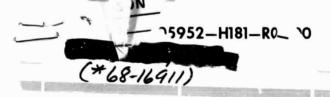# General Disclaimer

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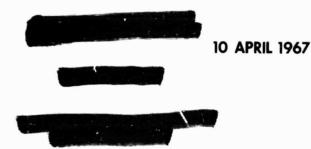
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10 APRIL 1967

COLLISION PROBABILITY
OF THE APOLLO SPACECRAFT
WITH OBJECTS IN EARTH ORBIT

Prepared by
Florida Operations
TRW Systems

MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

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By Florida Operations, TRW Systems

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MISSION PLANNING AND ANALYSIS DIVISION
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ABSTRACT

This report contains a discussion of two methods, and the results obtained by using these two methods, for computing the estimated collision probability for an Apollo spacecraft mission.

The first method utilizes the output of NORAD's COMBO program, a program that generates satellite position as a function of time. The appropriate output (closest approach distances of satellites relative to the Apollo spacecraft) is used as input to a process for calculating collision probability. In this method, the collision probability associated with each satellite is obtained by assuming a miss uncertainty of the satellite relative to the Apollo, as determined from the Rayleigh probability distribution.

In the second method, NORAD's Element Summary (Ephemeris data on all earth satellites) is utilized as input to a program that obtains the average collision probability over a relatively long period of mission duration time. Linear (i.e., proportional) scaling of both the number of pertinent satellites and the mission time duration is sufficient to modify the results of this program to account for differences in these factors.

The results of these two programs are in very good agreement. The 12-day, 600-mile altitude mission had a collision probability of 1.77×10^{-5} using the first method and 3.07×10^{-5} using the second method.

CONTENTS

TABLES

1. INTRODUCTION AND SUMMARY

1.1 TASK PURPOSE AND SCOPE

The task purpose and objectives, as outlined in Reference 1, are for convenience listed here in condensed form. There are, of course, deviations from this planned procedure. These changes came about when it was determined that an alternate approach was superior, particularly in regard to effecting a significant reduction in computer simulation time.

The purpose of Task MSC/TRW A-106 was as follows:

(1) to develop a mathematical model and procedure for estimating the collision probability between the Apollo spacecraft and all other earth orbiting satellites, and

(2) to estimate this collision probability for a hypothetical (worst case) Apollo spacecraft mission via the aforementioned procedure developed in (1) above.

This document defines a reference Apollo spacecraft mission and its relationship to the actual expected Apollo spacecraft missions. The reference Apollo spacecraft mission was chosen to be a reasonable "worst case", and therefore the collision probability obtained is a realistically pessimistic estimate of the actual.

Detailed descriptions are included on two methods that were developed for computing the expected collision probability associated with this "worst case" Apollo spacecraft mission. These two methods are the following:

(i) an approximate method utilizing NORAD COMBO data, and

(ii) a pseudo-collision method.

1.2 SUMMARY OF RESULTS

1.2.1 Methods for Computing Collision Probability

Two methods have been developed for calculating Apollo collision probability and are presented in this report.

The first method is an approximate method which makes use of the NORAD-originated COMBO program output (i.e., the closest approach distances). For each near miss (or closest j^{th} approach distance), the jth collision probability is calculated via use of a modified procedure developed from assuming a Rayleigh-distributed miss. (A Rayleigh probability distribution is a bi-variate Gaussian with equal sigmas, and zero correlation, of the two variates.) This process continues for all of the COMBO-listed satellites and the final mission collision probability is calculated therefrom. To obtain the COMBO data, of course, requires that NORAD personnel be given the Apollo spacecraft trajectory, orbital parameters and time of launch. The collision probability for a hypothetical Apollo spacecraft mission is contained in this report (and also contained in Reference 2). The hypothetical Apollo spacecraft mission assumed a "worst" altitude height of 600 statute miles in that a near maximum number of sightings were recorded for a 12-day Apollo mission.

The second method is a pseudo-collision method which assumes a random launch time and which approximates the geometric relationship between a pair of satellites by accounting for the secular changes in the orbital parameters due to the oblate spheroid (earth). Secular changes due to drag are accounted for via estimating the lifetimes of the satellites. This method is based upon the knowledge that the geometrical relationship between two orbits is primarily dependent upon their Right Ascension and Argument of Perigee angles. For a given period of time, these angles will orient the orbit pair such as to cause a number of coincidences ("intersections") of the two satellite paths. During the time interval when the two trajectory paths are in close proximity to each other (i.e.,

2

in coincidence), the jth satellite will make a number of passes through this portion of space. The total number of passes is modified (reduced) by a factor ("beat frequency" factor) to account for the effects of phasing of the jth satellite relative to the Apollo spacecraft, and the resulting number is multiplied by the probability of the Apollo spacecraft's being at the intersection when the jth satellite passes through.

The NORAD Element Summary dated 30 September 1966 was used as input for both methods.

1.2.2 Collision Probability

The collision probabilities obtained from the application of the two methods to the Apollo spacecraft reference mission (viz., a 12-day mission with orbital altitude of about 600 statute miles) are as follows:

(i) 1.77×10^{-5} for the approximate method utilizing NORAD COMBO data and a specific Apollo booster launch time, and

(ii) 3.07×10^{-5} for the pseudo collision method (which assumes a random launch time).

The pseudo collision method also gives the following collision probability for a 12-day, 400-statute mile orbital altitude mission (a "really worst" case) of the Apollo spacecraft:

$$3.68 \times 10^{-5}.$$

It is concluded that these collision hazards are very low. If the phasing of the satellites relative to the Apollo spacecraft is such as to lead to a maximum number of close passes, then the Apollo spacecraft would be subjected (during a 12-day, 400-statute mile orbital altitude mission) to a collision probability of about twice the estimates given above. This upper limit to the collision probability is still very low.

The above estimates of collision probabilities, as obtained from the two methods, are in excellent agreement, especially when one considers the inherent uncertainties involved in the estimation of phenomena of this type. Both methods are considered valid, and the usage of the more pessimistic (viz., the pseudo collision method and its results) is recommended.

The above collision hazards are to be interpreted as follows:

> In 100,000 Apollo spacecraft missions of 12 days duration at a 400-statute mile orbital height, the number of collisions expected would be approximately three or four; that is, an average of one collision would be expected in every 27,000 such missions.

1.2.3 Universality and Longevity of Results

Methods which are easy to use are provided in this eport whereby the results given herein can be appropriately scaled to account for the following: different mission durations; future changes (i.e., updates) in the number of satellites which bracket the orbital altitude of the mission under consideration; and different dimensions of the spacecraft under consideration.

