NASA TECHNICAL NOTE

NASA TN D-5954

C. 1

NASA TN D-5954

OPTIMIZATION OF SELF-ACTING THRUST BEARINGS FOR LOAD CAPACITY AND STIFFNESS

by Bernard J. Hamrock

Lewis Research Center

Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1970

1. Report No. NASA TN D-5954	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle OPTIMIZATION OF SELF-ACTING THRUST BEARINGS FOR LOAD CAPACITY AND STIFFNESS		5. Report Date August 1970
		6. Performing Organization Code
7. Author(s) Bernard J. Hamrock		8. Performing Organization Report No. E-5648
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		10. Work Unit No. 129-03
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

A linearized analysis of both infinite-width and finite-width (rectangular) step thrust bearings was performed. The dimensionless load capacity and stiffness of the finite bearing are expressed in terms of a Fourier cosine series. A digital computer program was developed which determines optimal bearing configuration for maximum load capacity or stiffness. Simple design curves are presented. Results are shown for compressible and incompressible lubrication. Through a parameter transformation, the results are directly usable in designing optimal step sector thrust bearings.

17. Key Words (Suggested by Author(s)) Lubrication; Self-acting step thrust bearing	18. Distribution Statement Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	28	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

OPTIMIZATION OF SELF-ACTING THRUST BEARINGS FOR LOAD CAPACITY AND STIFFNESS

by Bernard J. Hamrock

Lewis Research Center

SUMMARY

A linearized analysis of both infinite- and finite-width (rectangular) step thrust bearings was performed. Dimensionless load capacity and stiffness of the finite-width bearing are expressed in terms of a Fourier cosine series. The dimensionless load capacity and stiffness are a function of the dimensionless bearing number Λ, the pad length-to-width ratio λ, the film thickness ratio k, the step location parameter ψ, and the feed groove parameter η.

The equations obtained in the analysis were verified. The assumptions imposed were substantiated by comparison of the results with an existing exact solution for the infinite-width bearing. A digital computer program was developed which determines optimal bearing configuration for maximum load capacity or stiffness. Simple design curves are presented. Results are shown for both compressible and incompressible lubrication. Through a parameter transformation, the results are directly usable in designing optimal step sector thrust bearings.

INTRODUCTION

One of the first to apply the step film to a gas-lubricated bearing was Kochi (ref. 1). For the infinitely wide single thrust bearing, Kochi obtained an exact numerical solution. The expressions for the pressure were contained in a set of transcendental equations. Graphical methods were used to obtain the results.

In practical applications, one must use finite-width bearings. The finite-width step thrust bearing can appear in the shape of a rectangular pad or as a sector. For both the rectangular and the sector step thrust bearings, there is a definite need to know the optimal step configurations for maximum load capacity or maximum stiffness.

Ausman (ref. 2) in 1961 analyzed the gas lubricated step sector thrust bearing. He applied linearization assumptions to the Reynolds equation, thereby enabling the pressure to be determined. With the pressure known, the load capacity was obtained. The expression for the load capacity appeared in terms of eigenvalues and Bessel functions. Ausman's results do not lend themselves readily to obtaining optimal step configurations for maximum load capacity or maximum stiffness when various bearing operating conditions are considered. The reason for this is the way in which parameters were made dimensionless and the nature of the resulting equations.

In this report, a rectangular step thrust bearing is analysed. Linearization assumptions comparable to those imposed by Ausman (ref. 2) are used. The sector bearing results are obtained directly from the rectangular step bearing results since curvature effects are shown to be very small. Because of the simplified nature of the resulting equations, a computer program was developed which optimizes step parameters for maximum load capacity or maximum stiffness for a wide range of bearing operating conditions. Results are shown for both compressible and incompressible lubrication. Therefore, the objective of this report is to present easily usable design information for finding optimal step bearings of rectangular or sector shape. The results are valid for a wide range of operating conditions.

SYMBOLS

A, B, D, E	integration constants
AA, BB, CC, DD, EE, FF, GG, HH	constants defined in eqs. (A1) to (A8)
b	width of rectangular thrust bearing
C	film thickness in ridge region
G	constant
h	film thickness
I_m	Fourier coefficient
J	separation constant
K	dimensionless stiffness, $-C(\partial W/\partial C)$
k	film thickness ratio, $(C + \Delta)/C$
L	$l_r + l_s + l_g$
l	length of ridge, step, or feed groove region depending on subscript
M	last odd positive integer used in evaluation of Fourier cosine series

m	odd positive integers
N	number of pads placed in overall length
P	dimensionless pressure, $(p - p_a)/p_a$
p	pressure
p_a	ambient pressure
Q	mass flow rate
R_i	inner radius of sector thrust bearing
R_o	outer radius of sector thrust bearing
s	film thickness ratio as defined by Kochi (ref. 1), $1/(k - 1)$
U	velocity of bearing surface
W	dimensionless load capacity of finite-width bearing, $w/p_a b(l_r + l_s + l_g)$
W_∞	dimensionless load capacity of infinite-width bearing, $w_\infty/p_a(l_r + l_s + l_g)$
w	load capacity
w_∞	load capacity per unit width
X	dimensionless length coordinate, x/b
X_∞	x/L
x	coordinate in direction of motion
Y	dimensionless width coordinate, y/b
y	coordinate in direction of width of bearing
β	dimensionless bearing number used by Kochi (ref. 1), $3\mu U(l_s + l_r)\Big/p_a \Delta^2 = \Lambda/(k - 1)^2$
Δ	depth of step
η	feed groove parameter, $(l_r + l_s)/(l_r + l_s + l_g)$
Λ	dimensionless bearing number, $6\mu Ub\Big/p_a C^2$
Λ_∞	$6\mu U(l_s + l_r + l_g)\Big/p_a C^2$
λ	ratio of length to width of pad, $(l_r + l_s + l_g)/b$
μ	viscosity of fluid
ξ_r	$\sqrt{(\Lambda/2)^2 + m^2\pi^2}$

ξ_s $\sqrt{\left(\Lambda/2k^2\right)^2 + m^2\pi^2}$

ρ_a mass density of lubricant

ψ step location parameter, $l_s/(l_r + l_s + l_g)$

ω angular velocity

Subscripts:

g denotes feed groove region

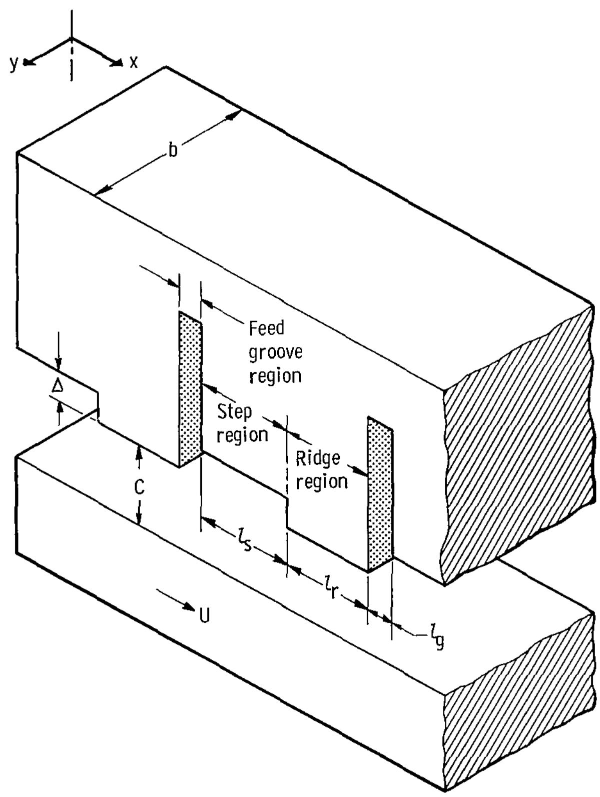
r denotes ridge region

s denotes step region

∞ denotes infinite-width analysis

BEARING DESCRIPTION

Sketch (a) shows the rectangular step thrust bearing to be studied. The ridge region



(a)

is where the film thickness is C, and the step region is where the film thickness is $C + \Delta$. The feed groove is the deep groove separating the end of the ridge region and the beginning of the next step region. Although not shown in this sketch, the depth of the feed groove is orders of magnitude deeper than the film thickness C. A pad is defined as the section that includes a ridge, a step, and feed groove regions. The length of the feed groove is small relative to the length of the pad. It should be noted that each pad acts independently since the pressure profile is broken at the lubrication feed groove.

