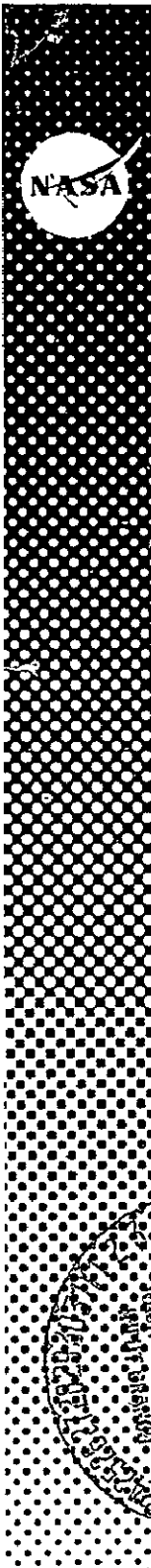


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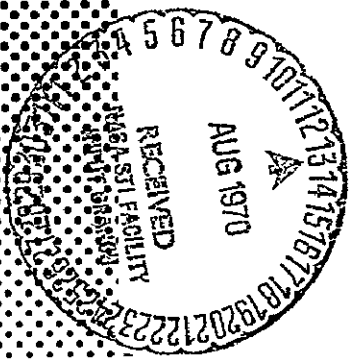
METHODS FOR DETERMINING THE ORBITAL TRANSFER REQUIREMENTS FOR NEAR-EARTH ORBITS

By Samuel L. Miller

and

David J. Griffith,

Orbital Mission Analysis Branch



MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER

HOUSTON, TEXAS



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HOUSTON, TEXAS

Approved: *Edgar C. Lineberry*
Edgar C. Lineberry, Chief
Orbital Mission Analysis Branch

Approved: *John P. Mayer*
John P. Mayer, Chief
Mission Planning and Analysis Division

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METHODS FOR DETERMINING THE ORBITAL TRANSFER

REQUIREMENTS FOR NEAR-EARTH ORBITS

By Samuel L. Miller and David J. Griffith

SUMMARY AND INTRODUCTION

The Apollo C mission has several SPS test objectives that depend on both fixed and variable burn durations for the rendezvous and deorbit maneuvers. In the process of defining the mission, the rendezvous maneuvers and the duration of SPS burns are subject to change, thus altering all planned maneuvers. This suggests a need for a method that quickly and accurately estimates the changed maneuver. This document provides both a graphical (plots) and an analytical method (Olivetti programs) for estimating the delta V requirements for the orbital transfer maneuvers. In addition to the total delta V requirements, the components of delta V in the external delta V system can be determined.

With the exception of those for the rendezvous, the parametric data presented can be used to determine the orbital maneuver requirements. The use of the parametric data should thus reduce considerably the numbers of computer runs required to obtain an exact solution. Both the Olivetti programs and the plotted data assume impulsive maneuvers; however, a good comparison of results with a finite burn can be obtained by placing the center of the burn arc at the impulsive point.

Use of both methods indicates that either gives comparatively accurate external delta V components. Thus, the mission planner has two independent sources of determining the orbital transfer requirements.

Although this document was designed using the Apollo C mission profile, the data presented may be used for general mission planning.

ABBREVIATIONS

Symbols

a	semimajor axis
e	eccentricity
h	altitude, n. mi.
r	radius
V	inertial velocity, fps
ΔV_t	total ΔV magnitude, fps
ΔV_X	components of ΔV in the external ΔV system, ft
ΔV_Y	
ΔV_Z	
β	pitch, i.e., the angle between the velocity vector and the thrust vector measured in the orbital plane, positive up, deg
γ	inertial flight-path angle, deg
θ	orbital central angle between perigee and vehicle position, deg
μ	earth's gravitational constant, 1.407648×10^{16} ft ³ /sec ²
ψ	yaw, i.e., the angle between the orbital plane and the projection of the thrust vector in a plane normal to the orbit plane, positive to the right, deg

Subscripts

a	apogee
E	earth
p	perigee
1	initial
2	desired

DISCUSSION OF METHODS

The following parameters will be available for any specified ground elapsed time: h , h_a , h_p , θ , V , γ .

With the preceding parameters available, the targets for a particular burn may be obtained either by the plotted data or by the Olivetti analytic programs.

Figures

The figures were generated using analytic equations.

Figure 1 presents the total velocity magnitude, ΔV_t , as a function of spacecraft weight and burn time. Also included in figure 1 is pitch as a function of ΔV_t and ΔV_Z .

Figure 2 presents the inertial velocity and inertial flight-path angle as a function of h_p , h_a , and θ for various spacecraft altitudes ranging from 75 n. mi. to 265 n. mi., in increments of 10 n. mi.

Figure 3 presents several nomograms giving the Z component of velocity as a function of present inertial velocity and flight-path angle; figure 4 gives the X component of velocity as a function of the same parameters.

Figure 5 presents the lateral velocity component, ΔV_Y , as a function of yaw and pitch for various ΔV_t 's and longitudinal velocity components, ΔV_X 's. The magnitudes of ΔV_t presented are representative of those associated with the SPS burns for the 205/101 mission.

Olivetti Programs

The following Olivetti programs may be used to obtain targets for a particular SPS burn without reference to plotted data.

Programs 1 and 2 calculate the velocity components in the orbital plane required for a particular transfer. Program 1 uses both altitude and velocity vector inputs. Program 2 uses only altitude inputs.

Program 3 calculates the lateral velocity component and the yaw and pitch attitudes using the total velocity component and the in-plane velocity components as input.

Program 4 calculates the pitch orientation using the total velocity magnitude and the vertical velocity component as input.

Program 5 uses the pitch attitude, the total velocity magnitude, and the in-plane velocity component to calculate the lateral velocity component and the yaw attitude.

These programs are presented in appendices A through E, respectively. The appendices provide instruction listings, input and output, equations used in each program, and the restrictions on the use of each program. Some equations given are not exact, but they do yield an accurate approximation of the true value.

Typical Example

To illustrate the use of the plotted data and the Olivetti programs, an example problem is presented. Suppose the following is known

Present position and ellipse	Transfer ellipse and position desired
$h = 175$ n. mi.	$h = 175$ n. mi.
$h_{p_1} = 114$ n. mi.	$h_{p_2} = 121$ n. mi.
$h_{a_1} = 228$ n. mi.	$h_{a_2} = 277$ n. mi.
$\theta = 97^\circ$	$0^\circ < \theta_2 < 180^\circ$
$\Delta V_t = 230$ fps	
$V_1 = 25\,291$ fps	
$\gamma_1 = .898924^\circ$	

Find ΔV_X , ΔV_Y , ΔV_Z , β , and ψ .

Graphical solution. - The following method uses plotted data only:

Step 1 - The inertial velocity and inertial flight-path angle on each ellipse is obtained from figure 2(k). (These values are then used to determine ΔV_X and ΔV_Z .)

$$\begin{aligned} V_1 &= 25\,296 \text{ fps} & V_2 &= 25\,395 \text{ fps} \\ \gamma_1 &= .91^\circ & \gamma_2 &= 1.15^\circ \end{aligned}$$

Step 2 - Figures 3(c) and (d) are used to determine the velocity components in the Z direction, while figures 4(b) and (c) give the velocity components in the X direction.

$$\begin{aligned} V_{Z_1} &= 400 \text{ fps} & V_{X_1} &= 25\,286 \text{ fps} \\ V_{Z_2} &= 510 \text{ fps} & V_{X_2} &= 25\,391 \text{ fps} \\ \Delta V_Z &= V_{Z_1} - V_{Z_2} & \Delta V_X &= V_{X_2} - V_{X_1} = 105 \text{ fps} \\ &= -100 \text{ fps} \end{aligned}$$

Step 3 - The absolute value of pitch associated with the above ΔV_Z and ΔV_t can be obtained from figure 1.

$$|\beta| = 29^\circ$$

Step 4 - From figure 5(a), the ΔV_Y component and subsequently the yaw magnitude can be determined.

$$|\Delta V_Y| = 173 \text{ fps}$$

$$|\psi| = 59^\circ$$

Analytic solution. - The following method uses Olivetti programs 1 and 2 to calculate a solution.

Programs 1 and 2 (appendices A and B) can be used to determine the ΔV_X and ΔV_Z components directly from the data available from the initial conditions.

Program 1

Input: $h = 175$ n. mi.
 $h_{p_2} = 121$ n. mi.
 $h_{a_2} = 277$ n. mi.
 $A = 1$
 $\gamma_1 = .898924^\circ$
 $V_1 = 25\ 291$ fps

Output: $\Delta V_X = 98.6026$ fps
 $\Delta V_Z = -120.9121$ fps

Program 2

Input: $h = 175$ n. mi.
 $h_{p_1} = 114$ n. mi.
 $h_{a_1} = 228$ n. mi.
 $h_{p_2} = 121$ n. mi.
 $h_{a_2} = 277$ n. mi.

Output: $V_1 = 25\ 296$ fps
 $|\sin \gamma_1| = 0.0157384$
 $V_2 = 25\ 392$ fps
 $|\sin \gamma_2| = 0.0203887$
 $\Delta V_X = 96$ fps
 $\Delta V_Z = -120$ fps

Using ΔV_X and ΔV_Z from program 1 as input, program 3 (appendix C) can now determine ΔV_Y , pitch and yaw magnitudes.

Input: $\Delta V_t = 230$ fps
 $\Delta V_X = 98.6026$ fps
 $\Delta V_Z = -120.9121$ fps

Output: $|\Delta V_Y| = 168.9906$ fps
 $|\psi| = 59.7274^\circ$
 $|\beta| = 31.7158^\circ$

Comparison with numerical integration.— The example problem was solved using numerical integration, and a comparison with both plotted data and Olivetti programs is presented below.

COMPARISON OF THE THREE METHODS

[h = 175 n. mi.; $h_{p_1} = 114$ n. mi.;

= 228 n. mi.; $h_{p_2} = 121$ n. mi.; $h_{a_2} = 277$ n. mi.]

Parameter	Methods		
	Integration	Figures	Olivetti programs
θ , deg	96.6	95.	
V_1 , fps	25 291	25 296	25 291
γ_1 , deg	0.898924	0.91	0.898924
θ_2 , deg	74.3	73.	
V_2 , fps	25 388	25 395	25 391
γ_2 , deg	1.17905	1.15	1.16818
ΔV_x , fps	98	105	98.6
ΔV_y , fps	166.346	173	168.99
ΔV_z , fps	-125	-108	-120.9
ΔV_t , fps	230	230	230
ψ , deg	59.4	59.	59.7
β , deg	32.8	28.	31.7

APPENDIX A

PROGRAM 1 - TRANSFER VELOCITY COMPONENTS REQUIRED BASED ON
BOTH ALTITUDE AND VELOCITY VECTOR INPUTS

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APPENDIX A

PROGRAM 1 - TRANSFER VELOCITY COMPONENTS REQUIRED BASED ON
 BOTH ALTITUDE AND VELOCITY VECTOR INPUTS

A program is presented that calculates the velocity change necessary in the X and Z direction, to transfer impulsively from the present ellipse (h, V_1, γ_1) to the desired ellipse (h_{a_2}, h_{p_2}) at some altitude h .

This program requires a decimal wheel setting of 7 with V being the entry point of the program. The order of the input and output quantities is as indicated.

Input: h, h_{p_2}, h_{a_2}, A (explained below), γ_1, V_1

The quantity A takes on the values of +1 or -1. If $A = 1$, then the sign of γ on the desired ellipse will be positive (i.e., the vehicle will be on the upleg of the desired ellipse after the impulsive maneuver). If $A = -1$, then the sign of γ on the desired ellipse will be negative (i.e., the vehicle will be on the downleg of the desired ellipse after the impulsive maneuver).

Output: $\Delta V_X, \Delta V_Z$

Equations:

$$r = h + r_E$$

$$r_a = h_a + r_E$$

$$r_p = h_p + r_E$$

$$a = \frac{r_a + r_p}{2}$$

$$e = 1 - \frac{r_p}{a}$$

$$V = \sqrt{\left(\frac{2}{r} - \frac{1}{a}\right)\mu}$$

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$$\sin \gamma = \sqrt{1 - \frac{a^2(1 - e^2)}{r(2a - r)}}$$

$$\cos \gamma = \sqrt{\frac{a^2(1 - e^2)}{r(2a - r)}}$$

$$\Delta V_X = v_2 \cos \gamma_2 - v_1 \cos \gamma_1$$

$$\Delta V_Z = v_1 \sin \gamma_1 - v_2 \sin \gamma_2$$

Restrictions:

1. Neither the present ellipse nor the desired ellipse can be circular.

2. This program should not be used when the flight-path angle of either ellipse exceeds 5° at the point of transfer, since a small angle approximation is used for both the sine and cosine of γ .

INSTRUCTION LISTING OF PROGRAM 1

AV	b↑	bX	R*	R◇
a↑	b÷	B÷	R÷	R+
R↓	A÷	b↓	R÷	R-
r↓	-	A+	R-	R↓
R+	A÷	B-	R*	R↓
R+	C↑	b↓	R↑	rX
D↓	b÷	b÷	r÷	R÷
b↑	c↓	A÷	D-	RS
S	B÷	b↑	÷	DX
↓	c-	-	AX	c↑
b+	a↑	A/	B↑	c÷
B↓	r↓	CX	a↑	C↓
S	R-	S	d↓	C-
↓	R÷	X	↓	cX
b+	R↑	b↓	-	A◇
b↓	DX	A/	A/	b↓
S	X	CX	B↓	cX
+	A/	C↓	S	B↓
b+	C↓	S	X	B-
a↑	AX	↓	B↓	A◇
d↑	A÷	a↑	X	V
c↑	-	R↓	a↑	
c÷	bX	R-	RX	

APPENDIX B

PROGRAM 2 - TRANSFER VELOCITY COMPONENTS REQUIRED
BASED ON ALTITUDE INPUTS

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APPENDIX B

PROGRAM 2 - TRANSFER VELOCITY COMPONENTS REQUIRED

BASED ON ALTITUDE INPUTS

This program calculates the velocity change necessary in the X and Z direction to transfer from the present ellipse (h_{p_1} , h_{a_1}) to the desired ellipse (h_{p_2} , h_{a_2}) at an altitude h . A decimal wheel setting of 7 is required for this program with V being the entry point of the program. The order of the input and output quantities is as given.

Input: $h, h_{p_1}, h_{a_1}, h_{p_2}, h_{a_2}$

Output: $V_1, |\sin \gamma_1|, V_2, |\sin \gamma_2|, \Delta V_x, \Delta V_z$

Equations:

$$r = h + r_E$$

$$r_p = h_p + r_E$$

$$r_a = h_a + r_E$$

$$a = \frac{r_a + r_p}{2}$$

$$e = 1 - \frac{r_p}{a}$$

$$v = \sqrt{\left(\frac{2}{r} - \frac{1}{a}\right)\mu}$$

$$|\sin \gamma| = \sqrt{1 - \frac{a^2(1 - e^2)}{r(2a - r)}}$$

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$$\Delta V_X \approx V_2 - V_1$$

$$\Delta V_Z = V_1 |\sin \gamma_1| - V_2 |\sin \gamma_2|$$

Restrictions:

1. Neither the present ellipse nor the desired ellipse can be circular.
2. The transfer from the present to the desired ellipse must occur on the same side of the line of apsides.
3. ΔV_X will have the correct sign, but will have an error in its magnitude. The error in the magnitude of ΔV_X will increase as γ increases in absolute value, since the cosine of γ is not used in the calculation.
4. ΔV_Z will have the correct magnitude and sign on the ascending side of the ellipse. On the descending side of the ellipse, the magnitude of ΔV_Z will be correct but its sign should be negated.
5. This program should not be used when the flight-path angle of either ellipse exceeds 5° at the point of transfer, since a small angle approximation is used for the sine of γ .

INSTRUCTION LISTING OF PROGRAM 2

AV	↓	b÷	X	C◇
D↓	b+	c÷	A/	A-
a↑	B+	A÷	ā↑	D↓
R↑	a↑	-	R◇	-
r↓	d↑	A/	R+	A◇
R↑	÷	C↑	R-	↑
R+	b↑	/Z	R↓	bX
D↑	B↑	b↓	R↓	D↑
b↑	B÷	/W	rX	CX
/Y	A÷	aZ	R÷	D↑
S	-	b↑	RS	D-
↑	AX	aW	DX	A◇
b+	A÷	B÷	X	c*
c↑	-	c÷	D↑	C*
aY	b↑	a↑	/V	D*
S	BX	r↓	V	V
↑	c-	R-	aV	
b+	b↑	R÷	A◇	
B↑	BX	R↑	b◇	
S	X	DX	D◇	

APPENDIX C

PROGRAM 3 - LATERAL VELOCITY COMPONENT AND THE YAW AND
PITCH ORIENTATIONS USING THE IN-PLANE VELOCITY
COMPONENTS AND THE TOTAL VELOCITY COMPONENT AS INPUT

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APPENDIX C

PROGRAM 3 - LATERAL VELOCITY COMPONENT AND THE YAW AND
 PITCH ORIENTATIONS USING THE IN-PLANE VELOCITY
 COMPONENTS AND THE TOTAL VELOCITY COMPONENT AS INPUT

The ΔV_Y and the yaw and pitch are calculated from ΔV_t , ΔV_X , and ΔV_Z . This program requires a decimal wheel setting of 9 with V being the entry point of the program. The order of the input and output quantities is as given.

Input: ΔV_t , ΔV_X , ΔV_Z

Output: $|\Delta V_Y|$, $|\psi|$, $|\beta|$

Equations:

$$|\Delta V_Y| = \sqrt{\Delta V_t^2 - \Delta V_X^2 - \Delta V_Z^2}$$

$$|\beta| = \sin^{-1} \left(\frac{\Delta V_Z}{\Delta V_t} \right)$$

$$|\psi| = \sin^{-1} \frac{\Delta V_Y}{\sqrt{\Delta V_X^2 + \Delta V_Y^2}}$$

Restrictions:

1. $\Delta V_t > \sqrt{\Delta V_X^2 + \Delta V_Z^2}$ If $\Delta V_t \leq \sqrt{\Delta V_X^2 + \Delta V_Z^2}$, then the program will output a negative number and return to the beginning.

2. It must be noted that the ΔV_Y , yaw and pitch output are absolute values. This requires the user to place the proper sign on these values. Also, the yaw and pitch output takes on the following range of values: $0 \leq \psi \leq 90$, $0 \leq \beta \leq 90$. As an example, if the yaw was output as 83° , but a retrograde component of velocity is desired, then the user knows the yaw desired is 97° . A similar situation holds true for pitch.

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INSTRUCTION LISTING OF PROGRAM 3

AV	↓	b÷	A√	R-
a↑	b÷	a↑	B+	R*
R◇	AX	dS	A√	R÷
R+	C↑	c↑	A+	R÷
R-	c+	AW	A+	R-
R↓	B↑	a↑	A+	R*
R↓	AX	d↓	+	R↑
rX	d↑	B↑	a↑	r÷
R÷	B-	AX	r↑	D-
RS	/Z	B-	D↓	X
DX	A◇	A√	+	A◇
b↑	V	a↑	A√	c↓
S	aZ	d↑	a↑	/V
↓	A√	B↑	dX	C↓
b÷	bX	A+	-	d÷
B↓	A◇	B+	A√	W
S	b÷	A√	a↑	aV
↓	AX	B+	r↑	c*
b÷	b↑	A√	D↑	V
AX	c+	B+	X	
c↓	A√	A√	a↑	
S	b↓	B+	R↑	

APPENDIX D

PROGRAM 4 - PITCH ORIENTATION BASED ON THE TOTAL VELOCITY
MAGNITUDE AND THE VERTICAL VELOCITY COMPONENT

APPENDIX D

PROGRAM 4 - PITCH ORIENTATION BASED ON THE TOTAL VELOCITY
MAGNITUDE AND THE VERTICAL VELOCITY COMPONENT

This program calculates pitch and the sine of the pitch as a function of ΔV_t and ΔV_z . A decimal wheel setting of 9 is required for this program with V being the entry point of the program. The order of the input and output is as given.

Input: $\Delta V_t, \Delta V_z$

Output: $|\sin \beta|, |\beta|$

Equations:

$$|\sin \beta| = \frac{\Delta V_z}{\Delta V_t}$$

$$|\beta| = \sin^{-1} \left(\frac{\Delta V_z}{\Delta V_t} \right)$$

Restrictions:

1. $\Delta V_z \leq \Delta V_t$
2. The proper sign must be placed on the pitch.
3. The range of values taken on by pitch are: $0 \leq \beta \leq 90$. As an example, if the pitch is output as 83° , but a retrograde component of velocity is desired then the user knows that the pitch desired is 97° .

INSTRUCTION LISTING OF PROGRAM 4

AV	B÷	B↑	+	R↓
a↑	C↓	A+	a↑	R-
RX	S	B+	r↑	R*
R◇	↓	A√	D↓	R÷
R+	B÷	B+	+	R÷
R-	C÷	A√	A√	R-
R↓	A◇	B+	a↑	R*
R↑	a↑	A√	dX	R↑
rX	d↓	B+	-	r÷
R÷	B↑	A√	A√	D-
RS	AX	B+	a↑	X
DX	B-	A√	r↑	A◇
B↑	A√	A+	D↓	/◇
S	a↑	A+	X	V
↓	d↑	A+	a↑	

· APPENDIX E

PROGRAM 5 - LATERAL VELOCITY COMPONENT AND ORIENTATION
BASED ON IN-PLANE VELOCITY COMPONENT AND THE
TOTAL VELOCITY COMPONENT AS INPUT

APPENDIX E

PROGRAM 5 - LATERAL VELOCITY COMPONENT AND ORIENTATION
 BASED ON IN-PLANE VELOCITY COMPONENT AND THE
 TOTAL VELOCITY COMPONENT AS INPUT

The ΔV_Y , $\sin \psi$, and ψ are calculated from the $\sin \beta$, ΔV_t , ΔV_X . This program requires a decimal wheel setting of 9 with V being the entry point of the program. The order of the input and output quantities is as given.

Input: $|\sin \beta|$, ΔV_t , ΔV_X

Output: $|\Delta V_Y|$, $|\sin \psi|$, $|\psi|$

Equations:

$$\Delta V_Z = \Delta V_t \sin \beta$$

$$|\Delta V_Y| = \sqrt{\Delta V_t^2 - \Delta V_Z^2 - \Delta V_X^2}$$

$$|\sin \psi| = \frac{\Delta V_Y}{\sqrt{\Delta V_X^2 + \Delta V_Y^2}}$$

$$|\psi| = \sin^{-1} \left(\frac{\Delta V_Y}{\sqrt{\Delta V_X^2 + \Delta V_Y^2}} \right)$$

Restrictions:

1. $\Delta V_t^2 > \Delta V_X^2 + \Delta V_t^2 \sin^2 \beta$. If this condition is violated then the program will output a negative number and return to the beginning.

