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A GUIDE FOR THE APPLICATION OF THE BRUCETON METHOD TO ELECTRO-EXPLOSIVE DEVICES

By

Fred M. Speed

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
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A GUIDE FOR THE APPLICATION OF THE BRUCETON METHOD TO ELECTRO-EXPLOSIVE DEVICES

Prepared by: Fred M. Speed

Fred M. Speed
Theory and Analysis Office

Approved by: Eugene L. Davis

Eugene L. Davis, Jr.
Chief, Theory and Analysis Office

Approved by: Eugene H. Brock

Eugene H. Brock
Chief, Computation and Analysis Division

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TABLE OF CONTENTS

INTRODUCTION

Because of the extensive use of Electro-explosive devices (EED's) in Apollo subsystems, there is a need for methods of man rating these devices. One such method is the "Bruceton Method," developed at Bruceton, Pennsylvania, in 1944, by the Statistical Research Group (1), Princeton University. Since that time, there have been many extensions and modifications of the original Bruceton Method. While a few of these changes were for the better, the overall effect was the production of a number of contradictory documents. As a consequence, many experimenters have misused or misapplied the Bruceton Method; therefore, the results they obtained were invalid.

The purpose of this report is to provide a reliable up-to-date guide for the application of the Bruceton Method to EED's. A number of areas that have caused confusion in the past are discussed, for example: the concept of the all-fire and no-fire point, the response curve and what it means, the sample size required, the concept of confidence limits, and the critical assumption of normality.

SYMBOLS

u	the mean of the normal distribution
σ	the standard deviation of the normal distribution
m	an estimate of u
s	an estimate of σ
A.F.	estimate of the All-Fire point
A.F.C.	estimate of the All-Fire point with 90% confidence
N.F.	estimate of the No-Fire point
N.F.C.	estimate of the No-Fire point with 90% confidence
S_m	estimate of standard deviation of μ
S_s	estimate of the standard deviation of σ

