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LINEAR ACCELERATION GUIDANCE FOR LUNAR LANDING AND LAUNCH TRAJECTORIES

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NASA PROJECT APOLLO WORKING PAPER NO. 1087

LINEAR ACCELERATION GUIDANCE FOR LUNAR LANDING AND LAUNCH TRAJECTORIES

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF SKETCHES

LIST OF FIGURES

LIST OF FIGURES

LINEAR ACCELERATION GUIDANCE FOR LUNAR LANDING AND LAUNCH TRAJECTORIES

SUMMARY

An approximate analytical solution to the problem of maneuvering a spacecraft to reach specified end conditions by finite thrusting in the vicinity of the moon is presented. The analysis includes all three dimensions and the acceleration terms in the equations are linearized. The examples presented are limited to the planar case. Although in general the thrusting scheme is variable, a constraining relation is introduced to allow burning time for constant thrust to be calculated. Similarly, a constraining relation may be introduced which allows a burning time for a constant thrust angle to be calculated.

The solution is versatile in that it allows variable thrust, constant thrust, and constant thrust angle trajectories between specified end conditions.

INTRODUCTION

In order that a spacecraft such as the Apollo Lunar Excursion Module (LEM) can be guided during landing, launch, abort or rendezvous to a set of specified end conditions in a manner that is near fuel optimal, a set of guidance equations must be mechanized on board the spacecraft that will predict the necessary thrust and/or thrust angles. This paper will present the derivation of a set of equations that are suitable for this task from the standpoint of guidance and fuel optimum performance.

In obtaining a solution for the guidance equations, it is desirable that several criteria be met:

(1) The equations must be computationally simple.

(2) The equations must be suitable for use in as many operational modes as possible.

(3) The equations must yield a solution that is near the fuel optimum.

In order to obtain such a set of guidance equations, the first step was choosing a suitable approximation to linearize the equations of motion. To accomplish this, it was assumed that the change in altitude of the spacecraft is small compared to the initial radius of the spacecraft. This is similar to assuming a constant gravitational field. The next step was to solve the two-point boundary value problem explicitly.

The approach taken in this paper in solving the problem is to prescribe that the radial, tangential, and out-of-plane components of the acceleration vary linearly with time. The equations of motion can then be solved in closed form.

Six parameters are introduced in the three acceleration components which can be determined in closed form in terms of the six specified end conditions. These parameters constitute the guidance equations which will always insure that the spacecraft's trajectory will meet the specified terminal conditions.

For this acceleration scheme, the equation for the thrust angle is the same as that for the fuel optimum for a flat central body or constant gravity field approximation where the final position and velocity are constrained (ref. 1).

It is also shown that by introducing other constraining relations, constant thrust and constant pitch angle trajectories can be generated.

LIST OF SYMBOLS

A_1, A_2, A_3, A_4, A_5	constants defined on page
a	$\alpha_r^2 + \alpha_\phi^2 + \alpha_\psi^2$
b	$2(\alpha_r\beta_r + \alpha_\phi\beta_\phi + \alpha_\psi\beta_\psi)$
c	$\beta_r^2 + \beta_\phi^2 + \beta_\psi^2$
$F(\tau)$	condition for constant thrust
$G(\tau)$	condition for constant pitch angle
g_E	$32.2 \frac{\text{feet}}{\text{sec}^2}$, constant relating mass to weight in earth pounds

h	altitude, feet
I_{sp}	specific impulse, seconds
K_1, K_2, K_3, K_4	arbitrary constants of integration related to initial conditions
m	mass
R	radius of attracting body, feet
r, ϕ, ψ	polar coordinates defined in Sketch 1
T	thrust, pounds
t	time, seconds
v	velocity, feet per second
V_c	characteristic velocity, V_c, feet per second
W	weight of spacecraft, pounds
X	arc length or range along surface of attracting body, feet
Z	out-of-plane range, feet
α	initial level of applied acceleration in a given direction, ft/sec^2
β	thrust azimuth angle, degrees
β (with subscripts)	rate of change of applied acceleration in a given direction, ft/sec^3
$\delta(\ )$	denotes variation
θ	angle between thrust vector and local horizontal; the pitch angle, degrees

μ	universal gravitational constant times mass of attracting body, ft^3/sec^2
ν	$T/g_E I_{sp} m_c$, seconds^{-1}
τ	burning time, seconds
$\dot{( )}$	derivative of ( ) with respect to t
$( )'$	derivative of ( ) with respect to τ

Subscripts

0	conditions when t = 0
1, 2, 3, 4, 5	denotes different A's and K's
r	pertaining to radial direction
τ	conditions when $t = \tau$
ϕ	pertaining to circumferential, or x, direction
ψ	pertaining to direction perpendicular to r, ϕ plane

DERIVATION OF EQUATIONS

The equations of motion of a thrusting spacecraft in a gravitational field are:

$$\ddot{r} - r(\dot{\phi}^2 + \dot{\psi}^2 \sin^2 \phi) + \frac{\mu}{r^2} = \frac{T_r}{m} \tag{1}$$

$$\frac{d}{dt}(r^2\dot{\phi}) - r^2\dot{\psi}^2 \sin\phi \cos\phi = r\frac{T_\phi}{m} \tag{2}$$

$$\frac{d}{dt}(r^2\dot{\psi} \sin\phi) + r^2\dot{\psi}\,\dot{\phi} \cos\phi = r\frac{T_\psi}{m} \tag{3}$$

See reference 2.

These equations may be solved analytically by prescribing linear acceleration components: (See sketch 1.)

$$\frac{T_r}{m} = \frac{T}{m} \sin \theta \qquad = \alpha_r + \beta_r t \tag{4}$$

$$\frac{T_\phi}{m} = \frac{T}{m} \cos \theta \cos \beta = \alpha_\phi + \beta_\phi t \tag{5}$$

$$\frac{T_\psi}{m} = \frac{T}{m} \cos \theta \sin \beta = \alpha_\psi + \beta_\psi t \tag{6}$$

and assuming that the change in altitude is small compared to the initial radius r_o, and that the out-of-plane angle and its time derivative, ψ and $\dot{\psi}$. are small.

In equation (2), assume that $r = r_o$ = constant and that: $r^2\dot{\psi}^2 \sin \phi \cos \phi << \frac{d}{dt} (r^2\dot{\phi})$. This equation, using (5) then becomes,

$$\frac{d}{dt} (r_o^2\dot{\phi}) = r_o(\alpha_\phi + \beta_\phi t)$$

Integrating this equation twice,

$$\dot{\phi} = \frac{1}{r_o}\left[K_1 + \alpha_\phi t + \frac{1}{2} \beta_\phi t^2\right] \tag{7}$$

$$\phi = \frac{1}{r_o}\left[K_2 + K_1 t + \frac{1}{2} \alpha_\phi t^2 + \frac{1}{6} \beta_\phi t^3\right] \tag{8}$$

Before applying initial conditions to (7) and (8), make a change of variables by defining the range $X = R\phi$. Equations (7) and (8) then become:

$$\dot{X} = \frac{R}{r_o}\left[K_1 + \alpha_\phi t + \frac{1}{2} \beta_\phi t^2\right] \tag{9}$$

$$X = \frac{R}{r_o}\left[K_2 + K_1 t + \frac{1}{2} \alpha_\phi t^2 + \frac{1}{6} \beta_\phi t^3\right] \tag{10}$$

where K_1 and K_2 are constants of integration.

Assuming that $r = r_o$ = constant and that $\dot{\psi}^2 \sin^2\phi << \dot{\phi}^2$, equation (1) becomes,

$$\ddot{r} - r_o \dot{\phi}^2 + \frac{\mu}{r_o^2} = \alpha_r + \beta_r t \tag{11}$$

Square both sides of equation (7), and then substitute $r_o\dot{\phi}^2$ from (11),

$$\dot{\phi}^2 = \frac{1}{r_o}\left[\ddot{r} + \frac{\mu}{r_o^2} - \alpha_r - \beta_r t\right]$$

$$= \frac{1}{r_o^2}\left[K_1^2 + 2K_1\alpha_\phi t + (\alpha_\phi^2 + K_1\beta_\phi)t^2 + \alpha_\phi\beta_\phi t^3 + \frac{1}{4}\beta_\phi^2 t^4\right]$$

Solving for $\ddot{r}$,

$$\ddot{r} = \left(\frac{K_1^2}{r_o} + \alpha_r - \frac{\mu}{r_o^2}\right) + \left(\frac{2K_1\alpha_\phi}{r_o} + \beta_r\right) t + \frac{1}{r_o}\left(\alpha_\phi^2 + K_1\beta_\phi\right)t^2 + \frac{\alpha_\phi\beta_\phi}{r_o} t^3 + \frac{\beta_\phi^2}{4r_o} t^4 \tag{12}$$

Now, $r = h + R$, where h is the altitude of the spacecraft and R is the radius of the attracting body. Using $\ddot{r} = \ddot{h}$ and by defining,

$$A_1 = \frac{K_1^2}{r_o} + \alpha_r - \frac{\mu}{r_o^2}$$

$$A_2 = \frac{2K_1\alpha_\phi}{r_o} + \beta_r$$

$$A_3 = \frac{1}{r_o}\left(\alpha_\phi^2 + K_1\beta_\phi\right)$$

$$A_4 = \frac{\alpha_\phi \beta_\phi}{r_o}$$

$$A_5 = \frac{\beta_\phi^{\;2}}{4 r_o}$$

equation (12) becomes:

$$\ddot{h} = A_1 + A_2 t + A_3 t^2 + A_4 t^3 + A_5 t^4 \tag{13}$$

Now integrate (13),

$$\dot{h} = K_3 + A_1 t + \frac{1}{2} A_2 t^2 + \frac{1}{3} A_3 t^3 + \frac{1}{4} A_4 t^4 + \frac{1}{5} A_5 t^5 \tag{14}$$

$$h = K_4 + K_3 t + \frac{1}{2} A_1 t^2 + \frac{1}{6} A_2 t^3 + \frac{1}{12} A_3 t^4$$

$$+ \frac{1}{20} A_4 t^5 + \frac{1}{30} A_5 t^6 \tag{15}$$

where K_3 and K_4 are constants of integration.

