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RTCC REQUIREMENTS FOR MISSION G: LUNAR MODULE ATTITUDE DETERMINATION USING ONBOARD OBSERVATION

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Mathematical Physics Branch

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MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

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PROJECT APOLLO

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SUMMARY AND INTRODUCTION

The orientation of the LM is recorded just after lunar landing with respect to a moon fixed coordinate system and stored onboard. The preferred nominal mode of surface inertial measurement unit (IMU) alignment uses optical sightings on two stars with the alignment optical telescope. If it is found that an alignment cannot be made with the alignment optical telescope and if the stored alignment has changed due to LM settling, a separate LM attitude determination method must be available. This note presents a method for determining LM attitude on the lunar surface by processing rendezvous radar shaft and trunnion angle measurements. These angles relate the CSM-LM line of sight to the LM body axes. The ground Real-Time Computer Complex (RTCC) will process this data and a telemetered gravity vector, in body coordinates, to determine the attitude.

This note presents the revised formulation (basic requirements) for the RTCC program and supersedes reference 1. This is a separate program from the Manned Space Flight Network (MSFN) data processor used for orbit determination. However, the data batching (preprocessor) is identical to the data batching for the Mission G landing site determination program which is described in detail in reference 2. The MSFN orbit determination processor and predictor (ref. 3) will be used to determine the CM ephemeris over the landing site.

PROCEDURE FOR PROCESSING ONBOARD RENDEZVOUS RADAR OBSERVATIONS TO DETERMINE LM ATTITUDE

The LM body orientation will be defined with respect to a local vertical coordinate system by three rotations about the local vertical system axes. A knowledge of both CSM and LM positions is assumed. The rendezvous radar must track the CSM, and the rendezvous radar shaft and trunnion angles must be transmitted via downlink to earth. In addition, the astronaut will determine a gravity vector in body coordinates by monitoring the IMU accelerometers at two special orientations of the stable member. This, too, must be transmitted to earth. The three rotations will be determined using a weighted least squares, three-element state by solving the following basic equation:

$$\Delta F = \left(\sum_{i=1}^{n} A^T W A \right)^{-1} \left(\sum_{i=1}^{n} A^T W \Delta y \right)$$

where

$F = (\alpha_1, \alpha_2, \alpha_3)$ three rotations about the local vertical system axes

y = observation (shaft or trunnion)

$A = \frac{\partial y}{\partial F}$

W = observation weight matrix

Δy = observation residual (observed - computed)

i = observation frame index

Flow charts 1 and 2 present the detailed logic for the program supervisor and convergence processor, respectively.

PREPROCESSOR TO HANDLE TELEMETERED DATA

A preprocessor is required to handle the telemetered data since this data will not be handled by the preprocessor program used for normal ground tracking. The function of this routine is to multiply the incoming telemetered rendezvous radar data by the correct granularity constants and store the data into batches suitable for subsequent use by the attitude processor. This preprocessor and these data batches are the same as used for the LM position determination and are explained in detail in reference 1. Data determined invalid by editing will be preceded by a minus sign. From these data batches, working batches will be generated which will have the following format.

Working Data Batch

Batch ID	No. of Obs. frames	
Time of observations		Observation frame no. 1
Shaft	Trunnion	

ATTITUDE START ROUTINE

The operator must select one of two modes for this routine. In one mode only the first rotation (azimuth) is determined from rendezvous radar data. The two other rotations are computed as direct functions of a gravity vector in LM body coordinates. This gravity vector is determined by the pilot and transmitted to earth by telemetry. The solutions of the second and third rotation angles from this gravity vector are:

$$\alpha_2 = \sin^{-1}(-g_z)$$

$$\alpha_3 = \tan^{-1}\left(-\frac{g_y}{g_x}\right)$$

where $(g_x, g_y\ g_z)$ = unit gravity vector in body coordinates. This vector has its origin at the LM and points in the direction of the lunar center of mass.

The other mode uses rendezvous radar data to determine all three rotations, and the gravity vector is not used.

