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MSC-EE-R-68-15

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

AN ANALYSIS OF POINT-TO-POINT COMMUNICATION

for

APPLICATION TO THE LUNAR FLYER PROGRAM

MANNED SPACECRAFT CENTER

HOUSTON, TEXAS

MSC INTERNAL NOTE MSC-EE-R-68-15

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APPLICATION TO THE LUNAR FLYER PROGRAM

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Manned Spacecraft Center
Instrumentation and Electronic Systems Division
August 1968

This study is concerned with an analysis to determine the optimum frequency for point-to-point communication in the vicinity of the line-of-sight horizon on the lunar surface. For this analysis, the transmitter output is assumed to be 1 watt (+30 dbm) and the antenna heights are 6 and 24 feet.

The circuit margin parameters used in the communication link analysis include the free-space transmission loss, the Bremmer Series loss, the ground proximity loss, antenna gains, transmitter power and receiver sensitivities. Each parameter is briefly explained.

The free-space transmission loss is defined as the ratio of the power received to the power transmitted assuming two isotropic antennas (0 db gain) such that

$$L_o = \frac{\lambda^2}{(4\pi d)^2} \tag{1}$$

where λ = the wavelength and d = distance between antennas. The Bremmer Series loss represents the loss which occurs as a result of blockage and diffraction by the lunar surface. The Bremmer Series loss is defined as

$$L_B = 8\pi \mathcal{J} \left| \sum_{n=1}^{\infty} \frac{e^{-\tau_n \mathcal{J}}}{\delta'' + 2\tau_n} f_n(h_1) f_n(h_2) \right|^2 \tag{2}$$

where $\mathcal{J}$ is a distance function, τ_n is the mode number, δ'' is a ground parameter and $f_n(h)$ is the height-gain function. The free-space loss along with the Bremmer Series diffraction loss is tabulated in Table 1 for frequencies ranging between 200 KHz and 2200 MHz and distances between 2.5 KM and 30 KM. A more detailed explanation of the Bremmer loss is given in Burrows (1949). A computer analysis was used to determine both the Bremmer Series and free-space losses. The first seven terms of the Bremmer Series were used. The program was formulated under the direction of Mr. Ray Thompson by Mr. Jim Pawlowski of the Digital Techniques Section and is included in the Appendix.

A tabulation of the total transmission loss is shown in Table II and a plot is given in Figure 1 depicting the changes in transmission loss as a function of frequency and range. It is noted that as the frequency decreases the transmission loss becomes smaller. Also it is noted that the dielectric parameters of the moon are more significant at the lower frequencies; whereas, at VHF and above the transmission loss is practically independent of the moon's dielectric parameters. In general, it is found that higher conductivity contributes to a longer communication range and this is especially true at lower frequencies.

The next factor to be considered is that of the ground proximity loss or that loss which affects the radiation resistance of an antenna because of its close proximity to the ground. These losses have been taken from graphs given by (Vogler, 1964) and are tabulated in Table III. As may be seen in Table III, this loss becomes important only at low frequencies when the antenna is electrically close to the lunar surface. Also shown in Table III are typical antenna gains for the frequencies involved. On the right side of Table III is listed the combined effects of antenna gains and the ground proximity loss.

The final frequency variable factor to be considered is receiver sensitivity which is shown in Table IV. An RF bandwidth of 40 KHz is assumed with a received signal-to-noise ratio of 18 db. Also, shown in Table IV is the allowed antenna port-to-antenna port loss which includes +30 dBm (1 watt) for the transmitter and the assumed receiver sensitivities at the various frequencies.

With the assumption of a 1 watt (30 dbm) transmitter one may determine from the combined effects tabulation in Table V and Table IV that the astronaut may go to a range of 10 kilometers and still communicate at a frequency of 10 MHz with a data bandwidth of 14 KHz (RF bandwidth of 40 KHz) and an output signal-to-noise ratio of 18 db. The maximum range is plotted in Figure 2 as a function of frequency. For application with television bandwidths, the 10 MHz carrier frequency would not be sufficient. A frequency band of VHF or higher would be required and a shorter maximum range would result.

For direction finding, the 10 MHz frequency could be used out to a range of approximately 20 kilometers by using a small RF bandwidth of 1 KHz and assuming a 10 db less efficient direction finding antenna. The astronaut could carry a low power receiver and a direction finding antenna with a gain of -15 db with respect to isotropic. A 1 watt 10 MHz transmitter could be used on the Lunar Module yielding a range of 20-23 kilometers depending on the lunar surface conductivity.

The optimum frequency for point-to-point communication with 6 foot and 24 foot antenna heights is found to be in the 1 to 10 MHz range depending on the conductivity of the lunar surface.

This analysis has not considered antenna size. Low frequency antennas become very large compared to VHF antennas of equivalent characteristics. In a lunar surface communication system this factor could weigh heavily in frequency selection.