It is recommended that the pseudo collision program be rerun in approximately three to five years in order to update the collision hazards resulting from earth orbiting satellites.

2. METHOD OF APPROACH

2.1 SIMULATION OF PROBLEM

A theoretically desirable approach to estimating the collision probability of satellites with respect to the Apollo spacecraft is to mathematically model the physical problem and to exercise this model sufficiently until all variables are taken into account (that is, simulate the collision occurrences).

A difficulty arises in that, when simulating only one satellite in motion, the amount of computation time increases in proportion to the complexity of the mathematics involved. For example, calculation of the "average" orbital position is relatively easy when only first order secular variations are incorporated in the model, but a numerical integration technique is required when accounting for the addition of both short and long period oscillatory variations.

Also, the amount of computation time increases in proportion to the number of satellites.

It is therefore necessary to consider a relatively simple scheme, one that will approximate the physical problem sufficiently to provide valid results. The original intent of this study was to find this acceptable simulation scheme and generate equivalent collision probabilities.

2.2 EPHEMERIS GENERATION (APPROXIMATE COLLISION PROBABILITY METHOD)

A valid approach is to utilize the actual catalogue (data bank) of satellite Element Summary data, choose a reference orbit for the Apollo spacecraft and simulate the Apollo spacecraft and other satellites for a predetermined time duration. This is done with the NORAD COMBO program.

An alternate to the same approach involves the following: forming histograms of the classical parameters from the catalogue of satellite orbits; sampling from these histograms of parameters to obtain a large number of pseudo satellites (Monte Carlo); and simulating the mission duration of the Apollo spacecraft and these satellites in a similar manner as does COMBO. If the histograms of the classical parameters remain the same shape (except for the total number of satellites), then the latter approach is essentially equivalent to the present COMBO. This latter approach has the advantage of accounting for a different total number of satellites (say, for one year hence). In either case, upon considering the "envelope" of variations of all of the orbital parameters for the Apollo spacecraft, a one-mission simulation via the present COMBO is much more desirable. That is, the placing of the desired confidence limits on the collision probability to appropriately account for the envelope of Apollo spacecraft missions may require that there be several times as many simulated Apollo spacecraft missions as there are actually planned.

The COMBO runs by NORAD simulated the Apollo spacecraft mission in the manner as outlined above. The output data which is the close approach distances of satellites with respect to the Apollo spacecraft was utilized as input to a computation process that is outlined in Appendix I (APPROXIMATE COLLISION PROBABILITY METHOD). Appendix I also contains the collision probability results from a COMBO run.

2.3 PSEUDO COLLISION METHOD

In light of the aforementioned considerations, it appears that if the physical problem is reasonably constrained such as to produce an acceptable simulation and at the same time reduce computational effort, there can be provided a valid estimate of collision probability both now and also on a continuing basis in the future.

The pseudo collision method is a modification to (or development from) the concept of mean time to collision. If τ_o is the mean time to collision and t* is the mission duration time, then the collision probability P_c is as follows:

$$P_c = 1 - e^{-t*/\tau_o} \; .$$

An initial examination was made to determine the computer simulation time for a mean time to collision program. An excessive amount of computer time was required. The pseudo collision method was then developed. Appendix III contains the analytic proof of the applicability of the pseudo collision method.

In the solution of the problem, some major decisions were made; these are described as follows:

(1)   In all of the methods considered, the Apollo spacecraft orbit was assumed to be circular. This assumption is compatible with some planned Apollo spacecraft missions, and it permits considerable simplification to the mathematics of keeping track of the Apollo spacecraft and the large number of satellites. In addition, this assumption simplified the parametric study of the Apollo spacecraft relative to the catalogue of satellites in that only two parameters (namely, Apollo spacecraft orbit height and orbit inclination angle) need be the variables which must be considered. Actually, the circular orbit assumption provides pessimistic results (i.e., larger collision probabilities).

(2)   It was decided that any mathematical model should simulate the physical event over a relatively long period of time (i.e., mission duration). This would provide an averaging effect, and hence would eliminate such questions

as the phasing (i.e., launch time) effects on the
Apollo spacecraft collision probability. Another
point of concern is that the effect of parameters
such as the angles, Right Ascension and Argument
of Perigee, would thereby be averaged out.

(3) The mathematics should be as simple as possible
(thus enabling fast and economic computation of
the problem). Therefore, it was decided that the
changing geometry would reflect only the first
order secular effects due to the oblate spheroid
(earth). Secular effects, due to atmospheric drag,
are assessed separately. It is recognized that in
any ephemeris-generating programs, these two effects
must be handled together (not separately). With the
scheme as outlined above, a separate accounting gives
negligible error.

Appendix II contains the description of the pseudo collision pro-
gram along with the results for Apollo spacecraft hypothetical orbits
at 200, 400, 600, 800, and 1,000 statute miles altitude. The incli-
nation angle employed is 30 degrees.

2.4 SUPPORTING ANALYSIS

Several important areas required investigation and, at least, a
modicum of analysis. These are presented in references 6 through 11.

Breakup of satellites because of explosion, etc., has been examined.
This was done to provide an estimate of the number of untrackable pieces.
The sizes of the pieces due to fragmentation, based on analysis of similar
phenomena, appear to follow a negative exponential probability distribution.
The analysis and supporting data appear in Reference 6. The analysis indi-
cates that the number of untrackable pieces does not significantly increase
the collision hazards.

Reference 7 contains the listing of the maximum dimension of the known satellites and, if available, the maximum dimension of the associated orbiting boosters. This listing was analyzed in order to determine a representative dimension of these orbiting objects, since the collision probability depends on the dimensions of both the Apollo spacecraft and other orbiting objects. In fact, the collision probability varies directly as the square of the sum of the representative lengths of the Apollo spacecraft and the jth satellite. A representative maximum satellite dimension and the length (i.e., the longest of the dimensions) of the Apollo were utilized in the calculations of collision probability. This eliminated extensive calculations and thereby provided a realistically pessimistic estimate of collision probability.

The lifetime of the satellites in earth orbit is dependent upon, among other things, the ballistic coefficient of the satellite and the perigee and apogee of the orbit. Average lifetimes of satellites were investigated by utilizing the Satellite Situation Report data (Reference 5).

Reference 8 contains the analysis of the histograms of the following classical parameters--semi-major axis (a), eccentricity (e), and inclination angle (i)--for the historical data abstracted from the NASA Satellite Situation Reports. In addition, the histograms of the above parameters plus those for Right Ascension (Ω) and Argument of Perigee (ω) are also included for the NORAD Element Summary dated 30 September 1966.