INITIALIZATION

In setting up onboard data for processing a single pass of data the operator specifies the following:

1. Batch ID to be processed - must be rendezvous radar batches.

2. LM position vector ID.

   (a) Computed estimate from landing site determination routine.

   (b) Primary navigation and guidance system vector.

(c) Abort guidance system vector.

3. Initial attitude - must be entered as defined below:

(a) For mode II, enter gravity vector ID.

(b) For mode I, default

Reference 4 should be consulted for details on the above general input description.

The operator can process a maximum of two batches of data at one time under the following conditions:

1. The MSFN determination of the CSM orbit should be equally good for both passes.

2. The LM must not have moved during the time between the batches. This will be checked by comparing and displaying gravity vectors. A minimum of three gravity vectors will probably need to be downlinked for the following times:

(a) Prior to the first batch.

(b) Prior to the second batch but following the first.

(c) Following the second batch.

For processing two batches together the operator selects:

1. The two batch ID's.

2. The LM vector.

3. The initial attitude:

(a) For mode II, enter gravity vector ID.

(b) For mode I, default.

STATE VECTOR

The three-element state F for this problem will be defined as three positive rotations about a local vertical coordinate system. The local vertical system is centered at the LM and has axes along the local vertical, in the direction of lunar north, and in the direction of lunar east. The three positive rotations are ordered as follows.

1. About local vertical, α_1.
2. About displaced east, α_2.
3. About displaced north, α_3.

The transformation from local vertical to LM body coordinates is then defined.

$$R_b = \begin{pmatrix} X_b \\ Y_b \\ Z_b \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha_3 & \sin\alpha_3 & 0 \\ -\sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{pmatrix} \begin{pmatrix} X_{LV} \\ Y_{LV} \\ Z_{LV} \end{pmatrix}$$

or $R_b = F(\alpha) R_{LV}$

where

$$F(\alpha)_{11} = \cos \alpha_3 \cos \alpha_2$$

$$F(\alpha)_{12} = \sin \alpha_3 \cos \alpha_1 + \cos \alpha_3 \sin \alpha_2 \sin \alpha_1$$

$$F(\alpha)_{13} = \sin \alpha_3 \sin \alpha_1 - \cos \alpha_3 \sin \alpha_2 \cos \alpha_1$$

$$F(\alpha)_{21} = -\sin \alpha_3 \cos \alpha_2$$

$$F(\alpha)_{22} = \cos \alpha_3 \cos \alpha_1 - \sin \alpha_3 \sin \alpha_2 \sin \alpha_1$$

$$F(\alpha)_{23} = \cos \alpha_3 \sin \alpha_1 + \sin \alpha_3 \sin \alpha_2 \cos \alpha_1$$

$$F(\alpha)_{31} = \sin \alpha_2$$

$$F(\alpha)_{32} = -\cos \alpha_2 \sin \alpha_1$$

$$F(\alpha)_{33} = \cos \alpha_2 \cos \alpha_1$$

and R_{LV} is the LM local vertical state.

OBSERVATION WEIGHTS

Shaft and trunnion weights (W_S, W_T) will be computed by the program as functions of the computed observations using the following formulation:

$$W_S = \frac{C_1}{\sigma_S^2}$$

and

$$W_T = \frac{C_2}{\sigma_T^2}$$

where

$$\sigma_S = k_1 + |k_2 \dot{S}|$$

$$\sigma_T = k_3 + |k_4 \dot{T}|$$

and C_1, C_2 are weight coefficients explained below.

For $i > 1$,

$$\dot{S}_i = \frac{S_i - S_{i-1}}{t_i - t_{i-1}}$$

and

$$\dot{T}_i = \frac{T_i - T_{i-1}}{t_i - t_{i-1}}$$

S_i and T_i are the computed shaft and trunnion at time t_i. The constants k_1, k_2, k_3, and k_4 will be provided by MPB after analysis of the filter and instrument capabilities.

Due to the nature of this formulation, the first set of angles S_1, T_1 will not be incorporated in the state correction equation; however, they will be used to determine the first set $\dot{S}$, $\dot{T}$ (i.e., $\dot{S}_2$ and $\dot{T}_2$).

