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EFFECTS OF TRANSMITTED NOISE ON RECEIVER SIGNAL-TO-NOISE RATIOS

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ABBREVIATIONS, ACRONYMS, AND SYMBOLS

AGC	Automatic Gain Control
dB	Decibel
FM	Frequency Modulation
k	Boltzmann's Constant ($1.38\text{x}10^{-23}$ joules/°K)
L_{α}	Signal power degradation factor for the non-peak-power-limited channel
$L_{\alpha p}$	Signal power degradation factor for the peak-power-limited channel
N_r	Received noise power
N_x	Transmitted noise power
N_{th}	Thermal noise power in receiver bandwidth
NBW_x	Transmitted signal noise bandwidth
NBW_{ro}	Receiver output noise bandwidth
NSD_{act}	Actual receiver noise spectral density
P_{rt}	Total received power
P_{xt}	Total transmitted power
S_x	Transmitted signal power
S_r	Received signal power
S_{r-eff}	Effectived received signal power
SNR	Signal-to-Noise Ratio
SNR_{r-act}	Actual signal-to-noise ratio at receiver
SNR_{r-app}	Apparent signal-to-noise ratio at reciever

ABBREVIATIONS, ACRONYMS, AND SYMBOLS (CONT'D)

SNR_x	Transmitted signal-to-noise ratio
T	Effective system temperature of receiver
T_r	Effective system temperature resulting from received noise only
VCO	Voltage-Controlled Oscillator
α	Ratio of actual receiver noise spectral density to receiver noise spectral density due only to thermal noise

$$\alpha = \frac{NSD_{act}}{kT}$$

SECTION 1

INTRODUCTION

1.1 PURPOSE

The purpose of this report is to present methods of calculating receiver signal-to-noise ratios (SNR's) for communications channels over which noise, in addition to signal, is transmitted. In most well-designed systems, the transmitted noise is insignificant (transmit SNR > 30 dB) and only thermal noise is considered at the receiver. This, however, may not always be the case, as is shown by considering a relay communication system. Since the signal-to-noise ratio received at the relay point may be low, a significant amount of noise may be transmitted from the relay to the end point.

1.2 SCOPE

For discussion purposes, the noise transmission is divided into two types. The first type of transmission involves a channel that has no limit on power. That is, the total channel power increases as the transmitted noise power increases. The second type of transmission is that of a peak-power-limited channel in which signal power is decreased as noise power is added, resulting in a constant total transmitted power.

Sections 3 and 4 present methods for calculating actual receiver SNR's for the above types of transmission. Section 3 presents methods for combining the transmitted signal-to-noise ratio with an apparent receiver signal-to-noise ratio (that receiver SNR that would be present if no noise were transmitted) to give the actual receiver signal-to-noise ratio. Section 4 presents methods for combining the transmitted and apparent receiver signal-to-noise ratios to obtain a signal power degradation factor. After applying this signal power degradation factor, the resulting effective signal power is divided by the thermal noise power to give the actual receiver signal-to-noise ratio.

1.2 SCOPE (CONT'D)

The following notations will be used throughout this report:

Transmitter

P_{xt} = Total transmitted power

S_x = Transmitted signal power

N_x = Transmitted noise power

$SNR_x = \frac{S_x}{N_x}$ = Transmitted signal-to-noise ratio

Receiver

P_{rt} = Total received power

S_r = Received signal power

N_r = Received noise power

N_{th} = Thermal noise power in receiver bandwidth

$SNR_{r\text{-}act} = \frac{S_r}{N_r + N_{th}}$ = Actual signal-to-noise ratio at receiver

$SNR_{r\text{-}app}$ = Apparent signal-to-noise ratio at receiver

$$= \begin{cases} \frac{S_r}{N_{th}} \left(\text{or } \frac{P_{rt}}{N_{th}} \text{ if } N_x = 0 \right), & \text{for the non-peak-power-limited channel} \\ \frac{P_{rt}}{N_{th}}, & \text{for the peak-power-limited channel} \end{cases}$$

SECTION 2

SUMMARY AND CONCLUSIONS

2.1 GENERAL

Methods of calculating receiver signal-to-noise ratios are developed for channels over which noise, in addition to signal, is transmitted. It is common practice to assume an infinite transmitted signal-to-noise ratio and, therefore, to calculate receiver signal-to-noise ratios based on total received power levels and thermal noise. It is shown, however, that the transmitted noise may indeed be significant. For some types of channels, the transmitted noise level may even exceed the transmitted signal level.

2.2 CONCLUSIONS

The effects of transmitted noise on receiver signal-to-noise ratios become more significant as the transmitted signal-to-noise ratio (SNR_X) becomes smaller. The effects are somewhat more severe for the peak-power-limited channel, since noise is transmitted only at the expense of signal.

Furthermore, as mentioned previously, transmitted signal-to-noise ratios may be significantly small, and the use of classical equations, which assume infinite transmitted signal-to-noise ratios, may not suffice for accurate communication system analysis. These classical equations may, however, be used with the equations presented in this report for more accurate analysis.