LINEARIZATION ASSUMPTIONS

The Reynolds equation for the steady-state isothermal gas-lubricated thrust bearing can be written as

$$\frac{\partial}{\partial x}\left(p \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(p \frac{\partial p}{\partial y}\right) = \frac{6\mu U}{h^2} \frac{\partial p}{\partial x} \tag{1}$$

Expanding and rearranging terms results in

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \frac{6\mu U}{ph^2} \frac{\partial p}{\partial x} = -\frac{1}{p}\left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right] \tag{2}$$

In order to obtain an analytic solution to the foregoing equation, linearization assumptions will be imposed. The first linearization assumption states that the right side of equation (2) is zero. The validity of this assumption was shown by Scheinberg (ref. 3): For $U \rightarrow \infty$, the terms in equation (2) other than the one containing U become insignificant. Therefore, they can be neglected. Since the right side of equation (2) becomes zero for the two limiting cases, he concludes that it is a reasonable approximation to neglect these terms for all values of U. A second and final linearization assumption required is that p, where it appears as a coefficient, be replaced by the ambient pressure p_a. When these assumptions are applied, equation (2) becomes

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{6\mu U}{p_a h^2} \frac{\partial p}{\partial x} \tag{3}$$

PRESSURE ANALYSIS OF FINITE-WIDTH BEARING

From equation (3), the linearized Reynolds equation can be written separately for the ridge and step regions of the finite-step thrust bearing:

$$\frac{\partial^2 p_r}{\partial x^2} + \frac{\partial^2 p_r}{\partial y^2} = \frac{6\mu U}{p_a C^2} \frac{\partial p_r}{\partial x}$$

$$\frac{\partial^2 p_s}{\partial x^2} + \frac{\partial^2 p_s}{\partial y^2} = \frac{6\mu U}{p_a (C + \Delta)^2} \frac{\partial p_s}{\partial x}$$

The subscripts r, s, and g refer to the ridge region (see sketch (a)), the step region, and the feed groove region, respectively. With $x = bX$, $y = bY$, $p_r = p_a(P_r + 1)$, and $p_s = p_a(P_s + 1)$, the foregoing equations can be rewritten as

$$\frac{\partial^2 P_r}{\partial X^2} + \frac{\partial^2 P_r}{\partial Y^2} = \Lambda \frac{\partial P_r}{\partial X} \tag{4}$$

$$\frac{\partial^2 P_s}{\partial X^2} + \frac{\partial^2 P_s}{\partial Y^2} = \frac{\Lambda}{k^2} \frac{\partial P_s}{\partial X} \tag{5}$$

where

$$\Lambda = \frac{6\mu Ub}{p_a C^2}$$

and

$$k = \frac{C + \Delta}{C}$$

Using a separation-of-variables technique on equations (4) and (5) gives

$$P_r = \exp\left(\frac{\Lambda X}{2}\right)\left[A_r \exp\left(X\sqrt{\frac{\Lambda^2}{4} + J_r^2}\right) + B_r \exp\left(-X\sqrt{\frac{\Lambda^2}{4} + J_r^2}\right)\right]\left[D_r \sin(J_r Y) + E_r \cos(J_r Y)\right] \tag{6}$$

$$P_s = \exp\left(\frac{\Lambda X}{2k^2}\right)\left[A_s \exp\left(X\sqrt{\frac{\Lambda^2}{4k^4}+J_s^2}\right)+B_s \exp\left(-X\sqrt{\frac{\Lambda^2}{4k^4}+J_s^2}\right)\right]\left[D_s \sin(J_s Y)+E_s \cos(J_s Y)\right] \tag{7}$$

The boundary conditions can be written as

(1) $P_s = 0 \quad$ when $\quad X = 0$

(2) $P_r = 0 \quad$ when $\quad X = \dfrac{l_s + l_r}{b} = \left(\dfrac{l_s + l_r}{l_s + l_r + l_g}\right)\left(\dfrac{l_s + l_r + l_g}{b}\right) = \eta\lambda$

(3) $P_r = P_s = \displaystyle\sum_{m=1,3,\ldots}^{\infty} I_m \cos(m\pi Y) \quad$ when $\quad X = \dfrac{l_s}{b} = \psi\lambda$

where I_m is a Fourier coefficient

(4) $\dfrac{\partial P_r}{\partial Y} = \dfrac{\partial P_s}{\partial Y} = 0 \quad$ when $\quad Y = 0$

(5) $P_r = P_s = 0 \quad$ when $\quad Y = \dfrac{1}{2}$

(6) $Q_r = Q_s \quad$ when $\quad X = \psi\lambda$

With the use of boundary conditions (1) to (5), equations (6) and (7) can be written as

$$P_r = \sum_{m=1,3,\ldots}^{\infty}\left\{\frac{I_m \cos(m\pi Y)\exp\left[\frac{\Lambda}{2}(X - \psi\lambda)\right]}{\exp(-\lambda\psi\xi_r) - \exp\left[-\lambda\xi_r(2\eta - \psi)\right]}\right\}\left\{\exp(-X\xi_r) - \exp\left[-\xi_r(2\lambda\eta - X)\right]\right\} \tag{8}$$

$$P_s = \sum_{m=1,3,\ldots}^{\infty}\left\{\frac{I_m \cos(m\pi Y)\exp\left[\frac{\Lambda}{2k^2}(X - \psi\lambda)\right]}{\exp(-\psi\lambda\xi_s) - \exp(\psi\lambda\xi_s)}\right\} \left. \exp(-X\xi_s) - \exp(X\xi_s)\right] \tag{9}$$

where

$$\xi_r = \sqrt{\left(\frac{\Lambda}{2}\right)^2 + m^2\pi^2}$$

and

$$\xi_s = \sqrt{\left(\frac{\Lambda}{2k^2}\right)^2 + m^2\pi^2}$$

The linearized equations describing the mass flow across the ridge and step regions can be written as

$$Q_r = \frac{\rho_a}{p_a}\left(\frac{p_r UC}{2} - \frac{p_a C^3}{12\mu}\frac{\partial p_r}{\partial x}\right)$$

$$Q_s = \frac{\rho_a}{p_a}\left[\frac{p_s U(C + \Delta)}{2} - \frac{p_a(C + \Delta)^3}{12\mu}\frac{\partial p_s}{\partial x}\right]$$

These equations may be made dimensionless by letting $p_r = p_a(P_r + 1)$, $p_s = p_a(P_s + 1)$, and $x = bX$ as was done for the Reynolds equations. Therefore,

$$Q_r = \frac{\rho_a p_a C^3}{12\mu b}\left[\Lambda(P_r + 1) - \frac{\partial P_r}{\partial X}\right]$$

$$Q_s = \frac{\rho_a p_a (C + \Delta)^3}{12\mu b}\left[\frac{\Lambda}{k^2}(P_s + 1) - \frac{\partial P_s}{\partial X}\right]$$