The operator may manually enter a two-element weight coefficient (C_1, C_2) which adjusts the weights relative to each other (shaft and trunnion). Nominally these coefficients will be unity.

Observation Computations

The following equations are used to compute values to compare with rendezvous radar raw observations for residual computations. This requires the availability of a six-point CM ephemeris in selenographic coordinates. The procedure is as follows for each observation time.

1. Define the LM (R_{LM}) state in a moon-centered local vertical system by

$$R_{LM} = \begin{pmatrix} X_{LV} \\ Y_{LV} \\ Z_{LV} \end{pmatrix} = \begin{pmatrix} r_{LM} \\ 0 \\ 0 \end{pmatrix}$$

where r = LM radius in the $\phi\lambda r$ system.

2. Compute the CM state (R_{CM}) in this system by the following transformation.

$$R_{CM} = \begin{pmatrix} X_{LV} \\ Y_{LV} \\ Z_{LV} \end{pmatrix} = \begin{pmatrix} \cos\phi \cos\lambda & \cos\phi \sin\lambda & \sin\phi \\ -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi \cos\lambda & -\sin\phi \sin\lambda & \cos\phi \end{pmatrix} \begin{pmatrix} X_{SG} \\ Y_{SG} \\ Z_{SG} \end{pmatrix}$$

where R_{SG} = selenographic coordinates of the CM.

3. Determine by interpolation the range vectors in this local vertical system for each LM rendezvous radar observation time.

The time tagging of the rendezvous radar data is done when the CDU's are read which is 5 to 10 milliseconds after the Doppler count is completed; however, the observation set will be time tagged in the middle of the Doppler count. For this reason an offset of about 50 milliseconds will be added.

4. Compute a unit range vector for each rendezvous radar observation time.

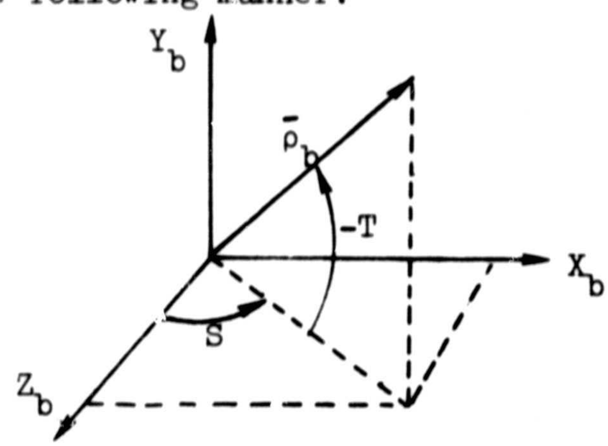
$$\bar{\rho}_{LV} = \frac{(R_{CM} - R_{LM})}{|R_{CM} - R_{LM}|}$$

5. Compute these vectors in the body system:

$$\bar{\rho}_b = \begin{pmatrix} X_b \\ Y_b \\ Z_b \end{pmatrix} = F(\alpha)\bar{\rho}_{LV}$$

where $F(\alpha)$ is the transformation defined by the three Euler rotations $\alpha_1, \alpha_2, \alpha_3$.

6. With this body vector, the observations may be computed. The LM rendezvous radar rotates about two axes, the shaft (S) axis and the trunnion (T) axis. They are defined for a LM-CSM line-of-sight direction in the following manner.



$\tan S = \frac{X_b}{Z_b}$, when S is in quadrants II or III.

$\sin T = -Y_b$, when T is in quadrants III or IV.