SECTION 3

DETERMINATION OF ACTUAL SIGNAL-TO-NOISE RATIOS AT THE RECEIVER

3.1 GENERAL

This section develops the equations required to calculate actual receiver signal-to-noise ratios when noise as well as signal is transmitted. Two types of transmission conditions will be discussed in this section.

3.2 NON-PEAK-POWER-LIMITED CHANNEL

The first type of noise transmission involves a channel with no peak power limitation; that is, a channel for which the total transmitted power increases as the noise power increases. An example of this type of noise addition is an FM system in which VCO jitter adds noise power over that of the modulating signal. An equation by which actual received signal-to-noise ratios may be calculated is derived in the following paragraphs.

The total transmitted power, P_{xt}, is given by

$$P_{xt} = S_x + N_x \tag{1}$$

where

S_x = the transmitted signal power (constant),

N_x = the transmitted noise power.

The apparent signal-to-noise ratio at the receiver (or the receiver SNR calculated assuming that the total received power is signal only and that the noise power is thermal only) is given by

$$SNR_{r-app} = \frac{S_r}{N_{th}} \tag{2}$$

3.2 NON-PEAK-POWER-LIMITED CHANNEL (CONT'D)

where

S_r = the received signal power,

N_{th} = the thermal noise power present in the receiver bandwidth (NBW_{ro}).

The transmitted signal power is independent of the transmitted noise power, so the received signal power is independent of the received noise power. If signal only is transmitted ($N_x = 0$), then the received signal power is equal to the total received power (P_{rt}). Therefore,

$$SNR_{r\text{-}app} = \frac{P_{rt}}{N_{th}}, \quad \text{if } N_x = 0 . \tag{3}$$

When noise is transmitted, the actual signal-to-noise ratio at the receiver must be calculated using both received noise power (N_r) and thermal noise power (N_{th}), or

$$SNR_{r\text{-}act} = \frac{S_r}{N_r + N_{th}} = \frac{1}{\frac{N_r}{S_r} + \frac{N_{th}}{S_r}} \tag{4}$$

But

$$\frac{N_r}{S_r} = \frac{N_x}{S_x} = \frac{1}{SNR_x} \tag{5}$$

where SNR_x is the transmitted signal-to-noise ratio; and, by equation (2),

$$\frac{N_{th}}{S_r} = \frac{1}{SNR_{r\text{-}app}} \tag{6}$$

Therefore,

$$SNR_{r\text{-}act} = \frac{1}{\frac{1}{SNR_x} + \frac{1}{SNR_{r\text{-}app}}} = \frac{1}{\frac{SNR_{r\text{-}app} + SNR_x}{(SNR_x)(SNR_{r\text{-}app})}} \tag{7}$$

or

$$SNR_{r\text{-}act} = \frac{(SNR_x)(SNR_{r\text{-}app})}{SNR_x + SNR_{r\text{-}app}} \tag{8}$$

Note that in the preceding equations, it is assumed that the receiver output filters pass all of the received noise, i.e., $NBW_x \leq NBW_{ro}$. If this is not the case, then the transmitted SNR must be calculated for a noise bandwidth equal to NBW_{ro}. To summarize,

$$\left.\begin{array}{l} N_x \text{ is calculated in } NBW_x \\ N_{th} \text{ is calculated in } NBW_{ro} \end{array}\right\} \quad \text{for } NBW_x \leq NBW_{ro}$$

$$\left.\begin{array}{l} N_x \text{ is calculated in } NBW_{ro} \\ N_{th} \text{ is calculated in } NBW_{ro} \end{array}\right\} \quad \text{for } NBW_x \geq NBW_{ro}$$

Equation (8) is plotted as a family of curves in Figure 3-1. After calculating the received signal-to-noise ratio assuming noisless transmission ($SNR_{r\text{-}app}$) and the actual transmitted signal-to-noise ratio (SNR_x), the actual received signal-to-noise ratio ($SNR_{r\text{-}act}$) may be found from Figure 3-1.

3.3 PEAK-POWER-LIMITED CHANNEL

The second type of noise transmission involves a channel with a limitation on transmitted power. That is, as noise power is added to the channel, signal power is decreased such that the total power is constant. An example of this type of transmission