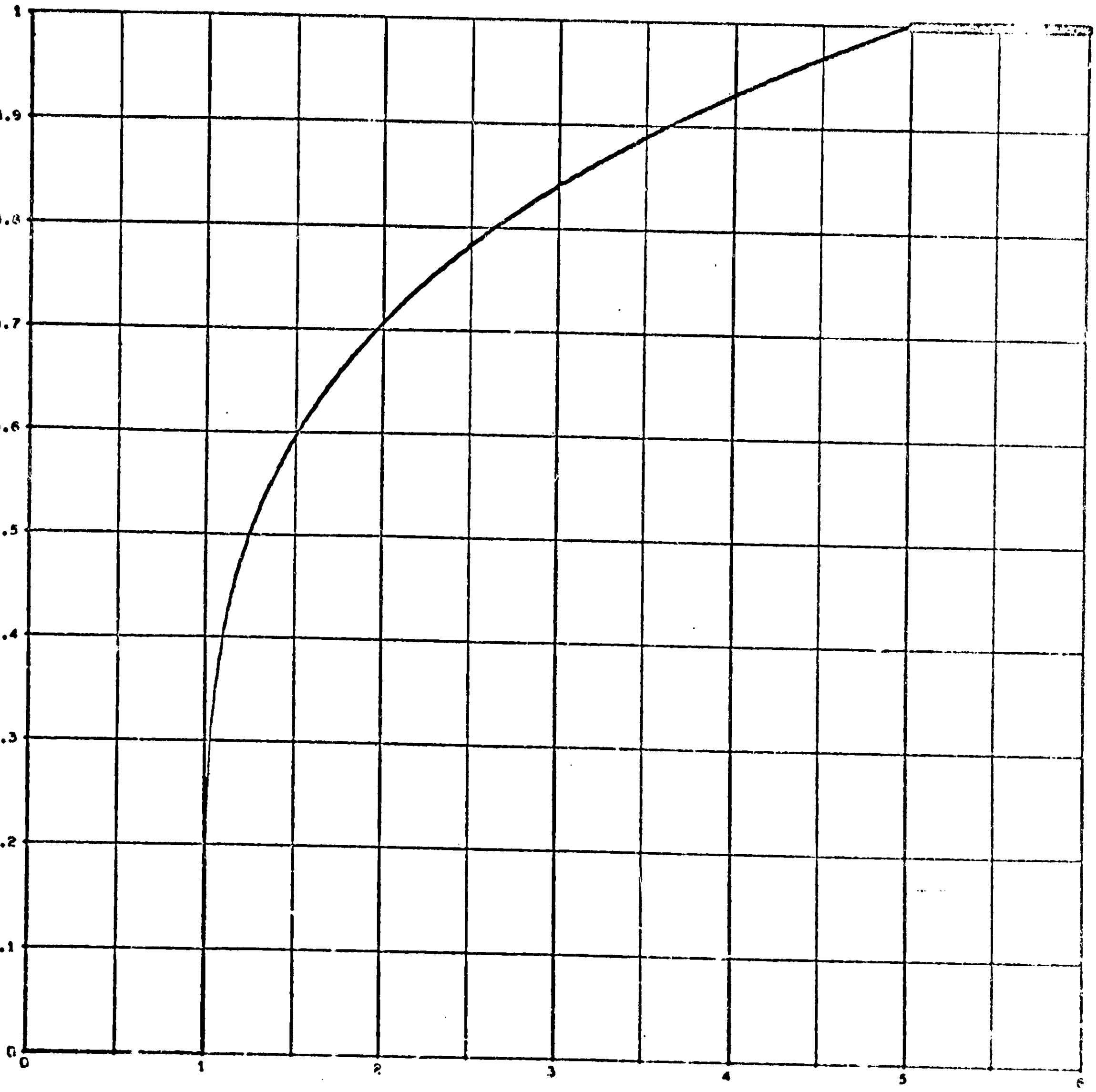
RESPONSE CURVE

The purpose of the Bruceton test is to estimate a response curve. The response curve represents the percent of units that should fire at a given current. Typical response curves are shown in GRAPHS 1-4. The Bruceton test requires that the response curve be a cumulative normal curve. This curve, shown in GRAPH 5, is characterized by two parameter μ, the mean, and σ, the standard deviation; i.e., if μ and σ are known the entire curve can be generated. Hence, the purpose of the Bruceton test can be stated more explicitly: the purpose of the Bruceton test is to obtain an estimate of μ, namely m; and an estimate of σ, namely s; and with these estimates generate an estimated response curve.

All-Fire and No-Fire Points

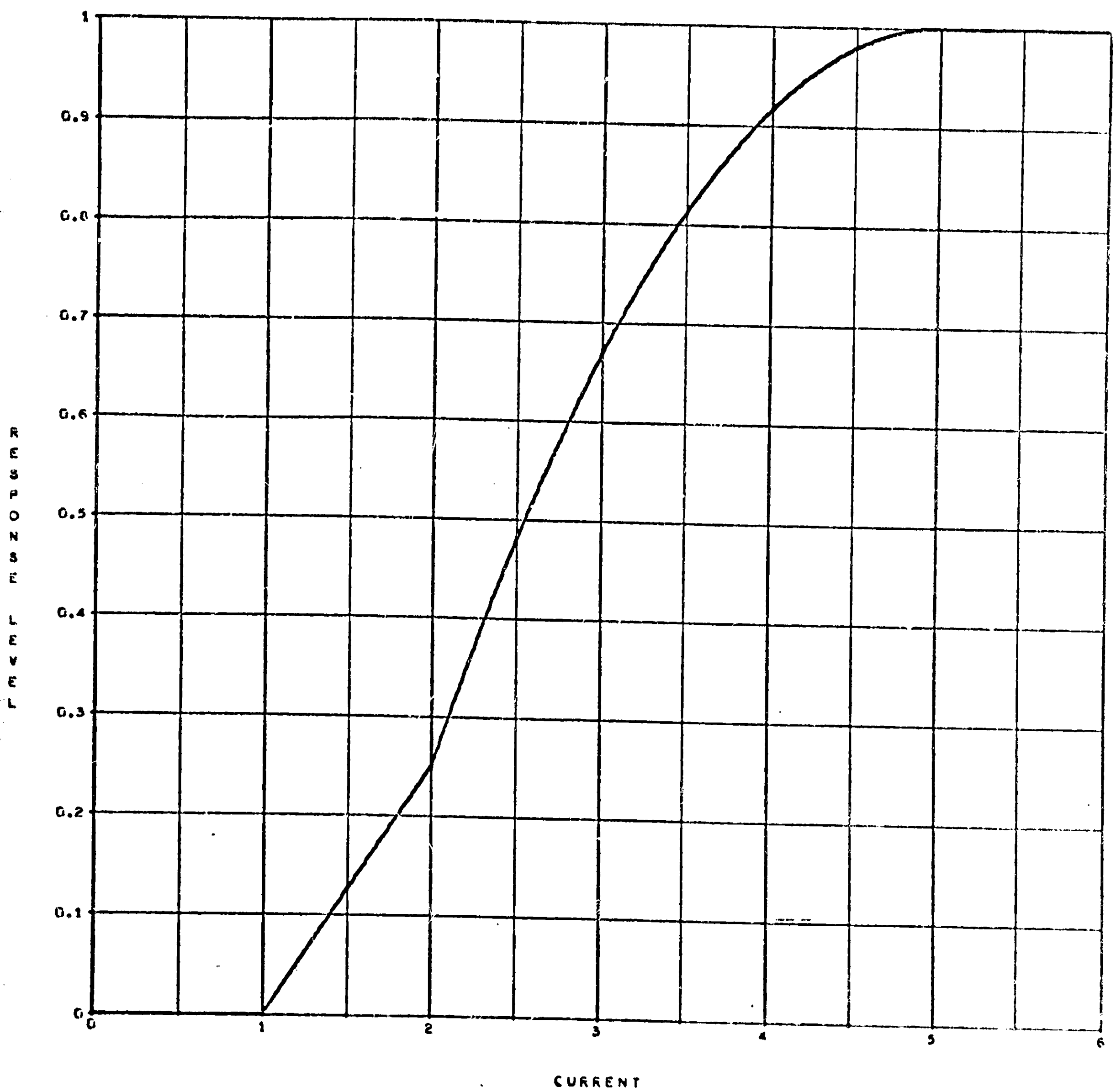
The term all-fire point has come to mean that current at which 99.9 percent of the units will fire and the no-fire point has come to mean that point at which .1 percent of the units will fire. Once estimates of μ and σ are obtained, an estimated response curve can be generated; thus, these two points can be located. For example, in GRAPH 5 it is seen that the all-fire point is 1.04 and the no-fire point is 3.56. The above definition of all-fire and no-fire points should be used regardless whether the experimenter is interested only in the no-fire point and not the all-fire point and vice-versa. In this connection, this paper will not engage in semantics in trying to define a "success" or a "failure"; these two words have been left out of the text due to the confusion they have caused in the past. If the experimenter is consistent in his definition of all-fire and no-fire, there can be no confusion.