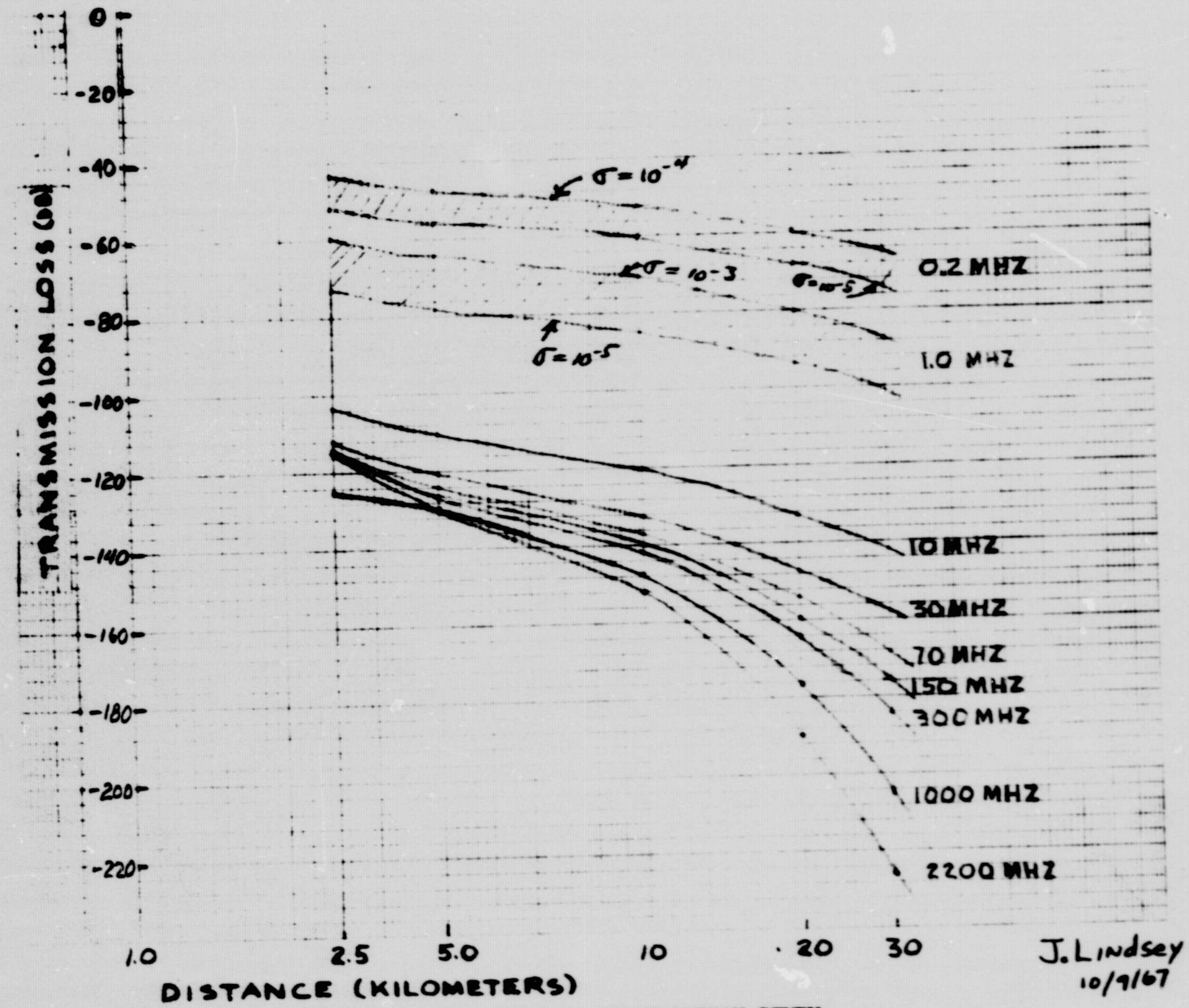

Figure 1. LUNAR COMMUNICATION STUDY

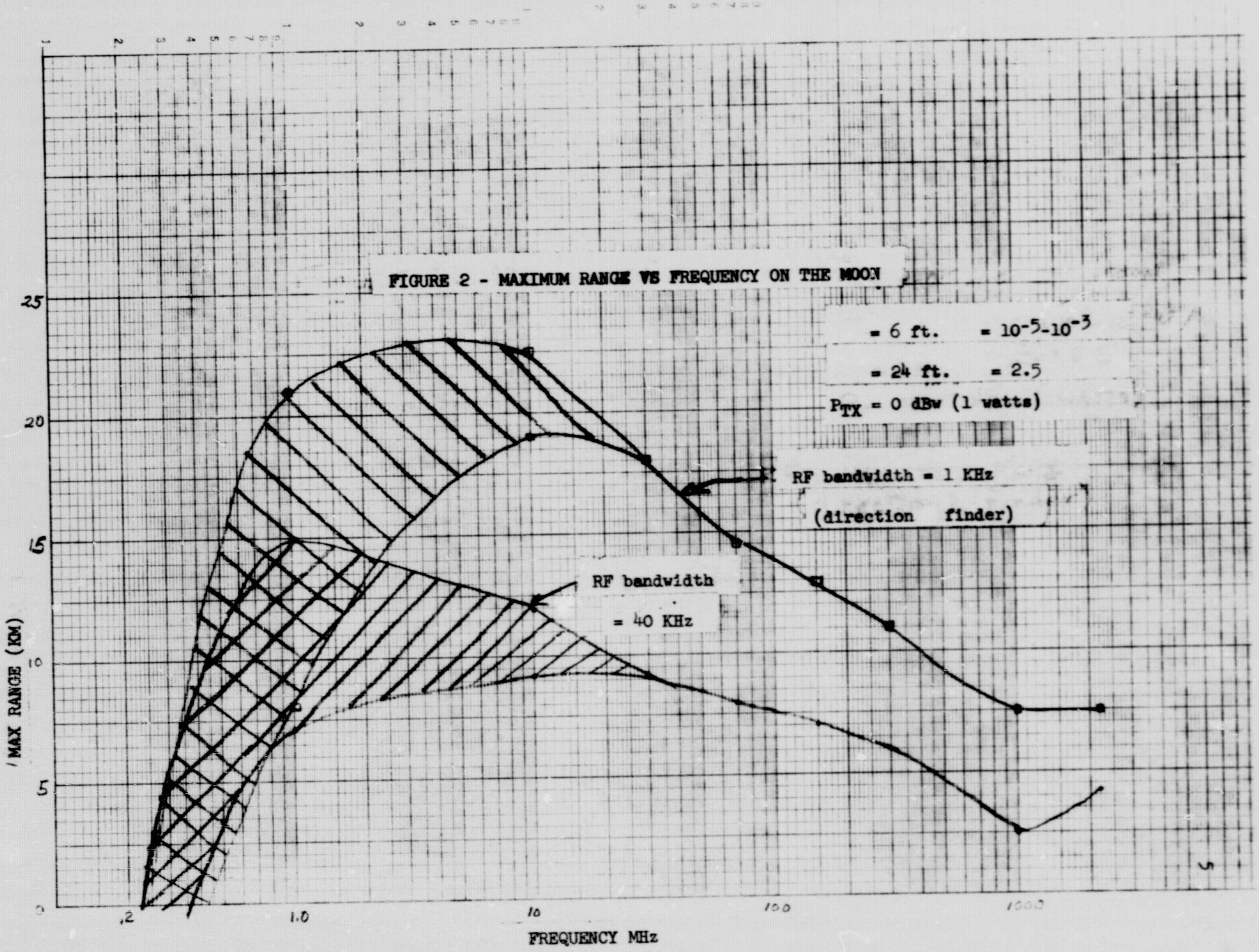

FIGURE 2 - MAXIMUM RANGE VS FREQUENCY ON THE MOON

TABLE I - FREE SPACE AND BREMMER LOSSES

Frequency	Conductivity	2.5 Km		5.0 Km		← Range
(MHz)	℧/M	Free Space Loss(db)	Bremmer Loss(db)	Free Space Loss(db)	Bremmer Loss(db)	Type of Loss ←
2200	10^{-5}	-107.2	-8.0	-113.2	-17.2	
2200	10^{-4}	-107.2	-8.0	-113.2	-17.16	
2200	10^{-3}	-107.2	-8.0	-113.2	-17.16	
1000	10^{-5}	-100.4	-14.5	-106.4	-22.4	
1000	10^{-4}	-100.4	-14.5	-106.4	-22.4	
1000	10^{-3}	-100.4	-14.5	-106.4	-22.4	
300	10^{-5}	-89.9	-25.1	-95.9	-31.3	
300	10^{-4}	-89.9	-25.1	-95.9	-31.2	
300	10^{-3}	-89.9	-25.1	-95.9	-31.2	
150	10^{-5}	-83.9	-31.5	-89.9	-36.5	
150	10^{-4}	-83.9	-31.5	-89.9	-36.5	
150	10^{-3}	-83.9	-31.5	-89.9	-36.5	
70	10^{-5}	-76.8	-37.9	-82.8	-41.5	
70	10^{-4}	-76.8	-37.9	-82.8	-41.5	
70	10^{-3}	-76.8	-37.9	-82.8	-41.5	
30	10^{-5}	-69.9	-42.3	-75.9	-44.6	
30	10^{-4}	-69.9	-42.3	-75.9	-44.6	
30	10^{-3}	-69.9	-42.3	-75.9	-44.9	
10	10^{-5}	-60.4	-42.75	-66.4	-43.6	
10	10^{-4}	-60.4	-42.8	-66.4	-43.6	
10	10^{-3}	-60.4	-44.0	-66.4	-44.9	
1	10^{-5}	-40.4	-32.8	-46.4	-31.7	
1	10^{-4}	-40.4	-32.8	-46.4	-31.9	
1	10^{-3}	-40.4	-19.03	-46.4	-17.9	
0.2	10^{-5}	-26.4	-25.5	-32.4	-23.6	
0.2	10^{-4}	-26.4	-16.6	-32.4	-14.6	

TABLE I (cont.)