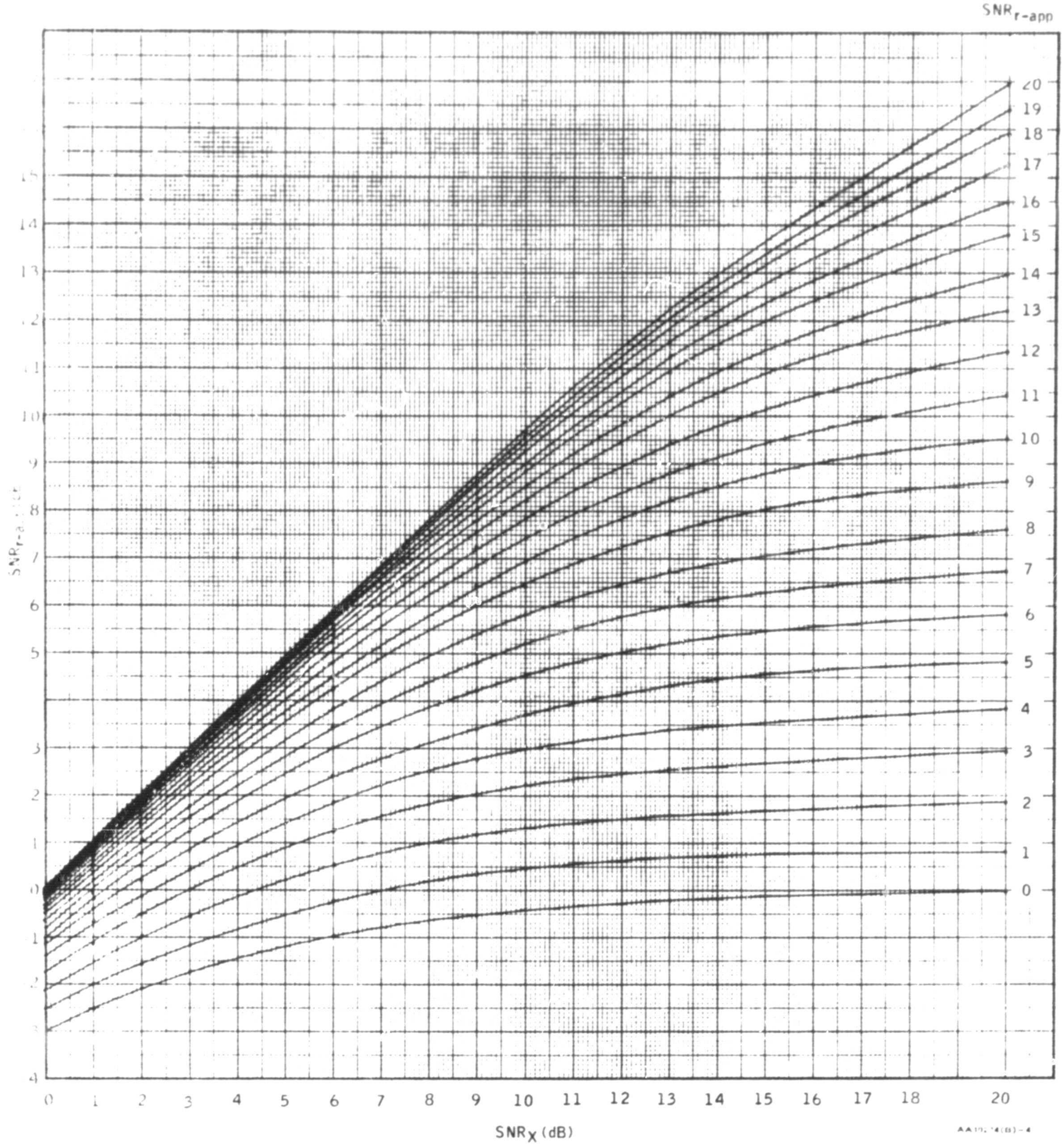

Figure 3-1 Actual Receiver Signal-to-Noise Ratio (SNR$_{r-act}$) vs Transmitted (SNR$_X$) and Apparent Received (SNR$_{r-app}$) Signal-to-Noise Ratios (Non-Peak-Power-Limited Channel)

is a relay communication system in which received signal and noise are passed through an ideal automatic gain control (AGC) amplifier before modulating a carrier (or subcarrier). Total transmitted and received powers may be expressed as

$$P_{xt} = S_x + N_x = \text{constant} \tag{9}$$

$$P_{rt} = S_r + N_r \tag{10}$$

Again defining $SNR_{r\text{-}app}$ as the apparent signal-to-noise ratio at the receiver, calculated using P_{rt} (assuming an infinite transmitted SNR),

$$\begin{aligned} SNR_{r\text{-}app} &= \frac{P_{rt}}{N_{th}} \\ &= \frac{S_r + N_r}{N_{th}}, \text{ when noise is transmitted.} \end{aligned} \tag{11}$$

The actual signal-to-noise ratio at the receiver may be expressed as

$$\begin{aligned} SNR_{r\text{-}act} &= \frac{S_r}{N_r + N_{th}} \\ &= \frac{1}{\frac{N_r}{S_r} + \frac{N_{th}}{S_r}} \\ &= \frac{1}{\frac{N_r}{S_r} + \left[\frac{(N_{th})\left(1 + \frac{N_r}{S_r}\right)}{(S_r)\left(1 + \frac{N_r}{S_r}\right)}\right]} \\ &= \frac{1}{\frac{N_r}{S_r} + \left(\frac{N_{th}}{S_r + N_r}\right)\left(1 + \frac{1}{SNR_x}\right)} \end{aligned} \tag{12}$$

