Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY

Theoretical Considerations of Some Nonlinear Aspects of Hypersonic Panel Flutter

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Fourth Annual Report
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NOMENCLATURE

a	Panel chord or wavy-wall wavelength
$\bar{a}_k$	Modal amplitude for transverse displacement
a_k	Dimensionless modal amplitude, $\bar{a}_k/h$
A	Wavy-wall amplitude
$\bar{b}_0, \bar{b}_R$	Modal amplitudes for in-plane displacement
c	Sound speed
c_1	Constant in momentum integral equation for wavy wall, $\beta/\alpha_1 R$ - See Eq. (3.49)
C_p	Pressure coefficient, $2(p-p_\infty)/\rho_\infty U^2$
D	Plate modulus, $Eh^3/12(1-\nu^2)$ (E here is modulus of elasticity)
$\bar{E}$	Total energy - See Eq. (2.5)
E	Dimensionless total energy, $a^3\bar{E}/Dh^2$
h	Panel thickness
$\bar{h}$	Enthalpy
h_0	Total enthalpy, $\bar{h}+(u^2+v^2)/2$
k	Reduced frequency, $\omega a/U$
k_0	Wave number, $2\pi/a$
K	Running spring constant, panel in-plane restraint spring
M	Free-stream Mach number
N	Number of assumed modes for panel transverse displacement
p	Pressure
$\overline{\Delta p}$	Static pressure difference across panel; positive if cavity pressure exceeds free-stream static pressure

Δp	Dimensionless static pressure difference, $\overline{\Delta p}\, a^4/Dh$
Pr	Prandtl number
q	Free-stream dynamic pressure, $\rho_\infty U^2/2$
$\bar{q}$	Heat-transfer rate
R	Reynolds number based on inverse wave number, $U/\bar{\nu}k_0$
$\overline{R_x}$	Applied in-plane load
R_x	Dimensionless applied in-plane load, $\overline{R_x}\, a^2/D$
s	Sheltering coefficient
t	Time
$\bar{u}$	Panel axial displacement
u	Local velocity component in x direction
U	Free-stream speed
v	Local velocity component in y direction
V	Viscous effect on pressure - see Eq. (3.53)
$\bar{w}$	Panel middle-surface transverse displacement
w	Dimensionless panel transverse displacement, $\bar{w}/h$
x	Axial coordinate
x_0	Lenght of flat-plate starting section
y	Transverse coordinate, Sec. III
z	Transverse coordinate, Sec. II
Z	Dimensionless transverse coordinate, $(y-y_s)/\Delta$
α	Constant in momentum integral equation for wavy wall, $2+\alpha_2/\alpha_1$ - see Eq. (3.49)
α'	In-plane restraint parameter, $K/[K+Eh/a(1-\nu^2)]$
α_1	Momentum-thickness parameter, θ/Δ - see Eq. (3.46)
α_2	Displacement-thickness parameter, δ^*/Δ - see Eq. (3.47)
β	Dimensionless surface velocity gradient - see Eq. (3.48)

γ	Gas constant for free stream ($\gamma = 1.4$)
δ	Boundary-layer edge
δ_1	Density-defect thickness - see Eq. (3.25)
δ^*, δ_1^*	Mass displacement thickness - see Eq. (3.22)
δ_2^*	Enthalpy displacement thickness - see Eq. (3.26)
Δ, Δ_1	Thickness of velocity boundary layer
Δ_2	Thickness of thermal boundary layer
ϵ	Dimensionless wavy-wall amplitude parameter, $k_0 A$
η	Displacement surface, $\overline{w} + \delta_1^*$
θ, θ_1	Momentum-defect thickness - see Eq. (3.23)
θ_2	Enthalpy-defect thickness - see Eq. (3.24)
λ	Dimensionless dynamic-pressure parameter, $2q\, a^3/MD$
μ	Dimensionless mass ratio, $\rho_\infty a/\rho_m h$
$\overline{\mu}$	Viscosity
ν	Poisson's ratio
$\overline{\nu}$	Kinematic viscosity, $\overline{\mu}/\rho$
ρ	Mass density
ρ_m	Panel mass density
τ	Dimensionless time, $t(D/\rho_m h a^4)^{\frac{1}{2}}$
$\overline{\tau}$	Shearing stress
Φ	Velocity potential
φ	Perturbation velocity potential
ω	Frequency
$(\dot{\ })$	Derivative of dimensionless quantity with respect to τ

Subscripts

e	Boundary-layer edge; also inviscid values on the displacement surface
s	Surface values on oscillating panel or wavy wall
∞	Free-stream values

I. INTRODUCTION

This report presents a summary of the fourth year's research activity under NASA Grant NGR 05-020-102, monitored technically by the Nonsteady Phenomena Branch of Ames Research Center. The research program has been divided into two more or less independent areas:

(1) A study of the effects of aerodynamic nonlinearities on panel flutter at hypersonic speeds. This study was begun in an attempt to explain certain nonlinear panel behavior observed experimentally; it has led to more general considerations of the impact of nonlinear aerodynamic loading on panel stability and postcritical response.

(2) An attempt to determine theoretically the effects of a turbulent boundary layer on the aerodynamic loading of an oscillating panel. It has already been demonstrated experimentally that the critical flutter dynamic pressure does depend on the thickness of the boundary layer over the panel, at least for the lower supersonic free-stream Mach numbers. The present study is aimed at improving existing theories to the extent of including all important effects while avoiding unnecessary complication in representing the boundary layer.

In Section II it is shown how nonlinear aerodynamic loading can cause an amplitude-sensitive instability, where the panel is unstable in a parameter region that is a stable one on the basis of linear theory, if the initial excitation is severe enough. Also discussed is an attempt to reproduce qualitatively the nonlinear experimental behavior alluded to above. Section III describes in some detail the fundamental parameters that govern the unsteady boundary-layer problem and presents a description of an integral formulation with a simple example. Section IV concludes this report with an outline of plans for continuing the research.

II. EFFECT OF AERODYNAMIC NONLINEARITIES ON PANEL STABILITY

2.1 Determination of an Amplitude-Sensitive Stability Boundary.

References 1 and 2 present the derivation of the equations of motion for the panel on hinged supports of Fig. 1, along with an assessment of the importance of various nonlinear aerodynamic terms for parameter ranges of practical interest. The panel middle-surface displacement $\bar{w}(x,t)$ is approximated by a series of assumed modes in space that satisfy the geometric boundary conditions, and the Rayleigh-Ritz technique is used to derive equations of motion in time for the modal amplitudes. These equations are then integrated by computer to produce the panel motion versus time. With variables and parameters as defined in the Nomenclature, these equations are

$$\frac{1}{2}\ddot{a}_k + \frac{1}{2}\pi^2 k^2 (R_x + \pi^2 k^2) a_k + \frac{3}{2}\pi^4 k^2 \alpha' a_k \sum_{n=1}^{N} n^2 a_n^2$$

$$+ \lambda \sum_{n=1}^{N} \frac{kn[1-(-1)^{k+n}]}{k^2-n^2} a_n + \frac{1}{2}\left(\lambda \frac{\mu}{M}\right)^{\frac{1}{2}} \dot{a}_k - \frac{[1-(-1)^k]}{k\pi} \Delta p$$

$$+ \frac{(\gamma+1)\pi k}{4} \lambda M \frac{h}{a} \sum_{m,n=1}^{N} \frac{mn(k^2-m^2-n^2)[1-(-1)^{k+m+n}]}{[k^2-(m-n)^2][k^2-(m+n)^2]} a_m a_n$$

$$+ \frac{(\gamma+1)\pi}{8} M \frac{h}{a}\left(\lambda \frac{\mu}{M}\right)^{\frac{1}{2}} \left[\sum_{\substack{m,n=1 \\ m+n=k}}^{N} m a_m \dot{a}_n + \sum_{\substack{m,n=1 \\ m-n=k}}^{N} (n a_n \dot{a}_m - m a_m \dot{a}_n) \right] = 0$$

$$k = 1,2,\ldots,N \qquad (2.1)$$

In these equations are two aerodynamic nonlinear terms, which come from the piston-theory aerodynamic terms proportional to $(\partial\overline{w}/\partial x)^2$ and $(\partial\overline{w}/\partial x)(\partial\overline{w}/\partial t)$. These two terms are the ones found to be significant for the parameter ranges of interest.

In Ref. 2 it was also demonstrated that energy levels capable of causing amplitude-sensitive instability could be generated by unstable panel motion near the linear stability boundary. In view of these results it was decided to condider how the linear stability boundary - presented, say, in the λ-R_x plane for other parameters fixed - would be changed for a given level of initial excitation.

The stability of nonlinear nonconservative systems has been the subject of several papers in the past few years; a recent example is the one by Dimantha and Roorda (Ref. 3). Their basic method of analysis is the direct method of Liapunov, with Zubov's procedure for constructing the Liapunov functional. It is not clear whether or not this method can be successfully applied to determine the panel stability boundary discussed above; however, the ideas of Ref. 3 do at least suggest a meaningful approach to the problem. In principle, one can determine a stability boundary for the panel in terms of initial values of the modal amplitudes and their time derivatives, for fixed values of the system parameters. This boundary can be viewed as a hypersurface S in the phase space made up of the modal amplitudes and their time derivatives. (Clearly, the origin of this phase space represents the panel in its flat undisturbed equilibrium position.) Any combination of initial values that plots on one side of S will result in unstable panel motion, while a combination that plots on the other side of S will result in stable panel motion.

Dimantha and Roorda then propose to calculate the minimum total panel energy on S ; this minimum value E_1 will determine a hypersphere that just touches S (at one or more points) and everywhere else is on the stable side of S. Hence E_1 provides an upper bound for the initial energy of the disturbance such that the resultant panel motion is stable. One can then calculate the maximum total panel energy on S ; this value E_2 determines another hypersphere that touches S from its unstable side, and it provides a lower bound on the initial disturbance energy in the panel for unstable motion. The reader is referred to Ref. 3 for the details. Suffice it to say here that these ideas suggest determining a nonlinear stability boundary in the λ-R_x plane for constant initial energy and comparing this boundary with the linear stability boundary, which is independent of the initial conditions.

The total energy of the panel system is obtained from Eqs. (2.4) and (2.5) of Ref. 1:

$$\overline{E} = \tfrac{1}{2}\rho_m h \int_0^a \left(\frac{\partial \overline{w}}{\partial t}\right)^2 dx + \tfrac{1}{2}\int_0^a \left\{ \frac{Eh}{1-\nu^2}\left[\frac{\partial \overline{u}}{\partial x} + \tfrac{1}{2}\left(\frac{\partial \overline{w}}{\partial x}\right)^2\right]^2 \right. \tag{2.2}$$

$$\left. + D\left(\frac{\partial^2 \overline{w}}{\partial x^2}\right)^2 \right\} dx - [\overline{R_x}(\overline{b}_R + \overline{b}_0) - \tfrac{1}{2}K\overline{b}_0^2]$$

Into this expression are substituted the assumed-mode series for $\overline{w}$ and $\overline{u}$:

$$\overline{w}(x,t) = \sum_{k=1}^{N} \overline{a}_k(t)\sin\frac{k\pi x}{a}$$

$$\overline{u}(x,t) = [\overline{b}_R + \overline{b}_0(t)]\frac{x}{a} + \sum_{k=1}^{2N} \overline{b}_k(t)\sin\frac{k\pi x}{a} \tag{2.3}$$

Then $\bar{b}_R$, $\bar{b}_0$, and $\bar{b}_k$ are eliminated in favor of $\bar{a}_k$ and other system parameters by making use of the panel equilibrium equations in the absence of aerodynamic loading (see Ref. 1):

$$\bar{b}_R = \frac{a(1-\nu^2)}{Eh}\bar{R}_x$$

$$\bar{b}_0 = -\frac{\pi^2}{4a}(1-\alpha')\sum_{k=1}^{N} k^2\bar{a}_k^2 \tag{2.4}$$

$$\bar{b}_k = -\frac{\pi}{4ak}\left(\sum_{\substack{m,n=1\\ m+n=k}}^{N} mn\bar{a}_m\bar{a}_n + 2\sum_{\substack{m,n=1\\ m-n=k}}^{N} mn\bar{a}_m\bar{a}_n\right)$$

After rearranging and cancelling some terms, the equation for the energy becomes

$$E = \frac{a^3}{Dh^2}\bar{E} = \frac{1}{4}\sum_{k=1}^{N}\dot{a}_k^{\,2} + \frac{3\pi^4}{8}\alpha'\sum_{k,\ell=1}^{N} k^2\ell^2 a_k^{\,2}a_\ell^{\,2}$$

$$+\frac{\pi^2}{4}R_x\sum_{k=1}^{N}k^2a_k^{\,2} + \frac{\pi^4}{4}\sum_{k=1}^{N}k^4a_k^{\,2} \tag{2.5}$$

It was decided at this point to choose an energy level corresponding to supercritical panel motion near the linear stability boundary in the λ-R_x plane, with the other system parameters fixed, and to use this energy level to determine initial conditions. There are, of course, many different combinations of the a_k and $\dot{a}_k$ that will give the same E ; it was further decided to set $a_1(0) = -a_2(0)$ and to let these be the only nonzero components of the initial state vector. This particular choice was a purely arbitrary one, and some consideration will be given later to determining more realistic forms of the initial conditions. Figure 2 presents the variation of $a_1(0) = -a_2(0)$ with R_x for $E = 1750$ and $\alpha' = 0.1$.