2. The user should note that ΔV_Y , $\sin \psi$, and yaw are output as absolute values, so this requires that he place the proper sign onto the computed value. The range of the value output for yaw is $0 \leq \psi \leq 90$. As an example, if the yaw was output as 83° , but a retrograde component of velocity is desired, then the user knows the yaw desired is 97° .

INSTRUCTION LISTING OF PROGRAM 5

AV	↓	B+	B+	D↓
S	bX	A✓	A✓	X
+	AX	b↓	B+	a↑
S	b↓	b÷	A✓	R↓
b↑	S	b*	B+	R-
b↓	+	B*	A✓	R*
a↑	C÷	A◇	A+	R÷
RX	AX	a↑	A+	R÷
R◇	B↓	d↓	A+	R-
R+	b-	B↑	+	R*
R-	B-	AX	a↑	R↑
R↓	/V	B-	r↑	r÷
R↓	A◇	A✓	D↓	D-
rX	-V	a↑	+	X
R÷	aV	d↑	A✓	A◇
RS	A✓	B↑	a↑	/◇
DX	CX	A+	dX	V
C↑	A◇	B+	-	
C÷	C÷	A✓	A✓	
AX	AX	B+	a↑	
B↓	b↑	A✓	r↑	

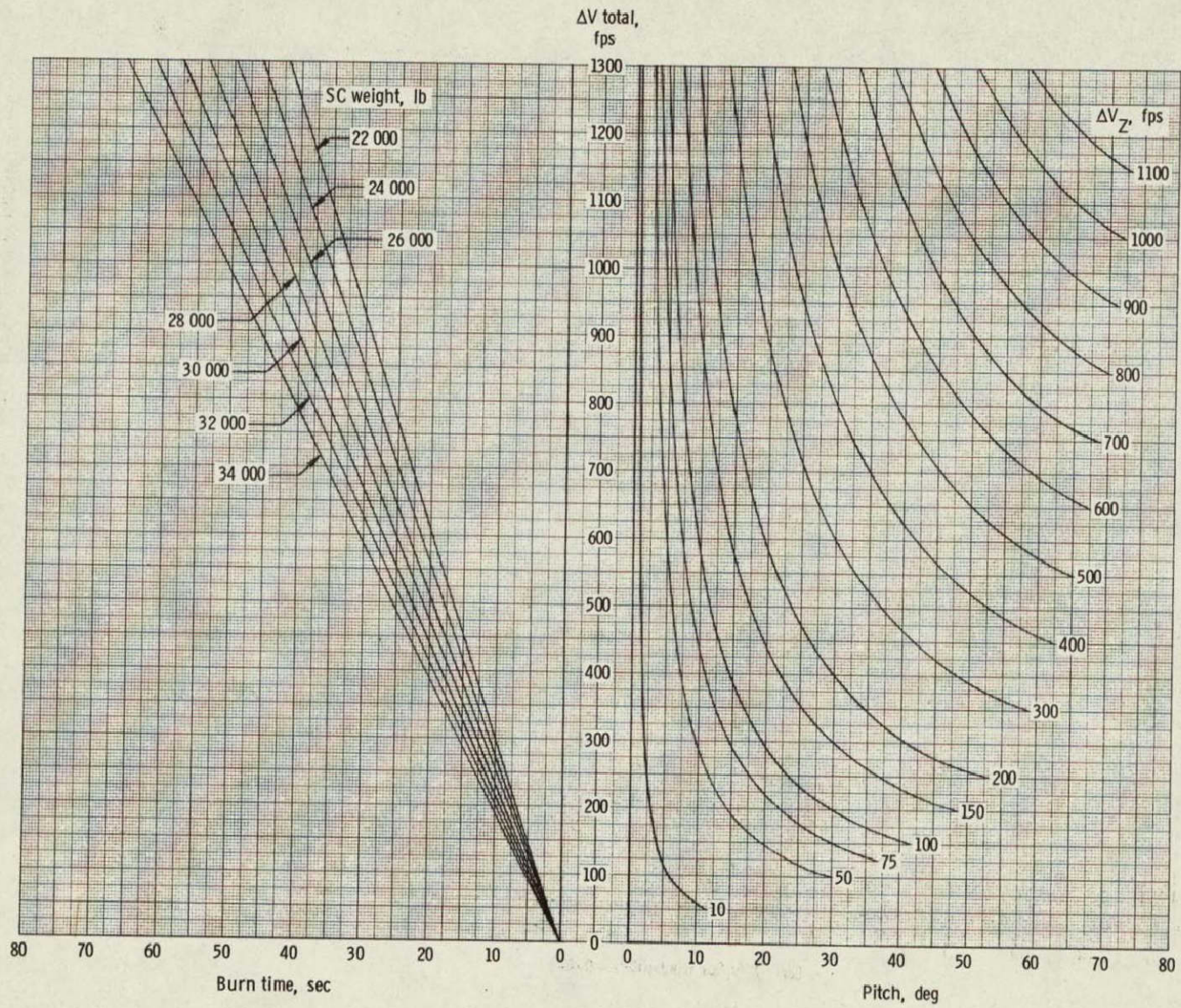
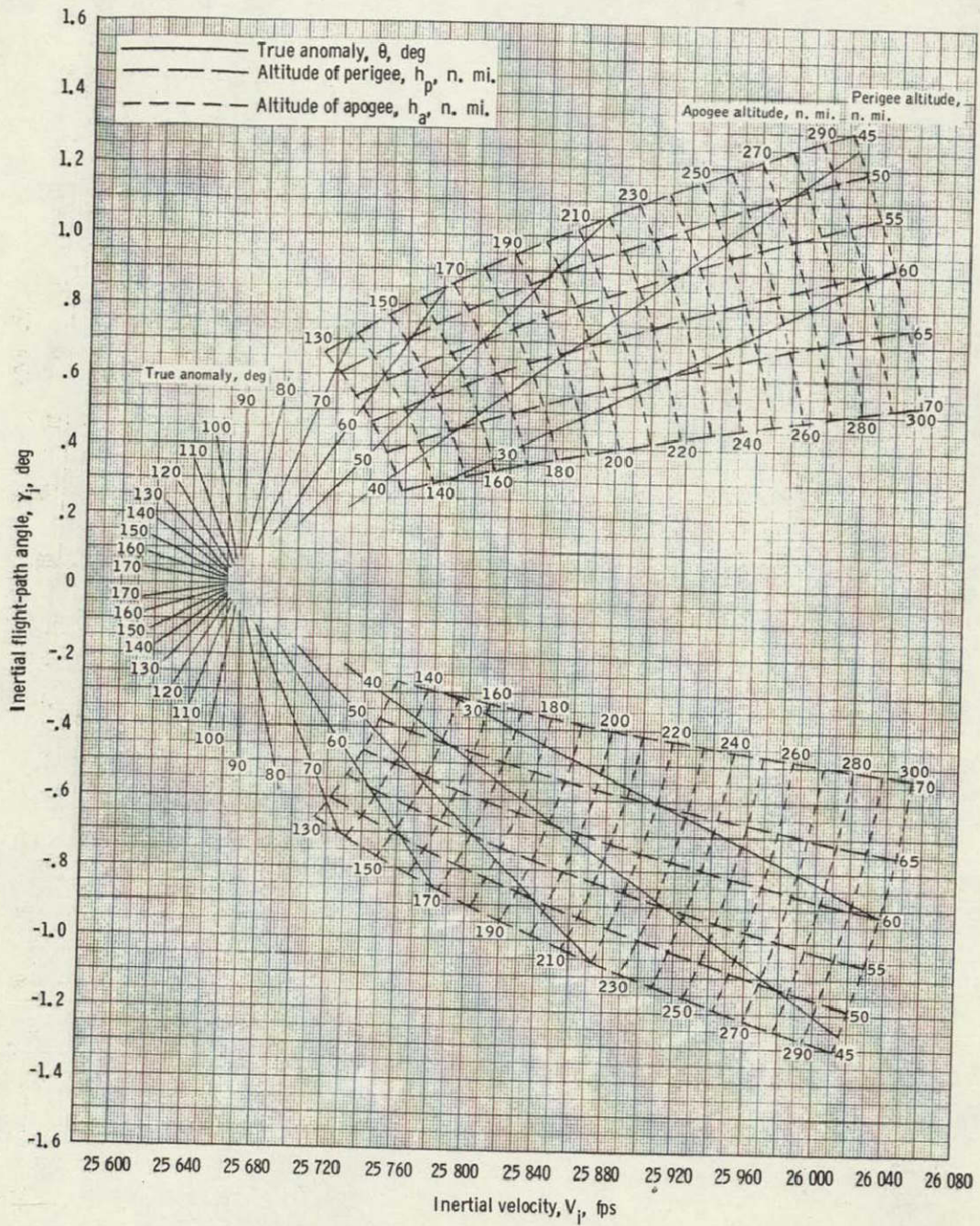
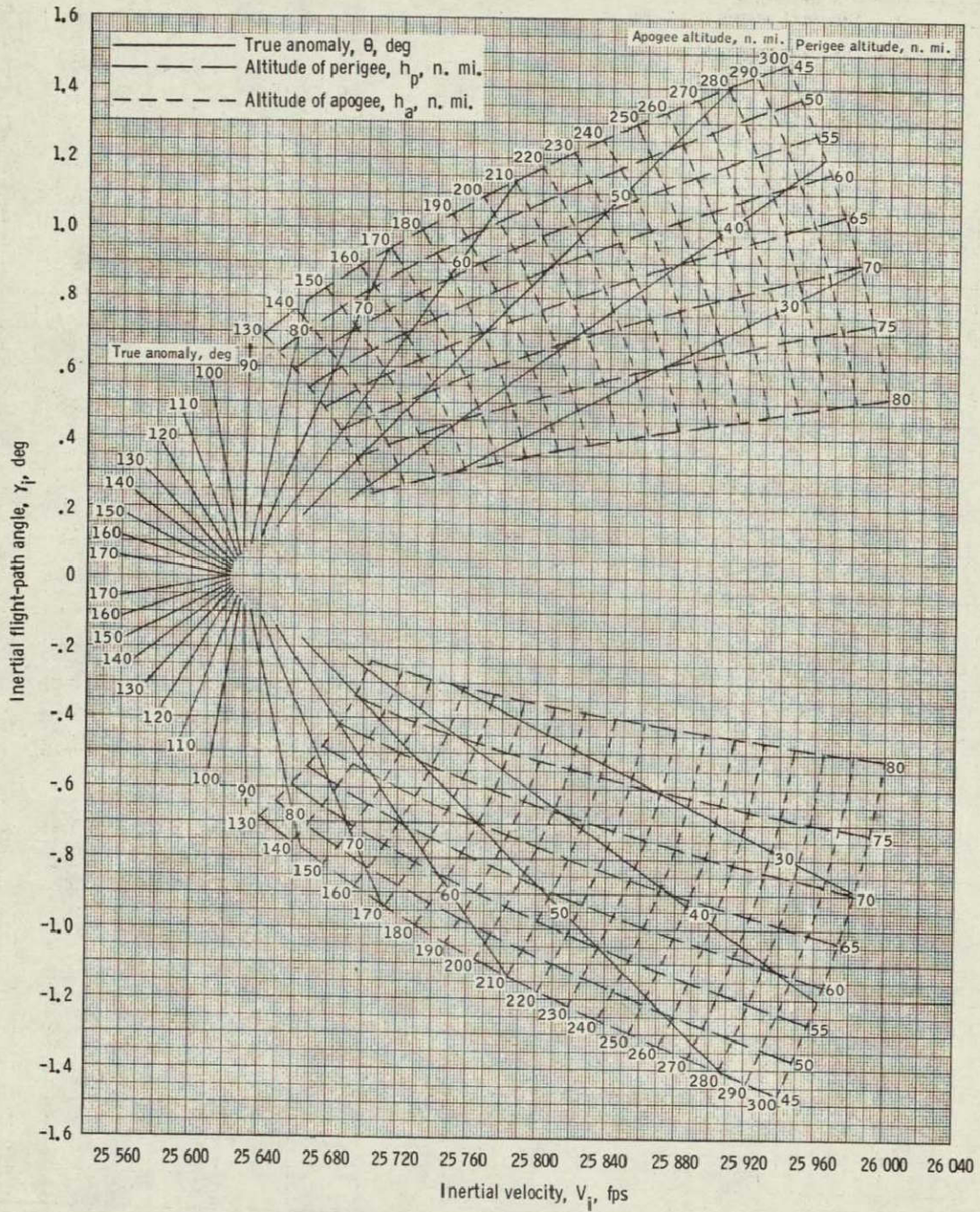


Figure 1. - Total velocity magnitude as a function of burn time and spacecraft weight; pitch attitude as a function of ΔV_t and ΔV_z .



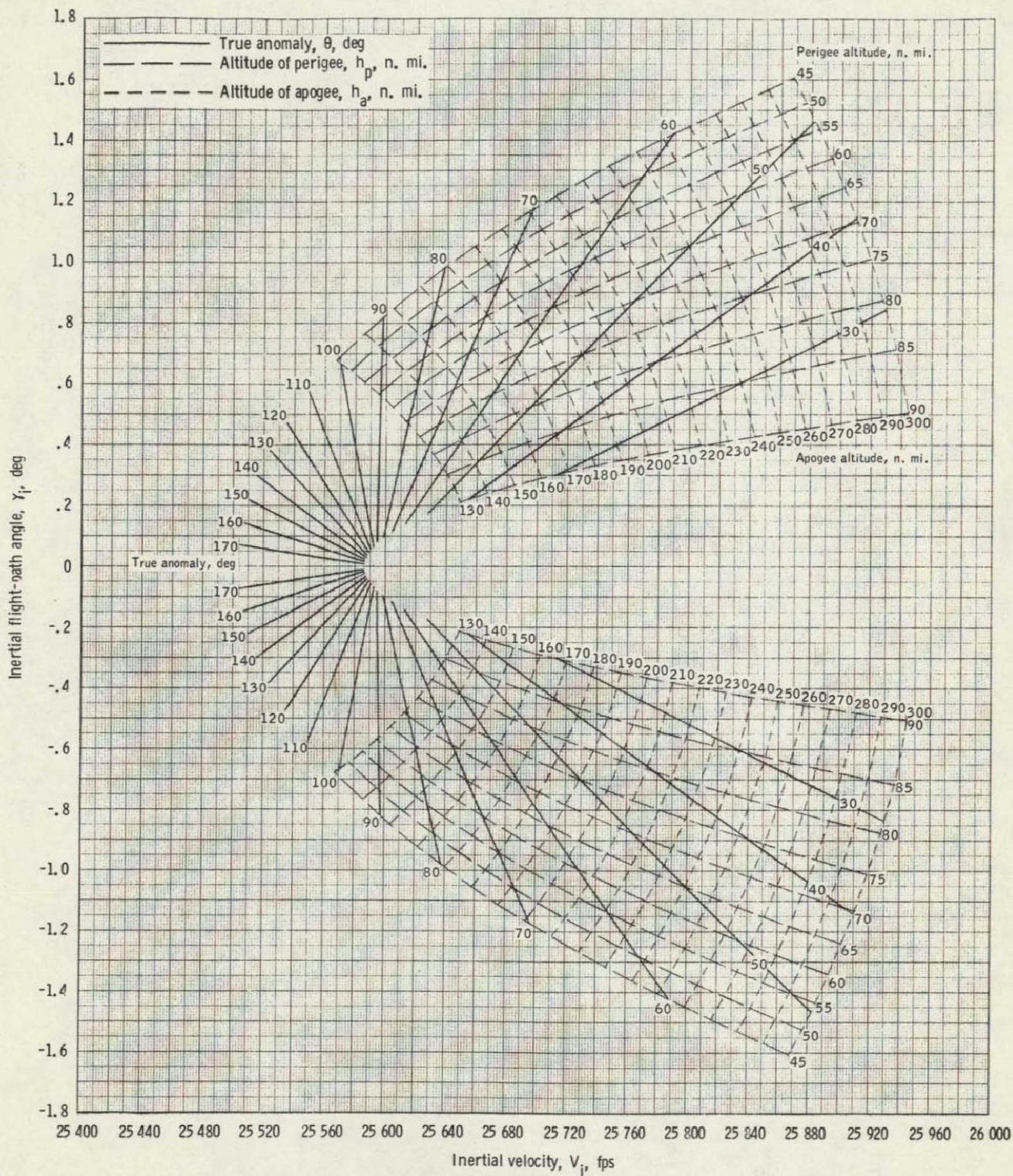
(a) Altitude = 75 n. mi.

Figure 2. - True anomaly, apogee altitude, and perigee altitude as a function of inertial velocity and inertial flight-path angle for various spacecraft altitudes.



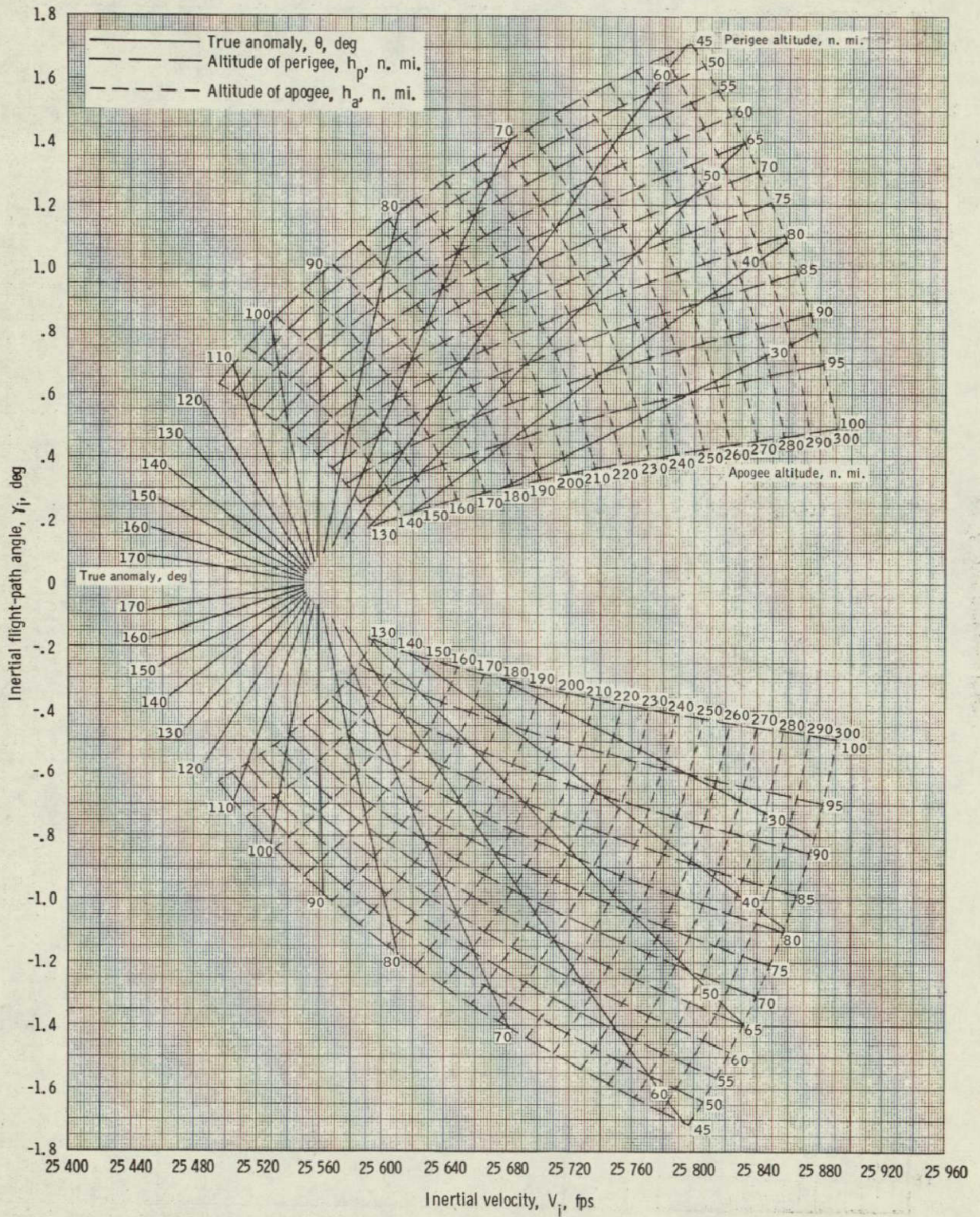
(b) Altitude = 85 n. mi.

Figure 2. - Continued.