Freq	Conductivity	10.0 Km		20.0 Km		30.0 Km		← Range
(MHz	$\mho/M$	Free Space Loss(db)	Bremmer Loss(db)	Free Space Loss(db)	Bremmer Loss(db)	Free Space Loss(db)	Bremmer Loss(db)	Type of Loss ←
2200	10^{-5}	-119.2	-33	-125.2	-64.7	-128.7	-97.5	
2200	10^{-4}	-119.2	-33	-125.2	-64.7	-128.7	-97.5	
2200	10^{-3}	-119.2	-33	-125.2	-64.7	-128.7	-97.5	
1000	10^{-5}	-112.4	-34.9	-118.4	-59.0	-121.9	-83.7	
1000	10^{-4}	-112.4	-34.9	-118.4	-59.0	-121.9	-83.7	
1000	10^{-3}	-112.4	-34.9	-118.4	-59.0	-121.9	-83.7	
300	10^{-5}	-101.9	-40.6	-107.9	-56.9	-111.4	-73.0	
300	10^{-4}	-101.9	-40.6	-107.9	-56.9	-111.4	-72.9	
300	10^{-3}	-101.9	-40.6	-107.9	-56.9	-111.4	-72.9	
150	10^{-5}	-95.9	-44.6	-101.9	-57.9	-105.4	-70.6	
150	10^{-4}	-95.9	-44.6	-101.9	-57.9	-105.4	-70.6	
150	10^{-3}	-95.9	-44.6	-101.9	-57.9	-105.4	-70.6	
70	10^{-5}	-88.8	-48.6	-94.8	-59.3	-99.3	-69.3	
70	10^{-4}	-88.8	-48.6	-94.8	-59.3	-99.3	-69.3	
70	10^{-3}	-88.8	-48.6	-94.8	-59.3	-99.3	-69.3	
30	10^{-5}	-81.9	-50.3	-87.9	-59.3	-91.4	-67.0	
30	10^{-4}	-81.9	-50.3	-87.9	-59.3	-91.4	-67.0	
30	10^{-3}	-81.9	-50.7	-87.9	-59.6	-91.4	-67.3	
10	10^{-5}	-72.4	-47.4	-78.4	-54.7	-81.9	-60.5	
10	10^{-4}	-72.4	-47.4	-78.4	-54.7	-81.9	-60.5	
10	10^{-3}	-72.4	-48.7	-78.4	-56.0	-81.9	-61.7	
1	10^{-5}	-52.4	-32.2	-58.4	-35.6	-61.9	-39.3	
1	10^{-4}	-52.4	-32.5	-58.4	-35.9	-61.9	-39.6	
1	10^{-3}	-52.4	-18.4	-58.4	-21.9	-61.9	-25.6	
0.2	10^{-5}	-38.5	-22.7	-44.5	-23.8	-48.0	-25.7	
0.2	10^{-4}	-38.5	-13.8	-44.5	-14.7	-48.0	-16.8	

TABLE II. TOTAL TRANSMISSION LOSS

FREQ (MHz)	σ ℧/M	2.5 KM Trans. Loss	5.0 KM Trans.Loss (db)	10 KM Trans.Loss (db)	20 KM Trans.Loss (db)	30 KM Trans.Loss (db)
2200	10^{-5}	-115.2	-130.4	-152.2	-189.7	-226.2
2200	10^{-4}	-115.2	-130.4	-152.2	-189.7	-226.2
2200	10^{-3}	-115.2	-130.4	-152.2	-189.7	-226.2
1000	10^{-5}	-124.9	-128.8	-147.3	-177.4	-205.6
1000	10^{-4}	-124.9	-128.8	-147.3	-177.4	-205.6
1000	10^{-3}	-124.9	-128.8	-147.3	-177.4	-205.6
300	10^{-5}	-115.0	-127.2	-142.5	-164.8	-184.4
300	10^{-4}	-115.0	-127.2	-142.5	-164.8	-184.4
300	10^{-3}	-115.0	-127.3	-142.5	-164.8	-184.4
150	10^{-5}	-115.4	-126.4	-140.5	-159.8	-176.0
150	10^{-4}	-115.4	-126.4	-140.5	-159.8	-176.0
150	10^{-3}	-115.4	-126.4	-140.5	-159.8	-176.0
70	10^{-5}	-114.7	-124.3	-137.4	-154.1	-168.6
70	10^{-4}	-114.7	-124.3	-137.4	-154.1	-168.6
70	10^{-3}	-114.7	-124.3	-137.4	-154.1	-168.6
30	10^{-5}	-112.2	-120.5	-132.2	-147.2	-158.4
30	10^{-4}	-112.2	-120.5	-132.2	-147.2	-158.4
30	10^{-3}	-112.6	-120.8	-132.6	-147.5	-158.7
10	10^{-5}	-103.2	-110.0	-119.8	-133.1	-142.4
10	10^{-4}	-103.2	-110.0	-119.8	-133.1	-142.4
10	10^{-3}	-104.4	-111.3	-121.1	-134.4	-146.3
1	10^{-5}	-73.2	-78.1	-84.6	-94.0	-101.2
1	10^{-4}	-73.5	-78.3	-84.9	-94.0	-101.
1	10^{-3}	-59.4	-64.3	-70.8	-80.3	-87.5
0.2	10^{-5}	-51.9	-56.0	-61.2	-68.3	-73.7
0.2	10^{-4}	-43.0	-47.0	-52.3	-59.2	-64.8