The other system parameters were then chosen, and a new stability boundary was determined as is shown in Fig. 3. The values of the system parameters are given in the caption. This boundary was obtained by integrating Eqs. (2.1), with initial conditions determined as discussed above, and observing whether or not the calculated panel motion persisted or died out past the initial transient. Note that there are some uncertainties in the stability boundary near $R_x = 0$, where it appears that the boundary has two branches. Clearly, the boundary should somehow be a closed one; at this time there are insufficient data to show just how the closure takes place. The panel-flutter motion versus time is shown in Fig. 4, for $\lambda = 295$ and $R_x = 0$. Values of the other parameters are given in the caption. The linear stability boundary gives the familiar value of 343 for λ, so the critical value of λ is decreased by 14%. The flutter mode shapes corresponding to the panel of Fig. 4 are given in Figs. 5-12. The mode shapes and time history are very similar to those discussed in Ref. 2 and are representative of the flutter motion all along the nonlinear stability boundary of Fig. 3. A strong traveling-wave component is evident in the flutter mode, even for positive (tensile) values of R_x.

The remarks in Ref. 3 concerning the accuracy of the numerical computation and the validity of the assumptions underlying the theoretical development of the equations of motion are also applicable here. The accuracy of the numerical integration was checked, and it was verified that the unstable panel motion did not violate the "moderate-rotation" assumption.

2.2 Comparison with Experiment.

Reference 4 describes some high-Mach-number panel-flutter experiments, where in-plane tension was used to stabilize the panels during tunnel starting. The tension was reduced until flutter was observed and then increased to stabilize the panel before it was damaged. It was found that in certain cases the final value of the tension had to be much greater than the initial value in order to stabilize the panel.

The aerodynamic nonlinear effects on stability discussed in Sec. 2.1 above suggest a possible explanation for this behavior. It was decided to attempt a qualitative comparison by looking for an amplitude-sensitive instability with Eqs. (2.1) and parameters corresponding to experimental conditions from Ref. 3. The following parameters were chosen: $R_x = 160$, $\lambda = 2000$, $M = 10$, $\mu = 0.1$, $\Delta p = 0$, and $h/a = 0.00054$. These values of R_x , λ , M , μ , and Δp give a point just on the stable side of the linear stability boundary for a panel on hinged supports. The experimental edge conditions would be much better represented by clamped supports, but in fact the theoretical differences between stability boundaries for these two edge conditions are not great at the relatively large value of 160 for R_x (see Ref. 4). The remaining unknown parameter is α' . Various initial amplitudes were used, appropriate to flutter amplitudes observed experimentally ($w \simeq 10$ in some cases), and α' was varied to see if these initial amplitudes would produce an unstable panel motion. The only unstable motion that could be produced was an oscillatory but divergent one, for values of α' near 10^{-3}. These results indicate that nonlinear aerodynamic influences are not the cause of the experimentally observed behavior, since the value of α' needed and the unstable motion calculated are not consistent with the experimental setup or observations.

III. EFFECT OF AN UNSTEADY BOUNDARY LAYER ON PANEL FLUTTER

3.1. Some Relevant Parameters.

Consider the flow past an oscillating panel (Fig. 13). We note that there are at least three characteristic lengths of interest - a , $\overline{w}$, and Δ - while there are three characteristic times associated with the problem - a/U , the mean time for flow of a fluid particle past the panel in the direction of the oncoming flow; $1/\omega$, the characteristic time for a panel oscillation; and Δ^2/ν , the characteristic response time of the boundary layer. Since analytical assumptions depend upon the relationship of these parameters, let us consider ratios that relate specifically to viscous effects on panel flutter in the low supersonic regime and some consequences of the various sizes. Characteristic values can be found in the data of Muhlstein et al. (Ref. 5).

The ratio Δ/a varies typically from .02 to .10 so that the boundary-layer thickness may not be constant along the panel chord. This variation is expected to be important when the boundary layer is comparatively thin (Ref. 3). One possibility would be to consider the effects of the quasi-steady increase of panel thickness due to the boundary-layer displacement thickness as well as the dynamic effect due to the rate of change of displacement thickness. For a laminar boundary layer the displacement thickness δ^* is

$$\frac{\delta^*}{a} = O(Re^{-\frac{1}{2}})$$

where the Reynolds number $Re = Ua/\nu$. Since this is true for turbulent boundary layers as well, the displacement effect may not be important for moderate Mach numbers; however, it can be of considerable importance for hypersonic Mach numbers, as shown by Orlik-Rückemann (Ref. 7).

Comparing the panel amplitude to the boundary-layer thickness, we note that the ratio $\overline{w}/\Delta$ is quite small, say $\sim 10^{-3}$, so that one might assume that the steady mean boundary-layer shear flow is not greatly affected by the panel motion. This was the basic assumption made by Dowell (Ref. 6) in his recent paper on the effect of a shear-flow profile in modifying the theoretical inviscid prediction of panel flutter. He solves a set of linearized unsteady perturbation equations where the direct effects of viscosity are neglected. His results for flutter dynamic pressure and frequency plotted versus boundary-layer thickness follow the trend of experimental data (Ref. 5) but predict flutter dynamic pressures consistently lower than those determined experimentally.

Looking at some of the characteristic time scales of the panel oscillation as compared to those of the flowing fluid, we note that the reduced frequency $k = \omega a/U$ compares the characteristic time for a panel oscillation with the characteristic time during which a fluid particle is in the vicinity of the moving panel. Hence, $k << 1$ is the requirement for the inviscid flow to be treated as quasi-steady. For panel flutter at low supersonic speed k is typically 0.5 to 1.0 so that the flow must be treated as unsteady. In considering the effect of the boundary layer we can compare the characteristic response time of the boundary layer, Δ^2/ν, which is a measure of the typical time for viscous diffusion through the boundary layer, with the characteristic time for a panel oscillation. Typically the ratio $\omega\Delta^2/\nu$ is $\sim 10^4$ so that one might conclude that the boundary layer is unaffected by the panel motion and can be treated as quasi-steady. However, if one assumes $\Delta/a = 0(Re^{-\frac{1}{2}})$ then the ratio $\omega\Delta^2/\nu$ is of the order of the reduced frequency from inviscid motion. Hence, one might conclude from this and the fact that $k \simeq 1$ that the boundary layer should be treated as an unsteady flow for the same reason as in the inviscid flow. The ambiguity of such reasoning coupled with the inability of a quasi-steady boundary-layer model (Ref. 3) to completely resolve the differences between experimental and theoretical flutter results leads one to conclude that an unsteady boundary-layer analysis is required.

3.2. Two-Dimensional Unsteady Viscous Flow Past an Oscillating Panel: Formulation of the Problem.

We would like to determine the pressure on the upper surface of an oscillating panel of chord a and of infinite span, imbedded in a rigid flat surface, the flow over which is known to be both unsteady and turbulent (Fig. 13). As described previously, the two-dimensional boundary-layer equations are a suitable starting point for the analysis (Ref. 2). We include only the shear stress as the dominant viscous stress and assume that the y-momentum equation reduces to $\partial p/\partial y = 0$. The later assumption can be demonstrated as follows:

The y-momentum equation is

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{1}{\rho}\frac{\partial \bar{\tau}}{\partial x} \tag{3.1}$$

Non-dimensionalizing as follows:

$$u = U\bar{u} \qquad p = \rho_\infty U^2 \bar{p}$$

$$v = \bar{w}\omega\bar{v} \qquad \rho = \rho_\infty \bar{\rho}$$

$$t = \omega^{-1}\bar{t} \qquad \bar{\tau} = \frac{\bar{\mu}U}{\Delta}\frac{\partial\bar{u}}{\partial\bar{y}}$$

$$x = a\bar{x}$$

$$y = \Delta\bar{y}$$

and substitution in Eq. (3.1) gives

$$A\frac{\partial\bar{v}}{\partial\bar{t}} + B\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + C\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}} = -\frac{1}{\bar{\rho}}\frac{\partial\bar{p}}{\partial\bar{y}} + \frac{1}{Re}\frac{1}{\bar{\rho}}\frac{\partial\bar{\tau}}{\partial\bar{x}}$$

where

$$A = \left(\frac{\bar{w}}{a}\right)\left(\frac{\Delta}{a}\right)k^2 \qquad k = O(1)$$

$$B = \left(\frac{\bar{w}}{a}\right)\left(\frac{\Delta}{a}\right)k \qquad \frac{\bar{w}}{a_\Delta}, \frac{\Delta}{a} << 1$$

$$C = \left(\frac{\bar{w}}{a}\right)k$$

We see that $\partial\bar{p}/\bar{p}\partial\bar{y}$ is $O(\bar{w}k/a)$ or $O(Re^{-1})$, whichever is larger. Since both quantities are small compared to unity, we can neglect the y pressure gradient and assume $p = p(x,t)$ in the boundary layer.

The boundary-layer equations are

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{3.2}$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) = -\frac{\partial p}{\partial x} + \frac{\partial\bar{\tau}}{\partial y} \tag{3.3}$$

$$\frac{\partial}{\partial t}(\rho h_0) + \frac{\partial}{\partial x}(\rho u h_0) + \frac{\partial}{\partial y}(\rho v h_0) = \frac{\partial p}{\partial t} + \frac{\partial}{\partial y}(\bar{\tau}u - \bar{q}) \tag{3.4}$$

The conservation equations are augemented by the definition of total enthalpy

$$h_0 = \bar{h} + (u^2+v^2)/2 \tag{3.5}$$

and an equation of state

$$\bar{h} = \bar{h}(p,\rho) \tag{3.6}$$

Relations can be given for the stress and heat transfer depending on the nature of the turbulent flow model chosen. Since we have chosen the laminar boundary-layer equations as a starting point for the analysis, we can present laminar-flow expressions simply for completeness at this point, keeping in mind that we are dealing with a turbulent flow. The integral analysis will be developed for turbulent flow where only the values of stress and heat transfer at the panel surface must be specified. For the present consider

$$\bar{\tau} = \bar{\mu}\frac{\partial u}{\partial y} \tag{3.7}$$

and

$$\bar{q} = \frac{\bar{\mu}}{P_r}\frac{\partial \bar{h}}{\partial y} \tag{3.8}$$

where $\bar{\mu}$ can be specified later in the analysis.

Equations (3.2) - (3.8) constitute a set of 7 equations for the 8 unknowns u , v , p , ρ , $\bar{h}$, h_0 , $\bar{\tau}$, $\bar{q}$; hence we need more information in order to solve this as a closed set of equations. We turn to the inviscid flow adjacent to the boundary layer for this additional information.

We assert that the viscous effects are concentrated in the flow layer next to the panel and that the flow outside this layer is essentially inviscid. The frictional effects are confined to a layer of thickness Δ_1, the velocity-layer thickness, while the heat-transfer effects are confined to a layer of thickness Δ_2, the thermal-layer thickness. The values of Δ_1 and Δ_2 must be determined as part of the problem. Let us assume for the moment that Δ_2 is the larger of the two at all positions along the surface so that the outer edge of the boundary layer is given by $y = \delta(x,t)$, where

$$\delta = \begin{cases} \bar{w} + \Delta_2 , & x_0 \le x \le x_0 + a \\ \Delta_2 , & 0 \le x \le x_0 , \quad x \ge x_0 + a \end{cases}$$

The inviscid flow is assumed to be adjacent to the boundary-layer flow at $y = \delta$. We have equations that describe the inviscid flow. In particular, at $y = \delta$, we have the x-momentum equation

$$\frac{1}{\rho_e}\frac{\partial p_e}{\partial x} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + v_e\frac{\partial u_e}{\partial y} \tag{3.9}$$

where u_e and v_e are the velocity components at the interface between the boundary layer and the outer inviscid flow. With the assumption that $\partial u/\partial y = 0$ at $y = \delta$ so that $\bar{\tau} = 0$ at $y = \delta$, this becomes

$$\frac{1}{\rho_e}\frac{\partial p_e}{\partial x} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} \tag{3.9a}$$

The equation for total enthalpy evaluated at $y = \delta$ reduces to the following when we assume $\partial h_{0_e}/\partial y = 0$:

$$\frac{1}{\rho_e}\frac{\partial p_e}{\partial t} = \frac{\partial h_{0_e}}{\partial t} + u_e\frac{\partial h_{0_e}}{\partial x} \tag{3.10}$$

We can simplify things somewhat by assuming that the outer inviscid flow is homentropic and irrotational, for which we may introduce a velocity potential $\Phi(x,y,t)$:

$$\vec{V} = \text{grad } \Phi \tag{3.11}$$

The following equation for the potential can be obtained:

$$c^2\nabla^2\Phi = \frac{\partial^2\Phi}{\partial t^2} + \nabla\Phi \cdot \nabla[2\frac{\partial\Phi}{\partial t} + \frac{(\nabla\Phi)^2}{2}] \tag{3.12}$$

where c , the speed of sound, can be determined from the compressible Bernoulli equation when the equation of state is given. For a thermally and calorically perfect gas c is determined from

$$\frac{\partial\Phi}{\partial t} + \frac{(\nabla\Phi)^2}{2} + \frac{c^2}{\gamma-1} = \text{const.} \tag{3.13}$$

The two adjacent flows are coupled by matching flow conditions at the boundary-layer edge. The pressure gradient in Eq. (3.9) depends upon the solution for Φ at $y = \delta$. Thus

$$u_e = \frac{\partial\Phi}{\partial x}(x,\delta,t) \tag{3.14}$$

Similarly, the time derivative of pressure in Eq. (3.10) may be evaluated in terms of the total enthalpy, given in terms of Φ as

$$h_{0_e} = h_{0_\infty} - \frac{\partial\Phi}{\partial t}(x,\delta,t) \tag{3.15}$$

and the density ρ_e is evaluated by using Eqs. (3.5), (3.6) and (3.15).

