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STRESS CONCENTRATION DUE TO A HYPERBOLOID CAVITY IN A THIN PLATE

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16. Abstract

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Results include the stress concentration factors for various ratios of hole depth to plate thickness and some plots of the spatial variation of stress to indicate the type of distribution that exists.

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NOTATION

a	radius of cavity at surface of plate
d	plate thickness
h	cavity depth
K	stress concentration factor at bottom of cavity
p	applied radial pressure
r	radial coordinate
σ_r	radial stress component tangent to surface
σ_θ	circumferential stress component

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SUMMARY

Space structures subjected to meteoroid impacts that do not penetrate the structure may still be locally overstressed because of the stress concentration of the surface cavity. Experimental and numerical results from the digital computer program NASTRAN are presented for the stress concentration of hyperboloid cavities in thin elastic plates statically loaded in uniform biaxial tension (or compression) in the plane of the plate. One hemispherical and two hyperboloid shapes are considered. The radius of the plate was large compared to that of the cavity (8-10 times) and the cavity depth ranged from 25 to 80 percent of the plate thickness. The plate material was assumed to be elastic, isotropic, and homogeneous.

Results include the stress concentration factors for various ratios of hole depth to plate thickness and some plots of the spatial variation of stress to indicate the type of distribution that exists.

INTRODUCTION

As the duration of manned and unmanned space flights increases, the effects of meteoroid impacts on space vehicles become an increasingly serious problem. Flights to other planets, extended lunar trips, reusable shuttle vehicles, and earth-orbiting laboratories will expose structures to the space environment for periods measured in months, thereby greatly increasing the chances of meteoroid impacts. Considerable work is being done on the mechanics of high velocity impact and penetration of structural elements to determine the structure needed to resist penetration by a given particle. However, even if the structure can be designed to resist penetration, the question still remains as to the effects of nonpenetrating cavities on the load-carrying part of the structure. The stress concentration around such a cavity may be more severe than that from a complete penetration and cyclic loadings or increased load levels could initiate a structural failure or induce leakage at a later time.

EXPERIMENTAL PROCEDURE

The experimental program[1] consisted of measuring surface strains, using resistance strain gages, on two 2024–T3 aluminum plates hydraulically loaded in axisymmetric radial compression

[1] The experimental program was started by Robert B. Clapper who designed, assembled, and partly debugged all the test fixtures and electronic measuring equipment.

as shown in figure 1. Figure 2 shows the details of the O-ring seals of the test fixture for holding the plate. The O-ring seals provided excellent sealing for pressures up to 5,000 psi although the data were taken at pressures of 1,000 to 3,000 psi. However, friction between the O-ring and the plate apparently caused some error in the strain readings. This was indicated by the stresses, calculated from the measured strain, near the outer edge of the plate. This stress consistently differed from the applied pressure by as much as 10 percent, depending on the pressure level and the plate thickness. The reason for this was probably the difference in radial stiffness between the O-ring and the plate. At low pressure levels, the O-ring would easily compress into its groove and as it compressed radially inward, some of its pressure load would be transferred to the plate, resulting in higher stresses in the plate. As the pressure level increased, this effect diminished (as the O-ring became firmly seated) and eventually appeared to reverse. That is, some of the pressure load applied to the plate was transferred to the O-rings. Consistent with this explanation, the largest difference occurred for the thinner plate where the O-ring diameter was a greater percentage of the thickness. Experimental data were taken at pressure levels that minimized this effect to about 2 percent.

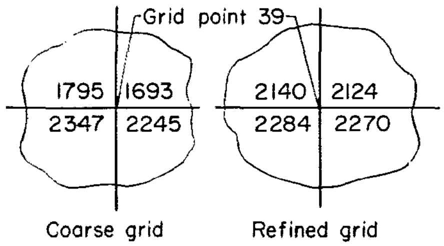
The strain gages were mounted along radial and circumferential lines on both sides and in the cavity of each plate. Twenty-six gages were mounted on the cavity side and 18 gages were mounted on the back surface. Small errors in strain readings were probably incurred by slight misalinement of the gages since the elements of a single foil gage do not remain parallel along their full length to either radial or circumferential lines. The basic electrical system used to measure the strains consisted of strain gage bridges powered by a 5 V DC supply. The output of the gages was amplified and displayed on a digital voltmeter by use of a channel selector.

Since the radial distance to a circumferential gage was slightly different than that to the corresponding radial gage, the radial and circumferential strain readings were plotted separately and a curve was fitted to each set of readings. The stresses were then calculated at discrete points from the curves.

The details of the plates and their corresponding machined cavities are shown in figures 3 and 4. The cavities are hyperboloids of revolution and the approximate equation for each shape is also given in figures 3 and 4. Since these cavities were intended to simulate meteoroid impact craters, the ratio of depth to radius, a basic descriptive parameter of impact craters, was selected to be 1.0 and 0.6, values in the range of those encountered in laboratory hypervelocity impact tests (ref. 1). The hyperboloid shape was convenient for the mathematical analysis, and a reasonable representation of the impact experiments.

COMPUTER ANALYSIS

The digital computer program NASTRAN was used to analyze the plates. This program was recently developed for common use by all NASA facilities and is based on the finite element displacement method of analysis (ref. 2). The present capability for three-dimensional analysis is limited to axisymmetric problems and uses toroidal elements with a triangular or trapezoidal cross section. The 60° hyperboloid experimental plate (fig. 3) was first modeled by 413 elements with 804 degrees of freedom (fig. 5). Smaller elements subsequently were used in the neighborhood of



Sketch (a) Element radial stress.

the cavity to avoid the large discontinuity in stresses for the same grid point of four adjoining elements, as shown for the coarse grid in sketch (a). Using smaller elements substantially reduced the discontinuity as shown by the refined grid-work in sketch (a). The second model contained 497 elements with 948 degrees of freedom (see fig. 6). The results presented are based on a grid network similar to this second model. Elements were either added or removed from the back surface of the plate to change the ratio of cavity depth to plate thickness. The thickest plate (cavity depth/plate thickness = 0.25) had 1,033 elements with 1,930 degrees of freedom. This case took 51 minutes to run on the IBM 360 Model 67. The program outputs were:

1. Displacements of all grid points
2. Stresses for all elements
3. Nodal forces at all grid points
4. Load vectors

DISCUSSION OF RESULTS

Figures 7 through 10 show a comparison of surface stresses obtained by NASTRAN and by experiment. For both plates, the agreement is good on the back surface as seen in figures 8 and 10, but some discrepancy appears in and around the cavities as seen in figures 7 and 9. Many factors could account for the differences between the computer solution and the experiment, but the differences were generally small so they were not investigated in detail. Figure 11 is a plot of the circumferential stress below the cavity for plates with the same cavity (60° hyperboloid) but different thicknesses. Note that as the ratio of cavity depth to plate thickness increases, the stress on the back surface decreases, demonstrating a basic difference between the three-dimensional problem and the two-dimensional notch problem. In the two-dimensional strip with a notch in the edge, the stress is required to increase as the notch becomes deeper since the total applied force must "flow under" the notch. In the three-dimensional problem, on the other hand, the applied force can "flow around" the cavity as well as under it. Also, with the thickest plate, the stress approaches the applied stress so this case is approaching the infinitely thick plate. The results are given in dimensionless form so they are applicable for thinner, but geometrically similar, plates used in space vehicle structures.