Reference 9 contains scatter diagrams and correlation coefficients of all combinations of the classical orbital parameters from the NORAD Element Summary data and includes an analysis and discussion of these scatter diagrams.

This supporting analysis was completed in preparation for an originally planned Monte Carlo program and is also pertinent to the analyses employed herein. The supporting analysis does show that the histograms of the classical parameters remain relatively unchanged (i.e., stationary) with respect to calendar time.

An early answer to the collision probability was presented in Reference 2. This paper contained a preliminary estimate of the collision probability. Additionally, it contained a parametric study of variations in the accuracy of NORAD tracking and also included the worst case, viz., that all close misses of satellites were measured by the tracker to be at a zero distance from Apollo. In other words, the NORAD-estimated uncertainty was utilized to compute collision probability assuming that one was trying to hit each close approach satellite with the Apollo spacecraft and furthermore that the Apollo spacecraft launch was exactly on time and that the Apollo spacecraft guidance errors were zero.

2.5 PREDICTION PROCEDURE

A prediction scheme which would employ the predicted number of satellites in the future to estimate the corresponding collision probability is now discussed. Histograms of classical parameters (see Reference 8) were examined. This examination showed that, in general, except for the total number of satellites, the histograms (i.e., frequency distributions) have retained essentially the same shape over the last two years. However, on further examination, if one is concerned about low altitude manned satellites, the predictability of satellite density is not very good. For example, satellites with low perigee (100 to 250 statute miles) tend to decay very rapidly. Hence, above about 250 statute miles the probability distributions of the classical orbital parameters can be assumed to be essentially stationary (i.e., unchanging) with respect to the calendar time for, at least, the next two years. Table 2.1 shows the stability (i.e., stationarity) over a two-year period (extending from 31 May 1964 to 30 June 1966) of the relative numbers of satellites which would bracket orbital heights above approximately 250 statute miles. For example, 450 statute miles is always bracketed by the largest number of satellites during the entire time period, 400 statute miles is always bracketed by the 2nd highest number of satellites, and 500 statute miles is always bracketed by the 3rd highest number of satellites.

Table 2.1 Summary of the Number of Satellites Which
Bracket Various Heights

Satellite ** Situation Report Date	HEIGHT (STATUTE MILES)								
	100	150	200	250	300	350	400	450	500
31 May 1964	0	8	21	31	40	48	83	88	73
28 Feb 1965	* 24	60	73	67	68	84	122	132	114
30 Jun 1965	0	20	55	71	89	121	155	156	147
31 Jan 1966	0	22	31	37	63	88	134	142	137
30 Jun 1966	0	23	43	54	81	134	189	204	189

* Cosmos 57 - 1-month life average.
** These data are from GSFC "Satellite Situation Reports" (Reference 5).

Table 2.2 Summary of NORAD Data, Satellites Which
Bracket Various Heights

Element Summary Date	HEIGHT (STATUTE MILES)								
	100	150	200	250	300	350	400	450	500
30 Sept 1966	23	64	115	151	221	330	426	531	451

An examination of the number of satellites whose perigees (HP's) and apogees (HA's) bracket a specified height (i.e., HP ≤ radius of earth plus specified height ≤ HA) shows a rather large fluctuation in numbers in the 100 to 250 statute mile altitudes (see Table 2.1). It appears that the optimum method of assessing the numbers and altitude distribution of satellites is to make use of the NASA-generated "Satellite Situation Reports" (cf. Reference 5). The Goddard Space Flight Center publishes on a bi-weekly basis "Satellite Situation Reports." These reports contain among other items, the catalogue satellite number, height of apogee (km), and height of perigee (km) for a large number of satellites.

A current count of satellites that bracket a specific reference height may be obtained from these reports. All satellites, unfortunately, are not listed; for example, 1961 Omicron has roughly 210 pieces not listed.

Once a current count is obtained, a linear extrapolation (i.e., proportional scaling) may be made to obtain the collision probability at the altitude in question.

For example, if an updated collision probability were required for a 400-statute mile orbital height and a 30-day mission for a spacecraft, and if there were, say, 365 satellites which bracket this height rather than the number listed in Table 2.2, then we would proceed as follows:

$$P_c = k_1 k_2 \left[\overline{P_c (400 \text{ sm})} \right],$$

where

$$sm \equiv \text{statute miles,}$$

$$P_c(400 \text{ sm}) = 11.16 \times 10^{-4} \text{ for a "one-year" mission duration,}$$
$$\text{(Appendix II, Table II.1),}$$

$$k_1 = \frac{30}{365} = \frac{\text{mission duration in days}}{\text{calendar days in one year}},$$

and

$$k_2 = \frac{365}{426} = \frac{\text{updated number of satellites at } h = 400 \text{ sm}}{\text{number of satellites at } h = 400 \text{ sm (from Table 2.2).}}$$

Therefore, the updated collision probability for a 30-day, 400-statute mile altitude orbit is:

$$P_c = 7.86 \times 10^{-5}.$$

As stated above, the updated number of satellites may be obtained from the GSFC Satellite Situation Reports. The number of satellites which are predicted to be in earth orbit at some future date could be estimated at the present time by deletion of those which are predicted to decay in the interim and by addition of those which are scheduled to be launched, making proper allowance for the following: predicted alterations in the launch schedule, the reliability of the launch vehicles, and future fragment-generating explosions of satellites while in earth orbit.

2.6 OTHER CONSIDERATIONS

Both methods calculated collision probability assuming a combined dimension of the Apollo spacecraft and satellite of 125 feet; the resulting collision probabilities are a function of the square of this dimension, i.e., $(125)^2$ ft^2. If one assumes a random orientation for the Apollo spacecraft and an average representative dimension of about 15 feet for all satellites, then the collision probabilities would be smaller by a factor of about 10 (i.e., the ratio of the assumed combined dimensions squared).

Initial computer runs were made assuming inclination angles of 30,40, and 50 degrees. The coincidence counts for the various altitudes at the 40- and 50-degree inclination angles differed at most by 3%, from those obtained at a 30-degree inclination.

The collision probability associated with the transfer trajectory (i.e., up and down legs of the trajectory) may be approximated by proportionately scaling by the flight times for these portions of the trajectory. The collision probability during transfer maneuvers is therefore infinitisimal compared to the mission (i.e., the duration in constant orbit).collision probability.