In summary, we have 12 equations - Eqs. (3.2)-(3.10), (3.12), (3.14), and (3.15) - for 13 unknowns - u, v, ρ, $\bar{h}$, h_0, $\bar{\tau}$, $\bar{q}$, Δ_1, Δ_2, p_e, u_e, h_{0_e}, Φ - so the problem for formulating the boundary layer simultaneously with the outer inviscid flow is not a determinate set of equations as it is presently posed. Let us consider an alternate formulation along more classical lines.

We represent the effects of viscosity on the outer flow by a single boundary-layer thickness, the unsteady displacement thickness, $\delta^*(x,t)$, defined as follows:

$$\rho_e u_e \delta^* = \int_{\bar{w}}^{\delta} (\rho_e u_e - \rho u)\,dy \tag{3.16}$$

Now let us solve for the unsteady inviscid flow past the displacement surface $\eta(x,t)$ defined as

$$\eta = \bar{w}(x,t) + \delta^*(x,t) \tag{3.17}$$

In principle, this problem can be solved with Eqs. (3.12) and (3.13) subject to the following boundary conditions:

$$\Phi_y(x,\eta,t) = \frac{\partial\eta}{\partial x}\Phi_x(x,\eta,t) + \frac{\partial\eta}{\partial t} \tag{3.18}$$

and $\left.\begin{array}{l}\Phi_x = U \\ \Phi_y = 0\end{array}\right\}$ at points far away from the surface.

This will yield a solution $\Phi(x,y,t)$ such that the result depends upon the unknown displacement surface. The surface pressure will be given by a version of Eq. (3.13) evaluated at $y = \eta(x,t)$:

$$C_p \equiv \frac{p_s - p_\infty}{\frac{1}{2}\rho_\infty U^2} = \frac{2}{\gamma M^2}\left\{[1-\frac{\gamma-1}{c_\infty^2}(\frac{\partial\Phi}{\partial t}+\frac{(\nabla\Phi)^2-U^2}{2})]^{\frac{\gamma}{\gamma-1}}-1\right\} \tag{3.19}$$

Now let us consider a solution of the boundary-layer equations that yields $\delta^*(x,t)$ as the output. Consider Eqs. (3.2)-(3.4). Since our intent is to determine $\delta^*(x,t)$ let us attempt to solve these equations in integral form. The integral equations of continuity, momentum, and energy have been derived (Ref. 1) by integrating Eqs. (3.3)-(3.5) with respect to y from $y = \overline{w}(x,t)$ to the edge to the thermal layer, $y = \delta(x,t)$. The latter two are as follows:

$$\frac{\partial}{\partial t}(\rho_e u_e \delta_1^*) - u_e \frac{\partial}{\partial t}(\rho_e \delta_1) + \rho_e u_e \delta^* \frac{\partial u_e}{\partial x} + \frac{\partial}{\partial x}(\rho_e u_e^2 \theta_1) = \overline{\tau}_s$$

$$+ \rho_e \Delta_2 [\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + \frac{1}{\rho_e}\frac{\partial p_e}{\partial x}] \tag{3.20}$$

$$\frac{\partial}{\partial t}[\rho_e u_e (h_{0_e} - h_{0_s})\delta_2^*] - (h_{0_e} - h_{0_s})\frac{\partial}{\partial t}(\rho_e \delta_1) + \rho_e \delta_1 \frac{\partial h_{0_s}}{\partial t} + \rho_e u_e \delta_1^* \frac{\partial h_{0_e}}{\partial x}$$

$$+ \frac{\partial}{\partial x}[\rho_e u_e (h_{0_e} - h_{0_s})\theta_2] = -\overline{q}_s + \rho_e \Delta_2 [\frac{\partial h_{0_e}}{\partial t} + u_e \frac{\partial h_{0_e}}{\partial x} - \frac{1}{\rho_e}\frac{\partial p_e}{\partial t}]$$

(3.21)

where

$$\delta_1^* = \delta^* = \int_{\bar{\bar{w}}}^{\delta} (1 - \frac{\rho u}{\rho_e u_e})\, dy \qquad (3.22)$$

$$\theta_1 = \theta = \int_{\bar{\bar{w}}}^{\delta} \frac{\rho u}{\rho_e u_e}(1 - \frac{u}{u_e})\, dy \qquad (3.23)$$

$$\theta_2 = \int_{\bar{\bar{w}}}^{\delta} \frac{\rho u}{\rho_e u_e}(1 - \frac{h_0 - h_{0_s}}{h_{0_e} - h_{0_s}})\, dy \qquad (3.24)$$

$$\delta_1 = \int_{\bar{\bar{w}}}^{\delta} (1 - \frac{\rho}{\rho_e})\, dy \qquad (3.25)$$

$$\delta_2^* = \int_{\bar{\bar{w}}}^{\delta} (1 - \frac{\rho(h_0 - h_{0_s})}{\rho_e(h_{0_e} - h_{0_s})})\, dy \qquad (3.26)$$

Equations (3.20) and (3.21) can be simplified, since the terms with Δ_2 are zero owing to Eqs. (3.9) and (3.10). These become

$$\frac{\partial}{\partial t}(\rho_e u_e \delta_1^*) - u_e \frac{\partial}{\partial t}(\rho_e \delta_1) + \rho_e u_e \delta_1^* \frac{\partial u_e}{\partial x} + \frac{\partial}{\partial x}(\rho_e u_e^2 \theta_1) = \bar{\tau}_s \qquad (3.27)$$

$$\frac{\partial}{\partial t}[\rho_e (h_{0_e} - h_{0_s})\delta_2^*] - h_{0_e} \frac{\partial}{\partial t}(\rho_e \delta_1) + \frac{\partial}{\partial t}(\rho_e \delta_1 h_{0_s}) + \rho_e u_e \delta_1 \frac{\partial h_{0_e}}{\partial x}$$

$$+ \frac{\partial}{\partial x}[\rho_e u_e (h_{0_e} - h_{0_s})\theta_2] = -\bar{q}_s \qquad (3.28)$$

As a means of coupling the outer inviscid flow to this boundary-layer problem we note that the flow variables ρ_e , u_e , and h_{0_e} are evaluated at $y = \delta(x,t)$, the boundary-layer edge. As an approximation (which might be verified by performing a matching of inner and outer expansions in a formal sense) let us assume that these variables are the values of ρ , u , and h_0 evaluated at the unsteady displacement surface $y = \eta(x,t)$ in the outer inviscid flow. Thus

$$u_e = \Phi_x(x,\eta,t) \tag{3.29}$$

Total enthalpy can be evaluated from the compressible Bernoulli equation

$$\frac{\partial\Phi}{\partial t}(x,\eta,t) + h_{0_e} = \text{const} = h_{0_\infty} \tag{3.30}$$

The density may be evaluated from the equation of state, Eq. (3.6), specialized to a perfect gas; thus

$$\rho_e = \frac{\gamma}{\gamma-1}\frac{p_e}{h_e} \tag{3.31}$$

where h_e is given by Eq. (3.5) evaluated at $y = \eta$,

$$h_{0_e} = h_e + \tfrac{1}{2}[\nabla\Phi(x,\eta,t)]^2 \tag{3.32}$$

and p_e is given by Eq. (3.19). Thus

$$\frac{\rho_e}{\rho_\infty} = \frac{\left\{1 - \frac{\gamma-1}{c_\infty^2}\left[\frac{\partial\Phi}{\partial t} + \frac{(\nabla\Phi)^2 - U^2}{2}\right]\right\}^{\frac{\gamma}{\gamma-1}}}{\left\{1 - \frac{1}{h_{0_\infty}}\left[\frac{\partial\Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2\right]\right\}} \tag{3.33}$$

where Φ is evaluated at $y = \eta(x,t)$.

Equations (3.29), (3.30), and (3.33) can be substituted in Eq . (3.27) and (3.28) to give a closed set of seven equations, Eqs. (3.22)-(3.28), for the seven boundary-layer thickness variables δ_1^* , δ_2^* , θ_1 , θ_2 , δ_1 , Δ_1 , and Δ_2 . The last two boundary-layer thicknesses appear implicitly in Eqs. (3.22)-(3.26). The set will be complete once the values h_{0_s} , $\overline{\tau}_s$, and $\overline{q}_s$ are specified at the panel surface. The five integrals (3.22)-(3.26) must be evaluated by a proper choice of velocity and enthalpy profiles (the density profile can be evaluated in terms of the enthalpy profile). The manner in which the surface values are chosen and the profiles specified is of key importance since the effects of the turbulent boundary layer will depend upon the model chosen.

3.3 Illustration of the Method - Incompressible Steady Flow Past a Semi-Infinite Wavy Wall.

In order to gain some familiarity with the formulation and solution of the unsteady flow past an oscillating panel, the steady flow past a rigid small-amplitude wavy wall was studied in some detail. Assume the viscous flow as shown in Fig. 149. The rigid surface is described by $y_s(x) = A \sin k_0 x$ and the condition of small amplitude is expressed by the smallness of the non-dimensional parameter $\epsilon = k_0 A$ in comparison with unity.

We assume that the viscous effects can be estimated by calculating the inviscid flow past the steady displacement surface $\eta(x)$ illustrated in Fig. 146:

$$\eta(x) = y_s(x) + \delta^*(x) \tag{3.34}$$

where

$$\delta^* = \int_{y_s}^{\delta} (1 - \frac{u}{u_e})\, dy$$

The governing equation for the flow is Laplace's equation

$$\nabla^2 \Phi = 0 \tag{3.35}$$

subject to the boundary condition at the surface

$$\Phi_y(x,\eta) = \Phi_x(x,\eta)\frac{d\eta}{dx}$$

and

$$\left.\begin{aligned} \Phi_x &= U \\ \Phi_y &= 0 \end{aligned}\right\} \text{ at points far away from the surface.}$$

This outer problem may be solved by perturbing the potential for a uniform free stream and linearizing the tangency condition. The problem is simplified by choice of non-dimensionalized variables, with velocities non-dimensionalized by U and lengths non-dimensionalized by the wave number k_0. The linearized problem for the perturbation potential is

$$\nabla^2 \varphi = 0 \tag{3.36}$$

$$\varphi_y(x,0) = \begin{cases} 0\,, & x<0 \\ \dfrac{d\eta}{dx} & x\geq 0 \end{cases}$$

$$\varphi_x, \varphi_y = 0 \qquad \text{as} \quad y \to \infty.$$

The solution obtained by thin-airfoil theory is

$$\varphi_x(x,y) = \frac{1}{\pi} \int_0^\infty \frac{\frac{d\eta}{dx}(x_1)(x-x_1)dx_1}{(x-x_1)^2+y^2} \tag{3.37}$$

The integral can be evaluated at the surface (y = 0 in the linearized theory) as follows:

$$\varphi_x(x,0) = \frac{1}{\pi} \oint_0^\infty \frac{\frac{dy_s}{dx}(x_1)dx_1}{(x-x_1)} + \frac{1}{\pi} \oint_0^\infty \frac{\frac{d\delta^*}{dx}(x_1)dx_1}{(x-x_1)} \tag{3.38}$$

The first integral is evaluated with $y_s(x) = \epsilon \sin x$ as

$$\frac{1}{\pi} \oint_0^a \frac{\frac{dy_s}{dx}(x_1)dx_1}{(x-x_1)} = \epsilon \left\{ \sin x + \frac{1}{\pi} [\mathrm{Ci}(x) \cos x + \mathrm{si}(x) \sin x] \right\} \quad , x>0 \tag{3.39}$$

where

$$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos t \, dt}{t}$$

$$\mathrm{si}(x) = -\int_x^\infty \frac{\sin t \, dt}{t}$$

are cosine and sine integrals. These integrals represent the effect of the leading edge of the wavy wall; both die out for large x , but Ci(x) is singular at x = 0 . We are therefore forced to exclude the vicinity of the leading edge from consideration, and as a first approximation let us use only the term $\epsilon \sin x$, which is appropriate for an infinite wavy wall. The full x-component of velocity at the displacement surface may be written therefore as

$$\Phi_x(x,0) = 1 + \epsilon \sin x + \frac{1}{\pi} \oint_0^\infty \frac{\frac{d\delta^*}{dx}(x_1)\, dx_1}{(x - x_1)} \tag{3.40}$$

and the linearized surface pressure coefficient is given by

$$Cp(x,0) = -2\varphi_x(x,0) = -2\epsilon \sin x - \frac{2}{\pi} \oint_0^\infty \frac{\frac{d\delta^*}{dx}(x_1)\, dx_1}{(x - x_1)} \tag{3.41}$$

We now consider the analysis of the boundary layer in order to calculate the unknown displacement thickness δ^*. The momentum integral equation for incompressible flow is

$$\frac{d}{dx}(u_e^2 \theta) + \delta^* u_e \frac{du_e}{dx} = \frac{\bar{\tau}_s}{\rho} \tag{3.42}$$

where the momentum thickness becomes

$$\theta = \int_{y_s}^{\delta} \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right) dy \tag{3.43}$$

and the axial velocity at the boundary-layer edge, u_e, is approximately, according to Eqs. (3.29) and (3.40),

$$u_e(x) = \Phi_x(x,\eta) \simeq \Phi_x(x,0) = 1 + \varepsilon \sin x + \frac{1}{\pi} \oint_0^{\infty} \frac{\frac{d\delta^*}{dx}(x_1)dx_1}{(x-x_1)} \qquad (3.44)$$