Table 1 shows the stress concentration factor K for all of the cases run (5 numerically, and 2 experimentally), as well as for a previous analysis (ref. 3). Neither the computer nor the experimental accuracies can be precisely defined, but the values of K as determined by the computer are believed accurate to within ±5 percent. The stresses obtained from the computer are average stresses at the center of the element. These values must be extrapolated to the edge of the element to obtain the stress on the surface of the plate, and relative accuracy is indicated by the rates of change of the stresses over several neighboring elements. The calculated stresses near the outer edge of the plate are within 2 percent of the applied stress. Note that K is relatively

TABLE 1.– STRESS CONCENTRATION FACTOR K

Cavity shape	$\frac{\text{Cavity depth}}{\text{Plate thickness}}$	K
60° hyperboloid (fig. 3)	0.80	3.5
	.67	3.5
	.51	3.5 (experimental 3.65)
	.25	3.4
45° hyperboloid (fig. 4)	.52	3.8 (experimental 3.65)
Hemispherical	.25	2.3 (ref. 3, 2.23)

independent of the ratio of cavity depth to plate thickness. Table 1 also shows that K is about the same for the two hyperboloid cavities. An analytical solution for the case of a hemispherical cavity in a semi-infinite solid (ref. 3) gives $K = 2.23$. Because of this large difference between the hemispherical and the hyperboloid cavities, it was felt that the hemispherical cavity should be checked. The value of K for this case is given in table 1 and is seen to compare well with reference 3. All of the values of K obtained are greater than 2.0, the value of K for the circumferential stress at the edge of a cylindrical hole in a thin plate. Therefore, procedures for designing pressure vessels and stressed plates subjected to meteoroid impacts should consider the additional stress concentration associated with surface cavities. Further studies should be made on the problem of yielding and fracture in the vicinity of surface cavities when the stress exceeds the yield or fracture limit.

If these results are used in predicting the stresses around a crater formed by a meteoroid impact, it must be remembered that an impact crater would not necessarily have a smooth shape, and residual stresses, cracks, and variable material properties could exist in the neighborhood of the crater. These effects are beyond the scope of this work but the results given here indicate the level of stress concentration that can exist.

CONCLUSION

A comparison of a hyperboloid cavity and a hemispherical cavity shows that the stress concentration may vary considerably depending on the specific shape of the cavity. As applied to meteorite impacts of space vehicles, this work shows that an impact that does not penetrate the structure may cause a significant stress concentration capable of initiating failure or causing leakage at a later time. A worthwhile area of investigation appears to be the study of crack propagation from a surface cavity in a plate under tension to see at what stress levels leakage will begin or catastrophic failure will occur.

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Moffett Field, Calif. 94035, April 21, 1970

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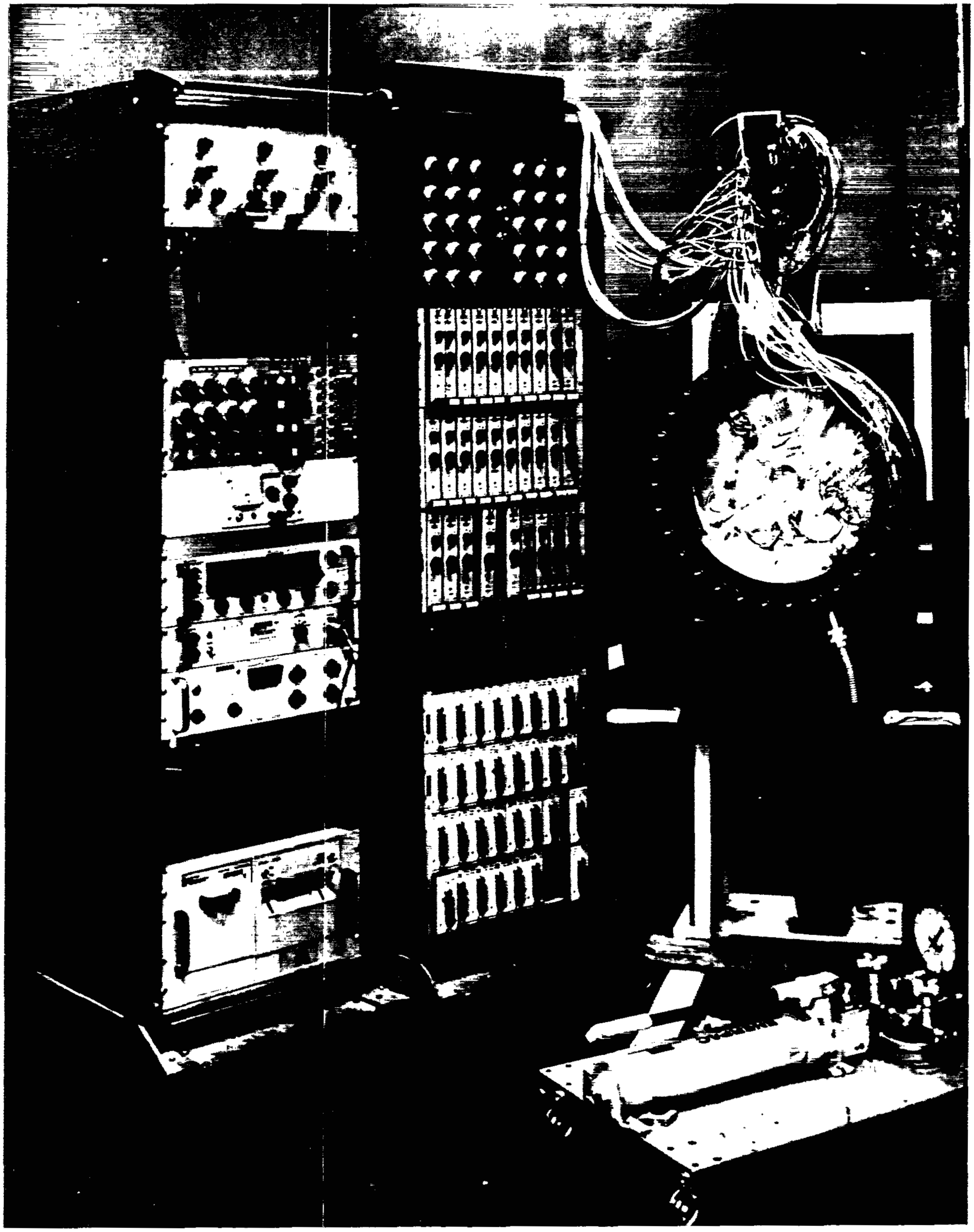