3. MAJOR RESULTS

3.1 ALTITUDE OF MAXIMUM NUMBER OF SATELLITES

The desire throughout the analysis was to use a realistically worst case for the orbital altitude of the Apollo spacecraft, even though it was realized that, in general, the planned Apollo spacecraft orbital altitudes would be different from this realistically worst case. For example, much of the emphasis in this analysis has been accorded to Apollo orbital altitudes of 400- and 600-statute miles, whereas there are orbital altitudes from 100- to 200-statute miles planned for Apollo spacecraft.

In regard to utilizing a worst case, the original COMBO I run by NORAD was, in fact, run with an Apollo orbital period of 105 minutes (which corresponds to an altitude of about 600 statute miles), because conversations with NORAD personnel indicated that an approximate maximum number of satellite orbits existed at this altitude.

Subsequent thereto, a special computer program was run as a preliminary to the detailed analysis of the pseudo collision method in order to precisely determine the altitude(s) at which the maximum number of satellites were concentrated. This program counted the number of satellites that satisfied the following equation:

$$HP \leq r_e + k\,\Delta h \leq HA,$$

where

$HP \equiv$ perigee height of satellite (statute miles),

$HA \equiv$ apogee height of satellite (statute miles),

$r_e \equiv$ radius of earth (statute miles),

$\Delta h = 50$ (statute miles),

and

$k = 1, 2, \ldots, 15.$

It was found that the count of the number of satellites was con-
centrated, as a maximum, at an altitude of 450 statute miles both for
those satellites listed in Reference 5 and also for the NORAD Element
Summary, dated 30 September 1966. It was also found that a near
maximum number of satellites were concentrated at an altitude of 400
statute miles.

As indicated above, the results listed in Appendix I (approximate
method using NORAD COMBO data) are for an equivalent altitude of
approximately 600 statute miles. The results shown in Appendix II
(pseudo collision method) are listed at 200-statute mile intervals,
extending from an altitude of 200 statute miles up to a maximum altitude
of 1,000 statute miles.

3.2   COLLISION PROBABILITY METHODS AND RESULTS

Two valid (but different) methods for obtaining collision prob-
abilities of the Apollo spacecraft with other objects in earth orbit
have been developed and used in this analysis.

The results of the two methods for computing collision probability
are also contained in Appendices I and II. In summary, the results
obtained using a 30-degree inclination angle and a 105-minute (approximate)
period for Apollo are given below.

From Appendix II (pseudo collision method), the "one-year" mission
collision probability (P_c) is:

$$P_c \approx 9.35 \times 10^{-4};$$

and the equivalent 12-day mission probability is:

$$P_c \text{ (12 days)} \approx (\frac{12}{365}) (9.35 \times 10^{-4})$$
$$= 3.07 \times 10^{-5}.$$

From Appendix I (approximate method using NORAD COMBO I data), the collision probability assuming a NORAD-tracking precision of $\sigma = 25,000$ ft is:

$$P_c = 1.77 \times 10^{-5}.$$

The ratio of these two estimates for a 12-day mission is about 1.7, which indicates a close agreement between the two methods. For example, upon examining the effects of σ on the latter estimate (see Reference 11), one can infer the two estimates are indeed compatible. That is, a larger σ than that assumed herein will bring these estimates into even better agreement. Also, an ensemble of different "launch" times for the Apollo spacecraft may tend to do the same (i.e., several NORAD simulations using different phasing of the Apollo spacecraft relative to the satellites).

3.3 MAXIMUM COLLISION PROBABILITY

The maximum collision probability (see Table II.1, Appendix II) may be taken as 1.116×10^{-3} (at an orbital altitude of 400 statute miles) per mission year. A linear interpolation (i.e., scaling factor) to account for actual mission duration is required. For comparison, the collision probability for a 12-day, 400-statute mile mission is:

$$\left(\frac{12}{365}\right) \times 1.116 \times 10^{-3} = 3.68 \times 10^{-5}.$$

3.4 EFFECT OF INCLINATION OF APOLLO

Several pseudo-collision runs were made wherein the inclination angle (i) of Apollo was varied. The total numbers of coincidence (intersections) experienced varied no more than 3% for inclinations from 30 to 50 degrees (cf. Reference 10).

3.5 UNTRACKABLES

An analysis indicated that the number of untrackable fragments, which result from explosions of satellites in earth orbit and whose radar cross

section areas are too small to be tracked by NORAD, constitutes an insignificant increase in the total number of objects in earth orbit and hence can be neglected in the calculation of collision probability (cf. Reference 6).

3.6 STATIONARITY OF CLASSICAL ORBITAL PARAMETERS

It was found that, in general, the probability distributions of the classical orbital parameters remained essentially unchanged (i.e., stationary) with respect to calendar time. The time period studied was about two years in length (cf. Reference 8). The exception to stationarity occurs at the lower altitudes (250 statute miles and less).

4. INTERPRETATION OF RESULTS, CONCLUSIONS AND RECOMMENDATIONS

4.1 COMPARISON OF THE TWO METHODS

The collision probability results which were obtained from the two methods are indeed compatible; in fact, when one considers the inherent uncertainties involved in the estimation of phenomena of this type, it is concluded that the results from the two methods are in close agreement and that both methods will provide valid predictions of collision probability and hence of the expectation of collision for a given mission duration.

It is recommended that the results given herein for the pseudo collision method be used, since these are the more conservative (i.e., the more pessimistic) and are based upon a random launch time (i.e., a launch time which follows a rectangular probability distribution).

It is also recommended that, in general, the pseudo collision method be used, in any future application, in preference to the approximate method. The exception to this recommendation occurs when it is desirable to estimate collision hazards just after an explosion of a satellite and just before the orbital parameters have stabilized.

In the future, say approximately three to five years, it is therefore recommended that the pseudo collision program be rerun in order to update the collision hazards from earth orbiting satellites, due to the possibility of a significant change in the form of the probability distribution of the classical orbital parameters.

4.2 REALISTICALLY PESSIMISTIC COLLISION HAZARDS

The in-orbit collision hazards of the Apollo spacecraft, as given in Sections 3.2 and 3.3 for the pseudo collision method, are concluded to be realistically pessimistic and valid estimates. These estimates are as follows:

$$P_c = 3.07 \times 10^{-5} \text{ for a 12-day, 600-statute mile}$$
$$\text{altitude mission,}$$

$$P_c = 3.68 \times 10^{-5} \text{ for a 12-day, 400-statute mile}$$
$$\text{altitude mission.}$$

These collision probabilities can be correctly interpreted as follows:

> In 100,000 Apollo spacecraft missions of 12 days
> duration at a 400-statute mile orbital altitude,
> the number of collisions expected would be approxi-
> mately three or four; that is, an average of one
> collision would be expected every 27,000 such missions.