By assuming that the velocity profiles are similar we can write

$$\frac{u}{u_e} = f\left(\frac{y-y_s}{\delta-y_s}\right) = f(Z) \qquad (3.45)$$

where $f(Z)$ is the non-dimensional velocity profile and $Z = (y-y_s)/\Delta$ is the distance from the wall referred to the boundary-layer thickness. Thus Eqs. (3.34) and (3.43) can be written

$$\theta = \alpha_1 \Delta \quad , \quad \alpha_1 = \int_0^1 \frac{u}{u_e}\left(1-\frac{u}{u_e}\right) dZ \qquad (3.46)$$

$$\delta^* = \alpha_2 \Delta \quad , \quad \alpha_2 = \int_0^1 \left(1-\frac{u}{u_e}\right) dZ \qquad (3.47)$$

For a laminar boundary layer the shearing stress at the wall is

$$\bar{\tau}_s = \bar{\mu} \frac{\partial u}{\partial y}\bigg|_w = \frac{\bar{\mu}\beta u_e}{\Delta} \qquad (3.48)$$

where

$$\beta = \frac{\partial(u/u_e)}{\partial Z}\bigg|_{Z=0}$$

When Eqs. (3.44) and (3.46)-(3.48) are substituted in Eq. (3.42) the result is the following integro-differential equation for $\Delta(x)$:

$$[1+\epsilon \sin x+\frac{\alpha_2}{\pi}\int_0^\infty \frac{\frac{d\Delta}{dx}(x_1)dx_1}{(x-x_1)}]\Delta\frac{d\Delta}{dx}+\alpha[\epsilon\cos x-\frac{\alpha_2}{\pi}\int_0^\infty \frac{\frac{d\Delta}{dx}(x_1)dx_1}{(x-x_1)^2}]\Delta^2 = c_1 \tag{3.49}$$

where

$$\alpha = \frac{2\alpha_1+\alpha_2}{\alpha_1}, \qquad c_1 = \frac{\beta}{\alpha_1 R}, \quad \text{and} \qquad R = \frac{U}{\nu k_0} .$$

This equation can be solved approximately when the effect of the displacement thickness on the outer flow is small; that is, when

$$\left| \int_0^\infty \frac{\frac{d\Delta}{dx}(x_1)dx_1}{(x-x_1)} \right| \ll \epsilon$$

Under the assumption that the wavy-wall portion of $d\Delta/dx$ is of $0(\epsilon x^{\frac{1}{2}} R^{-\frac{1}{2}})$, this will be reasonable whenever

$(\frac{x}{R})^{\frac{1}{2}} \ll 1$, or in other words, for x in the region of interest.

The solution to Eq. (3.49) subject to this restriction is essentially the solution of the momentum integral equation from classical boundary-layer theory since the effect of the boundary layer on the outer flow is neglected. The boundary-layer thickness subject to this equation and the initial condition $\Delta(0) = 0$ is to $0(\epsilon)$

$$\Delta(x) = (2c_1 x)^{\frac{1}{2}} \left\{1-\epsilon\alpha[\sin x + \frac{2\alpha-1}{2\alpha}(\frac{\cos x - 1}{x})]\right. \tag{3.50}$$

The viscous contribution to the pressure at the displacement surface can be calculated from Eqs. (3.41) and (3.47) as follows:

$$C_{p_v}(x,0) = \frac{-2\alpha_2}{\pi} \oint_0^{\infty} \frac{\frac{d\Delta}{dx}(x_1)\,dx_1}{(x-x_1)} \tag{3.51}$$

The sinusoidal part of this is found to be

$$\tilde{C}_{p_v}(x,0) = 2\epsilon\alpha_1\alpha_2(2c_1x)^{\frac{1}{2}}\Big\{[1-f(x)S_2(x)-g(x)C_2(x)]\sin x$$

$$+ [g(x)S_2(x)-f(x)C_2(x)]\cos x\Big\} \tag{3.52}$$

Equation (3.52) contains the Fresnel integrals

$$S_2(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \frac{\sin t\,dt}{\sqrt{t}}$$

$$C_2(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \frac{\cos t\,dt}{\sqrt{t}}$$

and also the functions

$$f = \tfrac{1}{2}(1+\frac{2-\alpha_1}{2\alpha_1 x} - \frac{1-\alpha_1}{2\alpha_1 x^2})$$

$$g = \tfrac{1}{2}(1-\frac{2-\alpha_1}{2\alpha_1 x} - \frac{1-\alpha_1}{2\alpha_1 x^2})$$

These results can be compared with the work of others. The displacement thickness predicted by Eqs. (3.47) and (3.51) agrees quite well with the $M = 0.2$ case of Flügge-Lotz and Fannelöp (Ref. 8) who solved the laminar compressible boundary-layer equations using an implicit finite-difference procedure. The surface pressure given by Eq. (3.52) agrees qualitatively with Benjamin's (Ref. 9) analytical result obtained by solving the Orr-Sommerfeld equation for the perturbation caused by the wavy wall. Benjamin's result for the sinusoidal viscous part of the surface pressure in laminar flow is

$$\tilde{C}_{p_V}(x,0) = 2\epsilon V[1.115 \sin x + 0.644 \cos x] \tag{3.53}$$

where $V = 6.3\, X^{5/6}/R^{1/2}$, X being a <u>fixed</u> distance measured from the start of a boundary layer formed in the absence of a pressure gradient. This agrees qualitatively with Eq. (3.52) in the dependence upon Reynolds number and the presence of a component proportional to the wave slope. This latter component is referred to as the sheltering coefficient and in Benjamin's theory is given by $s = .644V$. It is a measure of the ability of the air stream to supply energy to an already existing wave train, and it accounts for wave growth in the theory of water waves. The experimental work of Motzfeld (Ref. 10) in which measurements were made in a turbulent flow past a wavy wall of three cycles showed a sheltering effect with the experimental value of $s = .034$. Benjamin's theory predicts $s = .010$, and the present theory predicts values of S that are one order of magnitude smaller than these results.

3.4 Critique of Present Efforts and Discussion of Alternative Formulations.

The problem of determining the surface pressure on an oscillating panel in the presence of a turbulent boundary layer has been considered by attempting to solve the boundary-layer equations simultaneously with the inviscid equations for the outer flow, under the assumption that the former apply to the thin region adjacent to the panel, while the latter apply to an idealized inviscid flow adjacent to the thin boundary layer. This formulation has not provided a well-posed problem in the sense that the system of equations governing all of the unknown fluid quantities as well as the unknown boundary-layer thickness does not yield a determinate or closed set of equations. The indeterminacy is characterized by the existence of only 12 equations for 13 unknown quantities. This difficulty seems to be the result of formulating equations for two adjacent fluid regions without knowing a priori the location of the interface surface, in this case the edge of the boundary layer.

The alternative to this scheme follows (with a slight modification) the basic idea of Prandtl in his original formulation of the steady boundary layer over a flat plate. Prandtl's idea was to first solve the boundary-layer problem using information obtained from the inviscid flow over the same body. Specifically, the inviscid (outer) problem determines the pressure gradient which is impressed upon the boundary layer by the inviscid flow. The next step in Prandtl's procedure was to determine the inviscid flow around a displaced body, a parabolic cylinder for the case of flow past a flat plate, and in particular one would calculate the pressure distribution on this new surface. This is the main idea behind the present alternative to a simultaneous solution of the boundary layer and outer flow. The boundary-layer problem is formulated using information obtained from an inviscid flow over the thickened body or displacement surface rather than the original body. The impressed pressure gradient is that for inviscid flow over a modified body rather than that for inviscid flow over the original body. The aim of this alternate scheme is thus to solve the "displacement problem" simultaneously with the boundary-layer problem with the matching condition that fluid properties at the edge of the boundary layer will be approximately equal to fluid properties at the surface of the displaced body in inviscid flow.

This alternate scheme is illustrated for the case of steady incompressible flow along a semi-infinite wavy wall. The result here leads one to the same conclusion that Prandtl reached, namely, that displacement is a higher-order effect which can be calculated after a classical boundary-layer calculation has been performed using a matching condition based purely on the inviscid flow. This seems to point to the conclusion that the modified scheme is really equivalent to a classical boundary-layer analysis in the first approximation, at least for steady flow. This means that by lumping the viscous effects into one parameter, the displacement thickness, we can only hope to determine the effects of displacement and nothing else. The same conclusion cannot simply be taken as applying to the entire range of Mach number or to an unsteady flow without some justification. In the case of the latter, along with assuming no transverse pressure gradient in the boundary layer (as with steady flow), there appears to be only one additional effect, namely, the dynamic effect due to the rate of change of displacement thickness. This will produce an addition to the normal velocity of $\partial\delta^*/\partial t$, which should be added to the upwash produced by the unsteady panel motion. However, this effect will be important only when the static effect of the displacement thickness is important. Hence we must conclude that the displacement effect can only partially explain the unsteady-boundary layer effects on panel flutter. Furthermore, the boundary layer oscillations and the panel motion cannot be considered to be in phase with each other for the reduced frequencies near unity, and a quasi-steady treatment of the boundary layer is questionable (Ref. 7).

With this explanation of current understanding of the problem of formulating a system of equations to determine the unsteady boundary-layer effects on pressure, let us consider the alternatives for continued research on the problem. Clearly any approach must go beyond an analysis based on displacement effects alone. A possibility that could conceivably alleviate the indeterminacy caused by considering two separate regions of flow exists in considering the flow field as a single region. This leads us to consider the Navier-Stokes equations as the basic set of governing equations from which one would begin an analysis. For this approach one need only specify boundary conditions at the panel surface and at points far away from the panel. The analysis could be formulated in an integral fashion by integrating the Navier-Stokes equations to infinity in the transverse direction. This has been accomplished by Gerking (Ref. 11) for the case of steady incompressible flow around a flat plate of finite length. His formulation leads to a system with an infinte number of ordinary integro-differential equations that does not appear to present any great difficulty in solving. However, it is not known what additional complication would arise when compressibllity and unsteadiness are introduced. A further difficulty that was not encountered in Gerking's analysis arises in the correct choice of velocity and density profiles that adequately describe the unsteady turbulent boundary layer adjacent to the oscillating panel. An approximate method for treating an unsteady laminar boundary layer in incompressible flow has been developed by Teipel (Ref. 12) for oscillations that are parallel to the main flow and for which the pressure is assumed to be determined from the known outer flow. By assuming that the unsteady effects are small in comparison to the steady flow quantities he assumes the laminar velocity profiles to be of the form

$$u(x,y,t) = u_S(x,y) + u_i(x,y)\, e^{i\omega t}$$

where the unsteady component u_i is small in comparison to the steady component u_S. For the case of a turbulent boundary layer the importance of the background Reynolds stress must be considered in attempting to model the velocity profiles and in estimating the interaction between the turbulent region and the oscillating panel.

Reynolds (Ref. 13) formulated a set of equations for an incompressible turbulent shear flow that is used to study the flow over a surface with traveling waves moving in the downstream direction. The formulation is based on the Navier-Stokes equations with an eddy-viscosity model for the oscillating background Reynolds stress. Of particular interest is the value of the power input to the wavy surface from the fluid due to the pressure field on the surface wave. It appears that the difference between the power transfers predicted by turbulent theory, laminar theory, and inviscid theory is quite striking and can be explained by the fact that the presence of the turbulence alters the structure of the critical layer and greatly enhances the ability of the wave to draw energy from the mean field. In examining the wave-induced Reynolds stress across the flow it becomes apparent that the structure of the turbulent boundary layer is of great importance and must be considered in problems of this nature.

IV. CONCLUDING REMARKS

Further research activity in connection with the hypersonic panel-flutter problem will be devoted to completing and clarifying the amplitude-sensitive stability boundary discussed in Section II. Afterwards, some parametric studies will be performed, where the changes in the stability boundary due to changes in key parameters such as Mh/a , α' , or the initial energy can be determined. Also of interest are the effects of varying the modal content of the initial energy and the changes brought about by varying the edge rotational restraint. It is also apparent that to perform an exhaustive parametric survey with the "computer-experiment" method used so far would involve an inordinate amount of computer time and expense. There is still a need for a method (such as the Liapunov method) that can determine stability boundaries directly and accurately as a function of the system parameters. Some time will be devoted to monitoring the literature and to examining any proposed method that could be adopted to the present problem.

It is intended to continue the research on turbulent boundary-layer effects on oscillatory panel aerodynamic loads along the following lines:

(1) Formulate the integral approach differently by integrating the Navier-Stokes equations across the entire flow field in order to obtain a mathematically determinate set of equations.

(2) Continue the simple boundary-layer modification based on displacement-thickness effects for unsteady supersonic flow and compare results from this method with experimental observations.

(3) Consider an alternate approach to the problem, such as a perturbation method that takes into account the effects of viscosity on the perturbed flow; the turbulence can be accounted for by means of an eddy-viscosity model similar to one proposed by Reynolds (Ref. 13).

REFERENCES

1. McIntosh, S.C., "Theoretical Considerations of Some Nonlinear Aspects of Hypersonic Panel Flutter," Second Annual Report, NASA Grant NGR 05-020-102, 1 September 1966 to 31 August 1967, Dept of Aeronautics and Astronautics, Stanford Univ., Stanford, Calif.

2. McIntosh, S.C. and Lerner, J.I., "Theoretical Considerations of Some Nonlinear Aspects of Hypersonic Panel Flutter," Third Annual Report, NASA Grant NGR 05-020-102, 1 September 1967 to 31 August 1968, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, Calif.