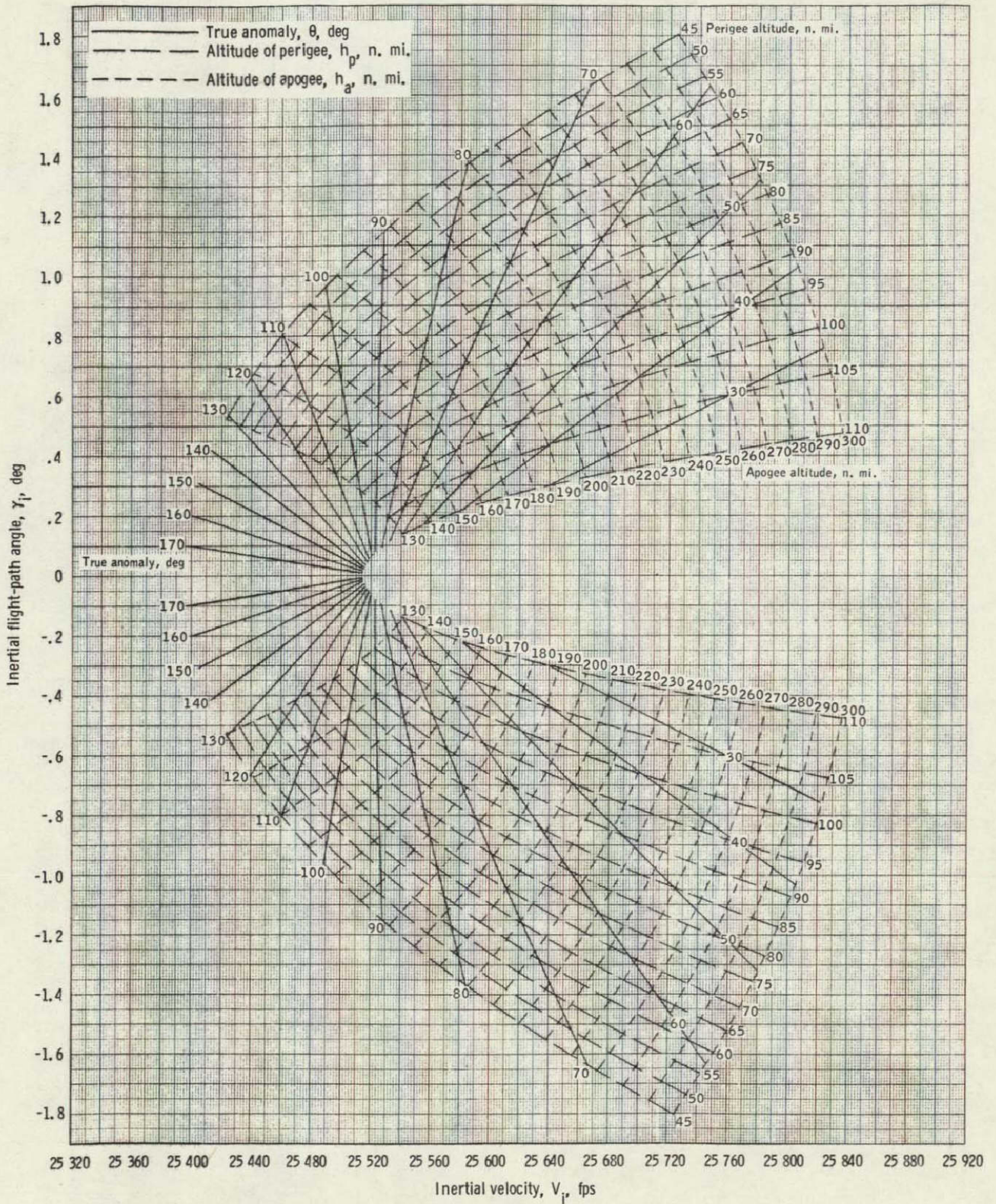
(c) Altitude = 95 n. mi.

Figure 2. - Continued.



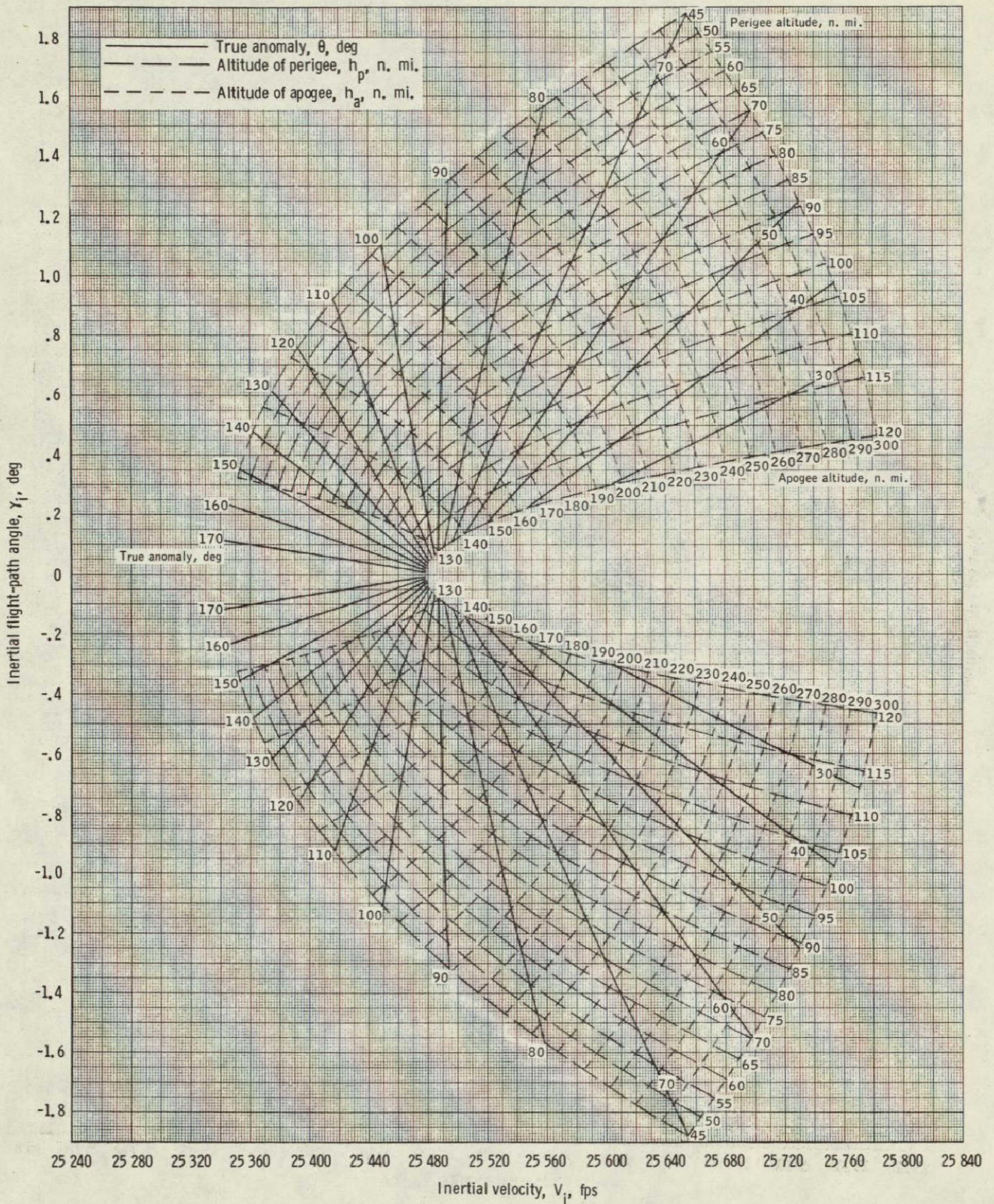
(d) Altitude = 105 n. mi.

Figure 2. - Continued.



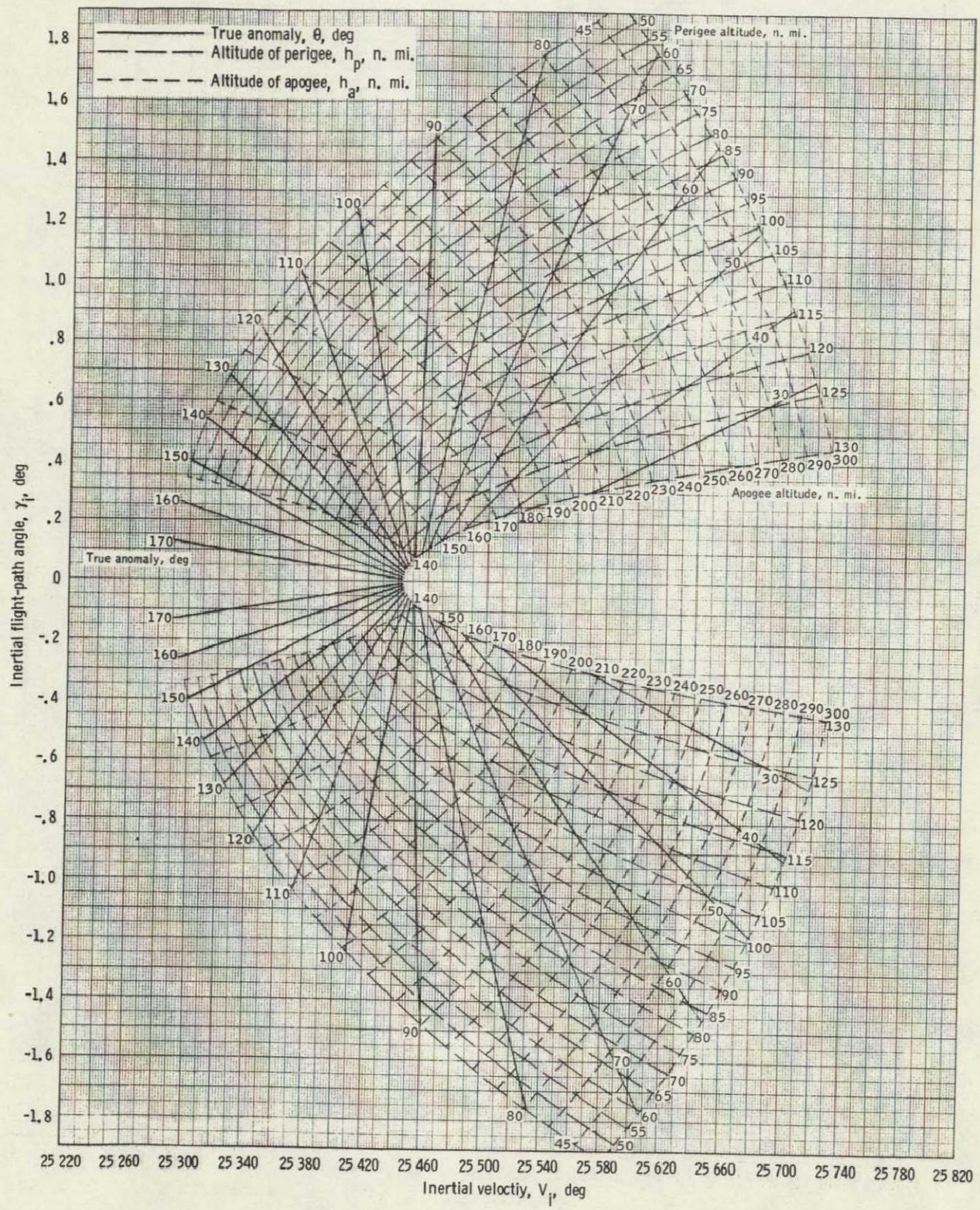
(e) Altitude = 115 n. mi.

Figure 2, - Continued.

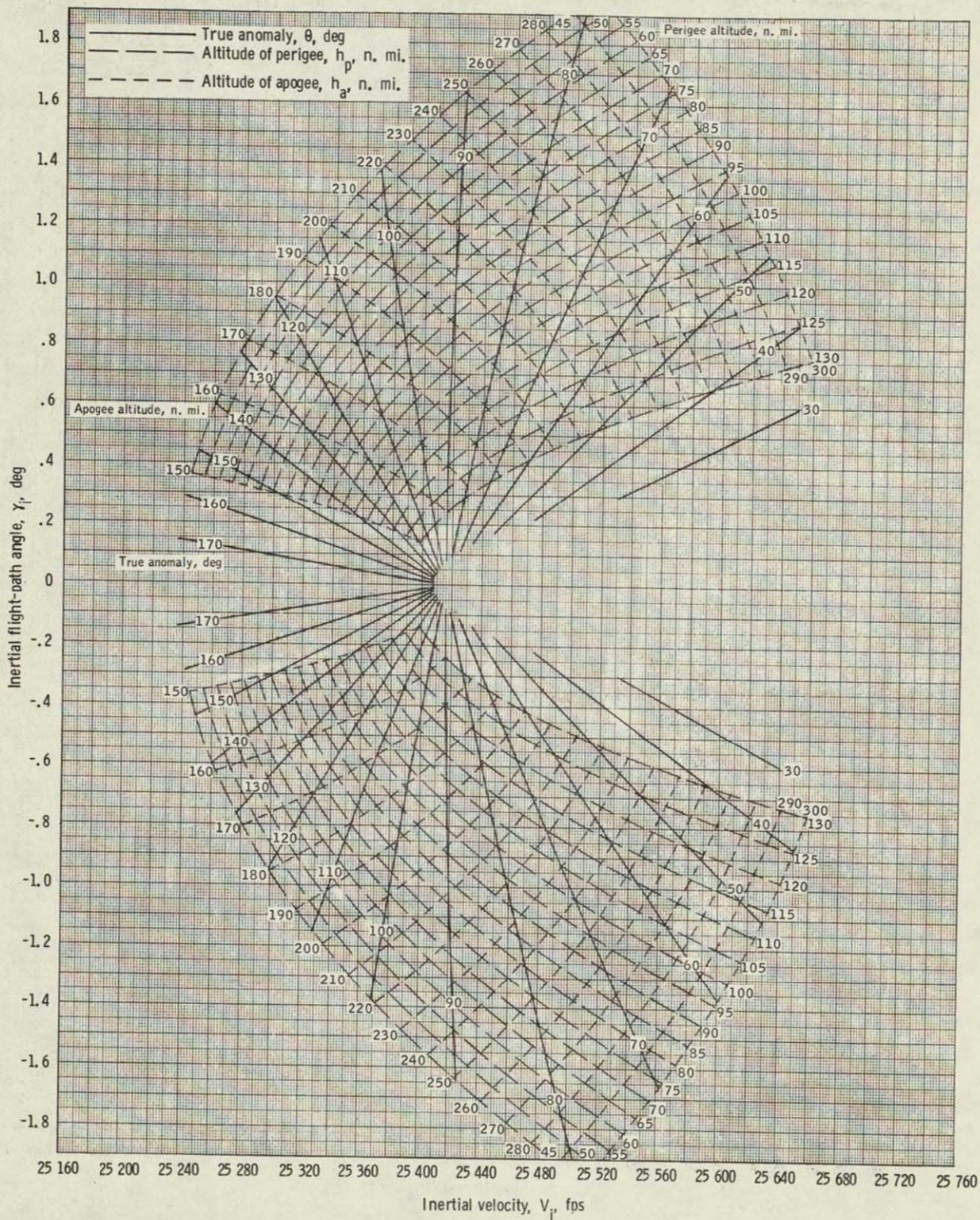


(f) Altitude = 125 n. mi.

Figure 2. - Continued.

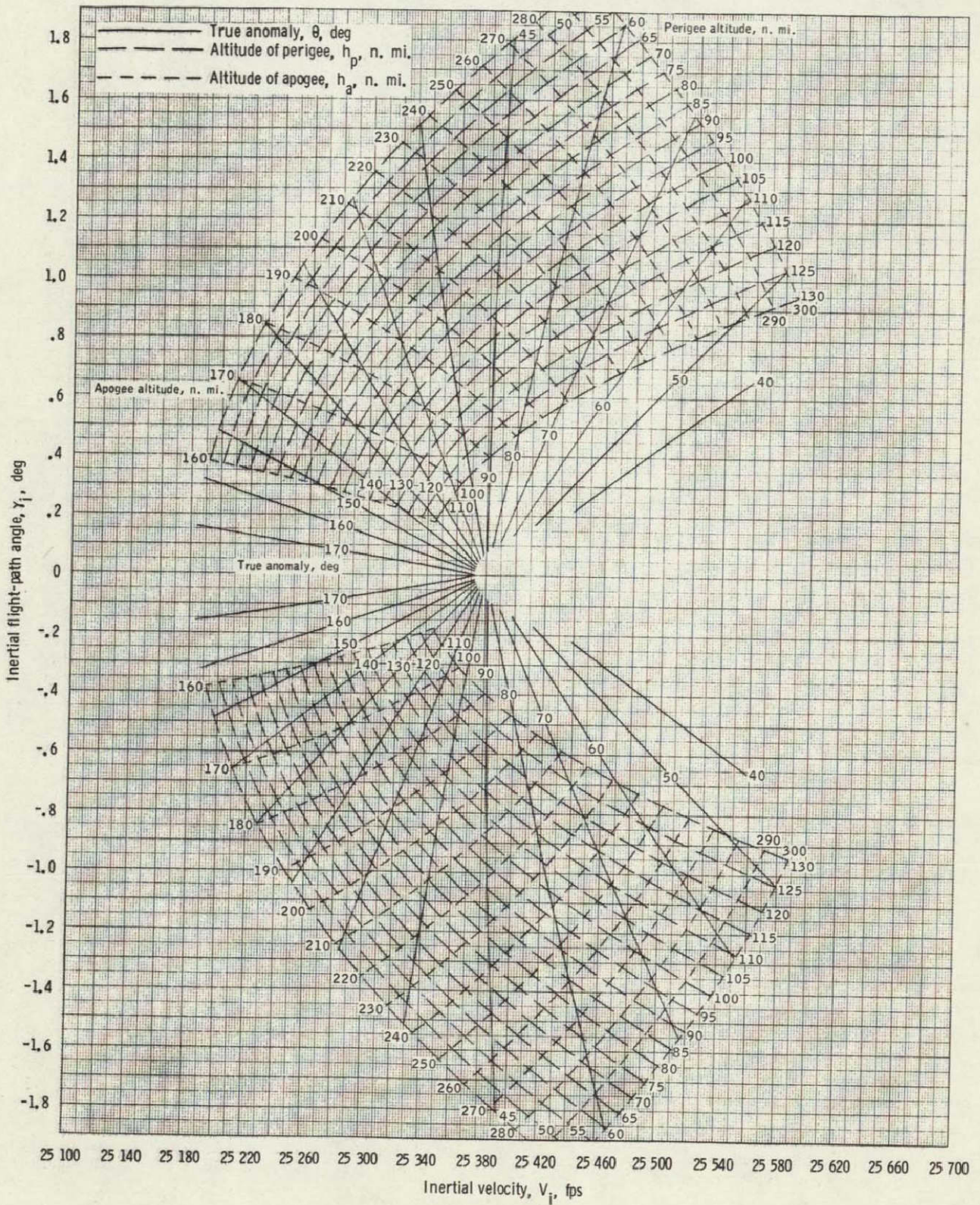


(g) Altitude = 135 n. mi.
Figure 2. - Continued.



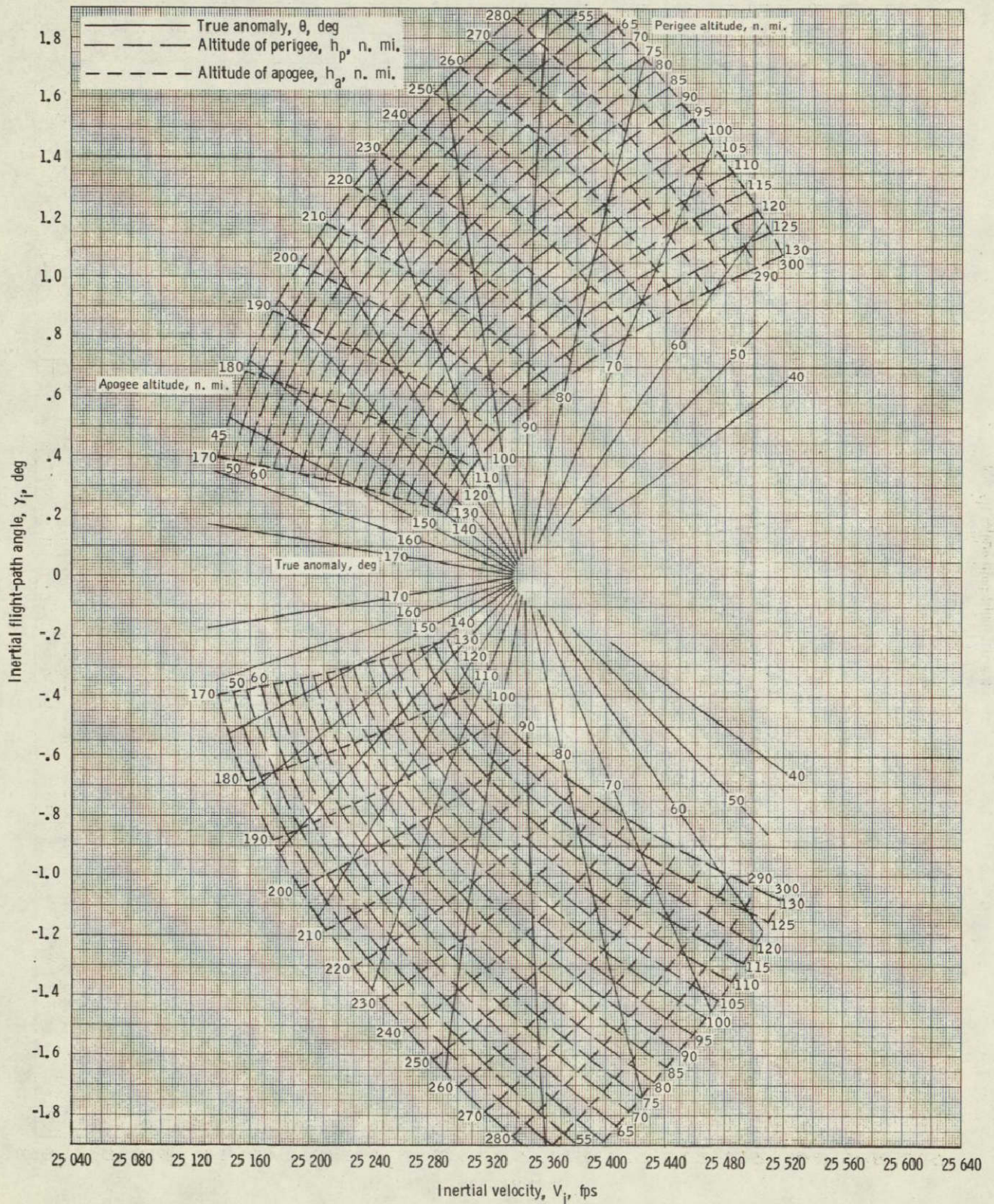
(h) Altitude = 145 n. mi.

Figure 2 - Continued.