GRAPH 1
TYPICAL RESPONSE CURVE FOR ELECTRO-EXPLOSIVE DEVICES



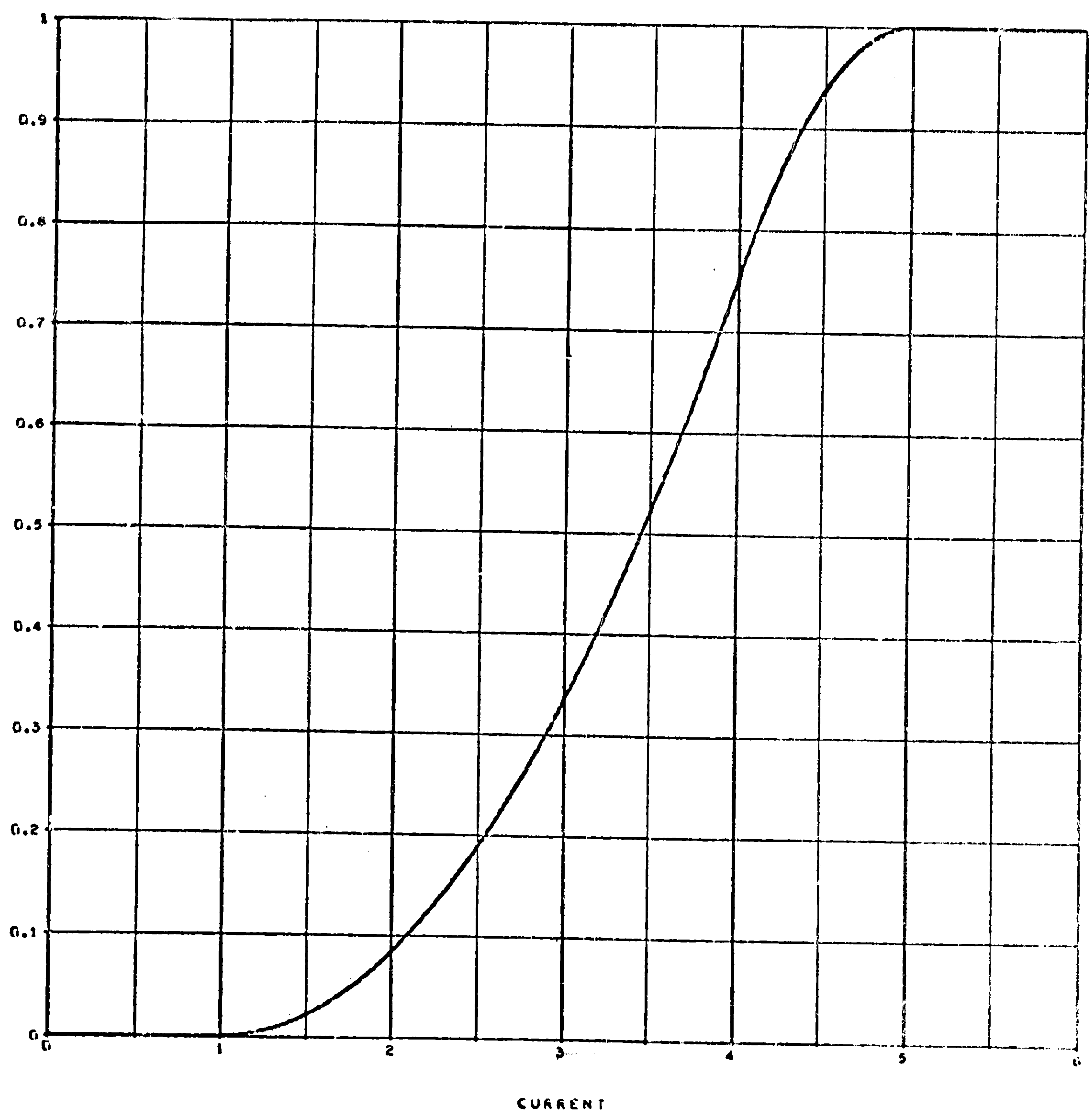
GRAPH 2

TYPICAL RESPONSE CURVE FOR ELECTRO-EXPLOSIVE DEVICES

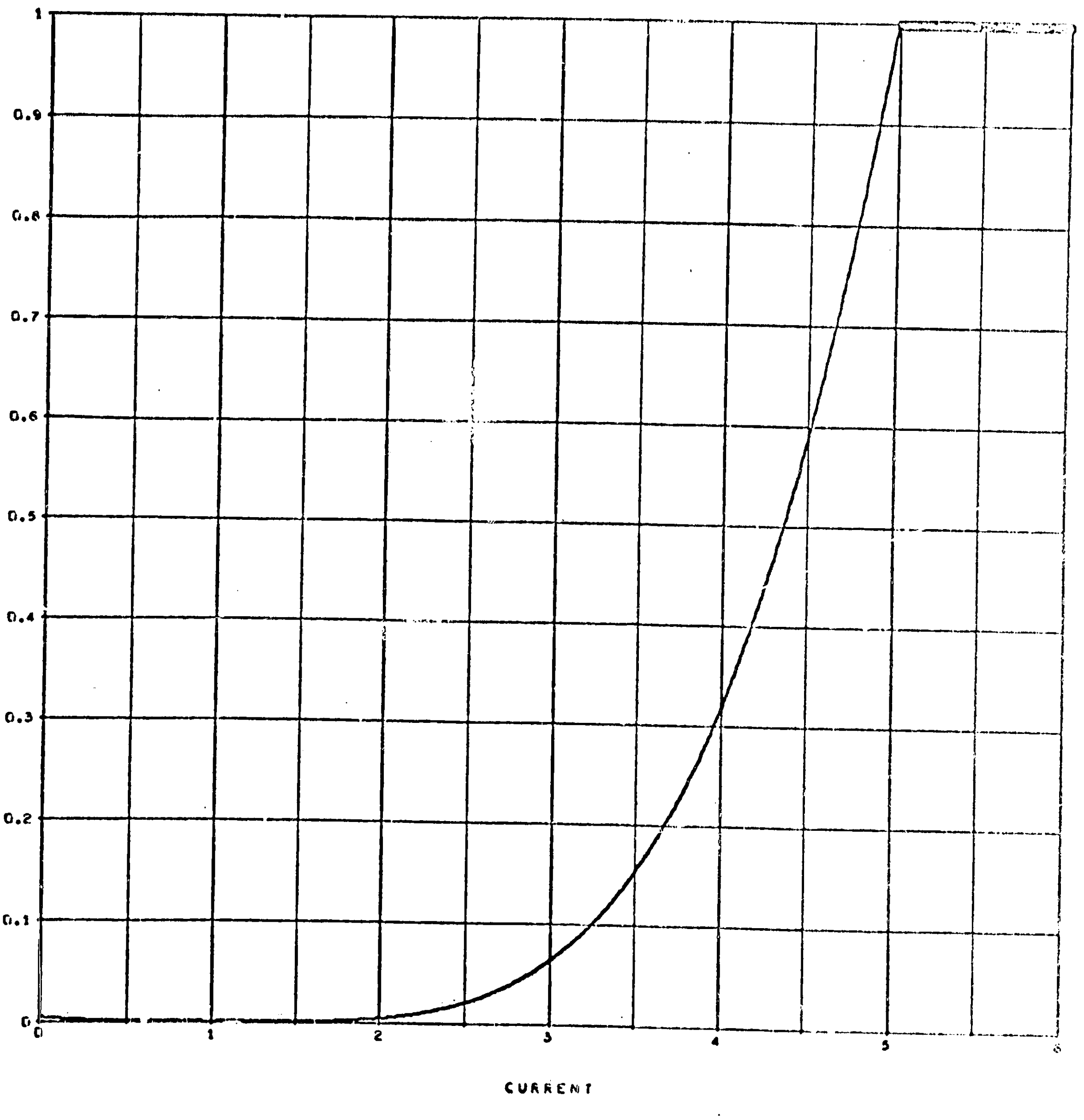


GRAPH 3

TYPICAL RESPONSE CURVE FOR ELECTRO-EXPLOSIVE DEVICES

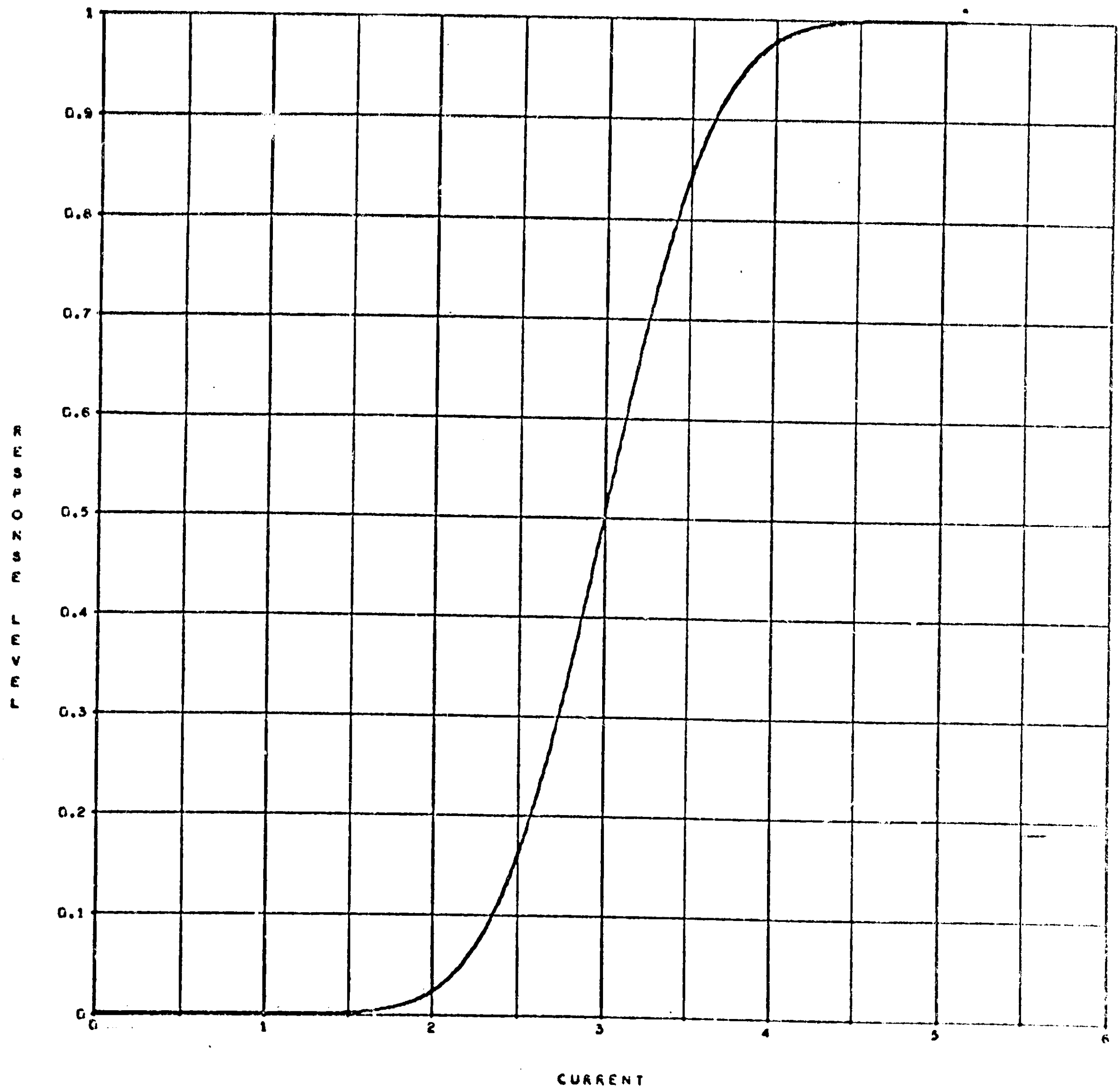


GRAPH 4
TYPICAL RESPONSE CURVE FOR ELECTRO-EXPLOSIVE DEVICES



GRAPH 5

NORMAL RESPONSE CURVE FOR ELECTRO-EXPLOSIVE DEVICES



Confidence Limits

Since m and s are only estimates of μ and σ respectively, the estimated response curve will differ from the true, but unknown, response curve. Hence, the estimates of the AF point and the NF point may be in error. In order to overcome these errors, the idea of confidence limits has been employed. This is explained best by an example. Suppose that in a metal container there were 10,000 white and black balls and that someone was interested in estimating the percentage of white balls. He could count the number of whites in the container, but that would be rather tedious. Rather he takes a sample of 300 and counts the number of white balls in his sample; this number divided by 300 represents an estimate of the true proportion p of white balls in the container. Note that it is only an estimate of the true value. It may be a very poor estimate if the sampling plan had defects in it. Now suppose he observed 235 white balls in his sample of 300. Then $\hat{p}$, the estimate of p would be equal to $\frac{235}{300} = .783$. But the experimenter is not content with knowing that $\hat{p} = .783$, he wants some confidence in this number. Depending upon what he wants, he can choose between three types of confidence limits.

They are:

(1) two-sided; i.e., he wants to know that with 100α percent confidence that the true proportion p lies between two numbers, say a_1 and a_2.

(2) one-sided to the left; i.e., he wants to know that with 100α percent confidence that the true proportion p is less than some number b .