∞

$\epsilon_r = 2.5$

		Ground Prox. Loss		Antenna Gain		Total Effect of $\Sigma L_p + \Sigma G$
		L_{p6}	L_{p24}	G_6	G_{24}	
f (MHz)	σ (℧/m)	6'	24'	$\frac{DB}{150}$	$\frac{DB}{150}$	DB
2200	10^{-5}	0	0	-4	-2	-6
2200	10^{-4}	0	0	-4	-2	-6
2200	10^{-3}	0	0	-4	-2	-6
1000	10^{-5}	0	0	-5	-2	-7
1000	10^{-4}	0	0	-5	-2	-7
1000	10^{-3}	0	0	-5	-2	-7
300	10^{-5}	0	0	-5	-2	-7
300	10^{-4}	0	0	-5	-2	-7
300	10^{-3}	0	0	-5	-2	-7
150	10^{-5}	0	0	-4	-1	-5
150	10^{-4}	0	0	-4	-1	-5
150	10^{-3}	0	0	-4	-1	-5
70	10^{-5}	0	0	-3	0	-3
70	10^{-4}	0	0	-3	0	-3
70	10^{-3}	0	0	-3	0	-3
30	10^{-5}	-1	0	-4	0	-5
30	10^{-4}	-1	0	-4	0	-5
30	10^{-3}	-1	0	-4	0	-5
10	10^{-5}	-2	0	-5	-5	-12
10	10^{-4}	-3	0	-5	-5	-13
10	10^{-3}	-5	0	-5	-5	-15
1	10^{-5}	-23	-10	-10	-10	-53
1	10^{-4}	-30	-17	-10	-10	-67
1	10^{-3}	-26	-13	-10	-10	-59
.2	10^{-5}	-53	-35	-17	-17	-112
.2	10^{-4}	-48	-35	-17	-17	-117
.2	10^{-3}	-43	-25	-17	-17	-102

L = Ground Proximity Loss

G = Antenna gain plus cable losses

TABLE III - GROUND PROXIMITY LOSS AND ANTENNA GAINS

		ALLOWED LOSS	
Freq. (MHz)	S_{rx} (dbm)'	RF_{BW} = 40 KHz	RF_{BW} = 1 KHz
.2	-67	97	106
1	-103	133	149
10	-104	134	150
30	-105	135	151
70	-105	135	151
150	-105	135	151
300	-105	135	151
1000	-101	131	147
2200	-101	131	147

'S/N_C = 18 db ≠ $R.F_{BW}$ = 40 kHz

'Obtained from A. Pajak and J. Fowler

TABLE IV - RECEIVER SENSITIVITIES

And Allowed Antenna port-to-antenna port

loss for P_{TX} = 30 dbm

		Distance(KM)				
Freq. (MHz)	Conductivi. $\mho$/M	2.5	5.0	10.0	20.0	30.0
2200	10^{-5}-10^{-3}	-121.2	-136.4	-158.2	-195.7	-232.2
1000	10^{-5}-10^{-3}	-131.9	-135.8	-154.3	-184.4	-212.6
300	10^{-5}-10^{-3}	-122.0	-134.2	-149.5	-171.8	-191.4
150	10^{-5}-10^{-3}	-120.4	-131.4	-145.5	-164.8	-181.0
70	10^{-5}-10^{-3}	-117.7	-127.3	-140.4	-157.1	-171.6
30	10^{-5}-10^{-3}	-117.2	-125.5	-137.2	-152.2	-163.4
10	10^{-5}	-115.2	-122.0	-131.8	-145.1	-154.4
10	10^{-4}	-116.2	-123.0	-132.8	-146.1	-155.4
10	10^{-3}	-119.4	-126.3	-136.1	-149.4	-158.6
1	10^{-5}	-126.2	-131.1	-137.6	-147.0	-154.2
1	10^{-4}	-140.5	-145.3	-151.9	-161.0	-168.2
1	10^{-3}	-118.4	-123.3	-129.8	-139.3	-146.5
.2	10^{-5}	-163.9	-168.0	-173.2	-180.2	-185.7
.2	10^{-4}	-160.0	-164.0	-169.3	-176.2	-181.8

TABLE V. Transmission Loss, Antenna Gains and Ground Proximity Loss

References

1. Burrows, C. R.: Radio Wave Propagation. Academic Press, Inc. 1949.
2. Volger, L. E.: A Study of Lunar Surface Communication. NBS Monograph 85, September 14, 1964.

APPENDIX

by J. Pawlowski

Computer Programs

for

Equations (1) and (2)

Analysis of Free Space Loss

The general formula to calculate Free Space Loss expressed in decibels is

$$L_o(db) = 10 \log_{10} \frac{|\hat{E}_o|^2}{|\hat{E}_s|^2} = 10 \operatorname{Log}_{10}\left[\frac{\lambda^2}{(4\pi d)^2}\right]$$

$\hat{E}_o$ = received signal

$\hat{E}_s$ = transmitted signal

λ = wave length

d = distance between antennas

It is necessary to express λ and d in the same units:

$$\lambda = \frac{\text{speed of light}}{\text{frequency}}$$

Therefore, it can be seen that the distance units of λ are the same as those of the speed of light. Hence, it is the same to say that the speed of light and d must be expressed in the same distance units.

$$\lambda(m) = \frac{300 \times 10^6 \ \frac{\text{meters}}{\text{sec}}}{\text{frequency (cps)}}$$

$$\lambda(m) = \frac{300 \ \frac{\text{Meg. meters}}{\text{sec}}}{\text{frequency (MHZ)}}$$

Likewise,

$$\lambda(f) = \frac{984 \ \text{Meg. feet/sec}}{\text{frequency (MHZ)}}$$

When d is expressed in meters, use $\lambda(m)$, and when d is expressed in feet use $\lambda(f)$. If it is necessary to express d in other units adjust λ by expressing the speed of light in the same units.

In order to include the loss and/or gain due to reflection and refraction we must compute the Geo-Optics or the Bremmer Series methods and add these results to the Free Space Loss. The choice of what method to use depends on the distance between the antennas.

If the distance between the antennas is greater than 25% of the line of sight then the Bremmer Series is used. If it is less than or equal to this value the Geo-Optic method is used.

$$\text{Line of Sight} = \sqrt{2kah_1} + \sqrt{2kah_2}$$

a = radius of the body

k = index of refraction

h_1 = height of one antenna

h_2 = height of the other antenna

CAUTION! a, h_1, and h_2 must be expressed in the same units of distance.

FREE SPACE LOSS

OPERATION OF PROGRAM

1. Load the Fortran operating system using the high speed reader.

2. Put the Free Space Loss Program object tape in the low speed reader and turn on the reader.

3. Load address 200 and depress start the tape will load.

4. When the tape stops, depress the continue switch. The message "Type Frequency" will be typed out on the teletype. Enter the frequency expressed in MHZ on the keyboard. Express all entries in Floating Point except the number of increments (Step 9).

5. "Type Speed of Light" is then typed. Enter the speed of light expressed in mega-units.

6. "Type index of refraction" is the next message. Enter this quantity.

7. "Type radius of Body" and "Type heights of antennas" are the next two messages type out. Remember that these quantities must be entered using the same units of distance.

8. The value for λ , the wave length, is calculated and typed out.

9. The message, "Type the number of increments", is typed. Enter in fixed point the total number of antenna distances that you wish to investigate.

10. The number +1 which labels the first distance will then be typed.

11. The message "Type the distance between antennas" is typed. Remember to enter this in the same units as the speed of light.