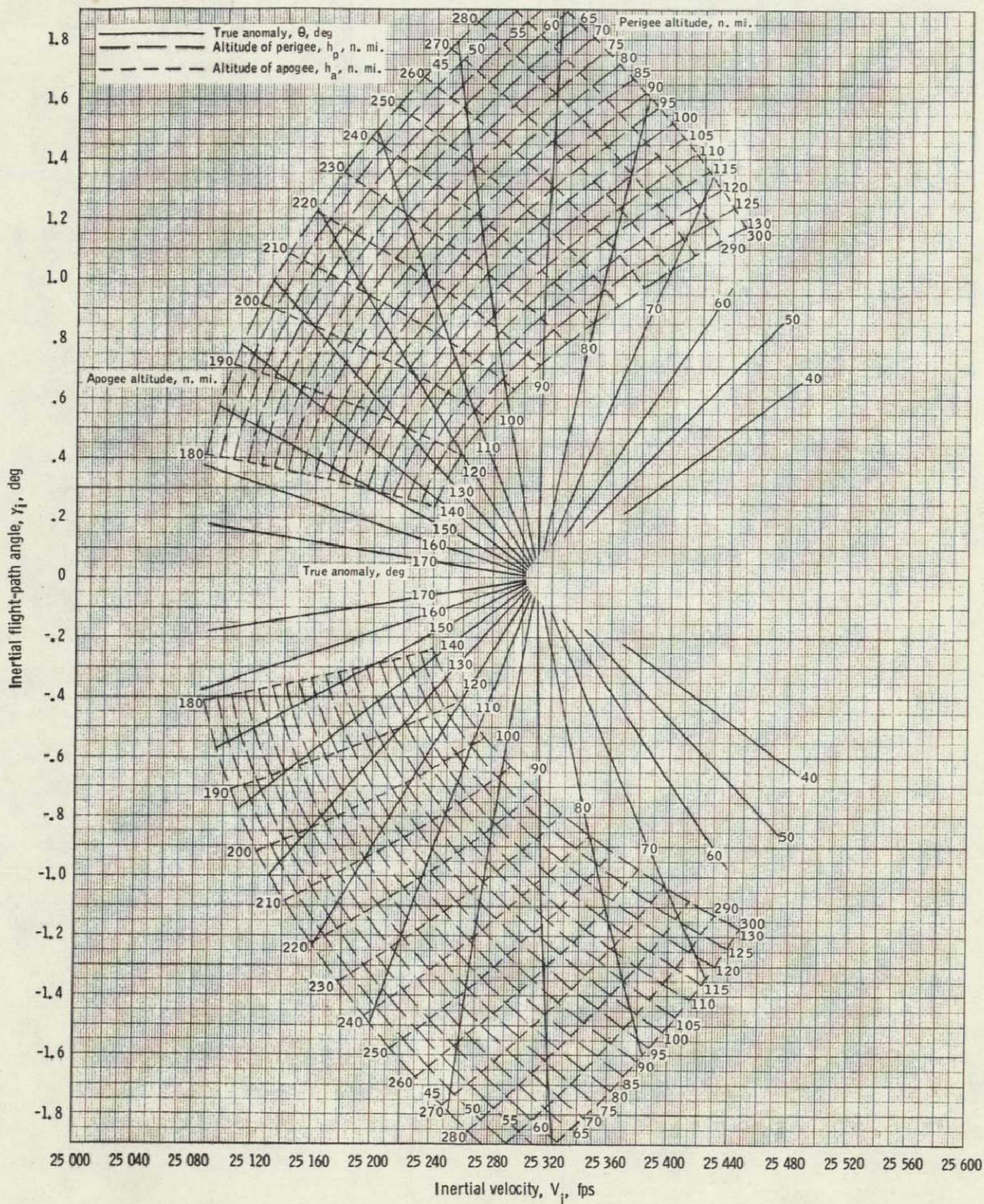
(i) Altitude = 155 n. mi.

Figure 2. - Continued.



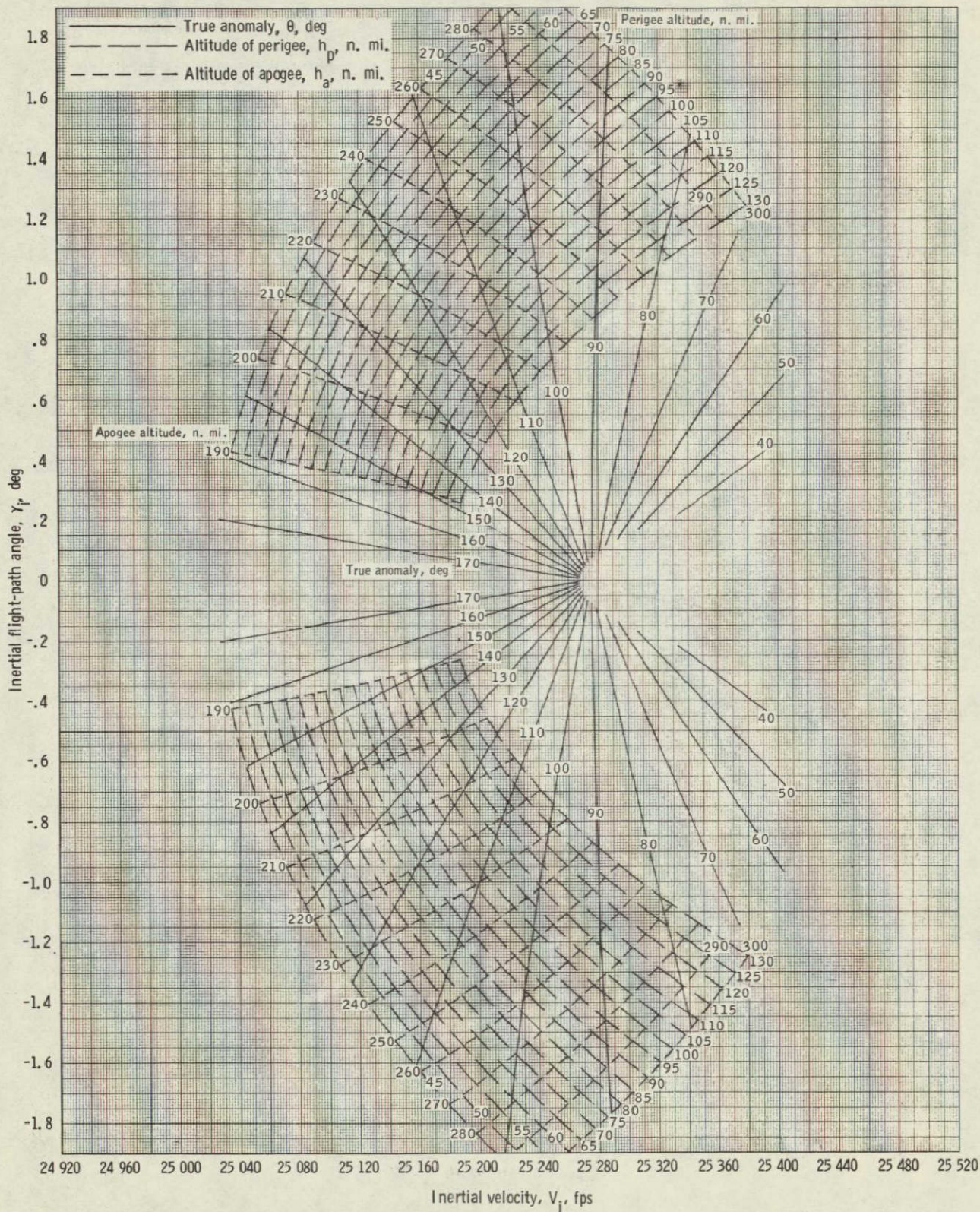
(j) Altitude = 165 n. mi.

Figure 2. - Continued.



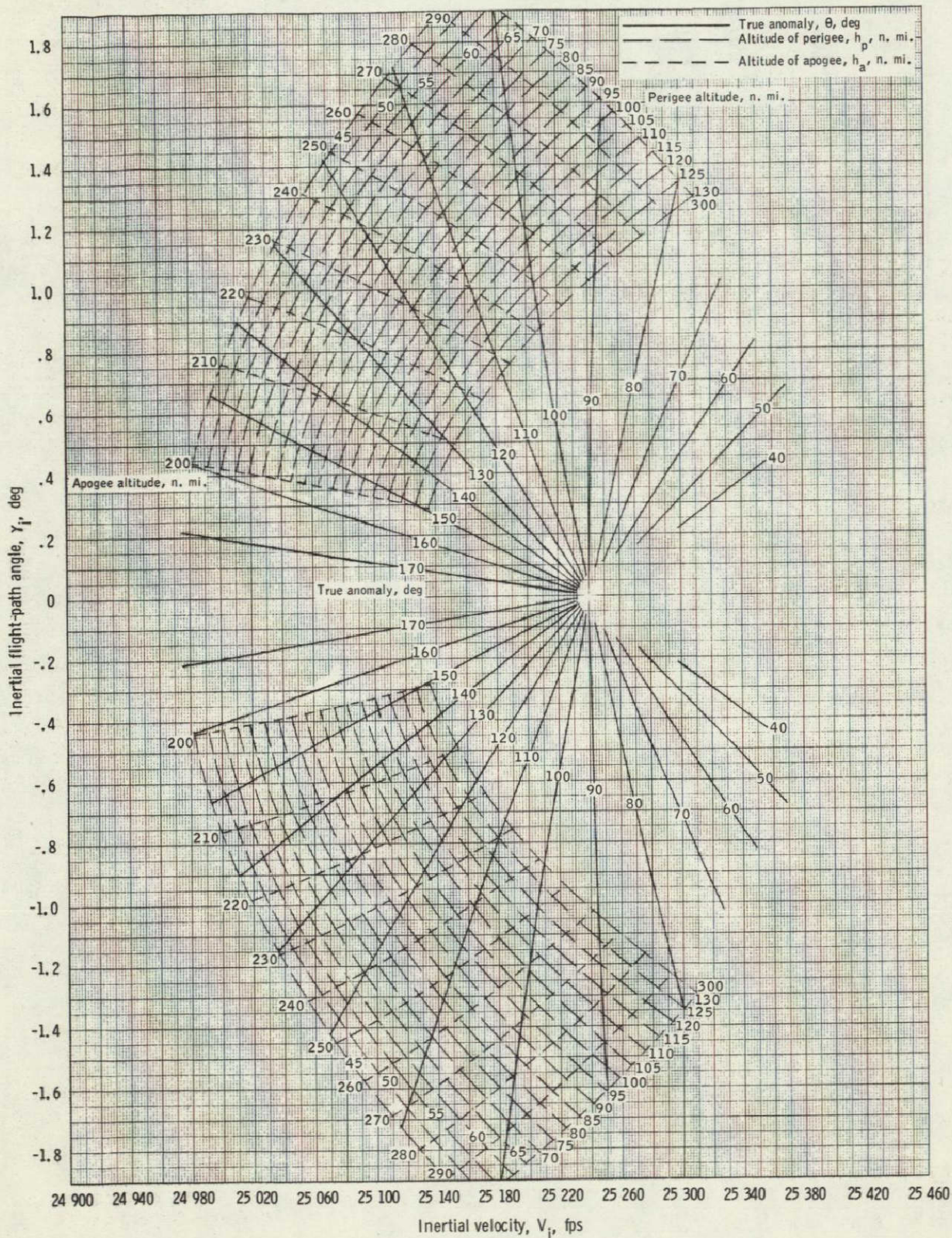
(k) Altitude = 175 n. mi.

Figure 2. - Continued.



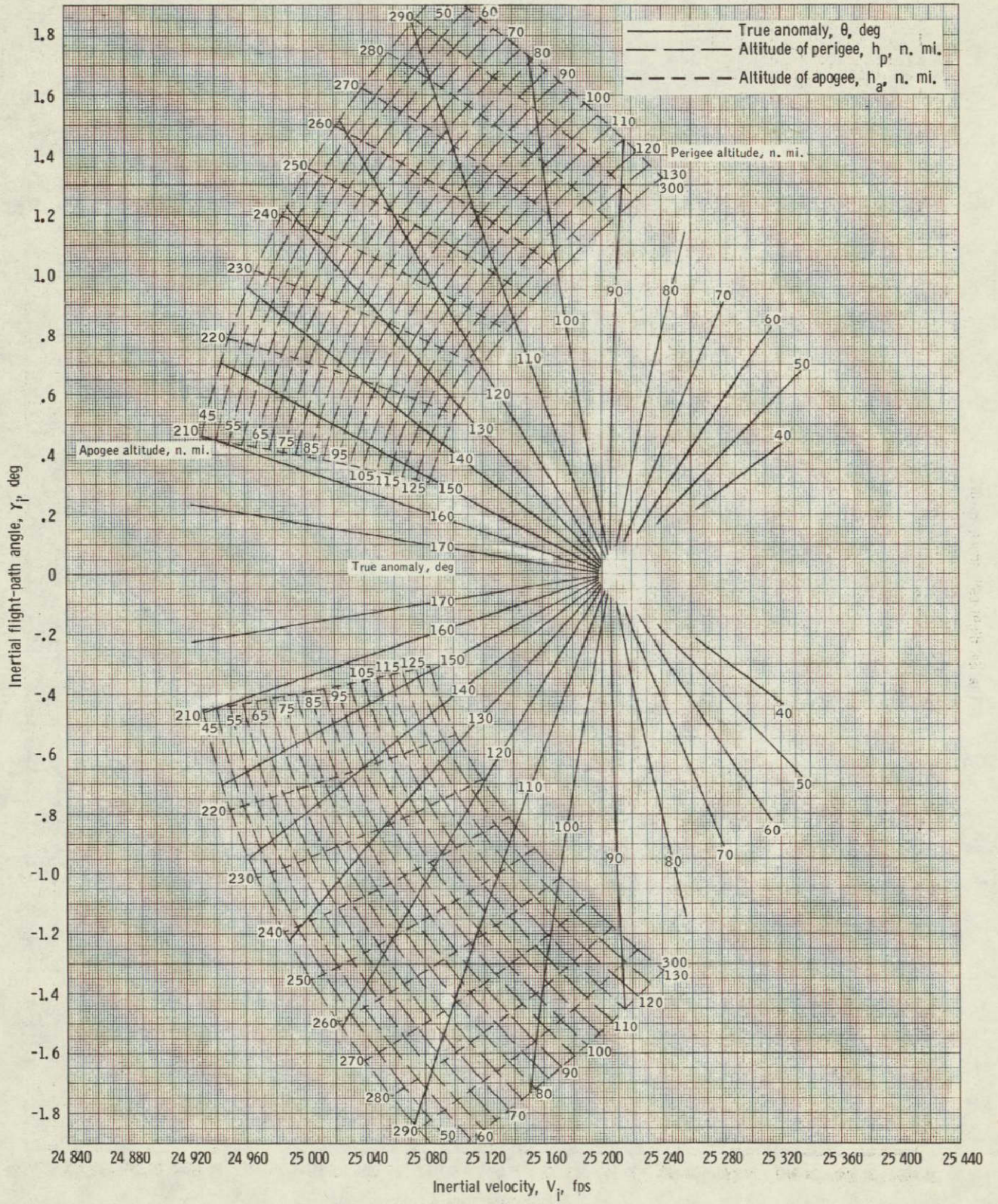
(I) Altitude = 185 n. mi.

Figure 2. - Continued.



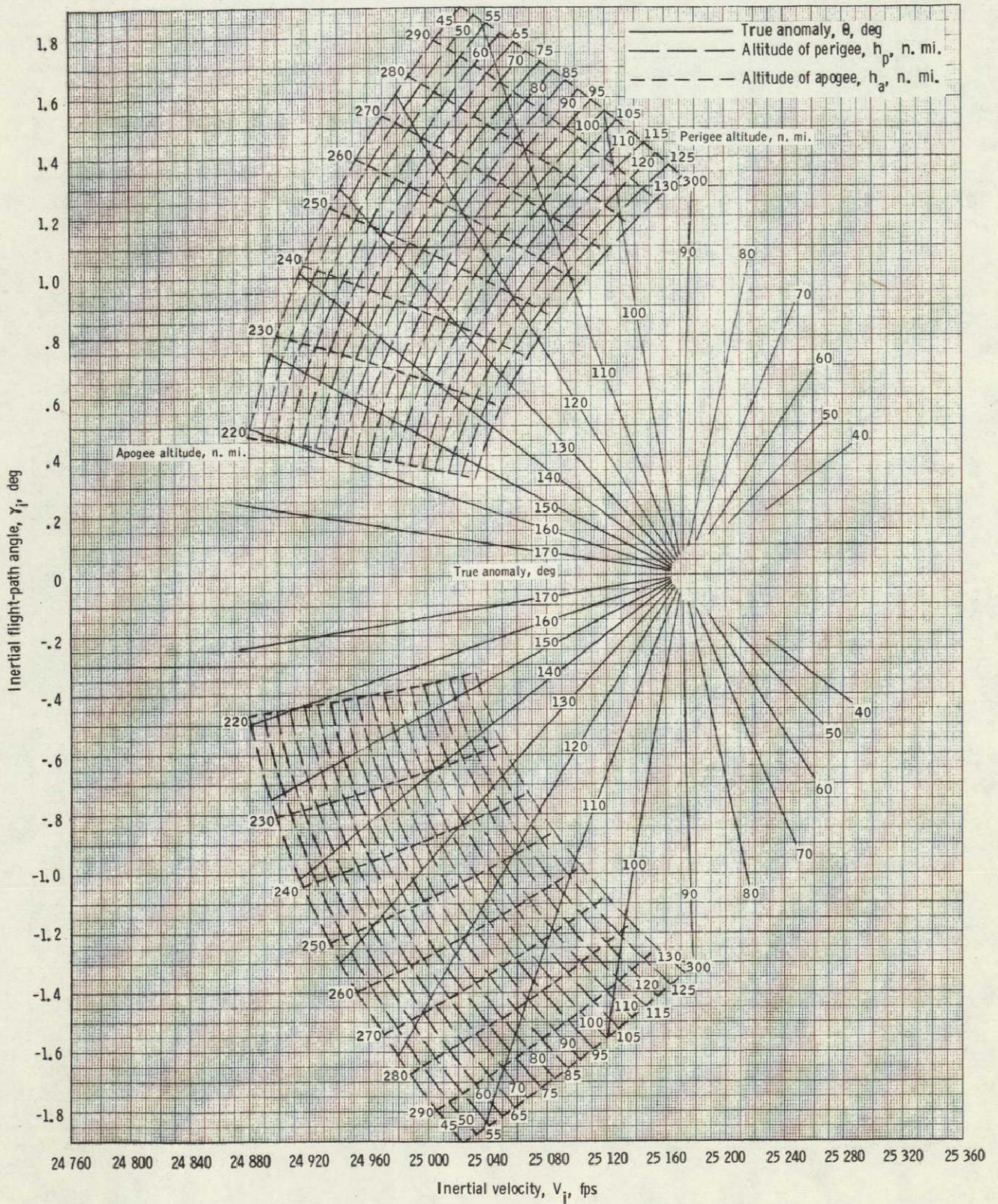
(m) Altitude = 195 n. mi.

Figure 2, - Continued.



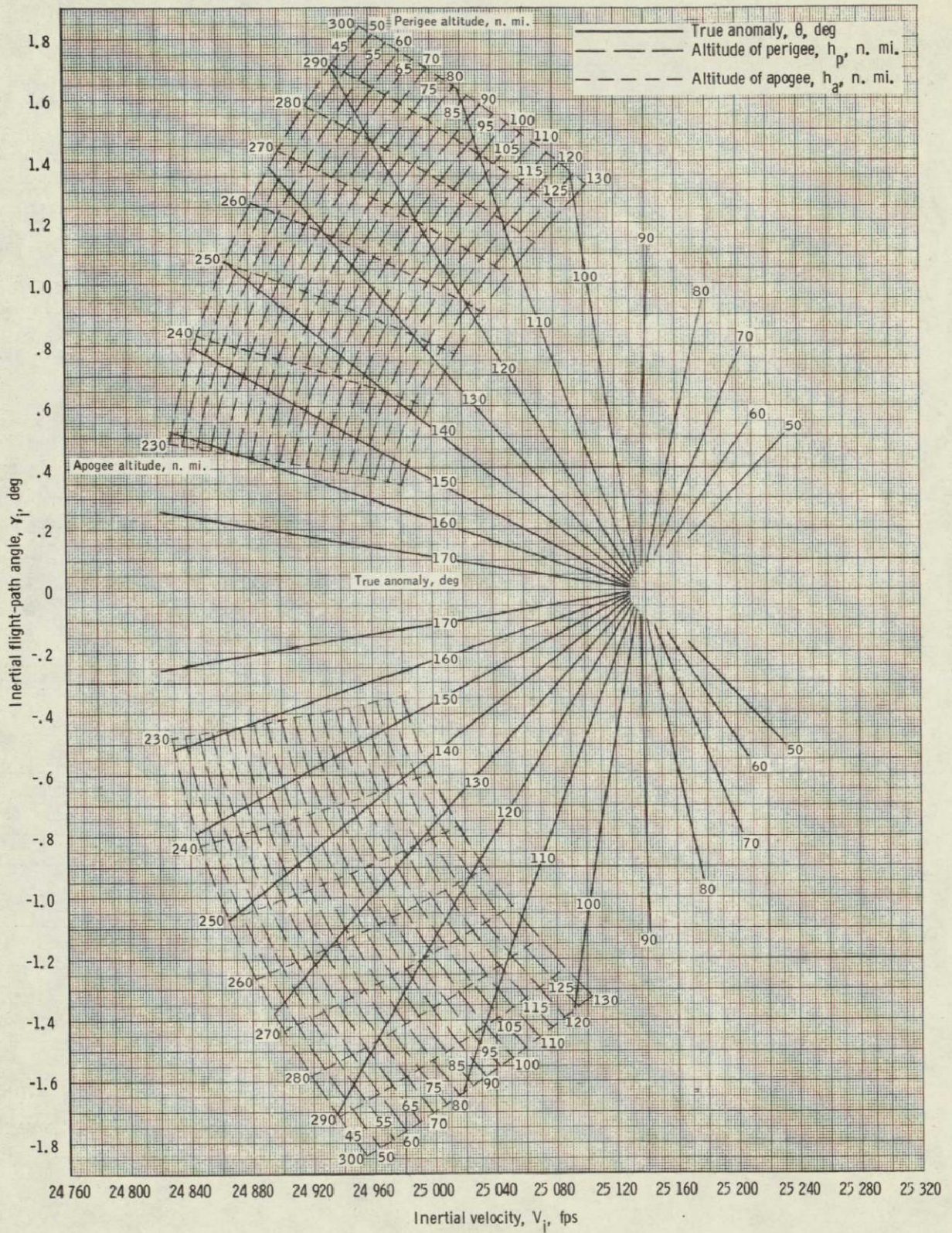
(n) Altitude = 205 n. mi.

Figure 2. - Continued.



