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INTERPOLATION OF BODE'S TABLES OF THE SOLAR CONTINUOUS ABSORPTION COEFFICIENT

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16. ABSTRACT

Knowledge of the total continuous absorption coefficient for a wide range of wavelengths is vital to many areas of solar physics, such as the construction of model atmospheres and line profile calculations. Recently, Gerhard Bode compiled a set of tables for the continuous absorption coefficient for the sun covering a wide range of wavelengths, electron pressures, and temperatures. To obtain the absorption coefficient as a function of electron pressure and temperature at an untabulated wavelength, one can employ standard interpolation formulas using the several wavelengths listed, which are between major absorption edges and span the desired wavelength. At a given wavelength the absorption coefficients at particular temperatures and pressures not given in the table can be obtained by a double linear interpolation scheme. This paper describes a computer subprogram which has been developed to calculate the solar continuous absorption coefficient for any given model atmosphere.

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TABLE OF CONTENTS

LIST OF TABLES

TECHNICAL MEMORANDUM X-64534

INTERPOLATION OF BODE'S TABLES OF THE SOLAR CONTINUOUS ABSORPTION COEFFICIENT

SUMMARY

Knowledge of the total continuous absorption coefficient for a wide range of wavelengths is vital to many areas of solar physics, such as the construction of model atmospheres and line profile calculations. Recently, Gerhard Bode [1] compiled a set of tables for the continuous absorption coefficient for the sun covering a wide range of wavelengths, electron pressures, and temperatures. To obtain the absorption coefficient as a function of electron pressure and temperature at an untabulated wavelength, one can employ standard interpolation formulas using the several wavelengths listed, which are between major absorption edges and span the desired wavelength. At a given wavelength the absorption coefficients at particular temperatures and pressures not given in the table can be obtained by a double linear interpolation scheme. This paper describes a computer subprogram which has been developed to calculate the solar continuous absorption coefficient for any given model atmosphere.

INTRODUCTION

The subprogram used to interpolate the Bode data employs a method of two-way differences. This method is described in detail by Scarborough [2].

It is hoped that an example problem of the technique of two-way differences will clarify the procedure used to interpolate the Bode tables by computer. A flow chart (Appendix A) is also used to indicate the processes undertaken by the computer; the subroutine is listed in Appendix B.

THE TECHNIQUE OF DOUBLE INTERPOLATION

The example problem uses a table of values of log (electron pressure/ absorption coefficient) at 5250. 20 Å vs log (electron pressure) and θ , where θ is 5040/absolute temperature.

It should be noted that there is no Bode table for 5250. 20 Å; rather Table 1 was compiled using a simple straight-line interpolation of a Bode table at 4500. 00 Å and one at 5698. 50 Å.

TABLE 1. VALUES OF LOG (Pe/K)

θ	Log (Pe)					
	-2. 5	-2. 0	-1. 5	-1. 0	-0. 5	0. 0
1. 30	24. 376	24. 569	24. 658	24. 693	24. 713	24. 758
1. 25	24. 412	24. 625	24. 727	24. 767	24. 784	24. 806
1. 20	24. 443	24. 681	24. 797	24. 884	24. 861	24. 872
1. 15	24. 465	24. 729	24. 865	24. 922	24. 942	24. 950
1. 10	24. 452	24. 767	24. 927	24. 999	25. 024	24. 033
1. 05	24. 313	24. 768	24. 985	25. 072	25. 105	25. 117
1. 00	23. 914	24. 627	25. 004	25. 138	25. 183	25. 202
0. 95	23. 298	24. 200	24. 879	25. 174	25. 255	25. 281

Suppose that a value of log (Pe/K) is needed for particular values of θ and log (Pe) not given in Table 1. The method of two-way differences would then be used as follows:

Problem: Log (Pe/K) needs to be calculated at a θ value of 1. 22 and a log (Pe) value of -2. 2 at 5250. 20 Å.

Solution: Tables 2A and 2B show forward differences for constant log (Pe) values, and Tables 3A and 3B show differences for constant θ values (hence, two-way differences). A general formula for double interpolation uses these differences in providing the desired value of log (Pe/K). Note that log (Pe/K) = 24. 412 at θ = 1. 25 and log (Pe) = -2. 5 is used as a "starting point" for taking differences. The starting point is that value of log (Pe/K) in Table 1 for which the corresponding values of log (Pe) and θ are closest to and above those which are being interpolated.