12. The "Free Space Loss" followed by the "Free Space Loss" expressed in db are typed out.

13. The message whether to use Geo Optic or use Bremmer series for the next calculations is typed.

14. The number +2 is typed and the steps 11 through 13 are repeated. Then the number +3, etc.

```
C;  FREE SPACE LOSS CALCULATIONS FOR ANTENNAS
1;  FORMAT (/,"TYPE FREQUENCY",/)
2;  FORMAT (/,"TYPE SPEED OF LIGHT",/)
3;  FORMAT (/,"TYPE INDEX OF REFRACTION",/)
4;  FORMAT (/,"TYPE RADIUS OF BODY",/)
5;  FORMAT (/,"TYPE HEIGHTS OF ANTENNAS",/)
8;  FORMAT (E,/)
9;  FORMAT (I,/)
  ; TYPE 1
  ; ACCEPT 8, F
  ; TYPE 2
  ; ACCEPT 8, CONST
  ; TYPE 3
  ; ACCEPT 8, REFR
  ; TYPE 4
  ; ACCEPT 8, A
  ; TYPF 5
  ; ACCEPT 8, H1, H2
  ; AMBA = CONST / F
  ; TYPE 15, AMBA
15; FORMAT ( /,"WAVE LENGTH = ",E,/)
10; FORMAT (/,"TYPE NUMBER OF INCREMENTS",/)
  ; TYPE 10
  ; ACCEPT 9, J
  ; DO 50 M=1,J
  ; TYPE 11, M
11; FORMAT (/,"I = ",I,/)
12; FORMAT (/,"TYPE DISTANCE BETWEEN ANTENNAS",/)
  ; TYPE 12
  ; ACCEPT 8, D
  ; FSL  = (AMBA / (4. * 3.1416 * D))**2
  ; DB   = 4.34294 * LOGF (FSL)
13; FORMAT (/,"FREE SPACE LOSS = ",E,/)
14; FORMAT (/,"FREE SPACE LOSS (DB) = ",E,/)
  ; TYPE 13, FSL
  ; TYPE 14, DB
  ;  DIST= SQTF(2. * REFR * A * H1)
  ;  DIST = DIST + SQTF(2. * A * H2)
  ; DIST = 0.25 * DIST
  ; IF (DIST - D)   21,20,20
18; FORMAT (/,"USE GEO-OPTIC",/)
19; FORMAT (/,"USE BREMMER",/)
20; TYPE 18
  ; GO TO 50
21; TYPE 19
50; CONTINUE
  ; STOP
  ; END
```

EXPLANATION OF PROGRAM

The program is broken into three sections:

Section I is composed almost entirely of commands to type messages and accept data. This section extends from the line labeled 1; through two lines after the line labeled 12;. However, the equation for wave length is solved in this section on the 11th line after 9;.

```
AMBA = CONST/F
```

corresponds to

$$\lambda = \frac{\text{speed of light}}{\text{frequency}}$$

Section II contains the calculations and outputs for the Free Space Loss and extends from the end of Section I to the second line after 14;

```
FSL = (AMBA/(4.*3.1416*D) )**2
```

corresponds to

$$L_o = \left(\frac{\lambda}{4\pi d}\right)^2$$

```
DB = 4.34294*LOGF  (FSL)
```

corresponds to

$$L_o\ (DB) = 10 \times LOG_{10}\ (L_o)$$

The factor of .434294 is used to equate the LOGF, which is $\log_e$, to $\log_{10}$.

Section III contains the test to see if the Bremmer Series or Geo-Optic method should next be used.

```
DIST = SQTF (2.*REFR*A*H1)
```

corresponds to

$$\sqrt{2 \cdot k \cdot a \cdot h_1}$$

```
DIST = DIST + SQTF  (2.*REFR*A*H2)
```

corresponds to

$$\text{Line of Sight} = \sqrt{2kah_1} + \sqrt{2kah_2}$$

DIST = 0.25*DIST

is 25% of the line of sight.

Next is the "IF" statement which determines whether 25% of the line of sight is greater than the distance between antennas or less than or equal to the distance between antennas.

Analysis of Bremmer Series

The general equation for the Bremmer Series expressed in decibels is

$$L_B(\text{db}) = 10 \log_{10} \frac{|\hat{E}|^2}{|\hat{E}_o|^2} = 10 \log_{10} \left[2(2\pi\rho)^{\frac{1}{2}} \left| \sum_{n=1}^{7} \frac{e^{-j\tau_n\rho}}{\delta + 2\tau_n} f(h_1) f(h_2) \right| \right]^2$$

This can be simplified to

$$(1) \quad L_B(\text{db}) = 10 \log_{10} \left[8\pi\rho \cdot |f(h_1)|^2 \cdot |f(h_2)|^2 \cdot \left| \sum_{n=1}^{7} \frac{e^{-j\tau_n\rho}}{\delta + 2\tau_n} \right|^2 \right]$$

$\hat{E}$ = received field

$\hat{E}_o$ = Free Space Field

ρ = distance factor

δ = ground parameter

h_1, h_2 = heights of antennas

τ_n = complex numbers which characterize the individual terms in equation (1)

For vertical polarization where $i = 1, 2$

$$f(h_i) = 1 + j\left(\frac{2\pi h_i}{\lambda} \frac{\sqrt{\epsilon_c - 1}}{\epsilon_c}\right) \qquad h_i < 30\lambda^{2/3}$$

For horizontal polarization where $i = 1, 2$

$$f(h_i) = 1 + j\left(\frac{2\pi h_i}{\lambda} \sqrt{\epsilon_c - 1}\right) \qquad h_i < 30\lambda^{2/3}$$

$$\hat{\epsilon}_c = \epsilon - j(60\lambda\sigma)$$

$$\hat{\epsilon}_c - 1 = (\epsilon - 1) - j(60\lambda\sigma)$$

$$\hat{\epsilon}_c - 1 = \sqrt{(\epsilon-1)^2 + (60\lambda\sigma)^2} \quad e^{j\tan^{-1}\left(\frac{-60\lambda\sigma}{\epsilon - 1}\right)}$$

ϵ = ddielectric constant of the spherical body

λ = wave length of signal

σ = conductivity of the spherical body

Let $W = 60\,\lambda\sigma$

Then $\sqrt{\epsilon_c - 1} = \left(\sqrt{(\epsilon-1)^2 + W^2}\right)^{\frac{1}{2}} \cdot e^{j\frac{\tan^{-1}\left(\frac{-60\lambda\sigma}{\epsilon-1}\right)}{2}}$

Let TINY = $\left(\sqrt{(\epsilon-1)^2 + W^2}\right)^{\frac{1}{2}}$

and $\alpha = \dfrac{\tan^{-1}\left(\frac{-60\lambda\sigma}{\epsilon-1}\right)}{2}$