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Figure 1.- Instrumentation and test setup; test plate and hydraulic loading fixtures are at the center right.

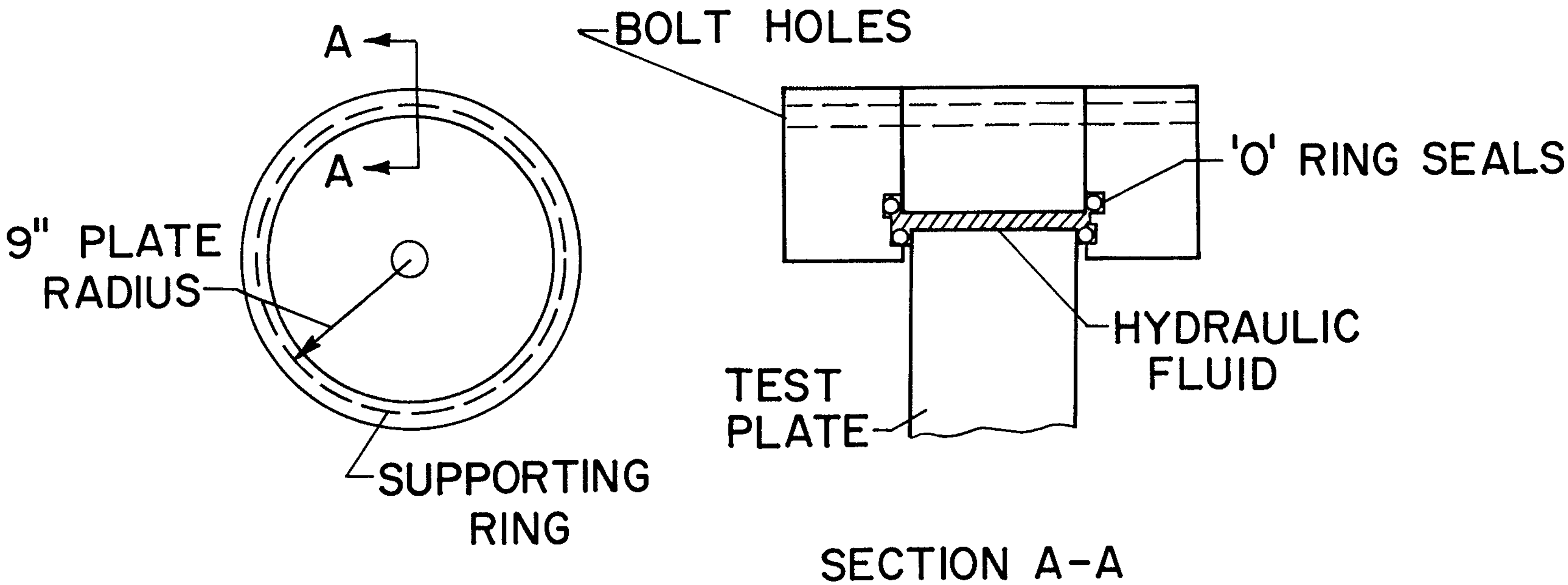

Figure 2.- Test fixture.