3. Dimantha, P.C. and Roorda, J., "On the Domain of Asymptotic Stability of Nonlinear Nonconservative Systems," Applied Scientific Research, Vol. 20, March 1969, pp. 272-288.

4. Ketter, D.J. and Voss, H.M., "Panel Flutter Analyses and Experiments in the Mach Number Range of 5.0 to 10.0," FDL-TDR-64-6, March 1964, Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio.

5. Muhlstein, L., Jr., Gaspers, P.A., Jr., and Riddle, D.W., "An Experimental Study of the Influence of the Turbulent Boundary Layer on Panel Flutter," TND-4486, March 1968, NASA.

6. Dowell, E.H., "Generalized Aerodynamic Forces on a Flexible Plate Undergoing Transient Motion in a Shear Flow With an Application to Panel Flutter," Paper No. 70-76, 1970, AIAA.

7. Orlik-Rückemann, K.J., "Stability Derivatives of Sharp Wedges in Viscous Hypersonic Flow," AIAA Journal, Vol. 4, No. 6, June 1966, pp. 1001-1007.

8. Flügge-Lotz, I. and Fannelöp, T., "The Laminar Compressible Boundary Layer Along a Wave-Shaped Wall," TR 136, 1963, Div. of Engineering Mechanics, Stanford Univ., Stanford, Calif.

9. Benjamin, T.B., "Shearing Flow Over a Wavy Boundary," Journal of Fluid Mechanics, Vol. 6, Part 2, August 1959, pp. 161-205.

10. Motzfeld, H., "The Turbulent Flow Past Wavy Walls," ZAMM, Vol. 17, No. 4, August 1937, pp. 193-212.

11. Gerking, L., "A Method for the Solution of the Navier-Stokes Equations With Integral Relations and its Application to the Flow Around a Flat Plate of Finite Length," Dr. Engineering Dissertation, 1967, Dept. of Mechanical Arts and Processing Skills, Technical University, Karlsruhe, Germany (IAA abstract A69-32993).

12. Teipel, I., "An Integral Method for the Calculation of Incompressible Oscillating Boundary Layer Flows," DLR-FB-69-09, March 1969, German Aerospace Research Institute, Aachen, Germany (STAR abstract N69-27771).

13. Reynolds, W.C., "The Mechanics of an Organized Wave in a Turbulent Shear Flow," Unnumbered Report, 1969, Dept. of Mechanical Engineering, Stanford Univ., Stanford, Calif.

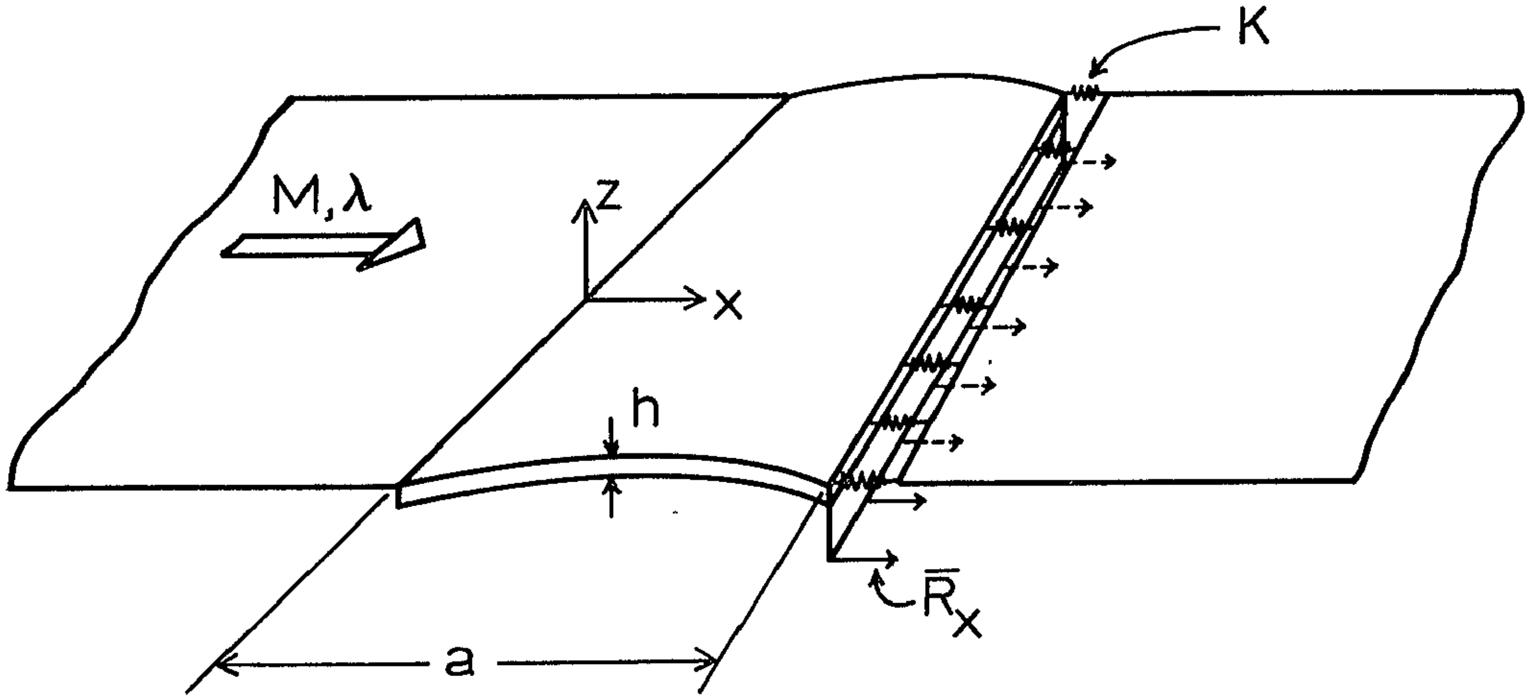


Figure 1. Two-dimensional panel (plate-column) on hinged supports.

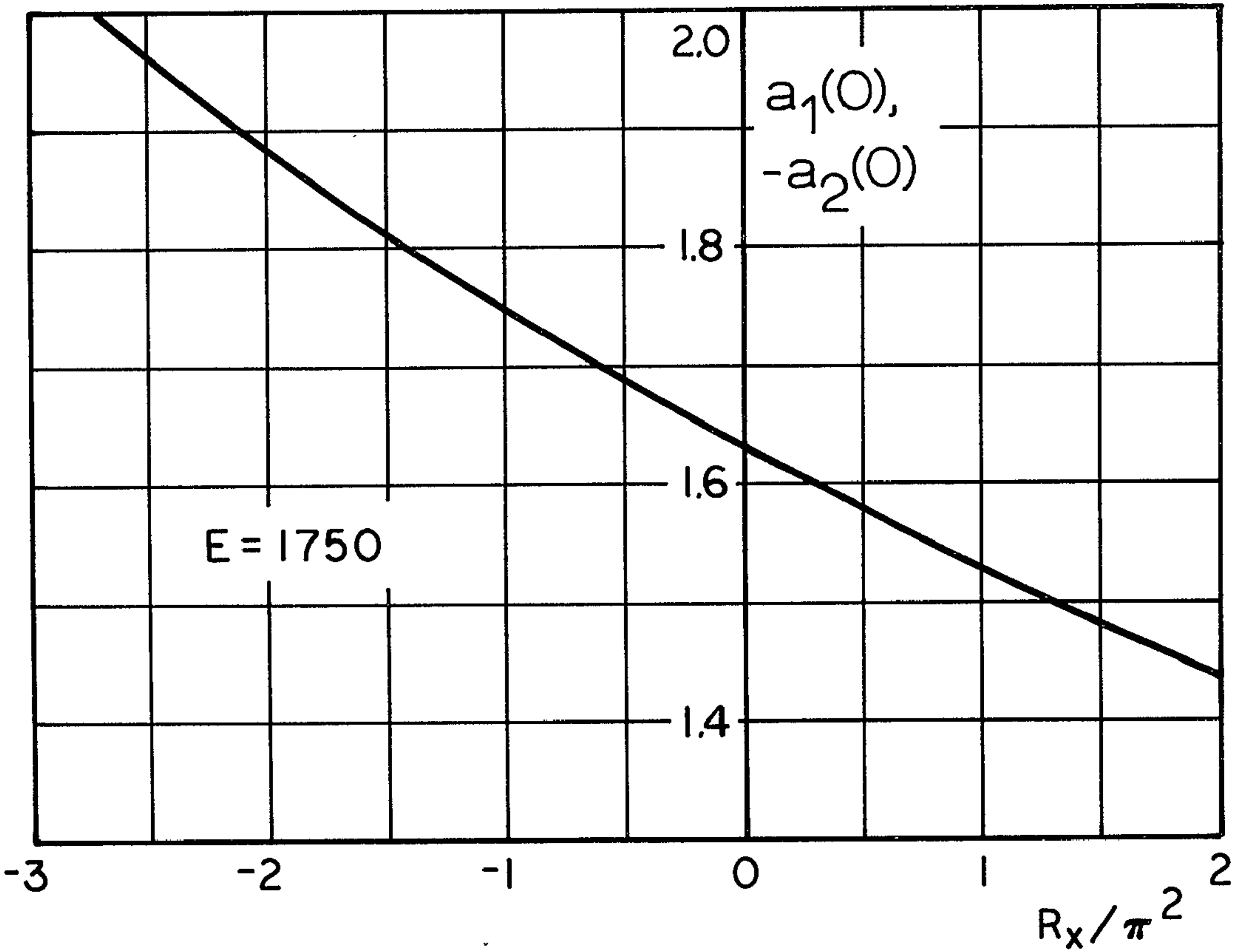


Figure 2. Variation of initial values $a_1(0) = -a_2(0)$ with dimensionless in-plane applied load R_x for $\alpha' = 0.1^2$, E = 1750 .

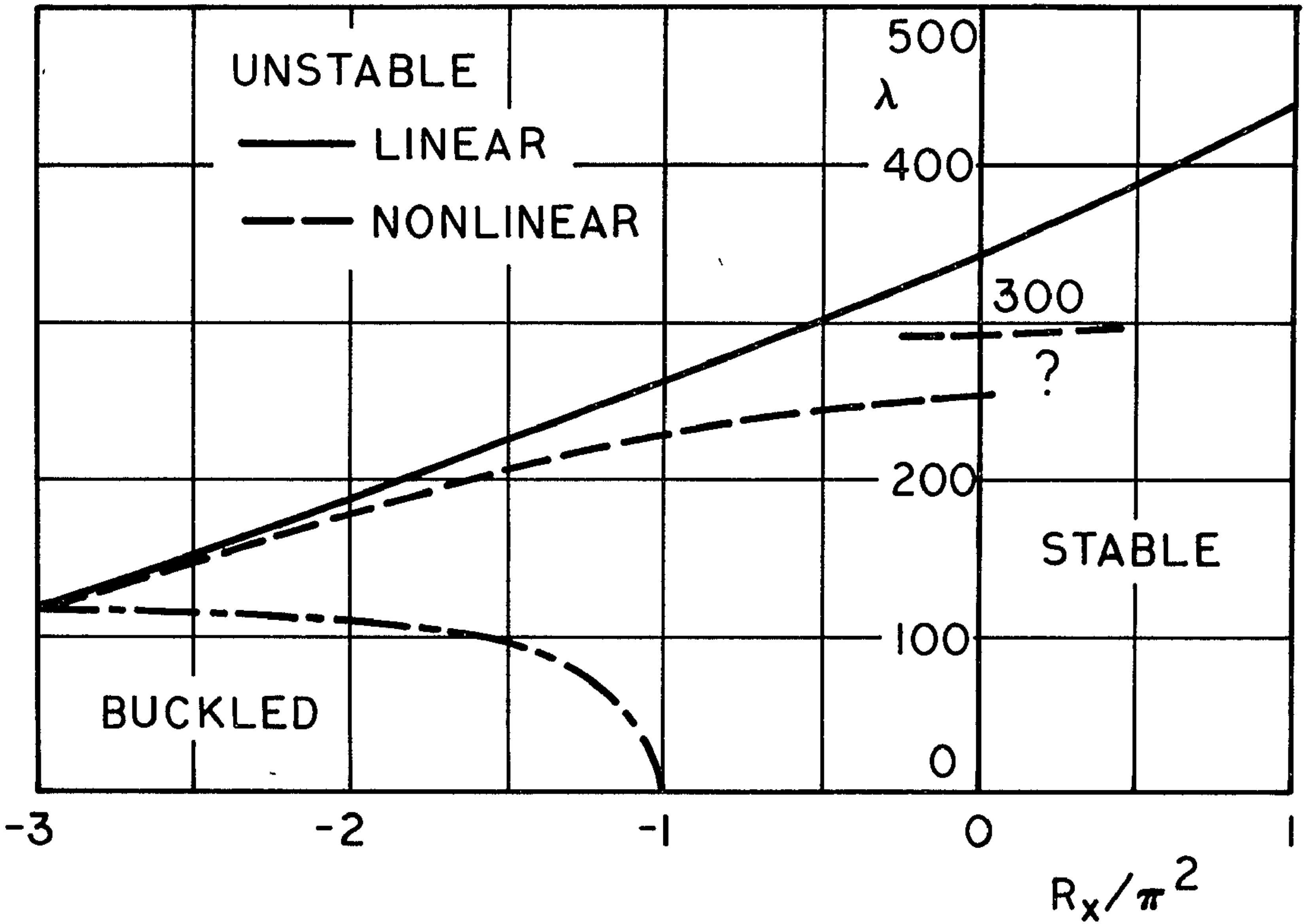


Figure 3. Linear stability boundary compared with nonlinear stability boundary for initial energy $E = 1750$, given by nonzero $a_1(0) = -a_2(0)$ as plotted in Fig. 2; $N = 6$, $\alpha' = 0.1$, $\mu/M = 0.01^2$, $\Delta p = 0$, $Mh/a = 0.05$.