3.3 PEAK-POWER-LIMITED CHANNEL (CONT'D)

But

$$\frac{N_{th}}{S_r + N_r} = \frac{N_{th}}{P_{rt}} = \frac{1}{SNR_{r-app}} \tag{13}$$

and

$$\frac{N_r}{S_r} = \frac{1}{SNR_x} \tag{14}$$

Therefore,

$$SNR_{r-act} = \frac{1}{\frac{1}{SNR_x} + \left(\frac{1}{SNR_{r-app}}\right)\left(\frac{SNR_x + 1}{SNR_x}\right)}$$

$$= \frac{1}{\frac{SNR_{r-app} + SNR_x + 1}{(SNR_x)(SNR_{r-app})}}$$

$$= \frac{(SNR_x)(SNR_{r-app})}{1 + SNR_x + SNR_{r-app}} \tag{15}$$

Again, consideration must be given the noise bandwidths of the transmitted signal and the receiver output filter as was done for the non-peak-power-limited channel. The family of curves representing this equation is plotted in Figure 3-2. Figure 3-2 is utilized in the same manner as that described for Figure 3-1.

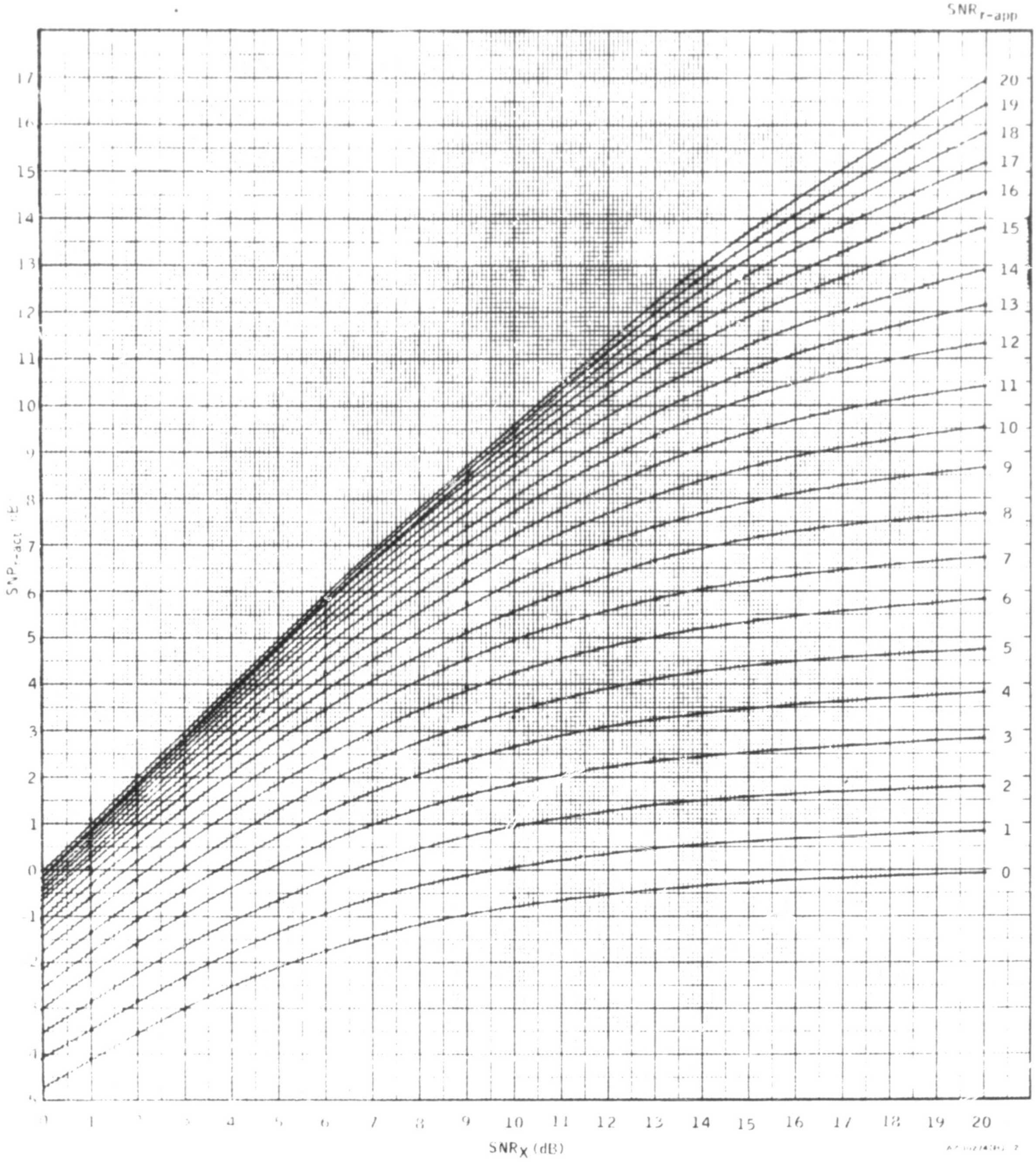

Figure 3-2 Actual Receiver Signal-to-Noise Ratio (SNR_{r-act}) vs Transmitted (SNR_X) and Apparent Received (SNR_{r-app}) Signal-to-Noise Ratios (Peak-Power-Limited Channel)