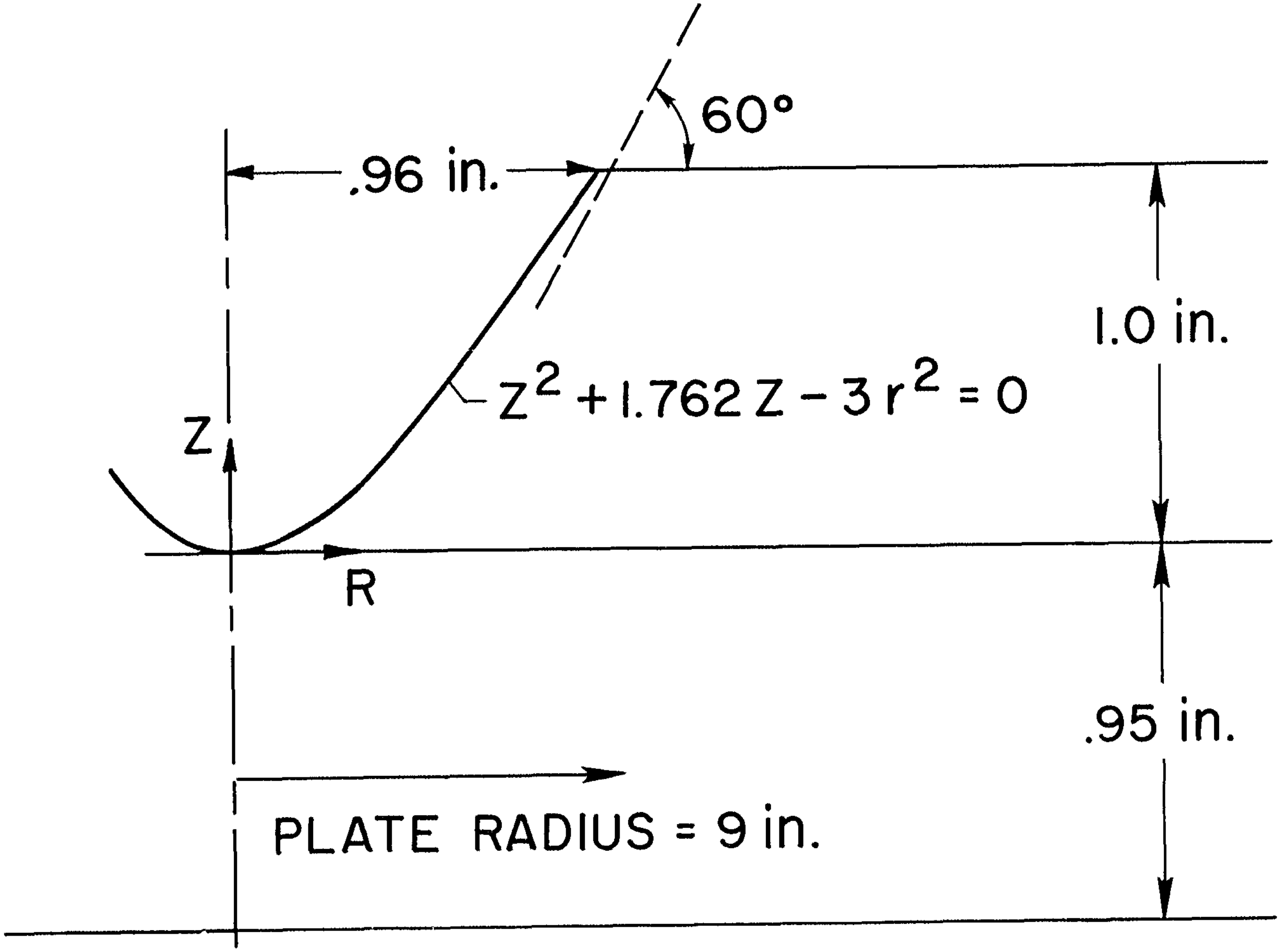

Figure 3.- 60° hyperboloid plate; h/d = 0.51.

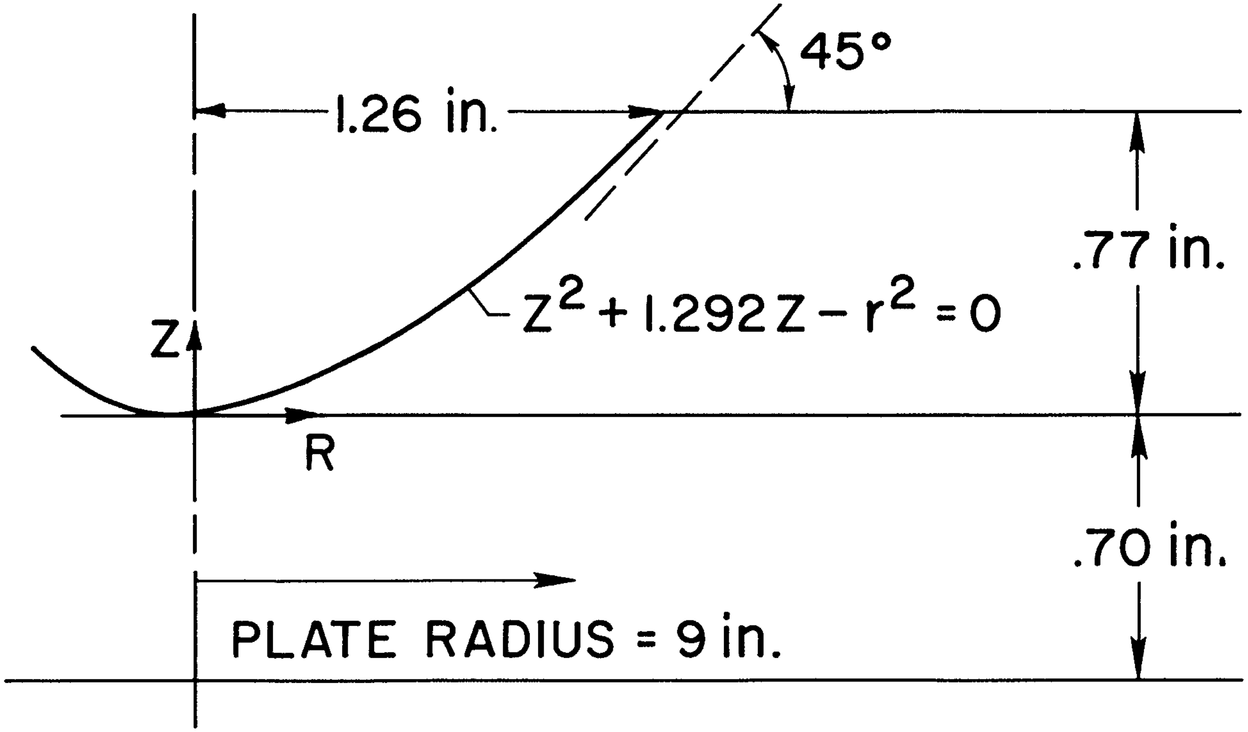

Figure 4.- 45° hyperboloid plate; h/d = 0.52.