Then $\sqrt{\epsilon_c - 1} = TINY\; e^{j\alpha}$

$$\frac{\sqrt{\epsilon_c - 1}}{\epsilon_c} = \frac{TINY\,(\cos\alpha + j\,\sin\alpha)}{\epsilon - jW}$$

$$= TINY\left[\frac{\epsilon\cdot\cos\alpha - W\cdot\sin\alpha + j\,(W\cdot\cos\alpha + \epsilon\cdot\sin\alpha)}{\epsilon^2 + W^2}\right]$$

Therefore, for vertical polarization, $i = 1, 2$

$$f(h_i) = 1 + j\left[\frac{2\pi h_i}{\lambda}\cdot TINY\cdot\frac{\epsilon\cdot\cos\alpha - W\cdot\sin\alpha + j(W\cdot\cos\alpha + \epsilon\cdot\sin\alpha)}{\epsilon^2 + W^2}\right]$$

REAL PART = $1 - \dfrac{2\pi h_i}{\lambda}\cdot TINY\cdot\dfrac{W\cdot\cos\alpha + \epsilon\cdot\sin\alpha}{\epsilon^2 + W^2}$

IMAGINARY PART = $\dfrac{2\pi h_i}{\lambda}\cdot TINY\cdot\dfrac{\epsilon\cdot\cos\alpha - W\cdot\sin\alpha}{\epsilon^2 + W^2}$

and for horizontal polarization, $i = 1, 2$

REAL PART = $1 - \dfrac{2\pi h_i}{\lambda}\cdot TINY\cdot\sin\alpha$

IMAGINARY PART = $\dfrac{2\pi h_i}{\lambda}\cdot TINY\cdot\cos\alpha$

For both cases

(2) $$|f(h_i)|^2 = (\text{REAL PART})^2 + (\text{IMAGINARY PART})^2$$

For vertical polarization

$$\delta = \left(\frac{2\pi k a}{\lambda}\right)^{\frac{2}{3}} \frac{\hat{\epsilon}_c - 1}{\epsilon_c^2}$$

k = index of refraction

a = radius of the spherical body

Let $H = \left(\frac{2\pi k a}{\lambda}\right)^{\frac{2}{3}}$

Then $$\delta = H \cdot \left[\frac{(\epsilon - 1) - jW}{(\epsilon^2 - W^2) - j(2\epsilon W)}\right]$$

Let $F = \epsilon^2 - W^2$ and $G = 2\epsilon W$

Then $$\delta = H \cdot \left[\frac{(\epsilon - 1) - jW}{F - jG}\right]$$

$$= H \cdot \left[\frac{(\epsilon - 1)F + GW}{F^2 + G^2} + j\,\frac{(\epsilon - 1)G - FW}{F^2 + G^2}\right]$$

Let $$\text{CEM} = H \cdot \left[\frac{(\epsilon - 1)F + GW}{F^2 + G^2}\right]$$

and $$\text{CAW} = H \cdot \left[\frac{(\epsilon - 1)G - FW}{F^2 + G^2}\right]$$

For horizontal polarization

$$\delta = H(\hat{\epsilon}_c - 1) = H\left[(\epsilon - 1) - jW\right]$$

Let CEM = H $(\epsilon - 1)$

and CAW = -HW

For both cases

$$\sigma = CEM + j\,CAW$$

The distance factor

(3) $$\mathcal{f} = \left(\frac{2\pi}{\lambda \cdot K^2 a^2}\right)^{\frac{1}{3}} \cdot d$$

d = distance between antennas

(4) $$\tau_n = \tau_{n,\infty} - \sigma^{-\frac{1}{2}} - \frac{2}{3}\,\tau_{n,\infty}\,\sigma^{-\frac{3}{2}}$$

where

$$\tau_{1,\infty} = 1.856\ e^{-j\frac{\pi}{3}} = 1.856\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$\tau_{2,\infty} = 3.245\ e^{-j\frac{\pi}{3}} = 3.245\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$\tau_{3,\infty} = 4.382\ e^{-j\frac{\pi}{3}} = 4.382\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

and for n = 4, 5, 6, 7

$$\tau_{n,\infty} = \frac{1}{2}\left[3\pi\left(n+\frac{1}{4}\right)\right]^{\frac{2}{3}}\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

Let TNR = real part of $\tau_{n,\infty}$

TNI = imaginary part of $\tau_{n,\infty}$

$\tau_{n,\infty}$ = TNR + j TNI

Since $\sigma = CEM + j\,CAW$

$$\sigma = \sqrt{CEM^2 + CAW^2}\ e^{j\tan^{-1}\left(\frac{CAW}{CEM}\right)}$$

$$\sigma^{-\frac{1}{2}} = \left(\sqrt{CEM^2 + CAW^2}\right)^{-\frac{1}{2}} e^{-j\frac{\tan^{-1}\left(\frac{CAW}{CEM}\right)}{2}}$$

Let SMALL = $\left(\sqrt{CEM^2 + CAW^2}\right)^{-\frac{1}{2}}$

and $$\gamma = \frac{\tan^{-1}\left(\frac{CAW}{CEM}\right)}{2}$$

Then $\quad -\delta^{-\frac{1}{2}} = -SMALL\,(\cos\gamma - j\,SIN\,\gamma)$

$$\delta^{-\frac{3}{2}} = \left(\sqrt{CEM^2 + CAW^2}\right)^{-\frac{3}{2}} e^{-j\,\frac{3}{2}\tan^{-1}\left(\frac{CAW}{CEM}\right)}$$

Let $\quad CALL = \left(\sqrt{CEM^2 + CAW^2}\right)^{-\frac{3}{2}}$

and $\quad \phi = \frac{3}{2}\tan^{-1}\left(\frac{CAW}{CEM}\right)$

Then $\quad \delta^{-\frac{3}{2}} = CALL\,(\cos\phi - j\,SIN\,\phi)$

$$-\frac{2}{3}\gamma_{n,\infty}\,\delta^{-\frac{3}{2}} = -\frac{2}{3}\left[TNR + j\,TNI\right]\cdot\left[CALL\,(\cos\phi - j\,SIN\,\phi)\right]$$

$$= -\frac{2}{3}\,CALL\left[TNR\,\cos\phi + TNI\,SIN\,\phi + j\,(TNI\,\cos\phi - TNR\,SIN\,\phi)\right]$$

Then, referring to equation (4)

$$\gamma_n = TNR + j\,TNI - SMALL\,(\cos\gamma - j\,SIN\,\gamma) - \frac{2}{3}\,CALL\left[TNR\,\cos\phi + TNI\,SIN\,\phi + j\,(TNI\,\cos\phi - TNR\,SIN\,\phi)\right]$$

and

real part of $\gamma_n = TNR - SMALL\,\cos\gamma - \frac{2}{3}\,CALL\,(TNR\,\cos\phi + TNI\,SIN\,\phi)$

imaginary part of $\gamma_n = TNI + SMALL\,SIN\,\gamma - \frac{2}{3}\,CALL\,(TNI\,\cos\phi - TNR\,SIN\,\phi)$