SECTION 4

ALTERNATE DETERMINATION OF SIGNAL-TO-NOISE RATIOS AT THE RECEIVER

4.1 GENERAL

In the preceding section, the actual signal-to-noise ratios at the receiver were calculated in terms of the transmitted signal-to-noise ratio and the apparent signal-to-noise ratio (assuming no transmitted noise) at the receiver. For the case of no peak power limitation on the transmitter, these calculations basically consist of adding the received noise power to the thermal noise power in the receiver bandwidth. Signal-to-noise ratio is then determined by dividing the received signal power (which is *independent* of the transmitted or received noise power) by the total noise power. For the peak power limited case, the calculations are slightly more involved, since the received signal power is *not* independent of the received noise power. Therefore, it is necessary to subtract the received noise power from the total received power to obtain received signal power, and then add the received noise power to the thermal noise power to obtain total noise power. The actual signal-to-noise ratio at the receiver is then calculated in the usual manner. For either case, receiver signal-to-noise ratios are determined by dividing actual signal power by actual noise power. The expressions for actual signal-to-noise ratio are eventually reduced until the only variables are transmitted signal-to-noise ratio and apparent received signal-to-noise ratio.

An alternate method of calculating actual receiver signal-to-noise ratios consists of calculating a *signal power degradation factor,* applying this either to the received signal power (for the non-peak-power-limited channel) or to the total received power (for the peak-power-limited channel) to determine an *effective signal power,* and then dividing by *thermal noise power* only.

4.2 NON-PEAK-POWER-LIMITED CHANNEL

For the case of no transmitter peak power limitation,

$$SNR_{r,act} = \frac{S_r}{N_r + N_{th}} \qquad (15)$$

SECTION 4

ALTERNATE DETERMINATION OF SIGNAL-TO-NOISE RATIOS AT THE RECEIVER

4.1 GENERAL

In the preceding section, the actual signal-to-noise ratios at the receiver were calculated in terms of the transmitted signal-to-noise ratio and the apparent signal-to-noise ratio (assuming no transmitted noise) at the receiver. For the case of no peak power limitation on the transmitter, these calculations basically consist of adding the received noise power to the thermal noise power in the receiver bandwidth. Signal-to-noise ratio is then determined by dividing the received signal power (which is *independent* of the transmitted or received noise power) by the total noise power. For the peak power limited case, the calculations are slightly more involved, since the received signal power is *not* independent of the received noise power. Therefore, it is necessary to subtract the received noise power from the total received power to obtain received signal power, and then add the received noise power to the thermal noise power to obtain total noise power. The actual signal-to-noise ratio at the receiver is then calculated in the usual manner. For either case, receiver signal-to-noise ratios are determined by dividing actual signal power by actual noise power. The expressions for actual signal-to-noise ratio are eventually reduced until the only variables are transmitted signal-to-noise ratio and apparent received signal-to-noise ratio.

An alternate method of calculating actual receiver signal-to-noise ratios consists of calculating a *signal power degradation factor,* applying this either to the received signal power (for the non-peak-power-limited channel) or to the total received power (for the peak-power-limited channel) to determine an *effective signal power,* and then dividing by *thermal noise power* only.

4.2 NON-PEAK-POWER-LIMITED CHANNEL

For the case of no transmitter peak power limitation,

$$SNR_{r\text{-}act} = \frac{S_r}{N_r + N_{th}} \qquad (15)$$

4.2 NON-PEAK-POWER-LIMITED CHANNEL (CONT'D)

But

$$N_{th} = kT(NBW_{ro}) \qquad (16)$$

$$N_r = kT_r(NBW_{ro}) \qquad (17)$$

where k = Boltzman's constant,

T = contribution to receiver effective system temperature resulting from thermal noise

T_r = contribution to receiver effective system temperature resulting from the received noise

NBW_{ro} = output noise bandwidth of the channel under consideration.

The total received noise in the channel bandwidth is given by

$$\begin{aligned} N_r + N_{th} &= k\,(T + T_r)NBW_{ro} \\ &= (NSD_{act})NBW_{ro} \\ &= \alpha(kT)(NBW_{ro}) \\ &= \alpha\, N_{th}, \qquad (18) \end{aligned}$$

where

NSD_{act} = the actual receiver noise spectral density resulting from both thermal and received noise, and

$$\alpha = \frac{NSD_{act}}{kT} = \frac{N_r + N_{th}}{N_{th}}.$$

4.2 NON-PEAK-POWER-LIMITED CHANNEL (CONT'D)

The actual receiver signal-to-noise ratio is

$$\begin{aligned} SNR_{r\text{-}act} &= \frac{S_r}{\alpha N_{th}} \\ &= \frac{\left(\frac{S_r}{\alpha}\right)}{N_{th}} \\ &= \frac{S_{r\text{-}eff}}{N_{th}} \; , \end{aligned} \qquad (19)$$

where $S_{r\text{-}eff}$ is the *effective* received signal power.

