FINAL REPORT FOR SECOND YEAR

(Sept. 16, 1968 to Sept. 15, 1969)

NASA GRANT NGL 15-004-026

''CONVOLUTIONAL CODING TECHNIQUES FOR DATA PROTECTION''

Department of

ELECTRICAL ENGINEERING

UNIVERSITY OF NOTRE DAME, NOTRE DAME, INDIANA

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ABSTRACT

This report describes the principal results obtained during the second year of research under NASA Grant NGL 15-004-026. These results are compiled into the following six categories:

(1) Contributions to the general theory of convolutional codes,

(2) Development of powerful quick-look-in nonsystematic convolutional codes,

(3) Contributions to sequential decoding,

(4) Unified analysis of decoders for convolutional codes,

(5) Inverses of linear systems, and

(6) Sub-baud coding.

This report also gives a list of the publications issuing from this research together with a list of personnel performing research under this grant.

I. Summary of Main Research Results

In the following sub-sections, we list the main results obtained from the research performed under this grant during the period from Sept. 16, 1968 to Sept. 15, 1969:

(1) Contributions to the General Theory of Convolutional Codes

During this period, D. Costello completed a major study of the capabilities and limitations of convolutional codes, together with the development of several techniques for synthesizing good codes. This work is fully described in his technical report (See Section II(2) for a complete reference) which was also submitted as his doctoral dissertation. Among the new contributions therein, we single out the following for special mention:

(a) The new concepts of the order j row distance and the order j column distance of a convolutional code, and the relation of these distance measures to the free distance of a convolutional code.

(b) The new concept of causal dominance of convolutional codes. This idea is central to Costello's important new result that systematic codes are as good as non-systematic codes (both in error-correcting power and ease of implementation) if one is willing to permit the use of feedback in the encoder.

(c) The general method for synthesizing the syndrome forming circuit for an arbitrary convolutional code.

(d) New and tight lower bounds on the free distance and the definite decoding minimum distance attainable with convolutional codes.

(e) Construction algorithms for synthesizing good long convolutional codes.

(2) Development of Powerful Quick-look-in Nonsystematic Convolutional Codes

As was reported in the Quarterly Progress Report of June 5, 1969, for the period Feb. 16, 1969, to May 15, 1969, the principal investigator proposed a class of non-systematic convolutional codes that permit quick recovery of the information bits (without decoding) for synchronization and other engineering purposes and which perform better than the best systematic codes of the same constraint length. D. Costello carried out a computer search for the best such codes and conducted comparison simulations. These results confirmed the superiority of the new class of non-systematic codes which are expected to find widespread application in space channels.

(3) Contributions to Sequential Decoding

During this period, the principal investigator and M. Sain reported on their simplified approach to analyzing the search performed by the Fano sequential decoding algorithm (See II(3) for complete reference) and also reported on their method for a special class of codes to obtain the exact distribution of the metric along the correct path in the code tree (See II(4)). These ideas were incorporated by J. Geist into an in-depth study of the algorithmic aspects of sequential decoding on which he embarked during this period. The March 4, 1969, Quarterly Progress Report for the period Nov. 16, 1968 to Feb. 15, 1969, describes one of the early results obtained from Geist's study, namely a geometric procedure to determine the "computational burst" resulting from closely spaced errors on the channel.

(4) Unified Analysis of Decoders for Convolutional Codes

During this period, the report of Morrissey's novel and powerful stochastic sequential machine approach to the decoding of convolutional codes was released (See II(5) for complete reference.) This work was described in detail in the report for the first year of this grant.

(5) Inverses of Linear Systems

During this period, the report of the fundamental work by the principal investigator and M.K. Sain on inverse linear systems was published (See II(6) for complete reference.) D. Costello (See II(2)) found this work and their earlier work on feed-forward inverses to be of fundamental importance in many phases of convolutional coding. In particular, he showed that the existence of a feed-forward inverse for the encoder guarantees that the order j row and column distances approach the free distance as j increases and attain this limit for j no greater than a small constant times the square of the memory order of the code. Moreover, Costello showed that the existence of a zero-delay inverse for the encoder is necessary and sufficient for the code to be causally equivalent to a systematic code (possibly with encoder feedback,) and it is precisely the case where the encoder has no zero-delay inverse which had defied past attempts to form causally equivalent systematic codes - a difficulty which Costello obviated by introducing the concepts of causal dominance rather than causal equivalence. Simply put, one encoder is causally dominant to another if the set of all output sequences which it produces from causal inputs properly includes the like set for the other encoder. Costello showed that the causally dominant encoder always has greater column distances (hence better error probability and less computation with sequential decoding) and showed that an encoder without a zero-delay inverse is causally dominated by a systematic code.