Define the out-of-plane range, $Z = r \sin \phi \sin \psi$

For small angles ψ, and assuming $r = r_o$ = constant this becomes,

$$Z = \psi \, r_o \sin \phi$$

and the out-of-plane velocity becomes,

$$\dot{Z} = \dot{\psi} \, r_o \sin \phi + \psi \, \dot{\phi} \, r_o \cos \phi$$

Substituting this last expression into (3), and again assuming $r = r_o$ = constant, (3) becomes,

$$\frac{d}{dt} (\dot{Z} - r_o \psi \dot{\phi} \cos \phi) + r_o \dot{\psi} \dot{\phi} \cos = \alpha_\psi + \beta_\psi t$$

Differentiating the first term and using the definition $X = R\phi$, the last equation reduces to:

$$\ddot{Z} + \frac{r_o}{R} \psi \frac{d}{dt} (\dot{X} \cos \phi) = \alpha_\psi + \beta_\psi t \tag{16}$$

Since ψ is a small angle and if $\dot{X}$ does not change too rapidly with time, the second term on the left may be neglected and (16) becomes:

$$\ddot{Z} = \alpha_\psi + \beta_\psi t \tag{17}$$

Equation (17) may now be integrated twice to give:

$$\dot{Z} = K_5 + \alpha_\psi t + \frac{1}{2} \beta_\psi t^2 \tag{18}$$

$$Z = K_6 + K_5 t + \frac{1}{2} \alpha_\psi t^2 + \frac{1}{6} \beta_\psi t^3 \tag{19}$$

Equations (9), (10), (14), (15), (18), and (19) are the equations of motion of the spacecraft. Now apply the initial conditions to evaluate K_1 through K_6. When $t = 0$, $X = X_o$, $\dot{X} = \dot{X}_o$, $h = h_o$, $\dot{h} = \dot{h}_o$, $Z = Z_o$, $\dot{Z} = \dot{Z}_o$. The constants of integration are seen to be:

$K_1 = \frac{r_o}{R} \dot{X}_o$, $K_2 = \frac{r_o}{R}$, $K_3 = \dot{h}_o$, $K_4 = h_o$, $K_5 = \dot{Z}_o$, and $K_6 = Z_o$.

The equations of motion are now determined except for the thrust parameters α_ϕ, β_ϕ, α_r, β_r, α_ψ, and β_ψ, which determine the thrust history; that is, the thrust magnitude and direction. The six equations of motion may be solved for these thrust parameters in terms of final conditions. When $t = \tau$, $X = X_\tau$, $\dot{X} = \dot{X}_\tau$, $h = h_\tau$, $\dot{h} = \dot{h}_\tau$, $Z = Z_\tau$, and $\dot{Z} = \dot{Z}_\tau$. After considerable manipulation the equations for the thrust parameters become:

$$\alpha_\phi = -\frac{2}{\tau} \frac{r_o}{R} \left[\frac{3}{\tau} (X_o - X_\tau) + 2\dot{X}_o + \dot{X}_\tau \right] \tag{20}$$

$$\beta_\phi = \frac{6}{\tau^2} \frac{r_o}{R} \left[\frac{2}{\tau} (X_o - X_\tau) + \dot{X}_o + \dot{X}_\tau \right] \tag{21}$$

$$\alpha_r = \frac{\mu}{r_o^2} - \frac{r_o}{R^2}\dot{X}_o^2 + \frac{1}{\tau}(\dot{h}_\tau - \dot{h}_o) - \frac{A_2}{2}\tau - \frac{A_3}{3}\tau^2 - \frac{A_4}{4}\tau^3$$

$$- \frac{A_5}{5}\tau^4 \qquad (22)$$

$$\beta_r = -2\alpha_\phi \frac{\dot{X}_o}{R} + \frac{12}{\tau^3}(h_o - h_\tau) + \frac{6}{\tau^2}(\dot{h}_o + \dot{h}_\tau) - A_3\tau$$

$$- A_3\tau - \frac{9}{10}A_4\tau^2 - \frac{4}{5}A_5\tau^3 \qquad (23)$$

$$\alpha_\psi = -\frac{2}{\tau}\left[\frac{3}{\tau}(Z_o - Z_\tau) + 2\dot{Z}_o + \dot{Z}_\tau\right] \qquad (24)$$

$$\beta_\psi = \frac{6}{\tau^2}\left[\frac{2}{\tau}(Z_c - Z_\tau) + \dot{Z}_o + \dot{Z}_\tau\right] \qquad (25)$$

For a specified set of initial and terminal conditions the boundary value problem is now completely solved.

The thrust azimuth angle, β, is given at any time by dividing equation (6) by (5),

$$\tan\beta = \frac{T_\psi/m}{T_\phi/m} = \frac{\alpha_\psi + \beta_\psi t}{\alpha_\phi + \beta_\phi t} \qquad (26)$$

The thrust pitch angle, θ, is given at any time by dividing (4) by (5) and using (26),

$$\tan\theta = \frac{1}{\cos\beta}\frac{T_r/m}{T_\phi/m} = \frac{1}{\cos\beta}\frac{\alpha_r + \beta_r t}{\alpha_\phi + \beta_\phi t} \qquad (27)$$

The total acceleration, $\frac{T}{m}$, is given at any time by,

$$\left(\frac{T}{m}\right)^2 = \left(\frac{T_r}{m}\right)^2 + \left(\frac{T_\phi}{m}\right)^2 + \left(\frac{T_\psi}{m}\right)^2$$

$$= (\alpha_r + \beta_r t)^2 + (\alpha_\phi + \beta_\phi t)^2 + (\alpha_\psi + \beta_\psi t)^2$$

or,

$$\frac{T}{m} = \sqrt{a + bt + ct^2} \tag{28}$$

where,

$$a = \alpha_r^2 + \alpha_\phi^2 + \alpha_\psi^2$$

$$b = 2(\alpha_r\beta_r + \alpha_\phi\beta_\phi + \alpha_\psi\beta_\psi)$$

$$c = \beta_r^2 + \beta_\phi^2 + \beta_\psi^2$$

The total acceleration in (28) is now used directly in the definition for characteristic velocity, or performance index, of a given trajectory.

$$V_c = \int_0^\tau \left(\frac{T}{m}\right) dt = \int_0^\tau \sqrt{a + bt + ct^2}\, dt \tag{29}$$

Performing the integration equation (29) becomes:

$$V_c = \frac{4ac - b^2}{8c^{3/2}} \ln\left[\frac{\sqrt{a + b\tau + c\tau^2} + \tau\sqrt{c} + \frac{1}{2}\frac{b}{\sqrt{c}}}{\sqrt{a} + \frac{1}{2}\frac{b}{\sqrt{c}}}\right]$$

$$+ \frac{2c\tau + b}{4c}\sqrt{a + b\tau + c\tau^2} - \frac{b\sqrt{a}}{4c} \tag{30}$$

Since a, b, and c are dependent only upon the initial and final conditions and the burning time τ, the characteristic velocity is related only to the burning time once the end conditions are specified. In most cases, it is desirable to minimize V_c subject to certain constraints.

THRUSTING MODES

Given a complete set of initial and terminal conditions, there exists an infinity of trajectories that will satisfy the end conditions. If a burning time is specified, the trajectory is uniquely defined. In some cases, a burning time will be chosen that will minimize V_c. Two important alternate cases of course are, (1) choosing the burning time so that a constant thrust magnitude trajectory can be obtained, and (2) choosing the burning time so that a trajectory with a constant value of θ is obtained.

Case a: Minimum V_c. To obtain the burning time for minimum V_c, the derivative of V_c with respect to burning time may be set equal to zero, and the resulting relation solved for τ. This value of τ must then be further investigated to determine if it yields a maximum, minimum, or an inflection point. This could be done but would be difficult, since V_c is a very complicated function of τ. It is much easier from the standpoint of mathematical complexity to solve (29) numerically by just evaluating it for a wide range of τ and then looking for the minimum on a plot of V_c versus τ. Once the optimum value of τ is obtained, α_r, β_r, α_ϕ, β_ϕ, α_ψ, β_ψ can be calculated from (20) through (25); then the trajectory with minimum V_c based on these values is calculated from (9), (10), (14), (15), (18), and (19).

Case b: Constant Thrust Magnitude. Equation (28) may be shown to yield a constant thrust magnitude by comparing it to the acceleration relation for constant thrust. The acceleration for constant thrust may be obtained by considering the basic relation between thrust and rate change of mass,

$$T = - g_E \, I_{sp} \, \dot{m} = m\dot{v}$$

For constant thrust,

$$m = m_o \, (1 - \nu t), \quad \dot{v} = \frac{T}{m} = \frac{T/m_o}{1 - \nu t}$$

$$\nu \equiv \frac{T/m_o}{g_E \, I_{sp}}$$

Equating this to (28),

$$\frac{T}{m} = \sqrt{a + bt + ct^2} = \frac{T/m_o}{1 - \nu t} \tag{31}$$

For a specified set of end conditions equation (31) may be solved for the burning time τ, since a, b, and c are functions of τ, at any time t. In particular at time $t = 0$, (31) becomes $\sqrt{a} = T/m_o$. But since $a = \alpha_\phi^2 + \alpha_r^2 + \alpha_\psi^2$, the relation to be solved for τ is,

$$a = \alpha_\phi^2 + \alpha_r^2 + \alpha_\psi^2 = (T/m_o)^2 = \left(\frac{T}{W_o}\right)^2 g_E^2 \qquad (32)$$

To determine the proper constant thrust level, equation (32) may be solved simultaneously with the characteristic velocity equation for constant thrust, $V_c = - g_E I_{sp} \ln (1 - \nu\tau)$.