Making use of boundary conditions (3) and (6) gives

$$k^3\left(\frac{\partial P_s}{\partial X}\right)_{X=\lambda\psi} - \left(\frac{\partial P_r}{\partial X}\right)_{X=\lambda\psi} = \Lambda(k - 1)\left[1 + \sum_{m=1,3,\ldots}^{\infty} I_m \cos(m\pi Y)\right] \qquad (10)$$

With the use of equations (8) to (10), the Fourier coefficient I_m can be solved:

$$I_m = \frac{4(k - 1)\sin\left(\frac{m\pi}{2}\right)}{m\pi\left(\frac{1 - k}{2} + \frac{\xi_s k^3}{\Lambda}\left[\frac{1 + \exp(-2\xi_s\lambda\psi)}{1 - \exp(-2\xi_s\lambda\psi)}\right] + \frac{\xi_r}{\Lambda}\left\{\frac{1 + \exp\left[-2\lambda\xi_r(\eta - \psi)\right]}{1 - \exp\left[-2\lambda\xi_r(\eta - \psi)\right]}\right\}\right)} \tag{11}$$

LOAD ANALYSIS OF FINITE-WIDTH BEARING

The dimensionless load capacity for the ridge and step region is

$$W_r = \frac{w_r}{p_a bL} = \frac{2}{\lambda}\int_0^{1/2}\int_{\lambda\psi}^{\lambda\eta} P_r \, dX \, dY$$

$$W_s = \frac{w_s}{p_a bL} = \frac{2}{\lambda}\int_0^{1/2}\int_0^{\lambda\psi} P_s \, dX \, dY$$

Substituting equations (8) and (9) into the foregoing equations and integrating gives

$$W_r = \sum_{m=1,3,\ldots}^{\infty} \frac{2I_m \sin\left(\frac{m\pi}{2}\right)}{m^3\pi^3\lambda}\left(\frac{\Lambda}{2} + \frac{\xi_r\left\{1 - 2\exp\left[-\lambda(\eta - \psi)\left(\xi_r - \frac{\Lambda}{2}\right)\right] + \exp\left[-2\lambda\xi_r(\eta - \psi)\right]\right\}}{1 - \exp -2\lambda\xi_r(\eta - \psi)}\right) \tag{12}$$

$$W_s = \sum_{m=1,3,\ldots}^{\infty} \frac{2I_m \sin\left(\frac{m\pi}{2}\right)}{m^3\pi^3\lambda}\left(-\frac{\Lambda}{2k^2} + \frac{\xi_s\left\{1 - 2\exp\left[-\lambda\psi\left(\xi_s + \frac{\Lambda}{2k^2}\right)\right] + \exp(-2\lambda\xi_s)\right\}}{1 - \exp(-2\lambda\psi\xi_s)}\right) \tag{13}$$

The total dimensionless load supported by the rectangular step slider bearing is

$$W = \frac{w_r + w_s}{p_a Lb} = W_r + W_s \tag{14}$$

STIFFNESS ANALYSIS OF FINITE-WIDTH BEARING

The equation for the dimensionless stiffness is

$$K = -C \frac{\partial W}{\partial C}$$

With the use of equations (12) to (14), the foregoing equation becomes

$$K = \sum_{m=1,3,\ldots}^{\infty} \left\{ W(AA) - \frac{2I_m \sin\left(\frac{m\pi}{2}\right)}{m^3\pi^3\lambda} \left[\Lambda\left(1 - \frac{1}{k^3}\right) - \xi_r(BB) - \xi_s(CC) + \frac{\Lambda^2}{2\xi_r}(DD) + \frac{\Lambda^2}{2\xi_s k^5}(EE) \right] \right\} \tag{15}$$

where AA, BB, CC, DD, and EE are constants defined in the appendix.

Therefore, with equations (11) to (15), the dimensionless load capacity and stiffness for a self-acting gas-lubricated finite-width step thrust bearing is completely defined. From these equations, it is evident that the dimensionless load capacity and stiffness are functions of the following parameters:

Dimensionless bearing number

$$\Lambda = \frac{6\mu Ub}{p_a C^2}$$

Length-to-width ratio of pad

$$\lambda = \frac{l_s + l_r + l_g}{b}$$

Film thickness ratio

$$k = \frac{C + \Delta}{C}$$

Step location parameter

$$\psi = \frac{l_s}{l_s + l_r + l_g}$$

Feed groove parameter

$$\eta = \frac{l_s + l_r}{l_s + l_r + l_g}$$

INFINITE-WIDTH-BEARING ANALYSIS

The linearized Reynolds equations for the ridge and step regions of an infinitely wide thrust bearing can be written in dimensionless form from equations (4) and (5) as

$$\frac{\partial^2 P_{r,\infty}}{\partial X_\infty^2} = \Lambda_\infty \frac{\partial P_{r,\infty}}{\partial X_\infty}$$

$$\frac{\partial^2 P_{s,\infty}}{\partial X_\infty^2} = \frac{\Lambda_\infty}{k^2} \frac{\partial P_{s,\infty}}{\partial X_\infty}$$

where

$$\Lambda_\infty = \frac{6\mu U(l_s + l_r + l_g)}{p_a C^2}$$

and

$$X_\infty = \frac{x}{l_s + l_r + l_g}$$

Proceeding in much the same manner as in the Finite-Analysis section of this report, one arrives at the following expressions for the dimensionless pressure for the ridge and step regions:

$$P_{r,\infty} = G\left\{1 - \exp\left[\Lambda_\infty(X_\infty - \eta)\right]\right\} \tag{16}$$

$$P_{s,\infty} = G\left\{\frac{1 - \exp\left[\Lambda_\infty(\psi - \eta)\right]}{1 - \exp\left(\frac{\Lambda_\infty \psi}{k^2}\right)}\right\}\left[1 - \exp\left(\frac{\Lambda_\infty X_\infty}{k^2}\right)\right] \tag{17}$$

where

$$G = \frac{k - 1}{1 + k\exp\left(\frac{-\Lambda_\infty \psi}{k^2}\right)\left\{\frac{1 - \exp\left[\Lambda_\infty(\psi - \eta)\right]}{1 - \exp\left(\frac{-\Lambda_\infty \psi}{k^2}\right)}\right\}}$$

With the pressure known, the resulting dimensionless load capacity of an infinite-width step thrust bearing can be easily formulated as

$$W_\infty = G\left(\eta - \psi\left\{\frac{1 - \exp\left(\frac{-\Lambda_\infty \psi}{k^2}\right)\exp\left[\Lambda_\infty(\psi - \eta)\right]}{1 - \exp\left(\frac{-\Lambda_\infty \psi}{k^2}\right)}\right\} + \frac{(k^2 - 1)}{\Lambda_\infty}\left\{1 - \exp\left[\Lambda_\infty(\psi - \eta)\right]\right\}\right) \tag{18}$$

The dimensionless stiffness can be written as

$$K_\infty = -C\frac{\partial W_\infty}{\partial C} = \frac{W_\infty(HH)}{G}$$

$$+ G\left(-\psi(GG) + \frac{2(k - 1)}{\Lambda_\infty}\left\{1 - \exp\left[-\Lambda_\infty(\eta - \psi)\right]\right\} - 2(k^2 - 1)(\eta - \psi)\exp\left[-\Lambda_\infty(\eta - \psi)\right]\right) \tag{19}$$

where FF, GG, and HH are constants defined in the appendix. For the infinite-width analysis, the dimensionless load and stiffness are functions of the following parameters:

Dimensionless bearing number for infinite-width bearing

$$\Lambda_\infty = \frac{6\mu U(l_s + l_r + l_g)}{p_a C^2}$$

Film thickness ratio

$$k = \frac{C + \Delta}{C}$$

Step location parameter

$$\psi = \frac{l_s}{l_s + l_r + l_g}$$

Feed groove parameter

$$\eta = \frac{l_s + l_r}{l_s + l_r + l_g}$$

VERIFICATION OF EQUATIONS

The equations for the dimensionless load capacity and stiffness for the finite- (eqs. (11) to (15)) and infinite- (eqs. (18) and (19)) width analyses were programmed on a digital computer. It should be recalled that linearization assumptions were imposed in order to obtain simplified results. Figure 1 shows that these assumptions are generally valid for the infinite-width bearing. This figure compares the results from the present work with Kochi's (ref. 1) exact solution. The agreement is good. Comparing equations (2) and (3) with and without the width coordinates y, one could further conclude that for a finite-width bearing the linearized analysis should be in good agreement with the exact results.

Table I shows that the solutions for the finite and infinite analyses approach each other when the length-to-width ratio approaches zero. The results, when 500 terms ($M = 1001$) are used in the Fourier cosine series, approach the infinite-width analysis much closer than when only 50 terms ($M = 101$) are used. Furthermore, the rate of convergence is much slower at large dimensionless bearing numbers ($\Lambda \rightarrow 500$) than at smaller values of Λ. Note the decrease in dimensionless load capacity W when Λ increases from 100 to 500. This is due to the fact that the step parameters are held constant; that is, the step parameters chosen are closer to the optimal for $\Lambda = 100$ than for $\Lambda = 500$.

Table II compares the dimensionless load capacity obtained from Ausman (ref. 2) with the present work for various dimensionless bearing numbers Λ and inner- to outer-radius ratios R_i/R_o. Ausman (ref. 2) considers curvature effects, whereas the present work does not. The equivalent length of a sector pad is assumed to be the arc length along the average radius. The width is the difference between inner and outer radii. For all inner- to outer-radius ratios, there is close agreement between the two analyses. Curvature effects are small. Therefore, the simplified equations of the present analysis are valid for evaluating the circular sector thrust bearing.

OPTIMIZING PROCEDURE

The problem as defined in the INTRODUCTION is to find the optimal step bearing for maximum load capacity or stiffness for various bearing numbers. This means, given the dimensionless bearing number Λ, finding the optimal length-to-width ratio λ, optimal film thickness ratio k, and optimal step location parameter ψ. The significance of the feed groove parameter η is much less than that of the other parameters. Therefore, for all evaluations, the feed groove parameter η will be set equal to 0.97.

The basic problem in optimizing λ, k, and ψ for maximum load and stiffness is essentially that of finding values of λ, k, and ψ that satisfy the following equations:

$$\frac{\partial W}{\partial \lambda} = \frac{\partial W}{\partial k} = \frac{\partial W}{\partial \psi} = 0 \tag{20}$$

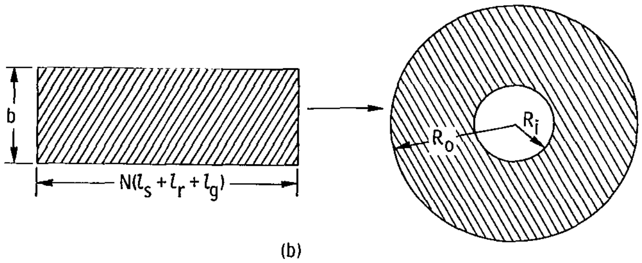
$$\frac{\partial K}{\partial \lambda} = \frac{\partial K}{\partial k} = \frac{\partial K}{\partial \psi} = 0 \tag{21}$$

The method used in solving equations (19) to (21) is the Newton-Raphson method for solving simultaneous equations. This method is described in reference 4 and in most other tests on numerical analysis.

Therefore, given the dimensionless bearing number Λ, the optimization computer program obtains optimum values of λ, k, and ψ for maximum dimensionless load capacity or stiffness. As a check on the optimization procedure, the following case was considered: $\Lambda = \lambda = 1\times10^{-5}$. This case approaches an infinitely-wide incompressibly lubricated step bearing for which the results are known. For this case, the computer program indicated that $k = 1.866$ and $\psi = 0.718$ were optimal for maximum load capacity. These results are in exact agreement with Archibald (ref. 5).

STEP SECTOR THRUST BEARING

For optimization of a step sector thrust bearing, parameters for the sector must be found that are analogous to those for the rectangular step bearing. Sketch (b) shows the transformation.



(b)

The following substitutions accomplish this transformation:

$$b \rightarrow R_o - R_i$$

$$N(l_s + l_r + l_g) \rightarrow \pi(R_o + R_i)$$

$$U \rightarrow \frac{\omega}{2}(R_o + R_i)$$

where N is the number of pads placed in the step sector. By use of the foregoing equations, the dimensionless bearing number can be rewritten as

$$\Lambda \rightarrow \frac{3\mu\omega\left(R_o^2 - R_i^2\right)}{p_a C^2} \tag{22}$$

The optimal number of pads to be placed in the sector is obtained from

$$N = \frac{\pi(R_o + R_i)}{(\lambda)_{opt}(R_o - R_i)} \tag{23}$$

where $(\lambda)_{opt}$ is the optimal value for the length-to-width ratio. The way $(\lambda)_{opt}$ is obtained is discussed in the next section. Since N will not be an integer normally, rounding it to the nearest integer is required.

DISCUSSION OF RESULTS

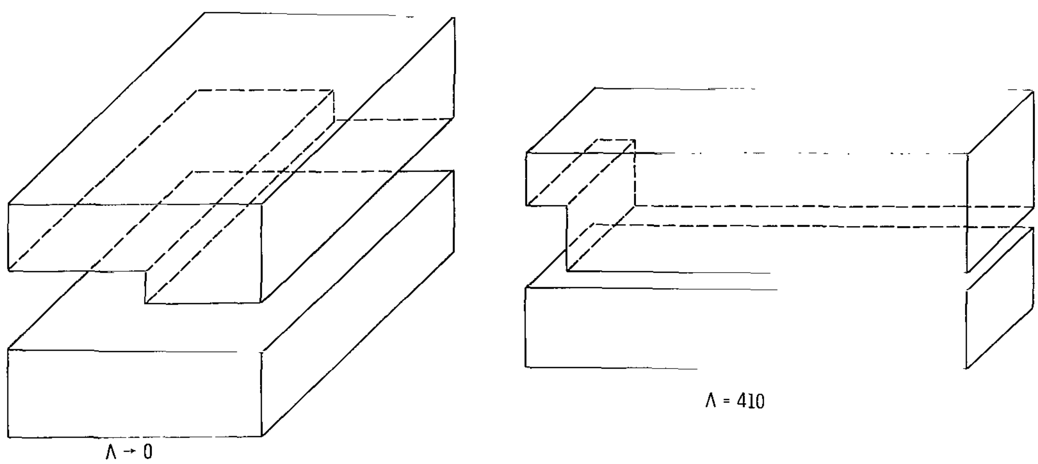
Tables III and IV give optimal step parameters (ψ, λ, and k) for resulting maximum load capacity and stiffness. The differences between these tables are that table III optimizes with respect to load capacity, whereas table IV optimizes with respect to stiffness. The following observations can be made about both tables III and IV as Λ (or bearing speed) is increased:

(1) The length-to-width ratio λ increases; that is, the length of the pad increases relative to its width.