Partials for Onboard Data Processing

Earlier in the basic equation the matrix A was defined as $\frac{\partial y}{\partial F}$, where y is the observation and F is the state. This matrix is a 2 × 3, and for observation of shaft and trunnion and a state of $\alpha_1, \alpha_2, \alpha_3$, the matrix is

$$A = \begin{pmatrix} \frac{\partial S}{\partial \alpha_1} & \frac{\partial S}{\partial \alpha_2} & \frac{\partial S}{\partial \alpha_3} \\ \\ \frac{\partial T}{\partial \alpha_1} & \frac{\partial T}{\partial \alpha_2} & \frac{\partial T}{\partial \alpha_3} \end{pmatrix}$$

The following equations are expressions for the six elements of this matrix, and S and T are computed values of the shaft and trunnion angles, respectively. The detailed derivation may be found in reference 5.

$$\frac{\partial S}{\partial \alpha_1} = \sin \alpha_3 \cos \alpha_2 - \tan T \cos S \sin \alpha_2 - \tan T \cos \alpha_3 \cos \alpha_2 \sin S$$

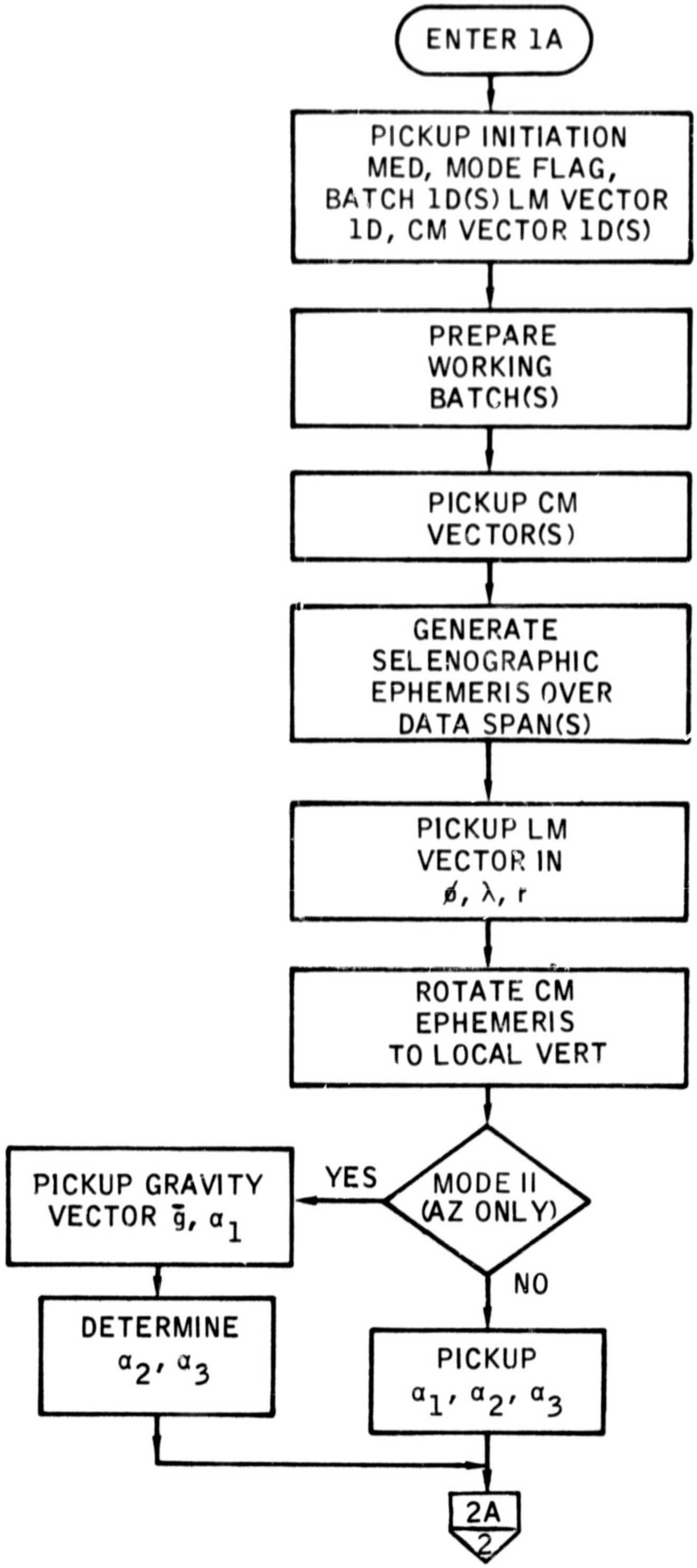
$$\frac{\partial S}{\partial \alpha_2} = -\cos \alpha_3 - \sin S \sin \alpha_3 \tan T$$