Equation (32) must be solved for τ by an iterative process. Define the function,

$$F(\tau) = \alpha_\phi^2 + \alpha_r^2 + \alpha_\psi^2 - \left(\frac{T}{W_o}\right)^2 g_E^2 = 0 \qquad (33)$$

and its derivative,

$$\frac{d}{d\tau} F(\tau) = F'(\tau) = 2(\alpha_\phi \alpha_\phi' + \alpha_r \alpha_r') \qquad (34)$$

The iteration process,

$$\tau_{n+1} = \tau_n - \frac{F(\tau_n)}{F'(\tau_n)} \qquad (35)$$

is then set up to determine τ.

Equation (35) has been observed to converge quite rapidly for the constant thrust burning time.

Case c: Constant θ. Assuming β is small so that $\cos \beta \approx 1$, the time derivative of equation (27) for θ is,

$$\dot{\theta} = \frac{\alpha_\phi \beta_r - \alpha_r \beta_\phi}{(\alpha_\phi + \beta_\phi t)^2} \cos^2 \theta \qquad (36)$$

For θ to be constant, $\dot{\theta} = 0$, so (36) yields,

$$\alpha_\phi \beta_r - \alpha_r \beta_\phi = 0 \qquad (37)$$

Now define the function,

$$G(\tau) = \alpha_\phi \beta_r - \alpha_r \beta_\phi = 0$$

and its derivative,

$$G'(\tau) = (\alpha_\phi \beta_r' + \alpha_\phi' \beta_r) - (\alpha_r \beta_\phi' + \alpha_r' \beta_\phi)$$

The iterative solution for the burning time for constant θ is,

$$\tau_{n+1} = \tau_n - \frac{G(\tau_n)}{G'(\tau_n)} \tag{38}$$

USE OF THE ANALYTICALLY DERIVED THRUST PARAMETERS FOR GUIDANCE

Equations (20) through (25) give analytical expressions for the thrust parameters. They were derived on the basis of a model that approximated the inverse square gravity field. These equations may be considered as guidance equations which will guide the spacecraft during various maneuvers. The performance of these guidance equations can be evaluated by a simulation that is programed on a digital computer. A trajectory program which numerically solves equations (1), (2), and (3) to predict the position and velocity of the spacecraft at any time is a basic component of the simulation. A guidance program which continually computes the thrust and its direction by solving equations (20) through (25) is the other basic component of the simulation. The instantaneous thrust vector computed by the guidance program depends only upon the present position and velocity of the spacecraft and the specified terminal conditions.

The problem of combining the two programs for variable thrust is straightforward. Both sets of equations are given the same initial conditions. The final conditions are also substituted into the thrust parameter relations, equations (20) through (25). α_r, β_r, α_ϕ, β_ϕ, α_ψ, β_ψ are computed with these conditions and are used to calculate θ, β, and (T/m) from equations (26), (27), and (28), for the first step. The output, x, $\dot{x}$, h, $\dot{h}$, Z, $\dot{Z}$, of the first integration step are then used as the initial conditions in equations (20) through (25) to compute θ, β, and (T/m), for the next integration step. The calculation thus proceeds to the final conditions of the problem.

If constant thrust or constant θ is desired the burning time is updated by equations (35) or (38), just as θ, β, and (T/m) are in the variable thrust case discussed above. τ is updated prior to each integration step using the output from the previous step as the initial conditions. The computation procedure can best be visualized by referring to the block diagram in Sketch 2.

EXAMPLES OF CALCULATIONS

Several examples will now be given to illustrate the various modes in which the computation scheme may be operated. In all of the following examples, the calculations were limited to the two-dimensional case.

(a) Variable thrust with minimum characteristic velocity:

This mode is illustrated by a descent trajectory from approximately 50,000 feet to 10,000 feet above the lunar surface. The burning time was 274 seconds and was calculated by solving $\frac{d\,V_c}{dt}$ numerically. The V_c was 4,833 feet per second and the total range was specified as 169.7 n.mi. Time histories of the variables are shown in figures (1a) through (1f).

(b) Constant thrust:

This mode is also illustrated by a lunar descent trajectory. The initial and final conditions are the same as in the variable mode discussed in (a) above. The burning time was 299.3 seconds and the V_c was 4,883 feet per second. The time histories of the variables are shown in figures (2a) through (2e). Also plotted for comparison is a trajectory having the same boundary conditions, that was calculated from a calculus of variations optimum program. This optimum trajectory had a burning time of 300.90 seconds with a V_c of 4,883 feet per second. The V_c of the guided constant thrust trajectory was less than that for the optimum because of the slight difference in burning time between the two. It should be noted here that the two trajectories flown are different, except for the boundary conditions. The thrust-to-initial-weight ratio was 0.4.

(c) Constant thrust launch with an error analysis for variations in the thrust vector:

This trajectory had an initial altitude of 1,000 feet and an initial vertical velocity of 100 feet per second. The final conditions were pericynthion altitude of 50,000 feet, with orbital velocity of about 5,558 feet per second. The total range was 92.92 n.mi. The guided trajectory is compared with the optimum in table I. Additional calculations of this trajectory were made perturbing independently the thrust magnitude T and thrust direction θ by small amounts. The effect of these perturbances are given in table II. It is evident from this table that the end conditions are not seriously affected by 1.0 to 5.0° misalinements in thrust direction, and 1.0 or 2.0 percent variations in thrust magnitude. The end condition most affected was the range.

CONCLUDING REMARKS

An approximate analytical solution to the problem of maneuvering a spacecraft to reach specified end conditions by finite thrusting has been presented. The feasibility of using this solution to guide the spacecraft to a specified final state has been demonstrated. Further, it has been shown that by the introduction of certain constraining relations that constant thrust and constant thrust angle trajectories may be flown. The guidance equations obtained from the analytical solution have been shown to be computationally simple; versatile, in that several operational modes are possible; and economical in that they guide the spacecraft along a trajectory that is near the fuel optimum.

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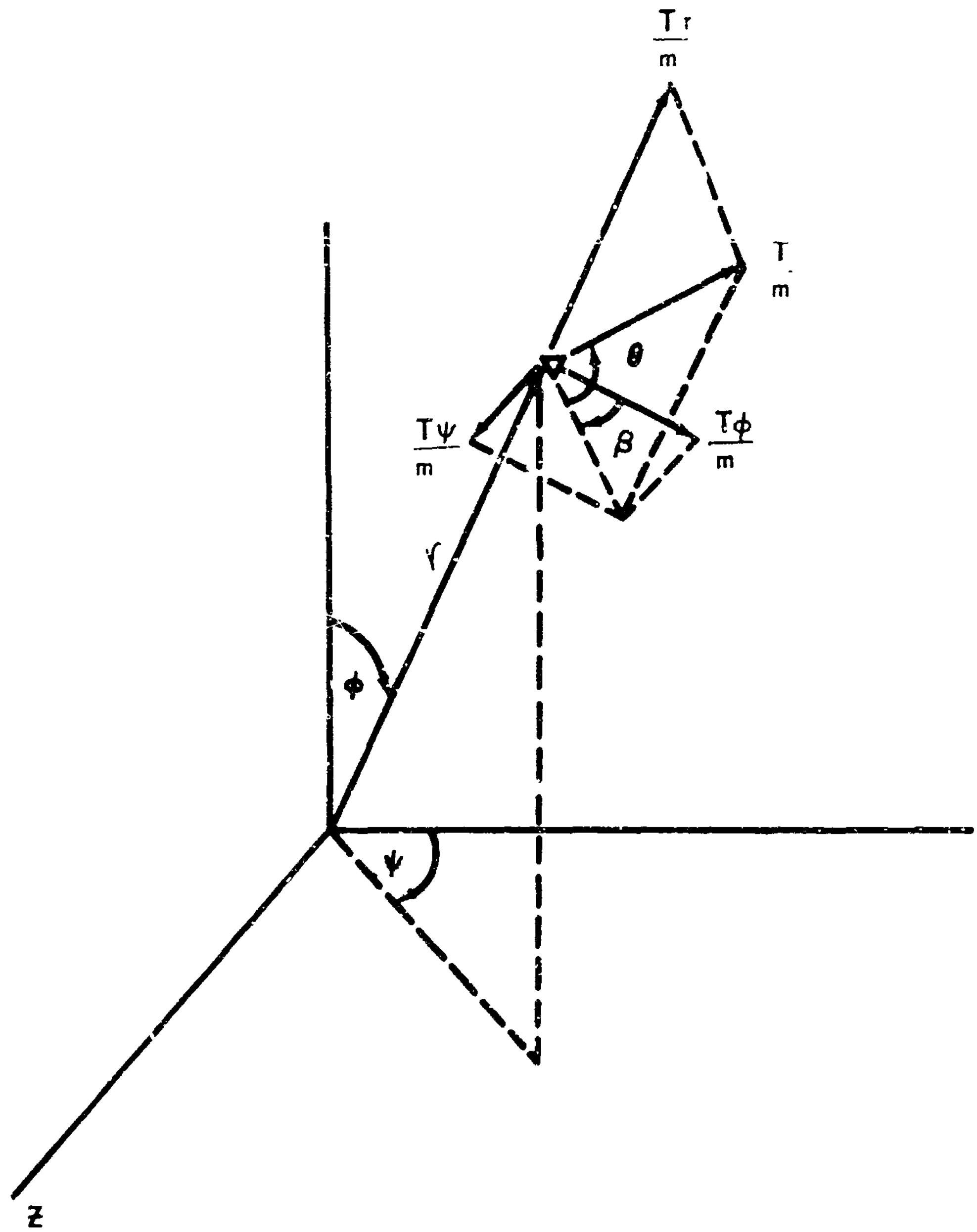
TABLE I.- COMPARISON OF FINAL CONDITIONS BETWEEN AN OPTIMUM CONSTANT THRUST LAUNCH AND A GUIDED LAUNCH; $T/W_o = 0.6$; $I_{sp} = 315$ sec.