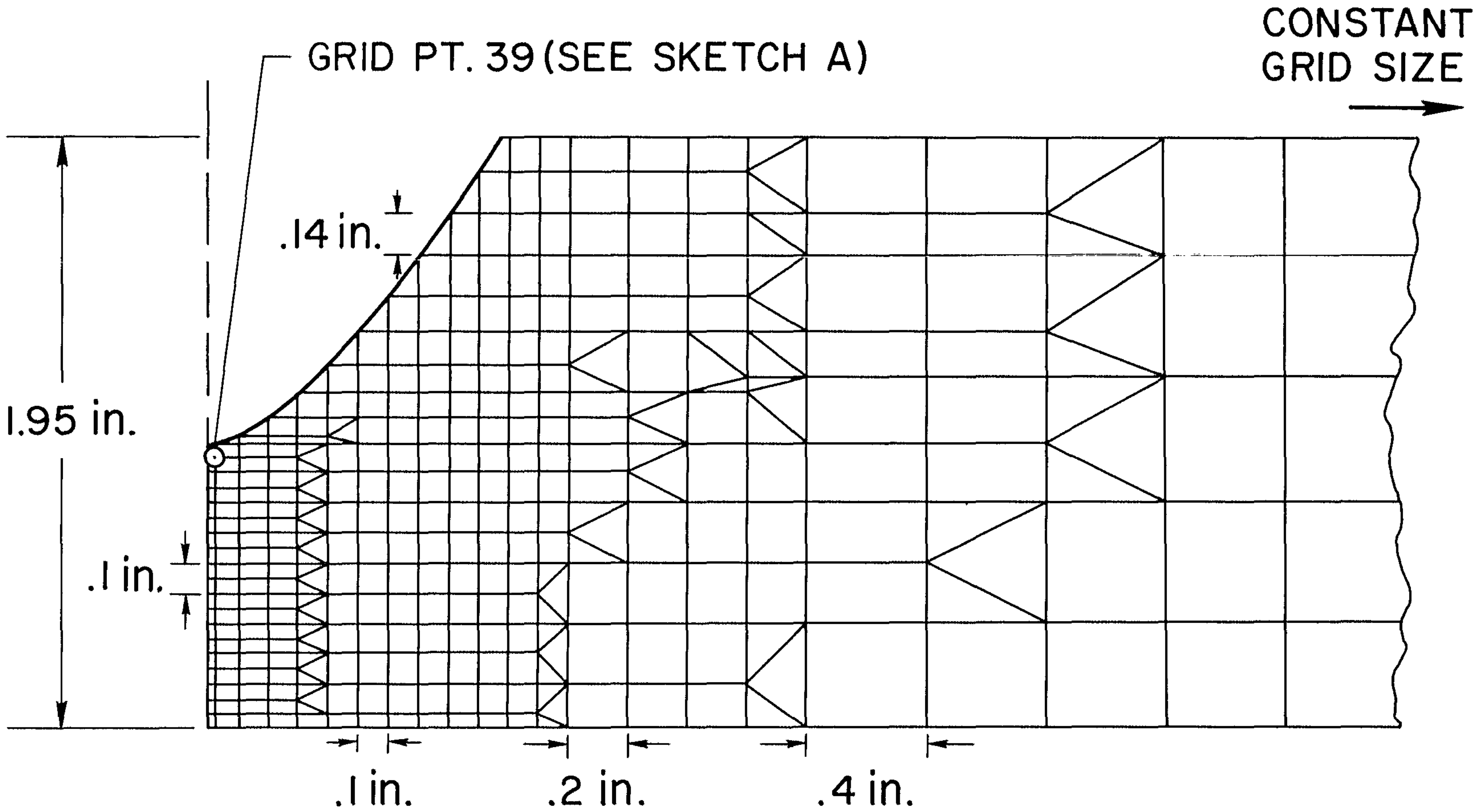

Figure 5.- Coarse grid of 60° hyperboloid plate; h/d = 0.51.

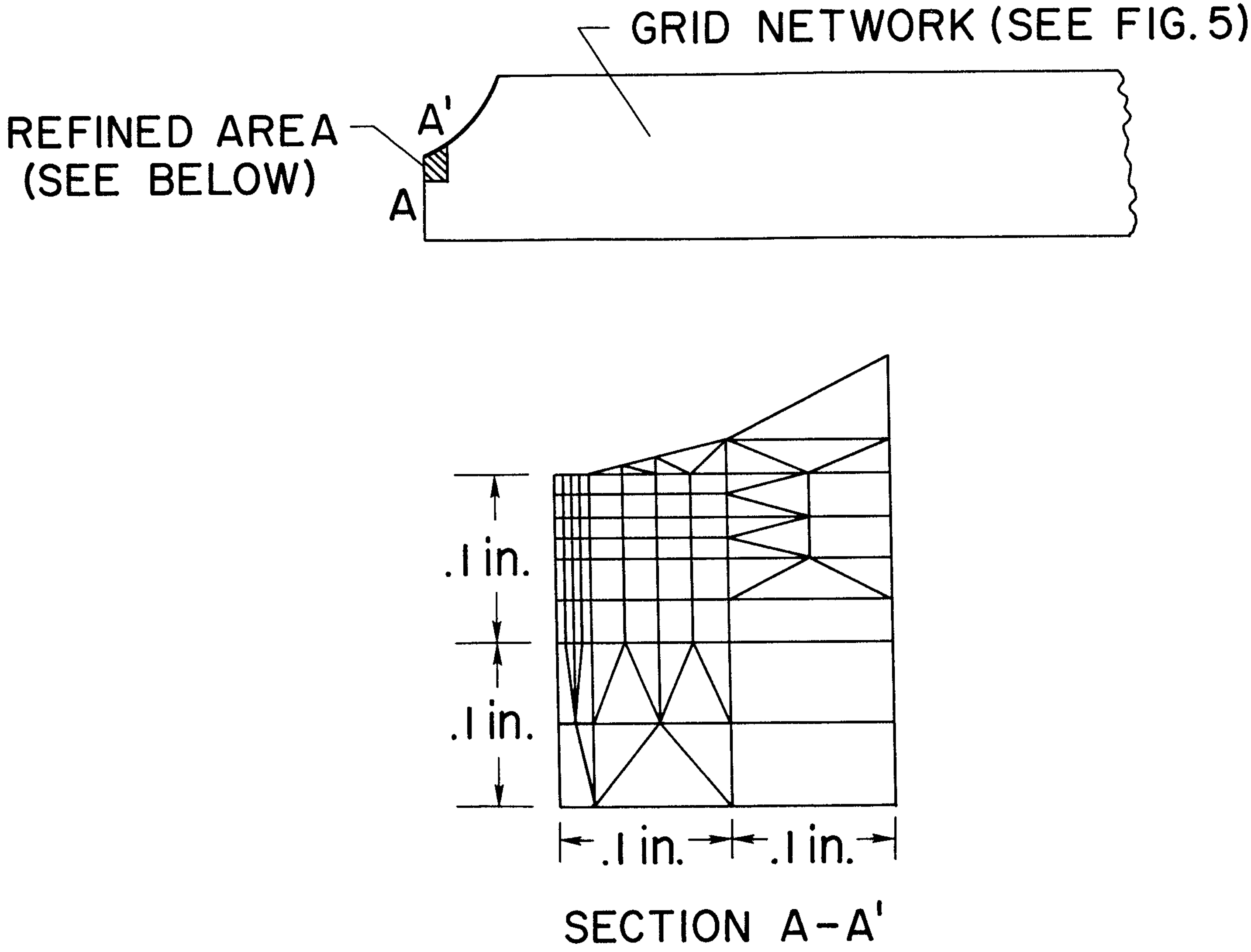

Figure 6.- Refined grid, 60° hyperboloid plate; h/d = 0.51.

