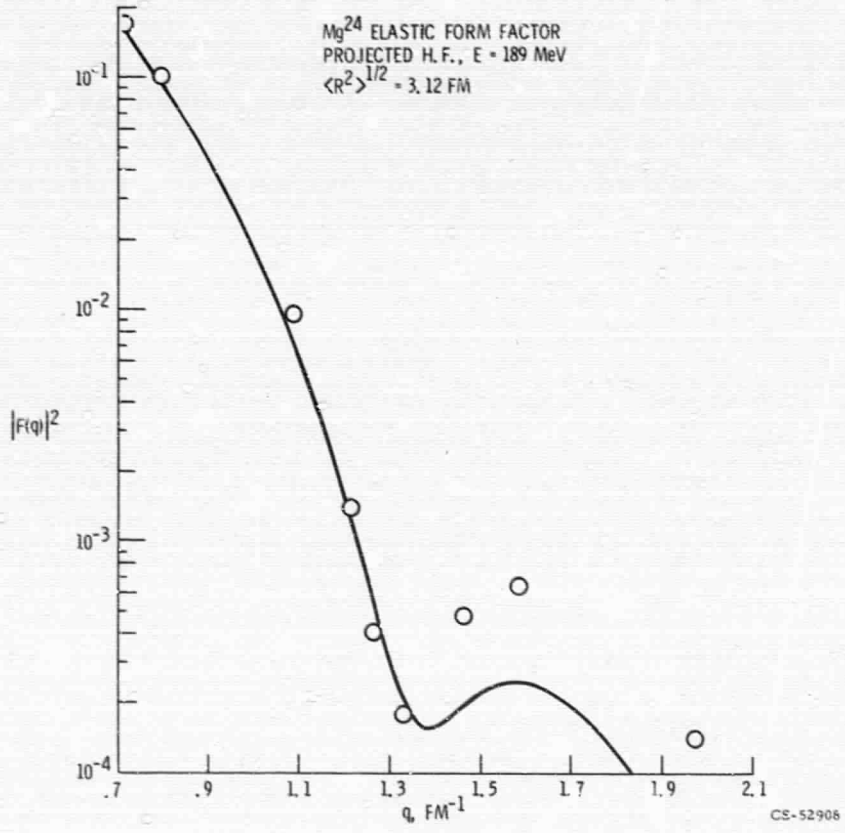
Mg^{24} ELASTIC FORM FACTOR
PROJECTED H.F., E = 189 MeV
$\langle R^2 \rangle^{1/2}$ = 3.12 FM
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$|F(q)|^2$
.7
.9
1.1
1.3
1.5
1.7
1.9
2.1
q, FM^{-1}
CS-52908

Figure 6

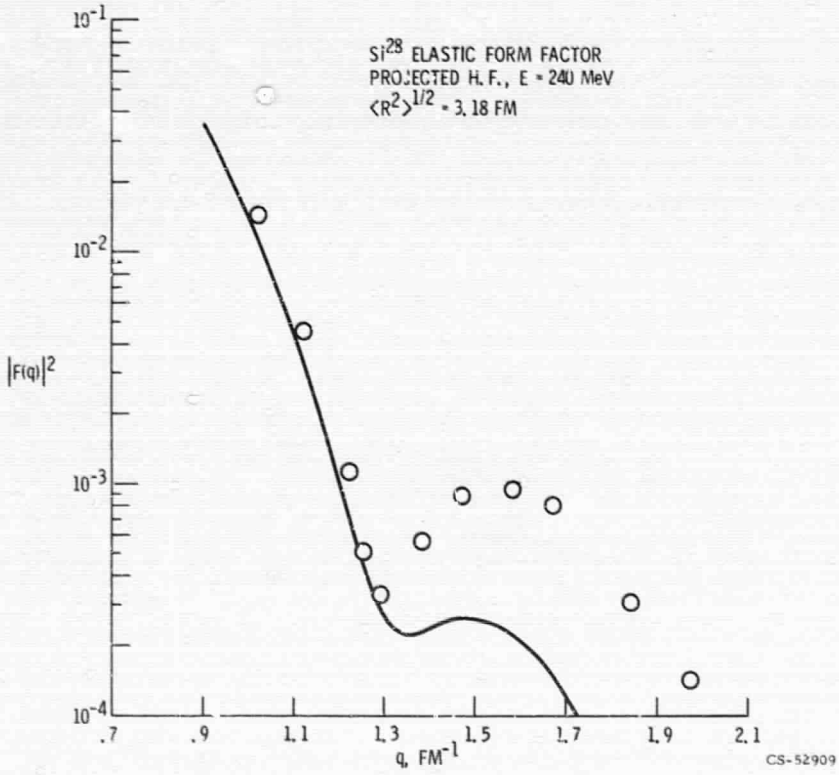
Si^{28} ELASTIC FORM FACTOR
PROJECTED H. F., E = 240 MeV
$\langle R^2 \rangle^{1/2}$ = 3.18 FM
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$|F(q)|^2$
.7
.9
1.1
1.3
1.5
1.7
1.9
2.1
q, FM^{-1}
CS-52909

Figure 7