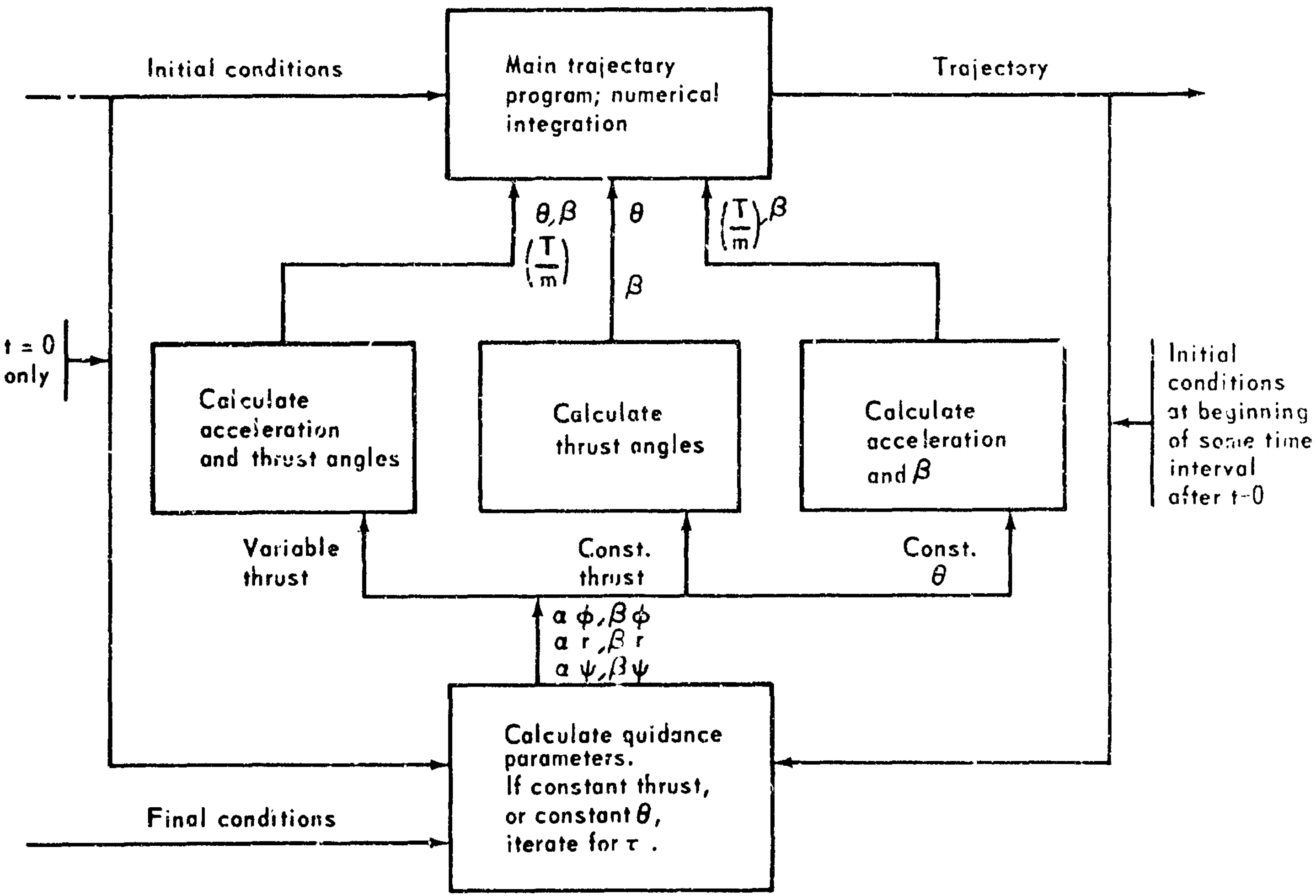
	Initial Conditions	Final Conditions Optimum	Guided
x, feet	0	564,573.4	562,982.0
h, feet	1,000	49,957.94	49,956.62
$\dot{x}$, fps	0	5,558.02	5,558.93
$\dot{h}$, fps	100	0.08257	0.26230
V_c, fps		5,775.25	5,778.12
τ, sec		227.916	228.00

TABLE II.- EFFECT OF ERRORS IN PITCH ANGLE AND THRUST ON THE FINAL CONDITIONS OF THE GUIDED LAUNCH IN TABLE I

Perturbation in thrust vector	Effect of perturbation on final conditions					
	x, ft	h, ft	$\dot{x}$, fps	$\dot{h}$, fps	V_c, fps	τ, secs
$\delta\theta = +1°$	-757.5	+4.36	-0.83	-1.444	+1.24	+0.036
$\delta\theta = -1°$	+643.9	-1.51	-0.90	-0.408	-1.60	-0.047
$\delta\theta = +5°$	-1650.8	+21.37	-1.00	-3.787	+22.63	+0.662
$\delta\theta = -5°$	+5798.4	-9.47	-0.94	-11.545	+11.15	+0.326
$\frac{\delta T}{T}$ = +1 percent	-5562.4	-5.14	-0.86	+2.279	-2.01	-2.31[illegible]
$\frac{\delta T}{T}$ = -1 percent	+7355.3	+4.63	-0.11	-0.437	+0.12	+2.307
$\frac{\delta T}{T}$ = +2 percent	-10883.6	-11.[illegible]	-0.70	+6.185	-3.15	-4.361
$\frac{\delta T}{T}$ = -2 percent	+10975.0	+10.02	-0.97	+ .0967	-1.23	+4.690



Sketch 1; Geometry of a thrusting trajectory in a gravitational field.