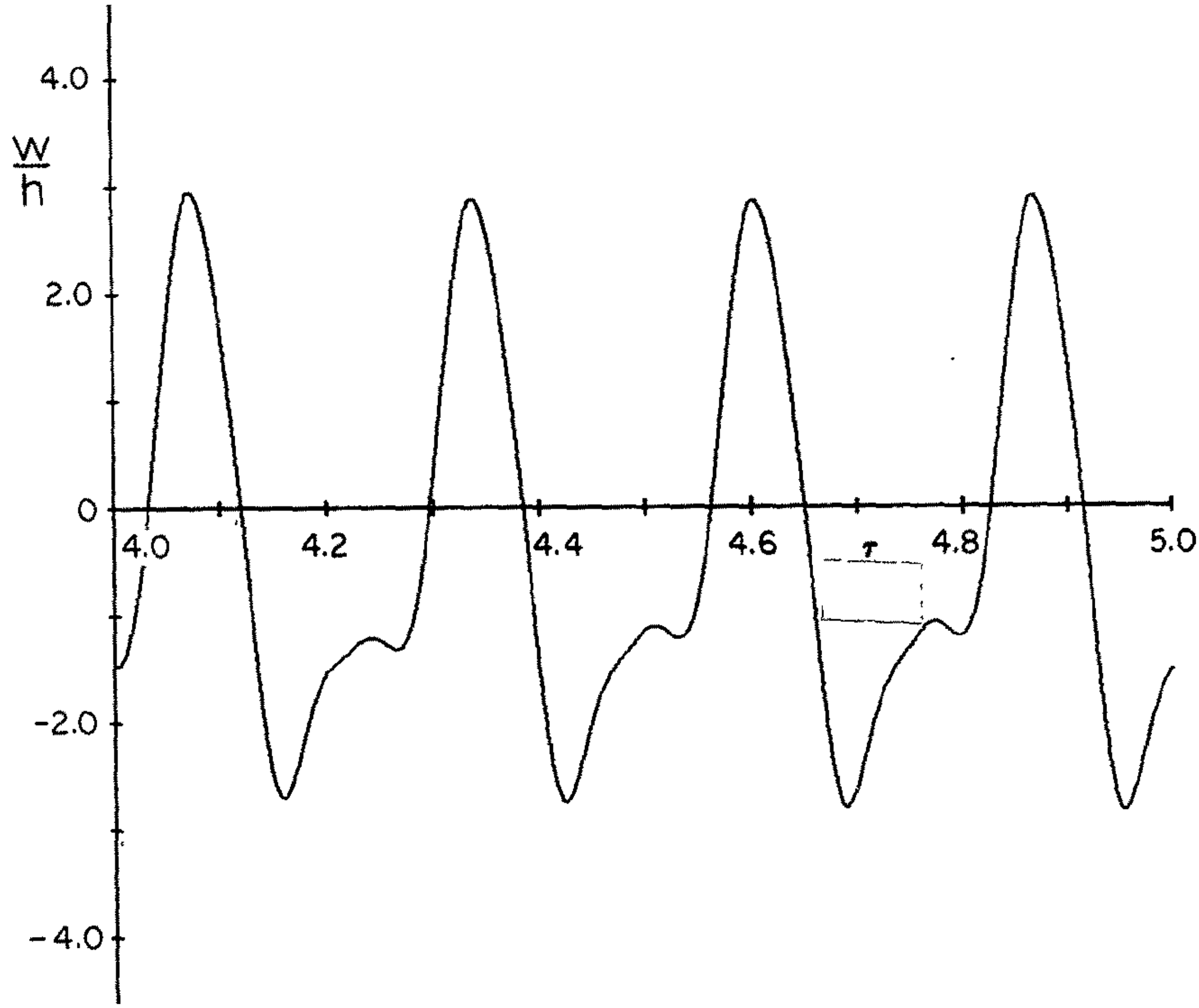


Figure 4. Dimensionless panel displacement w at $x/a = 0.75$ versus dimensionless time τ , initial energy $E = 1750$ given by nonzero $a_1(0) = -a_2(0)$, $\lambda = 295$, $R_x = 0$, other parameters as in Fig. 3.

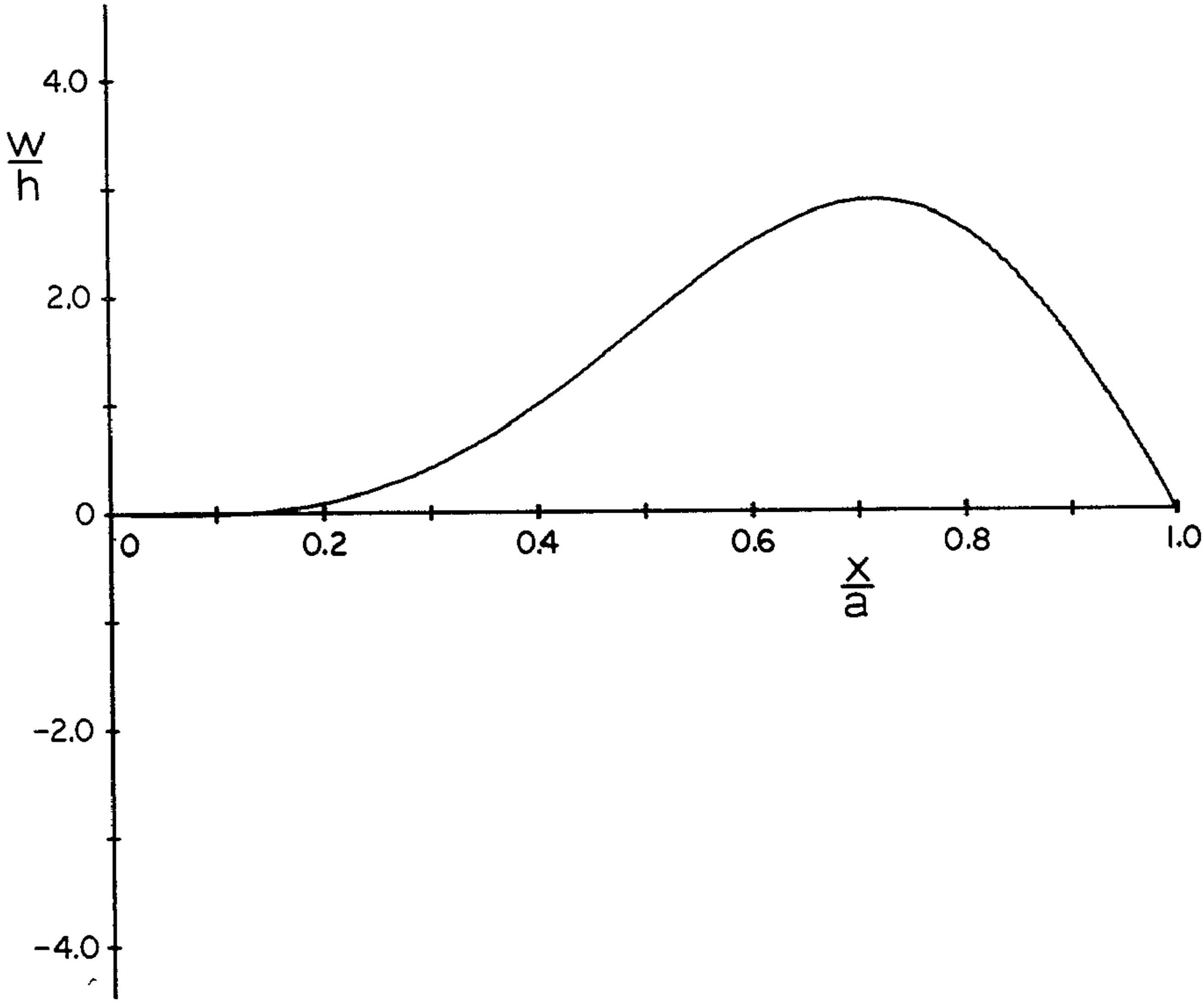


Figure 5. Panel displacement w versus chord distance x/a for panel of Fig. 4, $\tau = 4.60$..

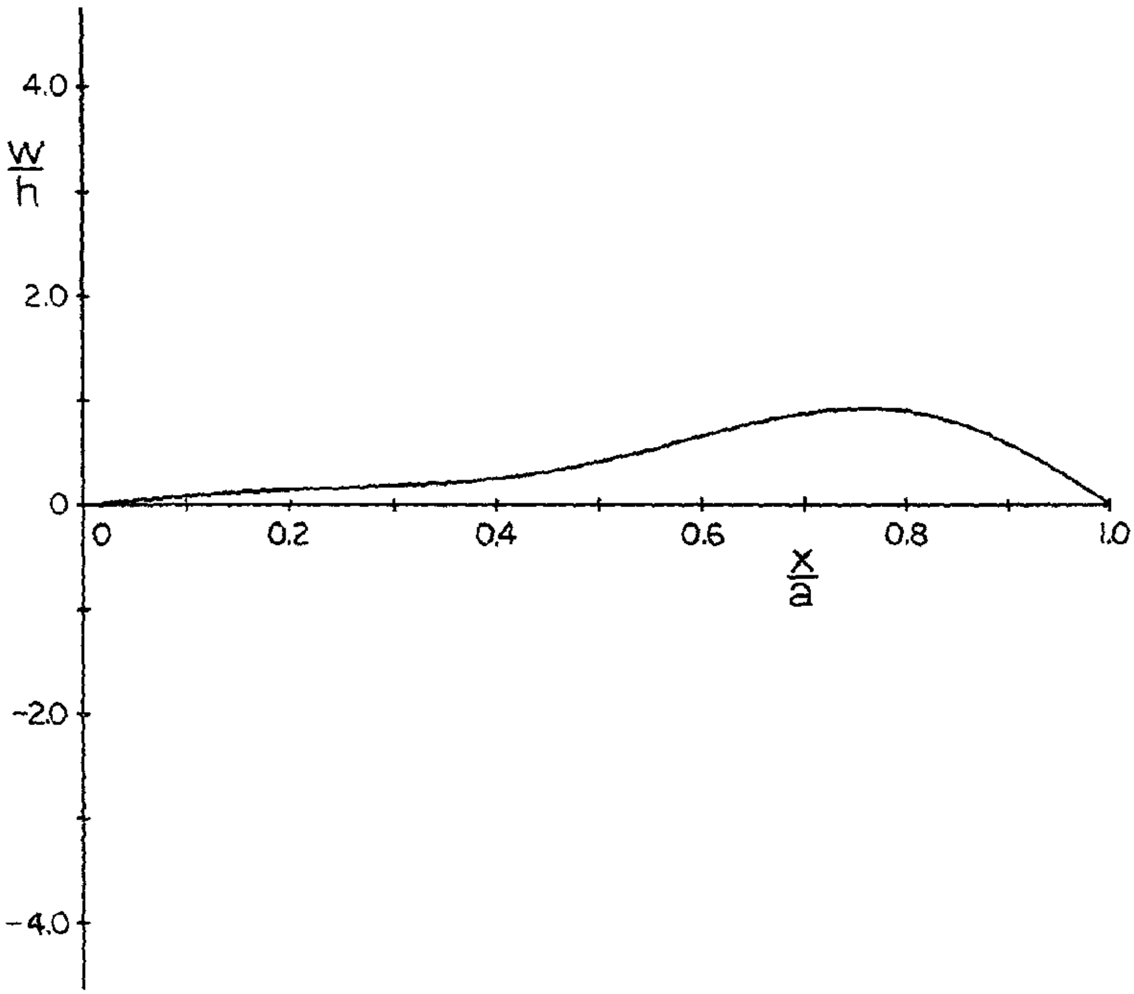


Figure 6. Panel displacement w versus chord distance x/a for panel of Fig. 4, $\tau = 4.64$.