TABLE 2A. CONSTANT LOG (Pe) VALUES; LOG (Pe) = -2.5

	$K(\theta, -2.5)$	$\Delta^{1+0}\kappa_{\theta 0}$ [a]	$\Delta^{2+0}\kappa_{\theta 0}$ [a]
$\theta_0 = 1.25$	24.412		
		0.031	
$\theta_1 = 1.20$	24.443		-0.009
		0.022	
$\theta_2 = 1.15$	24.465		

TABLE 2B. CONSTANT LOG (Pe) VALUES; LOG (Pe) = -2.0

	$K(\theta, -2.0)$	$\Delta^{1+0}\kappa_{\theta 1}$ [a]	$\Delta^{2+0}\kappa_{\theta 1}$ [a]
$\theta_0 = 1.25$	24.625		
		0.074	
$\theta_1 = 1.20$	24.699		-0.044
		0.030	
$\theta_2 = 1.15$	24.729		

a. K_{00} is the starting point for taking differences;

$K(\theta, \text{Log Pe}) = \text{Log (Pe/K)}$

$$\left.\begin{array}{l}\Delta^{1+0}\kappa_{\theta \mathrm{Pe}} \\ \\ \Delta^{0+1}\kappa_{\theta \mathrm{Pe}}\end{array}\right\} = \text{First-order difference,}$$

$$\left.\begin{array}{l}\Delta^{2+0}\kappa_{\theta \mathrm{Pe}} \\ \Delta \\ \Delta^{0+2}\kappa_{\theta \mathrm{Pe}}\end{array}\right\} = \text{Second-order difference.}$$

Also,

$$\Delta^{1+1}\kappa_{\theta \mathrm{Pe}} = \Delta^{1+0}\kappa_{\theta 1} - \Delta^{1+0}\kappa_{\theta 0} = \Delta^{0+1}\kappa_{1\mathrm{Pe}} - \Delta^{0+1}\kappa_{0\mathrm{Pe}}$$

TABLE 3A. CONSTANT θ VALUES; $\theta = 1.25$

	K(1.25, Log Pe)	$\Delta^{0+1}\kappa_{1Pe}$ [a]	$\Delta^{0+2}\kappa_{1Pe}$ [a]
Log $Pe_0 = -2.5$	24.412		
		0.213	
Log $Pe_1 = -2.0$	24.625		-0.111
		0.102	
Log $Pe_2 = -1.5$	24.727		

TABLE 3B. CONSTANT θ VALUES; $\theta = 1.20$

	K(1.20, Pe)	$\Delta^{0+1}\kappa_{1Pe}$ [a]	$\Delta^{0+2}\kappa_{1Pe}$ [a]
Log $Pe_0 = -2.5$	24.443		
		0.256	
Log $Pe_1 = -2.0$	24.699		-0.158
		0.098	
Log $Pe_2 = -1.5$	24.797		

a. K_{00} is the starting point for taking differences;

$K(\theta, \text{Log Pe}) = \text{Log}\,(Pe/K)$

$$\left.\begin{matrix} \Delta^{1+0}\kappa_{\theta Pe} \\ \\ \Delta^{0+1}\kappa_{\theta Pe} \end{matrix}\right\} = \text{First-order difference,}$$

$$\left.\begin{matrix} \Delta^{2+0}\kappa_{\theta Pe} \\ \\ \Delta^{0+2}\kappa_{\theta Pe} \end{matrix}\right\} = \text{Second-order difference.}$$

Also,

$$\Delta^{1+1}\kappa_{\theta Pe} = \Delta^{1+0}\kappa_{\theta 1} - \Delta^{1+0}\kappa_{00} = \Delta^{0+1}\kappa_{1Pe} - \Delta^{0+1}\kappa_{0Pe}$$

The general formula for double interpolation using two-way differences is:

$$K = K_{00} + u \cdot \Delta^{1+0}\kappa_{00} + v \cdot \Delta^{0+1}\kappa_{00} + \frac{1}{2!}\Big[u(u-1) \cdot \Delta^{2+0}\kappa_{00}$$

$$+ 2u \cdot v \cdot \Delta^{1+1}\kappa_{00} + v(v-1) \cdot \Delta^{0+2}\kappa_{00}\Big]$$

where

$$u = \frac{\theta - \theta_0}{\theta_1 - \theta_0}$$

and

$$v = \frac{\text{Log (Pe)} - \text{Log (Pe)}_0}{\text{Log (Pe)}_1 - \text{Log (Pe)}_0}$$