Sketch 2--Block diagram illustrating the trajectory calculation

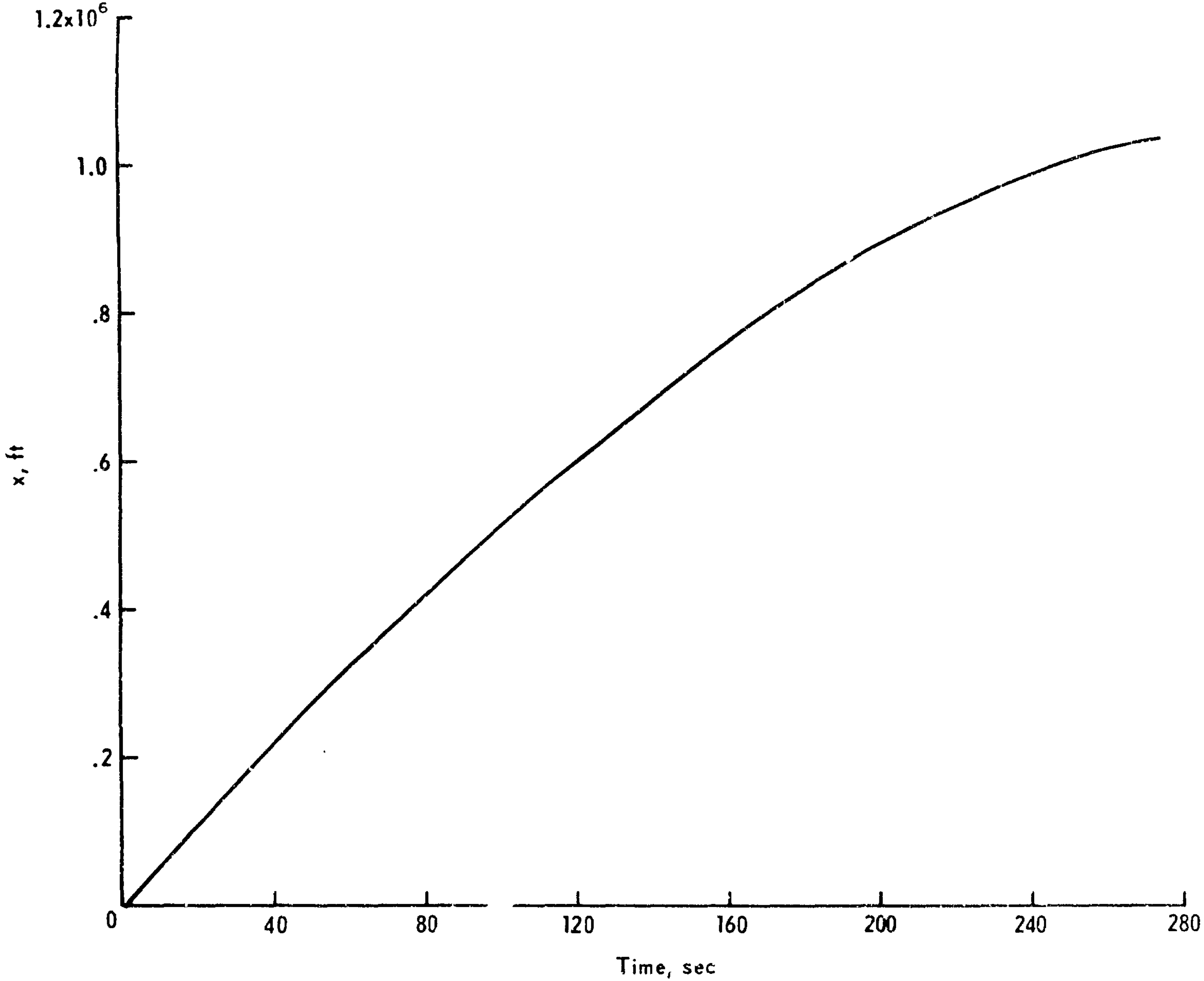


Fig 1: Variable thrust, minimum Vc solution
(a) Range as a function of time

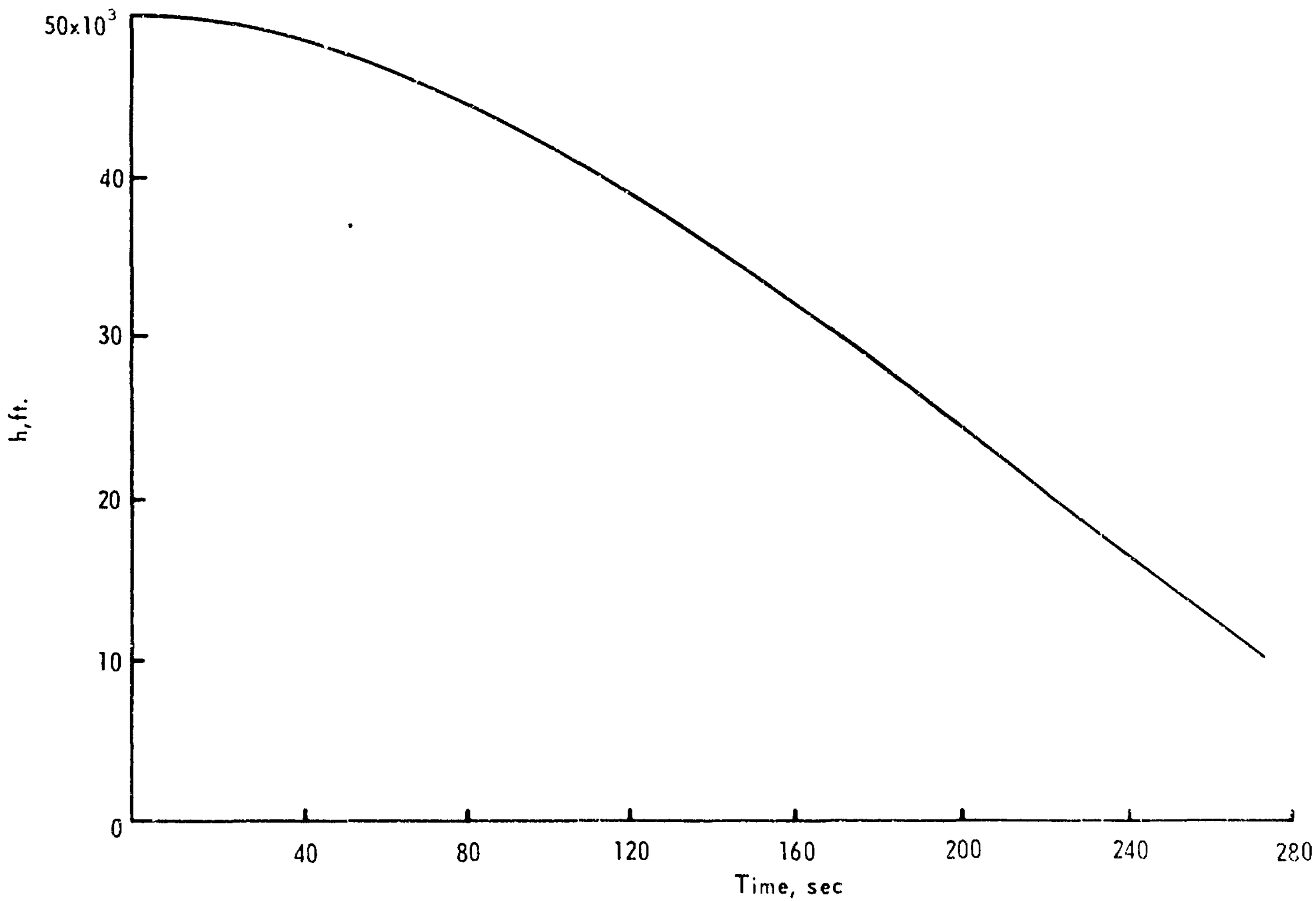


Fig 1: continued
(b) Altitude as a function of time