> The in-orbit collision hazards are thus concluded to be
> very low.

4.3 UPPER LIMIT TO COLLISION HAZARDS

If NORAD-tracking errors were zero, if the Apollo spacecraft trajectory error were zero, and if the Apollo spacecraft were launched exactly on time, the collision probability would be either zero or one. That is, the collision phenomena would then be purely deterministic (vis-a-vis probabilistic), even prior to launch time. The graph of collision probability versus launch time would therefore be a curve which would, in general, be equal to zero but which would every now and then, for a stretch of a few milliseconds, be equal to 1.0. The existence of NORAD-tracking errors, Apollo spacecraft trajectory errors, and launch time delays (even for a tenth of a second or so) make the collision phenomena a probabilistic event. Therefore, the maximum collision probability of 1.0 (if a zero-error Apollo space-craft were actually targeted for collision with a perfectly-tracked satel-lite) very rapidly decreases if trajectory (steering) errors are introduced and the uncertainty in position of the target satellite is admitted. Examination of the results of Appendix I and References 10 and 11 shows

that the upper limit to the collision probability (as a result of the initial geometrical relationships) is only about two times those estimated in Sections 1.2 and 3 (for a 12-day mission); and hence the upper limit to the collision hazards is also very low. For longer missions, the collision probabilities are those obtained in Appendix II properly scaled.

4.4 UTILIZATION OF SATELLITE SITUATION REPORTS

These reports, which are published semi-monthly by Goddard Space Flight Center, are useful in updating the collision probabilities. One deficiency exists in that all pieces of satellites, tankage, etc., are not listed.

4.5 EXPLODING SATELLITES

A satellite that has been fragmented by an explosion presents a problem in regard to application of the pseudo collision method. That is, immediately after this event occurs, the classical parameters--Right Ascension and Argument of Perigee--are highly correlated with one another and with those of inclination, etc.

Past history indicates that a few weeks (roughly a month or two) are required for these parts and pieces to distribute themselves such as to make the correlation insignificant.

Physically, it means that within a few hours after fragmentation, these pieces are concentrated around an average specific value of the classical parameters, i.e., in a group. Eventually, given enough time, these pieces distribute themselves in a band around the earth.

As an input to mission planning and launch scheduling, it is recommended that MSC request that NORAD inform them of the occurrences of, and pertinent data concerning, explosions.

4.6 DEGREE OF UNIVERSALITY OF RESULTS

The results given herein can be proportionally scaled with validity to account for different mission durations, future changes in the number of satellites which bracket the orbital height of the mission under consideration, and different dimensions of the spacecraft under consideration (by using the square of the ratio of the sum of the spacecraft and satellite dimensions). Also, the collision hazards associated with eccentric orbits can be assessed with validity by using the equivalent altitude of a circular orbit; the estimates of the hazards so resulting will be conservative (i.e., on the high side).

APPROXIMATE COLLISION PROBABILITY METHOD AND RESULTS

I.1 INTRODUCTION AND EXPLANATORY REMARKS

This section contains the equations for an approximate method (see Reference 11) for calculating the collision probability associated with any "reference" satellite mission. It also contains a sample set of calculations for a hypothetical Apollo spacecraft orbit. The only necessary data are the following: (a) NORAD-generated COMBO data or equivalent; and (b) a table of Gaussian (normal) ordinate values.

The reference mission assumes that the Apollo spacecraft is in a circular 105-minute period orbit (the height of the orbit is about 600 statute miles). The Apollo launch time was arbitrary, but specifically stated, and the inclination angle used was 30 degrees; the duration of the mission was 12 days; and the satellite catalogue employed was the NORAD Element Summary, dated 30 September 1966.

I.2 MISSION COLLISION PROBABILITY

The overall mission collision probability may be computed in a straightforward, although tedious, manner. If the probability of collision with the jth satellite is P_{c_j}, then the probability of miss is $1 - P_{c_j}$; and hence the probability of missing all (n) satellites is

$$\prod_{j=1}^{n} (1 - P_{c_j}),$$

since it is believed that these probabilities are independent. Finally, the probability of hitting at least one satellite is

$$P_c = 1 - \prod_{j=1}^{n} (1 - P_{c_j}).$$

Now if all $P_{c_j} \ll 1$, then the expression,

$$P_c \approx \sum_{j=1}^{n} P_{c_j} ,$$

will provide a good estimate of P_c . In general, the P_{c_j} are not equal,
so that each event probability must be computed individually.

To estimate the collision probability for each of the satellites, an
assumption about the uncertainties in the COMBO close approach distances is
required. That is, it is assumed that the uncertainties associated with
the three dimensions (coordinates) of these miss distances are Gaussian
(normal) with zero biases and equal variance, and are uncorrelated, in the
three dimensions. This assumption will apply to the position data of each
of the tracked satellites. NORAD personnel have indicated that equal vari-
ance, zero biases, and uncorrelated errors constitute a reasonable assump-
tion.

Now consider each close approach event. In a sufficiently small region
of space where Euclidean geometry holds, one can always compute a relative
velocity vector of the satellite with respect to the Apollo spacecraft, or
vice versa. Of more importance, a plane normal to this relative velocity
can always be found; and, as a consequence, the closest approach (relative)
vector will lie in this plane.

Utilizing the assumption concerning the position errors of each of
the satellites and the fact that the orientation of the coordinate system
is arbitrary, the tracking uncertainty of the position of the satellite
with respect to the Apollo spacecraft is tri-variate spherically normally
distributed; and consequently the projection of the uncertainty in any
arbitrary X, Y plane (and hence in the plane containing the closest
approach vector) is bi-variate circularly normally (Rayleigh) distributed.

The jth satellite collision probability can be computed by the expression,

$$P_{c_j} = P_{\Delta X} P_{\Delta Y} \, ,$$

where

$P_{\Delta X} \equiv$ the probability of the satellite's being within the collision range in an X-direction,

and

$P_{\Delta Y} \equiv$ the probability of the satellite's being within the collision range in a Y-direction.

The above has been shown to be (Reference 11):

$$P_{c_j} \approx \frac{(\Delta R)^2}{\sigma^2 \sqrt{2\pi}} \left[\psi(R_j') \right] \, ,$$

where

$\Delta R = \Delta Y = \Delta X =$ sum of the lengths of the Apollo spacecraft and the satellite (= 125 feet),

$\sigma \equiv$ uncertainty, in each coordinate, of NORAD satellite position data,

$R_j \equiv$ closest approach distance,

$R_j' \equiv \dfrac{R_j}{\sigma} = \dfrac{(X_j^2 + Y_j^2)^{\frac{1}{2}}}{\sigma} \, ,$

and $\psi(R_j') \equiv$ Gaussian ordinate value at R_j'.