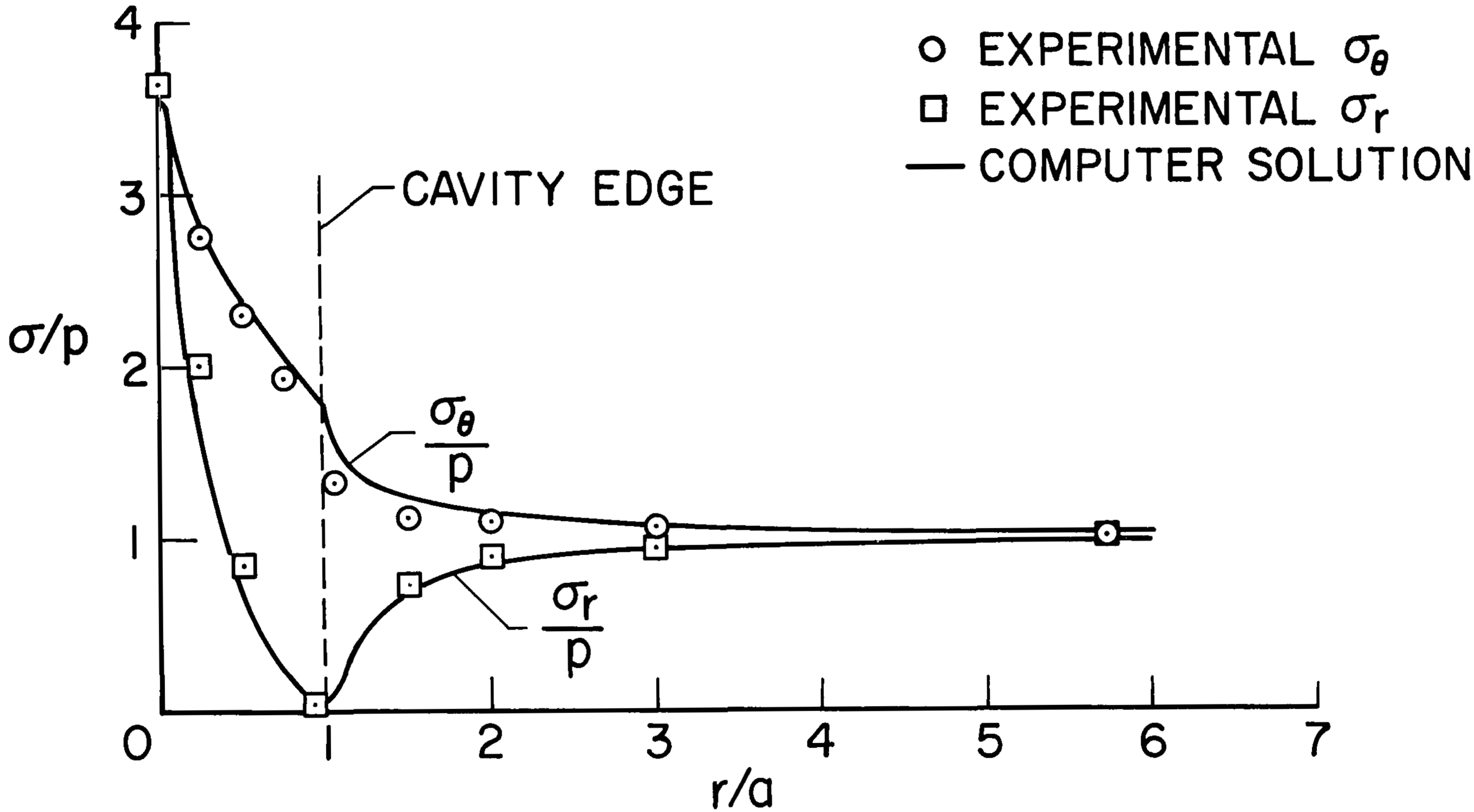

Figure 7.- Cavity surface stresses, 60° hyperboloid plate; h/d = 0.51.