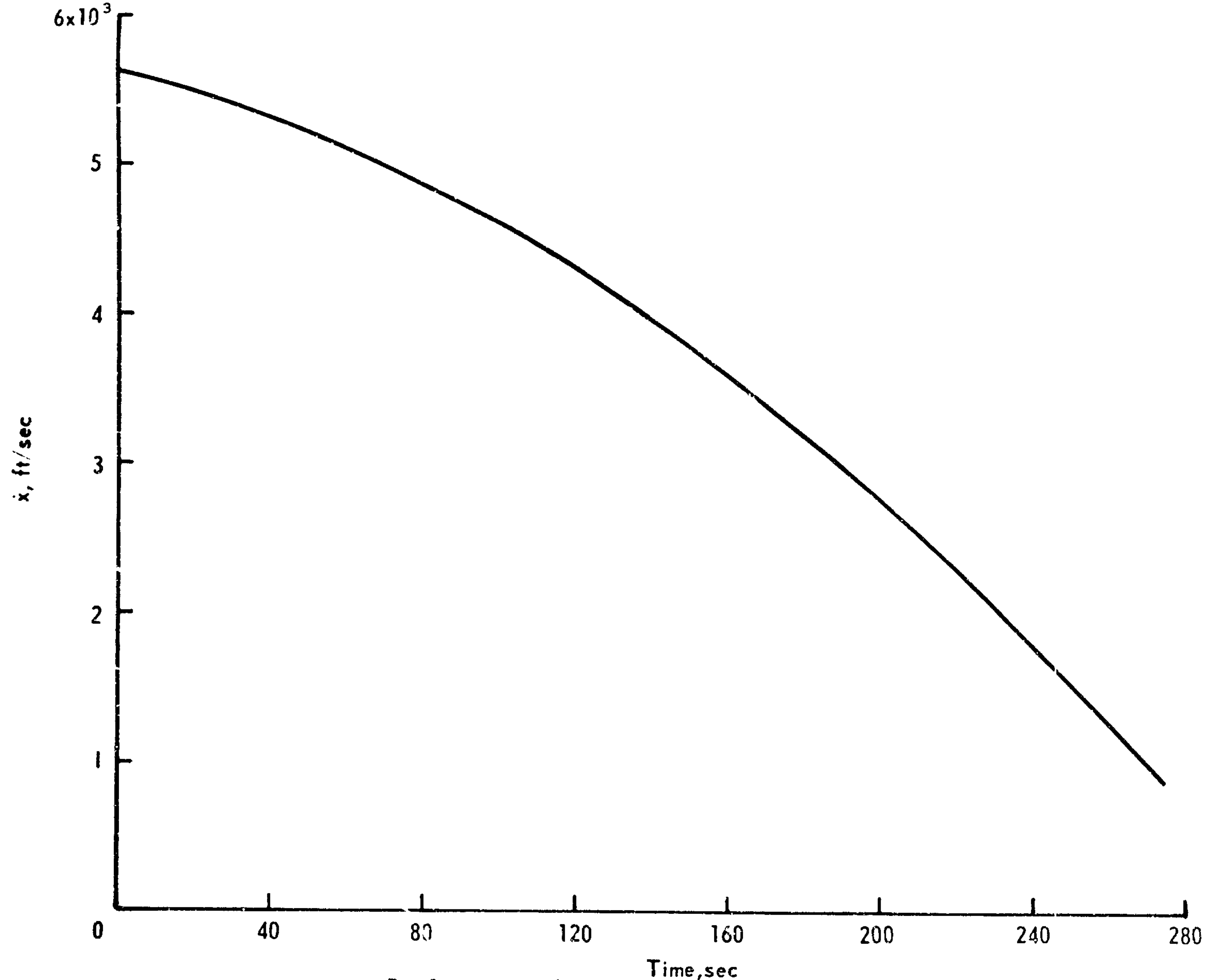


Fig 1: continued
(c) Horizontal velocity as a function of time

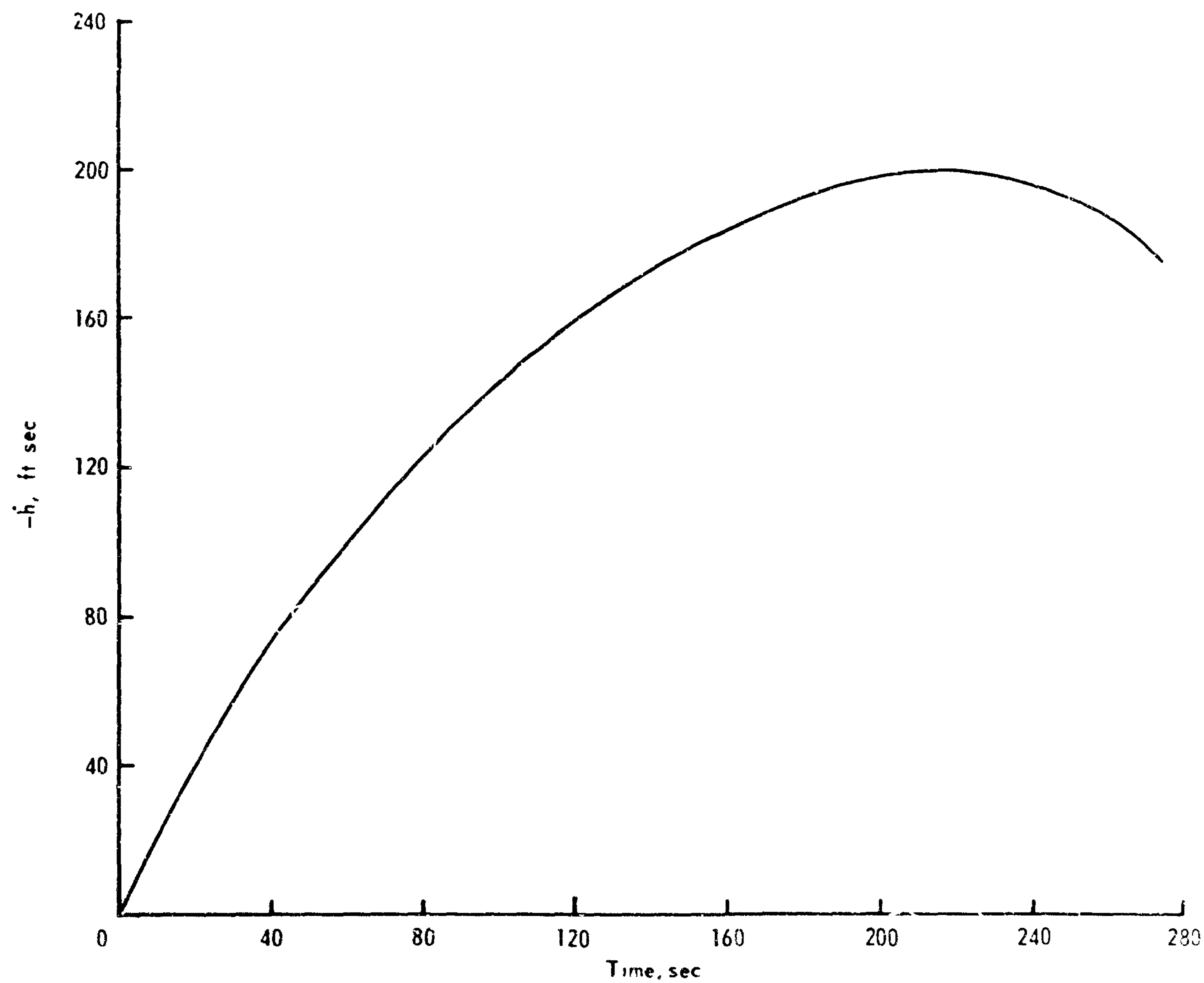


Fig 1: continued
(d) Vertical velocity as a function of time

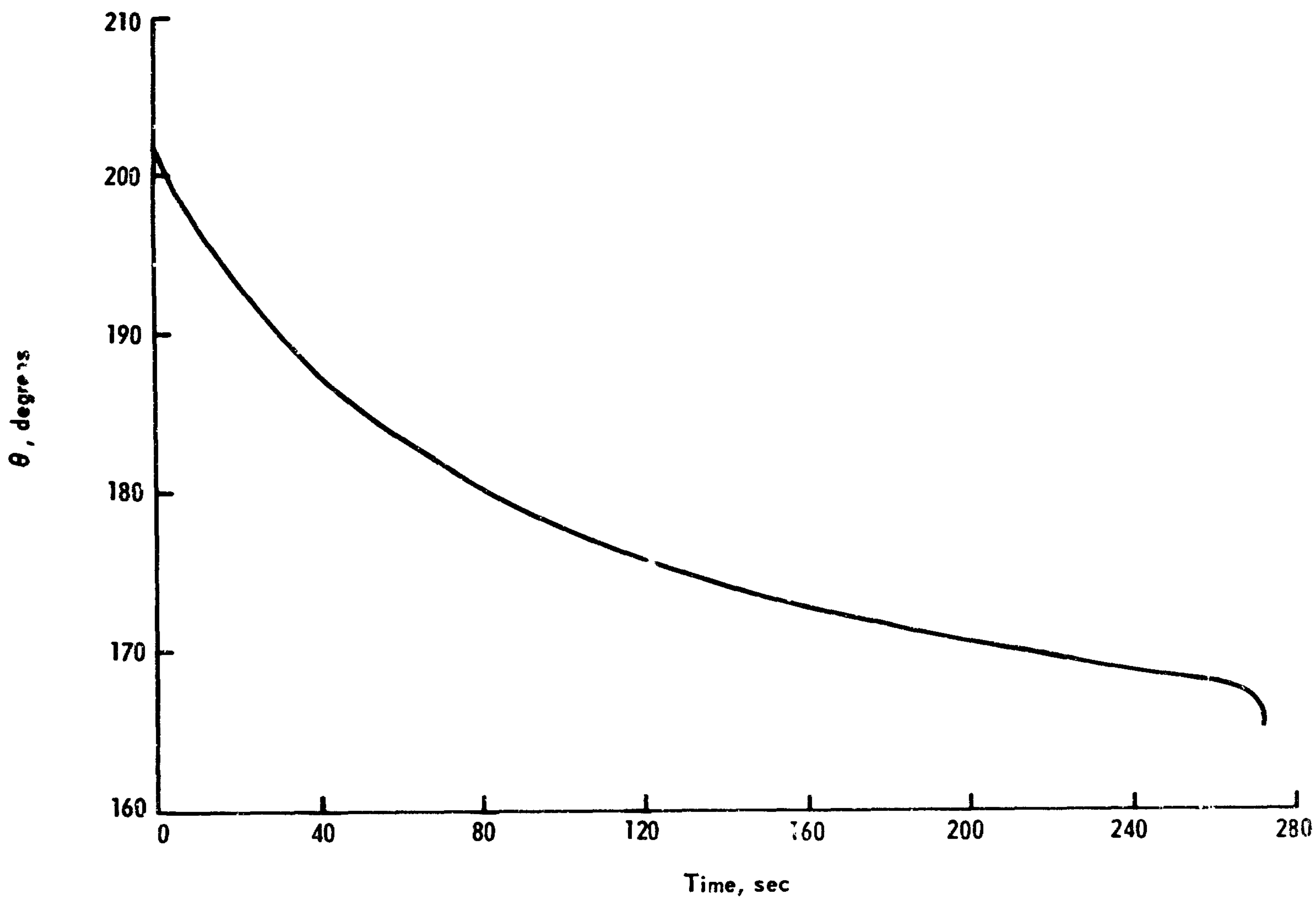


Fig 1: continued