From Table 1,

$$K_{00} = 24.412$$

$$\Delta^{1+0}\kappa_{00} = 0.031$$

$$\Delta^{0+1}\kappa_{00} = 0.213$$

$$\Delta^{2+0}\kappa_{00} = -0.009$$

$$\Delta^{0+2}\kappa_{00} = -0.111 \quad .$$

Also,

$$\Delta^{1+1}\kappa_{00} = \Delta^{1+0}\kappa_{\theta 1} - \Delta^{1+0}\kappa_{\theta 0} = 0.043 \quad .$$

For u and v :

$$u = \frac{1.22 - 1.25}{-0.05} = 0.6 \quad ,$$

$$v = \frac{-2.3 - (-2.5)}{0.5} = 0.4 \quad .$$

Then,

$$\begin{aligned} \text{Log}\,(\text{Pe/K}) = {} & 24.412 + (0.6)(0.031) + (0.4)(0.213) \\ & + \frac{1}{2}\Big[(0.6)(-0.4)(-0.009) + 2(0.6)(0.4)(0.043) \\ & + (0.4)(-0.06)(-0.111)\Big] = 24.541 \quad . \end{aligned}$$

Before the computer takes differences from Table 1, it first finds a suitable starting point. In the preceding example the computer would go to 24.412 to start taking differences. If a θ value of 1.12 and a log (Pe) value of -1.1 were read into the computer, the calculations would begin at 24.865 as a starting point. The computer scans those values of θ and log (Pe) in Table 1 until it finds the ones that are closest to but higher in the table than the θ and log (Pe) values read in. Then the log (Pe/K) value corresponding to these values of θ and log (Pe) is used as a starting point for taking differences. In this way the best possible interpolated value of log (Pe/K) is obtained.