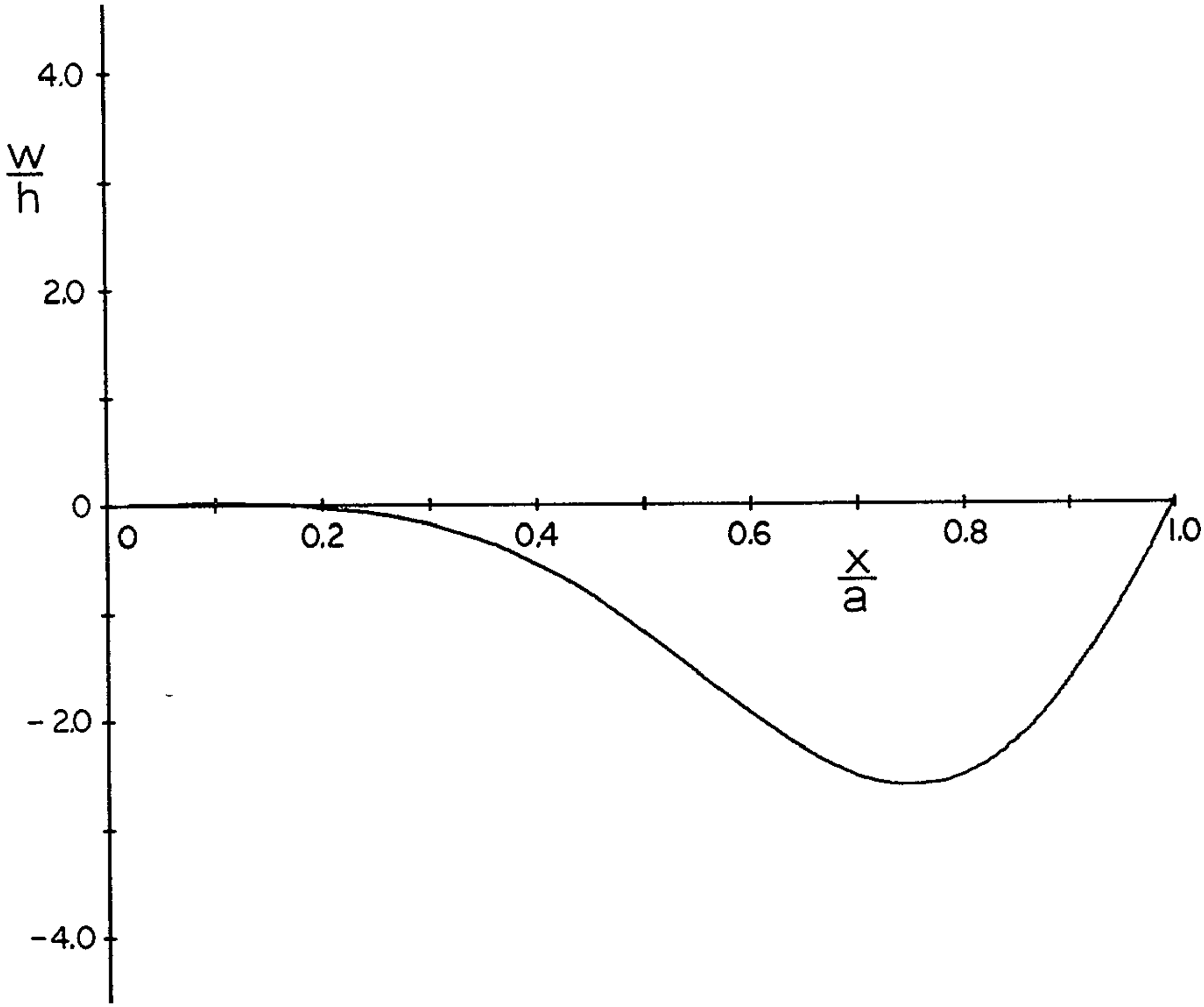


Figure 7. Panel displacement w versus chord distance x/a for panel of Fig. 4, $\tau = 4.68$.

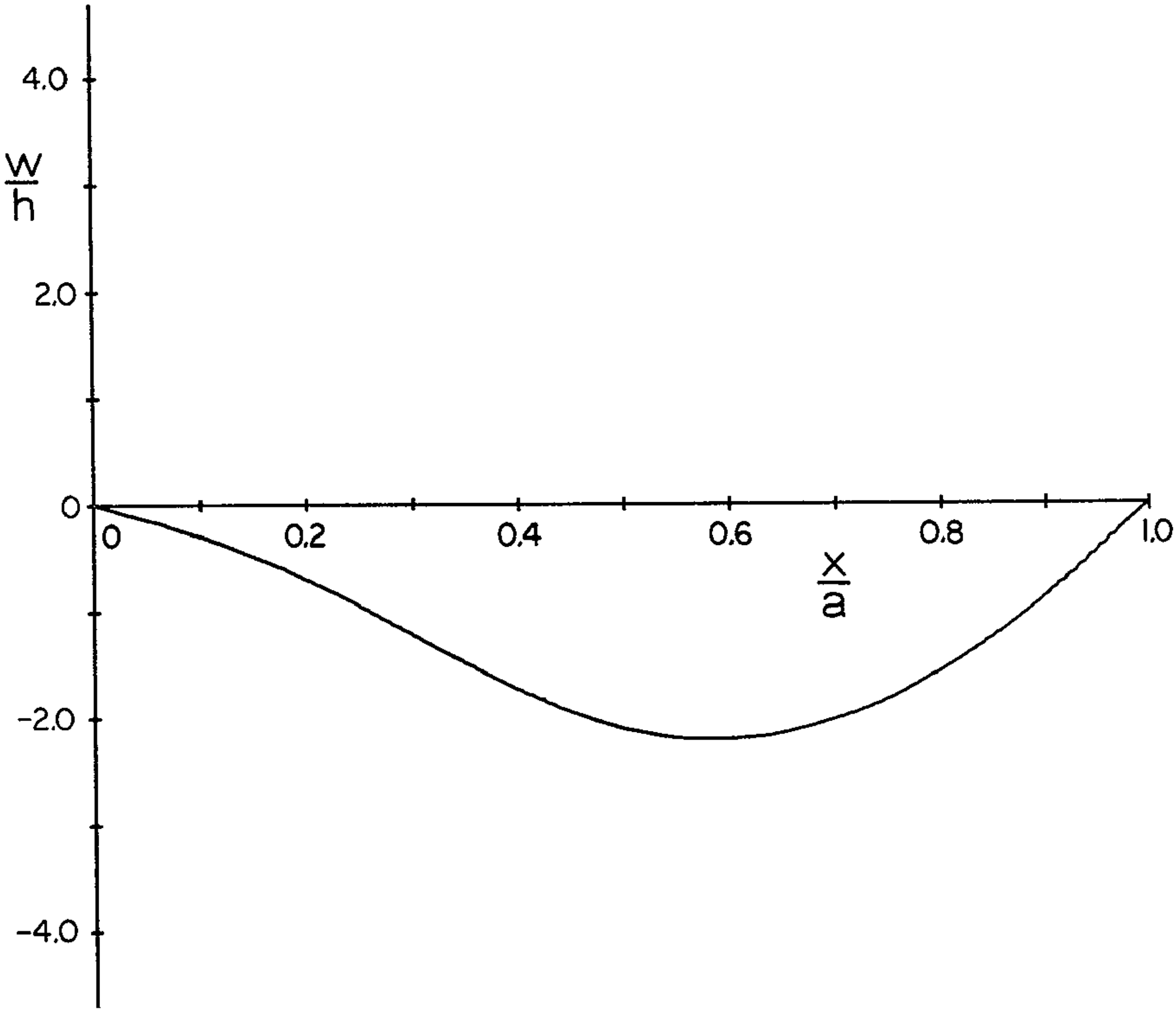


Figure 8. Panel displacement w versus chord distance x/a for panel of Fig. 4, $\tau = 4.72$.

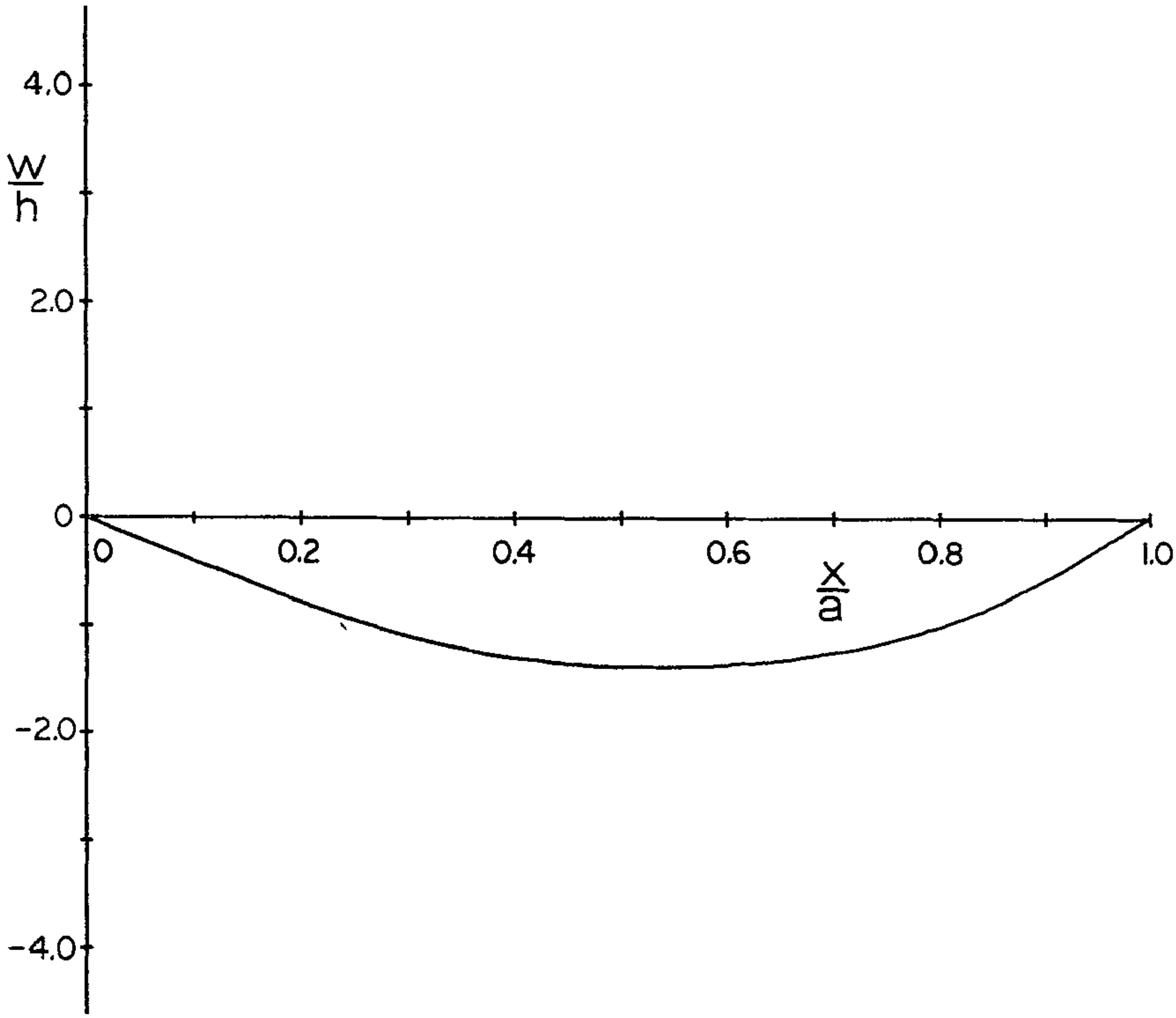


Figure 9. Panel displacement w versus chord distance x/a for panel of Fig. 4, $\tau = 4.76$.

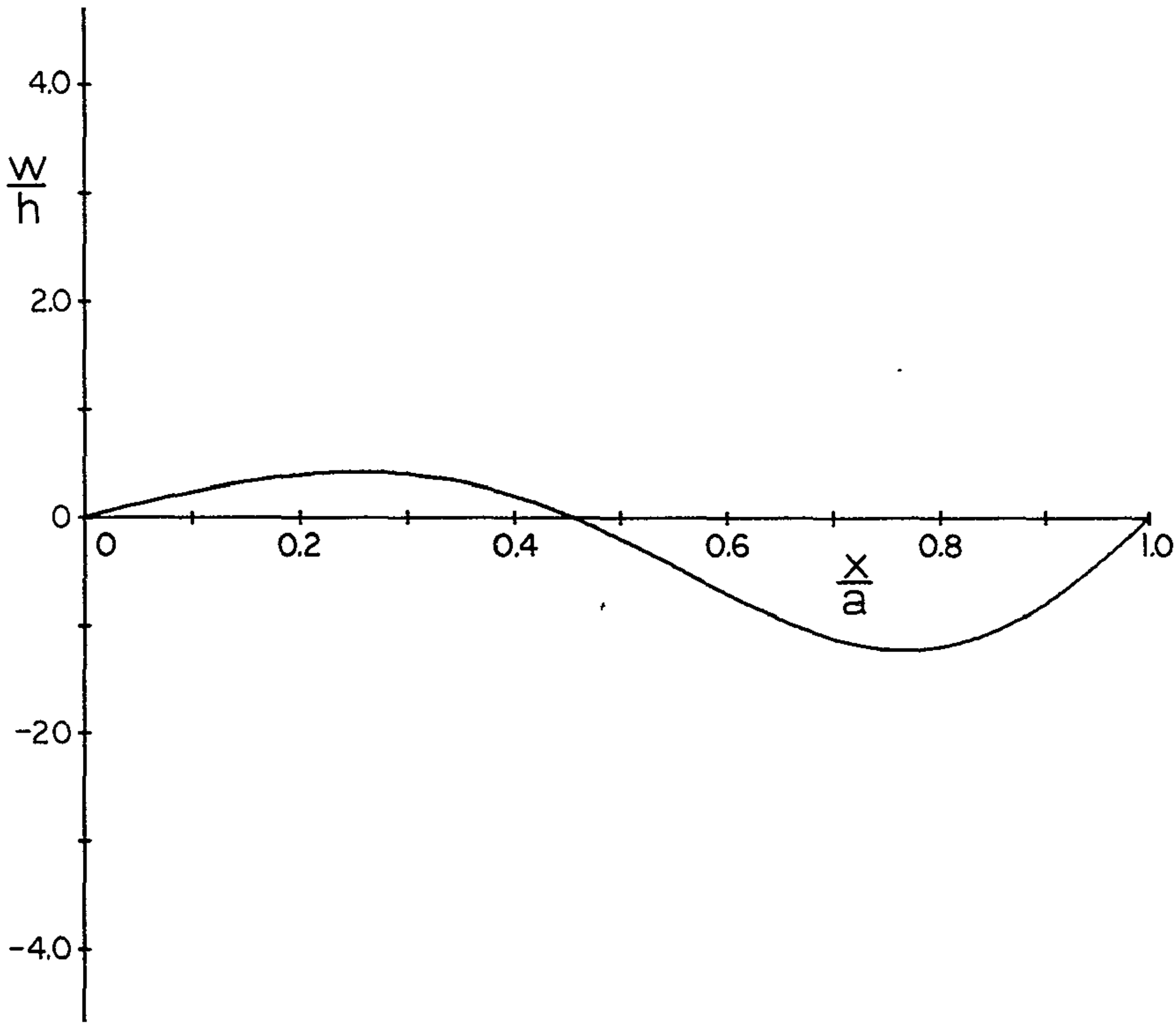


Figure 10. Panel displacement w versus chord distance x/a for panel of Fig. 4, $\tau = 4.80$.