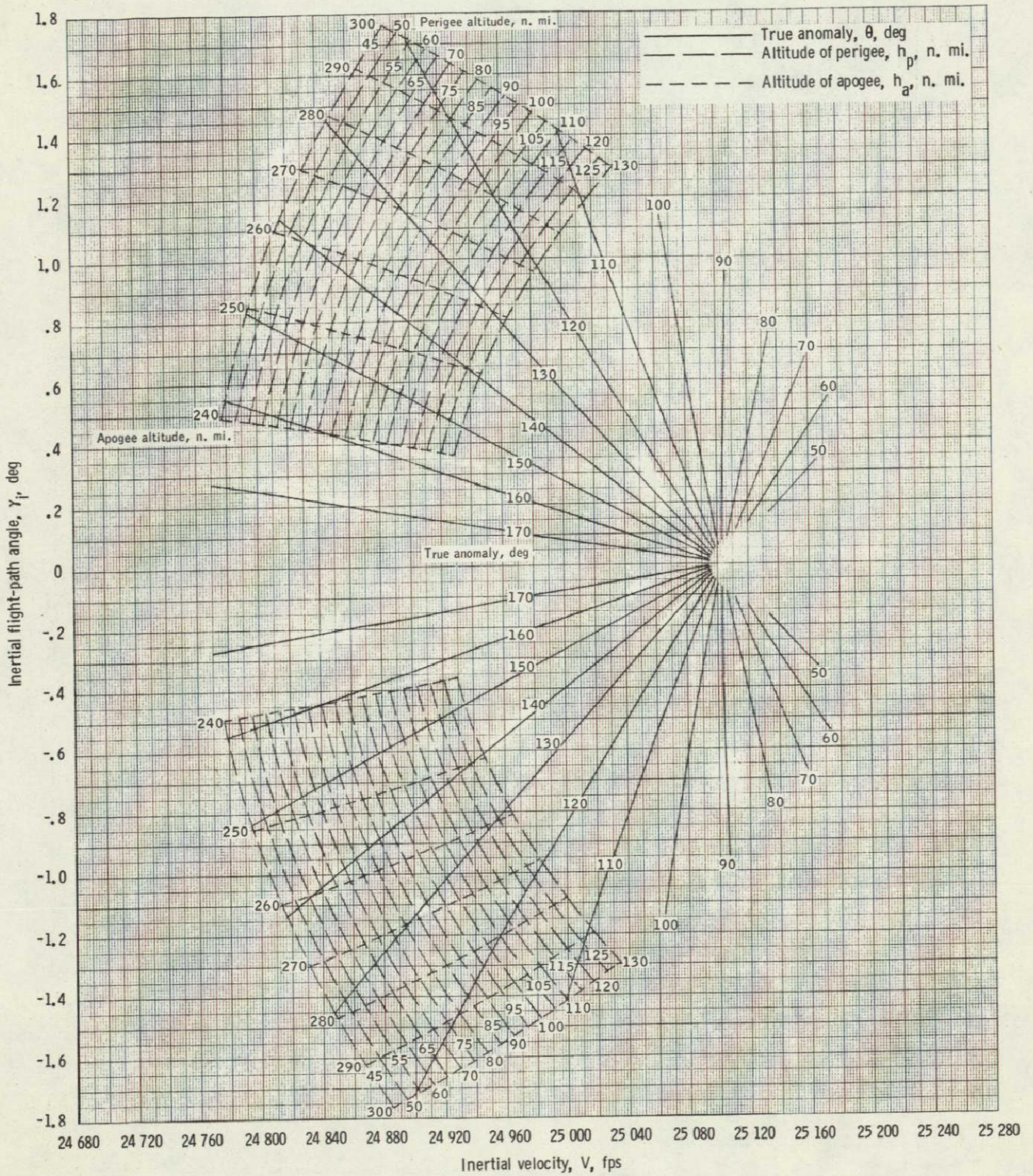
(c) Altitude = 215 n. mi.

Figure 2. - Continued.



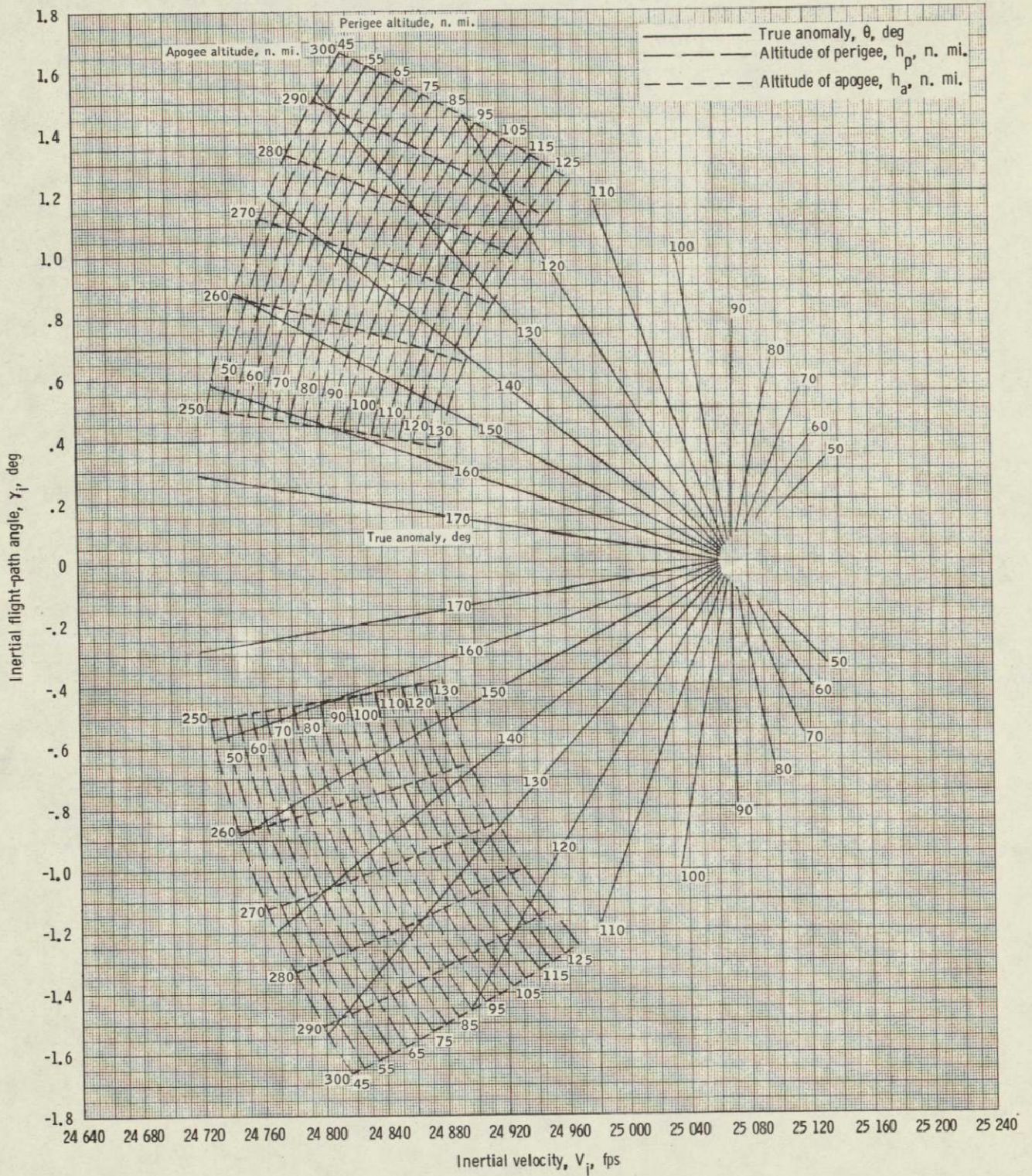
(p) Altitude = 225 n. mi.

Figure 2. - Continued.



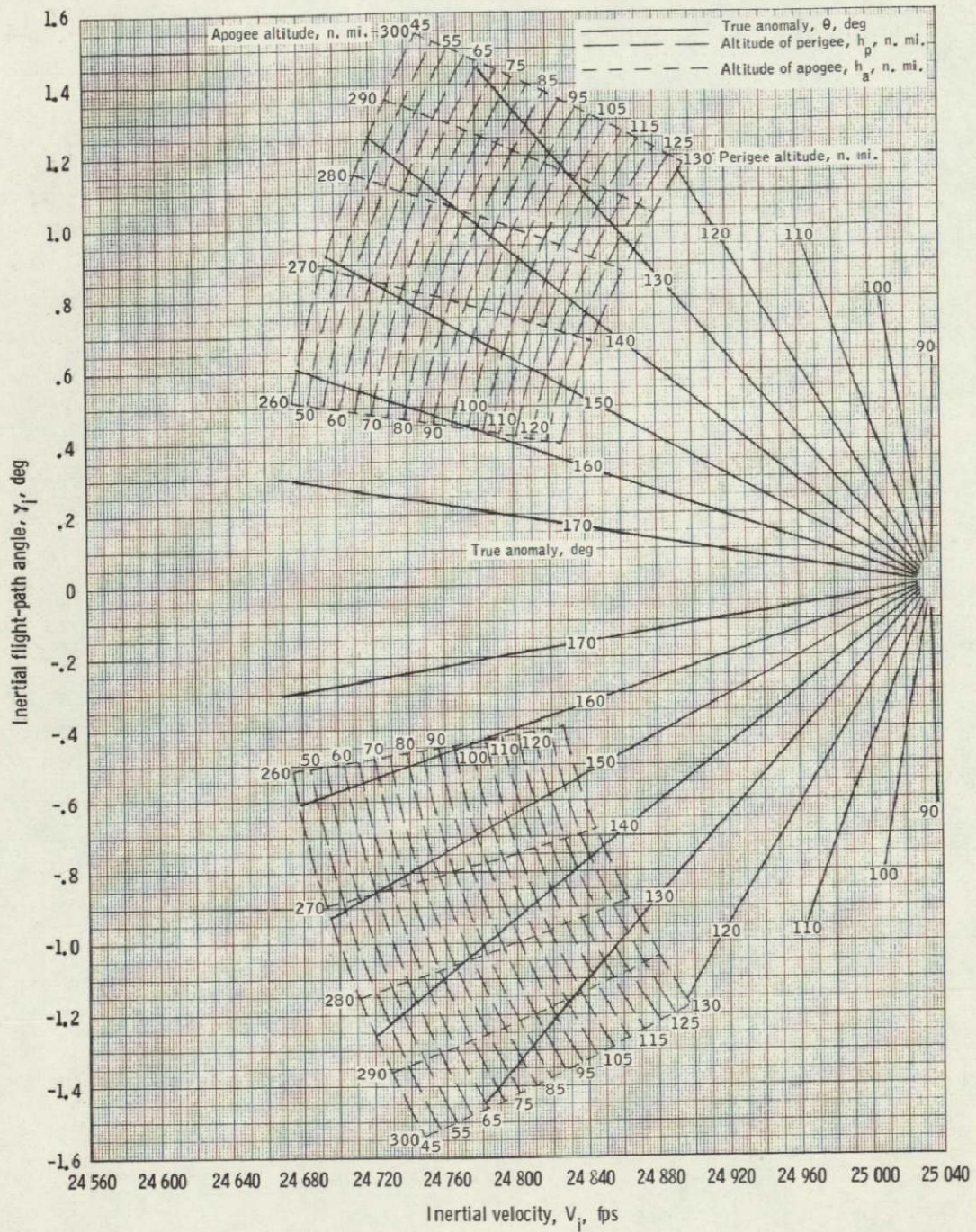
(q) Altitude = 235 n. mi.

Figure 2 - Continued.



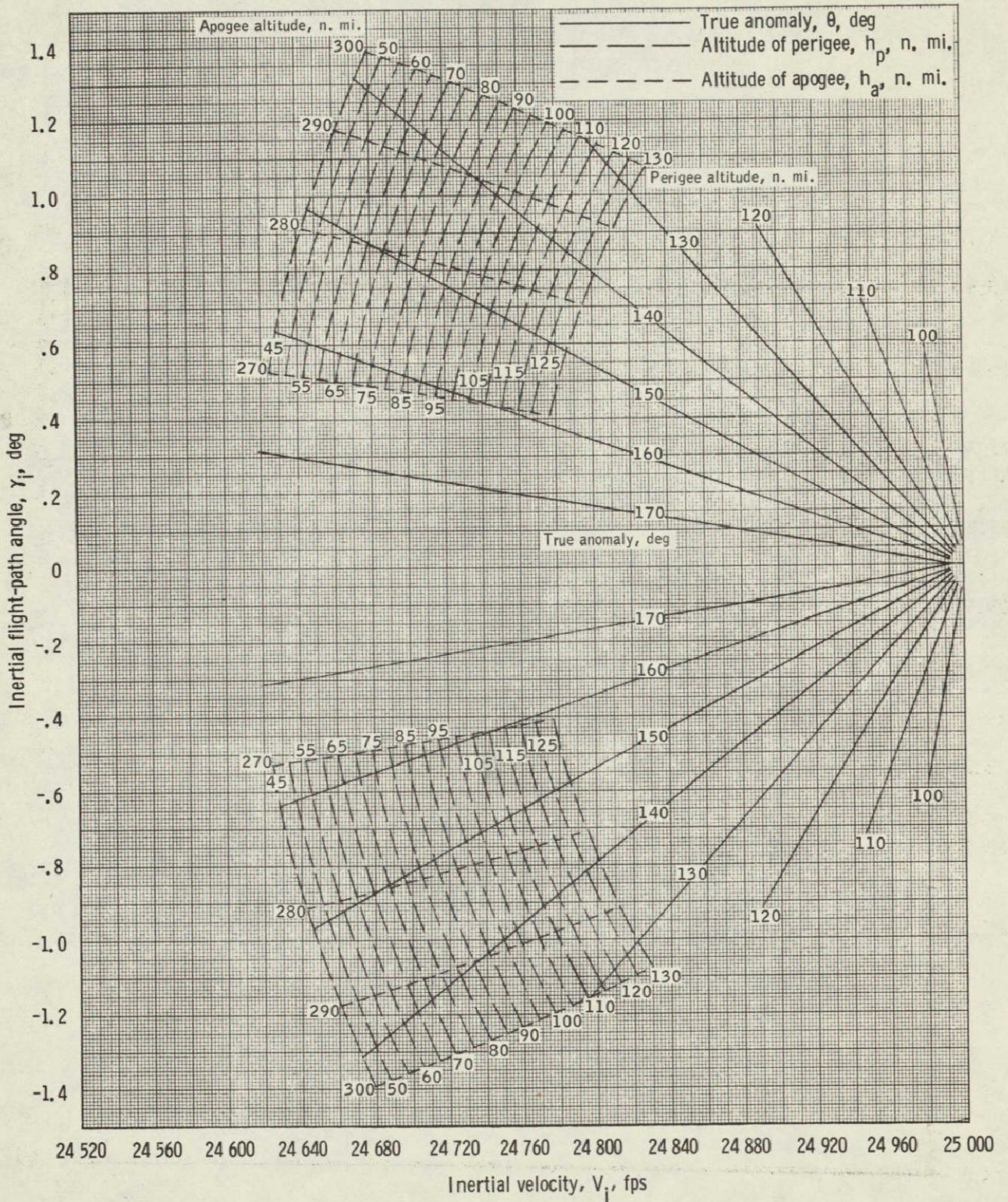
(r) Altitude = 245 n. mi.

Figure 2. - Continued.



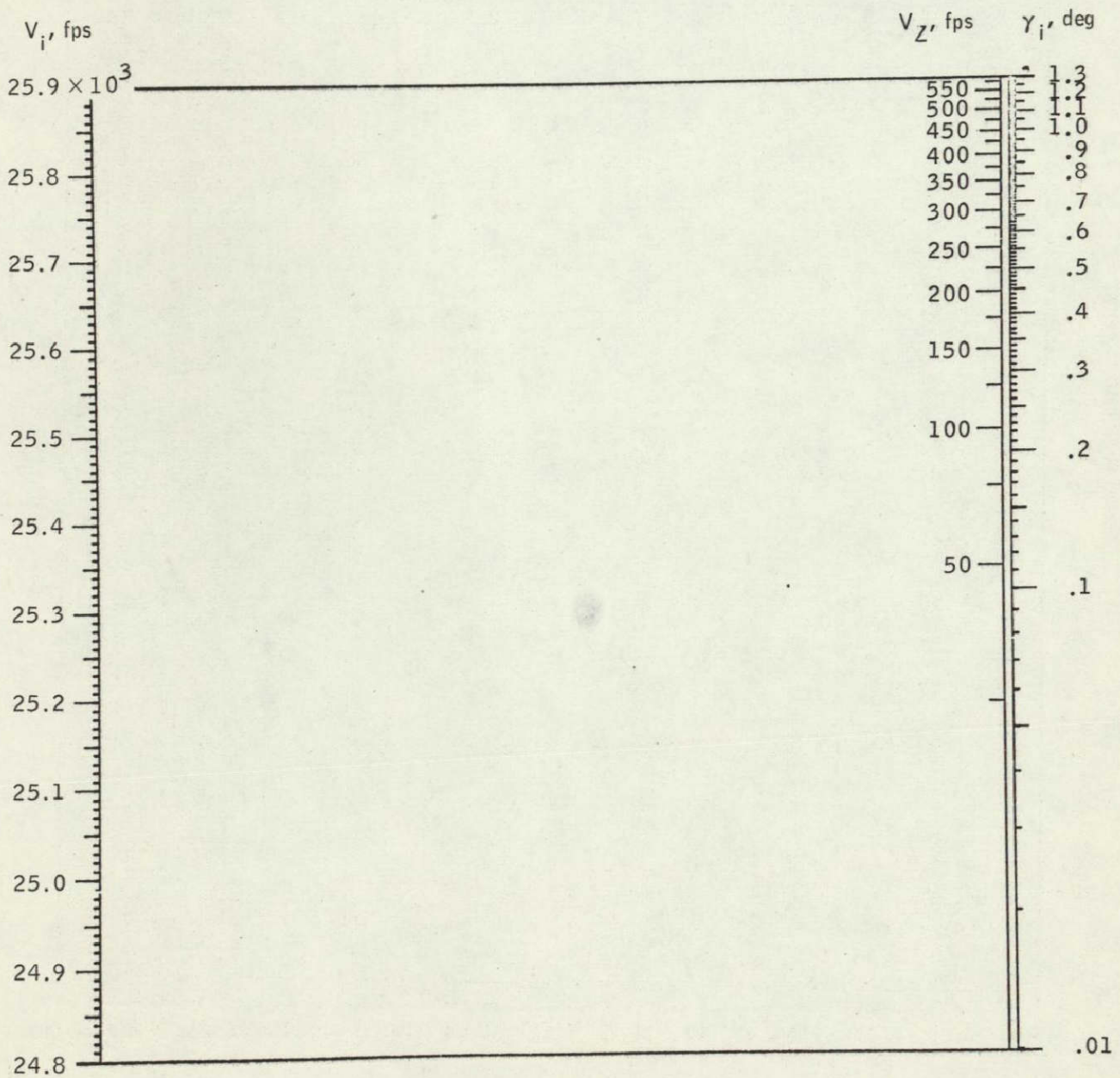
(s) Altitude = 255 n. mi.

Figure 2. - Continued.



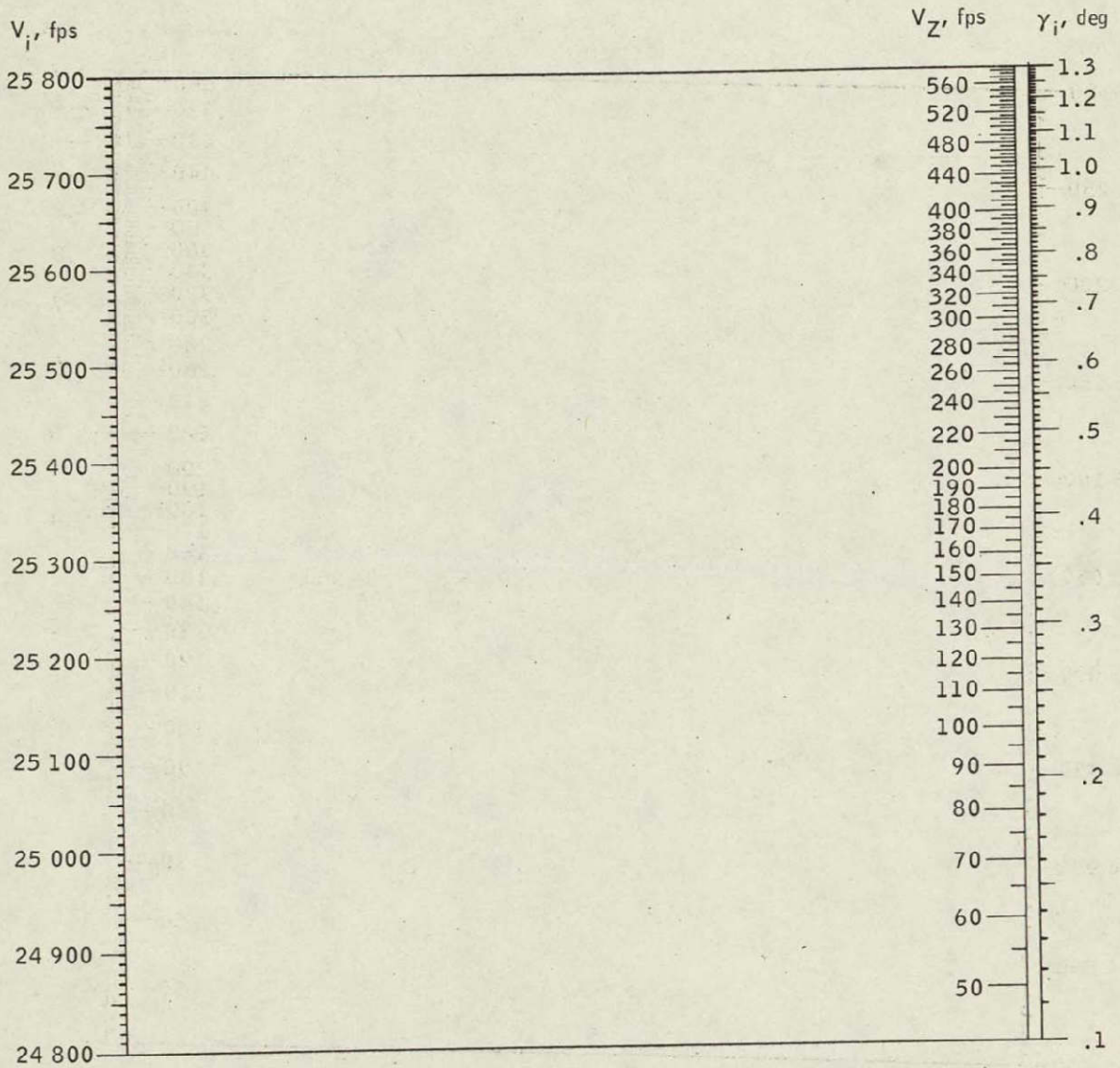
(t) Altitude = 265 n. mi.

Figure 2. - Concluded.