(2) The step location parameter ψ decreases; that is, the length of the step region decreases relative to the length of the pad.

(3) The film thickness ratio k increases; that is, the step depth increases relative to the clearance.

Sketch (c) shows optimal step bearings for two extremes of dimensionless bearing numbers.



(c)

Figures 2 to 4 are obtained directly from the data presented in tables III and IV. Figure 2 shows the effect of Λ on λ, k, and ψ for the maximum load capacity condition for a range of Λ from 0 to 410. The optimal step parameters (λ, k, and ψ) are seen to approach an asymptote as the dimensionless bearing number Λ becomes small. That is, for small $\Lambda (\Lambda \leq 0.1)$ the optimal step parameters are not a function of Λ. In the incompressible solution of a step bearing, the right side of equations (4) and (5) are zero. Therefore, it must be concluded that the asymptotic values that the step parameters approach in figure 2 correspond to the incompressible solution. These asymptotes are $\lambda = 0.918$, $\psi = 0.555$, and $k = 1.693$.

Figure 3 shows the effect of Λ on λ, k, and ψ for the maximum stiffness condition for a range of Λ from 0 to 410. As in figure 2, the optimal step parameters are seen to approach asymptotes as the incompressible solution is reached. The asymptotes are $\lambda = 0.915$, $\psi = 0.557$, and $k = 1.470$. Note that there is a difference in the asymptote for the film thickness ratio but virtually no change in λ and ψ when compared to those obtained from figure 2.

Figure 4 shows the effect of dimensionless bearing number Λ on dimensionless load capacity and stiffness. The difference in these figures is that the optimal step parameters are obtained in figure 4(a) for maximum load capacity and in figure 4(b) for maximum stiffness. Also shown in these figures are values of K and W when the step parameters are held fixed at the optimal values obtained for the incompressible solution. A significant decrease between the solid and dashed lines in W or K does not occur until $\Lambda > 8$.

Figures 2 to 4 contain all the necessary information for the design of an optimal rectangular step thrust bearing for maximum load capacity or stiffness. With the dimensionless bearing number Λ, the optimal values of λ, ψ, and k can be obtained from figure 2 or 3 depending on whether or not maximum load or stiffness is a major consideration. From figure 4(a) or (b), the resulting values for the dimensionless load and stiffness can be obtained. Furthermore, from figures 2 to 4 and equations (22) and (23), the optimal step sector thrust bearing with load capacity or stiffness considered can be obtained. The dimensionless bearing number Λ is obtained from equation (22). With Λ known, the optimal values of λ, ψ, and k can be obtained from figure 2 or 3 depending on whether maximum load or stiffness is a major consideration. From equation (23), the optimal number of pads placed in a sector can be determined. Finally, from figure 4, the resulting values for the dimensionless load capacity and stiffness can be obtained.

SUMMARY OF RESULTS

A linearized analysis of both infinite-width and finite-width (rectangular) step thrust bearings was performed. Dimensionless load capacity and stiffness of the finite-width bearing are expressed in terms of a Fourier cosine series. The equations obtained in the analysis were verified. The assumptions imposed were substantiated by comparison of the results with an existing exact solution for the infinite-width bearing. A digital computer program was developed which determines optimal bearing configuration for maximum load capacity or stiffness. Simple design curves are presented. Results are shown for both compressible and incompressible lubrication. Through a parameter transformation, the results are directly usable in designing an optimal step sector thrust bearing.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 15, 1970,
129-03.

APPENDIX - CONSTANTS OBTAINED IN EVALUATING DIMENSIONLESS STIFFNESS

Constants developed when deriving the dimensionless stiffness expressed herein. Equations (A1) to (A5) are obtained from the finite-width analysis, and equations (A6) to (A8) are obtained from the infinite-width analysis.

$$AA = \frac{-m\pi I_m}{4(k-1)\sin\left(\frac{m\pi}{2}\right)}\left[\frac{2\lambda\psi\Lambda}{k^2}\left\{\frac{\exp(-2\xi_s\lambda\psi)}{\left[1-\exp(-2\lambda\xi_s\psi)\right]^2}\right\}+\left(\frac{3k^2\xi_s}{\Lambda}-\frac{\Lambda}{2k^2\xi_s}\right)\left[\frac{1+\exp(-2\xi_s\lambda\psi)}{1-\exp(-2\xi_s\lambda\psi)}\right]\right.$$

$$\left.+\left(\frac{3\xi_r}{\Lambda}-\frac{\Lambda}{2\xi_r}\right)\left\{\frac{1+\exp\left[-2\lambda\xi_r(\eta-\psi)\right]}{1-\exp\left[-2\lambda\xi_r(\eta-\psi)\right]}\right\}+2\lambda\Lambda(\eta-\psi)\left(\frac{\exp\left[-2\xi_r\lambda(\eta-\psi)\right]}{\left\{1-\exp\left[-2\xi_r\lambda(\eta-\psi)\right]\right\}^2}\right)\right] \tag{A1}$$

$$BB = \frac{2\lambda\Lambda(\eta-\psi)}{\left\{1-\exp\left[-2\lambda\xi_r(\eta-\psi)\right]\right\}^2}\left[\frac{\Lambda}{2\xi_r}\left(2\exp\left[-2\lambda\xi_r(\eta-\psi)\right]-\exp\left[-\lambda(\eta-\psi)\left(\xi_r-\frac{\Lambda}{2}\right)\right]\right.\right.$$

$$\left.\left.\times\left\{1+\exp\left[-2\lambda\xi_r(\eta-\psi)\right]\right\}\right)+\exp\left[-\lambda(\eta-\psi)\left(\xi_r-\frac{\Lambda}{2}\right)\right]\left\{1-\exp\left[-2\lambda\xi_r(\eta-\psi)\right]\right\}\right] \tag{A2}$$

$$CC = \frac{2\lambda\psi\Lambda}{k^3\left[1-\exp(-2\lambda\psi\xi_s)\right]^2}\left(\frac{\Lambda}{2\xi_s k^2}\left\{2\exp(-2\lambda\psi\xi_s)-\exp\left[-\lambda\psi\left(\xi_s+\frac{\Lambda}{2k^2}\right)\right]\right.\right.$$

$$\left.\left.\times\left[1+\exp(-2\lambda\psi\xi_s)\right]\right\}-\exp\left[-\lambda\psi\left(\xi_s+\frac{\Lambda}{2k^2}\right)\right]\left[1-\exp(-2\lambda\psi\xi_s)\right]\right) \tag{A3}$$

$$DD = \frac{1 - 2\exp\left[-\lambda(\eta - \psi)\left(\xi_r - \frac{\Lambda}{2}\right)\right] + \exp\left[-2\lambda\xi_r(\eta - \psi)\right]}{1 - \exp\left[-2\lambda\xi_r(\eta - \psi)\right]} \tag{A4}$$

$$EE = \frac{1 - 2\exp\left[-\lambda\psi\left(\xi_s + \frac{\Lambda}{2k^2}\right)\right] + \exp(-2\lambda\psi\xi_s)}{1 - \exp(-2\lambda\psi\xi_s)} \tag{A5}$$

$$FF = \frac{2}{\left[1 - \exp\left(\frac{-\Lambda_\infty\psi}{k^2}\right)\right]^2}\left(\frac{\Lambda_\infty\psi}{k^3}\left\{1 - \exp\left[-\Lambda_\infty(\eta - \psi)\right]\right\}\exp\left(\frac{-\Lambda_\infty\psi}{k^2}\right)\right.$$