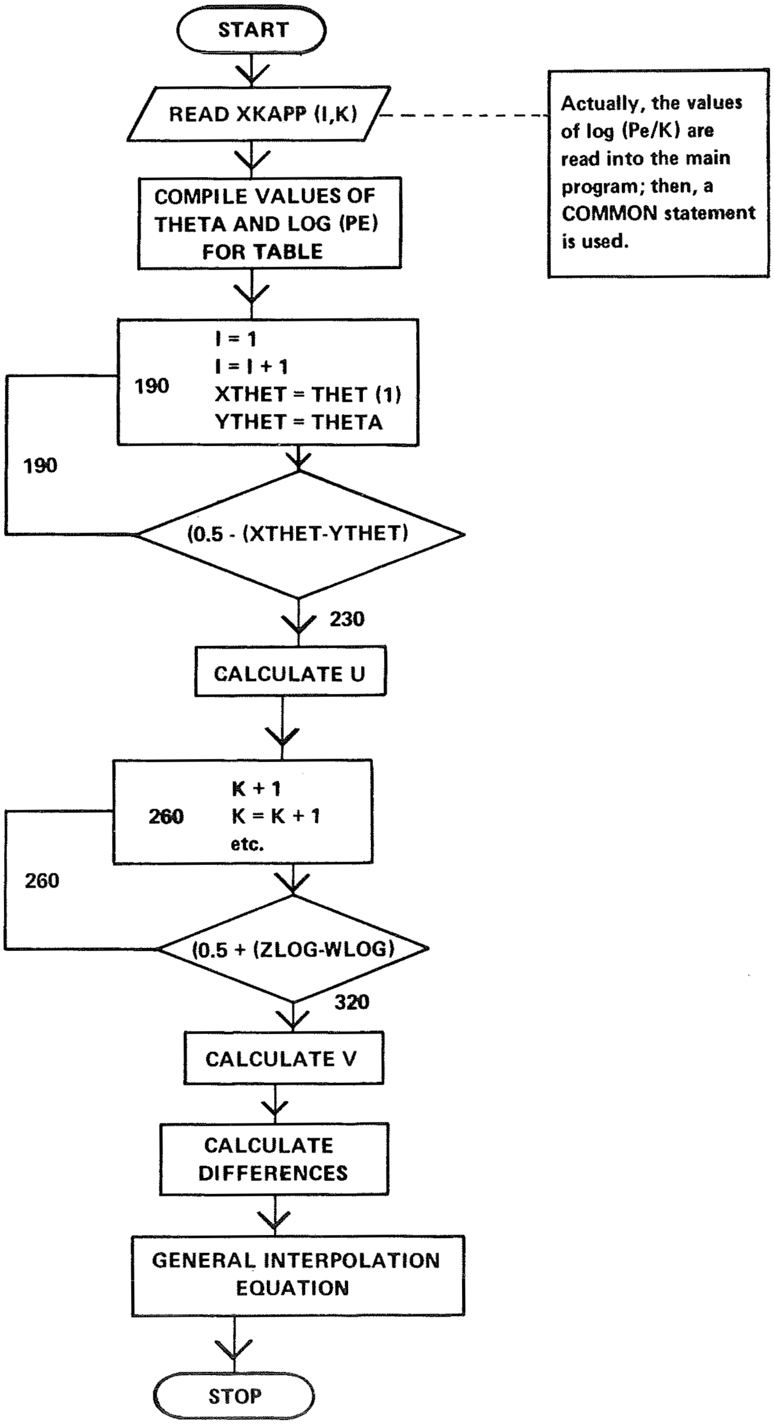
CONCLUSIONS

Some restrictions on using the subroutine should be mentioned. In Table 1, the values of θ must increase by equal amounts (-0.05 in this case); similarly, the values of log (Pe) must increase by equal steps.

For the subroutine to function, it must be able to take first- and second-order differences from Table 1. As long as the value of θ in the interpolation is nct equal to or smaller than 1.00 and the value of log (Pe) is not equal to or greater than -0.05, the calculations in the subroutine will be carried out. If this restriction is not observed, the table must be made larger, or the interpolation subroutine must be made to take only first-order differences.

APPENDIX A

COMPUTER FLOW CHART



APPENDIX B
COMPUTER SUBPROGRAM

```
&FOR,IS OPAC
        SUBROUTINE OPAC(X,THETA,PE,WV,AKCG)
        DIMENSION XKAPP(9,7),XLOG(7),THET(9)
        COMMON XKAPP
        THET(1)=1.35
 90     DO 100 I=2,9
100     THET(I)=THET(I-1)-.05
        CONTINUE
        XLOG(1)=-3.0
120     DO 130 K=2,7
130     XLOG(K)=XLOG(K-1)+.5
        CONTINUE
160     XTHET=0.0
170     YTHET=0.0
180     I=1
190     I=I+1
200     XTHET=THET(I)
        YTHET=THETA
220     IF (.05-(XTHET-YTHET)) 190,230,230
230     U=0.0
240     U=(XTHET-YTHET)/.05
250     K=1
260     K=K+1
270     ZLOG=0.0
280     ZLOG=XLOG(K)
290     WLOG=0.0
        WLOG= .434294*ALOG(PE)
310     IF (.5+(ZLOG-WLOG)) 260,320,320
320     V=0.0
330     V=(WLOG-ZLOG)/.5
340     FIRT=0.0
350     FIRT=XKAPP(I+1,K)-XKAPP(I,K)
360     SECT=0.0
370     SECT=XKAPP(I+2,K)-XKAPP(I+1,K)
380     FIRL=0.0
390     FIRL=XKAPP(I,K+1)-XKAPP(I,K)
400     SECL=0.0
410     SECL=XKAPP(I,K+2)-XKAPP(I,K+1)
420     SECTO=0.0
430     SECTO=SECT-FIRT
440     SECLO=0.0
450     SECLO=SECL-FIRL
460     DA=0.0
470     DA=XKAPP(I+1,K+1)-XKAPP(I,K+1)
480     DX=0.0
490     DX=DA-FIRT
500     XCAP=0.0
510     XCAP=XKAPP(I,K)+U*FIRT+V*FIRL+(SECTO*(U-1.)*U+2.*U*V*DX+SECLO*(V-1
       1.)*V)/2.
       XAKCG=0.0
       XAKCG=23.6203-(XCAP-WLOG)
       Y=0.0
        Y=XAKCG*2.30258
        AKCG=0.0
        AKCG=EXP(Y)
        RETURN
        END
```

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2. Scarborough, James B.: Numerical Mathematical Analysis. The Johns Hopkins Press, Baltimore, Md., 1950.

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APPROVAL

INTERPOLATION OF BODE'S TABLES OF THE SOLAR CONTINUOUS ABSORPTION COEFFICIENT

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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