(6) Sub-Baud Coding

As part of his effort under this grant, the principal investigator participated in a study of the multipath and multi-user interference problems associated with the proposed Teletype Data Relay Satellite system. His contribuiton to this study was a thorough investigation of sub-baud coding (See Chapter IV of the document cited in II(7).) Sub-baud coding was taken to mean the assignment of an n-bit binary codeword to each of the users to specify for each user the polarities of n adjacent pulses which form the baud signal for the user, i.e., either this composite signal or its negative is transmitted dpeending on the value of the information bit to be sent in that baud. The objective is to minimize the output of the matched filter for one user when the input is the signal of another user or a time-delay of the same user's signal. The principal investigator showed that the performance of such sub-baud codes is determined by the set of even and odd correlation functions for the binary code words. While the even correlation functions are familiar in communication studies, the necessity to consider the odd correlation functions seems not to have been noticed previously. He also showed that whereas the even correlation functions are unaffected by which phase of an n digit sequence is chosen for a codeword, the odd correlation functions are crucially phase-dependent. He also showed how cyclic error-correcting codes can be used to synthesize sub-baud codes with bounds on both the even and odd correlation functions determined by the minimum distance of the code. Finally, he showed that convolutional-type of sub-baud codes appear to have little practical merit.

II. List of Publications

The following technical reports and journal articles disclosing research performed under this grant were made public or submitted for publication during the period covered by this report (Sept. 16, 1968 to Sept. 15, 1969.) Reprints and/or preprints of all these documents were supplied to the National Aeronautics and Space Administration and to private investigators expressing an interest in receiving these publications.

(1) D.J. Costello, Jr., "A Construction Technique for Random Error Correcting Convolutional Codes," IEEE Trans. Info. Th., IT-15, pp. 631-636, Sept. 1969. (also released as Memo. EE-682, Dept. of Elec. Engr., Univ. of Notre Dame, Notre Dame, Ind., Nov. 14, 1968).

(2) D.J. Costello, Jr., "Construction of Convolutional Codes for Sequential Decoding," Tech. Rept. EE-692, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., August 1969.

(3) J.L. Massey and M.K. Sain, "Trunk and Tree Searching Properties of the Fano Sequential Decoding Algorithms," Proc. 6th Allerton Conf. on Circuit and System Th., Univ. of Ill., Urbana, Ill., pp. 153-160, Oct. 1968. (also released as Memo. EE-6817, Univ. of Notre Dame, Notre Dame, Ind., Oct. 1, 1968.

(4) J.L. Massey and M.K. Sain, "Distribution of the Minimum Cumulative Metric for Sequential Decoding," (abstract only) presented at IEEE International Symposium on Information Theory, Ellenville, N.Y., January 1969.

(5) T.N. Morrissey, Jr., "A Unified Markovian Analysis of Decoders for Convolutional Codes," Tech. Rpt. EE-687, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., October 24, 1968. (To appear in IEEE Trans. Info. Th.)

(6) M.K. Sain and J.L. Massey, "Invertibility of Linear Time-Invariant Dynamical Systems," IEEE Trans. Auto. Cont., AC-14, pp. 141-149, April 1969 (also released as Memo EE-687m Univ. of Notre Dame, Notre Dame, Ind., Aug. 8, 1968).

(7) J.L. Massey and J.J. Uhran, Jr., "Final Report for Multipath Study (NASA Contract NAS5-10786)," July 1, 1969 to Aug. 31, 1969, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., (The time devoted by J. Massey to this project was supported under this grant).

III. Personnel

Table I gives a complete list of all personnel engaged in research under this grant during the period (Sept. 16, 1969 to Sept. 15, 1969) covered by this report. The source of support for graduate students performing research under the direction of the principal investigator is shown only if different from this grant.

We are pleased to report that Mr. Daniel J. Costello, Jr., completed the requirements for the Ph.D. degree in electrical engineering with the support of this grant in September 1969 and is now an assistant professor of electrical engineering at the Illinois Institute of Technology, Chicago, Illinois.

	Name	Category	Dates of Affiliation	Source of Support if not this Grant
(1)	Dr. J. L. Massey	Principal Investigator	9-16-68 to 9-15-69	
(2)	Mr. J. K. Chang	Research Assistant	9-16-68 to 9-15-69	
(3)	Mr. D. J. Costello, Jr.	Research Assistant	9-16-68 to 9-15-69	
(4)	Mr. J. M. Geist	Research Assistant	9-16-68 to 9-15-69	NSF Traineeship

Table I. Personnel Involved in Research Under This Grant