$$\frac{\partial S}{\partial \alpha_3} = -\tan T \cos S$$

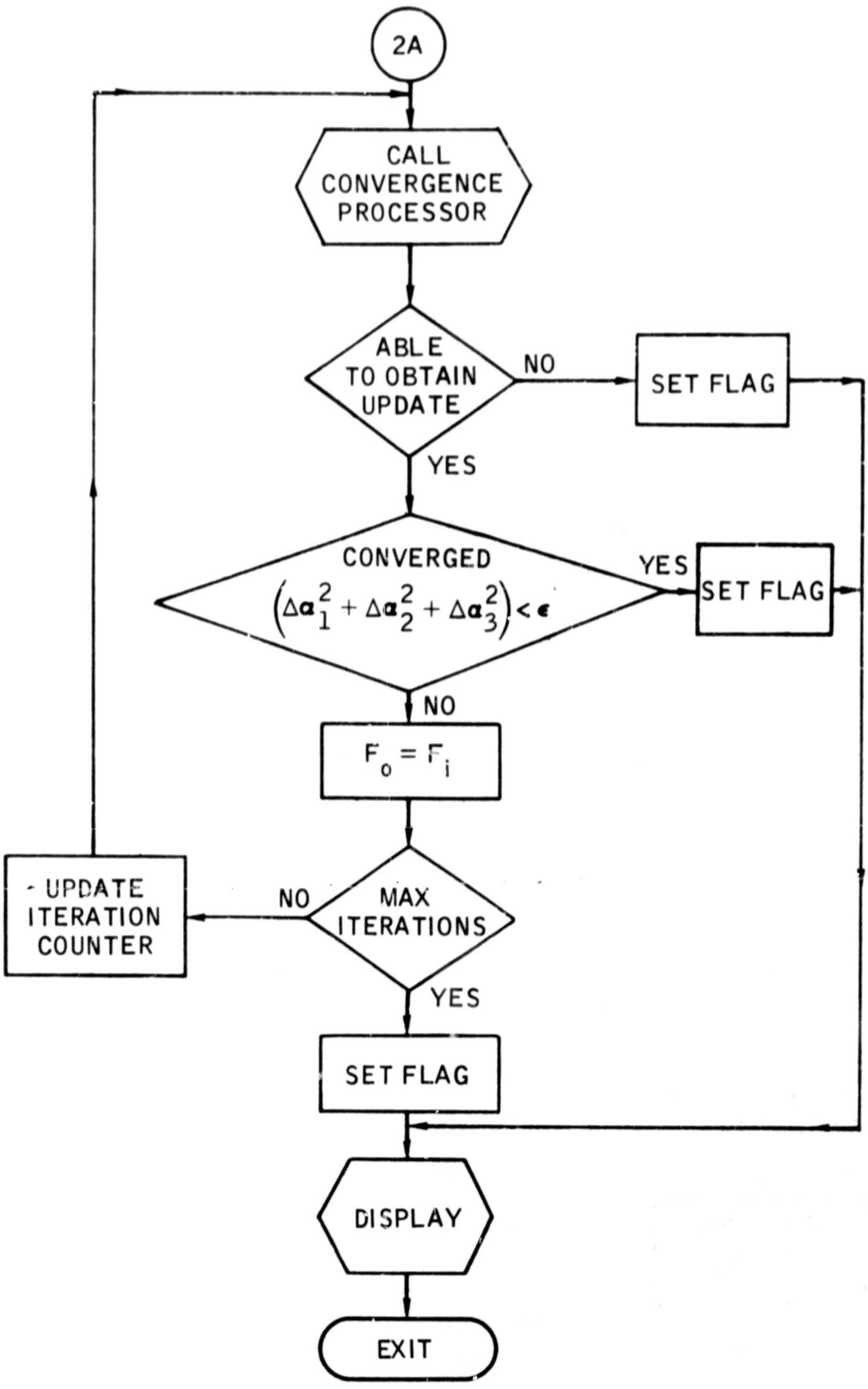
$$\frac{\partial T}{\partial \alpha_1} = \sin S \sin \alpha_2 - \cos S \cos \alpha_3 \cos \alpha_2$$

$$\frac{\partial T}{\partial \alpha_2} = -\sin \alpha_3 \cos S$$

$$\frac{\partial T}{\partial \alpha_3} = \sin S$$



Flow chart 1.- Supervisor logic.