Let the real part of $\gamma_n = RP$

and the imaginary part of $\gamma_n = IP$

Referring to equation (1)

$$\frac{e^{-j\tau_n\zeta}}{\delta + 2\tau_n} = \frac{e^{-j(RP + j\,IP)\zeta}}{(CEM + j\,CAW) + (2RP + j\,2IP)}$$

$$= \frac{e^{-j\,RP\,\zeta} \cdot e^{IP\,\zeta}}{(CEM + 2RP) + j(CAW + 2IP)}$$

Let $\alpha = IP\,\zeta$, $\beta = RP\,\zeta$

A = CEM + 2 RP, B = CAW + 2 IP

Then $$\frac{e^{-j\tau_n\zeta}}{\delta + 2\tau_n} = \frac{e^{-j\beta} \cdot e^{\alpha}}{A + jB}$$

$$= e^{\alpha}\left[\frac{A\cos\beta - B\sin\beta - j(B\cos\beta + A\sin\beta)}{A^2 + B^2}\right]$$

REAL PART = $\dfrac{e^{\alpha}(A\cos\beta - B\sin\beta)}{A^2 + B^2}$

IMAGINARY PART = $\dfrac{-e^{\alpha}(B\cos\beta + A\sin\beta)}{A^2 + B^2}$

Hence

$$\sum_{n=1}^{7} \frac{e^{-j\tau_n \delta}}{\delta + 2\tau_n} = \sum_{n=1}^{7} \text{REAL PART} + j \sum_{n=1}^{7} \text{IMAGINARY PART}$$

$$(5) \qquad \frac{e^{-j\tau_n \rho}}{\delta + 2\tau_n} = \left(\sum_{n=1}^{7} \text{REAL PART}\right)^2 + \left(\sum_{n=1}^{7} \text{IMAGINARY PART}\right)^2$$

Now the results of equations (2), (3), and (5) can be incorporated into equation (1).

BREMMER SERIES

OPERATION OF PROGRAM

1. Follow steps 1 through 3 for the operation of the Free Space Loss program.

2. Commands will be typed out as in the previous programs. Remember to be cautious and type in everything in floating point except the number of increments. Also type everything in the same units of distance.

3. This program is in two parts. At the end of the first part, four numbers will be typed out. These are real part of σ, imaginary part of σ, $|f(h_1)|^2$, $|f(h_2)|^2$, and $\left(\frac{2\pi}{\lambda} k^2 a^2\right)^{1/3}$

4. Load address 200 and start. The second half of the tape will load. Depress continue.

5. The command "Type the above numbers" will be typed. This refers to the five numbers typed from Part I.

6. More commands will be typed, so enter the appropriate data.

7. "Bremmer (db)" = the answer is typed and the next distance is requested.

```
C; BREMMER SERIES, A TWO PART PROGRAM.
1; FORMAT(/,"TYPE WAVE LENGTH",/)
2; FORMAT(/,"TYPE INDEX OF REFRACTION",/)
3; FORMAT(/,"TYPE RADIUS OF BODY",/)
4; FORMAT(/,"TYPE HEIGHTS OF ANTENNAS",/)
5; FORMAT(/,"TYPE DIELECTRIC CONSTANT",/)
6; FORMAT(/,"TYPE CONDUCTIVITY",/)
7; FORMAT(E,/)
 ; TYPE 1
 ; ACCEPT 7, AMBA
 ; TYPE 2
 ; ACCEPT 7, REFR
 ; TYPE 3
 ; ACCEPT 7, A
 ; TYPE 4
 ; ACCEPT 7, H1, H2
 ; TYPE 5
 ; ACCEPT 7, E
 ; TYPE 6
 ; ACCEPT 7, S
 ; H         = 6.2832 * REFR * A / AMBA
 ; H        = H * H
 ; H        = H ** .33333
 ; W         = 60. * AMBA * S
 ; Q1        = 2. * 3.1416 * H1 / AMBA
 ; Q2        = 2. * 3.1416 * H2 / AMBA
 ; ALFA      = -W / (E-1.)
 ; ALFA      = ATNF(ALFA)
 ; ALFA      =(ALFA) / 2.
 ; TINY      = (E-1.) * (E-1.) + W * W
 ; TINY      = SQTF(TINY)
 ; TINY      = SQTF(TINY)
33; FORMAT(/,"TYPE 1. FOR VERT. POL. OR 2. FOR HORIZ. POL",/)
 ; TYPE 33
 ; ACCEPT 7, COEF
 ; IF(2. - COEF)      33, 35, 34
34; F         = E *E - W * W
 ; G         = 2. * E * W
 ; DENOM     = (F * F + G * G) / H
 ; CEM       = ((E - 1.)*F + G*W) / DENOM
 ; CAW       = (G*(E - 1.) - (F*W)) / DENOM
 ; UND       = E*E + W*W
 ; X         = (W * COSF(ALFA) + E * SINF(ALFA)) / UND
 ; Y         = (E * COSF(ALFA) - W * SINF(ALFA)) / UND
 ; GO TO 19
35; CEM       = H * (E-1.)
 ; CAW       = - H * W
 ; Y         = COSF(ALFA)
 ; X         = SINF(ALFA)
19; FH1R      = 1. - (Q1 * TINY * X)
 ; FH1I      = Q1 * TINY * Y
 ; FH2R      = 1. - (Q2 * TINY * X)
```

```
; FH2I      = Q2 * TINY * Y
; SBA       = FH1R * FH1R + FH1I * FH1I
; ABS       = FH2R* FH2R + FH2I * FH2I
; XSE       = 6.2832 / (AMBA * REFR * REFR * A* A)
; XSE       = LOGF(XSE) / 3.
; XSE       = EXPF(XSE)
; TYPE 7, CEM, CAW, SBA, ABS, XSE
; STOP
; END
```

```
  C;       PART 2
 37; FORMAT(/,"TYPE THE ABOVE NUMBERS",/)
  7; FORMAT(E,/)
   ; TYPE 37
   ; ACCEPT 7, CEM, CAW, SBA, ABS, XSE
  8; FORMAT (I,/)
  9; FORMAT (/,"TYPE NUMBER OF INCREMENTS",/)
   ; TYPE 9
   ; ACCEPT 8, J
   ; DO 50  M=1,J
 10; FORMAT (/,"TYPE DISTANCE BETWEEN ANTENNAS",/)
 11; FORMAT (/,"I = ",I,/)
   ; TYPE 11, M
   ; TYPE 10
   ; ACCEPT 7, D
   ; AETA      = XSE * D
   ; EN        = 3.25
   ; SUMR      = 0.0
   ; RUMI      = 0.0
   ; DO 40  N = 1,7
   ; IF(N - 2)     12, 13, 14
 12; TNR       = .928
   ; GO TO 17
 13; TNR       = 1.6225
   ; GO TO 17
 14; IF(N-3)     50, 15, 16
 15; TNR       = 2.191
   ; GO TO 17
 16; EN        = EN + 1.
   ; TNR       = 3. * 3.1416 * EN
   ; TNR       = TNR * TNR
   ; TNR       = TNR ** .33333
   ; TNR       = .25 * TNR
 17; TNI       = -1.732 * TNR
   ; SMALL  = CEM*CEM + CAW*CAW
   ; SMALL  = SQTF(SMALL)
   ; SMALL  = SQTF(SMALL)
   ; SMALL  = 1./SMALL
   ; GAMMA  = CAW/CEM
   ; GAMMA  = ATNF(GAMMA)
   ; IF(CEM)   18,19,19
 18; GAMMA  = GAMMA + 3.1416
 19; GAMMA  = GAMMA/2.
   ; CALL   = SMALL*SMALL*SMALL
   ; PHI    = 3. * GAMMA
   ; REAL   = (2./3.)*CALL*(TNR*COSF(PHI) + TNI*SINF(PHI))
   ; REAL   = TNR - SMALL*COSF(GAMMA) - REAL
   ; EMAG   = (2./3.)*CALL*(TNI*COSF(PHI) - TNR*SINF(PHI))
   ; EMAG   = TNI + SMALL* SINF(GAMMA) - EMAG
   ; TNR    = REAL
   ; TNI    = EMAG
```

```
; ALFA        = AETA * TNI
  ; BETA       = AETA * TNR
  ; C          = EXPF(ALFA)
  ; ATE        =CEM + 2. * TNR
  ; BAT        = CAW + 2. * TNI
  ; C          = C/(ATE*ATE + BAT*BAT)
  ; PLUS       = C * (ATE*COSF(BETA) - BAT*SINF(BETA))
  ; SUMR       = SUMR + PLUS
  ; AWAY       = -C *(BAT*COSF(BETA) + ATE*SINF(BETA))
  ; RUMI       = RUMI + AWAY
40; CONTINUE
  ; VALUE      = SUMR * SUMR + RUMI * RUMI
  ; CONST      = 8. * 3.1416 * AETA
  ; BREM        = CONST * SBA * ABS * VALUE
20; FORMAT (/,"BREMMER (DB) = ",E,/)
  ; BREM   = 4.34294 * LOGF(BREM)
  ; TYPE 20, BREM
50; CONTINUE
  ; STOP
  ; END
```