Note that the factor $(\Delta R)^2 / \sigma^2 \sqrt{2\pi}$ is a constant.

I.3 RESULTS OF COMBO FIRST RUN

The original NORAD-generated COMBO data (COMBO I) simulated a 12-day Apollo spacecraft mission in a circular orbit of 600 statute miles altitude.

Utilizing the mathematical procedure listed above, the 12-day mission collision probability was

$$P_c = 1.77 \times 10^{-5} \text{ (see Table I.1).}$$

Table I.1 contains a count of all close-approach satellites with respect to the Apollo spacecraft. Column (1) is the mid-point of the close approach distances spanned by each cell, Column (2) is the normalized (i.e., standardized) mid-point of each cell, Column (3) is the Gaussian ordinate associated with the mid-point of each cell, Column (4) is the probability of collision for a single satellite in each cell, Column (5) is the number of satellites in each cell, Column (6) is the cell collision probability, and the summation of the entries in Column (6) is the mission collision probability.

TABLE 1.1
COLLISION PROBABILITY RESULTS FOR
σ = 25,000 FEET
(NORAD TRACKING SIGMA)

R_j (1) jth Interval Cell Midpoint (Kilometers)	R'_j (2) $\left[R_j\left(\frac{3280}{\sigma}\right)\right]$	$\psi(R'_j)$ (3) Gaussian Ordinate Value at R'_j	P_{c_j} (4) Note 1 $\left[C\,\psi(R'_j)\right]$	n_j (5) Number of Satellites In Interval	$n_j\,P_{c_j}$ (6) Collision Probability for Interval
5	0.656	.321709	0.32×10^{-5}	1	0.32×10^{-5} (Note 2)
15	1.968	.057529	0.57×10^{-6}	25	0.14×10^{-4}
25	3.280	.001839	0.18×10^{-7}	50	0.54×10^{-6}
35	4.592	$.1052 \times 10^{-4}$	0.11×10^{-9}	32	0.35×10^{-8}

Mission Collision Probability

$$P_c \approx \sum_{j=1}^{4} n_j\,P_{c_j} \approx 1.77 \times 10^{-5}$$

Note 1: $C \equiv (\Delta R)^2/\sigma^2 \sqrt{2\pi} = 1.0 \times 10^{-5}$.

Note 2: For comparison, the equivalent bi-variate normal (vis-a-vis Rayleigh) calculation is the following:

P[16,400 ft. $\leq$ x $\leq$ 16,525 ft.] $\cdot$ P[-62.5 ft. $\leq$ Y $\leq$ 62.5 ft.] = (0.0016)(0.0020) = 0.32×10^{-5}, which exactly agrees.

1.4 NORAD COMBO SECOND RUN

A second COMBO run (called COMBO II) was requested so as to rephase the Apollo spacecraft by approximately 30 minutes (later) with respect to the same NORAD data bank of satellites. The Apollo inclination of 30 degrees and a circular orbit with a 105-minute period were again assumed.

A tabulation of the number of satellites with close approach distance d, where $0 \leq d \leq 40$ km, is shown.

TABLE 1.2
NUMBER OF CLOSE APPROACHES PER DAY

Day	COMBO I (First COMBO Run)	COMBO II (Second COMBO Run)
1 October 1966	4	0
2	7	1
3	3	0
4	3	0
5	11	0
6	6	0
7	11	0
8	8	2
9	11	0
10	14	1
11	6	0
12	4	0
Total	88	4
Mean	7.3	0.3
Standard Deviation	3.5	0.6

As one can see, COMBO II is not at all from the same population as COMBO I.

A conversation with NORAD personnel did not in itself completely resolve the large differences in results between COMBO I and COMBO II; however, it was mentioned in the conversation that a change in the COMBO program had been made between the two runs. Further investigation by TRW revealed that different catalogues had been used for the two runs (NORAD Element Summary, dated 30 September 1966, for COMBO I; and NORAD Element Summary, dated 12 December 1966, for COMBO II). Furthermore, COMBO I required 21 minutes of computer time to simulate each day of the mission, and the COMBO II required only $2\frac{1}{2}$ minutes (approximately) of computer time to simulate each day of the mission giving rise to the suspecting that only a partial data bank was utilized in COMBO II.

The results of COMBO I have been retained because of this suspicion relative to COMBO II and because COMBO I provides a pessimistic (i.e., larger) value of collision probability and, more importantly, because it agrees with the results from the pseudo collision method.

The number of close approaches per day appears to approximately follow a Poisson probability distribution with mean 7.3 (for COMBO I) and hence to approximately follow a Gaussian probability distribution with the same mean. A one-sided 97.5% (two-sided 95%) upper confidence limit on this mean is approximately 10 which, if used instead of 7.3, would yield only a one-third increase (i.e., $\frac{10 - 7.3}{7.3} \approx \frac{1}{3}$) in the estimate of collision probability. In fact, upon using the maximum number of close approaches which occurred on any one day (viz., 14) as the true mean, the estimated collision probability would only be doubled (i.e., $\frac{14}{7.3} \approx 2$).

It can be pointed out that the Poisson probability distribution of the number of close approaches per day supports the assumption of a negative exponential probability distribution for the time to collision (cf. Section 2.3 and Appendix III).

The large variation in the daily number of close approaches from COMBO I (viz., from 3 to 14) indicates that any dependency of collision hazards on the exact time of launch is not significant for a 12-day mission. For example, Days 5 through 10 had approximately twice as many close approaches, on the average, as Days 1 through 4 and Days 11 and 12; thus, an appreciably different phasing of the Apollo spacecraft with respect to the satellites has indeed appeared to have occurred within the 12-day mission. This hence suggests that the effects, on collision hazards, of different launch times would tend to be averaged out for a mission as long as 12 days, but not for a significantly shorter mission duration (e.g., a one-day mission).

If a triangular probability distribution had been fitted to the COMBO I close approach data contained in Table 1.1, the estimate of the collision probability would have been about 50% higher (viz., 2.56×10^{-5} instead of 1.77×10^{-5}). This particular method of estimation was developed, by the Florida Operations of TRW Systems, in Reference 3.