(e) Thrust angle as a function of time

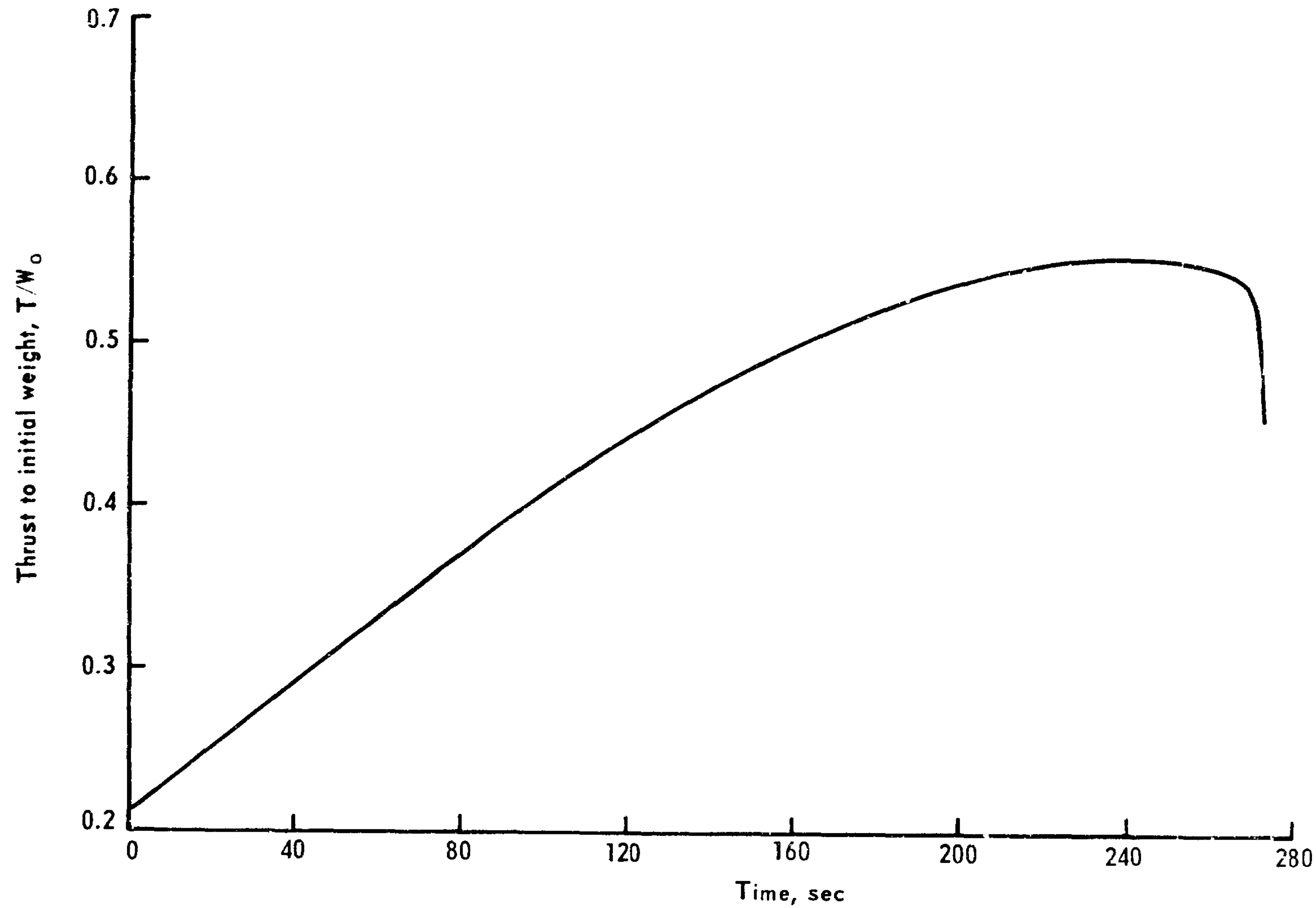


Fig 1: Concluded
(f) Thrust as a function of time

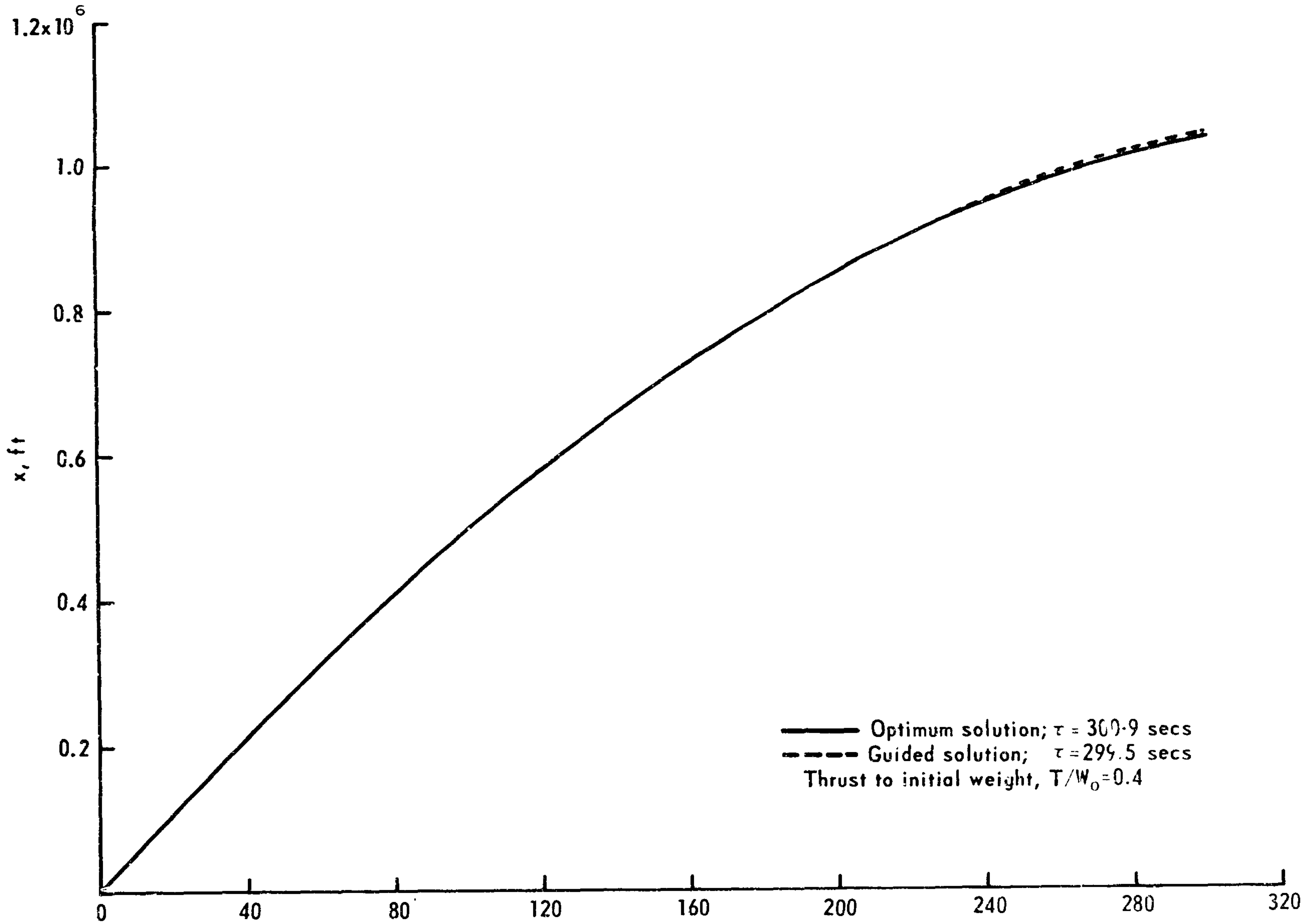


Fig 2: Comparison of optimum and guided constant thrust solutions
(a) Range as a function of time