(3) one-sided to the right; i.e., he wants to know that with 100α percent confidence that the true proportion p is greater than some number c_1 .

Note that α is some number between 0 and 1 that represents how confident the experimenter wishes to be. Suppose that he choose $\alpha = .90$. The three confidence limits would then be as follows:

The experimenter would be 90 percent confident that the true value of the proportion p would

(1) lie between .740 and .822 i.e., $.740 \leq p \leq .822$.

(2) b less than: i.e., $p \leq .817$

(3) be greater than: i.e., $p \leq 749$

The interpretation given to the above confidence limits is as follows: If the experimenter repeated his experiment 100 times and constructed his confidence limits, the true proportion p would lie within the prescribed limits 90 percent of the time. He would erroneously assume p is within the prescribed limits about 10 percent of the time.

When confidence limits are used for the all-fire and no-fire points, it is obvious that the most useful ones are the one-sided confidence limits. The experimenter does not want to know that the all-fire point lies between a_1 and a_2 . He usually has some maximum current that can be supplied and he wants to make sure the all-fire point is well below this level. The same logic holds true for the no-fire point. Many of the documents followed in the past explain only the two-sided confidence limits. This results in their use regardless of the situation.

SAMPLE SIZE

The number of specimens in the sample determines to a large extent the amount of information which can be derived. A limited amount of information can be obtained from small samples; precise conclusions require large samples. If one wishes to locate a mean only within wide limits a small sample will suffice. If, however, it is necessary to estimate the mean accurately, a large sample will be required.

The fact that sensitivity data do not consist of exact measurements reduces the amount of information contained in them. Roughly speaking, the amount of information is at best about half that which would be available in measured data. That is, a sample of 100 explosions and non-explosions will contain information equivalent to that in a sample of, at most, 50 measured critical currents. Sample sizes will therefore have to be larger than in statistical investigations where exact measurements may be made.

The method of statistical analysis of the data has been simplified considerably by the use of approximations which become poor for small samples. This puts a lower limit on the sample size if the present method of analysis is to be used.

In the original document on the Bruceton Method (1), the authors state repeatedly that their method is not valid for small samples, especially when the sample size is less than 100.

The next section which contains the Bruceton procedure has been taken for the most part from the original Bruceton report (1). However, a number of changes have been made in order to up-date the original report.

BRUCETON PROCEDURE

Preliminary Remark

A considerable amount of statistical investigation deals with quantities which are continuous variables, but which cannot be measured in practice. For example, in testing the sensitivity of electro-explosive devices to current, a common procedure is to test a number of EED's at a given current. There are currents at which some devices will fire and others will not, and it is assumed that those which did not fire would have fired had the current been sufficiently large. The assumption is, therefore, that different devices have different "critical" currents, and that if a device is tested at a given current it will or will not fire depending on whether the current at which it was fired is greater than or less than the critical current of that specimen. Furthermore, it is not possible to "creep up" on the critical current of a specimen by testing it at successively greater currents until it fires because this procedure alters the characteristics of the explosive mixture and thereby changes its critical current.

A current is a simple measurable variable but it is clear that the critical current of electro-explosive devices are not directly measurable in practice. All that one can do with a single device is determine whether its critical current is greater than or less than a chosen current at which the device is tested.

Methods will be provided for estimating parameters (mean, standard deviation, percentage points) and for testing statistical hypotheses. The methods will not be valid for small samples as numerous approximations will be made which rapidly become less accurate as the sample size decreases from 100. The methods are based on the assumption that the critical currents (or some known function of them) have a normal probability distribution. It is important that this assumption be reasonably well satisfied.

If the currents are not normally distributed, it is desirable to transform them so that they will have a normal distribution. In biological research, it is usually found that the logarithms of the critical concentrations themselves are normally distributed. There is some evidence in explosives research that logarithms of critical currents are more nearly normally distributed than the critical currents themselves, but the evidence is not so conclusive or so generally accepted in this field. This is a question, therefore, which will usually require some preliminary investigation. Any effort devoted to finding whether the critical currents are normally distributed, and if not, what function of the critical currents is normally distributed, will be well worthwhile.

The Experimental Procedure

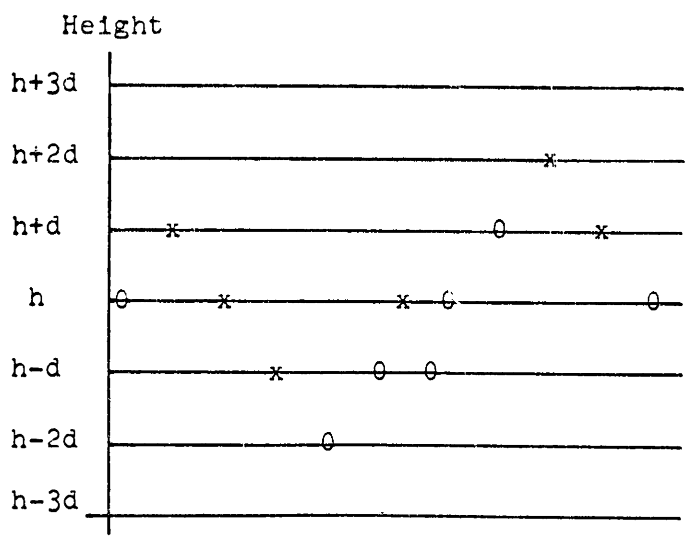
The procedure, described in terms of the explosives experiment, consists of the following steps:

(1) Choose a current h at which the first specimen will be tested, and an interval d which will be the distance between testing currents.