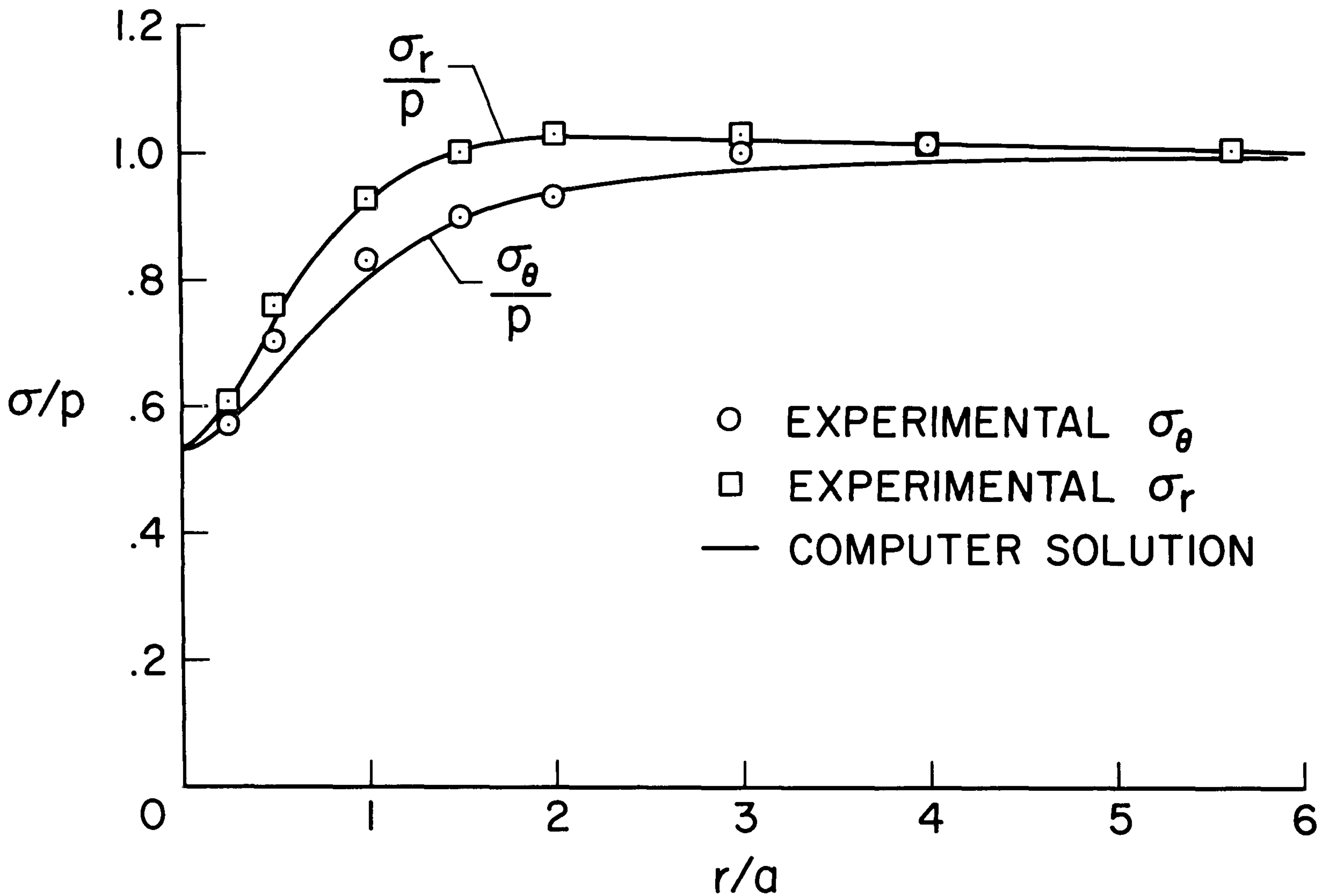

Figure 8.- Back surface stresses, 60° hyperboloid plate; h/d = 0.51.

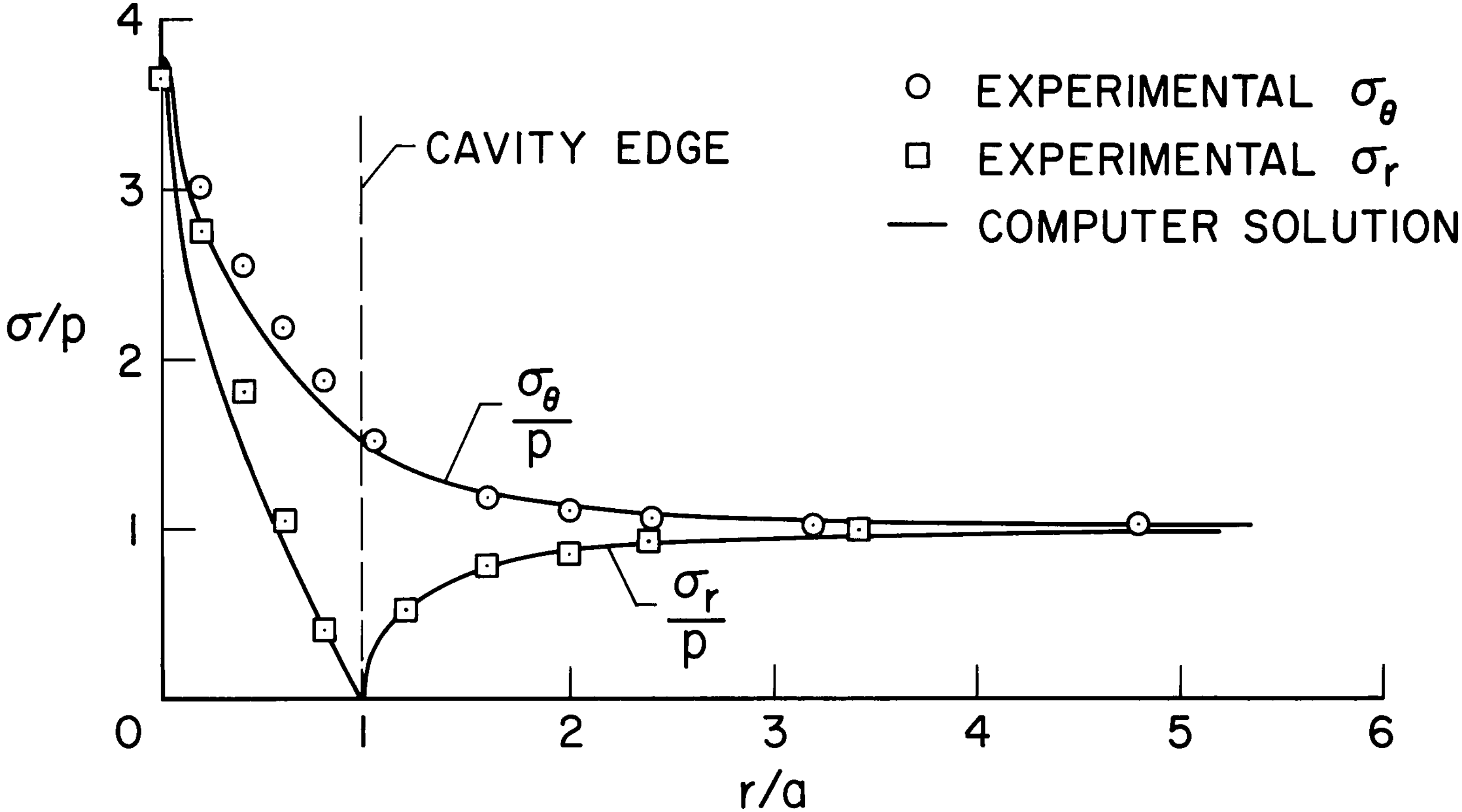

Figure 9.- Cavity surface stresses, 45° hyperboloid plate; $h/d = 0.52$.