But

$$\begin{aligned} S_{r\text{-}eff} &= \frac{S_r}{\alpha} \\ &= \frac{S_r}{\dfrac{N_r + N_{th}}{N_{th}}} \\ &= \frac{1}{\dfrac{N_r + N_{th}}{S_r N_{th}}} \\ &= \frac{1}{\dfrac{N_r}{S_r N_{th}} + \dfrac{1}{S_r}} \end{aligned}$$

$$= \frac{S_r}{S_r\left(\frac{1}{SNR_x N_{th}} + \frac{1}{S_r}\right)}$$

$$= \frac{S_r}{\frac{SNR_{r-app}}{SNR_x} + 1}$$

$$= \frac{S_r\ SNR_x}{SNR_x + SNR_{r-app}} \qquad (20)$$

The effective received signal power may be expressed as

$$S_{r-eff} = (S_r)(L_\alpha) \qquad (21)$$

where L_α is the signal power degradation factor,

$$L_\alpha = \frac{SNR_x{}^{*}}{SNR_x + SNR_{r-app}}. \qquad (22)$$

If all quantities are expressed in decibels, then

$$S_{r-eff} = S_r + L_\alpha \qquad (23)$$

Figure 4-1 contains plots of L_α in dB for various values of SNR_x and SNR_{r-app}.

It should be noted that the effective signal power is computed by applying the signal power degradation factor to the *received signal power*. The received signal power is, for the case under consideration, independent of transmitted noise power and is equal to the total received power (P_{rt}) only when no noise is transmitted. Once

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*Refer to page 3-3 for transmitted signal noise bandwidth considerations.