(2) If the first specimen explodes when tested at current h , the second specimen will be tested at current h-d . If the first specimen does not explode, the second specimen will be tested at current h+d .

(3) In general a specimen will be tested at a current d below the current at which the previous specimen was tested if that specimen exploded, and d above the current at which the previous one was tested if it did not explode.

In this manner one will obtain a sequence of fires and no-fires, where the x's denote fires and 0's denote no-fires.



Here the first specimen did not explode, so the second one was tested at h+d ; the second did explode, so the third was tested at h , the level just below h+d ; the third exploded, so the fourth was tested at h-d ; the fourth exploded, so the fifth was tested at h-2d ; the fifth did not explode, so the sixth was tested at h-d , the level just above h-2d.

Choice of h and d

On the basis of past experience with other explosives, it is usually possible to make rough estimates of the mean and standard deviation for the explosive to be tested. Such rough estimates will be useful in choosing h and d for the experiment to be performed.

If m and s are the estimated mean and standard deviation respectively, then good choices for h and d are simply $h = m$ and $d = s$. These are not the best choices for all circumstances, but they will serve very well for most purposes. Any great improvement in them would require that the preliminary estimates be very accurate.

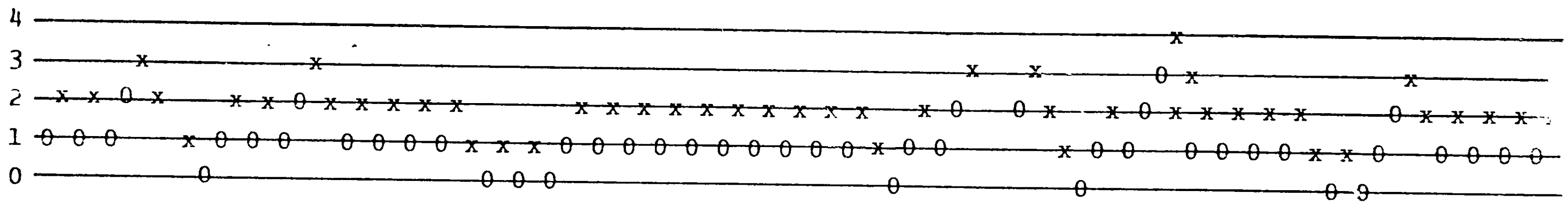
Computation Estimates of the Mean and Variance.

It is assumed that there is known or has been found by preliminary investigation a variable (the critical current, or some function of the critical current), which is normally distributed. Any reference to critical current in the following discussion will be to these "normalized" critical currents. The original testing current h and the interval d will be measured in terms of the normalized current. If, for example, the normalized current is the logarithm of the actual currents and the normalized interval is one, then corresponding to normalized current 0, 1, 2, 3, 4, would be actual testing currents of 0, 1, 10, 100, 1000 amperes.

Figure 6 is a chart showing the result of an actual test of 100 specimens of an explosive; here x denotes a fire and 0 a no-fire. On the left, the lines on which there were tests are numbered from zero to four. In the table below the chart are given the lines numbers, the actual testing currents (in amperes), the natural logarithms of the testing heights, the number of explosions, and the number of non-explosions at

Figure 6

The following is a chart showing the result of an actual test of 100 specimens of an explosive



i	h_i	$\ln h_i$	Frequencies x	Frequencies 0
4	3.67	1.300	1	
3	3.33	1.207	6	1
2	3.05	1.114	35	6
1	2.78	1.021	8	35
0	2.53	.928	0	8

each level. The set of numbers is the same in both columns as will always be the case when there are equal numbers of zeros and x's . When the total number of zeros is unequal to the total number of x's , the number of zeros at a given current will not differ by more than one from the number of x's at the next greater current.

The actual currents were chosen so that their natural logarithms were equally spaced, since previous investigations of similar explosives revealed that the natural logarithms of the critical currents could be assumed to be normally distributed. The interval $h = .093$ in ln units was used because another experiment with a similar explosive gave this value for the standard deviation.

The mean and standard deviation will be estimated from either the zeros or the x's depending on which occur in fewer numbers. In the particular example given in Figure 6, either the zeros or the x's may be used, but had there been 49 zeros and 51 x's, the zeros would have been used.