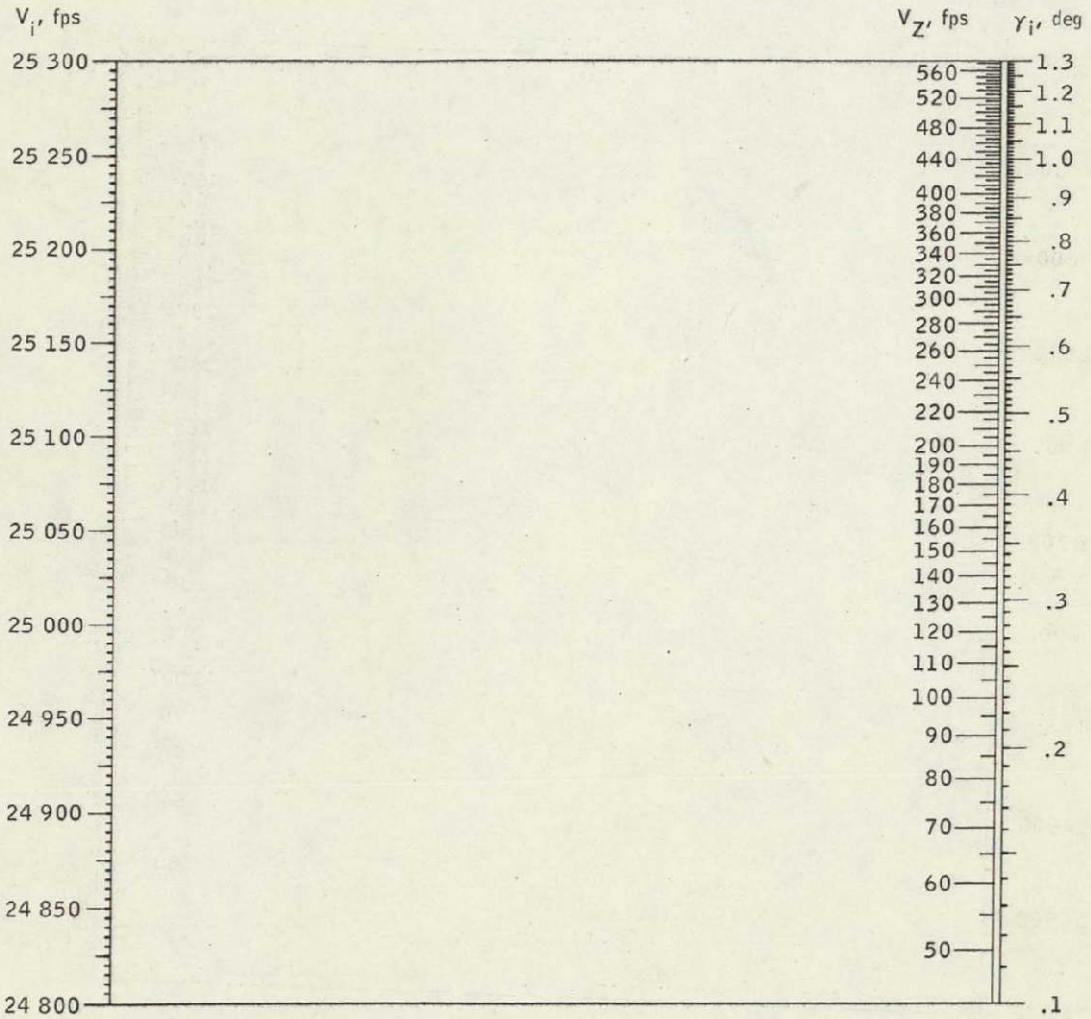
(a) $V_i = 24\,800 - 25\,900$ fps; $\gamma_i = .01 - 1.3$ deg.

Figure 3.- The Z component of velocity as a function of inertial velocity and inertial flight-path angle.



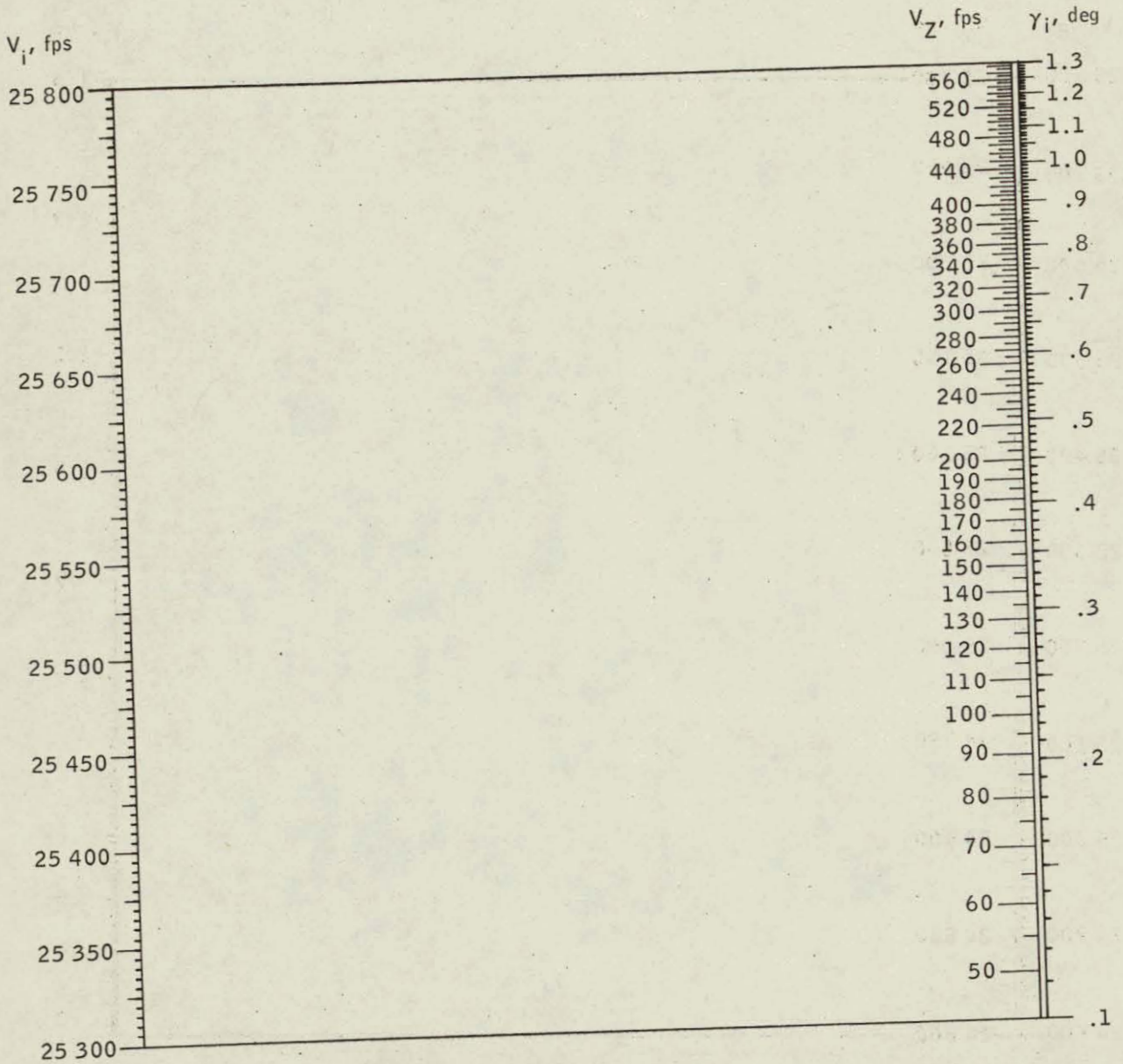
(b) $V_i = 24 800 - 25 800$ fps; $\gamma_i = .1 - 1.3$ deg.

Figure 3.- Continued.



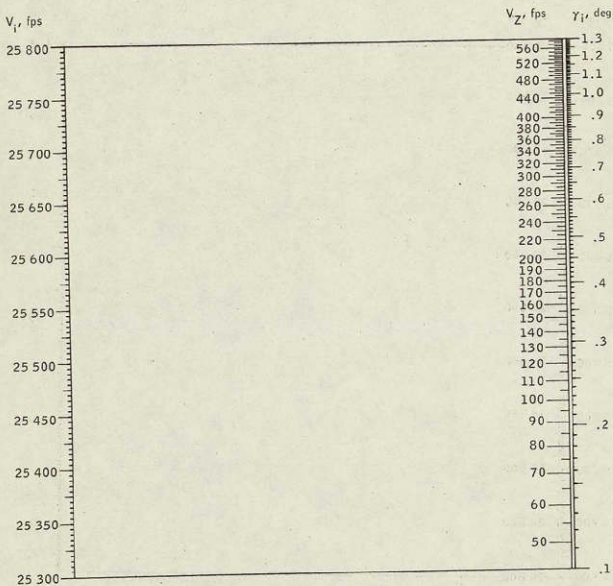
(c) $V_i = 24 800 - 25 300$ fps; $\gamma_i = .1 - 1.3$ deg.

Figure 3.- Continued.



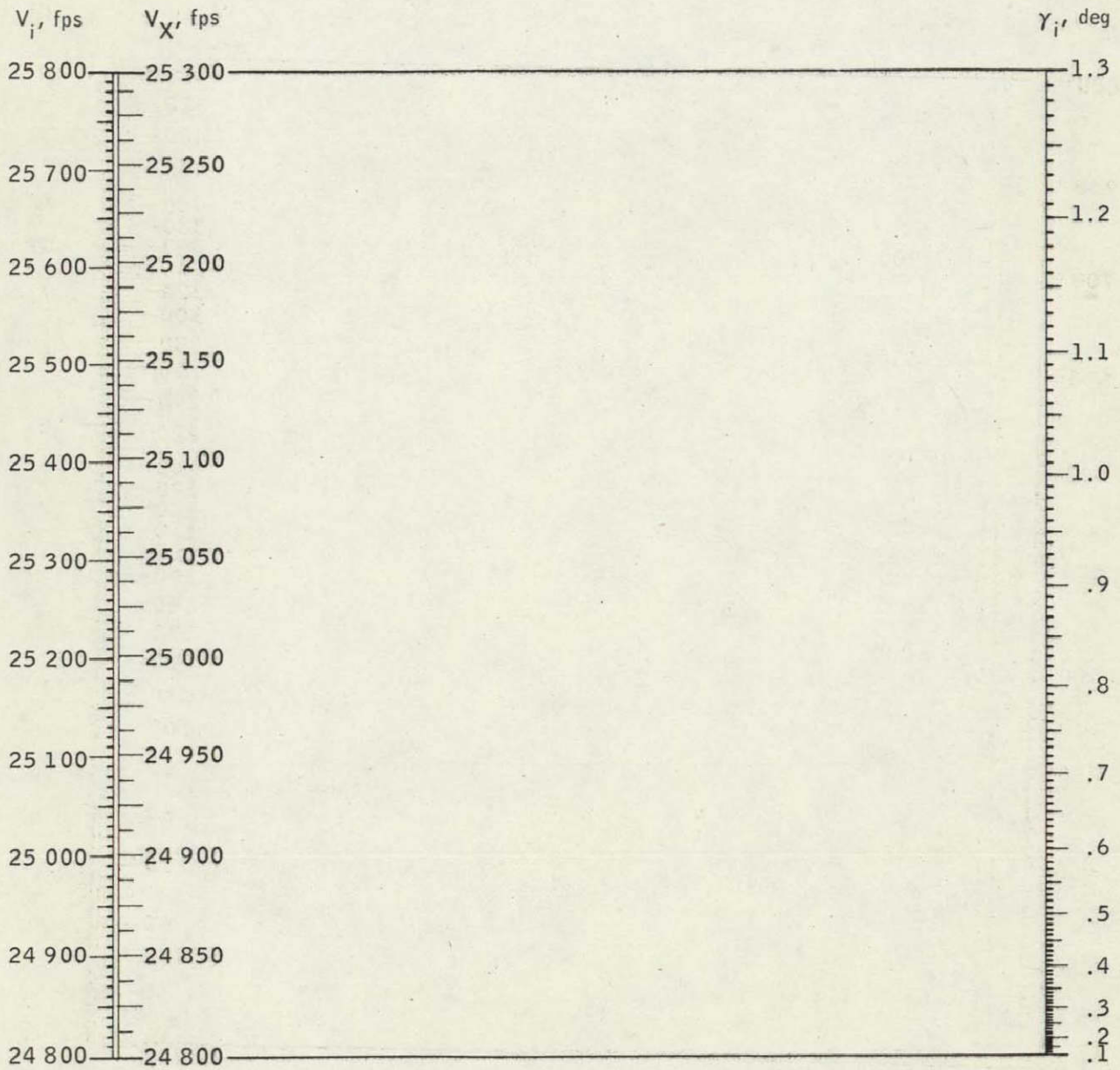
(d) $V_i = 25\ 300 - 25\ 800$ fps; $\gamma_i = .1 - 1.3$ deg.

Figure 3.- Concluded.



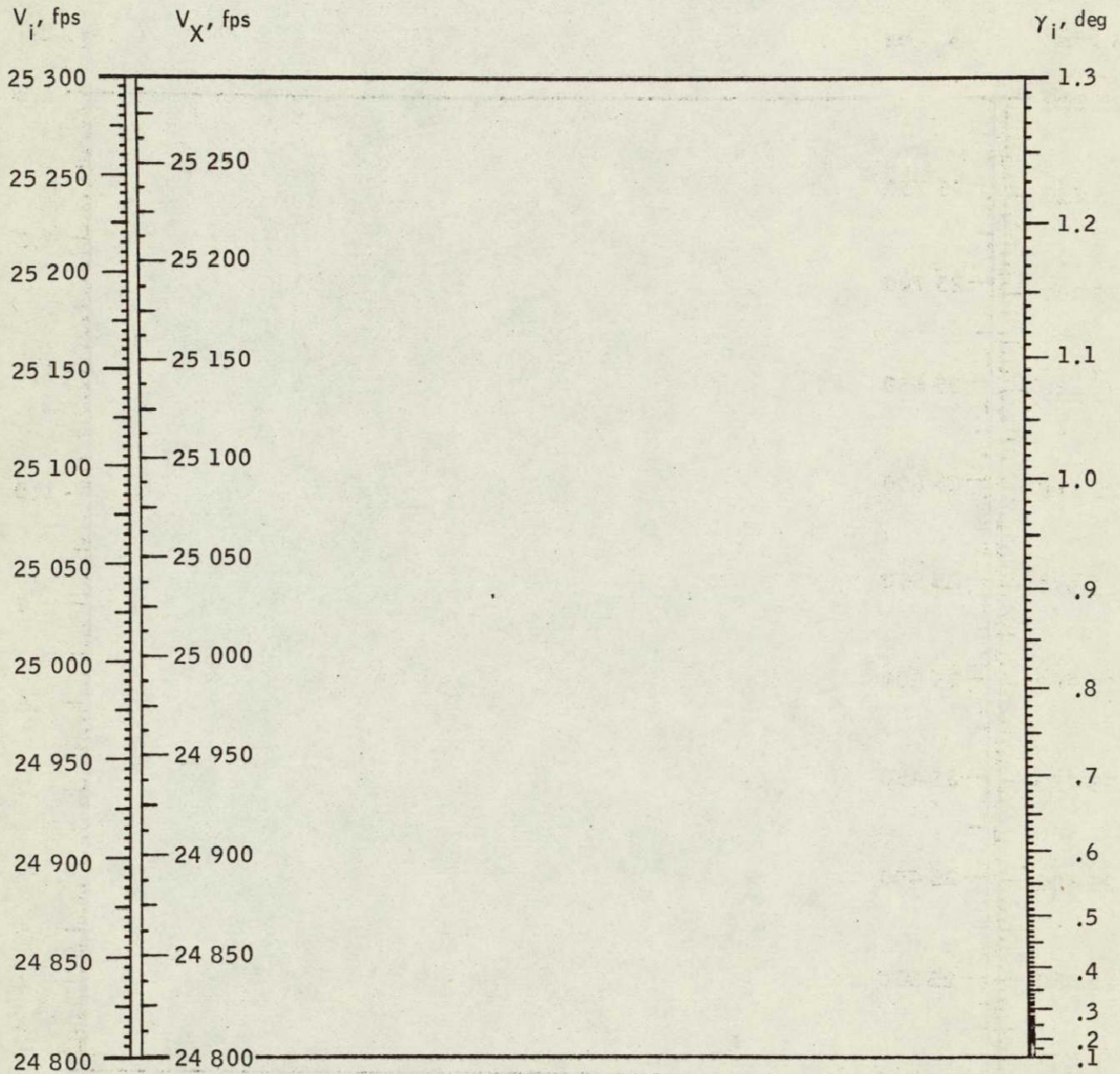
(d) $V_1 = 25\ 300 - 25\ 800$ fps; $\gamma_1 = .1 - 1.3$ deg.

Figure 3.- Concluded.



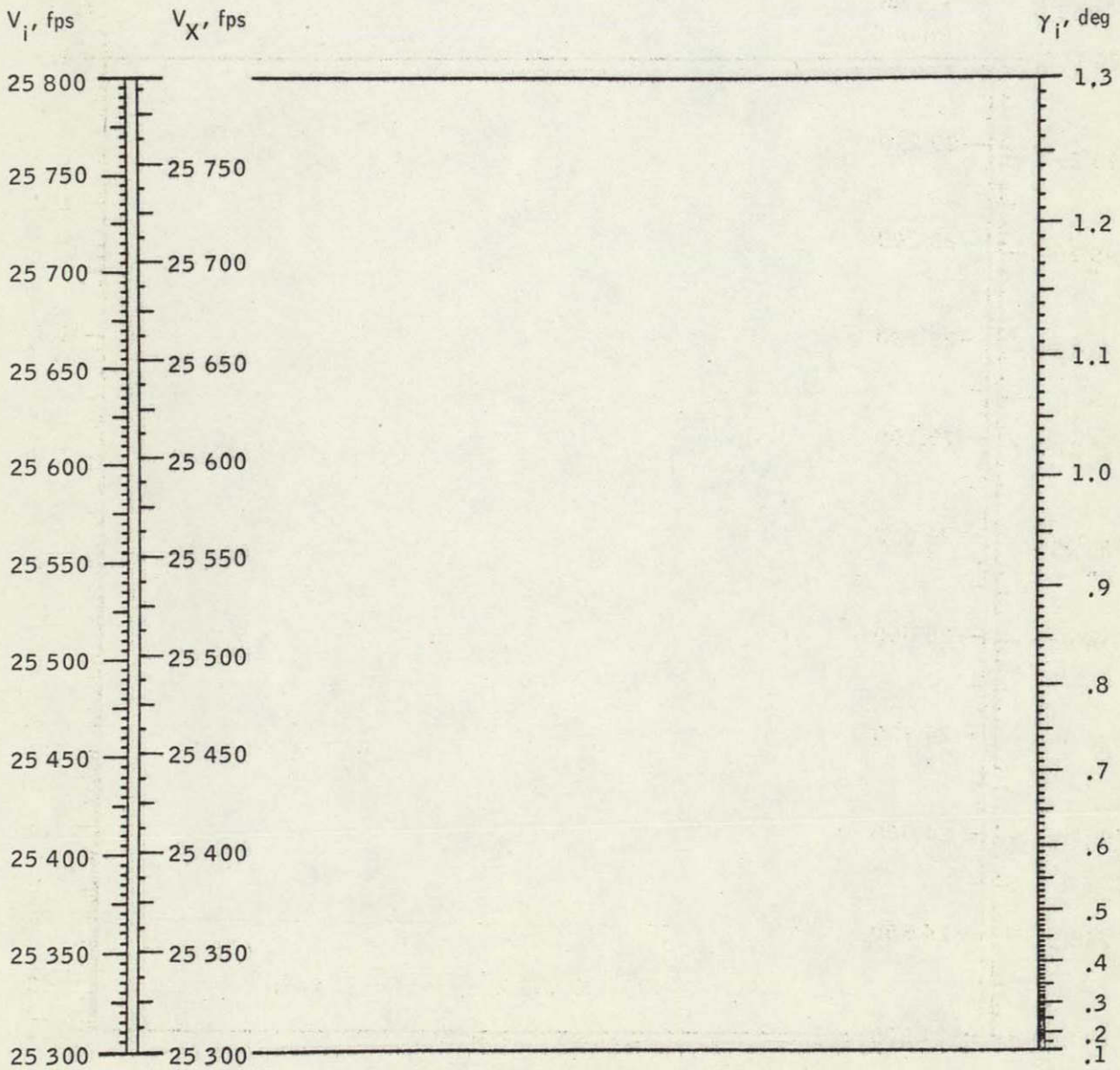
(a) $V_i = 24\,800 - 25\,800$ fps; $\gamma_i = .1 - 1.3$ deg.

Figure 4.- The X component of velocity as a function of inertial velocity and inertial flight-path angle.