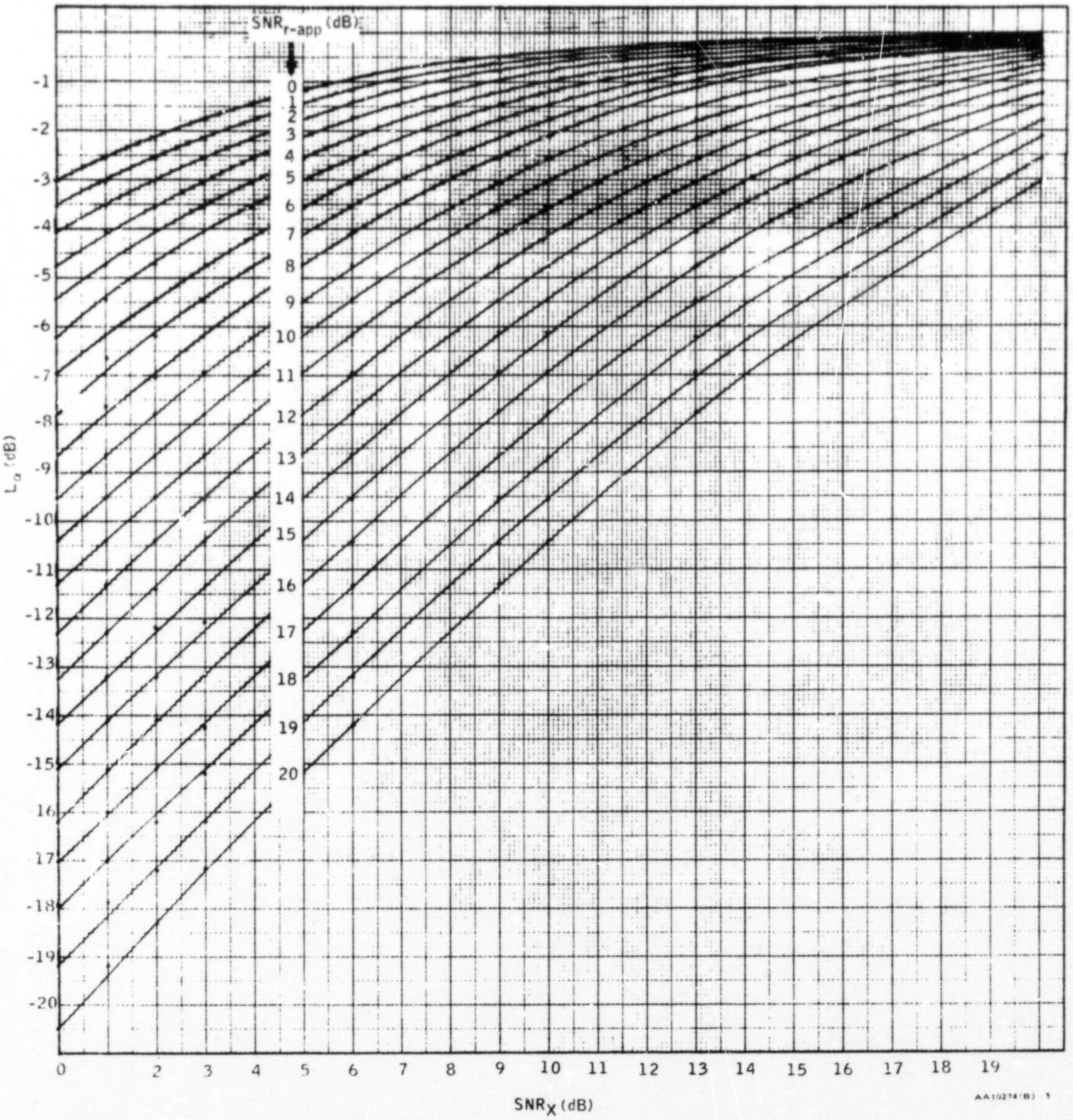

Figure 4-1 Signal Power Degradation Factor (L$_\alpha$) vs Transmitted (SNR$_X$) and Apparent Received (SNR$_{r\text{-}app}$) Signal-to-Noise Ratios (Non-Peak-Power-Limited Channel)

the effective signal power is determined, the receiver signal-to-noise ratio is found by dividing by the thermal noise power (N_{th}). The signal power degradation factor (L_α) may be applied to all receiver channels, so any channel signal-to-noise ratio may be determined by using only thermal noise power.

4.3 PEAK-POWER-LIMITED CHANNEL

For the case of a peak-power-limited transmitter,

$$SNR_{r\text{-}act} = \frac{S_r}{N_r + N_{th}} \tag{24}$$

But

$$N_{th} = kT(NBW_{ro}) \tag{25}$$

$$N_r = kT_r(NBW_{ro})$$

Then

$$\begin{aligned} N_r + N_{th} &= k\,(T + T_r)NBW_{ro} \\ &= (NSD_{act})(NBW_{ro}) \\ &= (\alpha)(kT)(NBW_{ro}) = \alpha N_{th} \end{aligned} \tag{27}$$

The actual receiver signal-to-noise ratio is given by

$$SNR_{r\text{-}act} = \frac{S_r}{\alpha N_{th}}$$

$$= \frac{\left(\frac{S_r}{\alpha}\right)}{N_{th}}$$

$$= \frac{S_{r\text{-}eff}}{N_{th}} \qquad (28)$$

where $S_{r\text{-}eff}$ is again defined as the *effective* received signal power.

The expression for effective received signal power may again be simplified as follows:

$$S_{r\text{-}eff} = \frac{S_r}{\alpha}$$

$$= \frac{S_r}{\frac{N_r + N_{th}}{N_{th}}}$$

$$= \frac{1}{\frac{N_r + N_{th}}{S_r\ N_{th}}}$$

$$= \frac{1}{\frac{N_r}{S_r N_{th}} + \frac{1}{S_r}}$$

$$= \frac{P_{rt}}{P_{rt}\left(\frac{1}{SNR_x N_{th}} + \frac{1}{S_r}\right)}$$

$$= \frac{\left(\frac{S_r}{\alpha}\right)}{N_{th}}$$

$$= \frac{S_{r\text{-}eff}}{N_{th}} \tag{28}$$

where $S_{r\text{-}eff}$ is again defined as the *effective* received signal power.

The expression for effective received signal power may again be simplified as follows:

$$S_{r\text{-}eff} = \frac{S_r}{\alpha}$$

$$= \frac{S_r}{\frac{N_r + N_{th}}{N_{th}}}$$

$$= \frac{1}{\frac{N_r + N_{th}}{S_r\ N_{th}}}$$

$$= \frac{1}{\frac{N_r}{S_r N_{th}} + \frac{1}{S_r}}$$

$$= \frac{P_{rt}}{P_{rt}\left(\frac{1}{SNR_x N_{th}} + \frac{1}{S_r}\right)}$$

$$= \frac{P_{rt}}{\dfrac{P_{rt}}{SNR_x N_{th}} + \dfrac{P_{rt}}{S_r}}$$

$$= \frac{P_{rt}}{\dfrac{SNR_{r-app}}{SNR_x} + \dfrac{S_r + N_r}{S_r}}$$

$$= \frac{P_{rt}}{\dfrac{SNR_{r-app}}{SNR_x} + 1 + \dfrac{1}{SNR_x}}$$

$$= \frac{P_{rt}\ SNR_x}{1 + SNR_x + SNR_{r-app}} \qquad (29)$$

The effective received signal power may be expressed as

$$S_{r-eff} = (P_{rt})(L_{\alpha p}), \qquad (30)$$

where

$$L_{\alpha p} = \frac{SNR_x{}^{*}}{1 + SNR_x + SNR_{r-app}} \qquad (31)$$

and is the signal power degradation factor. If all quantities are expressed in decibels, then

$$S_{r-eff} = P_{rt} + L_{\alpha p} \qquad (32)$$

Thus, the actual signal-to-noise ratio at the receiver may be determined by dividing the effective signal power (S_{r-eff}) by the

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*Refer to page 3-3 for transmitted signal noise bandwidth considerations.