Let $n_0, n_1, n_2, \cdots$ be the number of zeros (or x's as the case may be) on the 0^{th}, 1^{st}, 2^{nd} ... lines respectively and let N be the total number of zeros (or x's). Let c be the normalized current of the lowest line on which there was a test recorded. In the example the x's will be used and the zeros disregarded; the numbers just defined will then be: $N = 50$, $c = .928$, $n_0 = 0$, $n_1 = 8$, $n_2 = 35$, $n_3 = 6$, $n_4 = 1$.

The formula for the estimate of the mean is

$$(1) \qquad m = c + d\left[\frac{1}{N}\sum in_i + \frac{1}{2}\right]$$

if the no-fires are used; or

$$(2) \qquad m = c + d\left[\frac{1}{n}\sum in_i - \frac{1}{2}\right]$$

if the fires are used.

The standard deviation will be determined by computing

$$(3) \quad S = 1.620d\left\{\left[\frac{N\sum i^2 n - (\sum in_i)^2}{N^2} + 0.029\right]\right\}$$

The sums appearing in (1), (2), and (3) can easily be computed on a tabular form:

i	n_i	in_i	i^2n_i
0	0	0	0
1	8	8	8
2	35	70	140
3	6	18	54
4	1	4	16
	N = 50	A = 100	B = 218

where the data of the example in Figure 6 have been used, and the two sums $\sum in_i$ and $\sum i^2n_i$ have been represented by A and B . In terms of A and B the formulas (1), (2), and (3) may be written

(5) $$m = c + d\left[A/N \pm \frac{1}{2}\right]$$

(6) $$S = 1.620d\left[\frac{NB - A^2}{N^2} + 0.029\right]$$

where in (5) the plus sign is to be taken if the no-fires are used and the minus sign taken if the fires are used. Equation (6) is to be used only if $(NB - A^2)/N^2 > .3$.

All computations involving the currents are to be done in terms of the normalized currents and only final results transformed to actual currents. Thus for the particular example, the mean is

(7) $$m = .928 + .073\left[\frac{100}{50} - \frac{1}{2}\right]$$

$$m = 1.068$$

and the standard deviation is

(8) $$S = 1.620\ (.093)\left[\frac{50(218) - (100)^2}{50} + 0.029\right]$$
$$= .0586$$

In terms of actual currents, the mean $m = 1.068$, corresponds to 2.91 amperes; this is not the mean current but the median current, that is, the current at which there is an even chance that a particular specimen will or will not fire. The standard deviation S must always be used in normal units (ln units in this case), so there is no point in transforming it to amperes.

Estimate of the All-Fire and No-Fire Points.

The estimate of the All-Fire point (A.F.) is given by equation (9).

$$(9)\quad A.F. = m + 3.09S$$

the estimate of the No-Fire point (N.F.) is given by equation (10)

$$(10)\quad N.F. = m - 3.09S$$

One-sided confidence limits for A.F. and N.F.

In order to calculate the one-sided confidence limits, it is necessary to calculate the standard deviation of m and the standard deviation of s . These are obtained from equation (11) and (12).

$$(11)\quad S_m = (6s + \alpha)/7\sqrt{N} \text{ (valid only if } d < 3S)$$

$$(12)\quad S_s = (1.1s + .35^2/d)/\sqrt{N} \text{ (valid only if } d < 2S)$$

the 100α percent one-sided confidence limits for A.F. is given by equation (13).

$$(13)\quad A.F.C. = A.F. + t_{\alpha,N-1}\sqrt{S_m^2 + (3.09)^2\, S_s^2}$$

where $t_{\alpha,N-1}$ is a value obtained from cumulative t - dis tribution with n-1 degrees of freedom and probability α .

The 100α percent one-sided confidence limit for N.F. is given by equation (14)

$$(14) \quad N.F.C. = N.F. - t_{\alpha,N-1}\sqrt{S_m^2 + (3.09)^2\, S_s^2}$$

where $t_{\alpha,N-1}$ is a value obtained from a cumulative t - distribution with n-1 degrees of freedom and probability α .

In the example cited above:

$$S_m = \frac{6\ (.0586) + .093}{7\sqrt{50}} = .0090$$

$$S_s = \frac{1.1\ (.0586) + .3\ (.0586)^2/.093}{\sqrt{50}} = .0107$$

$$t_{.90,49} = 1.296$$

N.F. = 1.068 - 3.09 (.0586) = .8864 ; N.F. = 2.42 amperes

A.F. = 1.068 + 3.09 (.0586) = 1.2486 ; A.F. = 3.48 amperes

Estimate of the No-Fire point with 90 percent confidence
N.F.C. = 2.31 amperes
Estimate of the All-Fire point with 90 percent confidence
A.F.C. = 3.65 amperes

Maximum Likelihood Solutions

Since equations (3), (11), and (12) are valid only under certain conditions, it is advantageous to use the maximum likelihood method to obtain estimates of m , s , S_m and

S_s . However, this method requires a detailed computer program which is not available at the present time. This program should be available in the near future from:

The Computation and Analysis Division
Manned Spacecraft Center
Houston, Texas

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