Flow chart 1.- Supervisor logic - Concluded.

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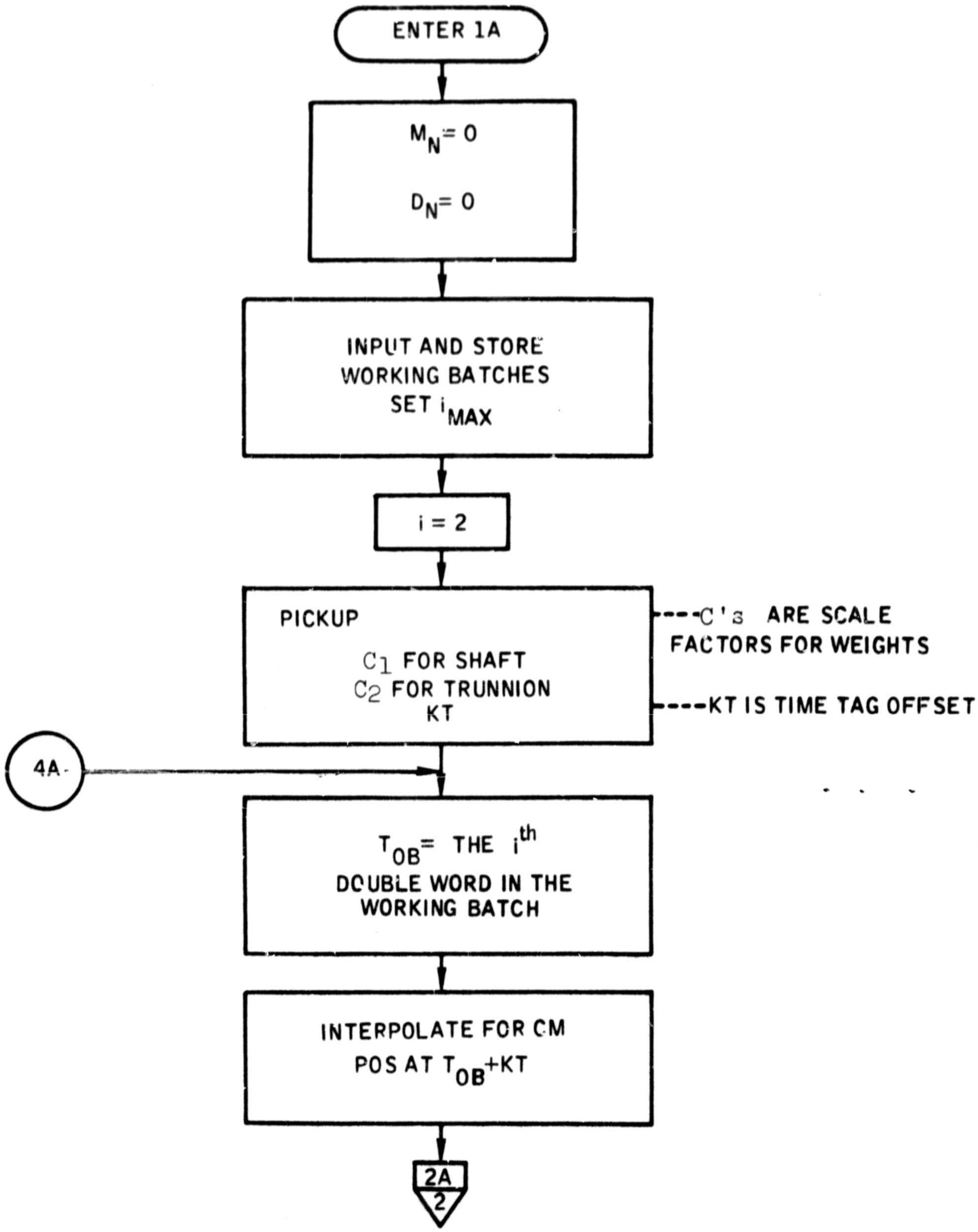