$$\left. - \Lambda_\infty(\eta - \psi)\left[1 - \exp\left(\frac{-\Lambda_\infty\psi}{k^2}\right)\right]\exp\left[-\Lambda_\infty(\eta - \psi)\right]\right) \tag{A6}$$

$$GG = \frac{2\exp\left(\frac{-\Lambda_\infty\psi}{k^2}\right)}{\left[1 - \exp\left(\frac{-\Lambda_\infty\psi}{k^2}\right)\right]^2}\left(-\Lambda_\infty(\eta - \psi)\exp\left[-\Lambda_\infty(\eta - \psi)\right]\left[1 - \exp\left(\frac{-\Lambda_\infty\psi}{k^2}\right)\right]\right.$$

$$\left. + \frac{\Lambda_\infty\psi}{k^3}\left\{1 - \exp\left[-\Lambda_\infty(\eta - \psi)\right]\right\}\right) \tag{A7}$$

$$HH = \frac{-G^2}{(k - 1)}\left[1 + \exp\left(\frac{-\Lambda_\infty\psi}{k^2}\right)\left(k(FF) + \left(1 + \frac{2\Lambda_\infty\psi}{k^2}\right)\left\{\frac{1 - \exp\left[-\Lambda_\infty(\eta - \psi)\right]}{1 - \exp\left(\frac{-\Lambda_\infty\psi}{k^2}\right)}\right\}\right)\right] \tag{A8}$$

REFERENCES

1. Kochi, Kikuo C.: Characteristics of a Self-Lubricated Stepped Thrust Pad of Infinite Width With Compressible Lubricant. J. Basic Eng., vol. 81, no. 2, June 1959, pp. 135-146.
2. Ausman, J. S.: An Approximate Analytical Solution for Self-Acting Gas Lubrication of Stepped Sector Thrust Bearings. ASLE Trans., vol. 4, no. 2, Nov. 1961, pp. 304-313.
3. Scheinberg, S. A.: Gas Lubrication of Sliding Bearings (Theory and Calculations). Friction and Wear in Machines, Inst. of Machine Sci., Acad. Sci. USSR, vol. 8, 1953, pp. 107-204.
4. Scarborough, James B.: Numerical Mathematical Analysis. Sixth ed., Johns Hopkins Press, 1966.
5. Archibald, F. R.: A Simple Hydrodynamic Thrust Bearing. Trans. ASME, vol. 72, no. 4, May 1950, pp. 393-400.

TABLE I. - COMPARISON OF DIMENSIONLESS LOAD CAPACITY FOR INFINITE-WIDTH SOLUTION AND LIMITING CASE OF FINITE-WIDTH SOLUTION FOR TWO-SERIES TRUNCATIONS

[Feed groove parameter, $\eta = 1.0$; step location parameter, $\psi = 0.45$; film thickness ratio, $k = 2.0$.]

Dimensionless bearing number, Λ	Dimensionless load capacity of infinite-width bearing, W_∞	Limiting case of dimensionless load capacity of finite-width bearing, $\lim_{\lambda \to 0} W$		Dimensionless stiffness of infinite-width bearing, K_∞	Limiting case of dimensionless stiffness of infinite-width bearing, $\lim_{\lambda \to 0} K$	
		M = 101 (50 terms)	M = 1001 (500 terms)		M = 101. (50 terms)	M = 1001. (500 terms)
10^{-4}	2.5516×10^{-6}	2.5414×10^{-6}	2.5505×10^{-6}	4.1825×10^{-6}	4.1659×10^{-6}	4.1808×10^{-6}
10^{-3}	2.5517×10^{-5}	2.5416×10^{-5}	2.5507×10^{-5}	4.1831×10^{-5}	4.1664×10^{-5}	4.1814×10^{-5}
10^{-2}	2.5533×10^{-4}	2.5431×10^{-4}	2.5522×10^{-4}	4.1887×10^{-4}	4.1720×10^{-4}	4.1870×10^{-4}
10^{-1}	2.5687×10^{-3}	2.5585×10^{-3}	2.5677×10^{-3}	4.2448×10^{-3}	4.2279×10^{-3}	4.2431×10^{-3}
1	2.7185×10^{-2}	2.7077×10^{-2}	2.7174×10^{-2}	4.7869×10^{-2}	4.7678×10^{-2}	4.7849×10^{-2}
10	3.2354×10^{-1}	3.2226×10^{-1}	3.2341×10^{-1}	5.8440×10^{-1}	5.8208×10^{-1}	5.8416×10^{-1}
100	5.7998×10^{-1}	5.7716×10^{-1}	5.7923×10^{-1}	5.6021×10^{-1}	5.5891×10^{-1}	5.6091×10^{-1}
500	5.5600×10^{-1}	5.4967×10^{-1}	5.5165×10^{-1}	5.5200×10^{-1}	5.8944×10^{-1}	5.6141×10^{-1}

TABLE II. - COMPARISON OF DIMENSIONLESS LOAD CAPACITY OF AUSMAN (REF. 2) WITH PRESENT ANALYSIS

Inner- to outer-radius ratio, R_i/R_o	Dimensionless bearing number, Λ				
	10	20	40	80	160
	Dimensionless load capacity of finite-width bearing, W (a)				
0.2	0.064 .063	0.141 .138	0.286 .280	0.470 .458	0.638 .619
0.3	0.059 .059	0.131 .129	0.270 .267	0.457 .450	0.622 .612
0.4	0.053 .053	0.119 .118	0.248 .246	0.431 .429	0.602 .598
0.5	0.046 .046	0.103 .102	0.219 .218	0.397 .396	0.572 .569
0.6	0.038 .038	0.084 .084	0.184 .183	0.349 .348	0.530 .531
0.7	0.029 .029	0.063 .063	0.141 .140	0.284 .284	0.466 .466
0.8	0.019 .019	0.041 .041	0.091 .091	0.195 .196	0.363 .363

[a] First value from ref. 2; second value from present analysis.

TABLE III. - OPTIMAL STEP PARAMETERS FOR RESULTING MAXIMUM DIMENSIONLESS LOAD CAPACITY FOR VARIOUS DIMENSIONLESS BEARING NUMBERS

[Resulting dimensionless stiffness also given when optimum step parameters are used.]

Dimensionless bearing number, Λ	Optimal value for length-to-width ratio of pad, $(\lambda)_{opt}$	Optimal value of step location parameter, $(\psi)_{opt}$	Optimal value of film thickness ratio, $(k)_{opt}$	Maximum value of dimensionless load capacity of finite-width bearing, $(W)_{max}$	Dimensionless stiffness, K
10^{-5}	0.918	0.555	1.693	1.181×10^{-7}	2.362×10^{-7}
2.5×10^{-2}	.919	.554	1.693	2.955×10^{-4}	5.913×10^{-4}
5.0×10^{-2}	.920	.553	1.693	5.914×10^{-4}	1.184×10^{-3}
0.1	.922	.552	1.693	1.184×10^{-3}	2.376×10^{-3}
.2	.925	.549	1.693	2.376×10^{-3}	4.779×10^{-3}
.4	.933	.544	1.694	4.779×10^{-3}	9.669×10^{-3}
.8	.948	.533	1.696	9.670×10^{-3}	1.979×10^{-2}
1.6	.980	.511	1.703	1.980×10^{-2}	4.145×10^{-2}
3.2	1.043	.471	1.723	4.144×10^{-2}	9.006×10^{-2}
6.4	1.145	.412	1.790	8.903×10^{-2}	1.974×10^{-1}
12.8	1.294	.344	1.949	1.878×10^{-1}	3.854×10^{-1}
25.6	1.575	.271	2.240	3.651×10^{-1}	6.514×10^{-1}
51.2	2.037	.204	2.698	6.492×10^{-1}	1.003
102.4	2.710	.151	3.359	1.072	1.456
204.8	3.642	.110	4.270	1.674	2.034
409.6	4.901	.080	5.501	2.502	2.770

TABLE IV. - OPTIMAL STEP PARAMETERS FOR RESULTING MAXIMUM DIMENSIONLESS STIFFNESS FOR VARIOUS DIMENSIONLESS BEARING NUMBERS

[Resulting dimensionless load capacity also given when optimal step parameters are used.]