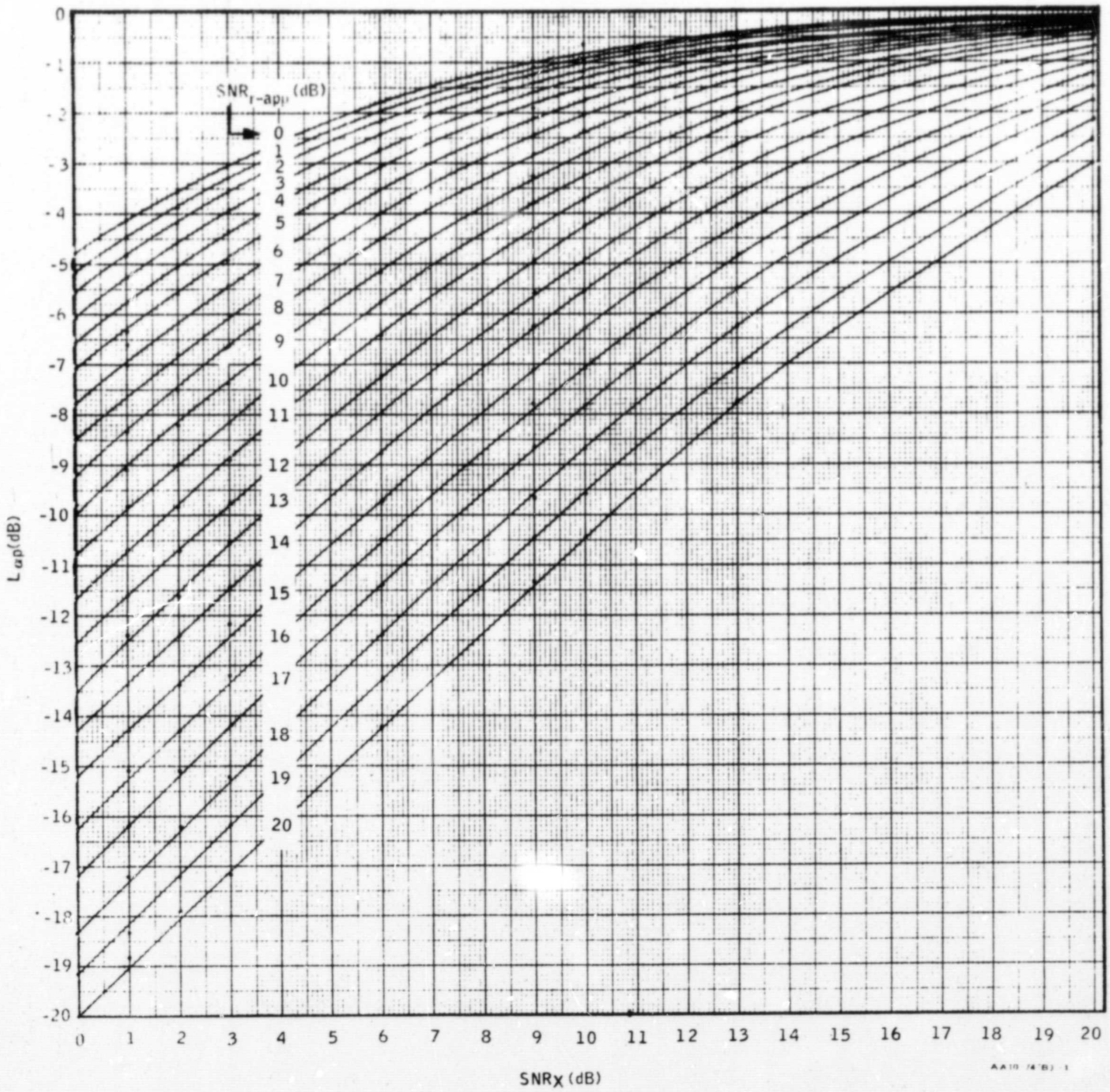

Figure 4-2 Signal Power Degradation Factor ($L_{\alpha p}$) vs Transmitted (SNR_x) and Apparent Received (SNR_{r-app}) Signal-to-Noise Ratios (Peak-Power-Limited Channel)

thermal noise power (N_{th}). Again, the signal power degradation factor ($L_{\alpha p}$) may be applied to all receiver channels, so any channel signal-to-noise ratio may be determined by using only thermal noise power. Figure 4-2 contains plots of $L_{\alpha p}$ for various values of SNR_x and SNR_{r-app}.