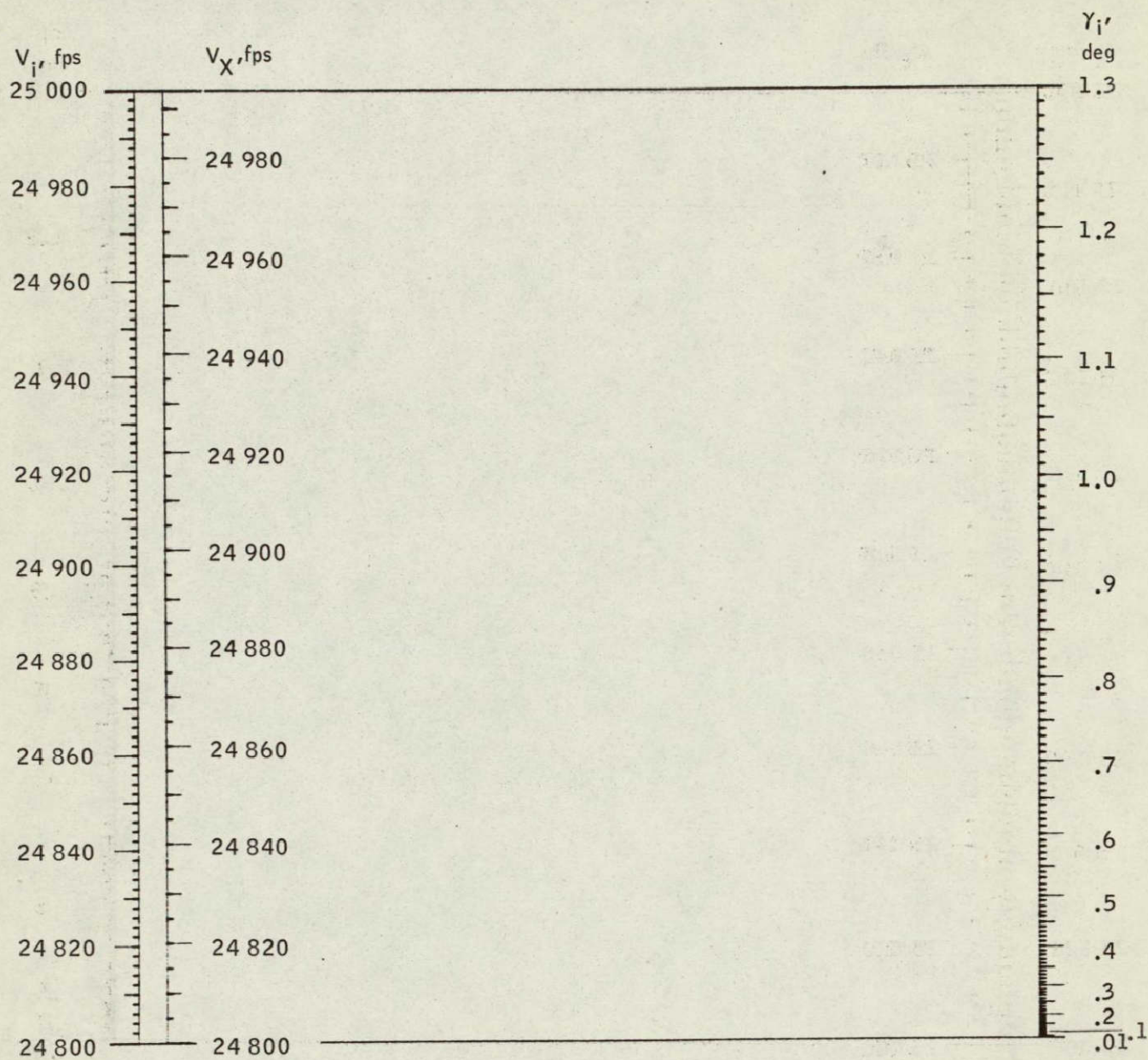
(b) $V_i = 24\,800 - 25\,300 \text{ fps}$; $\gamma_i = .1 - 1.3 \text{ deg}$.

Figure 4.- Continued.



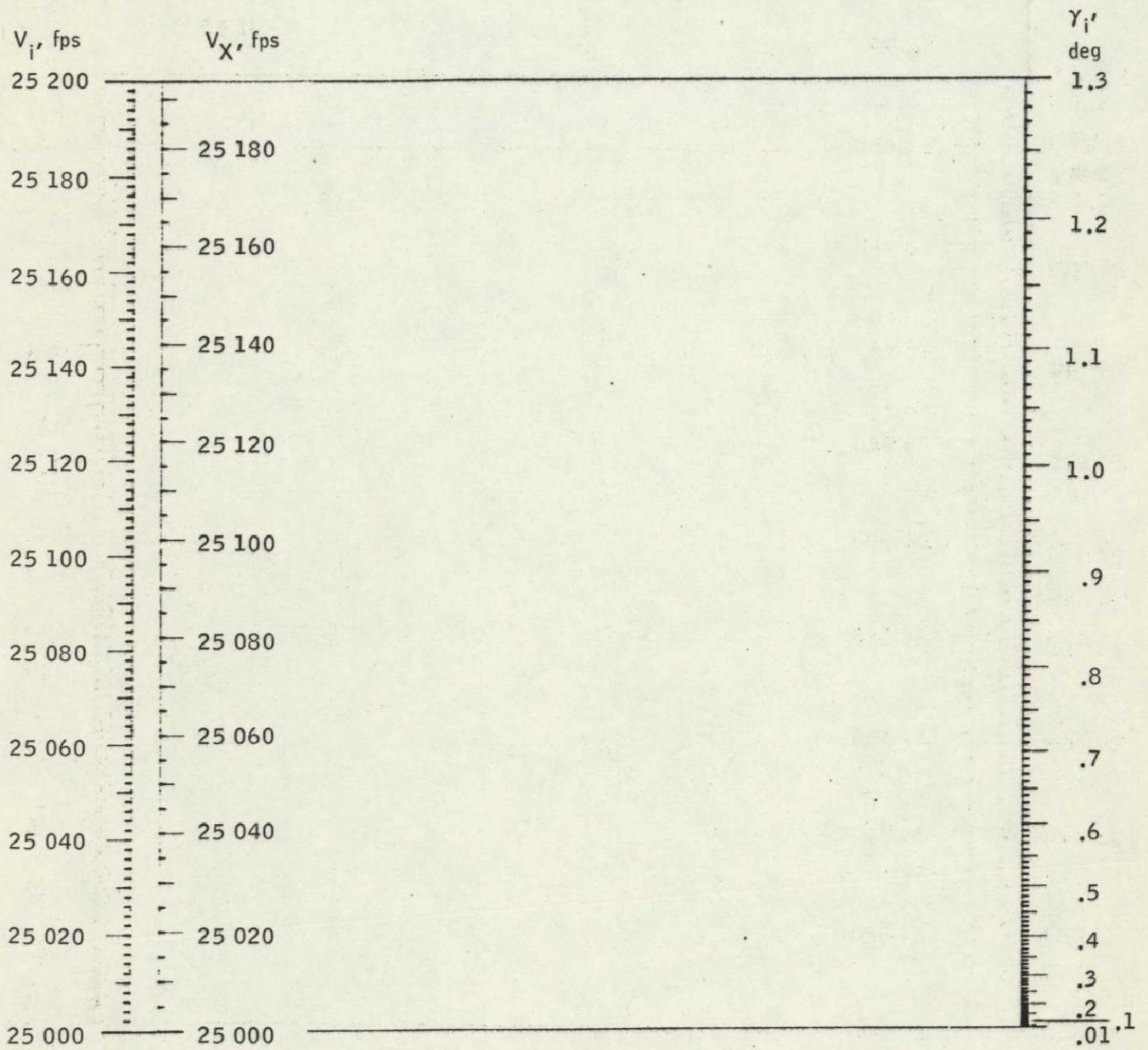
(c) $V_i = 25\ 300 - 25\ 800$ fps; $\gamma_i = .1 - 1.3$ deg.

Figure 4.- Continued.



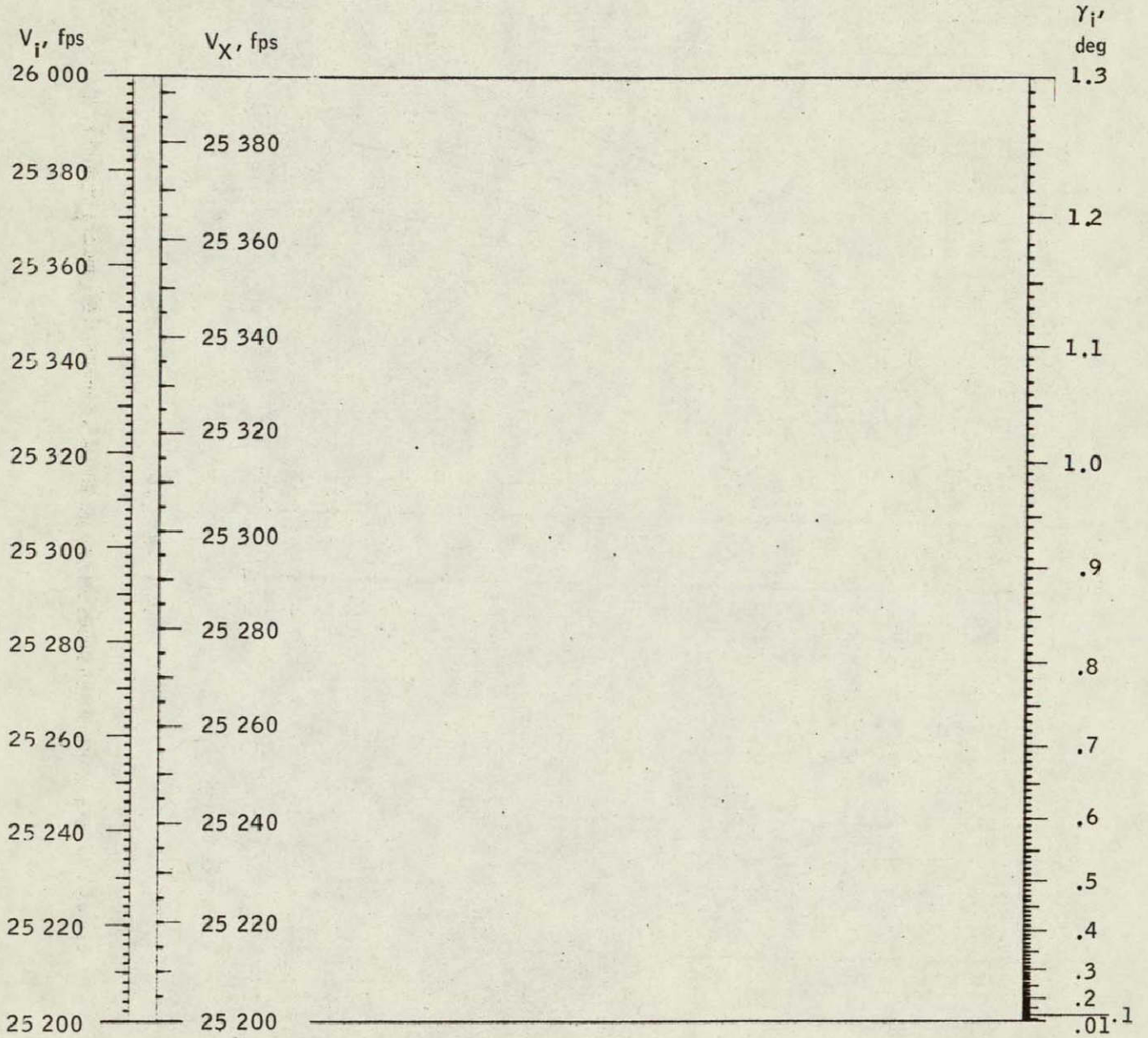
(d) $V_i = 24\,800 - 25\,000 \text{ fps}$; $\gamma_i = .01 - 1.3 \text{ deg}$.

Figure 4.- Continued.



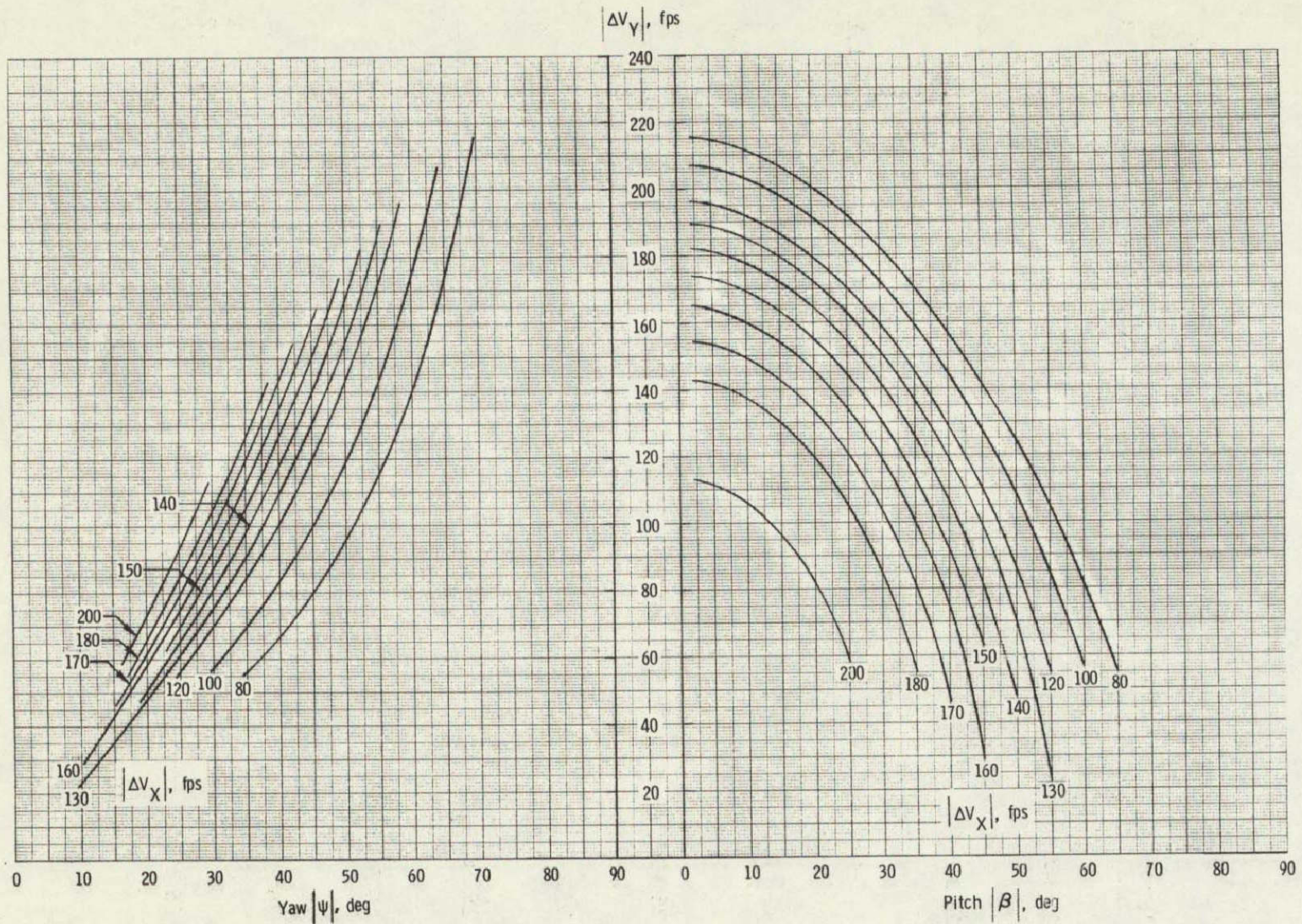
(e) $V_i = 25\ 000 - 25\ 200$ fps; $\gamma_i = .01 - 1.3$ deg.

Figure 4.- Continued.



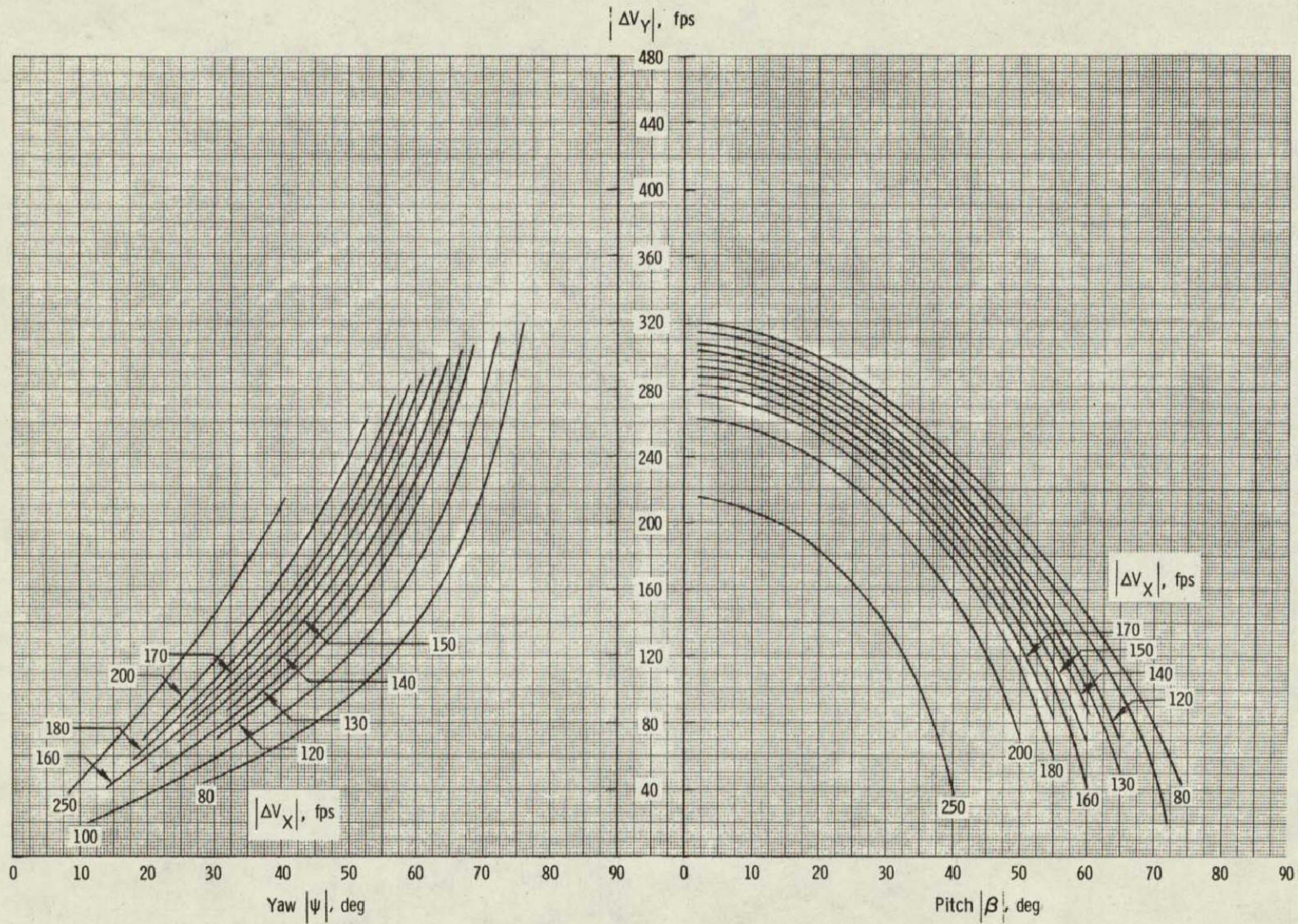
(f) $V_i = 25\ 200 - 26\ 000 \text{ fps}; \gamma_i = .01 - 1.3 \text{ deg.}$

Figure 4.- Concluded.



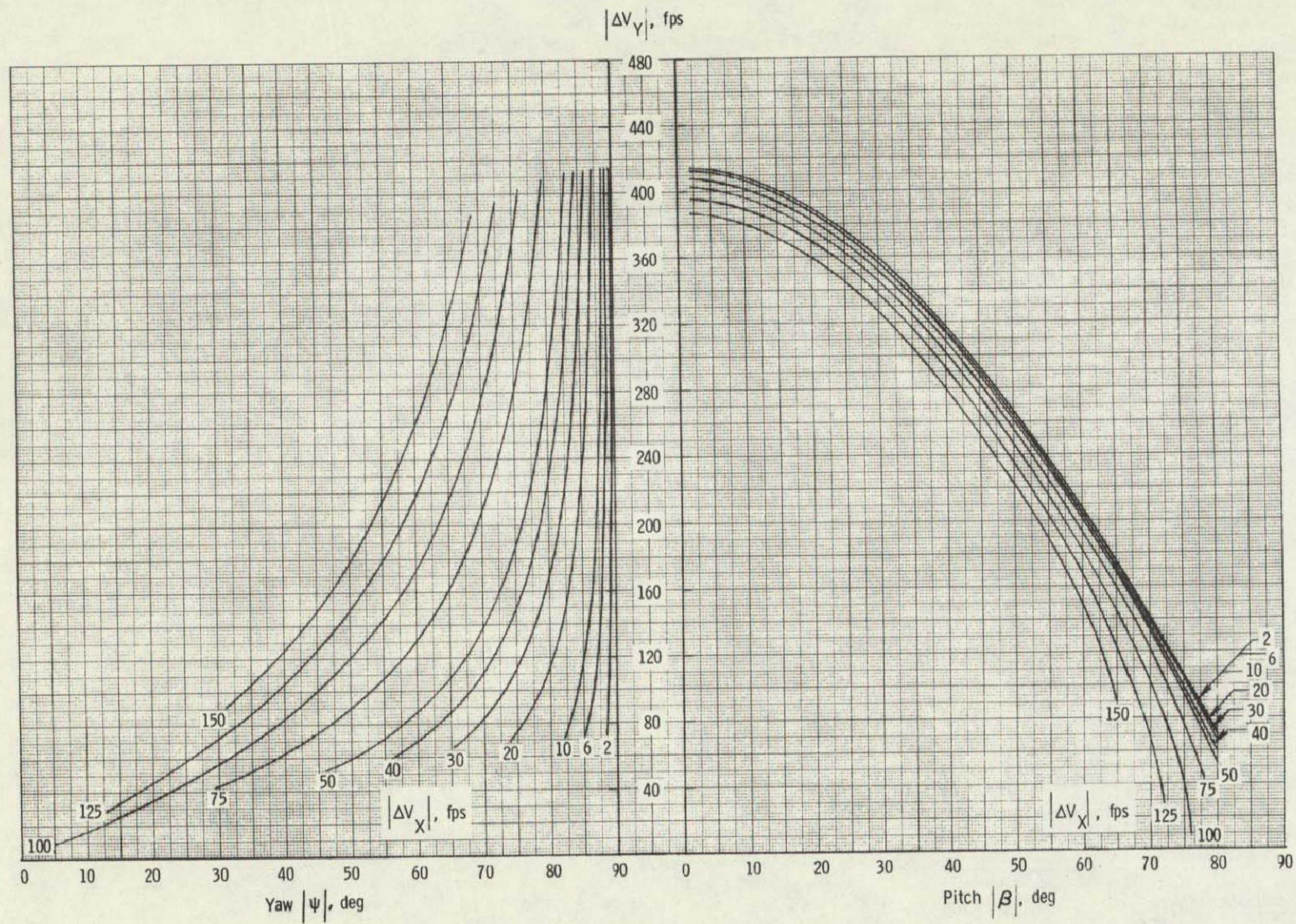
(a) ΔV total = 230 fps.

Figure 5. - ΔV_Y as a function of pitch attitude and ΔV_X ; yaw attitude as a function of ΔV_X and ΔV_Y for various ΔV_t .