Dimensionless bearing number, Λ	Optimal value for length-to-width ratio of pad, $(\lambda)_{opt}$	Optimal value of step location parameter, $(\psi)_{opt}$	Optimal value of film thickness ratio, $(k)_{opt}$	Maximum value of dimensionless stiffness, $(K)_{max}$	Dimensionless load capacity of finite-width bearing, W
10^{-5}	0.915	0.557	1.470	2.550×10^{-7}	1.115×10^{-7}
2.5×10^{-2}	.917	.555	1.471	6.334×10^{-4}	2.789×10^{-4}
5.0×10^{-2}	.919	.554	1.471	1.278×10^{-3}	5.582×10^{-4}
0.1	.922	.551	1.471	2.563×10^{-3}	1.118×10^{-3}
.2	.929	.546	1.472	5.152×10^{-3}	2.245×10^{-3}
.4	.943	.535	1.474	1.041×10^{-2}	4.520×10^{-3}
.8	.973	.514	1.479	2.125×10^{-2}	9.164×10^{-3}
1.6	1.035	.474	1.494	4.432×10^{-2}	1.882×10^{-2}
3.2	1.153	.408	1.537	9.561×10^{-2}	3.953×10^{-2}
6.4	1.353	.328	1.642	2.072×10^{-1}	8.518×10^{-2}
12.8	1.863	.232	1.849	4.070×10^{-1}	1.757×10^{-1}
25.6	2.952	.148	2.191	7.036×10^{-1}	3.289×10^{-1}
51.2	5.035	.088	2.687	1.105	5.624×10^{-1}
102.4	9.093	.051	3.368	1.627	8.931×10^{-1}
204.8	17.172	.030	4.274	2.293	1.338

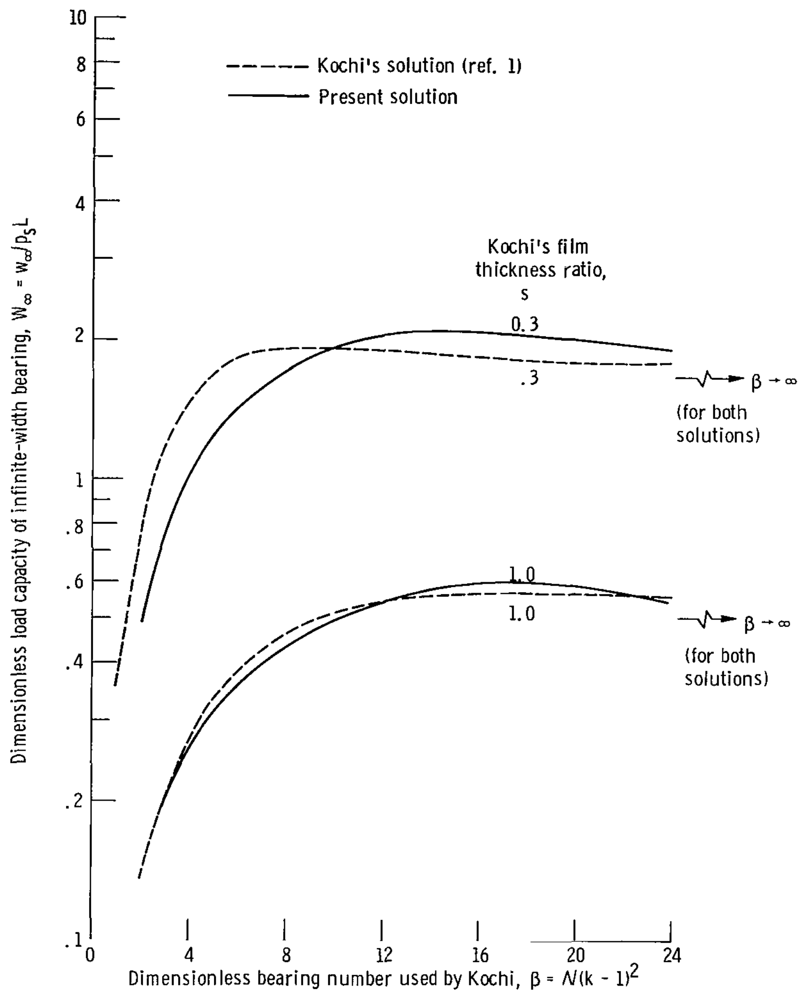


Figure 1. - Comparison of present linearized results with Kochi's exact results for infinite-width step slider bearing when step location parameter is 0.5.

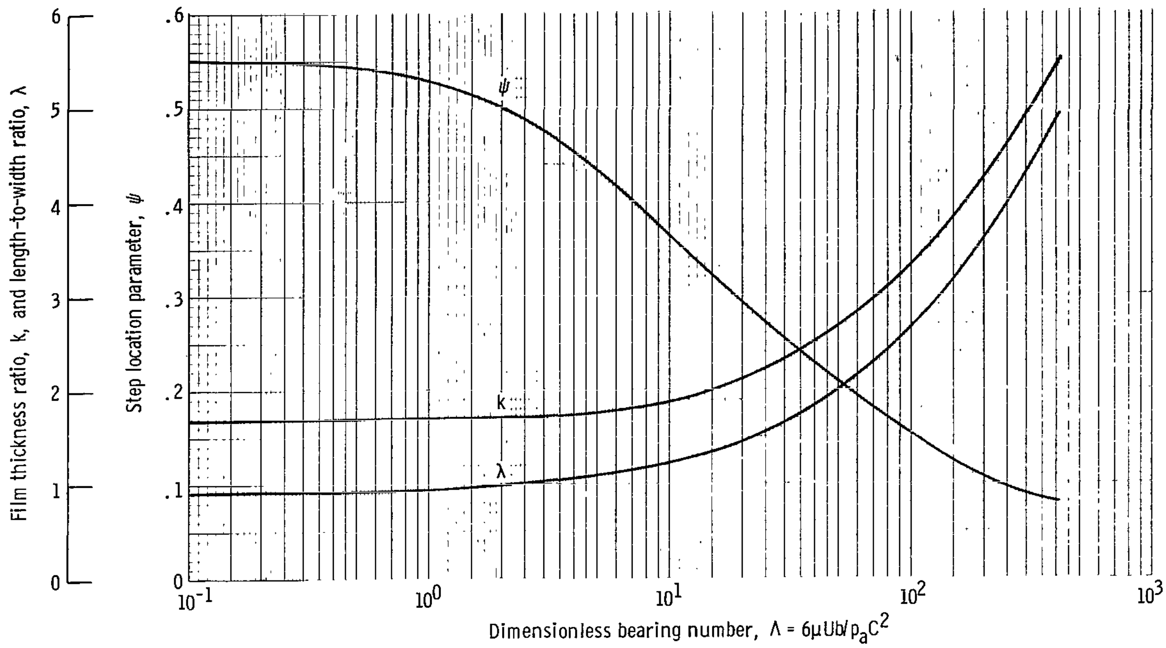


Figure 2. - Effect of dimensionless bearing number on optimum parameters for maximum dimensionless load.