EXPLANATION OF PROGRAM

The Bremmer Series Program begins by accepting

AMBA = λ , wave length

REFR = k, index of refraction

A = a, radius of body

H1, H2 = h_1, h_2, heights of antennas

E = ϵ , dielectric coefficient

S = σ, conductivity

From the beginning to label 33 the program computes values that are independent of polarization, distance, and n. They are, in the following order:

$$H = \left(\frac{2\pi k a}{\lambda}\right)^{\frac{2}{3}}$$

$$W = 60 \lambda \sigma$$

$$Q1 = \frac{2\pi h_1}{\lambda}$$

$$Q2 = \frac{2\pi h_2}{\lambda}$$

$$ALFA = \frac{\tan^{-1}\left(\frac{-W}{\epsilon - 1}\right)}{2}$$

$$TINY = \left(\sqrt{(\epsilon-1)^2 + W^2}\right)^{+\frac{1}{2}}$$

At label 33 the operator, depending on his requirements, can select horizontal or vertical polarization.

For vertical polarization the program goes to label 34. Here the real part and imaginary part of δ, CEM and CAW respectively, along with X and Y, factors in the calculation of $f(h_i)$, are computed. For horizontal polarization the program goes to label 35 where similar calculations are made.

The program then proceeds to label 19 where the previous calculations are used to compute the real part of $f(h_1)$, the imaginary part of $f(h_1)$, the real part of $f(h_2)$, the imaginary part of $f(h_2)$, $|f(h_1)|^2$, $|f(h_2)|^2$ and $\left(\frac{2\pi}{\lambda} k^2 a^2\right)^{1/3}$

The program then types out the values for CEM, CAW, $|f(h_1)|^2$, $|f(h_2)|^2$, $\left(\frac{2\pi}{\lambda} K^2 a^2\right)^{\frac{1}{3}}$ and stops. This is the end of the first part of the program.

Part 2 begins by requesting the operator to type the numbers typed at the end of Part 1. The program then goes into a "DO LOOP" which depends on the number of distances that are requested. The particular distance is then entered and

$$\rho = \left(\frac{2\pi}{\lambda K^2 a^2}\right)^{1/3} \cdot d \quad \text{is computed.}$$

The program then enters a nested "DO LOOP" in order to compute

$$\sum_{n=1}^{7} \frac{e^{-j\tau_n \rho}}{\delta + 2\tau_n}$$

SUMR and RUMI will eventually be the real and imaginary parts of the above formula.

During the first time through this nested "DO LOOP", TNR = .928 then the program goes to label 17 where TNI = $\sqrt{3}$ (.928). These values correspond to the real and imaginary parts of $\tau_{1,\infty}$.

The program then computes the real and imaginary parts of

$$\tau_1 = \tau_{1,\infty} - \delta^{-\frac{1}{2}} - \frac{2}{3}\tau_{1,\infty}\delta^{-\frac{3}{2}}$$

denoted by REAL and EMAG respectively.

Next the program uses these results to compute

$$\frac{e^{-j\tau_{1,\infty}\rho}}{\delta + 2\tau_{1,\infty}} = \frac{e^{\rho\, TNI} \quad e^{-j\rho\, TNR}}{(CEM + 2\,TNR) + j(CAW + 2\,TNI)}.$$

Then the real part is added to SUMR while the imaginary part is added to RUMI.

During the second and third time through the "DO LOOP" TNR = 1.6225 and 2.191 respectively, and the real and imaginary values are computed and added to SUMR and RUMI respectively. However, for the fourth through the seventh times, the equation

$$TNR = \frac{1}{4}\left(3\pi\left(n + \frac{1}{4}\right)\right)^{\frac{2}{3}} \qquad n = 4, 5, 6, 7 \qquad \text{is used}$$

EN corresponds to n + 1/4 and is incremented by one each pass through the loop. The program then goes to 17 where it proceeds as before.

After seven passes through the nested "DO LOOP". The program computes

$$\left|\sum_{n=1}^{7} \frac{e^{-j\tau_n \sigma}}{\sigma + 2\tau_n}\right|^2 = SUMR^2 + RUMI^2$$

BREM = CONST * SBA * ABS * VALUE is computed, and this result is then expressed in decibels by the formula

4.34294*LOGF (BREM) which corresponds to 10 LOG_{10} (BREM)

which corresponds to the general equation for the Bremmer Series expressed in decibels

$$L_B(db) = 10 \log_{10}\left(\frac{|\hat{E}|^2}{|E|_0^2}\right) = 10 \log_{10}\left[8\pi\rho \cdot |f(h_1)|^2 \cdot |f(h_2)|^2 \cdot \left|\sum_{n=1}^{7} \frac{e^{-j\tau_n \sigma}}{\sigma + 2\tau_n}\right|^2\right]$$