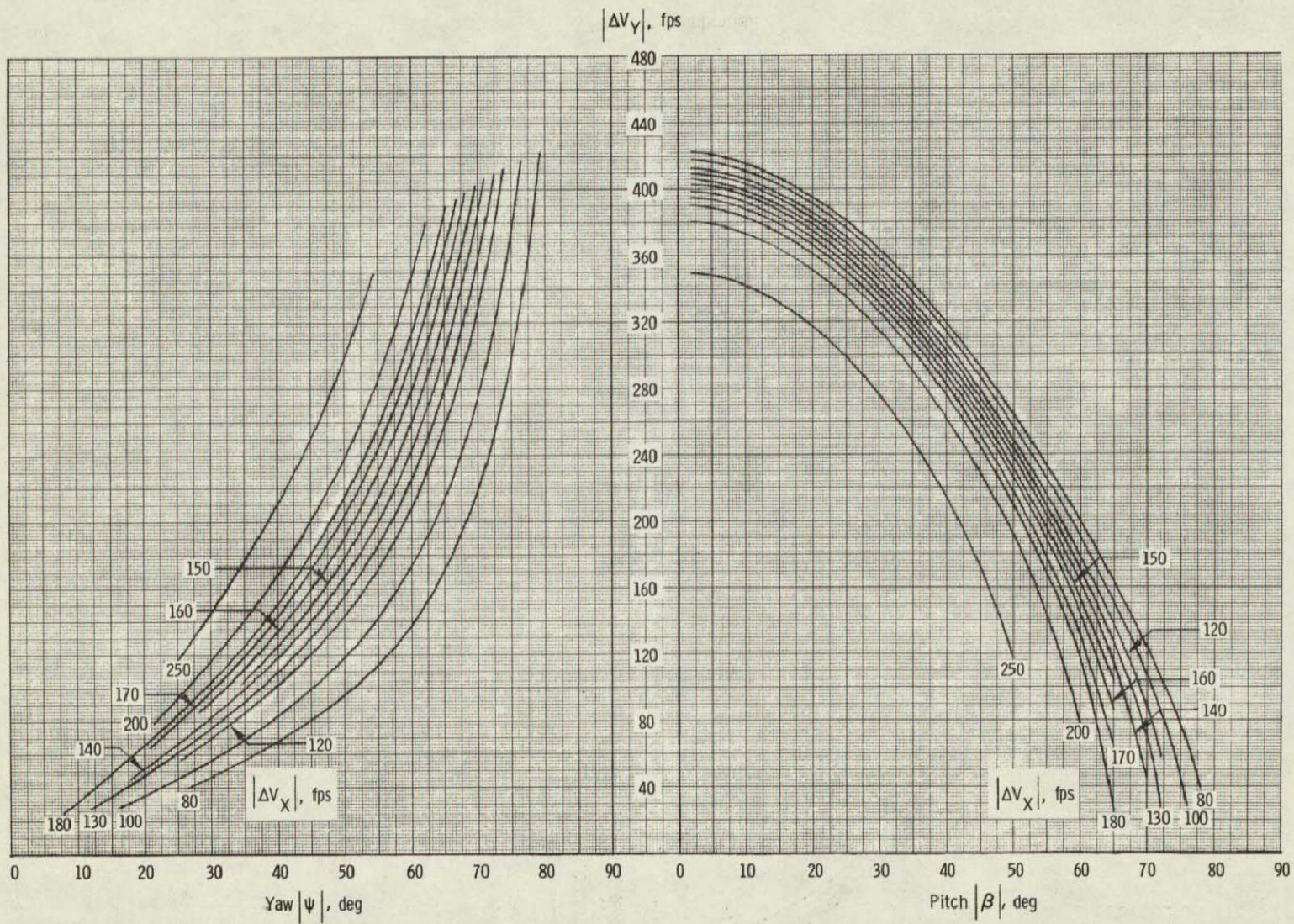
(b) $\Delta V \text{ total} = 330 \text{ fps}$.

Figure 5. - Continued.



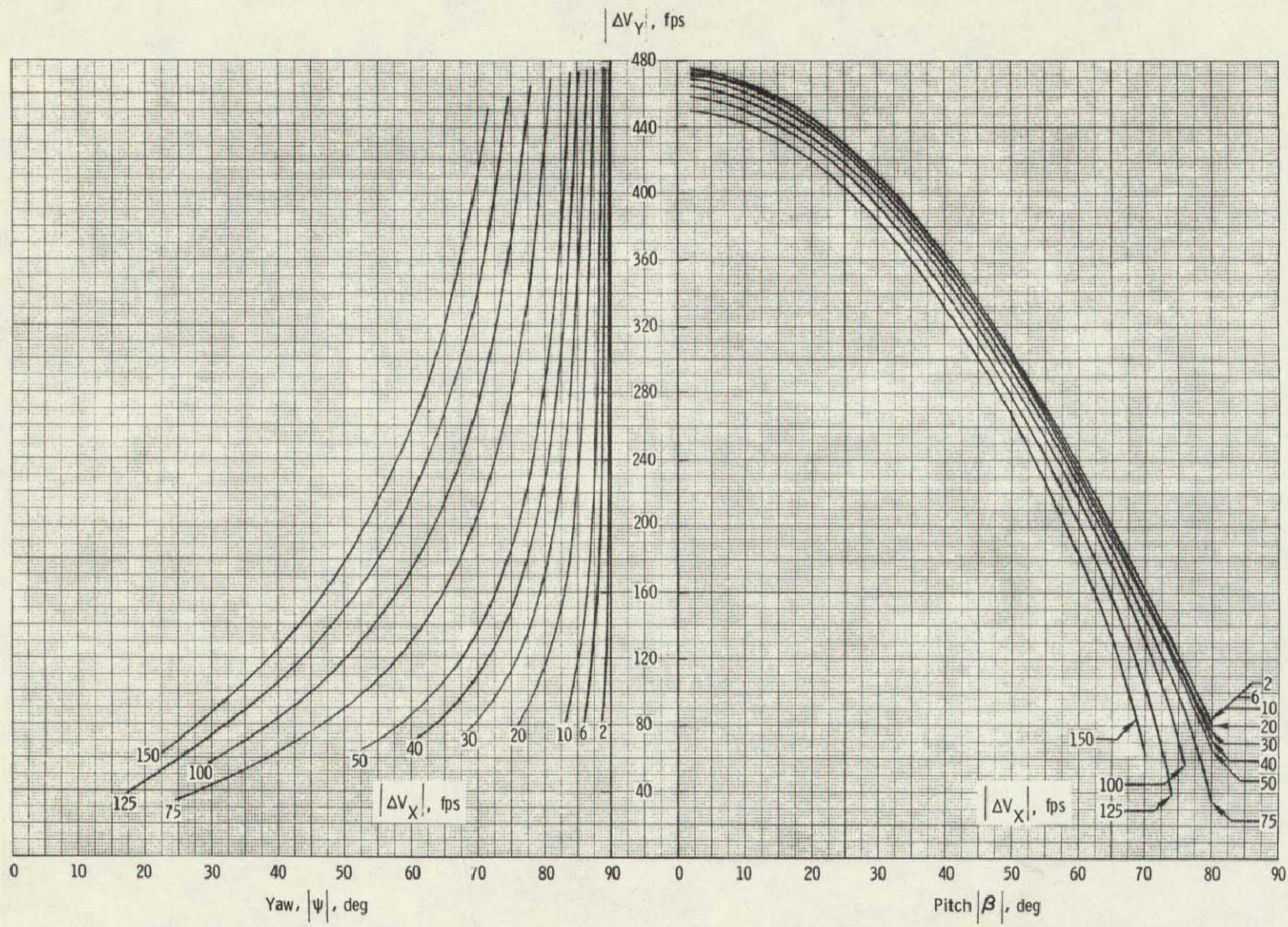
(c) ΔV total = 415 fps.

Figure 5. - Continued.



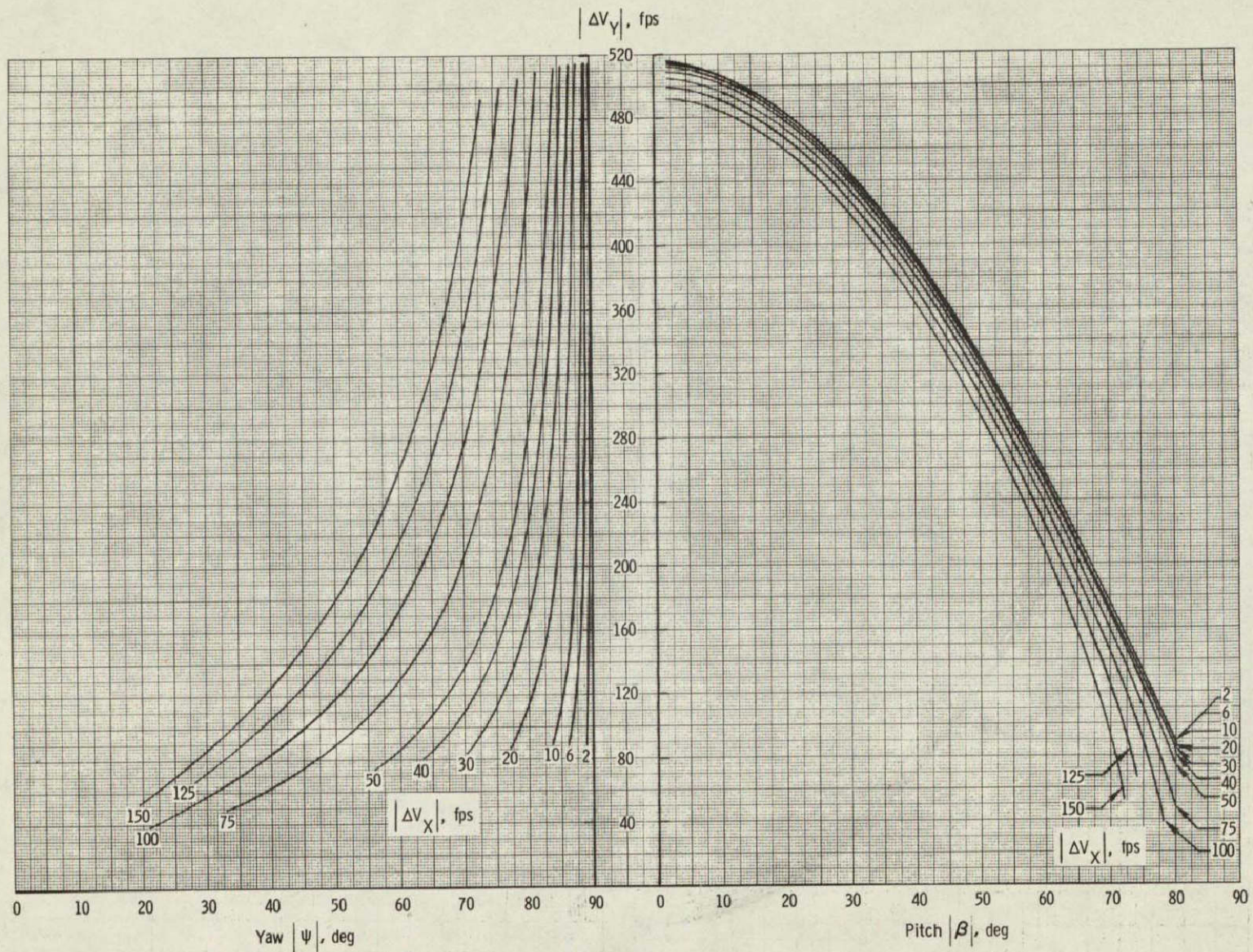
(d) ΔV total = 430 fps.

Figure 5. - Continued.



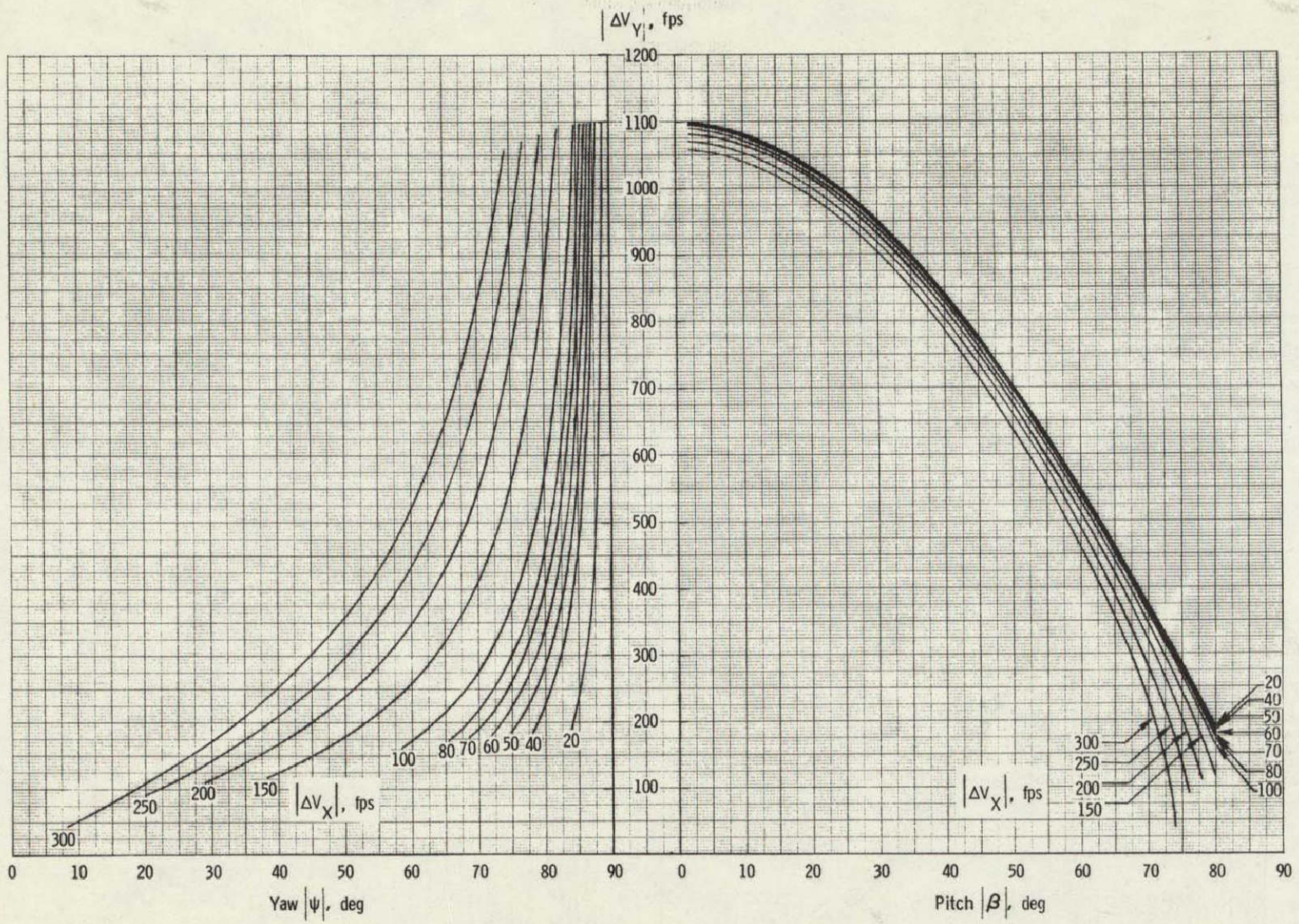
(e) ΔV total = 475 fps.

Figure 5. - Continued.



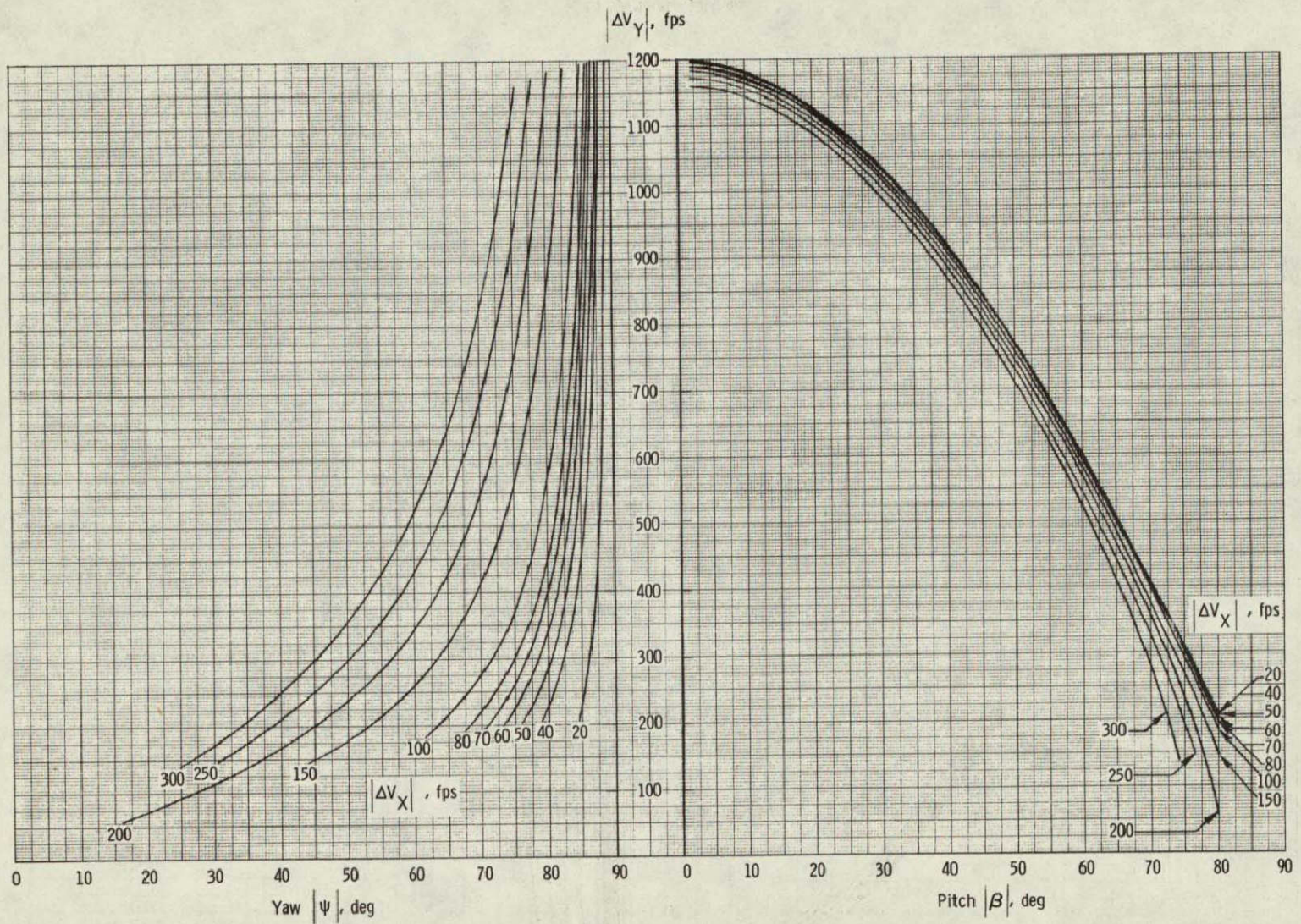
(f) ΔV total = 515 fps.

Figure 5. - Continued.



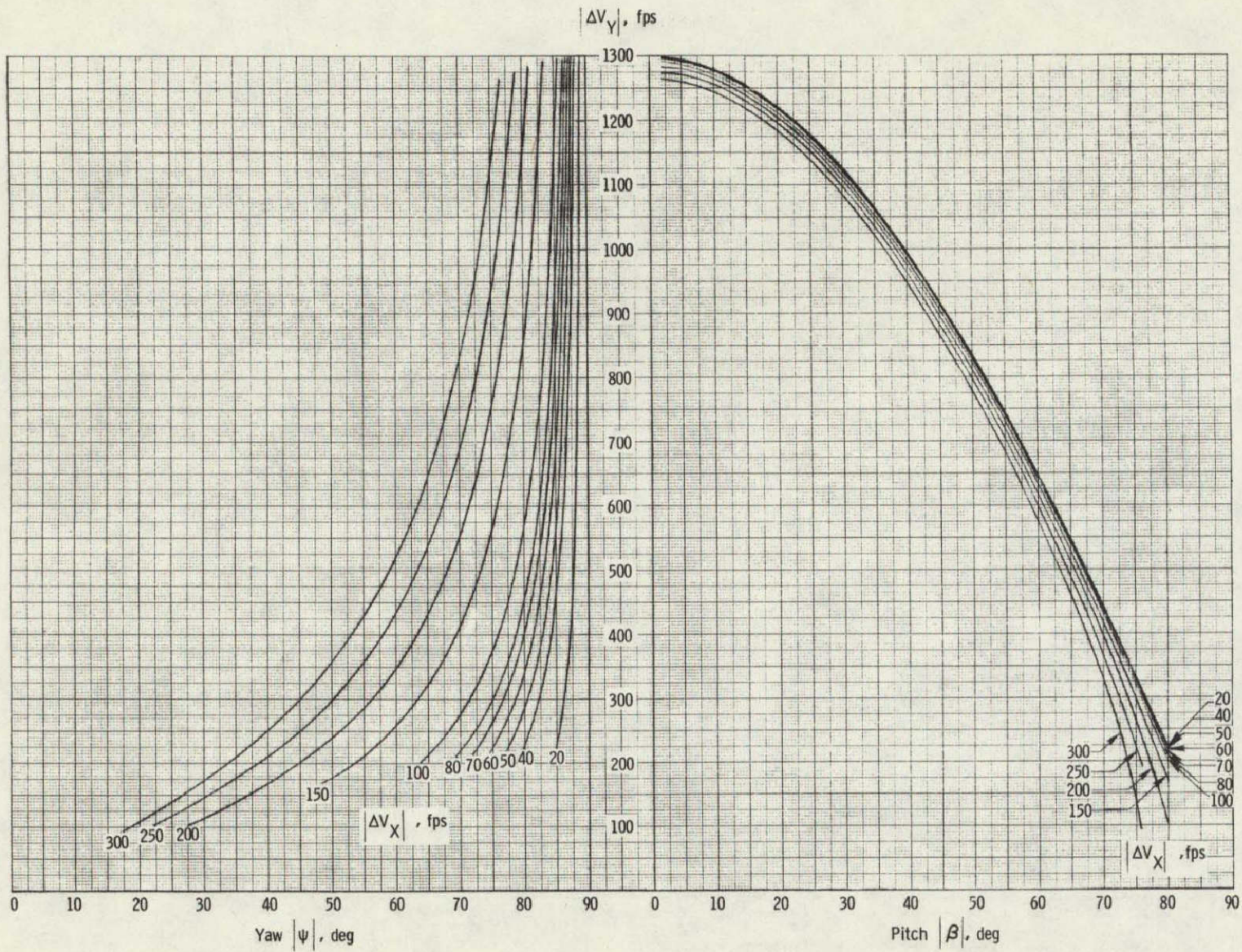
(g) ΔV total = 1100 fps.

Figure 5. - Continued.



(h) ΔV total = 1200 fps.

Figure 5. - Continued.



(i) ΔV total = 1300 fps.

Figure 5. - Concluded.