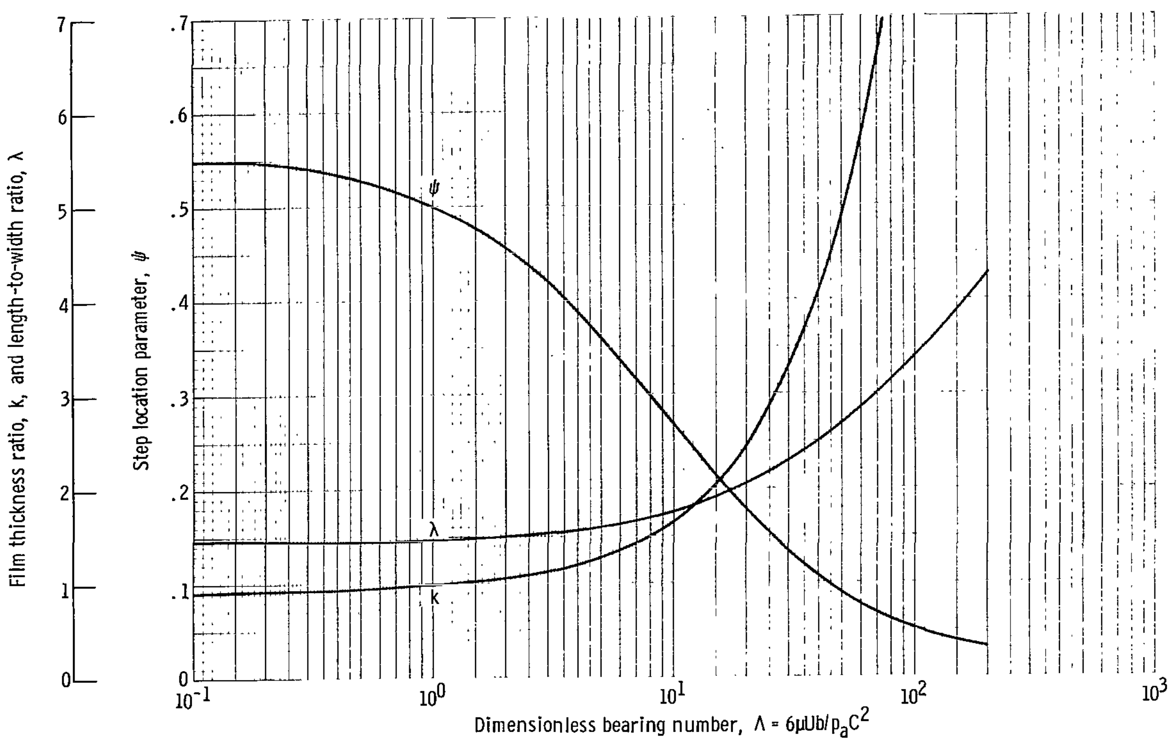
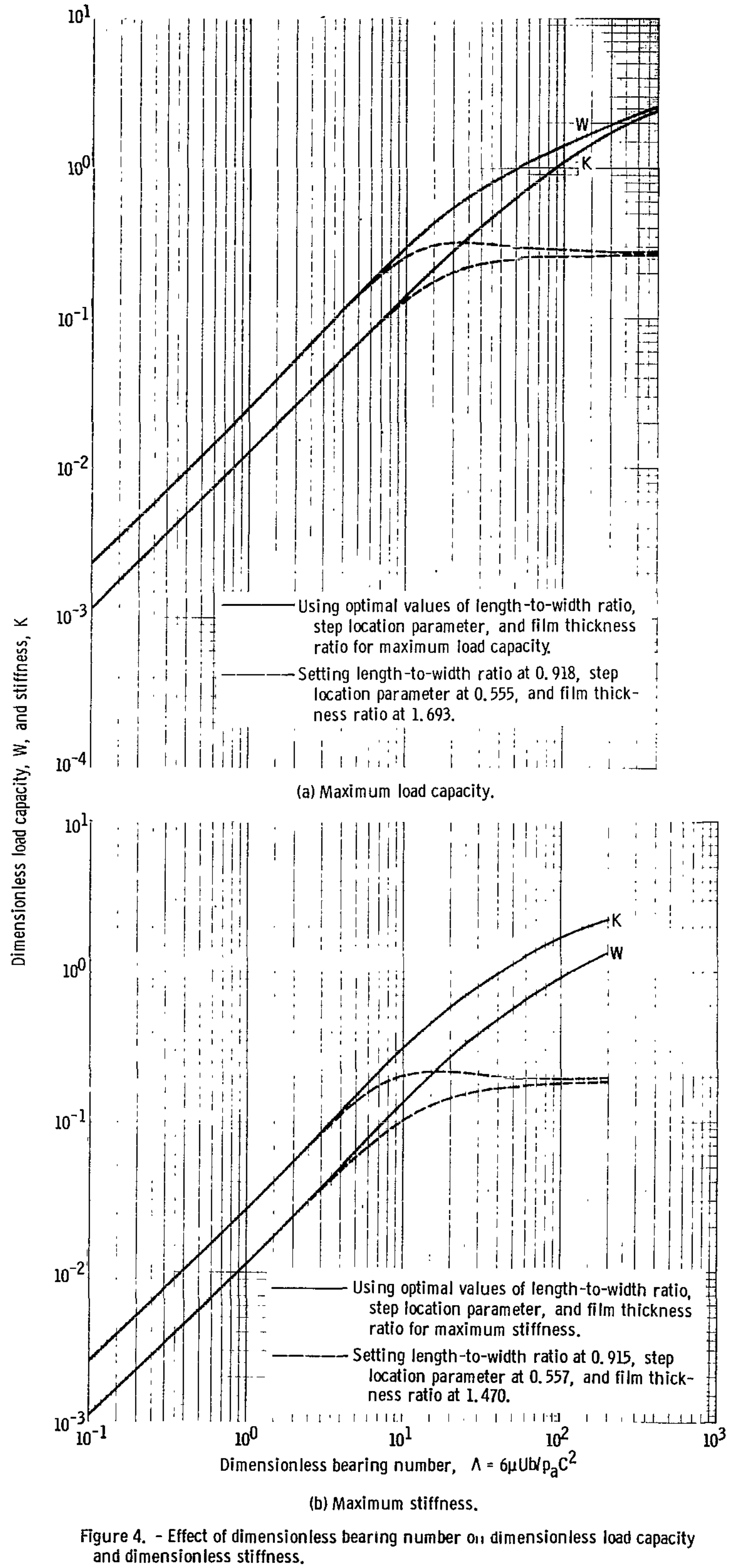


Figure 3. - Effect of dimensionless bearing number on optimal step parameters for maximum dimensionless stiffness.



(b) Maximum stiffness.

Figure 4. - Effect of dimensionless bearing number on dimensionless load capacity and dimensionless stiffness.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546

OFFICIAL BUSINESS

FIRST CLASS MAIL

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
: ADMINISTRATION

04U 001 40 51 3DS 70225 00903
AIR FORCE WEAPONS LABORATORY /WLOL/
KIRTLAND AFB, NEW MEXICO 87117

ATT E. LOU BOWMAN, CHIEF, TECH. LIBRARY

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546