APPENDIX II

PSEUDO COLLISION METHOD

II.1 CALCULATION OF COLLISION PROBABILITY

This Appendix describes the mathematical model and associated computer program used for determining the collision probability for a finite mission duration (see Reference 10).

For a given satellite pair, in this case the Apollo spacecraft and the jth satellite, the program computes the number of path coincidences (i.e., "intersections") for a given time interval Tot (say, one year). At each path coincidence there are one or more j^{th} satellite passes that occur within a small Δt time interval. These occur because the relative geometry between the two satellites changes slowly in time as compared to the period of the jth satellite. The total count of passes during the time interval Tot is simply the product of these two counts. It turns out that these counts are not independent but are highly correlated by the fact that the two satellites have an approximate beat frequency between them. A correction factor is therefore applied to the total count to provide an uncorrelated count for the time interval Tot. The collision probability per pass is then computed from the geometry at the intersection. The jth satellite collision probability is then formed from the product of the uncorrelated count and the pass collision probability. Finally, because these probabilities are very small, the overall collision probability for a Tot-duration mission is approximated by a simple summation.

II.2 SIMPLIFYING ASSUMPTIONS

Several simplifying assumptions are made. The first is that the Apollo spacecraft reference orbit is circular. This was done to minimize the solution difficulties and thereby minimize the computer running time. An analysis quickly shows that, if a satellite's apogee and perigee bracket the Apollo spacecraft orbit, the satellite subjects the Apollo spacecraft

to the possibility of collision. The reverse is also true. Furthermore, if the Apollo spacecraft orbit is slightly eccentric, the circular assumption is equivalent and finally, for a highly eccentric Apollo spacecraft orbit, the circular assumption provides a conservative (i.e., pessimistic) estimate of collision probability.

The second major simplifying assumption is that the changes in Right Ascension and Argument of Perigee are due to the first order secular perturbations arising from the earth's oblateness. Secular perturbations due to atmospheric drag were not considered here. This drag perturbation primarily affects the semi-major axis (a) and eccentricity (e). Section 2.5 of this report accounts for the drag via estimating the lifetimes of satellites that are close to the earth.

II.3 THE MATHEMATICS

The following mathematics constitute the actual program. All computations are for the jth satellite.

The input data are read from a tape that was generated from the NORAD Element Summary listing dated 30 September 1966 and are as follows:

$$I.D._j \equiv \text{identification of the } j^{th} \text{ satellite (catalogue number)},$$
$$\Omega_j \equiv \text{Right Ascension angle (radians)},$$
$$\dot{\Omega}_j \equiv \text{Right Ascension angle rate (radians/day)},$$
$$\omega_j \equiv \text{Argument of Perigee (radians)},$$
$$\dot{\omega}_j \equiv \text{Argument of Perigee rate (radians/day)},$$
$$HP_j \equiv \text{perigee (feet)},$$
$$HA_j \equiv \text{apogee (feet)},$$
$$i_j \equiv \text{inclination angle (radians)}.$$

The initialization loop is entered only once for each satellite. The satellite period (in seconds) is:

All computations are for the ith cycle.

The rotation angles are computed by:

$$\Omega_i = \Omega_{i-1} + \Delta\Omega,$$

and

$$\omega_i = \omega_{i-1} + \Delta\omega;$$

and they are corrected in magnitude to:

$$0 \leq |\Omega_i| < 2\pi,$$

and

$$0 \leq |\omega_i| < 2\pi.$$

The cosine of the angle between the position vector of the satellite and the vector normal to the Apollo spacecraft plane is computed by:

$$\cos \Phi_i = \left\{ a[\sin \Omega_i \cos \omega_i - b(\cos \Omega_i \sin \omega_i)] + cd \sin \omega_i \right\}.$$

The product,

$$\text{Product} = (\cos \Phi_i)(\cos \Phi_{i-1}),$$

is now tested.

If the condition,

$$\text{Product} > 0,$$

holds, the start of the sub-loop is reentered; if however, the condition,

$$\text{Product} \leq 0,$$

holds, a crossing count is added to N_j (where N_j is the total number of coincidences in the time interval under consideration) and then the start of the sub-loop is reentered.

Computations through this sub-loop are continued until $n \Delt t$ (n is some integer) exceeds some Tot (where Tot is the total simulated mission time).

An N_j count is formed in this sub-loop and, finally because there are two scalers that satisfy

$$|\vec{r}_j| = |\vec{r}_a|,$$

the N_j count is doubled, i.e.,

$$N'_j \equiv 2N_j.$$

(The vectors $\vec{r}_j$ and $\vec{r}_a$ are the position vectors of the jth satellite and the Apollo spacecraft, respectively.)

At the end of the sub-loop, the passes per coincidence are computed. Differentiating ν_j with respect to r (where $\Delta r \equiv \Delta L$) in the polar form of the ellipse, one obtains

$$\Delta \nu_j = \frac{a_j \left[(1 - e_j^2) \right]^{\frac{1}{2}} \Delta L}{r_a \left[2a_j\, r_a - r_a^2 - a_j^2\, (1 - e_j^2) \right]^{\frac{1}{2}}} \, .$$

Now letting $\Delta L \equiv$ sum of the lengths of the Apollo spacecraft and the satellite, the number of passes per coincidence is approximated by:

$$X_j = 2 \left(\frac{\Delta \nu_j}{\dot{\omega}_j} \right) \left(\frac{\text{time}}{T_j} \right) .$$

A whole number (integer) is formed by:

$$X'_j = \text{truncation } X_j + 1.$$

To account for the beat frequency effect, one computes:

$$T_{beat_j} = \frac{T_a\, T_j}{|\, T_a - T_j\,|} \, .$$

The collision probability for each pass is computed by:

$$P_a = \frac{\Delta L}{\pi (r_e + k \Delta r)} \quad .$$

The jth satellite collision probability is:

$$P_j = \left(\frac{T_a}{T_{beat_j}} \right) N_j' X_j' P_a \quad .$$

The next case is then accepted and computations continue until all satellites and Apollo spacecraft orbit heights are processed.

The outputs from this program (for each height) are as follows:

$I.D._j \equiv$ identification of satellite,

$N_j' \equiv$ total coincidences per time interval Tot (one year),

$X_j' \equiv$ total number of passes per coincidence,

$T_a/T_{beat_j} \equiv$ beat frequency correction factor,

$P_j \equiv$ collision probability with j^{th} satellite,

$\sum\limits_{j} N_j' X_j' \equiv$ total number of passes during the time interval Tot,

$\sum\limits_{j} P_j \equiv$ total collision probability for the entire time interval Tot for all satellites, and

$\sum\limits_{j} N_j' X_j' T_a/T_{beat_j} \equiv$ corrected number of passes per Tot.