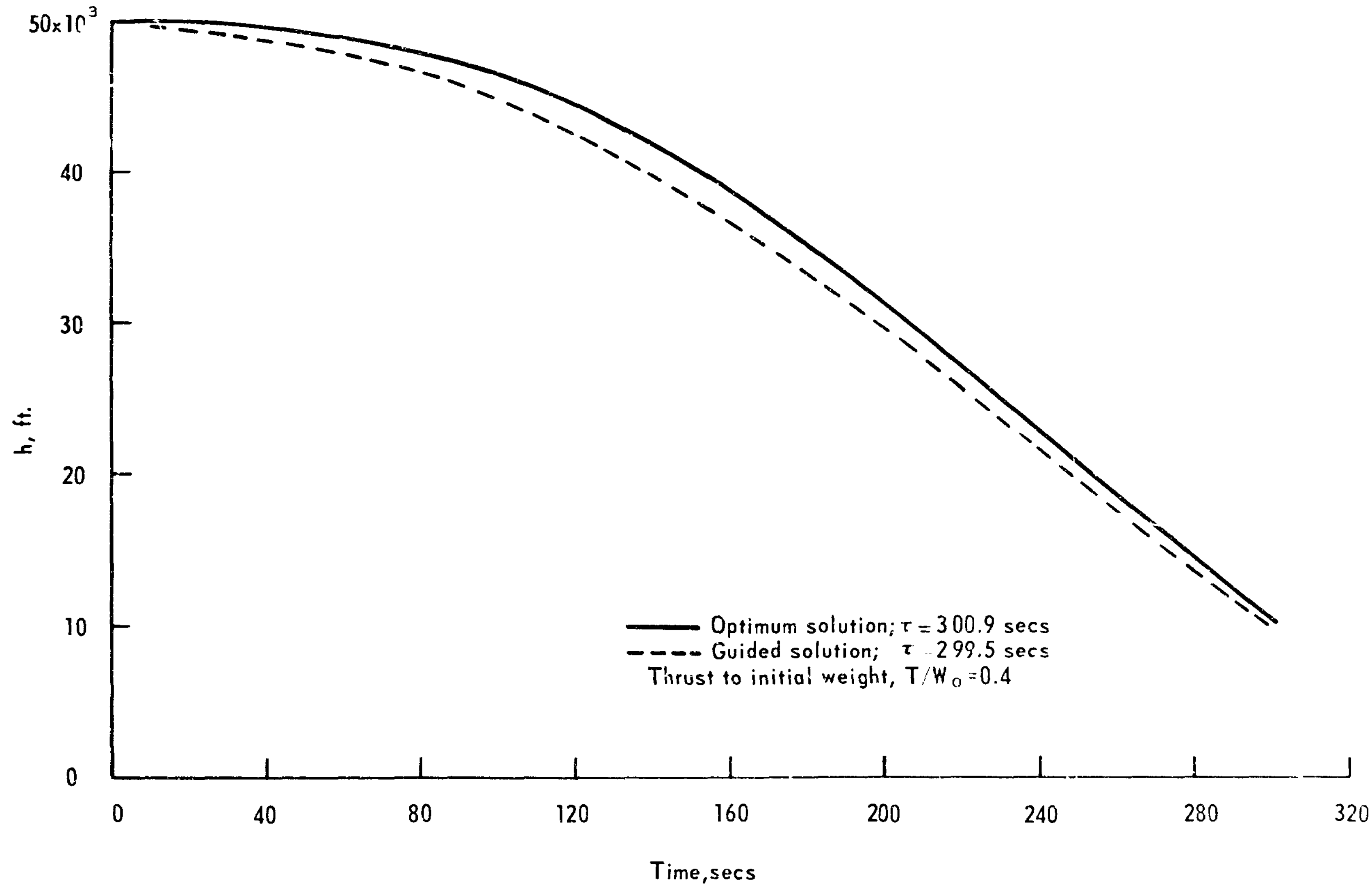


Fig 2: continued

(b) Altitude as a function of time

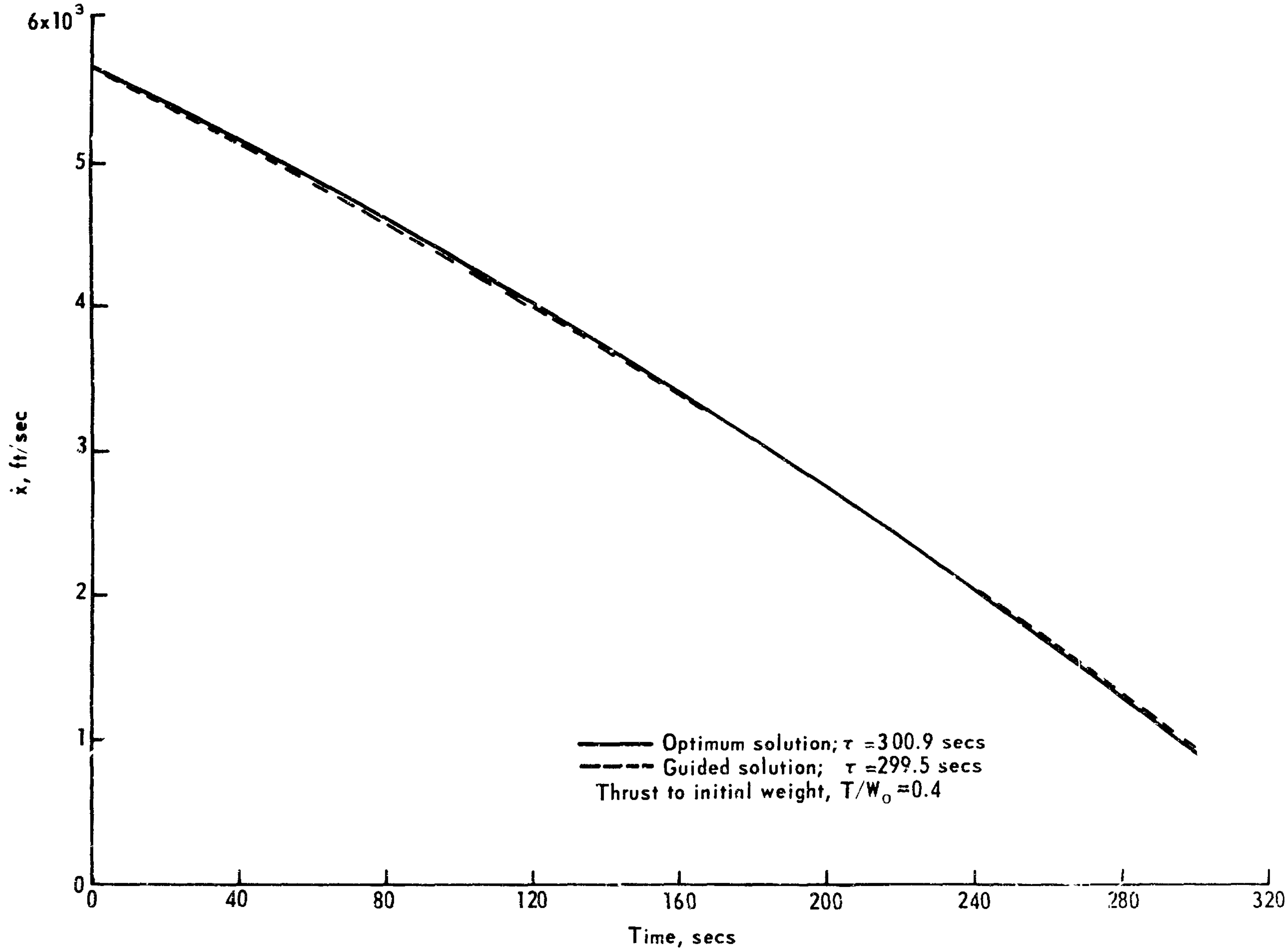


Fig 2: continued
(c) Horizontal velocity as a function of time

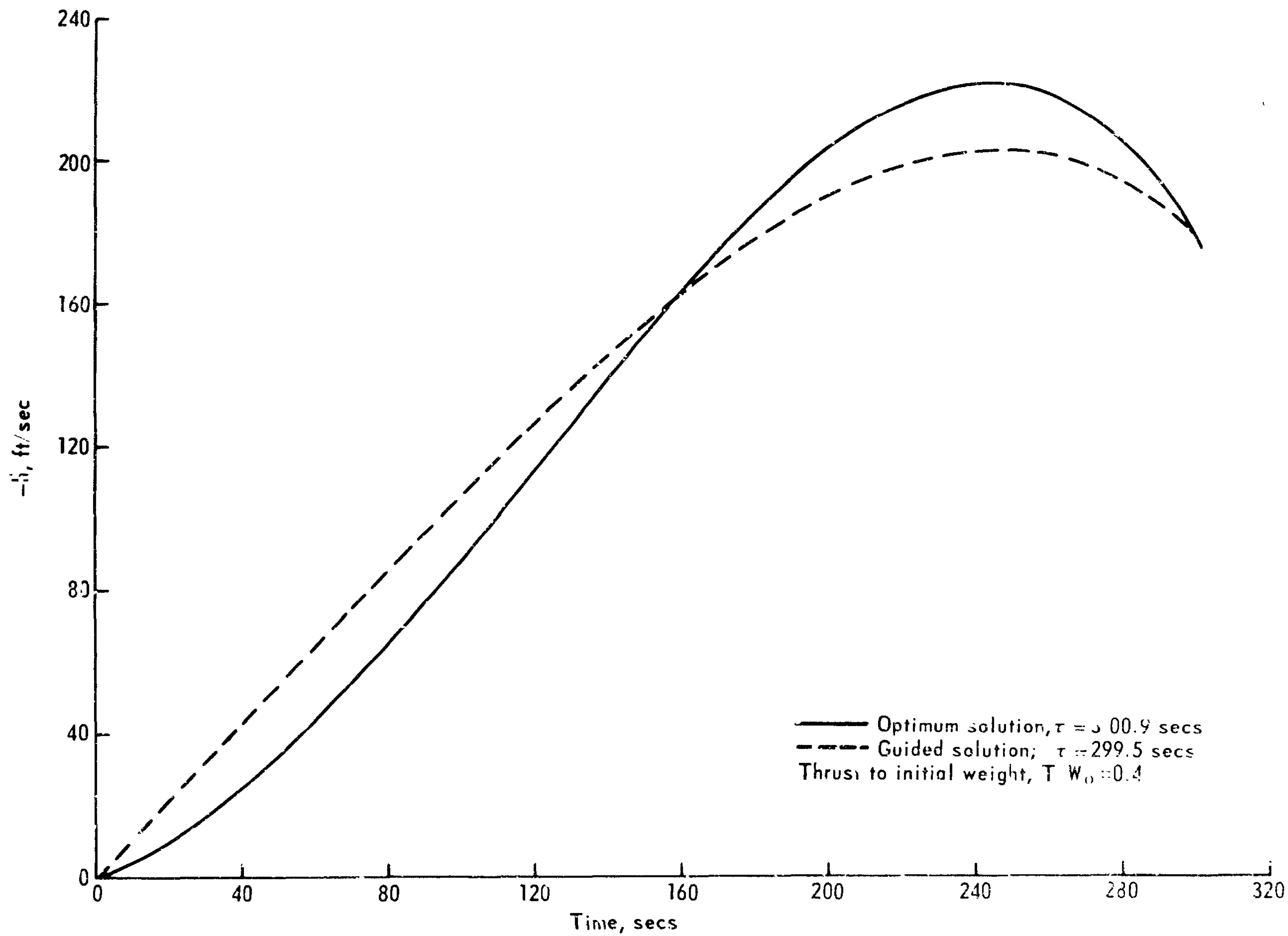


Fig 2: continued
(d) Vertical velocity as a function of time

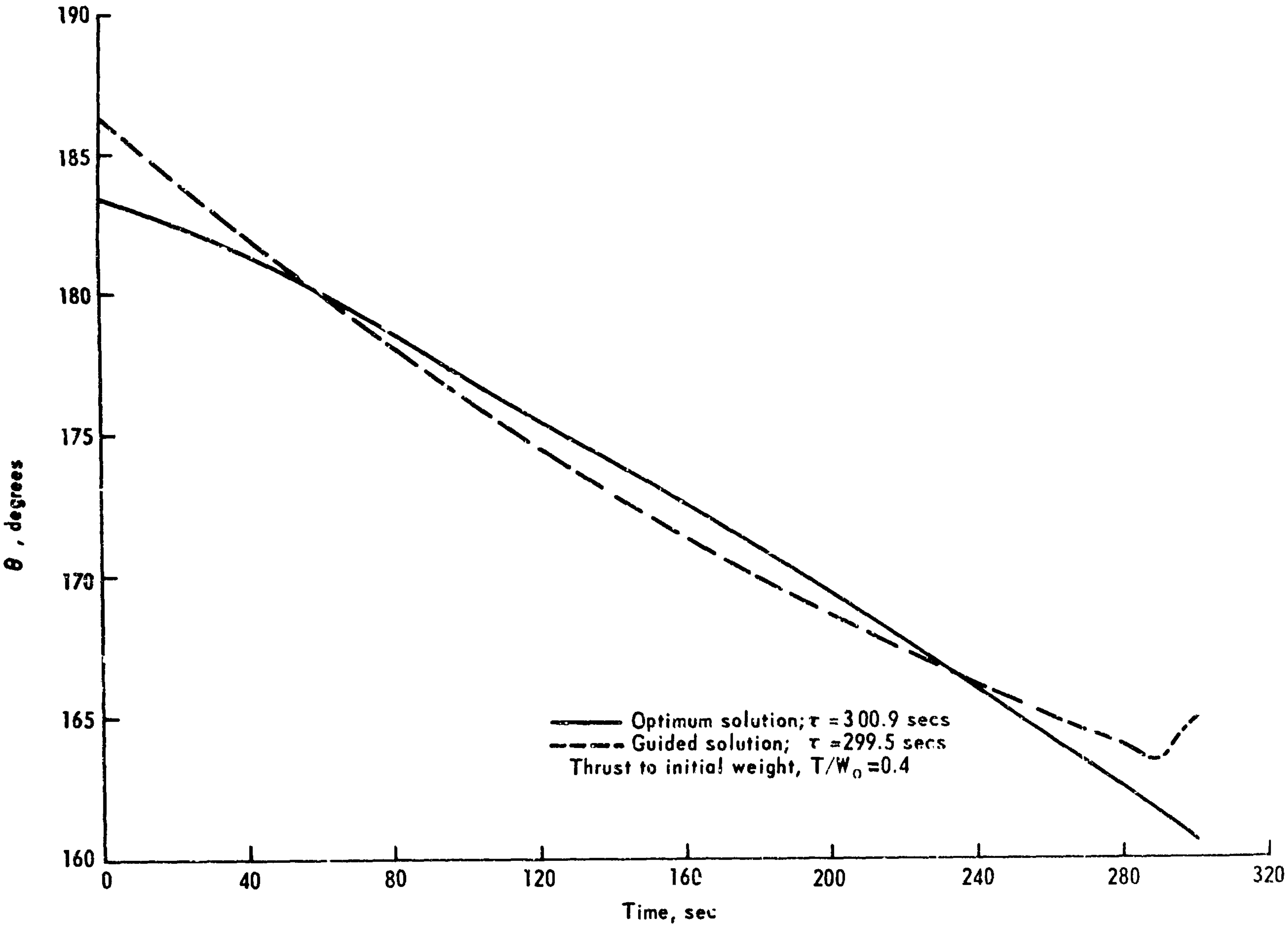


Fig 2: Concluded
(e) Thrust angle as a function of time