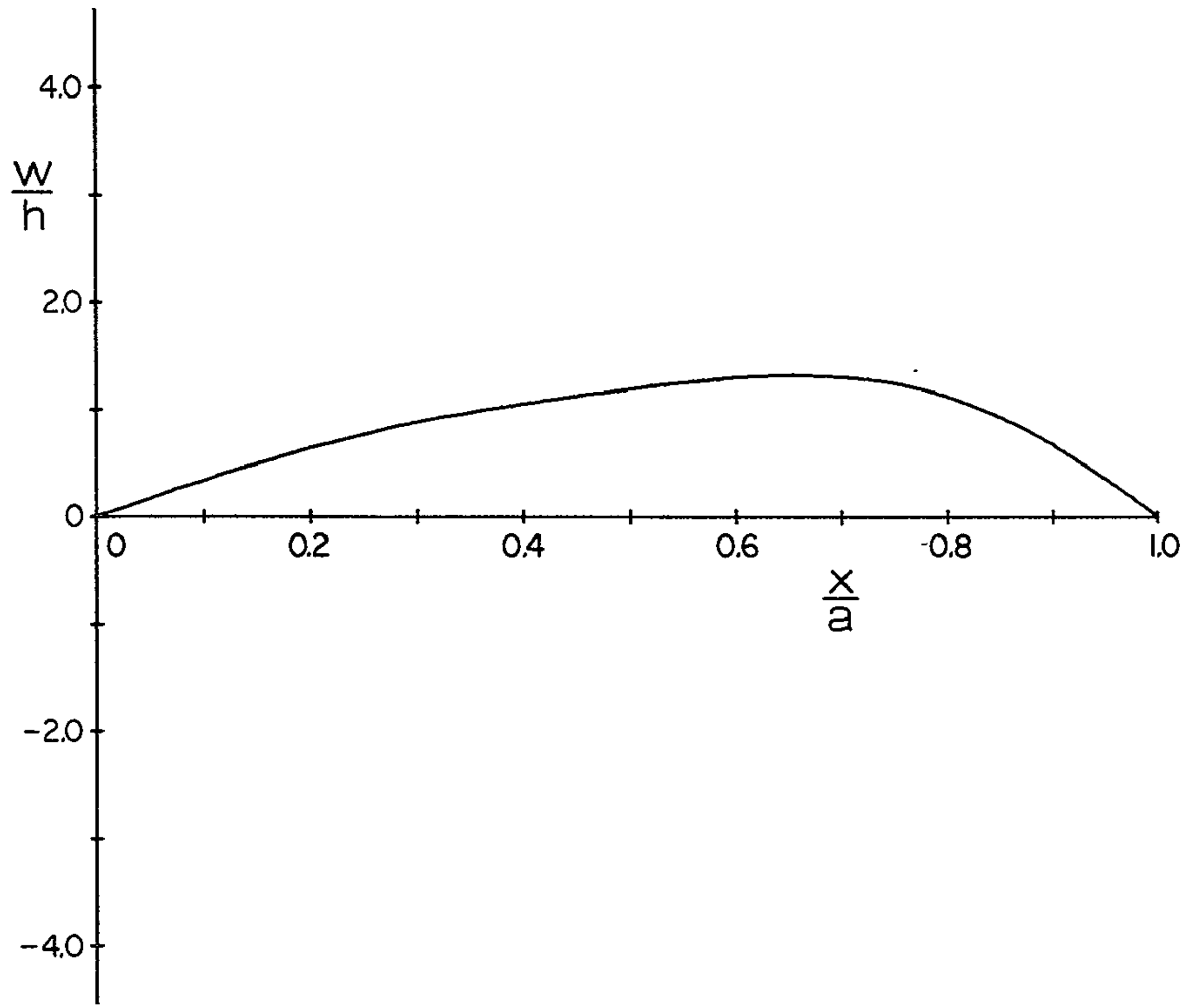


Figure 11. Panel displacement w versus chord distance x/a for panel of Fig. 4, $\tau = 4.84$.

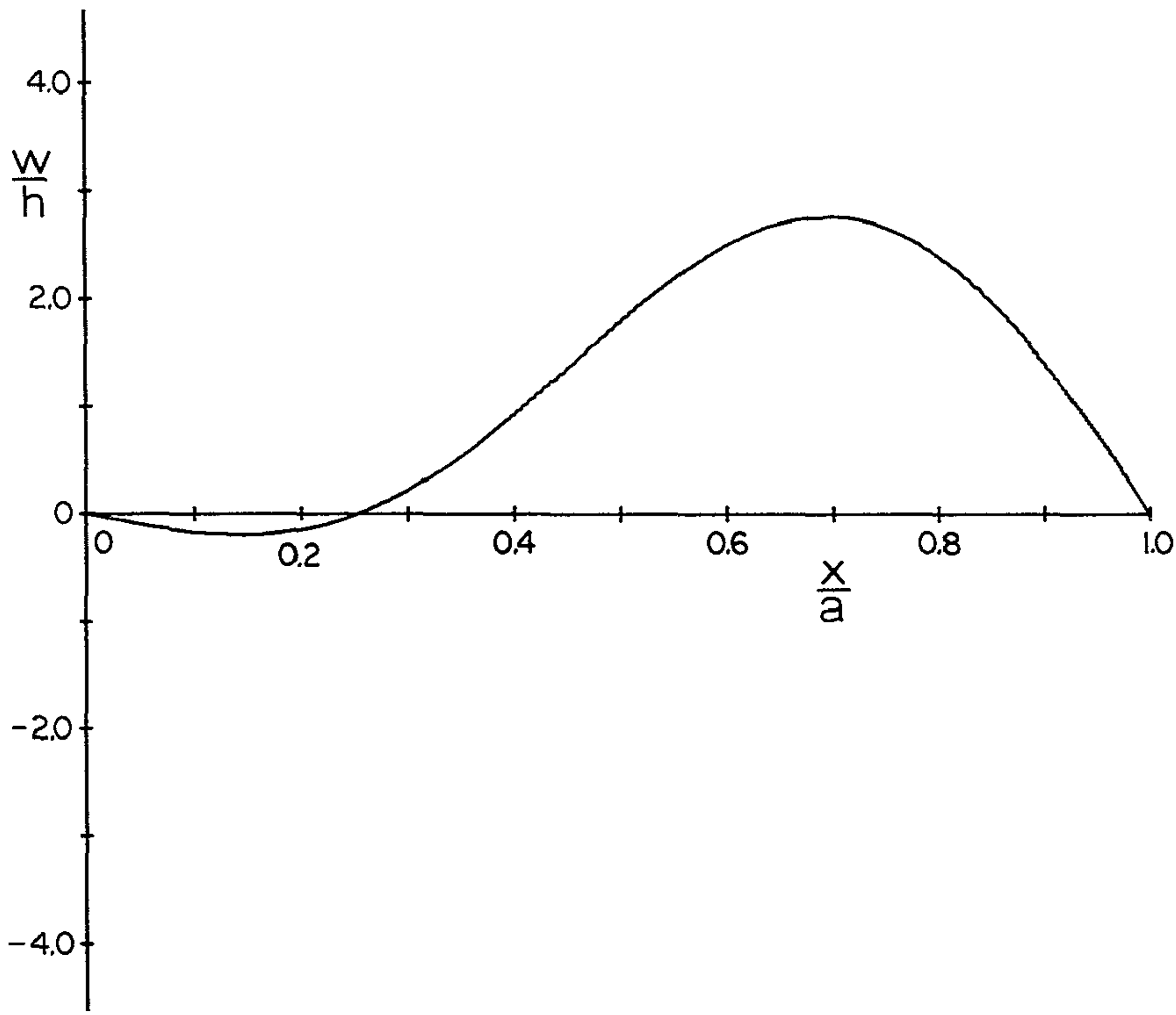


Figure 12. Panel displacement w versus chord distance x/a for panel of Fig. 4, $\tau = 4.88$.

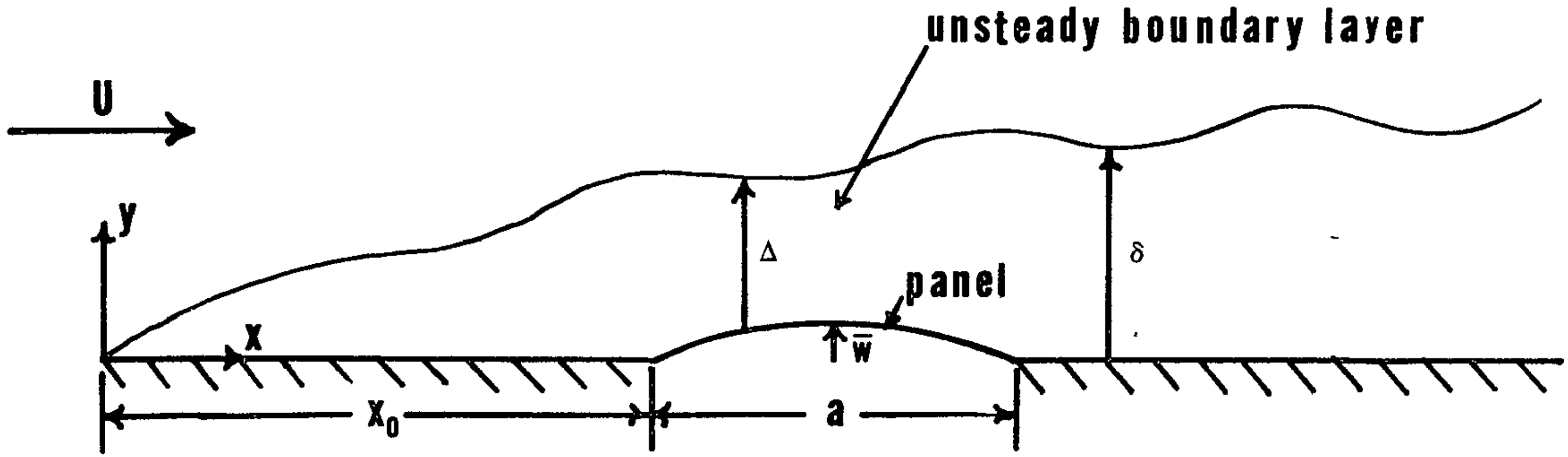
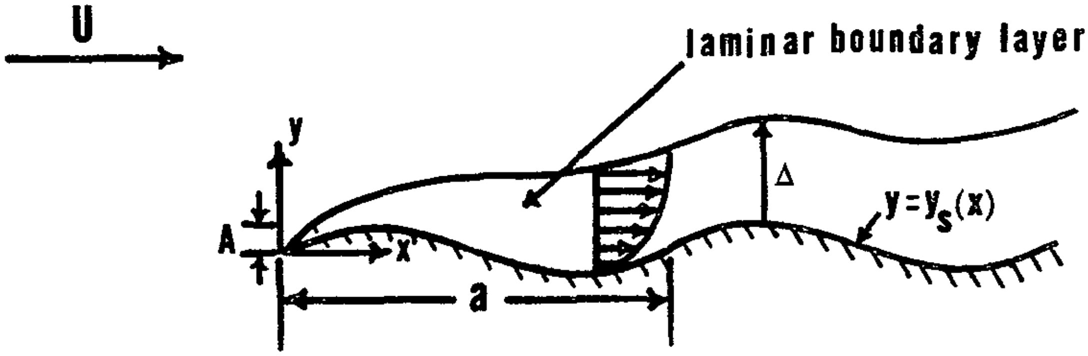
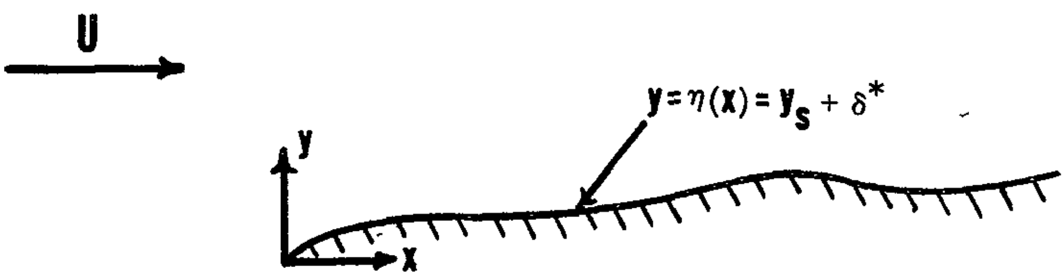


Figure 13. Viscous flow past an oscillating two-dimensional panel.



(a) Viscous flow past a semi-infinite wavy wall.



(b) Equivalent inviscid flow past the displacement surface.

Figure 14. Steady two-dimensional incompressible viscous flow past a wavy wall.