11.4 RESULTS

The one-year collision probability results are listed in Table 11.1.

TABLE 11.1

One Year Collision Probability P_c vs Height

Apollo Orbit Height (Statute Miles)	P_c (Collision Probability)
200	6.37×10^{-4}
400	11.16×10^{-4}
600	9.35×10^{-4}
800	4.47×10^{-4}
1,000	2.74×10^{-4}

As can be observed in the above table, the 400-statute mile orbit for Apollo would have experienced the highest collision probability, which is about 20% higher than the collision probability associated with the 600-statute mile orbit.

APPENDIX III

ANALYTIC EXPRESSION OF SIMULATION

The following is the development of the analytic expression of the pseudo collision simulation, that is, the obtaining of the average number of coincidences (N_j) of the paths of the jth satellite and Apollo spacecraft over a time period of length t and then the computing of the collision probability via the expression,

$$P_{c_j} = N_j P_j = N_j P(\Delta L),$$

where

$$N_j \equiv \frac{t}{\tau_j},$$

$$\tau_j \equiv \text{mean time to coincidence},$$

and

$$P(\Delta L) \equiv \text{the probability that the Apollo spacecraft is actually located at the coincidence of the two paths.}$$

Making use of the Poisson distribution (see Page 48, Reference 4), we obtain the following value for the probability of having exactly zero intersections (or coincidences):

$$P(0) = e^{-N_j},$$

where

$$N_j \equiv \frac{t}{\tau_j}.$$

The probability of experiencing exactly one coincidence becomes

$$P(1) = N_j e^{-N_j};$$

and the probability of experiencing exactly n coincidences is:

$$P(n) = \frac{N_j^n}{n!} e^{-N_j} \text{ for } n = 0, 1, 2, 3, \ldots .$$

Given that P_j is the collision probability conditional upon the occurrence of a single coincidence (that is, P_j is the conditional probability that both the jth satellite and the Apollo spacecraft are simultaneously located at a particular intersection), the collision probability conditional upon the occurrence of exactly k coincidences is therefore given by the expression,

$$1 - (1 - P_j)^k.$$

Consequently, the collision probability (P_{c_j}) of the Apollo with respect to the jth satellite for a t-day duration mission is as follows:

$$P_{c_j} = \sum_{i=1}^{\infty} \{P(i) [1 - (1 - P_j)^i]\}$$

$$= \sum_{i=1}^{\infty} \left\{\frac{N_j^i}{i!} e^{-N_j}\left[1 - (1 - P_j)^i\right]\right\}$$

$$= 1 - e^{-N_j P_j}.$$

Now, if $N_j P_j$ is very small, which is actually the case, then the jth collision probability (P_{c_j}) for the jth satellite can be approximated by the expression,

$$P_{c_j} \approx N_j P_j.$$

The overall t-day duration mission collision probability for m satellites, assuming that the collision probability attributed to each satellite is independent in the probability sense, is as follows:

$$P_c = 1 - \prod_{j=1}^{m} e^{-N_j P_j}$$

$$= 1 - e^{-\sum_{j=1}^{m} N_j P_j} .$$

This may be approximated by the following:

$$P_c \approx \sum_{j=1}^{m} N_j P_j .$$

Upon recalling that $N_j \equiv \dfrac{t}{\tau_j}$, so that a linear (i.e., proportional) scaling is required to convert from an arbitrarily chosen value of t, due to the actual mission duration, one obtains

$$P_c(\text{actual}) \approx \sum_{j=1}^{m} \left(\frac{t^*}{t}\right) \left(\frac{t}{\tau_j}\right) (P_j),$$

where

$$t^* \equiv \text{actual mission duration,}$$

or, equivalently,

$$P_c(\text{actual}) \approx \frac{t^*}{t} \sum_{j=1}^{m} \frac{t}{\tau_j} P_j .$$

REFERENCES

1. MTCP Task No. MSC/TRW A-106, "Detailed Technical Approach for Determination of the Collision Probability of the Apollo Spacecraft with Objects in Earth Orbit", TRW Systems, Florida Operations, dated 30 September 1966.

2. MTCP Task No. MSC/TRW A-106, "Approximate Collision Probabilities", TRW Systems, Florida Operations, dated December 1966.

3. "Gemini Mission GT-8 Orbital Collision Hazard Evaluation", Contract NAS9-4810, TRW Systems, Florida Operations, dated March 1966.

4. Weatherburn, C. E., "Mathematical Statistics", Second Edition, 1962, Cambridge University Press.

5. Goddard Space Flight Center, Space Operations Control Center, Satellite Situation Report(s):
   Vol. 4 No. 10, dated 31 May 1964,
   Vol. 5 No. 4, dated 28 Feb 1965,
   Vol. 5 No. 12, dated 30 Jun 1965,
   Vol. 6 No. 6, dated 31 Jan 1966, and
   Vol. 6 No. 12, dated 30 Jun 1966.

6. IOC, E. M. Simpson to C. R. Coates, Subject: "The Probability Distribution of the Weights (and/or Sizes) of the Pieces Generated by a Fragmented (Exploded) Earth Satellite, Task A-106", TRW Systems, Florida Operations, dated 13 March 1967.

7. IOC, E. M. Simpson to C. R. Coates, Subject: "Satellite Shapes and Dimensions, Task A-106", TRW Systems, Florida Operations, dated 13 March 1967.

8. IOC, E. M. Simpson to C. R. Coates, Subject: "Stationarity of the Classical Orbital Parameters, Task A-106", TRW Systems, Florida Operations, dated 13 March 1967.

REFERENCES (Cont'd)

9. IOC, E. M. Simpson to C. R. Coates, Subject: "Independence of the Classical Orbital Parameters, Task A-106", TRW Systems, Florida Operations, dated 13 March 1967.

10. IOC, E. M. Simpson to C. R. Coates, Subject: "Pseudo Collision Probabilities, Task A-106", TRW Systems, Florida Operations, dated 13 March 1967.

11. IOC, E. M. Simpson to C. R. Coates, Subject: "Approximate Collision Probabilities, Task A-106", TRW Systems, Florida Operations, dated 13 March 1967.

12. King-Hele D. G., et al, "Table of the Earth Satellites Launched in 1957-65", dated March 1966, Royal Aircraft Establishment, TR No. 66077.

13. Roy, A. R., "The Foundations of Astrodynamics", The MacMillan Company, New York, 1965.