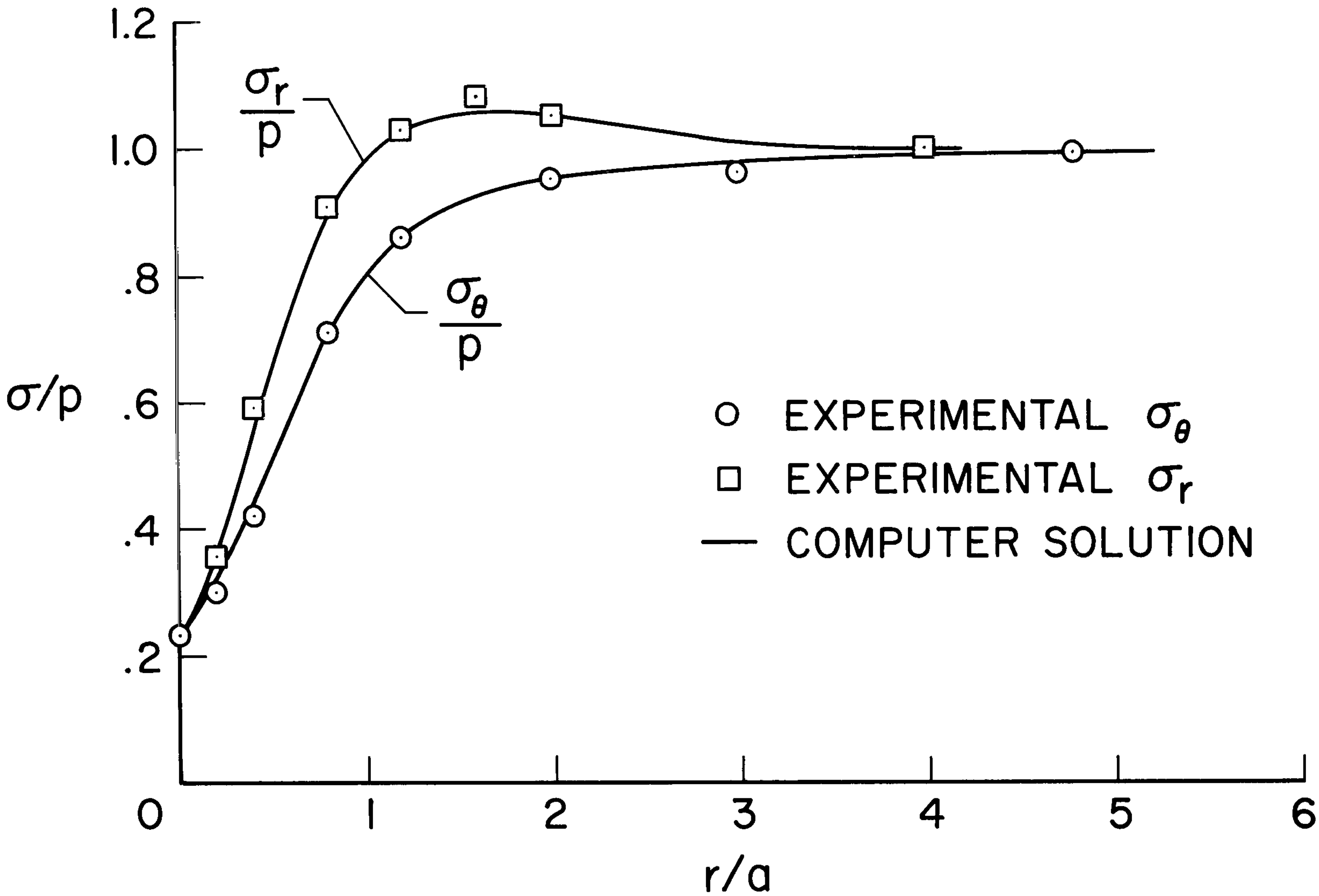

Figure 10.- Back surface stresses, 45° hyperboloid plate; h/d = 0.52.

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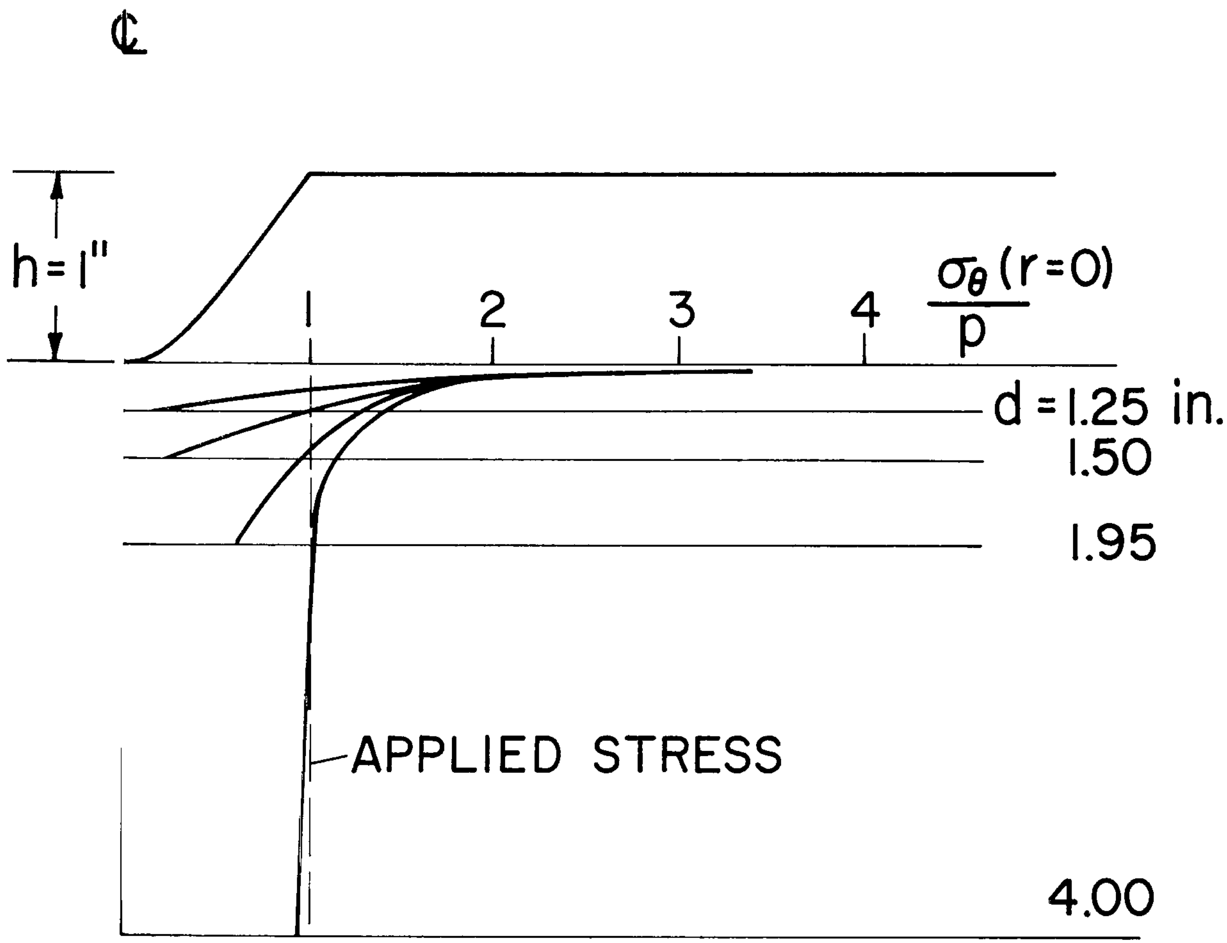

Figure 11.- Stress vs plate thickness, 60° hyperboloid cavity.

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