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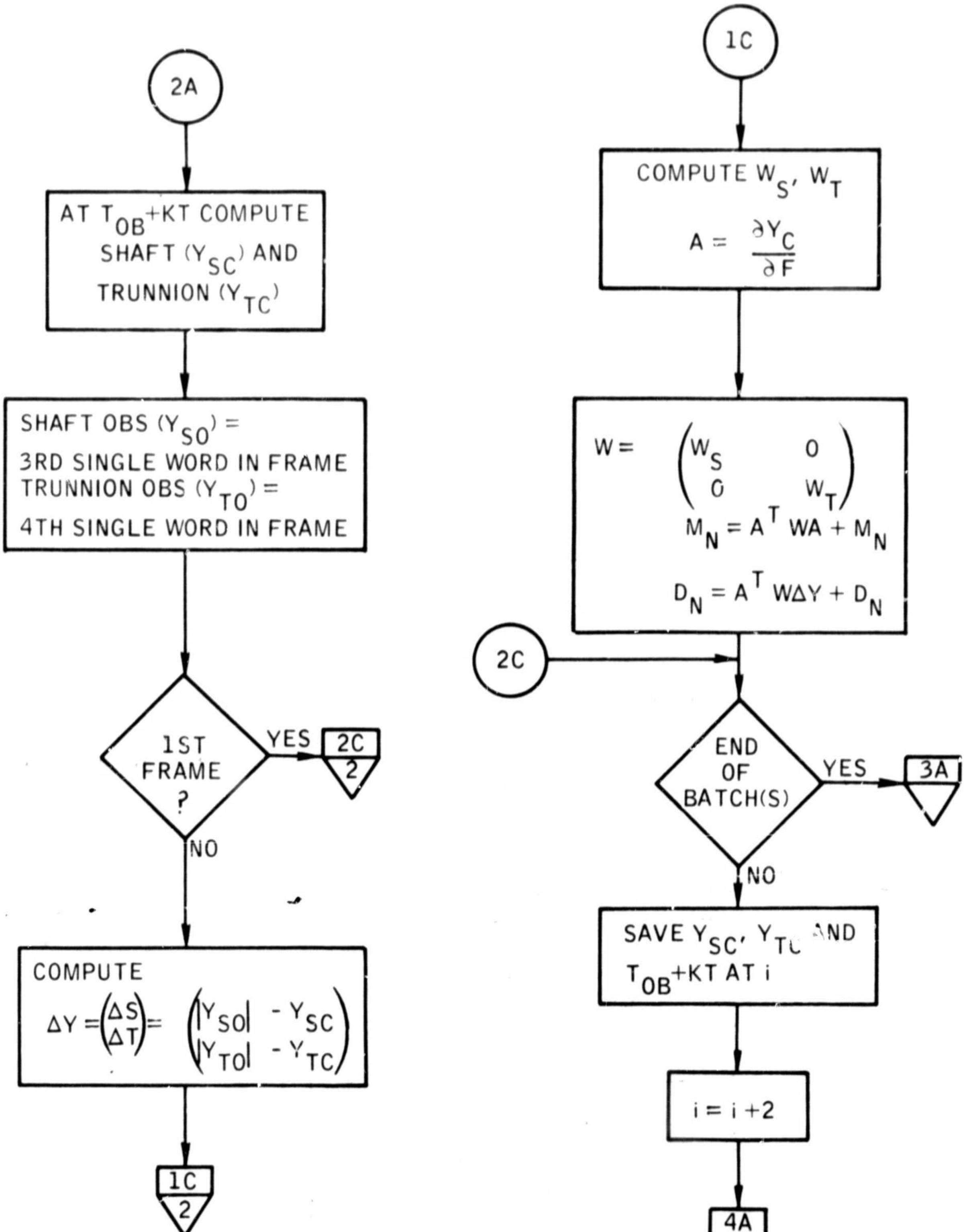
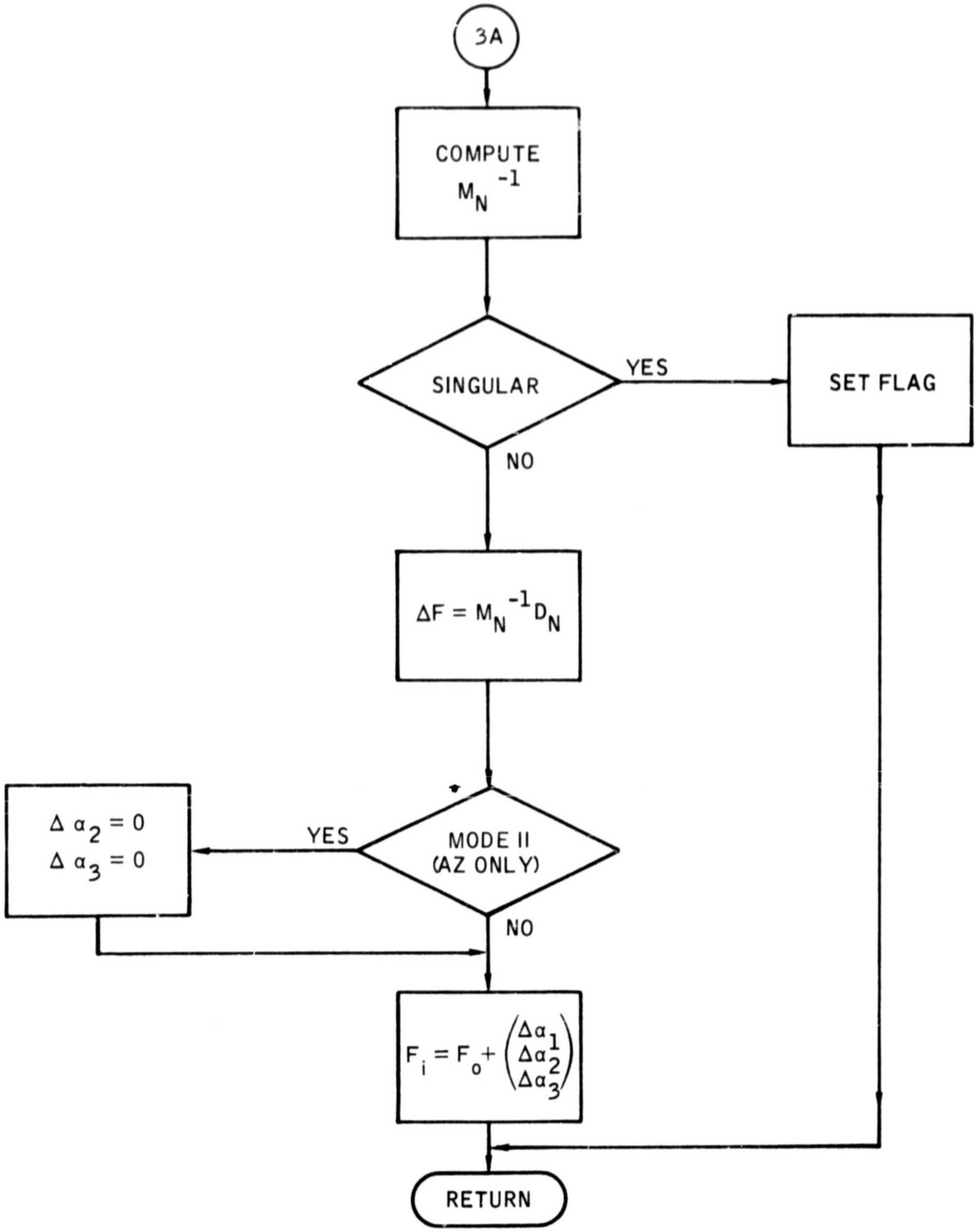
Flow chart 2.- Convergence processor.



Flow chart 2.- Convergence processor. - Continued.



Flow chart 2.- Convergence processor - Concluded.

REFERENCES

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