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MANNED SPACECRAFT CENTER
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AN EFFICIENT METHOD FOR CARTESIAN TRANSFORMATIONS

By Paul F. Flanagan and Samuel A. Pines
Mathematical Physics Branch

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AN EFFICIENT METHOD FOR

CARTESIAN TRANSFORMATIONS

By Samuel A. Pines and Paul F. Flanagan

1.0 SUMMARY AND INTRODUCTION

In this note, an efficient alternate method used to perform Cartesian coordinate transformations is derived. The method replaces the 3 by 3 matrix product transformation procedure by a simpler vector operation which requires less computer storage and less computer execution time. Also, for applications in which interpolation routines are applied to stored transformation matrices, the method outlined in this document is more efficient because it requires the interpolation and storage of only four elements, which results in a more accurate final computation.

2.0 CARTESIAN COORDINATE TRANSFORMATIONS BY MEANS OF RIGID VECTOR ROTATIONS

A 3 by 3 transformation matrix A is given which causes the Cartesian coordinate system I to be transformed to the Cartesian coordinate system II.

Thus, any vector R_I in the original coordinate system is carried into the second coordinate system by

$$R_{II} = AR_I \tag{1}$$

The same transformation can be achieved by a rigid rotation of R_I about a unit vector N through some angle w [eq. (2)]. The resultant vector equation may be found in reference 1.

$$R_{II} = R_I \cos w + N \cdot R_I (1 - \cos w) N + \sin w \; N x R_I \tag{2}$$

If the matrix is required, it can be generated from the unit vector N and the angle w by the following equation.

$$A = I \cos w + (1 - \cos w) NN^T + \sin w \, Nx \tag{3}$$

where

$$Nx = \begin{bmatrix} 0 & -N_3 & N_2 \\ N_3 & 0 & -N_1 \\ -N_2 & N_1 & 0 \end{bmatrix}$$

The vector N and the trigonometric function cos w are computed from the matrix as follows.

$$\cos w = \frac{\text{trace } A - 1}{2} \tag{4a}$$

$$\sin w = \sqrt{1 - \cos^2 w} \tag{4b}$$

$$N = [(a_{32} - a_{23})^2 + (a_{13} - a_{31})^2 + (a_{21} - a_{12})^2]^{-1/2} \begin{bmatrix} a_{32} - a_{23} \\ a_{13} - a_{31} \\ a_{21} - a_{12} \end{bmatrix} \tag{4c}$$

3.0 DERIVATION OF THE EQUIVALENT TRANSFORMATION EQUATION

In reference 1, it is shown that the rigid rotation matrix may be written as the sum of a symmetric and a skew symmetric matrix.

The symmetric matrix terms of equation (3) are

$$A \text{ sym} = I \cos w + (1 - \cos w) NN^T \tag{5a}$$

The skew symmetric terms are given by

$$A \text{ skew sym} = \sin w \, Nx \tag{5b}$$

The trace of A is given by the trace of its symmetric part; thus,

$$\text{trace } A = 3 \cos w + (1 - \cos w)(N_1^2 + N_2^2 + N_3^2) \tag{6}$$

Because N is a unit vector, equation (7) is obtained.

$$\cos w = \frac{\text{trace } A - 1}{2} \qquad (7)$$

Because a rigid rotation about N through an angle w greater than π may be replaced by a rigid negative rotation about the same vector N through $2\pi - w$, the angle w can be restricted to the first or second quadrants, and

$$\sin w = \sqrt{1 - \cos^2 w} \qquad (8)$$

To obtain the components of the vector N, note that the skew symmetric part of A is given by

$$A \text{ skew sym} = \frac{A - A^T}{2} \qquad (9)$$

This expression must be equal to the skew symmetric part of A (N, w) so that

$$\frac{A - A^T}{2} = \sin w \; Nx \qquad (10)$$

It follows that because N is a unit vector, equation 4(c) is obtained.

4.0 REFERENCE

1. Pines, S.; and Cockrell, B. F.: Partial Derivatives of Matrices Representing Rigid Body Rotations. MSC IN 68-FM-231, September 20, 1968.